TAXATION AND PRODUCTION STRUCTURE

By

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1. Introduction

Rather than presenting a detailed comment on the excellent paper by Bernstein and Nadiri (1986), we have decided to augment their analysis in two directions.

Bernstein and Nadiri's model of the structure of production is very general and they show how many particular production models (such as those involving endogenous depreciation, the adjustment of quasifixed factors, etc.) can be obtained as special cases of their general model. In section 2, we suggest an even more general model which contains their model as a special case. By postulating this very general model of intertemporal production structure, we hope to make applied researchers more aware of the assumptions that are required to simplify a "realistic" general model down to the relatively simple models that have been used in the empirical literature. Thus the first point that we make in this comment is that a general neoclassical model of intertemporal production is exceedingly complex and hence empirical results obtained from the relatively simple neoclassical models that have appeared in the literature thus far must be viewed with a certain amount of skepticism.

The second purpose of this comment is to emphasize the deadweight loss aspects of alternative systems of business taxation. This topic is addressed in section 3 where we indicate that nonneutral systems of business income taxation can lead to substantial losses in the real output of the economy. In
section 4, we show how subsidies and inefficient systems of commodity taxation can also lead to substantial losses of real output.

In contrast to previous papers which develop very general formulae for the loss of output due to inefficient systems of taxation,¹ the present paper illustrates the general methodology by considering very specific examples involving a two input Cobb-Douglas production function. The required technical results are developed in three Appendices. Appendix 1 derives the variable profit function that corresponds to a single output, one variable input and one capital input Cobb-Douglas production function. The results of this section are used in the subsequent Appendices. Appendix 2 derives an expression for the approximate loss of output due to a nonneutral system of business income taxation. We then evaluate the loss formula under the additional hypothesis of a Cobb-Douglas technology. Appendix 3 derives an expression for the approximate loss of net output in a multisector model where one sector faces a commodity tax or subsidy that does not apply to the other sectors (a very common situation in Canada). We then evaluate the loss under the additional hypothesis of a Cobb-Douglas technology.


Suppose that a firm starts up in period 0 and in period t, it is producing one output, using the services of N-1 variable inputs and using the services of M types of durable input which have been purchased over periods 0,1,...,t. The firm's technology may be represented by the period t general neoclassical production function \( f^t \); i.e., we have
(1) \[ x_1^t = f^t(x_2^t, x_3^t, \ldots, x_N^t, k_1^0, I_1^t, \ldots, I_t^t) \]

where \( x_1^t \) is the maximum number of units of output that can be produced; \( x_2^t, \ldots, x_N^t \) represent the amounts of the N-1 variable inputs that are utilized in period t; \( k_1^0, \ldots, k_M^0 \) is the firm's initial capital stock vector and \( I_s^r \equiv [I_1^s, \ldots, I_M^s] \) where \( I_m^s \) is the firm's purchases of capital good m in period s for \( m = 1, \ldots, M \) and \( s = 1, \ldots, t \).

There are two important things to note about (1). The first is that the production functions \( f^t \) may be totally different as t changes. Technological change may make the previous period's technology totally irrelevant and so \( f^t \) could be very different from \( f^{t-1} \). The second point is related to the first point: as t increases, the number of arguments in \( f^t \) increases. In general, we cannot assume that the efficiency of a period r capital good bears any simple relation to the efficiency of the corresponding period s capital good.

Of course, the general neoclassical production function framework defined by (1) is too general to be estimated econometrically. The traditional declining balance or geometric depreciation model is the following special case of (1):

(2) \[ x_1^t = f(t, x_2^t, \ldots, x_N^t, (I_M - \delta)^t k_1^0 + \sum_{s=1}^{t} (I_M - \delta)^{t-s} I_s^r) \]

where \( I_M \) is an identity matrix of dimension M and \( \delta \) is a diagonal matrix which has the exogenously given depreciation rates \( \delta_1, \delta_2, \ldots, \delta_M \) on its main dia-
gonal. Thus the period $t$ vintage production function is now only a function of $N+M$ arguments instead of $N-1+M(t+1)$ arguments. Note also that $f^t$ can no longer be totally unrelated to $f^s$ if $f$ is a continuous function of its arguments; i.e., the period specific production functions for nearby periods will now resemble each other.

Another special case of (1) is:

$$\begin{align*}
(3) \quad x^t_1 &= f(t, x^t_2, \ldots, x^t_N, I^t, (I^t_M - \delta)^t K^0 + \sum_{s=1}^{t-1} (I^t_M - \delta)^t S^s I^s) .
\end{align*}$$

In (3), the most recently purchased vector of investment goods, $I^t$, is not aggregated with previous period investment vectors. Thus (3) allows for a very general costs of adjustment model.

Bernstein and Nadiri (1986) consider many additional models of technology that are less general than (1) (which is of course too general to be implemented empirically) but that are more general than the usual neoclassical model 2. The point we wish to emphasize here is that we must always make some rather arbitrary aggregation assumptions in order to implement the reasonably general neoclassical model of production represented by (1).

We turn now to a discussion of the deadweight loss aspects of business taxation.
3. **Losses due to Nonneutral Business Income Taxation.**

Rather than attempting to be very general in this comment, we will try to explain the basic principles involved in the context of a very simple model.³

Instead of the general neoclassical period t production function \( f^t \) defined by (1), we will consider a simple one output, two input production function \( f \) where

\[
y = f(L, K)
\]

where \( y > 0 \) denotes the output produced during a period, \( L > 0 \) is the amount of variable input utilized during the period (call it "labour"), and \( K > 0 \) is the amount of a durable input purchased by the firm and utilized during the period (call it "capital").

Since capital is a durable good (i.e., its services last longer than one period), the price that should be charged for the use of the capital good in the current period is not its purchase price \( P \). The appropriate current period price for the use of a unit of capital services is called the **user cost** of capital, \( q \) say. It may be defined as the current price \( P \) minus next period's discounted opportunity cost; i.e.,

\[
q = P - Q/(1+r)
\]

where \( P > 0 \) is the current period purchase price, \( Q > 0 \) is next period's price for a unit of used capital and \( r \) is an appropriate discount rate.
It is possible to put the user cost formula (5) into a more familiar form. Let the one period combined depreciation and obsolescence rate δ be defined by:

\[ Q = (1 - \delta)p^+ \]

where \( p^+ \) is next period's expected price for one unit of a new capital good. Substituting (6) into (5) and simplifying yields the following expression for the user cost of capital:

\[ q = \left[ rP + \delta P^+ - (P^+ - P) \right] / (1+r). \]

Thus the user cost equals an interest payments component plus a depreciation component less an expected capital gains component. If there were no business income tax, the firm's undistorted profit maximization problem could be written as follows:

\[ \max_{y, L, K} \{py - wL - qK : y = f(L, K)\} \]

where \( p > 0 \) is the price of output, \( w > 0 \) is the wage rate and \( q \) is the user cost of capital defined by (5) or (7). Thus the firm wishes to choose output, labour and capital to maximize profits subject to its production function constraint.

If we substitute (5) into (8), it can be seen that the firm wishes to maximize its discounted cash flow (i.e., discounted revenues including sales
of used durable inputs minus expenditures on materials, labour and durable inputs) subject to the constraints of technology. This interpretation of the firm's undistorted profit maximization problem can readily be carried over to more complicated intertemporal profit maximization problems where we assume that the firm's period t technology can be characterized by general neoclassical production functions of the type defined by (1); see Diewert (1985c). 5

With a corporate or business income tax, the firm's tax distorted profit maximization problem may be written as:

\[ \max_{y,L,K} \{ [py - wL - qK] - \tau[py - wL - dK] : y = f(L,K) \} \]

where \( \tau (0 < \tau < 1) \) is the business income tax rate and \( d \) is the deduction from taxable income that the tax code allows for each unit of capital utilized in the period.

Note that if the deductibility parameter \( d \) equals \( q \), the user cost of capital, then the objective function in (9) collapses to \((1-\tau)(py - wL - qK)\) which is proportional to the objective function in the undistorted profit maximization problem (8). Thus when \( d = q \), the business income tax will be neutral: the firm's investment and production decisions will not be distorted by the tax system and there will be no deadweight loss.

The \( d = q \) case is equivalent to a system of cash flow taxation or immediate expensing of investment good purchases. The fact that making the business income taxation base equal to discounted cash flow leads to a neutral tax system is well known in the literature; see Brown (1948), Smith (1963),
Samuelson (1964), Boadway, Bruce and Mintz (1982a) (1982b), Diewert (1981) (1985c) and Boadway and Bruce (1984). However, the literature on measuring the costs of a nonneutral system is much smaller. In Appendix 2, we derive a methodology for measuring the approximate loss of real output that a nonneutral tax system will generate for a firm which faces the tax distorted maximization problem (9). In the Appendix, we replace the deductibility parameter d by q(1+ε). Thus if ε = 0, then the tax system is neutral and there will be no loss of output.

In Appendix 2, we show that a second order approximation to the loss of output (as a fraction of the optimal value of output) generated by a nonneutral system of business income taxation is:

\[ L_A = -(1/2) \tau^2 \varepsilon^2 q^2 / (1-\tau)^2 p y_0 \]  

where \( \tau, \varepsilon, q \) and \( p \) have already been defined (the business income tax rate, the deductibility nonneutrality parameter, the user cost of capital, and the price of output respectively) and \( y_0 \) is defined to be the optimal output solution to the undistorted maximization problem (8), \( \pi^0_{KK} \equiv \delta^2 \pi(p, w, K^0) / \delta K^2 \) where \( K^0 \) is the optimal capital solution to (8) and \( \pi \) is the firm's variable profit function defined by

\[ \pi(p, w, K^0) \equiv \max_{y, L} \{ p y - w L : y = f(L, K) \} \]

which is considered at greater length in Appendix 1.
In order to determine \( y^0 \) and \( \pi_{\text{KK}}^0 \), we need to know the functional form for the production function \( f \) or its dual variable profit function \( \pi \). Let us assume for simplicity that the production function has the following Cobb-Douglas form:

\[
y = f(L,K) \equiv cL^\alpha K^\beta, \quad \alpha + \beta < 1, \quad \alpha > 0, \quad \beta > 0, \quad c > 0.
\]

The parameters \( \alpha \) and \( \beta \) can be interpreted as labour and capital shares of the value of output; see Appendix 1.

With the Cobb-Douglas assumption (12), we show in Appendix 2 that the approximate loss formula (10) reduces to:

\[
L_A = (1/2)\tau^2(1-\tau)^2\varepsilon^2(1-\alpha)\beta(1-\alpha-\beta)^{-1}.
\]

Assuming that \( \tau = .5 \), Table 1 below lists the approximate loss \( L_A \) for various values of \( \varepsilon, \alpha \) and \( \beta \).

TABLE 1: LOSSES DUE TO A NONNEUTRAL INCOME TAX

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( L_A )</th>
<th>( \varepsilon )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( L_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.65</td>
<td>.25</td>
<td>.44%</td>
<td>.2</td>
<td>.70</td>
<td>.25</td>
<td>3.00%</td>
</tr>
<tr>
<td>.1</td>
<td>.60</td>
<td>.30</td>
<td>.60%</td>
<td>.2</td>
<td>.65</td>
<td>.30</td>
<td>4.20%</td>
</tr>
<tr>
<td>.1</td>
<td>.70</td>
<td>.25</td>
<td>.75%</td>
<td>.3</td>
<td>.65</td>
<td>.25</td>
<td>3.94%</td>
</tr>
<tr>
<td>.1</td>
<td>.65</td>
<td>.30</td>
<td>1.05%</td>
<td>.3</td>
<td>.65</td>
<td>.30</td>
<td>5.40%</td>
</tr>
<tr>
<td>.2</td>
<td>.65</td>
<td>.25</td>
<td>1.75%</td>
<td>.3</td>
<td>.70</td>
<td>.25</td>
<td>6.75%</td>
</tr>
<tr>
<td>.2</td>
<td>.60</td>
<td>.30</td>
<td>2.40%</td>
<td>.3</td>
<td>.65</td>
<td>.30</td>
<td>9.45%</td>
</tr>
</tbody>
</table>
Thus if $\epsilon = .1$ so that the tax code allows the business to deduct 110% of the user cost of capital from taxable income, labour's share is .65 and capital's share is .25, then the approximate loss of output generated by the tax system in a Cobb-Douglas world is .44%. On the other hand, if $\epsilon = .3$, $\alpha = .65$ and $\beta = .30$, then the approximate loss of output is 9.45%, a very large loss. The average loss over the twelve cases presented in Table 1 is 3.31%.

Of course, the key determinant of the size of the loss is the magnitude of $\epsilon$, the deductibility nonneutrality parameter. There appear to be many cases where the Canadian and U.S. tax codes will generate values of $\epsilon$ that are far greater than .3 in magnitude, so the losses due to the nonneutrality of the present system of business taxation are potentially very large indeed.

4. **Losses due to Inefficient Systems of Commodity Taxation.**

Elementary economic reasoning leads to the conclusion that there will be a loss in productive efficiency for the economy as a whole if different producers face different relative prices. These differences in prices are typically due to the actions of governments: local governments may give local producers preferential treatment, Provincial or Federal governments may subsidize certain sectors (e.g., agriculture), or governments may impose taxes on the output of one industry where that output is used as an intermediate input by other industries (e.g., gasoline taxes, the manufacturing sales tax). Finally, governments impose discriminatory taxes, tariffs or subsidies on internationally traded goods. For a small open economy, all of these distortions lead to a loss of productive efficiency.
The basic point made in the above paragraph is well understood. What is less well developed are methods for quantifying the loss in productive efficiency due to commodity tax distortions. There have been two broad approaches to this measurement problem. The first is the applied general equilibrium modelling approach which has been popularized by Shoven and Whalley. The second is a second order approximation approach which has its roots in the work of Harberger (1974). The latter approach is the one that we shall use. Unfortunately, in order to implement either approach, we require information on various second order parameters such as elasticities of substitution and this information is not easy to obtain. As in the previous section, we "solve" this last problem by assuming that the technology is Cobb-Douglas.

Initially, we assume that a producer's technology is characterized by a one output, two input production function \( f \) defined by (4). We assume that our producer faces the market wage rate \( w \) as before but its output \( y \) is sold at the price \( p+tp \) where \( tp \) is a subsidy to our first producer and \( p \) is the price other sectors face for the output. Alternatively, \( p \) is the world price of output and \( tp \) is the combined tax and tariff that is imposed on imports of the good.

For simplicity, we hold capital stocks fixed in this section. In Appendix 3, the sectoral loss in the economy's real output due to the distortion \( t \) (holding \( K \) fixed) is defined as the change in the value of output minus the value of variable input induced by the distortion, where output and variable input are priced at the social prices \( p \) and \( w \) respectively. A second order approximation to this loss, divided by the optimal value of output \( py^0 \).
(the output $y^0$ is that which would be produced if the distortion $t$ were zero),
is shown to be:

\begin{equation}
L_A = \frac{1}{2} t^2 p_0 \pi_{pp}^0 / y^0
\end{equation}

where $\pi_{pp}^0 = \frac{\partial^2 \pi(p, w, k^0)}{\partial p^2}$ and $\pi$ is the firm's variable profit function defined by (11) in the previous section.

Assuming that the producer's technology is Cobb-Douglas (recall (12)), we show in Appendix 3 that the approximate loss $L_A$ defined by (14) reduces to:

\begin{equation}
L_A = \frac{1}{2} a(1-a)^{-1} t^2
\end{equation}

where $a$ is the labour share parameter which appears in (12).

Table 2 below lists the approximate loss $L_A$ defined by (15) for various values of $a$ and $t$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$t$</th>
<th>$L_A$</th>
<th>$a$</th>
<th>$t$</th>
<th>$L_A$</th>
<th>$a$</th>
<th>$t$</th>
<th>$L_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.6</td>
<td>.1</td>
<td>.75%</td>
<td>.15</td>
<td>1.69%</td>
<td>.2</td>
<td>3.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.7</td>
<td>.1</td>
<td>1.16%</td>
<td>.15</td>
<td>2.62%</td>
<td>.2</td>
<td>4.67%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.8</td>
<td>.1</td>
<td>2.00%</td>
<td>.15</td>
<td>4.50%</td>
<td>.2</td>
<td>8.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.9</td>
<td>.1</td>
<td>4.50%</td>
<td>.15</td>
<td>10.12%</td>
<td>.2</td>
<td>18.00%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Losses Due to a Commodity Tax Distortion
Thus if \( t = .1 \) so that the tariff rate for example is 10\% and the share of variable input \( \alpha \) is .6, then the economy's loss of real output will be about .75\% of the output of the protected sector. However, if the share of variable input is .9 (so that there are very few fixed factors in the protected industry), then the approximate loss climbs up to 4.5\%. If \( t = .2 \) and \( \alpha = .9 \), then the approximate loss of output generated by the tax distortion is 18\% of the output of the protected sector if it has a Cobb-Douglas technology.

Hence we see that the potential loss of efficiency due to an inefficient, discriminatory system of business commodity taxes can be very large.

The examples presented in this section and the previous section indicate that the losses in the real output of the economy generated by an inefficient scheme of business income and commodity taxation can be large. Moreover, whatever the actual losses are, they are completely unnecessary. If governments wish to subsidize or penalize particular groups of consumers, this can always be done by setting up direct transfers or by imposing commodity taxes or subsidies between the production sector and the household sector. There is no need to destroy the productive efficiency of the economy by having a nonneutral system of business income taxation or by having discriminatory producer commodity taxes.

What would an efficient system of business taxation look like? This is our topic in the next section.

A system of business taxation is productively efficient if it leads to an allocation of resources which has the property that no additional units of any output could be produced using the aggregate amounts of inputs that are being utilized by all producers in the competitive, profit maximizing sector of the economy.

It seems useful to strive for an efficient system of business taxation since an inefficient system simply wastes output. An inefficient tax system is equivalent to an efficient economy which simply dumps a proportion of its output into the Great Lakes.

Drawing on the analysis presented in the previous two sections (and some extensions of this analysis), we make five points and then conclude.

Point 1. The tax base for a business income tax should be discounted cash flow. Purchases of assets should be immediately deducted from taxable income and current period negative cash flows should be carried forward at an appropriate interest rate until they can be offset by later period positive cash flows. Sales of durable assets should enter the tax base whenever they are sold. With these conventions, the tax base will turn out to be discounted pure profits (including capital gains on durable assets) plus the value of initial stocks of capital goods, inventories, land and natural resources. A possible objection to this proposal is that it will lead to a very uncertain stream of revenues for the government; e.g., if there is an investment boom, current period cash flow will decrease and so will government revenue. A solution to this problem is available although the solution involves additional accounting costs: instead of deducting the full cost of a capital
good in the current period, the firm would be given a schedule of deductions spread over a number of years which had the property that the present value of these deductions would equal the initial full cost of the capital good. However, if the asset (or the entire firm) were ever sold, the tax base would have added to it the sales price of the asset less an adjustment factor that would reflect the initial cost of the asset less the discounted sequence of deductions up to the period when the asset was sold. This adjustment factor would depend only on market data and would lead to a neutral tax system.

Point 2. To achieve productive efficiency, there should be no commodity taxes that fall unevenly on transactions within the production sector of an economy. Thus there should be: (i) no manufacturing sales tax, (ii) no industry specific subsidies (unless the output of the industry goes directly into the household or government sectors), (iii) no commodity taxes on intermediate inputs, (iv) no special enterprise zones and (v) no tariffs, taxes or hidden charges\textsuperscript{14} that fall only on foreign goods (assuming that we are a small country).\textsuperscript{15} After seeing this extensive list of prohibitions, it may appear to the reader that we are ruling out most taxes. However, this is not the case: governments would still be allowed to impose an arbitrary collection of taxes and subsidies on transactions that took place between the producer and consumer sectors. Thus sales taxes on consumer purchases and taxes on labour income would still be permitted in our ideal system of business taxation.\textsuperscript{16}

Point 3. A consumption based value added tax\textsuperscript{17} (or business transfer tax as it is called in Canada) is consistent with productive efficiency. The tax base here is equal to cash flow plus payments to labour. However, a GNP type value added tax that did not allow deductions for purchases of capital goods
would not be efficient and hence it should be avoided. The combination of a consumption based value added tax and cash flow taxation would be efficient and would answer a common criticism of cash flow taxation which is that it would not raise much revenue. Our answer to this criticism is that the combination of the two systems would raise an adequate amount of revenue in a productively efficient manner.

Point 4. Local governments may use property taxes to tax land and natural resources that fall under their jurisdiction without destroying the efficiency of a system of cash flow taxation combined with a consumption based value added tax. However, these property taxes should not exceed the rents or pure profits that the resources are earning and these property tax payments should be deductible from both the cash flow tax base and the value added tax base in order to preserve the overall efficiency of the tax system. Moreover, property taxes should not be levied on reproducible capital goods used in the business sector.

Point 5. If there are differential rates of business income taxation or value added taxation across business units, can we still have an efficient tax system? With respect to value added taxation, the answer to this question is no: to achieve efficiency, we must have the same tax rate across all sectors (assuming that labour is mobile between sectors). With respect to cash flow business income taxation, at first glance, the answer appears to be yes: we can achieve efficiency with different tax rates. However, a closer look at the problem yields a negative answer. Firms in high tax rate jurisdictions will have an incentive to set up subsidiaries in low tax rate jurisdictions. This multi-jurisdiction firm can then choose transfer prices for intermediate outputs and inputs that will minimize its overall tax bill. However, since
the firm's objective function will no longer coincide with the socially optimal objective function, transfer pricing will lead to a loss of productive efficiency. If we want to achieve overall productive efficiency with our trading partners, then an implication of this argument is that all trading partners should set the same business tax rates. Thus both value added and business income tax rates should be uniform across jurisdictions.

Our overall conclusion is that losses of real output due to inefficient schemes of business taxation can be substantial (recall sections 3 and 4). The basic principle that leads to an efficient scheme is that all producers should face the same relative prices for their inputs and outputs. This principle still leaves lots of leeway for governments to set up arbitrary patterns of taxes and subsidies that fall on transactions between the business and household sectors. Finally, it seems silly not to have an efficient system of business taxation: why should a country simply waste resources by having an inefficient system?

Appendix 1: A Cobb-Douglas Variable Profit Function.

Let \( y > 0 \) denote the quantity of output produced, \( L > 0 \) the amount of labour utilized during the time period under consideration and \( K > 0 \) the amount of capital services utilized. If the technology is Cobb-Douglas, the production function relation may be written as:

\[
y = f(L,K) = cL^\alpha K^\beta, \quad \alpha + \beta < 1, \quad \alpha > 0, \quad \beta > 0, \quad c > 0
\]

where \( \alpha, \beta \) and \( c \) are parameters which characterize the technology.
We assume that $\alpha + \beta < 1$ so that the technology exhibits diminishing returns to scale. Alternatively, we can assume that there is a third factor (e.g., land or managerial ability) that is being held fixed throughout the period under consideration. There may be increasing or constant returns to scale in all three factors, but when the third factor is held fixed, we assume that there are diminishing returns with respect to $K$ and $L$.

Let $p > 0$ and $w > 0$ be the price of output and the wage rate respectively. Then if capital is temporarily held fixed, a competitive manager will want to choose the output level $y$ and labour input $L$ that maximizes variable profits. This leads to the following constrained maximization problem:

$$\max_{y,L} \{py - wL : y = f(L,K)\}$$

(17) $$\quad = \max_L \{pc^\alpha L^\beta - wL\} \quad \text{substituting in (16)}$$

$$\quad = \gamma p^{1/(1-\alpha)} w^{-\alpha/(1-\alpha)} K^{\beta/(1-\alpha)} \quad \text{solving (17)}$$

(18) $$\quad \equiv \pi(p,w,K)$$

where $\gamma \equiv (c^\alpha)^{1/(1-\alpha)} [\alpha^{-1-1}]$ and $\pi(p,w,K)$ denotes the Cobb-Douglas variable profit function which is dual to the Cobb-Douglas production function $f$ defined by (16). This function $\pi$ tells us what the maximum revenue minus labour cost for the firm is, given that it has $K$ units of capital at its disposal and faces the output price $p$ and the wage rate $w$. 
Note that the first order necessary condition for an interior maximum for (17) may be written as

\[ \alpha p y L^{-1} - w = 0 \quad \text{or} \quad \alpha = wL/py. \tag{19} \]

Thus the parameter \( \alpha \) may be interpreted as the optimal labour share of revenue. We note that \( 0 < \alpha < 1 \) implies that the second order sufficient conditions for (17) are satisfied.

As was done in section 3, we may set up the firm's undistorted competitive profit maximization problem:

\[
\max_{y, L, K} \left\{ py - wL - qK : y = f(L, K) \right\}
= \max_{L, K} \left\{ pf(L, K) - wL - qK \right\}
= \max_K \left\{ \pi(p, w, K) - qK \right\} \tag{20} \] using (18)

where \( q \) is the appropriate user cost of capital defined by (5) in the main text.

The first order necessary condition for (20) is

\[ \frac{\partial}{\partial K} - q = 0 \tag{21} \]

where \( \frac{\partial}{\partial K} \equiv \frac{\partial \pi(p, w, 0)}{\partial K} \) and \( K^0 \) is the solution to (20).

Substitution of (18) into (21) yields the following equation which may be used to solve for the optimal capital stock \( K^0 \):
Using \( \pi(p,w,K^0) = py^0 - wL^0 \) where \( y^0 \) and \( L^0 \) solve (20) and also using (19), we can show that (22) is equivalent to:

\[
(23) \quad \beta = \frac{qK^0}{py^0}.
\]

Thus \( \beta \) may be interpreted as the optimal capital share of revenue.

Finally, using \( \alpha + \beta < 1 \), we can show that the second order sufficient conditions for (20) are satisfied.

**Appendix 2: The Approximate Loss due to an Inefficient Business Income Tax.**

The firm's tax distorted profit maximization problem is not (20) but (24) below:

\[
\max_{y,L,K} \left\{ [py - wL - qK] - \tau[py - wL - q(1+\varepsilon)K] : y = f(L,K) \right\}
\]

\[
= \max_{K} \{ [\pi(p,w,K) - qK] - \tau[\pi(p,w,K) - q(1+\varepsilon)K] \}
\]

using the definition of \( \pi \)

\[
(24) \quad = \max_{K} \{ (1-\tau) [\pi(p,w,K) - qK] + \tau q \varepsilon K \}
\]
where $\tau$ is the business income tax rate ($0 < \tau < 1$). The parameter $\varepsilon$ is to be interpreted in the context of the detailed structure of the tax code: it is assumed that the firm is allowed to deduct $q(1+\varepsilon)K$ as its capital expense for tax purposes for the current period, rather than the economic user cost, $qK$. If $\varepsilon = 0$, then the tax system is neutral and true economic user costs are deductible. Of course, the Canadian system of business income taxation is far from neutrality.

The first order necessary condition for (24) is:

\[
(25) \quad (1-\tau)[\frac{\partial \pi(p,w,K^*)}{\partial K} - q] + \tau q \varepsilon = 0
\]

where $K^*$ is the firm's optimal capital stock when it faces the nonneutral tax system that corresponds to the maximization problem (24).

Consider the following family of tax distorted optimization problems indexed by the scalar $z$ where $0 \leq z \leq 1$:

\[
(26) \quad \max_{\lambda} \{\pi(p,w,K) - qK + \tau(1-\tau)^{-1}q\varepsilon z K\}.
\]

The first order necessary condition for (26) is:

\[
(27) \quad \frac{\partial \pi(p,w,K(z))}{\partial K} - q + \tau(1-\tau)^{-1}q\varepsilon z = 0
\]

where $K(z)$ denotes the capital solution to (26) as a function of $z$.

Note that when $z = 0$, (27) reduces to (21), the first order condition for the undistorted profit maximization problem. When $z = 1$, (27) reduces to
(25), the first order condition for the tax distorted profit maximization problem (24). Thus as \( z \) travels from 0 to 1, \( K(z) \) travels from the undistorted capital stock \( K^0 = K(0) \) to the tax distorted capital stock, \( K(1) = K^* \).

Differentiate (27) with respect to \( z \) and evaluate the resulting equation at \( z = 0 \). We obtain the following expression for \( K'(0) \):

\[
K'(0) = \frac{\tau q \varepsilon}{(1-\tau)} \frac{\partial^2 \pi}{\partial K^2}.
\]

where \( \frac{\partial^2 \pi}{\partial K^2} = \frac{\partial^2 \pi(p, w, K^0)}{\partial K^2} \).

As \( z \) varies between 0 and 1, we may define the real output loss function \( L(z) \) as the optimal net value of output less the tax distorted net value of output where output is valued at the price \( p \), labour at the wage rate \( w \) and capital input at the user cost \( q \); i.e.,

\[
L(z) \equiv [\pi(p, w, K(0)) - qK(0)] - [\pi(p, w, K(z)) - qK(z)].
\]

The level and the first two derivatives of the loss function are:

\[
L(0) = 0;
\]

\[
L'(0) = -[\pi^0_K - q]K'(0) = 0 \quad \text{using (21)};
\]
(32) \[ L''(0) = -[\tau K' - q]K''(0) - \pi_{KK}^0 [K'(0)]^2 \]

\[ = \tau^2 q^2 \varepsilon^2 / (1-\tau)^2 \pi_{KK}^0 \text{ using (21) and (28).} \]

By Taylor's Theorem, \( L(1) \) may be approximated to the second order by

(33) \[ L(1) \approx L(0) + L'(0)(1-0) + (1/2)L''(1-0)^2 \]

\[ = (1/2)L''(0) \text{ using (30) and (31).} \]

Thus the approximate loss as a fraction of the optimal value of output \( \pi_0 \) is:

\[ L(1)/\pi_0 \approx (1/2)L''(0)/\pi_0 \]

\[ = -(1/2)\tau^2 q^2 / (1-\tau)^2 \pi_0 \pi_{KK}^0 \text{ using (32)} \]

(34) \[ \approx L_A \]

which is formula (10) in the main text.

In the case of a Cobb-Douglas technology, if we differentiate (18) twice, we get:

(35) \[ \pi_{KK}^0 = -\beta (1-\alpha-\beta) \pi_0^0 / (1-\alpha)^2 (K_0^0)^2 \]

where \( \pi_0^0 \equiv \pi(p, w, K_0^0) \). Substitution of (22) and (35) into (34) yields formula (13) in the main text.
Appendix 3: The Approximate Loss due to Commodity Taxes and Subsidies.

Assume that a producer in an industry faces the market wage \( w \) as before but it sells its output at the price \( p + tp \) where either: (i) \( p \) is the world price and \( tp \) is a tax or tariff that is imposed on imports of the good, or (ii) if the good is not traded internationally, then \( p \) is the price other domestic producers face for the good and \( tp \) is a subsidy (alternatively, if \( tp \) is negative, then the output of our first industry is taxed if it is used as an intermediate input in other industries). The point is that the producer we have singled out either does not face the world price or faces a different price for its output than other domestic producers. Either of these situations leads to a loss of productive efficiency for the economy as a whole.

In this Appendix, we hold the capital stock \( K \) of our producer fixed.

The maximum revenue minus labour cost that our producer can generate is given by the variable profit function, \( \pi(p, w, K) \), as usual.

From Hotelling's Lemma (see Diewert (1973)), we know that the producer's short run supply function, \( y(p, w, K) \), can be obtained by differentiating the profit function with respect to the output price; i.e., we have:

\[
(36) \quad y(p, w, K) = \frac{\partial \pi(p, w, K)}{\partial p} = \pi_p(p, w, K).
\]

Furthermore, the negative of the producer's short run labour demand function, \( L(p, w, K) \), can be obtained by differentiating the profit function with respect to the wage rate:
(37) \[-L(p,w,K) = \partial \pi(p,w,K)/\partial w \equiv \pi_w(p,w,K).\]

The sectoral loss in the economy's real output due to the distortion \(t\) (holding \(K\) fixed) is defined to be the change in the value of output minus the value of variable input induced by the distortion, where output and variable input are priced at the social prices \(p\) and \(w\) respectively:

\[
L(1) \equiv p[y(p,w,K) - y(p+tp,w,K)] - w[L(p,w,K) - L(p+tp,w,K)]
\]

\[
= py(p,w,K) - wL(p,w,K) - [py(p+tp,w,K) - wL(p+tp,w,K)]
\]

\[
= \pi(p,w,K) - [p\pi_p(p+tp,w,L) + w\pi_w(p+tp,w,L)]
\]

where (38) follows from the line above using (36), (37), the linear homogeneity property of \(\pi(p,w,K)\) in prices (see Diewert (1973)) and Euler's Theorem on homogeneous functions.

We now derive a second order approximation to \(L(1)\). For \(0 \leq z \leq 1\), define:

\[
L(z) \equiv \pi(p,w,K) - [p\pi_p(p+ztp,w,K) + w\pi_w(p+ztp,w,K)].
\]

Note that when \(z = 1\), \(L(z)\) becomes \(L(1)\) defined by (38). Also, when \(z = 0\), we find that the term in square brackets in (39) reduces to \(\pi(p,w,K)\), so we have.
Using the linear homogeneity property of the profit function in the prices $p, w$, it can be shown that (see Diewert (1974)):

$$\pi_{pp}(p+tp, w, K)[p+tp] + \pi_{pw}(p+tp, w, K)w = 0$$

where $\pi_{pp} \equiv \frac{\partial^2 \pi}{\partial p^2}$ and $\pi_{pw} \equiv \frac{\partial^2 \pi}{\partial p \partial w}$.

Now differentiate (39) with respect to $z$:

$$L'(z) = \left[ p \pi_{pp}(p+tp, w, K)tp + w \pi_{wp}(p+tp, w, K)tp \right]$$

using (41) and $\pi_{wp} = \pi_{pw}$. Thus

$$L'(0) = 0.$$

Differentiating (42) with respect to $z$ and evaluating the derivative at 0 yields:

$$L''(0) = t^2 p^2 \pi_{pp}(p, w, K).$$

A second order approximation to $L(1)$ is:
\begin{equation}
L(1) = L(0) + \mathcal{L}'(0) + (1/2)\mathcal{L}''(0)
\end{equation}

\begin{equation}
(45) \quad = (1/2)t^2p^0 \pi^0_{pp}
\end{equation}

using (40), (43) and (44).

Thus the approximate loss as a fraction of the sector's optimal value of output, $py^0$ is

\begin{equation}
(46) \quad L_A = (1/2)t^2p^0 \pi^0_{pp}/y^0
\end{equation}

which is formula (14) in the main text.

When the technology is Cobb-Douglas (recall (18)), we have:

\begin{equation}
(47) \quad \pi^0_{pp} = (1-\alpha)^{-1}[(1-\alpha)-1] \pi(p, w, \kappa)/p^2 = \alpha\pi^0/(1-\alpha)^2 p^2.
\end{equation}

Now divide both sides of $\pi^0 = py^0 - wL^0$ by $py^0$ and obtain:

\begin{equation}
(48) \quad \pi^0/py^0 = 1 - \alpha
\end{equation}

where we have also used (19).

Substitution of (47) and (48) into (46) yields the approximate loss formula (15) which was used in section 4 above.
Footnotes

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2. General models of this type have been considered by Hicks (1945) and Diewert (1985c). If the firm produces more than one output, then the additional outputs can be included as (negative) inputs.

3. General results are available in Diewert (1985c).

4. Formula (7) was derived in Diewert (1980) where an extensive discussion may be found. For further discussion and references to the literature on the user cost concept, see Broadway (1936) and Harper, Berndt and Wood (1985). Note that if $\delta = 1$, so that the capital good is actually a nondurable good, then $q = P$.

5. Our simplifying assumption of a static production function, homogeneous capital and well defined opportunity costs for used capital goods have allowed us to decompose the firm's intertemporal cash flow maximization problem into a series of one period maximization problems of the form (8).

6. In Diewert (1981) and (1985a), we develop a single sector approach and an economy wide approach respectively to measuring the loss of real output due to a nonneutral system of business taxation. These two papers rely on the Malinvaud (1953) and Dorfman, Samuelson and Solow (1958) model of capital accumulation where possibly unobservable capital stocks play a
large role. In Diewert (1985c), we develop an alternative single sector approach to measuring the deadweight loss of a nonneutral tax system which relies instead on the general neoclassical production function defined by (1). The advantage of the latter approach is that there are no unobservable stock variables in the model.

7. See the Department of Finance (1985), Hall (1981) and the references in Boadway (1986).

8. See Shoven and Whalley (1994) for references to the literature.


10. If \( t \) is negative, then the output of our producer is being subjected to a tax and other producers face the higher price \( p \).

11. In general, this will bias our loss estimates in a downward direction; see Diewert (1985a; 236).

12. See Brown (1943) for the details. For moral hazard reasons, we do not advocate current period cash payments by the government to offset current period negative cash flows.

13. On equity grounds, full or partial tax credits for these initial asset stocks could be given to deserving firms and businesses. This would not affect the efficiency of cash flow taxation.

14. An example of a well hidden tax is the differential markup on Canadian wines versus foreign wines imposed by Provincial Government Liquor monopolies in Canada.

15. Moreover, there should be no tariffs or subsidies on internationally traded goods if we wish to achieve international productive efficiency.

16. These taxes lead to a loss in Pareto optimality (or overall efficiency in the economy), but they do not lead to a loss in productive efficiency.
17. See Atkinson and Stiglitz (1980; 129) for an explanation of this form of taxation.

18. Efficiency would result whether or not business transfer taxes were or were not deductible from taxable income in the cash flow system of taxation.

19. This follows since a consumption based value added tax is equivalent to a tax on pure profits and labour earnings. Hence if different business units faced different tax rates, the effective cost of labour services would differ across these business units.

References


