NEUTRAL BUSINESS INCOME TAXATION REVISITED

by

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Discussion Paper No. 85-04

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* This research was supported by the SSHRC of Canada
1. Introduction

In a recent paper, Boadway and Bruce (1984) have proposed a new scheme of business taxation that would achieve **neutrality**; i.e. the firm's production, hiring and investment decisions under their scheme of taxation would be identical to the corresponding decisions made in a situation where there were no business taxes at all. As Boadway and Bruce note, there has been increased interest in the design of neutral tax systems since recent episodes of inflation have interacted with the nonneutral systems of business taxation currently in place in most western economies to dramatically distort the investment decisions of firms (see Jorgenson and Sullivan for empirical evidence on this point). Boadway and Bruce (1984;237) assert that their new system of business taxation generalizes the following two well known ways of levying a neutral business tax: (i) the **immediate expensing of investment expenditures** with provision for carrying forward any current period negative taxable income as an offset against future taxable income (this is sometimes called **cash flow taxation**) and (ii) allowing interest and depreciation deductions for investment goods that are equal to the asset's true loss of economic value during the period (the **deductability of economic user costs**). Thus it would appear that Boadway and Bruce have obtained an important new result that has practical policy implications.

However, there are at least two problems with the Boadway-Bruce system of business taxation that might hinder its adoption: (i) their scheme was derived in the context of a continuous time optimization model for the firm and it is not clear how accountants (who must work with discrete time data) would be able to implement their plan and (ii) their scheme assumes that the firm is infinitely long lived, and thus it is not clear how their plan should be modified if the firm is sold or goes bankrupt.
Thus a reexamination of the Roadway-Bruce plan for a system of neutral business taxation in the context of a discrete time, finite horizon model of the firm seems called for. Thus in section 2, we lay out such a model in a tax free environment. In section 3, we derive a discrete time, finite horizon counterpart to the Roadway-Bruce scheme for a neutral business tax.

In section 4, we derive an even more general system of neutral business taxation. The scheme presented in this section is a discrete time counterpart to the general system of neutral business taxation derived in continuous time by Smith (1963:85–89).

The model of the firm utilized in sections 2–4 is reasonably general: there are arbitrary numbers of variable inputs, outputs, and investment goods; adjustment and installation costs for investment goods are allowed for; depreciation for investment goods can be endogenously determined and finally, there are no differentiability assumptions made about the firm's production function. However, in section 5, we consider a less general model where depreciation of investment goods is assumed to be exogenous and we also make some differentiability assumptions. These restrictive assumptions are made in section 5 so that we can derive an expression for the approximate cost of a nonneutral system of business taxation. Our formula for the approximate loss (45), can be evaluated if we have information on: (i) the deductibility provisions of the tax code, (ii) the prices and interest rates that the firm faces (and expects to face) and (iii) the second order derivatives of the firm's one period variable profit function with respect to the various investment goods that it is holding at that time; i.e. we require information about various elasticities that are associated with the firm's production possibilities set, information that can be obtained by econometric investigations in principle.

Section 6 concludes.
2. The Firm's Undistorted Intertemporal Profit Maximization Problem

We suppose that the firm has a horizon of \( T+1 \) periods where no economic activity takes place in the last period other than the disposal of any investment goods that have not completely depreciated. We assume that there are \( M \) variable goods that the firm can produce or utilize as inputs in each period and \( y_m^t \) denotes the quantity of good \( m \) produced by the firm in period \( t \) if \( y_m^t > 0 \) or \( y_m^t \) denotes the quantity of good \( m \) used as an input in period \( t \) if \( y_m^t < 0 \). We assume that the price of one unit of good \( m \) in period \( t \) is \( p_n^t > 0 \). Define the firms period \( t \) net output vector \( y^t \) for \( t=1,\ldots,T \) by \( y^t = [y_1^t,\ldots,y_M^t] \) and define the corresponding period \( t \) price vector by \( p^t = [p_1^t,\ldots,p_M^t] \). We assume that any inputs which appear in the list of \( M \) variable goods are nondurable in nature; i.e. they are used up completely in the period under consideration. We assume that there are \( N \) durable inputs that are available to the firm during each period: \( I_n^t \geq 0 \) denotes the planned purchases of the \( n \)th durable input or investment good to be made during period \( t \) and \( p_n^t > 0 \) denotes the corresponding purchase price for one unit of the good that is expected to prevail in period \( t \). Define the firm's period \( t \) investment vector by \( I^t = [I_1^t,\ldots,I_N^t] \) and the corresponding price vector by \( p^t = [p_1^t,\ldots,p_N^t] \) for \( t=1,\ldots,T \). At the start of period 1, we assume that the firm has at its disposal a historically determined nonnegative vector of capital stocks denoted by \( K^0 = [K_1^0,\ldots,K_N^0] \).

For each period \( t \), the firm can choose a net output vector \( y^t \) and an investment vector \( I^t \) out of a technologically determined set \( S^t \) of \( N+M \) dimensional vectors. We assume that this set depends on the firm's initial capital stock vector \( K^0 \) and on the stream of investment purchases made from periods 1 to \( t-1 \), namely \( I_1^1, I_2^2,\ldots,I_{t-1}^{t-1} \). We also assume that the period \( t \) production possibilities set \( S^t \) depends on how intensively the firm has utilized durable
inputs during previous periods. This dependence can be captured by making $s^t$ depend on the firm's choice of variable outputs and inputs produced and utilized prior to period $t$. Thus we assume

$$s^t = S(k^0; I^1, I^2, \ldots, I^{t-1}; y^1, y^2, \ldots, y^{t-1}), \quad t = 1, 2, \ldots, T.$$  

In period $T+1$ when the dissolution of the firm is envisaged, it may be possible to sell the depreciated components of the initial capital stock and the various investment goods purchased during the first $T$ periods. Thus let $n^0_n \geq 0$ be the period $T+1$ scrap value for one unit of the $n$th initial capital stock and define the vector $\alpha^0 = [n^0_1, \ldots, n^0_N]$. Let $n^t_n \geq 0$ be the period $T+1$ scrap value for one unit of the $n$th investment good purchased in period $t$ and define the vector of scrap values for investment goods purchased in period $t$ by $\alpha^t = [n^t_1, \ldots, n^t_N]$ for $t = 1, 2, \ldots, T$. However, in general, these scrap value prices will not be independent of the capital stock utilization decisions of the firm. These endogenously determined utilization (or depreciation) effects can be modeled by making the scrap value price vectors depend on the firm's utilization of variable inputs. Thus if a capital good purchased in period $t$ is utilized more heavily than is usual, this increased utilization will be detected by increased consumption of electricity or increased hiring of labour and so on. In general, we allow the scrap value prices for investment goods purchased in period $t$ to depend on the net output vectors utilized by the firm from periods $t$ to $T$; i.e. we have

$$\alpha^0 = \alpha^0(y^1, \ldots, y^T); \quad \alpha^t = \alpha^t(y^t, y^{t+1}, \ldots, y^T) \quad \text{for} \quad t = 1, 2, \ldots, T.$$
We assume, following Hicks (1946), that the objective of the firm is to maximize expected discounted profits plus expected discounted scrap value over the horizon of the firm subject to the constraints of technology. Thus the firm’s objective function, regarded as a function of its net output and investment decision variables, may be defined by:

\[
\pi(y, \ldots, y, I, \ldots, I) = \sum_{t=1}^{T} R_t [\eta^t y_t - p^t I_t] + R_{T+1} [\sum_{t=1}^{T} 0^t I_t]
\]

where all variables have been defined above with the exception of the discount factors \( R_t \) which are defined by

\[
P_1 = 1, P_2 = \frac{1}{1+r_2}, \ldots, P_{T+1} = \frac{1}{(1+r_2)(1+r_3) \ldots (1+r_{T+1})}.
\]

The interest rate \( r_t > -1 \) is the one period interest rate used to discount period \( t \) earnings relative to the previous period. Alternatively, it is the one period lending (or borrowing) rate that the firm expects to face in period \( t-1 \). Recall that the scrap value vectors \( O^t \), in general, depend on the \( y^t \) decision values; recall (2).

Using definition (3), we may now write the firm’s Hicksian intertemporal expected profit maximization problem as

\[
\max_{y, \ldots, y, I, \ldots, I} T \{ \pi(y, \ldots, y, I, \ldots, I) : (y^t, I^t) \in R_t, t=1, \ldots, T \}
\]

\[
= \pi(y^*, \ldots, y^*, I^*, \ldots, I^*) \text{ say}
\]
so that we assume \( y^* \equiv [y^1, \ldots, y^T] \) and \( I^* \equiv [I^1, \ldots, I^T] \) solve (5).

Note that the period \( t \) technology set \( S^t \) which occurs in (5) is defined by (1). In view of (1) and (2), it can be seen that the finite horizon intertemporal profit maximization problem defined by (5) is flexible enough to capture costs of adjusting or installing capital equipment as well as endogenously determined depreciation of investment goods.

3. The Roadway-Bruce System of Business Taxation in Discrete Time

Consider the firm described in the previous section. Assume that the firm now has to pay a uniform tax on taxable income in each period at the rate \( r \) where \( 0 < r < 1 \). Taxable income in period \( t \leq T \) is defined to be net revenue \( p^t y^t \) less a deduction \( D^t \), which is determined by the firm's investment good purchases up to period \( t \). For simplicity, we take the firm's taxable income in the final period \( T+1 \) (when the firm is expected to be dissolved) to be \( 0^0 y^0 + \sum_{t=1}^{T} 0^t I^t \), the sum of the scrap values for the firm's initial capital stock \( K^0 \) plus the scrap values for the investment good purchases it made from periods 1 to \( T \).

We assume that the firm wishes to maximize after tax discounted profits. Defining the firm's intertemporal \( MT \) dimensional vector of planned outputs and variable inputs by \( y \equiv [y^1, y^2, \ldots, y^T] \) and the \( MT \) dimensional vector of planned investment purchases by \( I \equiv [I^1, I^2, \ldots, I^T] \), the firm's new objective function regarded as a function of its decision variables \( y \) and \( I \) may be defined as
\[
\begin{align*}
\mathcal{P}(y,I) & \equiv \gamma \sum_{t=1}^{T} R_t [p_t y_t - p_t I_t] + \sum_{t=1}^{T+1} \left( \alpha^0 \cdot r^0 + \sum_{t=1}^{T} \gamma^t \cdot I_t \right) \\
& - \gamma \sum_{t=1}^{T} R_t [p_t y_t - I_t] + \sum_{t=1}^{T+1} \left( \alpha^0 \cdot r^0 + \sum_{t=1}^{T} \gamma^t \cdot I_t \right) \\
(7) & \quad = (1-\gamma) \pi(y,I) - \gamma \sum_{t=1}^{T} R_t [p_t y_t - I_t]
\end{align*}
\]

where (7) follows using definition (3). Using the above definition of \( \mathcal{P} \), we may now express the firm's tax distorted profit maximization problem as

\[
\max_{y,I} \{ \mathcal{P}(y,I) : (y,I) \in S^t, t=1,...,T \}
\]

where the period \( t \) production possibilities set \( S^t \) is defined by (1).

Define a **tax scheme** to be a set of definitions for the capital account deductability allowances \( D_t \), \( t=1,...,T \), which appear in (7). A tax scheme is defined to be **neutral** if the solutions \( (\tilde{y}, \tilde{I}) \) to the firm's tax distorted inter-temporal profit maximization problem (7) are also solutions to the firm's undistorted maximization problem (5).

Now we are ready to describe a discrete time version of the Roadway and Bruce (1984) scheme for neutral business taxation. Define the period \( t \) deduction as

\[
D_t \equiv \delta_t A_t + r_t A_{t-1}, \quad t = 1,2,...,T.
\]

The interest rate \( r_t \) which appears in (9) is the same \( r_t \) which appears in (4); i.e., it is the firm's opportunity cost of lending or borrowing going from period \( t-1 \) to \( t \). The nonnegative numbers \( \delta_t \) are "accounting depreciation rates" to he
chosen by the firm. The number \( A_t \) defines the firm's "value of assets for tax purposes" in period \( t \). The initial value of assets \( A_0 \) may be chosen by the firm in the Roadway-Bruce scheme, but we shall see later that this does not lead to a very happy outcome for the government (see Proposition 3 below). In any case, given \( A_0 \) and the firm's choices for the accounting depreciation rates \( \delta_t \), the accounting asset values \( A_t \) for \( t \geq 1 \) are defined by

\[
A_t = \left( p_t I_t + A_{t-1} \right) / (1 + \delta_t), \quad t = 1, 2, \ldots, T.
\]

Given \( A_0 \) and the firm's choices for \( \delta_1, \ldots, \delta_T \), the accounting asset values \( A_t \) may be expressed in terms of \( A_0 \) and \( \delta_1, \ldots, \delta_T \) by using (10) repeatedly. We obtain

\[
A_t = (p_t I_t + A_0)(1 + \delta_t)^{-1},
\]

\[
A_2 = p_2 I_2(1 + \delta_2)^{-1} + p_1 I_1(1 + \delta_1)(1 + \delta_2)^{-1} + A_0(1 + \delta_1)^{-1}(1 + \delta_2)^{-1},
\]

\[
\vdots
\]

\[
A_T = \sum_{t=1}^{T} p_t I_t \left[ \prod_{i=1}^{t} (1 + \delta_i)^{-1} \right] + A_0 \prod_{i=1}^{T} (1 + \delta_i)^{-1}.
\]

Equations (11) may now be substituted into (9) in order to express the deductions \( D_t \) as functions of \( A_0 \) and \( \delta_1, \ldots, \delta_T \).

The following proposition is a discrete time, finite horizon counterpart to the continuous time, infinite horizon general neutrality proposition of Roadway and Bruce (1984; 234).

**Proposition 1:** Suppose the government implements the tax scheme defined by (9) and (10), where the firm is free to choose the nonnegative accounting
depreciation rates $\delta_1, \ldots, \delta_T$. In addition, suppose that the government chooses the initial accounting asset value $A_0$ and allows the firm an extra deduction from period T taxable income equal to $A_T$ defined in (11). Then the resulting scheme of business taxation is neutral irrespective of the firm's choices for the accounting depreciation rates.

**Proof:** The objective function for the firm's maximization problem is $P(y, I)$ defined by (7) plus the discounted period T tax rebate, $\tau R_T A_T$. Recalling that $\pi(y, I)$ is defined by (3), the firm's objective function is:

$$P(y, I) + \tau R_T A_T = (1-\tau)\pi(y, I) - \tau L_{t=1}^{T} R_t [p^t \cdot I^t - D_t] + \tau R_T A_T$$

using definition (9)

$$= (1-\tau)\pi(y, I) - \tau L_{t=1}^{T} R_t [p^t \cdot I^t - \delta_t A_t - \tau_{t-1} A_{t-1}] + \tau R_T A_T$$

using a rearrangement of (10)

$$= (1-\tau)\pi(y, I) - \tau L_{t=1}^{T} R_t [A_t - (1+\tau_{t-1}) A_{t-1}] + \tau R_T A_T$$

using $R_t \equiv 1$ and $(1+\tau_{t}) R_t = R_{t-1}$; see (4)

$$= (1-\tau)\pi(y, I) + \tau (1+\tau_{1}) A_0. \quad (12)$$

Hence for any choice of $\delta_1, \ldots, \delta_T$, maximizing the firm's tax distorted objective function defined by (12) subject to the constraints of technology will be equivalent to maximizing $\pi(y, I)$ with respect to the components of $y$ and $I$, since
the term \( r(1+r_1)A_0 \) does not depend on any of the firm's decision variables, \( y, I \) and \( \delta_1, \ldots, \delta_T \). Hence any \( \hat{y}, \hat{I} \) solution to the firm's tax distorted maximization problem will also be a solution to the undistorted intertemporal profit maximization problem (5), and thus the tax system is neutral.

\[ \hat{P}(y, I, \delta_1, \ldots, \delta_T) = (1-t)p(y, I) + r(1+r_1)A_0 - TR_A_T \]

where \( p \) is defined by (3) and \( A_T \) is defined in (11). Since \( A_T \) depends on \( I \) and \( \delta_1, \ldots, \delta_T \), it can be seen that for any given \( \delta_1, \ldots, \delta_T \), solutions to the tax distorted maximization problem (5) where \( P \) is replaced by \( \hat{P} \) defined by (13), will not in general be solutions to the tax free maximization problem (5).

The proof of Proposition 1 explains why we defined the period \( t \) deduction \( D_t \) by (9) and (10): using those definitions, we were able to show that the terms \( \Sigma_{t=1}^T [R_t^T - R_{t-1}^T] = \Sigma_{t=1}^T [R_t A_t - R_{t-1} A_{t-1}] = (1+r_1)^{A_0} - R_T A_T \). This is the discrete time counterpart to the technique used by Roadway and Bruce (1984, 234), who obtained the following continuous time identity:
(14) \[ \int_0^\infty R_t [P^t I - (r_t + \delta_t) A_t] \, dt = A_0 - \lim_{t \to \infty} P_t A_t. \]

Roadway and Bruce implicitly assumed that \( \lim_{t \to \infty} R_t A_t = 0. \) If they had restricted themselves to a finite horizon, they would have found that the term \( R_t A_t \) would have destroyed their neutrality result; i.e. they would have obtained the counterpart to our Corollary 1.1.

In Corollary 1.1, we did not allow the firm to choose arbitrarily large accounting depreciation rates. In the following Proposition, we allow firms to optimize with respect to their accounting depreciation rates, subject only to the nonnegativity constraints \( \delta_t \geq 0. \)

**Proposition 2:** Suppose the government implements the tax scheme described by (9) and (10) where the government chooses \( A_0 \) but firms are allowed to choose nonnegative (and arbitrarily large) accounting depreciation rates \( \delta_1, \ldots, \delta_T. \) Then a profit maximizing firm that also wishes to minimize the time stream of its tax payments will choose \( \delta_t \) to be arbitrarily large. In the limiting case where all \( \delta_t = +\infty, \) the resulting scheme of business taxation is neutral.

**Proof:** For any choice of \( y \) and \( I \) which satisfies the firm's technological constraints, the firm will want to maximize \( \tilde{P}(y, I, \delta_1, \ldots, \delta_T) \) defined by (13) with respect to the \( \delta_t. \) This is equivalent to minimizing \( A_t \) with respect to the \( \delta_t. \) From the last equation in (11), it can be seen that we may choose, \( \delta_1, \ldots, \delta_{T-1} \) arbitrarily and then choose \( \delta_T = +\infty \) which will make \( A_T = 0. \) The resulting tax scheme will be neutral.
If the firm wishes to minimize the time stream of its tax payments, then from (7) it can be seen that the firm will wish to make each deduction $D_t$ as large as possible. Substituting (10) into (9) yields

$$\delta_t = \frac{(1+\delta_t)^{-1} P^t I^t + \lambda_{t-1}^t r_t^t + \delta_t (1+\delta_t)^{-1} r^t I^t}{.}$$

Given $y$, $I$ and $A_0$, for each $t$, maximizing $D_t$ with respect to $\delta_t \geq 0$ yields the solution:

$$\delta_t = +\infty, \quad t = 1, 2, \ldots, T.$$ 

Substitution of (16) into (11) yields:

$$\lambda_1 = 0, \lambda_2 = 0, \ldots, \lambda_T = 0.$$ 

Substituting (16) and (17) into (15) yields:

$$D_t = \frac{1}{P^t I^t + A_0 (1+r_t) + \delta_t (1+\delta_t)^{-1} r^t I^t}, \quad t = 2, \ldots, T.$$ 

Thus all investment purchases made in period $t$ are immediately deducted from taxable income for period $t$.

Corollary 2.1: If the government sets the accounting value for the firm's initial capital stock $A_0$ equal to zero and the profit maximizing firm that also
wishes to minimize the time stream of tax payments is allowed to choose arbitrarily large accounting depreciation rates \( \delta_t \), then the tax scheme described by (9) and (10) is equivalent to a system of cash flow taxation (i.e. investment expenditures are expensed immediately).

The corollary follows from (18) upon setting \( A_0 = 0 \).

Thus taking into account the endogeneity of the firm's choice of accounting depreciation rates, we find that our discrete time, finite horizon version of the Boadway-Bruce system of business taxation reduces to cash flow taxation if the government sets \( A_0 = 0 \).

Boadway and Bruce show that their tax scheme is neutral even if firms are allowed to pick \( A_0 \). We can similarly show that our neutrality results stated in Propositions 1 and 2 above are still valid if firms are allowed to pick \( A_0 \). However, the revenue implications of allowing firms to pick \( A_0 \) are rather severe as the following Proposition shows.

**Proposition 3:** Suppose that the firm would make the nonnegative discounted profits \( \pi(y^*, I^*) \) defined by (6) if there were no business taxes. Suppose that the government implements the tax scheme described by (9) and (10) where the firm is allowed to choose arbitrary nonnegative \( A_0 \) and \( \delta_t \). Then the government will never collect a positive present value of tax revenue from the firm.

**Proof:** For any \( A_0 \), from the proof of Proposition 2, we see that if \( \delta_t = +\infty \) for each \( t \), then a solution \((y^*, I^*)\) to the tax distorted problem (9) will also be a solution to the firm's maximization problem (5) which has no business taxation. Then the before tax discounted net revenue of the firm is \( \pi(y^*, I^*) \) and the after tax discounted revenue of the firm is \( P(y^*, I^*) = (1-\tau)\pi(y^*, I^*) + \tau(1+r_c)A_0 \).
Hence government discounted tax revenues are

\[ R(y^*, I^*, A_0) = \pi(y^*, I^*) - (1-T)\pi(y^*, I^*) + \tau(1+r_1)A_0 \]

\[ = \tau[\pi(y^*, I^*) - (1+r_1)A_0]. \]

Hence the firm needs only to choose \( A_0 \geq \pi(y^*, I^*)/(1+r_1) \geq 0 \) in order to pay no taxes.

Thus a government desiring to raise revenue will have a strong incentive to force firms to set \( A_0 = 0 \), in which case, rational profit maximizing firms will cause our discrete time version of the Roadway-Brucne neutral business taxation scheme to collapse down to the well known cash flow (or immediate expensing of investment) system of taxation, which is, of course, much easier to administer than the Roadway-Brucne system.

In the following section.

Are there more general schemes for achieving a neutral business tax than the cash flow scheme or the discrete time Roadway-Brucne scheme which appeared in Proposition 1? We address this question in the following section.

4. General Neutrality Schemes in Discrete Time, Finite Horizon Models

Recall the firm's taxless intertemporal profit maximization problem defined by (5) and the firm's tax distorted profit maximization problem defined by (8).

The firm's objective function in (8) is the discounted after tax net revenue function \( P(y, I) \). Let us now define this function by:

\[ P(y, I) = \sum_{t=1}^{T} R_t [P_t \cdot y_t - P_t \cdot I_t] + P_{T+1} \left[ \sum_{t=1}^{T} R_t \cdot y_t - \pi_{T+1} \right] \]
where all variables have been defined in sections 2 and 3 except for the
deductability vectors \(d^s_1, \ldots, d^s_N\) and the final period taxable income
scrap value vectors \(q^t_1, \ldots, q^t_N\). Define:

\[
(21) \quad d^s_n = \text{deduction allowed from taxable income in period } t \text{ for one unit}
\quad \quad \quad \text{of the } n\text{th investment good purchased in period } s \leq t, \text{ where}
\quad \quad \quad 1 \leq s \leq t \leq T \text{ and } n = 1, 2, \ldots, N,
\]

\[
(22) \quad q^0_n = \text{the amount that must be added to taxable income in period } T+1 \text{ for}
\quad \quad \quad \text{each unit of the } n\text{th capital stock which was held at the}
\quad \quad \quad \text{beginning of period } 1 \text{ by the firm for } n = 1, 2, \ldots, N, \text{ and}
\]

\[
(23) \quad q^t_n = \text{the amount that must be added to taxable income in period } T+1 \text{ for}
\quad \quad \quad \text{each unit of the } n\text{th investment good which was purchased by the}
\quad \quad \quad \text{firm in period } t \text{ for } n = 1, 2, \ldots, N \text{ and } t = 1, 2, \ldots, T.
\]

The following Proposition is a discrete time counterpart to the continuous
time results of Smith (1963:85-89).

**Proposition 4:** Suppose the firm solves the tax distorted maximization problem
(8) where \(P(y,I)\) is defined by (20). The system of business taxation associated
with this problem is neutral over all possible technologies if for each period
\(t = 1, \ldots, T\) and each investment good \(n = 1, \ldots, N\), the following restrictions on
the deductabilities \(d^s_n\) and the taxable scrap values \(q^t_n\) hold:
\[(24) \quad \sum_{s=t}^{T} p_s d_s^t - R_{T+1} n^t = R_{t} n^t - R_{T+1} n^t.\]

If \( \cap^T_n \) is not independent of \((y,I)\), then the \( d_s^t \) and \( q^t \) must also be functions of \((y,I)\) and the TN relations \((24)\) must hold identically for all feasible values of \((y,I)\).

**Proof:** We may rearrange the firm's objective function \( P(y,I) \) defined by \((20)\) as follows:

\[
(25) \quad P(y,I) = (1-\tau) \pi(y,I) - \tau \left[ \sum_{t=1}^{T} \left( p_t - R_{t} - R_{T+1} q^t \right) \right] + \tau R_{T+1} \left[ 0^0 - q^0 \right] \cdot k^0.
\]

If the relations \((24)\) hold, \( P(y,I) = (1-\tau) \pi(y,I) + \tau R_{T+1} \left[ 0^0 - q^0 \right] \cdot k^0 \) so maximizing \( P(y,I) \) subject to the constraints of technology is equivalent to maximizing \( \pi(y,I) \). Thus the tax system is neutral.

O. P. D.

The relations \((24)\) have the following interpretation: for each unit of the \( n \)th investment good purchased in period \( t \), the present value of the deductions allowed for tax purposes from period \( t \) through \( T \) minus the discounted salvage or resale value allowed for tax purposes at the final period of our horizon must equal the discounted purchase cost of the asset less the discounted actual scrap value of the asset at the final period.

**Corollary 4.1** (discrete time counterpart to Smith (1963:86)): Immediate expensing and defining taxable scrap value to be actual scrap value is a neutral tax system; i.e. the following deductibility and scrap value provisions lead to a neutral system of business taxation:
\[
(26) \quad d_{tt}^n = p_t^t, \quad d_{ts}^n = 0 \text{ for } s = t+1, t+2, \ldots, T; \quad t = 1, 2, \ldots, T;
\]

\[
(27) \quad q_n^t \equiv \cap_n^t \text{ for } n = 1, 2, \ldots, N \text{ and } t = 1, 2, \ldots, T.
\]

Proof: It is easy to show that (26) and (27) satisfy (24).

The neutrality result in Proposition 4 contains the neutrality result of Proposition 1 as a special case.

We agree with Smith (1963;90) who observed that the government's lack of information will tend to prevent it from implementing a general system of neutral business taxation of the type described by Proposition 4, except for the immediate expensing system described by Corollary 4.1.

Roadway and Bruce (1984;232) note the following objection to the immediate expensing scheme: it may require (perhaps large) negative tax payments to firm's acquiring capital. However, Brown (1948) noted a solution to this problem: simply allow the firm experiencing negative tax payments in a period (due to heavy investment expenditures) to carry forward an appropriately appreciating tax credit which could be used in later periods to offset positive taxable incomes.

Thus if neutrality is the government's objective, the immediate expensing tax system described by Corollary 4.1 will do the job. In the following section, we ask the question: what are the costs of a nonneutral system of business taxation?
5. The Approximate Cost of a Nonneutral Business Tax

Given that the system of business taxation in a country or region is nonneutral, it seems worthwhile to investigate the costs of the nonneutrality. If the costs are high, then there will be a definite incentive on the part of the governments involved to reduce the tax distortions, whereas if the costs are demonstrated to be low, then the transactions costs of implementing any change in the tax code will prevent any tax reform.

Our attempt to measure the costs of a nonneutral tax system will follow in the footsteps of Harberger (1974), who used quadratic approximations to measure the effects of various tax distortions. However, we will not follow Harberger and attempt a general equilibrium approach to measuring the costs of nonneutrality; instead we will take a partial equilibrium approach which regards the price vectors facing the firm for variable goods $p^t$ and for investment goods $n^t$ as fixed. We regard the cost of having a nonneutral tax scheme as the reduction in the value of the firm’s discounted net outputs (treating investment good purchases as inputs) compared to the corresponding no tax situation, holding constant the reference prices $p^t$ and $n^t$ (as well as the scrap value prices $s^t$).

In order to make our subsequent computations more tractable, we make the following simplifications to our general model of firm behaviour explained in section 2:

\begin{equation}
\eta^t \equiv \left[ \eta_1^t, ..., \eta_n^t \right] \text{ is constant for } t = 1, 2, ..., T;
\end{equation}

\begin{equation}
s^t = S(r^0, I^1, I^2, ..., I^{t-1}) \text{ for } t = 1, 2, ..., T.
\end{equation}
Assumption (28) says that the vector of final period scrap values for investment goods purchased in period $t$, $\mathbf{0}^t$, does not depend on the utilization of the investment goods. Assumption (29) says that the firm's period $t$ short run production possibilities set $S^t$ depends only on the firm's initial capital stock vector $K^0$ and subsequent purchases of investment goods up to period $t$. The effect of assumptions (28) and (29) is to make depreciation of investment goods exogenous.

Define the firm's period $t$ gross profit function (see Gorman (1968) for this terminology) or variable profit function (see Diewert (1973) for this terminology) by

$$(30) \quad \pi(p^t, K^0, I^1, \ldots, I^t) = \max_{y} \left\{ p^t \cdot y : (y^t, I^t) \in S(K^0, I^1, \ldots, I^{t-1}) \right\}, \quad t = 1, \ldots, T.$$ 

Thus in the $t$th maximization problem defined by (30), the firm maximizes the value of outputs produced in period $t$ minus the value of variable inputs used, conditional on the period $t$ price vector $p^t$ and on the firm's investment decisions made from periods 1 to $t, I^1, \ldots, I^t$.

Using definitions (30), the firm's undistorted intertemporal profit maximization problem (5) may be rewritten as the following problem:

$$(31) \quad \max_{I} \left\{ \sum_{t=1}^{T} \left[ \pi(p^t, K^0, I^1, \ldots, I^t) - p^t \cdot I^t \right] + \sum_{t=1}^{T+1} \mathbf{0}^t \cdot I^t : I^t \geq \mathbf{0}_N, \right\}$$

$$= \max_{I} \left\{ \Pi(I) : I \geq \mathbf{0}_N \right\}$$

where $\Pi$ is the objective function in (31), $I \equiv [I^1, I^2, \ldots, I^T]$, and $\mathbf{0}_N$ and $\mathbf{0}_N$ are zero vectors of dimension $N$ and $NT$ respectively.
We now make some additional assumptions: we assume that \( I^* \) solves (37), that \( I^* \to 0 \) is a strictly positive vector so that the nonnegativity constraints in (31) are not binding, and the objective function in (31) is twice continuously differentiable at \( I^* \), so that the \( NT \) dimensional vector of first order partial derivatives \( V_I \Pi(I^*) \) exists and the \( NT \) by \( NT \) dimensional matrix of second order partial derivatives \( V_{II} \Pi(I^*) \) exists and is symmetric. Under these assumptions, the solution \( I^* = [I^*_1, ..., I^*_{NT}] \) to (31) satisfies the following first order necessary conditions:

\[
\begin{align*}
\sum_{t=1}^{T} V_{t} \Pi_t(p_t, K_t, I^*_{t-1}, ..., I^*) &= R_1 - R_{T+1}^1, \\
\sum_{t=2}^{T} V_{t} \Pi_{t-1}(p_t, K_t, I^*_{t-2}, ..., I^*) &= R_2 - R_{T+1}^2, \\
&\vdots \\
R_T V_T \Pi_T(p_T, K_T, I^*_{T-1}, ..., I^*) &= R_T - R_{T+1}^T.
\end{align*}
\]

Under the above assumptions and the additional assumption that the government knows the functional form for the variable profit functions \( \pi^t \), we may prove the following corollary to Proposition 4 above:

**Corollary 4.2** (counterpart to the Samuelson (1964) economic user cost rule for achieving neutrality): If the government chooses the vector of tax deductibilities in period \( t \) for investment goods purchased in period \( s \leq t \), \( \delta_t^s \), to be the schedules

\[
\delta_t^s = V_{ts} \Pi_{ts}(p_t, K_t, I^*_{t-1}, ..., I^*), \quad 1 \leq s \leq t \leq T,
\]
and does not require the firm to add any scrap value income to its taxable income in period T+1 so that

\[ q_t^T 
\]

then the resulting system of business taxation is neutral.

**Proof:** Substitute the definitions (33) into the first order conditions (32). The resulting equations are equivalent to the Smith neutrality conditions (24) upon using equations (34).

Obviously, the informational requirements for using the above method to achieve business tax neutrality are much higher than for the cash flow method, so we will not consider this Samuelsonian method any further.

Let us return to the primary problem in this section; the derivation of a second order approximation to the loss of output induced by a nonneutral system of business taxation. The first order conditions (32) may be written more succinctly as

\[ V_{I_{11}}(I^*) = 0. \]

We now assume that the solution \( I^* \) to the no tax maximization problem satisfies Samuelson's (1947;360) strong second order conditions for a regular maximum; i.e. we assume
(36) \( V_2^{1/2} \tilde{h}(I^*) \) is a negative definite symmetric matrix.

Consider now the firm's tax distorted intertemporal profit maximization problem (8), where the objective function is defined by (25). Let us neglect the constant term, \( r_{T+1} q_T^0 \cdot I^0 \) in (25). Then using definitions (30), problem (8) may be rewritten as

\[
\max_{I \geq 0} \left[ \|I\| - \tau \left( E_{t=1}^{T} R_t \left[ \sum_{s=1}^{T} d_{s,t} \right] + R_{T+1} q_T^0 \cdot I^0 \right) \right]
\]

\[
= \max_{I \geq 0} \left[ \left( 1-\tau \right) \|I\| - \tau \left( \sum_{t=1}^{T} \left( R_t p_t - R_{T+1} q_T^0 \right) I^t \right) \right]
\]

\[
= \max_{I \geq 0} \left[ \left( 1-\tau \right) \|I\| - \tau \left( \sum_{t=1}^{T} d^t \cdot I^t \right) \right]
\]

(37)

where the no tax objective function \( \|I\| \) is defined in (31) and the period \( t \) distortion vector \( d^t = [d^t_1, d^t_2, \ldots, d^t_N] \) is defined by

\[
d^t = R_t p_t - R_{T+1} q_T^0 - \sum_{s=t}^{T} R_s d_{s,t} + R_{T+1} q_T^0, \quad t = 1, 2, \ldots, T.
\]

Note that if the general neutrality conditions (24) are satisfied by the \( d_{s,t} \) and \( q_t \), then the distortion vectors \( d^t = 0_N \) for \( t = 1, \ldots, T \) and solving the tax distorted maximization problem (37) is equivalent to solving the no tax problem (31).

Our final assumption is that \( I^* \geq 0 \) solves (37) and that this solution satisfies the following first order necessary conditions for (37):
\[ (39) \quad (1 - \tau) \nabla I(1) - TD = 0_{NT} \]

where \( D = [D^1, D^2, \ldots, D^T] \) is \( NT \) dimensional overall distortion vector.

Consider the following system of equations implicitly defining the investment vector \( I(z) \) as a function of \( z \) for \( 0 < z < 1 \):

\[ (40) \quad (1 - \tau) \nabla I[I(z)] - zTD = 0_{NT}. \]

When \( z = 0 \), (40) reduces to (35), so \( I(0) = I^* \), the optimal investment vector when there are no tax distortions. When \( z = 1 \), (40) becomes (39), so \( I(1) = I^* \), the solution to the tax distorted intertemporal profit maximization problem (37). Assumption (36) and the Implicit Function Theorem guarantees the existence of \( I(z) \) and its derivatives in a neighborhood of \( z = 0 \). Differentiating (40) with respect to \( z \) and evaluating the derivatives yields the following expression for the vector of investment derivatives, \( I'(0) \):

\[ (41) \quad I'(0) = \tau(1 - \tau)^{-1} \left[ \nabla^2 I[I^*] \right]^{-1} D \]

where the distortion vector \( D \) is interpreted as an \( NT \) dimensional column vector.

As we mentioned earlier, the loss of discounted net output produced by the firm due to the nonneutrality of the tax system is

\[ (42) \quad L = H(I^*) - H(I) = H(I(0)) - H(I(1)). \]
Define \( f(z) \equiv \Pi(I(z)) \). Compute the first and second derivatives of \( f(z) \) evaluated at \( z = 0 \) as follows:

\[
(43) \quad f'(0) = V_I[I(0)]I'(0) = V_I(I^*)I'(0) = 0 \quad \text{using (35)}. 
\]

\[
(44) \quad f''(0) = I'(0) \cdot V^2_{II}[I(0)]I'(0) + V_I[I(0)]I''(0) \\
\quad = I'(0) \cdot V^2_{II}[I^*]I'(0) + Q_{NT} \cdot I''(0) \quad \text{using (35)}. 
\]

Now approximate \( \Pi(I(1)) \approx f(1) \) to the second order by \( f(0) + f'(0)[1-0] + (1/2) \cdot f''(0)[1-0]^2 \). Then using (42) and (43), \( L \) is approximated to the second order by the following approximate loss:

\[
(45) \quad L_A \equiv -(1/2) \tau^2(1-\tau)^2 D \cdot [V^2_{II}[I^*]]^{-1} D \geq 0 
\]

where the inequality follows from the negative definiteness of the Hessian matrix \( V^2_{II}[I^*] \); recall (36).

The elements of the distortions vector \( D \) can be determined from a knowledge of the prices of investment goods, interest rates and the relevant deductibility provisions in the tax code; see (38). The rate of business income taxation \( \tau \) is easy to determine. The remaining information required to implement the approximate loss measure \( L_A \) defined by (45) is information on the responses of
(inverse) demand functions for investment goods to marginal changes in the firm's utilization of investment goods; i.e. we require information on the derivatives $\nabla^2_{II} l(I^*)$.

Some interesting qualitative information on the size of the approximate loss can be obtained by examining formula (45) more closely: (i) the loss grows faster than a quadratic rate as the tax rate $\tau$ increases from 0 to 1, (ii) the loss grows quadratically in the elements of the distortions vector $D$, so if the elements of $D$ are small, the loss is likely to be small; and (iii) the loss decreases as the matrix $\nabla^2_{II} l(I^*)$ becomes more negative definite; i.e. the loss decreases as the (inverse) demand functions for investment goods become more negatively sloped (or more inelastic). Thus the loss increases if the demand functions are relatively flat or elastic, so that a small increase in the price of an investment good induces the firm to dramatically reduce its demand for that good.

The above second order approximation partial equilibrium approach to the measurement of the costs of a nonneutral system of business taxation can be extended to encompass the entire production sector of an open economy\textsuperscript{11} or it can be extended to a complete general equilibrium model.\textsuperscript{12}

6. Conclusion

Our main conclusion is that Roadway-Bruece schemes for achieving neutral business taxation have little to offer over a conventional cash flow scheme. Indeed, under some reasonable hypotheses, our discrete time Roadway-Bruece scheme collapses to the cash flow scheme; see Corollary 2.1.

Along with Brown (1949) and Smith (1963;91-92), we recommend that governments seriously consider implementing a system of cash flow taxation.
FOOTNOTES

1. The neutrality of this scheme of taxation has been noted by many authors, including Brown (1948), Smith (1963), Hall and Jorgenson (1967), Sandmo (1947), King (1975), roadway (1978) (1980), Schworm (1979) and Hall (1981).


3. Notation: $p_t \cdot y_t \equiv \sum_{m=1}^{M} p_t^m y_t^m$ denotes the inner product of the vectors $p_t$ and $y_t$.

4. The rate $r_1$ appears in (9) but it did not appear in (4); $r_1$ is the historically determined borrowing rate the firm faced in period 0 if its period 0 cash flow was not sufficient to finance its investment expenditures in period 0. If the firm’s cash flow was sufficient, then $r_1$ would be the relevant marginal lending rate that the firm faced in the period prior to period 1.

5. Compare our equation (10) with equation (3) in Roadway and Bruce.

6. For a recent econometric approach to measuring the costs of a nonneutral business tax system (and for references to other econometric and applied general equilibrium modelling approaches), see Jorgenson and Yun (1984).
7. The reader may well wonder what happened to the vectors of variable outputs and inputs \( y_t \) which appeared in (5) but not in the equivalent problem (31). If the variable profit functions are differentiable with respect to the components of the price vectors \( p_t \), then using Hotelling's lemma, the \( y_t^* \) which appear in (6) may be calculated as the following gradient vectors:

\[
y_t^* = \nabla \pi(p_t, k, I_t, ..., I_T) \quad \text{for } t = 1, ..., T.
\]

In the nondifferentiable case, the situation is more complex and the reader may consult Diewert (1973; 290) for the details.

8. This restrictive assumption may be relaxed using the techniques outlined in Diewert (1984a). However, the resulting notational complexities are too cumbersome for us to deal with in this paper.

9. Boadway and Bruce (1984; 231) call this the imputed income method for achieving neutrality. See footnote 2 for other references to this method.

10. Equation (41) may be used to analyze the effects on investment of changes in tax policy over an initial neutral system. For example, if in an initially neutral system, the present value of the deductions allowed for the purchase of units of the nth investment good purchased in period \( t \) were increased, then \( I_{n,t}^* \) would increase; i.e. (41) would imply \( I_{n,t}^* (0) > 0 \). For similar comparative statics results, see Sandmo (1974).


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