Bias Due to Input Source Substitutions: Can It Be Measured?

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Abstract

Once a business opts to purchase rather than produce an input, it can also change the source from which the product is procured. Producer price index programs face problems in dealing with price changes associated with sourcing changes. We present measures for price index bias due to sourcing substitutions. We begin with highly simplified cases to convey the rationale for our approach, and then show how the measures could be generalized. We also explain related aspects of the industry accounts. This material makes it clear that the growth of outsourcing and the related increases in domestic and foreign sourcing substitutions pose important challenges for statistics agencies.

117 Department of Economics, University of British Columbia and the University of Alberta School of Business. This is a revised version of a paper presented at the Conference on “Measurement Issues Arising from the Growth of Globalization,” sponsored by the W.E. Upjohn Institute and the National Academy of Public Administration and held in Washington, DC, November 6–7, 2009. The authors thank William Alterman, Robert Feenstra, Mike Horrigan, Susan Houseman, Emi Nakamura, and Jón Steinsson for helpful comments on various drafts, and the Social Sciences and Humanities Research Council of Canada (SSHRC) for partial funding. All errors and opinions are the sole responsibility of the authors.
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Offshoring, outsourcing, consumer price indexes, producer price indexes, intermediate input price indexes, bias in price indexes, Fisher index, total factor productivity growth, production accounts, System of National Accounts, exports and imports in the input-output accounts.
Producers utilize and make many thousands of products in a year. Index numbers help reduce and summarize this abundance of microeconomic information. Hence, index numbers intrude themselves on virtually every aspect of empirical research about firms and the business sector.

The price index programs of the U.S. Bureau of Labor Statistics (BLS) were put in place back when firms produced on an in-house basis more of the intermediate goods and services required as inputs. When a firm switches from producing to procuring an input, this is called outsourcing. Of course, once a firm has found one external source for an intermediate input, it could switch to another source. Thus, sourcing substitutions should rise in the wake of increased outsourcing. Unfortunately though, the main producer side measures of inflation, which feed into productivity and other national economic performance measures, miss much of the price change associated with sourcing substitutions, and this can cause sourcing substitution bias in the producer price indexes. This has become an important problem because finding cheaper input sources is a prevalent modern day business strategy for lowering production costs.

Intermediate input price competition is also believed to play a key role in the survival and growth of new firms. Foster, Haltiwanger, and Syverson (2008) make this point in an important empirical study. For firms that produce physically homogenous products, they are able to evaluate and compare a measure of physical productivity with the more conventional productivity measure computed using revenue, cost, and price index information. Foster, Haltiwanger, and Syverson find a strong inverse correlation between physical productivity and prices that is consistent with newer entrants having lower marginal costs, and with this allowing those businesses to charge lower prices. More generally, they and others document examples of physically identical producer inputs being available from different sources for different prices at the same point in time.

The derivations of our bias formulas in the main body of the paper are easy to understand because they are for a highly simplified economy. In an appendix, we show that these formulas can be extended to a more realistic case. Fortunately, the same main parameters emerge as the determinants of the bias size, though the derivations are more involved.

We also explain how price index bias problems undermine the validity of the industry accounts produced by the U.S. Bureau of Economic Analysis (BEA): accounts that support national productivity measurement and economic policy analysis. We explain how these bias problems are obscured by and impede efforts to understand the workings of the economy.

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118 In another recent AER paper, Bergin, Feenstra, and Hanson (forthcoming) develop a model in which the timing of when firms begin to offshore a variable-cost activity to foreign firms is endogenously determined in response to economic conditions, including unit cost price differences for input factors.
119 For example, Byrne, Kovak, and Michaels (2009) report that shifts in the location of production to lower-cost countries can deliver cost declines of up to 0.8 percent per year. Also, Klier and Rubenstein (2009) report that “The mass-produced aluminum wheel is a commodity that is sourced by carmakers on the basis of price.” Of course, firms sometimes purposely buy inputs from sources charging premium prices because of perceived benefits offered by the higher priced suppliers. In a later section in the paper, “Recommended Data Gap Fixes,” a potential procedural solution is proposed for dealing with quality differences that might be associated with price changes that occur together with sourcing substitutions.
120 Houseman et al. (2009) demonstrate the value of this formulation for empirical studies.
SETTING THE STAGE

We begin by considering an economy with just enough in it so that we can show how input source substitution bias arises and can be measured. We consider an economy with four firms, one truly homogeneous product, and no taxes or transport costs. To explore the importance of certain price index compilation protocols, we take a case where firms 1 and 2 are suppliers and firms 3 and 4 are purchasers of the homogeneous product, and where firms 1 and 3 are large and firms 2 and 4 are initially small, but grow substantially from period 0 to 1. In other words, we allow for the situation that Foster, Haltiwanger, and Syverson (2008) document where firms enter that have found ways of producing at lower cost and then gain market share by selling their products for less than the incumbent firms.

The activities of the four firms can be summarized as follows:

- Firm 1 is a higher-cost producer that sells exclusively to firm 3 in periods 0 and 1.
- Firm 2 is a lower-cost producer that is small and only sells to small firm 4 in $t = 0$. However, in $t = 1$, firm 2 moves out of the small firm category by selling to firm 4, which has grown, and by also winning a contract to supply part of the intermediate product needs of large firm 3.
- Firm 3 only purchases the output of the higher-cost supplier, firm 1, in period 0, but shifts some of its purchases to firm 2 in period 1: a sourcing substitution.
- Firm 4, which is small in $t = 0$, only uses the output of the lower cost firm 2 in $t = 0,1$.

We let $p^t_{jk}$ and $q^t_{jk}$ denote the price and quantity for sales of the one product from firm j to firm k in period $t = 0,1$. The firm value flows in Table 1 have a positive sign for outputs and a negative sign for inputs. Firm 1 is always the higher-cost supplier. Thus we always have

\[
(1) \quad p^0_{13} > p^0_{24} > 0; \quad p^1_{13} > p^1_{24} > 0; \quad \text{and} \quad p^1_{13} > p^1_{23} > 0.
\]

Table 1. Value Flows between the Four Firms

<table>
<thead>
<tr>
<th>Output flows</th>
<th>Input flows</th>
</tr>
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<tbody>
<tr>
<td><strong>Firm 1</strong></td>
<td><strong>Firm 2</strong></td>
</tr>
<tr>
<td><strong>Period 0 Value Flows</strong></td>
<td></td>
</tr>
<tr>
<td>$p^0_{13}q^0_{13}$</td>
<td>$p^0_{24}q^0_{24}$</td>
</tr>
<tr>
<td><strong>Period 1 Value Flows</strong></td>
<td></td>
</tr>
<tr>
<td>$p^1_{13}q^1_{13}$</td>
<td>$p^1_{23}q^1_{23} + p^1_{24}q^1_{24}$</td>
</tr>
</tbody>
</table>

\[\text{121} \quad \text{Normally we would expect} \quad p^1_{23} \quad \text{to be close to} \quad p^1_{24}, \quad \text{but we allow for possible price discrimination by firm 2.}\]
It is important to understand some specifics of the price collection and index compilation practices of the BLS. The Producer Price Index (PPI) measures inflation in the prices of the outputs of domestic producers: products sold to other domestic businesses as intermediate inputs as well as final demand outputs. The Import Price Index (MPI) measures inflation for the products imported by U.S. businesses and residents. The Export Price Index (XPI) measures inflation for U.S. export products.\textsuperscript{122} When a producer changes from buying an intermediate input from a domestic source to buying that same input from a foreign producer, the product price will drop out of the domain of definition for the PPI and will become part of the domain of definition for the MPI, and any price change associated with that sourcing substitution will not be included in either the PPI or the MPI.

For the PPI and XPI, price collection is on the sellers’ side, whereas for the MPI, price collection is on the buyers’ side.

As the first step in the procedures for collecting prices from producers, the BLS first selects establishments. Products are then chosen and prices are collected at the chosen establishments. The BLS collects price data from establishments roughly in proportion to their transaction volumes. Prices are collected from some small establishments for both the PPI and the International Prices Program (responsible for the MPI and the XPI). However, the proportion of establishments selected for price collection falls steeply with establishment size below a threshold value for each program.

BLS protocols specify that price collection from businesses should focus on the price forming units. Firms often centrally determine the selling prices for all of their establishments. Some price collection is now carried out in a “head office” format, but most still involves interactions with the establishments where the productive activities are carried out. Of course, it is only for single establishment firms that the establishment and the firm are the same. Nevertheless, for convenience, hereafter we will usually refer to the production units as firms.

Abstracting from some of the finer points, the current BLS price index compilation practices for the PPI and MPI have the consequence that the prices used for index compilation are only for ongoing supply contracts (see Nakamura and Steinsson [2008, 2009]). The objective of these BLS practices is to ensure that only identical products, of identical quality, are priced period to period. This is how the BLS has been implementing the matched model methodology for comparing price change over time. However, one important reason why firms make sourcing substitutions is to benefit from lower prices. When firms change the input suppliers they are using, this inevitably results in the initiation of new supply contracts that, we would think, usually involve lower prices. Hence, a statistical agency practice of using only the prices from ongoing supply contracts in index compilation will miss a substantial and systematic source of price change.

Summarizing the key points from the above discussion, BLS procedures miss price changes that accompany producer sourcing switches for three main reasons:

\textsuperscript{122} See Chapter 14 on the PPI Chapter 15 on the MPI and the XPI in the \textit{BLS Handbook of Methods}, available at http://www.bls.gov/opub/hom/homtoc.htm . See also Diewert (2007a,b).
When a producer switches from a domestic to a foreign supplier (or vice versa), any price change for the input falls into a price collection gap between the PPI and MPI programs.

In an effort to control for quality differences in products and possible bundled services and amenities, the BLS typically only uses prices from ongoing contracts for index number compilation purposes. This practice was instituted to avoid treating quality-related price differences as true price change.

The lower-cost suppliers that producers switch to are often new, small suppliers trying to gain market share by undercutting the prices of established firms. Existing BLS price collection practices mean there is relatively little price collection from small firms, which will tend to result in missing the lower-price transactions for purchases from newer firms.

We want to focus attention on the above problems that are believed to be the biggest causes of sourcing substitution bias in the PPI and MPI. Thus, for now, we do not bother with specifying the relevant BLS program or with whether the price collection is from sellers or purchasers for the relevant BLS price index program: issues we return to briefly later in the section titled “Price Collection from Sellers versus Buyers.”

Table 2 shows unit values, given the value flows in Table 1. The unit value for a homogeneous product is the total cost of all purchases, or the total revenue from all sales, of the product divided by the number of items transacted. Table 3 shows inflation measures for specified cases. These inflation measures are ratios of the period 1 to the period 0 unit values for the different cases in Table 2.

For panel 1 in Tables 2 and 3, only the prices from the sales or purchases of large firms are included. In contrast, for panel 2, transactions for firms of all sizes are included. Now there are no blank cells. However, firm 2 sales to firm 3 and firm 3 purchases from firm 2 are still ignored. In the rest of this paper, we focus on this panel 2 case. (Actual BLS practice falls between the panel 1 and panel 2 cases in the sense that the BLS does collect prices from some small firms.)

For panels 3 and 4, all transactions for all firms are included. In panel 3, firm-specific expressions are shown whereas panel 4 gives the corresponding economy wide aggregate expressions. We refer subsequently to the expressions in these panels 3 and 4 as true targets. The “true” or “target” firm indexes in panel 3 of Table 3 can be compared with the incorrectly measured price indexes given in panels 1 and 2 of Table 3. The entries in Tables 2 and 3 are used in the following section for showing how the sourcing substitution bias could be measured.

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123 Since the product being traded between firms is assumed to be homogeneous in all of the following scenarios, the methodological advice given in the *Producer Price Index Manual* applies, and unit value prices are the appropriate prices to insert into index number formulas in deriving the true target price index measures. See IMF et al. (2004, pp. 509–510), Reinsdorf (1993), and Dievert (1995). The idea that a unit value for homogeneous items is the appropriate price to use in a bilateral index number formula can be traced back to Walsh (1901, p. 96) (1921, p. 88), and Davies (1924, 1932). Other index formulas are used in the appendix.
when all four firms in the hypothetical economy are domestic. Then in the next two sections, we extend the analysis to cover cases where some of the firms are foreign.\textsuperscript{124}

\begin{footnote}
\textsuperscript{124} The “optimal” procedures may simply be too expensive for the agency to implement. But it is good to have our analysis so that rough estimates of bias could be made.
\end{footnote}
### Table 2. Unit Values for Domestic Firms under Alternative Measurement Conditions

<table>
<thead>
<tr>
<th>Output flows</th>
<th>Input flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>Firm 2</td>
</tr>
<tr>
<td>Period</td>
<td>Large in $t = 1$</td>
</tr>
<tr>
<td>0</td>
<td>$\frac{p_{13}q_{13}}{q_{13}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$\frac{p_{13}q_{13}^{1}}{q_{13}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 1. Firm unit values; based on prices for continuing contracts at large firms only.

### 2. Firm unit values; based on prices for continuing contracts at all firms.

### 3. Firm unit values; based on all prices at all firms.

### 4. Economy wide unit values; based on all prices at all firms.

Note: The unit value expressions shown above are only observable for domestic firms.
Table 3. Price Indexes for Domestic Firms under Alternative Measurement Conditions

| Output flows |  | Input flows |
|--------------|  |            |
| Firm 1       | Firm 2 | Firm 3      | Firm 4 |
| Large in     | Small in | Large in | Small in |
| t = 0        | t = 0   | t = 0      | t = 0 |

1. Firm unit values; based on prices for continuing contracts at large firms only.

\[
p_{13}^1 \quad \text{no price} \quad p_{13}^1 \quad \text{no price} \\
p_{13}^0 \quad \text{collection} \quad p_{13}^0 \quad \text{collection}
\]

2. Firm unit values; based on prices for continuing contracts at all firms.

\[
p_{13}^1 \quad p_{24}^1 \quad p_{13}^1 \quad p_{24}^1 \\
p_{13}^0 \quad p_{24}^0 \quad p_{13}^0 \quad p_{24}^0 
\]

3. Firm unit values; based on all prices at all firms.

\[
p_{13}^1 \quad p_{24}^{13}q_{23} + p_{24}^{13}q_{24} \quad p_{13}^{13}q_{13} + p_{23}^{13}q_{23} \quad p_{24}^1 \\
p_{13}^0 \quad p_{24}^{0}q_{24} \quad p_{13}^{0}q_{13} \quad p_{24}^0 
\]

4. Economy wide unit values; based on all prices at all firms.

\[
p_{13}^{13}q_{13} + p_{23}^{13}q_{23} + p_{24}^{13}q_{24} \quad p_{13}^{13}q_{13} + p_{23}^{13}q_{23} + p_{24}^{13}q_{24} \\
p_{13}^{0}q_{13} + p_{24}^{0}q_{24} \quad p_{13}^{0}q_{13} + p_{24}^{0}q_{24}
\]

Note: The inflation measures shown are only observable for domestic firms.

DOMESTIC SOURCING SUBSTITUTION BIAS

We first take up sourcing substitutions among domestic sources: the most prevalent case. For the Table 1 value flows, we consider the case for which results are given in panel 2 of Tables 2 and 3.\(^{125}\) We consider the situation in which the only reason some changes in the price for the homogeneous product are missed is because only prices for continuing contracts are used in index compilation. For reasons explained later, we focus on the measures for the purchasing firms 3 and 4 rather than the corresponding measures for the selling firms 1 and 2.

The firm 4 price index that would be computed, denoted here by \(P_4^{(4)}\) and shown in panel 2 of Table 3, is compared with the true target index for firm 4, \(P_T^{(4)}\), shown in panel 3 of Table 3.

\(^{125}\) In Appendix A, we extend the analysis in this section to the case where we are dealing with the simultaneous outsourcing of N products instead of just a single product. The methods we use to measure sourcing substitution bias are similar to, but not the same as, the method used by Diewert (1998) to measure outlet substitution bias in the CPI.
Both are simply the ratio of the firm 4 purchase price for the homogeneous product in period 1, $p_{24}^1$, to its price in period 0, $p_{24}^0$. Thus, for firm 4, we have\textsuperscript{126}

\[ p^{(4)}_I \equiv \frac{p_{24}^1}{p_{24}^0} = p^{(4)}_I. \]

The sourcing substitution bias is defined as the measured index minus the true target index. This difference is zero for firm 4, so there is no sourcing substitution bias problem for this firm.

The analysis for firm 3 is more complex. There are two transactions in period 1 for this firm at potentially different prices. Since the product being traded is assumed to be homogeneous, as already noted, the unit value is the appropriate price to use for price (and also quantity) index compilation. The true firm 3 unit value, $u^1_3$, for period 1 is

\[ u^1_3 = \frac{[p_{13}^1q_{13}^1 + p_{23}^1q_{23}^1]}{[q_{13}^1 + q_{23}^1]}, \]

as also shown in panel 3 in Table 2. We find it convenient to define share parameters for the proportions of firm 1 and firm 2 output sold to firm 3 in period 1; i.e.,

\[ S_{13}^1 = \frac{q_{13}^1}{[q_{13}^1 + q_{23}^1]} \text{ and} \]

\[ S_{23}^1 = \frac{q_{23}^1}{[q_{13}^1 + q_{23}^1]}, \]

with $S_{13}^1 + S_{23}^1 = 1$. The unit value expression in Equation (3) can now be rewritten as

\[ u^1_3 = p_{13}^1S_{13}^1 + p_{23}^1S_{23}^1. \]

Using Eqs. (3)–(6), the target firm 3 price index, shown in panel 3 of Table 3, is

\[ p^{(3)}_I \equiv u^1_3 / p_{13}^0 = (p_{13}^0 / p_{13}^1)S_{13}^1 + (p_{23}^0 / p_{13}^1)S_{23}^1. \]

However, the formula given in Equation (7) is not what would be evaluated for firm 3 if the period 1 sales of firm 2 to firm 3 are ignored because this is the first period for a new contract. As given in panel 2, the measured price change component for firm 3 would be

\[ p^{(3)}_I \equiv p_{13}^1 / p_{13}^0. \]

\textsuperscript{126} We assume that the corresponding true quantity index is obtained by deflating the value ratio by the true price index. Thus $Q^{(1)}_I = [p_{13}^1q_{13}^1 / p_{13}^0q_{13}^0] / p_{13}^0 = [q_{13}^1 / q_{13}^0] = q_{13}^1 / q_{13}^0$ and $Q^{(4)}_I = [p_{24}^1q_{24}^1 / p_{24}^0q_{24}^0] / p_{24}^0 = q_{24}^1 / q_{24}^0$.  

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The numerator in this incorrect index is the period 1 price from the high-cost supplier, $p^{1}_{13}$, rather than the unit value price, $u^{1}_{3}$, which is an average price for firm 3 input purchases in period 1 from both the high- and low-cost suppliers.

Before specifying a formula for the bias for the firm 3 price index, it is helpful, for interpretive purposes, to introduce a bit more notation. We will let $i$ be the rate of price inflation for deliveries from the high-cost supplier, firm 1, to firm 3.

$$ (9) \quad (1 + i) \equiv \frac{p^{1}_{13}}{p^{0}_{13}} = P^{(3)}_{1}, \quad \text{using Equation (8)}. $$

Also, we let $0 \leq d < 1$ be a discount factor reflecting the proportional price discount for firm 3 purchases from firm 1 versus firm 2. Thus, we have

$$ (10) \quad (1 + i)(1 - d) = \frac{p^{1}_{23}}{p^{0}_{13}}, $$

where $(1 - d) = \frac{p^{1}_{23}}{p^{1}_{13}} \leq 1$. From Equation (9), Equation (10), and Equation (7), we now have

$$ (11) \quad P^{(3)}_{1} = (p^{1}_{13} / p^{0}_{13})S^{1}_{13} + (p^{1}_{23} / p^{0}_{13})S^{1}_{23} $$

$$ = (1 + i)S^{1}_{13} + (1 + i)(1 - d)S^{1}_{23} $$

$$ = P^{(3)}_{1} - (1 + i)dS^{1}_{23} \quad \text{also using Equation (8) and the property} $$

$$ S^{1}_{13} + S^{1}_{23} = 1. $$

Therefore, for the case of a purchasing substitution from one domestic producer to another, the sourcing substitution bias for firm 3 is defined as the index that is computed, given in Equation (8), less the true target index given in Equation (7):

$$ (12) \quad B^{(3)} \equiv P^{(3)}_{1} - P^{(3)}_{1} = (1 + i)dS^{1}_{23} > 0. $$

From Equation (12) we see that the sourcing bias is positive since $d > 0$, and the product of three factors:

1) The rate of price inflation for the high-cost supplier; i.e., $(1 + i) \equiv \frac{p^{1}_{13}}{p^{0}_{13}}$;

2) The proportional cost advantage of the low-cost supplier over the high-cost supplier; i.e.,

$$ d = 1 - [(p^{1}_{23} / p^{0}_{13})/(p^{1}_{13} / p^{0}_{13})] > 1; \text{ and} $$

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3) The share of deliveries to sector 3 in period 1 that are from the new low-cost supplier; i.e.,
\[ S_{23}^1 = q_{13}^1 / [q_{13}^1 + q_{23}^1]. \]

If rough guesses can be made for the cost advantage of the low-cost supplier and the input shares displaced, then a rough approximation of the bias in the intermediate input price index could be made using Equation (12).127

We conclude this section with some observations regarding two extensions of the above results for domestic sourcing substitutions: 1) an extension to the whole economy level, and 2) an extension to the case where the higher cost supplier, firm 1, shuts down in period 1 because of the competition from firm 2, so that \( p_{13}^1 \) and \( q_{13}^1 \) are not available.

Regarding the whole economy extension, since the firms 1 and 2 are selling a homogeneous commodity, the true period 1 unit value for the combined economy-wide inputs of firms 3 and 4 is
\[ u^1 = -[p_{13}^1 q_{13}^1 + p_{23}^1 q_{23}^1 + p_{24}^1 q_{24}^1] / [q_{13}^1 + q_{23}^1 + q_{24}^1], \]
and the corresponding period 0 whole economy unit value is
\[ u^0 = -[p_{13}^0 q_{13}^0 + p_{24}^0 q_{24}^0] / [q_{13}^0 + q_{24}^0], \]
as in panel 4, Table 2. Thus the target price index is
\[ p_{13}^{(3+4)} = u^1 / u^0, \]
which equals the expression in panel 4 of Table 3.

If the statistical agency fails to include the price information for the new period 1 contract that firm 2 has to supply firm 3, then the computed economy-wide index would be
\[ [p_{13}^1 q_{13}^1 + p_{24}^1 q_{24}^1] / [p_{13}^0 q_{13}^0 + p_{24}^0 q_{24}^0], \]
which will yield a higher value than the target index. Thus the computed index for the combined firms will have an upward bias.

Moving on to the second extension, suppose firm 1 shuts down in period 1 because of the competition from firm 2, so that \( p_{13}^1 \) and \( q_{13}^1 \) are not available. Now it is not possible to define an observable true output price index for firm 1 in period 1. However, the true input price index for firm 3 can still be defined by Equation (7) with \( S_{13}^1 = 0 \) and \( S_{23}^1 = 1 \), so \( P_{13}^{(3)} \) could be specified to be the (unmatched) price ratio \( p_{23}^1 / p_{13}^0 \), and the rest of the algebra above goes through.

However, although the true input price index for firm 3, \( p_{23}^1 / p_{13}^0 \), could be evaluated, note that the period 0 and period 1 price observations come from different suppliers. Given currently accepted practices, a statistical agency would be more likely to measure the price change by \( p_{24}^1 / p_{24}^0 \). With low inflation for the prices charged by any one firm, this price ratio would be close to one and hence generally larger than the mixed price ratio, \( p_{23}^1 / p_{13}^0 \). Thus, in general,

127 In Appendix A, we show that the bias formula becomes more complex when we generalize the above one commodity case to the case of many commodities, but Equation (12) is valuable as a rough approximation to the bias.
the incorrect index will again have an upward bias, though this bias is not given by Equation (12). 128

OFFSHORE SOURCING SUBSTITUTION BIAS

The extension of the analysis in the previous section to cover the case of a domestic firm switching among foreign suppliers is straightforward. We simply reinterpret firms 1 and 2 as foreign suppliers. Now none of the column 1 and 2 expressions can be evaluated in Table 2, so we focus on the prices that can be observed for firms 3 and 4. Inflation measurement for imports must be carried out using prices collected from purchasers. (Comparability with the results here is one reason we chose to focus on price indexes for firms 3 and 4 in the previous section too.)

All of the algebra in the previous section can be applied. There are no sourcing substitution bias problems for firm 4. However, the statistical agency will be likely to compute the incorrect price index for firm 3 given by Equation (8) instead of the true target index given by Equation (7), for the same reasons explained above. Thus, the bias Equation (12) is again operative.

Now imports are in the picture, for each firm in the domestic economy, input purchases should be distinguished by their point of origin so it can be determined whether a purchase should be classified as a domestic or an imported input. We further discuss this issue in the next to last section which is on the BEA Industry Accounts.

DOMESTIC TO FOREIGN SOURCING SUBSTITUTIONS

Finally, we take up the case where firm 1 is a domestic supplier and firm 2 is a lower-cost foreign supplier of the same homogeneous product. Now it is just the column 2 expressions in Tables 2 and 3 that cannot be evaluated. However, since firms 3 and 4 are domestic, the column 3 and 4 expressions can still be evaluated. The sourcing substitution for firm 3 is now the switch from exclusively using a domestic supplier (firm 1) in period 0 to importing some of the firm’s requirements for the homogeneous input from the foreign firm 2 in period 1.

All of the algebra in section 3 for firm 3 remains valid. Thus the bias analysis in the third section carries over to the present context in a straightforward manner.

PRICE COLLECTION FROM SELLERS VERSUS BUYERS

With the producer price indexes currently produced by the BLS, the price collection is on the purchasers’ side for the MPI since it is only the purchasers that are domestic, but it is on the sellers’ side for the PPI. We ignored this institutional information in the previous sections and proceeded as though all price collection were on the buyers’ side. We did this partly for expositional convenience, but also because doing this allowed us to demonstrate that the

128 The new bias formula will be $B = p_{1}^{(3)} - p_{1}^{(3)} = \left(\frac{p_{24}}{p_{24}} - \frac{p_{24}}{p_{23}}\right)^{(3)} - \left(\frac{p_{24}}{p_{23}} - \frac{p_{23}}{p_{13}}\right)$.
sourcing substitution bias problem cannot be corrected solely by creating a comprehensive input price index with price collection on the purchasers’ side.\textsuperscript{129}

As of now, the United States does not have a comprehensive input price index. Rather, the components of the PPI program that are for intermediate products sold to other businesses and the price index components of the MPI for purchases by businesses are combined to form a pseudo input price index. The BLS hopes to begin producing a true Input Price Index (IPI) in the future, as explained in Alterman (2009), but has not yet been given the funding to do so. We view this as an important step forward. However, to deal with the sourcing substitution bias problem, the BLS also needs to utilize the initial observation when a new procurement contract is started, relating this price to the price paid under the previous contract, and to collect prices from all firms rather than mostly from large ones. The purchaser knows, presumably, whether the product is truly the same despite a price change.\textsuperscript{130} On the other hand, a seller typically has no information about the prior purchasing choices of their customers or their reasons for making sourcing changes. So, new sources of supply should be flagged and the statistical agency must decide, case by case, on whether the product from a new source of supply is closely comparable to the product from the old source of supply. This is the information that would make the BLS feel comfortable about including prices for first time purchases under new procurement contracts in producer price index calculations. Making these changes will not be easy, but our bias Equation (12) makes it clear the changes are important!

THE BEA INDUSTRY ACCOUNTS

One important use of producer price indexes is for use by the BEA for producing industry accounts that are corrected for pure price change over time. The BEA industry accounts, which include the benchmark input-output (I-O) accounts and annual GDP-by-industry accounts, are heavily used for economic policy analyses.\textsuperscript{131} Benchmark I-O accounts are produced every five years using data from the quinquennial economic census of businesses. These include make, use, and requirements tables. A make table shows the value of each product produced by each industry in a given year.\textsuperscript{132} A use table shows the uses of products by intermediate and final users.\textsuperscript{133}

Starting with the initial benchmark make and use tables, the BEA produces balanced make and use tables using a procedure that sequentially changes row and column figures until the recorded

\textsuperscript{129} Almost 50 years ago, the 1961 report of the NBER Price Statistics Review Committee (the Stigler Report) also recommended that the BLS should rely on purchasers’ prices rather than the sellers’ prices.

\textsuperscript{130} If the purchaser is acting on behalf of another party, then the question might need to be referred to that other party.

\textsuperscript{131} See, for example, Streitwieser (2009) on the ongoing sorts of public policy uses of the BEA industry accounts.

\textsuperscript{132} Each industry gets a row in a make table, and each product gets a column. The values in a row sum to the current value of output for the stated industry. Each column total equals the total output for the given product.

\textsuperscript{133} Each product gets a row, and each row sum is the gross output for the given product. There are columns for the industry intermediate uses and columns for final uses. The value added for an industry is the sum of the values of total industry sales (gross output) minus the value of purchases from other industries (aggregate intermediate industry input). Industry value added summed over industries equals the business sector GDP for the nation.
total use of each product equals its recorded total supply, and the recorded final demands equal the values given in the U.S. national income and product accounts (NIPAs).

Requirements tables are derived from the balanced make and use tables. The coefficients in the requirements tables represent interindustry linkages that, in turn, link output and final demand. For example, the entries in the employment requirements table show the estimated direct and indirect impacts of a change in final demand on employment in the different industries. Requirements tables are used for policy impact analyses.

For the purpose of producing constant price real I-O accounts from nominal I-O accounts, price indexes are needed for deflating the outputs and also the inputs. Unfortunately, as explained above, we believe the input price indexes for producers are distorted by sourcing substitution biases. In addition, there are gaps in intermediate product price collection. Imputed rather than direct pricing is used for numerous products. Moreover, the same economy-wide producer price deflator is generally used to deflate the value flows between and from each of the industries. Diewert (2007c) notes that this procedure is correct only if each industry produces the same mix of micro commodities within each of the broad commodity classes and micro commodity prices move together across industries: conditions unlikely to be satisfied for a nation!

Moving to the annual industry accounts (AIAs), these consist of the annual GDP-by-industry accounts and the annual I-O accounts. The annual GDP-by-industry estimates are based on annual survey and administrative data from several sources. In contrast, the annual I-O accounts are produced by updating the benchmark I-O accounts utilizing the assumption that the real (constant-price) use of intermediate inputs relative to each industry’s real gross output does not change year-to-year; see Strassner, Yuskavage, and Lee (2009). The estimates of an industry’s real intermediate inputs are first updated based on projected changes in the industry’s real gross output from the GDP-by-industry accounts and using the observed ratios of intermediate input use to gross output. The AIAs are then “integrated” internally and with the NIPAs. The integration achieved is referred to as partial integration.

The balancing process can be viewed, Parker (2004) suggests, as using the relative strengths of the available data to produce the best possible data for users. However, an initial imbalance, say, in the initial use table in the form of too little, or too much, supply of a product after the estimated intermediate input and final uses have been accounted for represents either an inconsistency in the data or in the framework for the accounts. The balancing process renders less visible the initial imbalances; see Meade (2006).

Filling the intermediate products value and price data gaps would help the BEA to move toward full rather than just partial integration of the industry accounts. Lawson, Moyer, and Okubo (2006) note that the methods developed by the BEA to achieve partial integration were never seen as an adequate substitute for the needed improvements to the source data. They explain that, with full integration, the measures of value added by industry would be independent of the NIPAs and could provide a useful “feedback” loop that would improve the estimates of the product composition of GDP.

134 The techniques used for this balancing are explained in Appendix B of Lawson, Moyer, and Okubo (2006). See also Horowitz and Planting (2006) and Yuskavage, Strassner, and Medeiros (2008).
Also, policymakers would like to know what factors account for current U.S. offshoring. They would like to understand and be able to foresee and perhaps influence the major effects of offshoring on U.S. workers and the economy.\(^{135}\) The I-O tables serve as the framework for combining the available data for estimated GDP, and are essential to empirical studies of how outsourcing and offshoring and inshoring are affecting the U.S. economy. However, to properly allow for foreign engagement, the commodity classification that is used in the I-O tables must be expanded. A gross output that is being produced by a particular industry in a particular commodity category must be further distinguished as being supplied to the domestic market or as an export while an intermediate input that is being used by a particular industry in a particular commodity category must be further distinguished as being purchased from a domestic or a foreign supplier. Making these changes would not be a dramatic methodological leap since the 1993 System of National Accounts (SNA93) guidelines already suggested this treatment of intermediate inputs as a supplementary table.\(^{136}\) However, this change would be expensive.

If it is deemed important to have information on the exports produced and the imports demanded by each industrial sector, it will be necessary to construct unit value output prices for both exports and deliveries to domestic demanders and to construct unit value input prices for both imports and deliveries to the sector from domestic suppliers. If we take this approach as the ideal, the dimensionality of the supply and use tables would be expanded beyond what could conceivably be implemented in terms of needed data collection, \textit{given present survey data collection methods} and business concerns about confidentiality. However, the suggested approach could be partially implemented at least, as suggested by Diewert (2001).\(^{137}\)

If our purpose is to measure industry productivity, or to measure industry level product or labor demand impacts, then the answer is reasonably straightforward. When calculating the constant dollar I-O matrices, each value cell for outputs and each value cell for inputs needs to be deflated by a price index that matches up with the value flows in that cell. At present, however, there simply are no adequate surveys on the interindustry flows of services. Even in manufacturing, where information on commodity flows is relatively complete thanks to explicit surveys of manufacturing industries, relatively little information on the flows of purchased services is collected. More attention also needs to be given to the development of basic prices by industry and by commodity—i.e., we need accurate information on the exact location of indirect taxes (and commodity subsidies) by commodity and industry on both outputs and inputs.

\(^{135}\) See Dey, Houseman, and Polivka (2006); Freenstra and Bradford (2009); Jarmin, Krizan and Tang (2009); Kletzer (2009); and Norwood et al. (2006).

\(^{136}\) See Table 15.5 in Eurostat et al. (1993). For a more detailed discussion of how exports and imports could be introduced into the production accounts, see Diewert (2007a,b) and IMF et al. (2009).

\(^{137}\) See Diewert (2005, 2007c) for a treatment of these problems in a closed economy context, and Diewert (2007a,b) for an open economy treatment.
RECOMMENDED DATA GAP FIXES

As explained following Table 1, the present BLS price collection procedures miss price changes of three sorts that accompany producer sourcing switches: 1) price changes associated with domestic to foreign sourcing substitutions (or vice versa), 2) price changes associated with the start of new procurement contracts, and 3) price changes involving sales of or purchases from small firms for one of the two time periods involved for the price change. The resulting price data gaps lead to sourcing substitution bias problems.

Changes in statistical agency practices could fix these data-based problems, though these are changes that a statistical agency like the BLS would need additional funding to implement. A comprehensive Input Price Index (IPI) is needed of the sort the BLS would like to have the funds to produce. Price collection should be from purchasers rather than sellers whenever possible, since it is only the purchasers who are in a position to state whether there were quality changes as well as price changes associated with sourcing changes. Also, it is important for data to be collected from all firms, or at least from higher proportions of small firms.

We believe that an important step toward achieving the needed expansion of data collection from firms is for the present survey data collection methods to progressively be replaced with full electronic price and value data capture for all transactions of all businesses large enough to have electronic information systems.\textsuperscript{138} Many firms have taken advantage of the low cost of computing and now have detailed data on all their financial transactions (e.g., they have the value of each sale and the quantity sold by commodity).\textsuperscript{139} This information could enable industry and firm research like the scanner data studies that have proven to be so useful in the context of the Consumer Price Index.\textsuperscript{140} This information would permit the construction of true microeconomic price and quantity indexes at the firm level and the evaluation of more accurate firm and industry productivity indexes. The 2008 \textit{American Economic Review} study of Foster, Haltiwanger, and Syverson makes it clear that having this sort of information could result in a revolutionary rewrite of our understanding of how productivity growth takes place in the U.S. economy. Some additional tentative conclusions also follow from our analysis:

- Basic index number theory and statistical agency practice has not paid enough attention to the problems that arise when new firms enter and some grow, and some established firms shrink and then exit.
- It seems likely that statistical agency operating procedures have led to upward sourcing substitution biases in the intermediate product components of the PPI and MPI. Upward biases in input price indexes lead to downward biases in the corresponding quantity indexes and have the effect of overstating total factor productivity improvements.

\textsuperscript{138} Abraham and Spletzer argue convincingly that economies of scale would be realized if data were collected, when possible, for entire firms from their head offices. The context in which Abraham and Spletzer (2009) present these options is different, but the options they outline seem relevant for our context too.

\textsuperscript{139} For more on new ways of collecting price data, see also Gudmundsdottir, Gudnason, and Jonsdottir (2008) and Grimm, Moulton, and Wasshausen (2002).

\textsuperscript{140} See, for example, Reinsdorf (1994a,b); Nakamura (2008), Ivancic, Diewert, and K.J. Fox (forthcoming); and Nakamura, Nakamura, and Nakamura (forthcoming).
Since the value of international trade as a proportion of GDP has mostly increased over time, it seems likely that sourcing substitution bias has also increased over time.

There are other data gaps in the U.S. economic statistics that compound the problems resulting from sourcing substitution bias. In particular, currently in the United States, value information for intermediate products is only collected every five years, and there is no direct price collection for large numbers of intermediate products of economic importance.

N. Gregory Mankiw (2006, p. 44) quotes John Maynard Keynes as stating:

“If economists could manage to get themselves thought of as humble, competent people on a level with dentists, that would be splendid.”

Mankiw explains that Keynes was expressing a hope that macroeconomics would evolve into a useful sort of engineering and that “avoiding a recession would be as straightforward as filling a cavity.” He goes on to lament a paucity of those in our profession who are willing to commit their time and energies to helping to achieve the objectives that Keynes enunciated. However, dentists work on the teeth of their patients with the aid of lights, special mirrors, patients who willingly open their mouths, and dental X rays. In our view, for economists to make greater progress on understanding the economy and for economists and government policymakers to be able to do better on managing the economy, we need better “equipment” for seeing the economy. The data improvements recommended in this article would help in this regard.
Appendix A: Outsourcing Bias when there are N Commodities Being Outsourced

The analysis in the main text associated with Table 1 deals only with the case of a single homogeneous product. In this appendix, we define indexes as aggregates over N products instead of a single product. The overall message remains the same but the details are more complex.

In this more general setup, there are four groups of firms. We will assume that group 3 simultaneously switches their sourcing arrangements for N homogeneous commodities. In particular, group 3 switches some of its input procurement contracts for N commodities group 3 firms need as inputs from group 1 (the higher-cost supplier) to group 2 (the lower-cost supplier) for N commodities. Thus, the flows shown in Table 1 are still applicable except that now each price and quantity shown in the table is interpreted as a vector and the old ordinary price and quantity products must now be interpreted as inner products of the corresponding price and quantity vectors. Thus, the old value flow of supplies from group i to group j in period t, $t_{ij}^{qp}$, is replaced by $\sum_{n=1}^{N} t_{ijn}^{t} q_{ijn}^{t}$ for $t = 0,1$ where now $t_{ij}^{t} = [p_{ij1}^{t}, p_{ij2}^{t}, ..., p_{ijN}^{t}]$ is a price vector and $q_{ij}^{t} = [q_{ij1}^{t}, q_{ij2}^{t}, ..., q_{ijN}^{t}]$ is a quantity vector.

Let $p^{t}$ and $q^{t}$ be generic price and quantity vectors pertaining to a group for periods $t = 0,1$. Then the Laspeyres and Paasche price indexes, $P_L$ and $P_P$, are defined as follows:

\[
(A1) \quad P_L(p^0, p^1, q^0, q^1) = \frac{p^1 \cdot q^0}{p^0 \cdot q^0} \quad \text{and} \quad P_P(p^0, p^1, q^0, q^1) = \frac{p^1 \cdot q^1}{p^0 \cdot q^1}.
\]

The Fisher (1922) ideal price index, $P_F$, is defined as the geometric mean of the Laspeyres and Paasche price indexes:

\[
(A2) \quad P_F(p^0, p^1, q^0, q^1) = \left[ P_L(p^0, p^1, q^0, q^1) P_P(p^0, p^1, q^0, q^1) \right]^{1/2}.
\]

The Fisher price index can be justified as a “best” index from multiple perspectives\(^{141}\) and will be used for forming the “true” target index in what follows.

Looking at Table 1, it is straightforward to define the true output price index for group 1, $P_T^{(1)}$, as the Fisher index $P_F(p^0_{13}, p^1_{13}, q^0_{13}, q^1_{13})$ and the true input price index for group 4, $P_T^{(4)}$, as the usual Fisher index $P_F(p^0_{24}, p^1_{24}, q^0_{24}, q^1_{24})$. There are no sourcing substitution complications here. However, in period 1, group 2 firms began selling the same product to group 3 as well as group 4 firms, possibly at different prices. Thus, proper measurement of inflation for group 2 and group 3 transactions is more complicated.

For groups 2 and 3, the unit value prices for the N commodities $(n = 1, ..., N)$ in period 1 are

\(^{141}\) See Diewert (1976, 1992) and the Producer Price Index Manual.
(A3) \[ u_{2n} = \frac{p_{23n}q_{23n} + p_{24n}q_{24n}}{q_{23n} + q_{24n}} = p_{23n}S_{23n} + p_{24n}S_{24n}; \]

(A4) \[ u_{3n} = \frac{p_{13n}q_{13n} + p_{23n}q_{23n}}{q_{13n} + q_{23n}} = p_{13n}S_{13n} + p_{23n}S_{23n}; \]

where the group 2 and 3 (physical) quantity shares for product \( n \) in period 1 are given by

(A5) \[ S_{23n} = \frac{q_{23n}}{q_{23n} + q_{24n}}; \quad S_{24n} = \frac{q_{24n}}{q_{23n} + q_{24n}}; \quad S_{23n} + S_{24n} = 1; \]

(A6) \[ S_{13n} = \frac{q_{13n}}{q_{13n} + q_{23n}}; \quad S_{23n} = \frac{q_{23n}}{q_{13n} + q_{23n}}; \quad S_{13n} + S_{23n} = 1. \]

Let the vector of group 2 unit value prices for period 1 be \( u_{2} = [u_{21}, u_{22}, ..., u_{2N}] \) where the \( u_{2n} \) are defined by (A3) and let the vector of group 3 unit value prices for period 1 be \( u_{3} = [u_{31}, u_{32}, ..., u_{3n}] \) where the \( u_{3n} \) are defined by (A4). The period 1 quantity vectors that correspond to these unit value vectors in period 1 are \( q_{23} + q_{24} \) for group 2 and \( q_{13} + q_{23} \) for group 3. Thus our true output index for group 2 and our true input index for group 3 are defined to be the following Fisher ideal price indexes, respectively,

\[
\begin{align*}
P^{(2)}_T &= P_F(p_{24}, u_{2}, q_{24}, q_{23} + q_{24}) \\
&= [P_L(p_{24}, u_{2}, q_{24}, q_{23} + q_{24})P_P(p_{24}, u_{2}, q_{24}, q_{23} + q_{24})]^{1/2}; \\
P^{(3)}_T &= P_F(p_{13}, u_{3}, q_{13}, q_{13} + q_{23}) \\
&= [P_L(p_{13}, u_{3}, q_{13}, q_{13} + q_{23})P_P(p_{13}, u_{3}, q_{13}, q_{13} + q_{23})]^{1/2}.
\end{align*}
\]

In principle, a statistical agency could compute the true output price index \( P^{(2)}_T \) defined by (A7) and the true input price index, \( P^{(3)}_T \) defined by (A8). In practice, there are likely to be problems. Of particular relevance here, group 3 has switched to a new supplier in period 1 for the \( N \) commodities under consideration, so there will be no matching price for these supplies in period 0. Given current BLS practices, it is likely that the statistical agency will use the following “matched model” incorrect intermediate input price index for group 3:

\[
P^{(3)}_I = [P_L(p_{13}, p_{3}, q_{13}, q_{13})P_P(p_{13}, p_{3}, q_{13}, q_{13})]^{1/2}.
\]

The incorrect index \( P^{(3)}_I \) is the geometric mean of the Laspeyres and Paasche price indexes, \( P_L(p_{13}, p_{3}, q_{13}, q_{13}) \) and \( P_P(p_{13}, p_{3}, q_{13}, q_{13}) \), that use only the price and quantity data for the incumbent supply source (group 1 firms) for both periods. Since group 1 is a high-cost supplier, we can expect \( P^{(3)}_I \) to be higher than the true index, \( P^{(3)}_T \). Here we will develop formulae that will enable us to determine the magnitude of this upward bias.
It is cumbersome to develop a bias formula for the difference of \( p_{13}^{(3)} \) less \( p_{T}^{(3)} \) but it is fairly easy to develop bias formulas for the differences between the Laspeyres and Paasche components of these indexes. Thus, we start our analysis by expressing the high-cost supplier price relatives, \( p_{13n}^{1} / p_{13n}^{0} \), as the following high-cost supplier product specific inflation rates:

\[
(A10) \quad p_{13n}^{1} / p_{13n}^{0} \equiv (1 + i_{n}) ; \quad n = 1, \ldots, N ;
\]

Next, the lower-cost supplier prices in period 1 relative to the corresponding high-cost supplier prices in period 0 are expressed as:

\[
(A11) \quad p_{23n}^{1} / p_{13n}^{0} \equiv (1 - d_{n})(1 + i_{n}) , \quad n = 1, \ldots, N
\]

where \( d_{n} \) is a *proportional discount factor* for the low- versus the higher-cost supplier for product \( n \) that satisfies

\[
(A12) \quad 0 < d_{n} < 1 ; \quad n = 1, \ldots, N
\]

We start our analysis by looking at the *Laspeyres component* \( p_{TL}^{(3)} \) of the true input price index defined by (A8)

\[
(A13) \quad p_{TL}^{(3)} = \frac{u_{3}^{1} \cdot q_{13}^{0}}{p_{13}^{0} \cdot q_{13}^{0}} = u_{3}^{1} \cdot \frac{q_{13}^{0}}{q_{13}^{0}}
\]

\[
= \sum_{n=1}^{N} \left[ p_{13n}^{1}S_{13n}^{1} + p_{23n}^{1}S_{23n}^{1} \right] q_{13n}^{0} / p_{13n}^{0} q_{13n}^{0}
\]

\[
= \sum_{n=1}^{N} \left[ (p_{13n}^{1} / p_{13n}^{0})S_{13n}^{1} + (p_{23n}^{1} / p_{13n}^{0})S_{23n}^{1} \right] p_{13n}^{0} q_{13n}^{0} / p_{13n}^{0} q_{13n}^{0}
\]

\[
= \sum_{n=1}^{N} \left[ (l + i_{n})S_{13n}^{1} + (1 - d_{n})(l + i_{n})S_{23n}^{1} \right] p_{13n}^{0} q_{13n}^{0} / p_{13n}^{0} q_{13n}^{0}
\]

\[
(A11)
\]

using (A10) and

\[
= \sum_{n=1}^{N} \left[ (l + i_{n})S_{13n}^{1} + (1 - d_{n})(l + i_{n})S_{23n}^{1} \right] q_{13n}^{0}
\]

where the *period 0 expenditure shares on the \( N \) commodities in group 3* are defined as follows:

\[
\]
It is straightforward to show that \( \sum_{n=1}^{N} (1 + i_n) s_{13n}^0 \) is the incorrect Laspeyres component, \( P_L(p_{13}^0, p_{13}^1, q_{13}^0, q_{13}^1) \), in the incorrect Fisher price index for group 3 defined by (A9) above; so,

(A15) \[ P_L^{(3)} = P_L(p_{13}^0, p_{13}^1, q_{13}^0, q_{13}^1) = \sum_{n=1}^{N} (1 + i_n) s_{13n}^0. \]

Thus if we define the bias \( B_L \) in the incorrect Laspeyres index as the difference between \( P_L^{(3)} \) and \( P_L^{(3)} \), using (A13) and (A15), we have the following expression for this bias:

(A16) \[ B_L = P_L^{(3)} - P_L^{(3)} = \sum_{n=1}^{N} d_n (1 + i_n) s_{23n}^1 s_{13n}^0 > 0, \]

where the inequality follows from the nonnegativity of the physical shares \( S_{23n}^1 \) (with at least one of these shares being positive), the positivity of the base period expenditure shares \( s_{13n}^0 \) and the positivity of the discount factors \( d_n \). The bias formula (A16) has the same general structure as the bias formula (12) in the main text except that now it is the base period expenditure shares \( s_{13n}^0 \) of the group making the N-product sourcing switch that enter the formula.

We now need to repeat the above analysis for the Paasche component of the true index \( P_T^{(3)} \) defined by (A8) and the Paasche component of the incorrect index \( P_T^{(3)} \) defined by (A9). Define these Paasche components as follows:

(A17) \[ P_T^{(3)} = P_T(p_{13}^0, p_{13}^1, q_{13}^0, q_{13}^1, q_{23}^1) = u_3^1(q_{13}^1 + q_{23}^1)/p_{13}^0(q_{13}^1 + q_{13}^1); \]

(A18) \[ P_T^{(3)} = P_T(p_{13}^0, p_{13}^1, q_{13}^0, q_{13}^1) \equiv p_{13}^1 q_{13}^1 / p_{13}^0 q_{13}^1. \]

In place of the base period expenditure shares \( s_{13n}^0 \), for our Paasche analysis, we require two sets of expenditure weights that use the prices of period 0 but quantities that pertain to period 1. These hybrid expenditure shares are given by:

(A19) \[ s_{n}^{01} = p_{13n}^0 q_{13n}^1 / p_{13}^0 q_{13}^1; \]

(A20) \[ s_{n}^{0*} = p_{13n}^0 (q_{13n}^1 + q_{23n}^1) / p_{13}^0 (q_{13}^1 + q_{13}^1); \]

The expenditure shares \( s_{n}^{01} \) use the base period prices for group 3, \( p_{n}^{01} \), and the deliveries of the high cost group to group 3 in period 1, \( q_{13n}^1 \), whereas the expenditure shares \( s_{n}^{0*} \) use the base
period prices for group 3, $p_{13}^0$, as the price vector and the sum of all deliveries to group 3 in period 1, $q_{13}^1 + q_{23}^1$, as the quantity vector.

We must now look at the *Paasche component* $P_{TP}^{(3)}$ of the true input price index defined by (A8). Using definition (A17), we have

(A21)  
\[
P_{TP}^{(3)} = u(q_{13}^1 + q_{23}^1)/p_{13}^0 \cdot (q_{13}^1 + q_{23}^1)
\]

\[
= \sum_{n=1}^{N} \left[ p_{13n}^1 S_{13n}^1 + p_{23n}^1 S_{23n}^1 \right] \left[ q_{13n0}^1 + q_{23n0}^1 \right] / p_{13}^0 \left[ q_{13}^1 + q_{23}^1 \right] \quad \text{using (A4)}
\]

\[
= \sum_{n=1}^{N} \left[ (p_{13n}^0 / p_{13}^0) S_{13n}^1 + (p_{23n}^0 / p_{13}^0) S_{23n}^1 \right] s_{n}^{01v}
\]

\[
= \sum_{n=1}^{N} \left[ (1+i_n) S_{13n}^1 + (1-d_n)(1+i_n) S_{23n}^1 s_{n}^{01v} \right] \quad \text{using (A10) and (A11)}
\]

Moreover, for the *Paasche component* $P_{IP}^{(3)}$ of the incorrect input price index defined by (A9), using definition (A18), we have

(A22)  
\[
P_{IP}^{(3)} = p_{13}^1 \cdot q_{13}^1 / p_{13}^0 \cdot q_{13}^1
\]

\[
= \sum_{n=1}^{N} (p_{13n}^0 / p_{13}^0) p_{13n}^0 q_{13n}^1 / p_{13}^0 q_{13}^1
\]

\[
= \sum_{n=1}^{N} (1+i_n) s_{n}^{01v} \quad \text{using (A10) and (A19)}.
\]

Define the *bias* $B_p$ in the incorrect *Paasche index* as the difference between $P_{IP}^{(3)}$ and $P_{TP}^{(3)}$. Using (A21) and (A22), we have the following expression for this bias:

(A23)  
\[
B_p = P_{IP}^{(3)} - P_{TP}^{(3)}
\]

\[
= \sum_{n=1}^{N} (1+i_n) s_{n}^{01v} - \sum_{n=1}^{N} (1+i_n) s_{n}^{01v} + \sum_{n=1}^{N} d_n (1+i_n) S_{23n}^1 s_{n}^{01v}
\]

\[
= \sum_{n=1}^{N} (1+i_n) [s_{n}^{01v} - s_{n}^{01v}] + \sum_{n=1}^{N} d_n (1+i_n) S_{23n}^1 s_{n}^{01v}.
\]
Under normal conditions, the first term in the last line of (A23) will be close to zero\textsuperscript{142} so the second term, $\sum_{n=1}^{\infty} d_n (1 + i_n) s_{i_{23n}}^{1} s_{i_{31n}}^{01}$, will dominate. Since this second term is positive under our assumptions, the Paasche component of the bias, $B_p$, will usually be positive. This second term has the same general form as the Laspeyres bias component $B_L$ defined above by (A16).

We can approximate the true Fisher index $P_{T}^{(3)}$ defined by (A8) by the arithmetic mean of its Laspeyres and Paasche components. Also, we can approximate the incorrect Fisher index $P_{I}^{(3)}$ defined by (A9) by the arithmetic mean of its Laspeyres and Paasche components. Expressions (A16) and (A23) can be used to form an overall bias estimate for the incorrect Fisher index.

\textsuperscript{142} If all of the commodity specific inflation rates for the high cost producer are equal (i.e., the $i_n$ are all equal), then it can be seen that the first term on the right hand side of (A23) will vanish since the two sets of shares sum to one. This term will also be zero if the correlation between the vector of commodity specific inflation rates $i_n$ and the vector of differences in the shares is zero.
References


