Capital Controls, Global Liquidity Traps and the International Policy Trilemma

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Abstract

The zero interest rate bound introduces a new dimension to the international policy trilemma. International financial market openness may render monetary policy ineffective, even within a system of fully flexible exchange rates, because shocks that lead to a ‘liquidity trap’ in one country are propagated through financial markets to other countries. But monetary policy effectiveness may be restored by the imposition of capital controls. We derive an optimal monetary policy response to a global liquidity trap in the presence of capital controls. We show that, even though capital controls may facilitate effective monetary policy, capital controls are not generally desirable in welfare terms.

Keywords: zero lower bound; financial openness; international transmission

JEL codes: E2; E5; E6

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I Introduction

Following the financial crisis of 2008-2009, the monetary authorities of many countries simultaneously reduced policy interest rates to record low levels, or even zero. But these aggressively accommodative monetary policies failed to prevent substantial declines in GDP and increases in unemployment. Many countries found themselves in a ‘liquidity trap’, where the zero bound on interest rates exposed the limits of accommodative monetary policy. Unlike previous experiences where monetary policy was constrained by a liquidity trap, most notably Japan in the early 2000’s, in the more recent cases the liquidity trap phenomenon took on a global character. In particular, countries tightly linked by trade and financial markets, such as the US, the UK, Canada and the Eurozone, experienced simultaneous downturns matched by declines in policy rates to near zero levels.

This paper explores the characteristics of the zero lower bound on policy interest rates within a multi-country model where shocks that precipitate a liquidity trap tend to be propagated through international financial markets. In our model, open trade and financial markets give rise to global liquidity traps. This introduces a new dimension to the classic open economy policy ‘trilemma’. In the standard interpretation of this phenomenon, countries which attempt to peg their exchange rate cannot also have both effective monetary policy and open capital markets. We point out a new aspect of the policy trilemma that is implied by the zero bound constraint. Even with flexible exchange rates, a country that is fully open to international capital markets may find itself limited by constraints on effective monetary policy actions. This is because external shocks can cause the zero bound constraint to apply domestically as well. In fact, in the simple model of this paper, with fully open capital markets, all liquidity traps are global, independent of the source of the shock. A large external shock which pushes down ‘natural interest rates’ (defined as the interest rate which supports zero inflation and the flexible price level of GDP) in the global economy exposes the limits of effective monetary policy when interest rates are linked by integrated financial markets. In this sense, the constraints on effective policy actions are implied by open capital markets rather than by any desire to influence exchange rates.

We then explore the impact of introducing capital controls on policy independence. In the classic open economy model, capital controls help to sustain monetary independence despite a goal of exchange rate stability. In our model, where the key constraint on monetary independence relates to the zero bound, capital controls can act so as to sustain effective
monetary independence, even without any exchange rate target. In the presence of capital controls, natural interest rates diverge across countries and the zero bound constraint on policy no longer applies simultaneously across all countries. This implies that monetary policy effectiveness can be sustained in the face of external shocks that would otherwise push the monetary authority to the zero bound constraint.

We develop a model to explore the way in which capital controls may operate to insulate a country from shocks which would precipitate a liquidity trap in the case of fully open financial markets. We also characterize optimal monetary policy in the presence of capital controls. We focus on optimal policy under discretion. A particular implication of our results is that optimal monetary policy may involve raising interest rates, even when conventional analysis would indicate that interest rates are stuck at the zero bound.¹

Given that capital controls can sustain effective monetary independence, it might be expected that some level of capital market restrictions would be desirable relative to a situation of free and open capital mobility. Surprisingly, our results show that this is not the case. Capital controls are always welfare reducing, when monetary policy is chosen optimally, even though capital controls can support a situation where the monetary policy (of one country) remains effective. The intuition behind this result is quite simple. Capital controls reduce risk sharing by driving a wedge between natural interest rates in the two economies. By doing so, they break the perfect correlation between the incidence of the zero-bound constraint in both countries. But in each country, the policy rate is chosen optimally, conditional on the degree of capital controls. We show that there are two regimes in which the zero lower bound is binding. When capital controls are absent or limited in scope, both countries are constrained by the zero bound. With substantial capital controls, one country remains constrained by the zero bound while the other, which is not, sets an optimal positive policy interest rate. Within each regime, an increase in the severity of capital controls will reduce risk sharing and reduce welfare. Since an increase in the severity of capital controls can also lead to a shift from the first to the second regime, it

¹The logic for this is explained in Cook and Devereux (2013). In the absence of the zero bound constraint, the optimal policy is clearly for both countries to set policy rates equal to their ‘natural interest rates’, defined as the interest rate that would obtain in a flexible price, zero inflation equilibrium. But when one country is more constrained by the zero bound than another, it may be the case that this country experiences a terms of trade and exchange rate appreciation, leading to expenditure switching towards the other country. As a result, it may be optimal for that other country to raise interest rates in order to dampen the depreciation in the terms of trade and exchange rate affecting its economy.
might be thought that this can increase overall welfare. But this shift from one regime to
the other happens endogenously, when one country optimally chooses to move away from
the zero-bound on policy rates as capital controls are tightened. At this threshold level
of capital controls, where the country chooses to raise its interest rate above zero, welfare
is identical across regimes. It follows that the welfare benefit from shifting from the first
regime to the second regime, which allows for effective monetary policy independence, is
always offset by the direct costs in terms of reduced cross country risk-sharing associated
with increased capital controls. An equivalent interpretation is that the move from the
zero bound regime to the positive interest regime is in fact an endogenous response to the
effects of the tightening of capital controls. This policy response can ameliorate the negative
welfare costs of capital controls, but cannot completely undo them.

The paper is related to a recent literature on the economics of liquidity traps. In par-
ticular, with the experience of Japan in mind, Krugman (1998), Eggertsson and Woodford
(2003, 2006), Jung et al (2005), Svensson (2003), Auerbach and Obstfeld (2005) and many
other writers explore how monetary and fiscal policy could be usefully employed even when
the authorities have no further room to reduce short term nominal interest rates. Recently,
a number of authors have revived this literature in light of the very similar problems recently
encountered by the economies of Western Europe and North America. Papers by Christiano
possibility of using government fiscal expansions, tax cuts and monetary policy when the
economy is in a liquidity trap.

A recent literature also addresses the international dimensions of optimal policy in a
liquidity trap. Bodenstein et al (2009) is one example of a fully specified two country DSGE
model used to examine the international transmission of standard business cycle shocks
when one country is in a liquidity trap. Jeanne (2009) examines whether either monetary
policy or fiscal policy can implement an efficient equilibrium in a global liquidity trap in a
use numerical results to describe optimal monetary policy responses to asymmetric interest
rate shocks. The present paper describes natural interest rates as a function of demand
shocks and characterizes optimal policies analytically. Nakajima (2008) and Fujiwara et al
(2011) examine optimal policy responses to technology shocks in a model with a zero bound
constraint. We focus on the role of capital controls in creating asymmetries in optimal
policies amongst countries differentially affected by a demand contraction in the tradition of Eggertsson (2010). Cook and Devereux (2011) and Fujiwara and Ueda (2013) examine the fiscal policy multiplier in an open economy at the lower bound. Cook and Devereux (2013) explore how home bias in preferences generates asymmetries in the optimal policy response to liquidity trap shocks. In the model of the present paper, we abstract entirely from home bias in preferences. This implies that, if there were no controls on capital flows, all countries would be affected equally by demand shocks.

A related paper on capital controls is Farhi and Werning (2012). They argue that, in a fixed exchange rate regime, instituting capital controls may be beneficial as a means of ensuring monetary policy independence, for example in response to risk premium shocks. In contrast, we focus on the case of a flexible exchange rate regime where the loss of monetary policy independence is a result of the zero lower bound rather than a fixed exchange rate. And, in that context, we find that capital controls are never optimal. The main differences between the two analyses are as follows. In our paper, capital controls are not state contingent responses to shocks, but act directly as a tax on capital flows which reduces financial openness across countries. So capital controls cannot act as a substitute for monetary policy. Secondly, in our model, monetary policy is chosen optimally with and without capital controls and, as argued above, the negative risk-sharing effects of capital controls always dominate the endogenous response of monetary policy in welfare terms. Finally, our welfare assessment is based on an optimal cooperative policy measure, rather than focusing on welfare for a small open economy.

The rest of the paper is organized as follows. The next section develops the basic model and defines the way in which capital controls restrict financial risk-sharing. Section III defines natural interest rates and examines an equilibrium with sticky prices. Section IV examines the determination of optimal monetary policy subject to a zero bound constraint and the role of capital controls in affecting policy. Section V looks at the welfare effects of capital controls. Some conclusions are then offered.

II A two country model with preference shocks

We assume there are two countries in a general equilibrium economy. Households in each country consume goods and supply labour. Firms hire labour and sell to households, but are constrained by the inability to adjust prices instantaneously. The countries are called ‘home’
and ‘foreign’, with foreign variables marked with an asterisk. The population measure is one
in each country, and each country produces a range of differentiated goods. Governments
have access to lump sum taxation.

**Households**

Utility of the infinitely lived home household evaluated from date zero is:

\[
U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t (U(C_t, \xi_t) - V(N_t)),
\]

where \( U \) and \( V \) represent, respectively, the utility of the composite home consumption
bundle \( C_t \), and disutility of labour supply \( N_t \). We define \( \xi_t \) as a shock to preferences (or a
‘demand’ shock), and assume that \( U_{12} > 0 \). Hence, a temporary, positive, \( \xi_t \) shock implies
that agents in the home country have an increase in their valuation of today’s consumption
relative to future consumption. A negative \( \xi_t \) shock implies agents wish to defer consumption
to the future, and so will wish to increase their desired savings.

Composite consumption is defined as:

\[
C_t = \frac{1}{2} C_{Ht}^{1/2} C_{Ft}^{1/2}.
\]

\( C_H \) is the consumption of the home country composite good, and \( C_F \) consumption of the
foreign composite good, by the home household. In this specification, we assume no home
bias in consumption preferences. This assumption could easily be relaxed without substantial
consequence to the results. The special case of identical preferences highlights the tight
links between natural interest rates and the occurrence of liquidity traps in the analysis
below.

Consumption aggregates \( C_H \) and \( C_F \) are composites, defined over a range of home and
foreign differentiated goods, with elasticity of substitution \( \theta \) between goods. Price indices
for home and foreign consumption are:

\[
P_H = \left[ \int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad P_F = \left[ \int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},
\]

while the aggregate (CPI) price index for the home country is \( P = P_H^{1/2} P_F^{1/2} \) and for the
foreign country is $P_t^* = P_F^{*1/2} P_H^{*1/2}$.

Demand for each differentiated good ($j = H, F$) is:

$$\frac{C_j(i)}{C_j} = \left( \frac{P_j(i)}{P_j^*} \right)^{-q}.$$  (4)

The law of one price holds for each good so $P_j(i) = S P_j^*(i)$, where $S_t$ is the nominal exchange rate (home price of foreign currency).

The household’s implicit labour supply at nominal wage $W_t$ is:

$$U_C(C_t, \xi_t) W_t = P_t V'(N_t).$$  (5)

We make a special assumption on the existence of financial markets. We assume first that there is a complete set of financial assets that can be traded between countries so that, in principal, there can be complete cross-country risk sharing. But, in addition, we assume that countries may tax the returns on these financial assets, effectively meaning that the returns to home and foreign consumers may differ. At this point, we take these taxes as arbitrarily determined. In a later section we will investigate the welfare effects of these taxes. It is assumed that the revenue (cost) of the tax (or subsidy) is financed by lump-sum transfers (taxes). In the presence of the tax, optimal risk sharing implies:

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*) \frac{S_t P_t^*}{P_t} (1 + t_t) = U_C(C_t^*, \xi_t^*)(1 + t_t),$$  (6)

where $t_t$ represents the state contingent tax on securities. We assume without loss of generality that the tax (or transfer) is levied only by the home government. Since we focus only on optimal cooperative policy outcomes, this assumption is without force.

In what follows, we assume that the specification for the optimal tax on security returns is given by:

$$(1 + t_t) = \left( \frac{P_t C_t}{P_H Y_H} \right)^{(1-\lambda) \lambda}.$$  (7)

This implies that the tax or subsidy depends on the home country trade balance. If $(P_t C_t)/(P_H Y_H) = 1$ then the trade balance is zero, and the tax (subsidy) is zero. But if $(P_t C_t)/(P_H Y_H) > 1 (< 1)$ then there is a trade deficit in that state, and there is a positive tax on security returns (or a trade surplus, with a subsidy on security returns).
value of $\lambda$ can be adjusted to vary the equilibrium allocation between unrestricted security trade with complete markets ($\lambda = 1$) and effectively zero security trade with a zero trade balance ($\lambda = 0$). This is made clearer below.\footnote{Our assumption is that the tax is imposed only by the home country and the proceeds are rebated in a lump-sum fashion (or the cost of subsidies is raised by a lump-sum tax). This is sufficient to characterize the impact of capital controls on optimal policy, since it is necessary to have only one country imposing taxes in order to drive a wedge between natural interest rates across countries. Note that the paper does not characterize a state contingent optimal tax schedule. If taxes were imposed by both countries and chosen separately, then there would be a clear strategic interaction in tax setting, and the characterization of equilibrium would be considerably more complex.}

Nominal bonds pay interest, $R_t$. Then the consumption Euler equation for the home country is:

$$\frac{U_C(C_t, \xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}}.$$  \hspace{1cm} \text{(8)}

The nominal interest rate on bonds is the monetary authority’s instrument for monetary policy.

Foreign household preferences and choices are defined exactly symmetrically. The foreign representative household has identical weights of one-half on the foreign (home) composite good in their preferences.

\textit{Firms}

Each firm $i$ employs labour to produce a differentiated good, with the production function:

$$Y_t(i) = N_t(i).$$  \hspace{1cm} \text{(9)}

Profits are written as $\Pi_t(i) = P_{Ht}(i)Y_t(i) - [(\theta - 1)/\theta]W_tN_t(i)$. This implicitly involves an optimal subsidy, financed by lump-sum taxation, designed to eliminate steady-state first-order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability of adjusting prices equal to $1 - \kappa$. Firms that adjust their price set a new price given by $\tilde{P}_{Ht}(i)$:

$$\tilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0}^{m_{t+j}} \kappa^j W_{t+j} Y_{t+j}(i)}{E_t \sum_{j=0}^{m_{t+j}} \kappa^j Y_{t+j}(i)},$$  \hspace{1cm} \text{(10)}

where the stochastic discount factor is $m_{t+j} = [P_t U_C(C_{t+j}, \xi_{t+j})]/[P_{t+j} U_C(C_t, \xi_t)]$. In the aggregate, the price index for the home good then follows:

$$P_{Ht} = [(1 - \kappa)\tilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{1/\theta}.$$

\hspace{1cm} \text{(11)}
The behaviour of foreign firms and the foreign good price index is exactly analogous.

**Monetary policy**

We characterize monetary policy as a targeting rule, where the rule represents the solution to an optimal monetary policy problem described below. Operationally, optimal targeting is achieved by adjustments in the policy interest rate, subject to the policy rate satisfying the zero bound constraint. As we describe below, in some instances it is optimal for one or both monetary authorities to set their policy rates to zero – the zero bound constraint – as this is the best that discretionary monetary policy can do.

**Market clearing**

Equilibrium in the market for good \( i \) is given by:

\[
Y_{Ht}(i) = \left( \frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta} \left[ \frac{1}{2} \frac{P_t}{P_{Ht}} C_t + \frac{1}{2} \frac{S_t P^*_t}{P_{Ht}} C^*_t \right],
\]

and aggregate market clearing in the home good by:

\[
Y_{Ht} = \frac{1}{2} \frac{P_t}{P_{Ht}} C_t + \frac{1}{2} \frac{S_t P^*_t}{P_{Ht}} C^*_t.
\]

Here \( Y_{Ht} = V_t^{-1} \int_0^1 Y_{Ht}(i)di \) is aggregate home country output, where we have defined \( V_t = \int_0^1 (P_{Ht}(i)/P_{Ht})^{-\theta} di \). It follows that home country employment (employment for the representative home household) is given by \( N_t = \int_0^1 N(i)di = Y_{Ht}V_t \).

An equilibrium with positive nominal interest rates is described by the equations (5), (8), (10), (11) and (13) for the home country, and the analogous equations for the foreign country. Together with (6), and for given values of \( V_t \) and \( V^*_t \), given monetary rules (to be described below), these equations determine an equilibrium sequence for the variables \( C_t, C^*_t, W_t, W^*_t, S_t, P_{Ht}, P^*_{Ft}, \bar{P}_{Ht}, \bar{P}^*_{Ft}, R_t, R^*_t, N_t \) and \( N^*_t \).

**III The effects of demand shocks**

We now explore the impact of shocks which push one or both monetary authorities towards the zero bound on interest rates. These are shocks which reduce the marginal utility of today’s consumption, relative to the marginal utility of future consumption, leading to a
fall in equilibrium real interest rates. If real interest rates that would accommodate the fall in current marginal utility for a given path of consumption fall below zero, then the monetary authority may be constrained by the zero lower bound. As a short-hand, we refer to these as demand or savings shocks.

Define \( \sigma \equiv -U_{CC}\bar{C}/U_C \) as the inverse of the elasticity of inter-temporal substitution in consumption, and \( \phi \equiv -V''\bar{H}/V' \). Also, \( \epsilon_t = U_C\xi \ln(\xi_t)/U_C \) is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country. A temporary fall in \( \epsilon_t \) will raise savings and reduce real interest rates. We assume that savings shocks are independent across countries. In the analysis below, we make more specific assumptions about the stochastic distributions of \( \epsilon_t \) and \( \epsilon_t^* \).

**The flexible price allocation**

First consider the flexible price allocation, expressed in terms of linear approximations around a steady state. \( x_t = \ln(X_t/\bar{X}) \). The home and foreign goods market clearing conditions (here \( \tau \) is the home terms of trade in terms of linear approximation) are given by:

\[
y_t = \frac{1}{2}(c_t + c_t^*) + \frac{1}{2}\tau_t, \tag{14}
\]

\[
y_t^* = \frac{1}{2}(c_t + c_t^*) - \frac{1}{2}\tau_t. \tag{15}
\]

The linear approximation of (6) is given by:

\[
\lambda(\sigma(c_t - c_t^*) - \epsilon_t + \epsilon_t^*) + (1 - \lambda)(c_t - y_t + \frac{1}{2}\tau_t) = 0. \tag{16}
\]

This expression represents a convex combination of the complete markets condition (first expression in brackets) and the financial market autarky condition (final expression in brackets). With \( \lambda = 1 \) (0) we have complete markets (financial autarky). Note that financial autarky doesn’t rule out goods trade, so the terms of trade still plays a role in the dynamics of the model.

Combining (5) with (10) in the case where \( \kappa = 0 \), the flexible price equilibrium allocation,
gives optimal production for the home and foreign countries:

\[ \phi y_t + \sigma c_t - \epsilon_t + \frac{1}{2} \tau_t = 0, \]  
(17)

\[ \phi y_t^* + \sigma c_t^* - \epsilon_t^* - \frac{1}{2} \tau_t = 0. \]  
(18)

Solving (14)-(18), we get:

\[ y_t = \omega \frac{1}{2} \epsilon_t + \frac{1}{2} \epsilon_t^* + (1 - \omega) \frac{1}{2} \frac{(1 + \sigma + 2\phi)\epsilon_t - (\sigma - 1)\epsilon_t^*}{(1 + \phi)(\sigma + \phi)}, \]  
(19)

\[ y_t^* = \omega \frac{1}{2} \epsilon_t + \frac{1}{2} \epsilon_t^* + (1 - \omega) \frac{1}{2} \frac{(1 + \sigma + 2\phi)\epsilon_t^* - (\sigma - 1)\epsilon_t}{(1 + \phi)(\sigma + \phi)}, \]  
(20)

\[ \tau_t = \frac{1 - \omega}{1 + \phi} (\epsilon_t - \epsilon_t^*), \]  
(21)

\[ c_t = \omega \frac{1}{2} \frac{(\phi + 2\sigma)\epsilon_t - \phi \epsilon_t^*}{\sigma(\phi + \sigma)} + (1 - \omega) \frac{1}{2} \frac{\epsilon_t + \epsilon_t^*}{\sigma + \phi}, \]  
(22)

\[ c_t^* = \omega \frac{1}{2} \frac{(\phi + 2\sigma)\epsilon_t^* - \phi \epsilon_t}{\sigma(\phi + \sigma)} + (1 - \omega) \frac{1}{2} \frac{\epsilon_t + \epsilon_t^*}{\sigma + \phi}, \]  
(23)

where \( \omega = (2\sigma \lambda)/(2\sigma \lambda + 1 - \lambda) \). So the values of output, the terms of trade and consumption under fully flexible prices are linear combinations of their values under complete markets and under autarky. With complete markets, output responses under flexible prices are identical across countries. There are no terms-of-trade movements but, due to country-specific preference shocks, consumption differs between the home and foreign countries. With incomplete markets, the trade balance is zero in each country and output responses differ across countries: a home demand shock causes a terms-of-trade deterioration, but consumption responses are identical.\(^3\)

**Natural interest rates**

Now we can work out the natural interest rates in our model. These are the real interest rates

\(^3\)Note that even in the case of identical preferences and unit elasticity of substitution across home and foreign goods, it is not true that financial autarky and complete markets support identical allocations, as might be surmised from the well-known Cole and Obstfeld (1991) result. Cole and Obstfeld’s result applies to productivity shocks. Here, the shocks are due to changes in effective rates of time preference. In a complete markets environment, these would not require any movement in the terms of trade. Thus, the terms of trade cannot act as a proxy for establishing effective risk-sharing, in contrast to Cole and Obstfeld.
rates that would hold with fully flexible prices. Natural interest rates are an object of interest because, with an unrestricted monetary policy, setting policy rates equal to the natural interest rate in both countries achieves a first-best allocation with zero inflation. The question then arises of how policy rates should be set when natural interest rates dip below zero, and policy rates cannot follow.

The natural interest rate is given by \( \tilde{r}_t = E_t(c_{t+1} - c_t) - E_t(\epsilon_{t+1} - \epsilon_t) \) for the home country, and similarly for the foreign country, when consumption is evaluated at the flexible price equilibrium. In order to compute natural interest rates we assume for now that the shocks \( \epsilon \) and \( \epsilon^* \) can be characterized as Markov processes with continuation probability \( \mu \). Specifically, given any shock \( \epsilon_t \), assume this shock continues with probability \( \mu \) and goes to zero with probability \( 1 - \mu \). We make the same assumption regarding \( \epsilon^*_t \), and assume that the home and foreign shocks are independent.\(^4\) Since there are no state variables in the economy this means that, with a shock \( \epsilon_t \neq 0 \), all variables have expected persistence given by \( \mu \).

Using these assumptions, and substituting the solutions for consumption, implies that the natural interest rates are given by:

\[
\tilde{r}(\lambda, \epsilon_t, \epsilon^*_t) = \rho + \omega \frac{1}{2} \frac{(1 - \mu)\phi(\epsilon_t + \epsilon^*_t)}{(\phi + \sigma)} + (1 - \omega) \frac{1}{2} \frac{(1 - \mu)((\sigma + 2\phi)\epsilon_t - \sigma\epsilon^*_t)}{(\phi + \sigma)},
\]

\[
\tilde{r}^*(\lambda, \epsilon_t, \epsilon^*_t) = \rho + \omega \frac{1}{2} \frac{(1 - \mu)\phi(\epsilon_t + \epsilon^*_t)}{(\phi + \sigma)} + (1 - \omega) \frac{1}{2} \frac{(1 - \mu)((\sigma + 2\phi)\epsilon^*_t - \sigma\epsilon_t)}{(\phi + \sigma)},
\]

where \( \rho = \ln(\beta^{-1}) \) is the rate of time preference, which in this model equals the real interest rate in a non-stochastic steady state.

Note that for \( \lambda = 1 \) we have \( \omega = 1 \) and natural interest rates are equated. Despite the presence of country specific preference shocks, because financial markets are fully integrated and consumer price indices are identical, real interest rates must be equated. We might then suspect that, with complete financial markets, countries will be simultaneously constrained by the zero bound constraint on interest rates. Indeed, we show below that, with \( \lambda = 1 \), the occurrence of a liquidity trap (where the monetary authorities set the policy rate to zero) is always simultaneous in the two countries.

\(^4\)Formally, we take the shock \( \epsilon_t \) as following a two-state Markov process with probability matrix

\[
\begin{bmatrix}
\mu & 1 - \mu \\ 1 - \mu & \mu
\end{bmatrix}.
\]
On the other hand, for $\lambda = 0$, natural interest rates are negatively correlated. In this case, home and foreign consumption respond to an $\varepsilon$ shock identically. For a positive home shock $\varepsilon$, this implies that the foreign natural interest rate falls, because consumption is expected to fall over time. But in the home country, the fall in expected consumption growth is combined with a rise in effective time preference, so that the home natural interest rate rises overall.

Given that natural interest rates are imperfectly correlated when $\lambda < 1$, it is no longer necessarily the case that the zero bound on policy interest rates will occur simultaneously in both countries. As we show below, the optimal monetary policy may involve optimal policy rates deviating from one another. In effect, the presence of imperfect financial markets generates a degree of effective monetary policy independence when monetary policy is chosen as an optimal targeting policy.

To illustrate the determination of natural interest rates, take the example where there is a negative shock emanating from the home country alone, so that $\varepsilon_t < 0$ and $\varepsilon_t^* = 0$. Moreover, assume the shock is such that, for $\lambda = 1$, $\bar{r}_t = \bar{r}_t^* < 0$. Then, from (24) and (25), there must exist a value $\bar{\lambda}$ where $0 < \bar{\lambda} < 1$ such that, for $\lambda \leq \bar{\lambda}$, $\bar{r}_t^* > 0$. Figure 1 illustrates such a case. As $\lambda$ falls below 1, the home natural real interest rate declines, but the foreign natural real interest rate rises. At $\bar{\lambda}$, the foreign natural real interest rate is zero. We pursue this as a leading example below, when we examine the determination of optimal monetary policy.

(Figure 1 about here)

Inflation and output gaps with sticky prices

Now we use the model of section 2 to derive the solution under sticky prices. Standard linear approximations (see, for example, Cook and Devereux 2011) allow us to derive the following equations, which jointly determine the dynamics of inflation and output gaps in the two countries when natural interest rates are governed by (24) and (25). We define PPI inflation for the home (foreign) country as $\pi_t$ ($\pi_t^*$). Then:

$$\pi_t = k((\phi + \frac{1 + \sigma}{2})\dot{y}_t + \frac{\sigma - 1}{2}\dot{y}_t^*) + \beta E_t \pi_{t+1},$$  \hspace{1cm} (26)

$$\pi_t^* = k((\phi + \frac{1 + \sigma}{2})\dot{y}_t^* + \frac{\sigma - 1}{2}\dot{y}_t) + \beta E_t \pi_{t+1}^*,$$  \hspace{1cm} (27)
\[ E_t \left( \frac{\sigma + 1}{2} (\hat{y}_{t+1} - \hat{y}_t) + E_t \frac{\sigma - 1}{2} (\hat{y}_{t+1}^* - \hat{y}_t^*) \right) = r_t - E_t \pi_{t+1} - \tilde{r}_t, \]  

\[ E_t \left( \frac{\sigma + 1}{2} (\hat{y}_{t+1} - \hat{y}_t^*) + E_t \frac{\sigma - 1}{2} (\hat{y}_{t+1} - \hat{y}_t) \right) = r_t^* - E_t \pi_{t+1}^* - \tilde{r}_t^*, \]

where \( \hat{x} \) represents the log deviation of \( x \) from its flexible price equilibrium value; hence \( \hat{x} \) represents a ‘gap’ term.

If there were no constraints on interest rates then monetary authorities could close all gaps by setting policy rates equal to natural interest rates and achieve zero output gaps and zero inflation. This would achieve the ‘divine coincidence’ in an international setting, as defined by Blanchard and Gali (2007). But this is not possible when nominal interest rates are constrained by the zero bound, and at least one of \( \tilde{r} \) and \( \tilde{r}^* \) is negative.

How preference shocks impact on inflation and output gaps at the zero bound depends not only on the degree of capital mobility, and the implied path of natural interest rates, but also on the policy response. We now turn to the analysis of optimal policy.

IV Optimal monetary policy

We now characterize optimal monetary policy, where policy is chosen in a cooperative fashion to maximize an equal weighted sum of home and foreign welfare. We focus on a discretionary monetary policy environment. This means that policy-makers take all expectations of future economic variables and future policy choices as given. Furthermore, in determining optimal monetary policy, we explicitly take account of the zero bound on policy interest rates.

Initially, we will take the value of \( \lambda \) as given. The focus is then on how different degrees of capital mobility affect the optimal monetary policy response in a liquidity trap environment.

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5 An optimal monetary policy with commitment, in face of the zero bound constraint, is defined and explored by Eggertsson and Woodford (2003). This requires that the central bank engage in ‘forward guidance’ by announcing a path of interest rates to hold in the aftermath of the liquidity trap. Without credible commitment, however, this policy is not time-consistent. Our focus on the case of discretionary monetary policy can be defended on two grounds: a) this is the situation where the zero bound has the greatest real effects, as discussed in Eggertsson and Woodford (2003) and Adam and Billi (2007), and thus the potential gains from escaping the zero bound are likely to be very high; and b) unlike the discretionary case in the closed economy, where the central bank follows a passive zero-interest policy, in the global context outlined here it is shown that discretion allows for activism in monetary policy so long as some capital controls are in place. Note that besides focusing on discretion, we also restrict ourselves to Markov equilibria. By doing so, we only condition on current shocks and ignore their history. And because our model has no (endogenous) state variables, we do not need to include the impact of current policy choices on future policy choices. It is possible that better outcomes are possible if the central bank responds to past shocks (as well as current shocks), even under discretion. We briefly discuss how commitment in policy would affect our results in the conclusions.
In a later section, we examine the effects of $\lambda$ on welfare both directly and indirectly through its influence on optimal monetary policy.

**Welfare approximation**

In order to formulate optimal policy, we first have to determine the welfare function. Following the literature on welfare approximations developed by Woodford (2003), Benigno and Woodford (2012) and, for open economies, Benigno and Benigno (2006) and Engel (2011), we can compute social welfare as a second order approximation to an equal weighted sum of home and foreign utility. Unlike most of this literature, however, the relevant measure of welfare is not simply a function of ‘gaps’, or deviations of the critical aggregate variables from their flexible price equilibrium values. This is because the value of $\lambda$, representing the degree of capital market integration, will affect the distribution of flexible price equilibrium variables. Thus, in determining the welfare effects of $\lambda$, we cannot ignore the welfare approximation of the flexible price equilibrium economy. While this is not affected by monetary policy, it is clearly dependent on the degree of capital market openness.

Cook and Devereux (2011) describe the details involved in computing the welfare objective. Taking a second-order approximation to an equal-weighted sum of home and foreign utility around a non-stochastic steady state, we derive an approximate welfare function. Since we focus on a discretionary policy equilibrium, only the contemporaneous terms in the welfare function are relevant. These are given by:

$$V_t = (1 - \sigma) \left( \frac{\hat{y}_t + \hat{y}_t^*}{2} \right)^2 - \frac{(\phi + 1)}{2} \hat{y}_t^2 - \frac{(\phi + 1)}{2} \hat{y}_t^{*2} - \frac{\theta}{2k} \pi_t^2 - \frac{\theta}{2k} \pi_t^{*2} + V_{Ft}, \quad (30)$$

where:

$$V_{Ft} = \frac{(1 - \sigma)}{2} c_t^2 + \epsilon c_t - \frac{(\phi + 1)}{2} y_t^2 + \frac{(1 - \sigma)}{2} c_t^{*2} + \epsilon c_t^* - \frac{(\phi + 1)}{2} y_t^{*2}. \quad (31)$$

Here $V_{Ft}$ is the second order approximation to the social welfare function evaluated at the

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6 As in Engel (2011), we focus on cooperative monetary policy design. The non-cooperative monetary policy setting involves a substantial increase in complexity, related to the point of approximation of the welfare function, the initial conditions and the definition of strategy spaces – see, in particular, Benigno and Benigno (2006).

7 For this approach to a discretionary equilibrium see, for instance, Clarida et al. (1999). By definition in such a discretionary equilibrium policy-makers take future policy instruments and expectations of future variables as given. Hence, the optimal discretionary policy can be described by focusing only on the maximization of current period utility. In the case where the zero lower bound binds for both countries, the optimal discretionary policy is simply to set nominal interest rates to zero for the period.
fully flexible price equilibrium. Clearly, from (19)-(23), this will depend on the value of $\lambda$.

The other expressions indicate that welfare is a function of the squares and cross products of output gaps and squares of inflation rates.

The optimal cooperative monetary policy under discretion may be described as the solution to the following problem:

$$\begin{align*}
\text{P1} & \quad \max_{\hat{y}_t, \hat{y}_t^*, \pi_t, \pi_t^*, r_t, r_t^*} L_t = V_t + \psi_{1t}\left[\pi_t - k((\phi + \frac{1}{2}\sigma)\hat{y}_t + \frac{(\sigma - 1)}{2}\hat{y}_t^*) - \beta E_t\pi_{t+1}\right] \\
& \quad + \psi_{2t}\left[\pi_t^* - k((\phi + \frac{1}{2}\sigma)\hat{y}_t^* + \frac{(\sigma - 1)}{2}\hat{y}_t) - \beta E_t\pi_{t+1}^*\right] \\
& \quad + \psi_{3t}\left[\frac{1}{2}E_t(\hat{y}_{t+1} - \hat{y}_t) + \frac{(\sigma - 1)}{2}E_t(\hat{y}_{t+1}^* - \hat{y}_t^*) - E_t(r_t - \bar{r}_t - \pi_{t+1})\right] \\
& \quad + \psi_{4t}\left[\frac{1}{2}E_t(\hat{y}_{t+1} - \hat{y}_t) + \frac{(\sigma - 1)}{2}E_t(\hat{y}_{t+1} - \hat{y}_t) - E_t(r_t^* - \bar{r}_t^* - \pi_{t+1})\right] \\
& \quad + \gamma_{1t}r_t + \gamma_{2t}r_t^*,
\end{align*}$$

where the $\psi_{jt}$ variables are the multipliers on the four constraints and the last line represents the two non-negativity constraints on policy interest rates reflected by the multipliers $\gamma_{1t}$ and $\gamma_{2t}$. Formally, a cooperative equilibrium is defined by the sequence $\hat{y}_t, \hat{y}_t^*, \pi_t, \pi_t^*, r_t, r_t^*, \psi_{jt}, \gamma_{jt}$, $i = 1..4$, $j = 1,2$, that maximizes (30) subject to (26)-(29) and the two non-negativity constraints on nominal interest rates, $r_t \geq 0$, $r_t^* \geq 0$.

The solution to this problem depends solely on the characteristics of the natural interest rates, which in turn depend on the individual preference shocks and the capital mobility parameter $\lambda$. If both $\bar{r}_t$ and $\bar{r}_t^*$ are greater than zero, then the solution is simply $r_t = \bar{r}_t$ and $r_t^* = \bar{r}_t^*$, and all gaps are closed. But we focus on the case where at least one of $\bar{r}_t$ and $\bar{r}_t^*$ are less than zero. In this case, at least one of the non-negativity constraints will be binding.

In the appendix, we show that the solution to problem (P1) may be reduced to two conditions relating policy interest rates to natural interest rates, and the multipliers $\gamma_{1t}$ and $\gamma_{2t}$. These are:

$$\begin{align*}
r_t &= \bar{r}_t + \frac{1}{2}(\Omega_\sigma + \Omega)\gamma_{1t} + \frac{1}{2}(\Omega_\sigma - \Omega)\gamma_{2t}, \\
r_t^* &= \bar{r}_t^* + \frac{1}{2}(\Omega_\sigma + \Omega)\gamma_{2t} + \frac{1}{2}(\Omega_\sigma - \Omega)\gamma_{1t}.
\end{align*}$$

These conditions must be combined with the comparative slackness conditions: a) $r_t \geq 0$, $\gamma_{1t} \geq 0$ and $r_t \gamma_{1t} = 0$; and b) $r_t^* \geq 0$, $\gamma_{2t} \geq 0$ and $r_t^* \gamma_{2t} = 0$. The expressions $\Omega_\sigma$ and $\Omega$ are
defined in the appendix, and satisfy the conditions \( \Omega_\sigma > (\geq, <) \Omega \) when \( \sigma > (\geq, <) 1 \), and \( \Omega_\sigma > 0, \Omega > 0 \). Note that when \( \sigma = 1 \), \( \Omega_\sigma = \Omega \), and conditions (32) and (33) dichotomize so that the solution is \( r_t = \max(0, \tilde{r}_t) \) and \( r_t^* = \max(0, \tilde{r}_t^*) \). But, in general, \( r_t \) and \( r_t^* \) will be simultaneously determined.

First note from (24) and (25) that when \( \lambda = 1 \), so that there is complete risk sharing, \( r_t = r_t^* \). In this case, the only solution that is consistent with (32), (33) and the comparative slackness conditions is \( r_t = r_t^* = \max(0, \tilde{r}_t) \). Thus, without capital controls, if one country is in a liquidity trap, then both must be so simultaneously.

Now, to characterize the optimal monetary policy choice, we focus again on the example of Figure 1 where \( \tilde{r}_t < 0 \) and \( \tilde{r}_t \leq \tilde{r}_t^* \). We can then use the solutions to (32) and (33) to establish the following Proposition.

**Proposition 1.** When natural interest rates satisfy the conditions \( \tilde{r}_t < 0 \) and \( \tilde{r}_t \leq \tilde{r}_t^* \), the optimal monetary policy is characterized by the conditions:

\[ a) \quad r_t = 0, \]
\[ b) \quad r_t^* = \max(0, \tilde{r}_t^* - \frac{\Omega_\sigma - \Omega}{\Omega_\sigma + \tilde{r}_t}). \]

**Proof.** There are four cases to consider: (i) \( r_t > 0, r_t^* > 0 \); (ii) \( r_t > 0, r_t^* = 0 \); (iii) \( r_t = 0, r_t^* > 0 \); and (iv) \( r_t = 0, r_t^* = 0 \). We consider each in turn. In case (i), if both policy rates are positive then, since \( \gamma_{1t} = \gamma_{2t} = 0 \), we have \( r_t = \tilde{r}_t \) and \( r_t^* = \tilde{r}_t^* \), which is a contradiction, for at least \( r_t \). Case (ii) implies that \( \gamma_{1t} = 0 \), so combining (32) and (33) we have:

\[ r_t = \tilde{r}_t - \tilde{r}_t^* - 2 \frac{\Omega}{\Omega_\sigma + \tilde{r}_t}. \]

Again, this is a contradiction, since the right hand side of this equation is negative, and therefore \( r_t > 0 \) cannot hold.

In case (iii), solving (32) and (33), we get:

\[ r_t^* = \tilde{r}_t^* - \frac{\Omega_\sigma - \Omega}{\Omega_\sigma + \tilde{r}_t}. \]

This is feasible so long at the right hand side is non-negative.
In case (iv) we solve (32) and (33) to obtain:

\[ \gamma_{1t} = \frac{-\tilde{r}_t (\Omega + \Omega) + \tilde{r}_t^* (\Omega - \Omega)}{\Delta \gamma}, \]

\[ \gamma_{2t} = \frac{-\tilde{r}_t^* (\Omega + \Omega) + \tilde{r}_t (\Omega - \Omega)}{\Delta \gamma}, \]

where \( \Delta \gamma = (1/2)[(\Omega + \Omega)^2 - (\Omega - \Omega)^2] > 0 \). Here \( \gamma_{1t} > 0 \) is always satisfied. This case is feasible so long as \( \gamma_{2t} > 0 \). But this is only satisfied if case (iii) is not satisfied. Hence, the foreign policy rate must be the maximum of zero or \( \tilde{r}_t^* - [(\Omega - \Omega)/((\Omega + \Omega)]\tilde{r}_t \).

Proposition 1 has immediate implications for the effect of capital controls on optimal monetary policy. To illustrate the effect of \( \lambda \) on the optimal choice of monetary policy, now return to the example of Figure 1, where \( \epsilon_t < 0 \) and \( \epsilon_t^* = 0 \), and assume \( \epsilon_t \) is large enough in absolute terms such that \( \tilde{r}_t = \tilde{r}_t^* < 0 \) when \( \lambda = 1 \). Then, Proposition 1 immediately implies that the optimal discretionary interest rate rule is zero, for both countries, when \( \lambda = 1 \). In addition, from the analysis of Figure 1, it is clear that \( r_t = 0 \) for all possible values of \( \lambda \). Thus, the home country is always constrained by the zero bound. But the analysis for the foreign country depends on the value of \( \lambda \). From Figure 1 we know that there exists a value \( \bar{\lambda} \) such that \( \tilde{r}_t^* > 0 \) for \( \lambda < \bar{\lambda} \). But Proposition 1 says that the optimal foreign interest rate is not always equal to \( \tilde{r}_t^* \), even when \( \tilde{r}_t^* > 0 \). The value of \( r_t^* \) depends both on \( \tilde{r}_t^* \) and the parameter \( \sigma \). Take first the case where \( \sigma > 1 \). Then from Proposition 1 it is clear that \( r_t^* \geq \tilde{r}_t^* \). More generally, from Proposition 1 part (iii), it is clear that there exists a value of \( \lambda \), denoted \( \lambda_H \), where \( \lambda_H > \bar{\lambda} \), such that \( r_t^* > 0 \) for \( \lambda < \lambda_H \). Thus, for \( \sigma \geq 1 \), there is always a value of \( \lambda \) such that the foreign country is outside the liquidity trap zone. Moreover, in this case, there is a range in which the foreign country chooses a positive interest rate, even though its natural interest rate is negative.\(^8\)

In the special case of \( \sigma = 1 \), then Proposition 1 implies that \( r_t = \max(0, \tilde{r}_t^*) \), and the foreign policy rate is exactly as in a closed economy. It is still the case that the foreign monetary policy response depends on the degree of capital mobility, with the foreign policy rate above zero for all \( \lambda < \bar{\lambda} \).

Finally, in the case \( \sigma < 1 \), then from the optimal policy of Proposition 1 we cannot say

\(^8\)A case similar to this (without capital controls but with home bias in preferences) is examined by Cook and Devereux (2013).
whether there exists any value of $\lambda$ such that the foreign policy rate is positive. On the one hand, for any $\lambda < \bar{\lambda}$, the $\tilde{r}_t^*$ component of the monetary rule of Proposition 1 is positive. But the second component is always negative since, for $\sigma < 1$, $\Omega_\sigma < \Omega$.

The situation with $\sigma > 1$ is illustrated in Figure 2. The figure shows the case where $\epsilon_t < 0$ and $\epsilon_\sigma^* = 0$, and describes the value of $\tilde{r}_t^*$, and the optimal foreign interest rate $r_t^*$, for different values of $\lambda$. The general implication is that increasing capital controls reduces the likelihood that the demand shock originating in the home country pushes the foreign country into a liquidity trap. Equivalently, increasing capital controls preserves effective monetary independence in the sense that optimal monetary policy is not constrained by the zero bound.

(Figure 2 about here)

The intuition for the link between capital controls and effective monetary independence is not a direct application of the well-known policy trilemma which says that, in the absence of exchange rate adjustment, monetary independence can only be attained under the cover of capital controls. In the present case, we have not imposed any restrictions at all on exchange rate adjustment. Each country is assumed to have an independent interest rate policy, and the exchange rate can therefore adjust freely. But interest rate adjustment can be followed only so long as it does not violate the zero lower bound. The key feature of the model is that, even under fully flexible exchange rates, optimal monetary adjustment to a demand shock coming from the rest of the world may push the monetary policy authority into a region where monetary policy is ineffective, and the best that can be done is to set a zero interest rate. It is clearly an option to have a higher interest rate, but it is suboptimal.

When there is full capital mobility ($\lambda = 1$), large negative demand shocks are propagated world-wide because natural interest rates are equated across countries. These shocks, then, lead to a global liquidity trap so that monetary policy is ineffective even though there is no restriction on exchange rate adjustment. By contrast, the presence of capital controls leads to a divergence among natural interest rates, and hence may restore effective monetary independence for a country reacting to an external shock.

We noted from Figure 2 that the foreign country’s optimal response to an external shock may be to raise the policy interest rate, even when its own natural interest rate is negative. The logic behind this comes from (24) and (25). When $\lambda < 1$, natural interest rates are more likely to be negatively correlated than positively correlated across countries. Then
a negative home demand shock may create an expansionary shock in the foreign country, leading optimal monetary policy to be more contractionary than it would were it a closed economy.\footnote{In the closed economy, the optimal policy rate would always equal the natural interest rate.}

To see this more clearly, we may use conditions (26)-(29) and the solution to (14)-(16) to track the response of the exchange rate to a demand shock originating in the home country, when both countries are constrained by the zero bound on interest rates. By definition, the exchange rate is equal to the terms of trade multiplied by the relative price of the home good to the foreign good, \( P_H / P_F \). Thus, we may describe the dynamics of the exchange rate by:

\[
s_t - s_{t-1} = \tau_t - \tau_{t-1} + \pi_{Ht} - \pi_{Ft}.
\]

Conditions (14)-(15) imply that, in all cases, the response of the terms of trade to a demand shock is given by \( \tau_t = y_t - y_t^* \). Using this, and the solutions to (26)-(29), we can show that, when neither country’s interest rate responds to a home country shock \( \epsilon_t \), the unanticipated response of the exchange rate is:

\[
s_t - E_{t-1}s_t = \frac{(1 - \omega)(1 - \mu)(1 - \beta \mu + k(1 + \phi))\epsilon_t}{(1 - \beta \mu)(1 - \mu) - k\mu(1 + \phi)}.
\]

From (35) we see that, for a negative demand shock, the home country exchange rate appreciates whenever \( \omega < 1 \) (that is, whenever there are restrictions on capital markets). As we saw, in the limit with completely free capital flows, natural interest rates are identical and there is no relative price change at all. But with some restrictions on capital mobility, the foreign natural interest rate falls by less than the home natural interest rate. Consequently there is a greater decline in home output relative to foreign output, and a home terms-of-trade appreciation, mirrored in an exchange rate appreciation. This implies that there is expenditure switching away from the home country and towards the foreign country. At the point \( \lambda_H \) in Figure 2, the optimal monetary policy rule indicates that it is desirable to raise interest rates in the foreign country, which acts to limit the degree of home country appreciation (or foreign depreciation). Figure 3 illustrates the response of the nominal exchange rate for a given value of the shock \( \epsilon_t \), for different values of \( \lambda \): at lower values of \( \lambda \), the home exchange rate appreciates more. For \( \lambda < \lambda_H \), the exchange rate is still falling in \( \lambda \), but the degree of response is less, because declining \( \lambda \) is matched by a rising foreign
The importance of the inter-temporal elasticity of substitution in determining the foreign policy response to a home country shock comes from the role of $\sigma$ in governing the international transmission of shocks. When $\sigma > 1$ then, ceteris paribus, a home output contraction tends to be expansionary in the foreign country. Intuitively, this comes from the mixture of wealth and substitution effects on marginal cost. A fall in home output, given (17), leads to a fall in home consumption and a decline in marginal cost. But, simultaneously, this is countered by a real appreciation of the home terms of trade, which is a depreciation of the foreign terms of trade, increasing marginal cost for the foreign country. When $\sigma = 1$, these two factors exactly balance out and the net effect is zero. When $\sigma > 1$, foreign marginal cost tends to fall, leading the effect to be expansionary. When $\sigma < 1$, foreign marginal cost rises, leading the effect to be contractionary. This logic then translates into the transmission of demand shocks, when capital mobility is imperfect. When $\sigma > 1$ and $\lambda < \lambda_H$, the imperfect correlation of demand shocks tends to be associated with small or even negative international transmission of the home $\epsilon$ shock to the foreign output gap. Whereas, when $\sigma < 1$, the demand shocks tend to foster positive transmission between output gaps across countries. Consequently, the optimal monetary policy response in the foreign country to a negative $\epsilon$ shock is more likely to be contractionary in the $\sigma > 1$ case, as illustrated in Figure 2.

V Welfare effects of capital controls

In the previous section, we showed that the introduction of capital controls could restore effective monetary independence. Without capital controls, large negative demand shocks precipitated a uniform global liquidity trap in which neither country could gainfully employ countercyclical monetary policy. But, in response to the same shock, with capital controls such that $\lambda < \lambda_H$, a country will find it optimal to directly respond to external demand shocks. The size of the policy response depends upon both the degree of capital market restrictions and the value of $\sigma$.

This raises the question of the welfare consequences of capital market restrictions. Absent the zero bound on interest rates, monetary policy can be used independently in either country: there is effective monetary independence (assuming no other preconditions on
monetary policy, such as an exchange rate peg). In this case, capital controls would have no compensating benefit in terms of allowing independent monetary policy, and so should be unambiguously welfare reducing. But, in the case where shocks can be large enough to push one or both countries into a liquidity trap, it might be anticipated that, by supporting effective monetary policy independence, some degree of capital market restrictions may be welfare enhancing. We now investigate this proposition.

Initially, however, we establish that in the absence of the zero lower bound, capital controls as represented by $\lambda < 1$ must always be welfare reducing. We can show this very easily in the following proposition.

**Proposition 2.** When monetary policy is not constrained by the zero bound, capital controls are welfare reducing.

**Proof.** When shocks are such that neither country is ever constrained by the zero bound in setting optimal monetary policy, then clearly the optimal monetary response for both countries is to set the policy rate equal to their respective natural rates. Thus optimal monetary policy is described by $r_t = \tilde{r}_r$ and $r_t^* = \tilde{r}_r^*$. In this case, it is obvious from (26)-(29) that all output gaps are zero, and each country’s inflation rate is zero. Then welfare may be approximated by (31), which is $V_{Ft}$, the welfare approximation under flexible prices.

Under the assumption that $\epsilon_t$ and $\epsilon_t^*$ are independent and have identical variance $Var_{\epsilon}$, we may show that

$$E_{t-1}V_{Ft} = \frac{1}{2} \frac{(\sigma - 1)}{(1 + \phi)(\sigma + \phi)} Var_{\epsilon} + 2 \frac{(\sigma \lambda \phi + \phi + \sigma + 1 + \sigma^2 \lambda) \lambda}{(\phi + 1)(2\sigma \lambda - \lambda + 1)^2} Var_{\epsilon}. \quad (36)$$

Then it is straightforward to show that

$$\frac{dE_{t-1}V_{Ft}}{\lambda} = 2 \frac{(\lambda \phi + \phi + 1 + \sigma + \lambda - \sigma \lambda)}{(\phi + 1)(2\sigma \lambda - \lambda + 1)^3} Var_{\epsilon} > 0. \quad (37)$$

Therefore, not surprisingly, a rise in $\lambda$, indicating increased risk-sharing, unambiguously increases global welfare when monetary policy is unconstrained by the zero bound constraint.

The more interesting case arises when capital controls may play a role in supporting effective monetary independence. Somewhat surprisingly, we can show that, even in this
case, capital controls still cannot be welfare-enhancing. While the presence of capital controls may successfully protect a country from being pushed into a liquidity trap, the welfare loss from decreased risk-sharing always dominates the welfare benefit from a more active monetary policy.

To see this, we proceed as follows. From (26)-(29) and (30), it is clear that, given $V_{Ft}$, capital controls impact on welfare only by affecting the natural interest rates (24) and (25). Moreover, we have shown above that, absent the zero bound, capital controls can affect welfare only through the term $V_{Ft}$. Thus, to show the welfare consequence of capital controls for given $V_{Ft}$, we have to focus on cases where the zero bound is binding for one or both countries. We do this by examining welfare across the different segments of the value of $\lambda$ which generate different degrees of monetary policy response.

For ease of interpretation, we take the case where $\epsilon_t < 0$ and $\epsilon^*_t = 0$ and $\epsilon_{t+j} = \{\epsilon_t, 0\}$, with probability $\mu$ and $1 - \mu$ respectively so that, if capital controls can support effective monetary independence, they do so for the foreign country.\footnote{This is for ease of exposition only. The argument presented below holds more generally, so long as monetary policy can react optimally to changes in $\lambda$, as will be seen below. For simplicity also, we focus on ex-post welfare, given an $\epsilon_t$ shock.} Thus, from (24) and (25), we have $r(\lambda, \epsilon_t, 0) < 0$ and $r(\lambda, \epsilon_t, 0) \leq r^*(\lambda, \epsilon_t, 0)$. If capital controls do support effective monetary independence, then there exists a value of $\lambda = \lambda_H$ such that, for $\lambda \geq \lambda_H (< \lambda_H)$, $r^*_t = 0$ ($r^*_t = \bar{r}^*_t - [(\Omega_\sigma - \Omega)/(\Omega_\sigma + \Omega)]\bar{r}_t$).

Now, we can decompose welfare into two segments. First, for $\lambda \geq \lambda_H$, we can solve (26)-(29) to obtain inflation and output gaps when both countries are constrained by the zero lower bound. We can then substitute these into (30) to obtain the following expression for welfare as a function of natural interest rates:

$$V_{1t}(\lambda, \epsilon, 0) = -V_{Ft} - \Lambda(\bar{r}(\lambda, \epsilon_t, 0)^2 + \bar{r}^*(\lambda, \epsilon_t, 0)^2)$$

$$- \Gamma(\bar{r}(\lambda, \epsilon_t, 0) + \bar{r}^*(\lambda, \epsilon_t, 0))^2,$$

where:

$$\Lambda = \frac{1}{2} \frac{(1 + \phi)((1 - \beta \mu)^2 + \theta k(1 + \phi))}{(1 - \mu)((1 - \beta \mu) - \mu k(1 + \phi))} > 0,$$

and $\Gamma > 0$ is a function of the underlying parameter values. How does $V_{1t}$ depend on $\lambda$?

Abstracting from the $V_{Ft}$ term, take the final right-hand-side term of (38). From (24) and
(25), we see that:
\[
\bar{r}(\lambda, \epsilon, 0) + \bar{r}^*(\lambda, \epsilon, 0) = 2\rho + \frac{(1 - \mu)\phi}{\sigma + \phi} \epsilon_t.
\]
Hence, this term is independent of \(\lambda\). While capital controls do affect the response of each country’s individual natural interest rate to a demand shock, they do not affect the *average world response* of the natural interest rate.

Now turn to the middle term on the right hand side of (38). Define \(R_2 \equiv \bar{r}(\lambda, \epsilon, 0)^2 + \bar{r}^*(\lambda, \epsilon, 0)^2\). In general, this will be sensitive to the value of \(\lambda\). Using (24) and (25), we may show that:
\[
\frac{dR_2}{d\lambda} = 2\frac{(\mu - 1)^2(\lambda - 1)^2\epsilon^2\sigma}{(2\sigma\lambda - \lambda + 1)^3} < 0.
\]
This is decreasing in \(\lambda\), which implies that the second term on the right hand side of (38) is increasing in \(\lambda\). Hence, starting at \(\lambda < 1\), an increase in the degree of capital controls (fall in \(\lambda\)) unambiguously reduces welfare when both countries are constrained by the zero bound. This is not surprising since, when both countries are at the zero bound, capital controls have no compensating effect in facilitating effective monetary independence.

To see the potential benefit of capital controls, now take the case where \(\lambda < \lambda_H\) so that the foreign policy interest rate is given by part (iii) of Proposition 1. In this case, we may show that:
\[
V_{2t}(\lambda, \epsilon, 0) = -V_{Ft} - \Gamma_1 \bar{r}(\lambda, \epsilon, 0)^2,
\]
where \(\Gamma_1 > 0\) is again a function of the underlying parameter values (and independent of \(\lambda\)). Note that because \(\lambda < \lambda_H\), Proposition 1, case (iii), applies and \(V_{2t}\) is independent of \(\bar{r}^*\). The optimal foreign interest rate policy completely offsets any movements in the foreign natural interest rate. So, in the region \(\lambda < \lambda_H\), welfare depends on capital controls only to the extent that \(\lambda\) affects the home country natural interest rate.

How does (39) depend on \(\lambda\)? Again, abstracting from the \(V_{Ft}\) term, which is common to both (38) and (39), we have
\[
\frac{d\bar{r}(\lambda, \epsilon, 0)^2}{d\lambda} = -\bar{r}(\lambda, \epsilon, 0)^{\frac{\sigma(1 - \mu)\epsilon_t}{(2\lambda\sigma - \lambda + 1)^2}}.
\]
This is negative when \(\epsilon_t < 0\), implying that a rise in \(\lambda\) raises welfare, conditional on \(V_{Ft}\), from (38). So, again, more binding capital controls reduce welfare even when the capital controls are supporting effective monetary independence for the foreign country. Hence, a
tightening of capital controls reduces welfare whether policy is in the region of \( \lambda \geq \lambda_H \), and both countries are at the zero bound, or in the region \( \lambda < \lambda_H \), so that the foreign country has effective monetary independence.

But is it possible that \( V_{2t} > V_{1t} \), so that an increase in capital controls which pushes the policy environment from the first region to the second region increases welfare? The answer is no. The reason is straightforward. Because \( \lambda_H \) is defined by the condition

\[
V_{1t}(\lambda_H, \epsilon, 0) = V_{2t}(\lambda_H, \epsilon, 0),
\]

it must be that, at \( \lambda = \lambda_H \), welfare in the first region is identical to that in the second region. Hence, welfare must be continuously increasing in \( \lambda \) across both regions. So while a tightening of capital controls acts so as to facilitate effective monetary independence as we move from region 1 to region 2, it cannot increase welfare.

Putting all these arguments together, we can state the following proposition

**Proposition 3.** *An increase in capital controls reduces welfare when one or both countries are constrained by the zero lower bound, even if the increase leads one country to move out of the zero lower bound region.*

*Proof. See above argument.*

The intuition behind this result is very clear. The zero bound constraint is not a physical restriction on policy, but a policy choice. When \( \lambda \geq \lambda_H \), each country could choose a positive nominal interest rate but prefers not to, given their natural interest rates \( \tilde{r}_t \) and \( \tilde{r}_t^* \). As \( \lambda \) falls, so that capital controls become more binding, it can be advantageous for one country to move out of the zero lower bound region. This leads welfare to be higher than it would be were the monetary authority to maintain a zero interest rate. But, because it chooses interest rates optimally for every value of \( \lambda \), there is no discrete increase in welfare as policy moves out of the zero interest rate region. Essentially, the tightening of capital controls leads to an optimal policy which raises interest rates in the foreign country, precisely to offset the negative welfare effects of these controls. But the interest rate response can only ameliorate the negative welfare effect of the increased controls. It cannot overturn it. So while tightening capital controls can help to increase the effectiveness of monetary policy,
any gain from this is more than offset by the cost of reduced risk sharing.

Figure 4 illustrates the relationship between \( \lambda \) and welfare. Welfare is increasing in \( \lambda \). For \( \lambda < \lambda_H \), the slope of the relationship becomes less steep because, in this region, the foreign country is following an activist monetary rule given by case (iii) of Proposition 1. But it is still the case that overall welfare is lower for all values of \( \lambda < \lambda_H \) than for \( \lambda > \lambda_H \). Welfare is monotonically increasing in \( \lambda \), for \( \lambda < 1 \).

(Figure 4 about here)

**Suboptimal monetary policy and gains from capital controls**

Is there any case in which an increase in capital controls is associated with higher welfare due to the improved effectiveness of monetary policy? Proposition 3 establishes that this cannot be the case, so long as policy adjusts continuously to changes in \( \lambda \), for a given demand shock \( \epsilon_t \). But it could be that monetary policy does not adjust continuously to the external environment. In particular, the optimal rule in Proposition 1 requires that the foreign country follows a positive interest rate even in cases where its natural interest rate is negative. From a communications perspective, this rule may be hard to implement. Here, we briefly explore the implications of an alternative, suboptimal rule that represents a variation of Proposition 1 but rules out an activist policy when \( \tilde{r}_t^* \) is negative. Focusing on the case where \( \sigma > 1 \), consider the policy rule given by:

\[
\begin{align*}
    r_t^* &= I_t(\tilde{r}_t^* - \frac{\tilde{r}_t^* \Omega - \Omega}{\Omega + \Omega}); \\
    I_t &= 1 \quad \text{if} \quad \tilde{r}_t^* > 0; \\
    I_t &= 0 \quad \text{otherwise.}
\end{align*}
\]

Under this rule, the foreign policymaker follows an activist policy characterized by Proposition 1, but only when the natural interest is positive.

The effect of this alternative rule is that the policy interest rate is no longer continuous in the degree of capital mobility for a given \( \epsilon_t \) shock. For \( \lambda < \bar{\lambda} \), the policy rate increases discretely to the rate implied by Proposition 1. This has the implication that welfare can be improved by a small decrease in \( \lambda \) in the region of \( \bar{\lambda} \). We can establish that using the same example of Proposition 3, in the following proposition.
Proposition 4. If monetary policy follows rule (40) then there exists a value $\lambda_I$, where $\bar{\lambda} < \lambda_I < \lambda_H$, such that, for $\lambda \in (\bar{\lambda}, \lambda_I)$, a reduction in $\lambda$ to $\bar{\lambda}$ will increase welfare.

Proof. For a given $\epsilon_t$ shock, rule (40) gives the same welfare as (38) in the region $\lambda \in (\bar{\lambda}, \lambda_H)$. Thus, for $\lambda$ in this region, $V_{1t}(\lambda, \epsilon_t, 0) \leq V_{2t}(\lambda, \epsilon_t, 0)$. In addition, since $V_{1t}$ and $V_{2t}$ are continuous in $\lambda$, there must be a $\lambda_I$ such that $V_{2t}(\bar{\lambda}, \epsilon_t, 0) = V_{1t}(\lambda_I, \epsilon_t, 0)$, where $\bar{\lambda} < \lambda_I$. Because the policy rule (40) implies that (39) applies for $\lambda \leq \bar{\lambda}$, and because $V_{1t}$ is increasing in $\lambda$, it follows that there is a region of $(\lambda_L, \bar{\lambda})$, where $\lambda_L < \bar{\lambda}$, such that reducing $\lambda$ from anywhere in the region $(\bar{\lambda}, \lambda_I)$ to this region will increase welfare.

Figure 5 illustrates this case. Unlike the optimal policy rule, rule (40) implies that the policy rate stays zero until $\lambda$ falls below $\bar{\lambda}$. This means that welfare jumps discretely as $\lambda$ moves from just above to just below $\bar{\lambda}$. As a result, tightening capital controls in this region may be welfare improving. Note that, even in this case, Proposition 4 cannot be an unconditional argument for capital controls. Compared to unrestricted financial flows, the policy given by rule (40) is still inferior. Rather, the case for capital controls is limited to the argument that, for a given status quo for capital restrictions, if monetary policy is set sub-optimally, increasing capital controls can sometimes be welfare improving.

(Figure 5 about here)

VI Conclusions

The recent experience of simultaneous liquidity traps in many of the world’s major economies has raised questions about ways in which monetary policy effectiveness can be restored when monetary authorities are faced with the problem of the zero lower bound. The zero bound introduces an extra dimension to the classical policy trilemma in an open economy. This paper focuses on the transmission of shocks that cause liquidity traps across countries, and shows that the existence of capital controls can act so as to restore effective monetary policy independence when external shocks would otherwise push a country into a liquidity trap. But, despite the effectiveness of capital controls in this regard, in general they are not desirable from a welfare criterion.

We have assumed that monetary policy is purely discretionary. The effectiveness of precommitment in monetary policy at the zero bound has been an active area of debate in recent times. Woodford (2012) discusses the effectiveness of ‘forward guidance’ in US
monetary policy. In our model, optimal monetary policy with commitment would have a feature familiar from Eggertsson and Woodford (2003). The worst hit country (in the presence of capital controls) would promise to maintain a more expansionary monetary policy for a longer time after the conditions leading to the zero bound in nominal interest rates have elapsed. In this way, the effects of future monetary commitments on expectations would ameliorate the immediate appreciation of that country’s exchange rate. Nevertheless, because optimal policy under a quadratic welfare objectives will trade off losses across all times and states of the world, in general it is not the case that the ability to credibly commit to future monetary policy will completely remove the impact of contemporaneous shocks.\footnote{For instance, Eggertsson and Woodford (2003) find that it is never optimal to use forward guidance to the extent that it eliminates the contemporaneous zero bound constraint.} As a result, the qualitative features of the discretionary equilibrium solution would still obtain, even in the case of monetary policy with credible commitment.

Of course, the paper has abstracted away from many other reasons that capital controls may play a role in policy-making in open economies. In particular, if there are other distortions in financial markets that lead to excessive borrowing, capital controls may be welfare enhancing (e.g. Bianchi and Mendoza (2011); Uribe and Schmitt-Grohe (2012)). Combining these distortions with the additional problem of the zero bound on monetary policy may be an interesting avenue for future research.
Appendix

Derivation of conditions (32) and (33)

Problem P1 gives the following 6 first order conditions:

\[-\frac{1}{2}(1+\sigma)(\hat{y}_t + \hat{y}_t^*) - \phi \hat{y}_t - \psi_{1t} k \phi - \frac{1}{2}(\psi_{1t} - \psi_{2t}) k (1+\sigma) - \frac{1}{2} \psi_{3t}(1+\sigma) + \frac{1}{2} \psi_{4t}(1-\sigma) = 0, \quad (A1)\]

\[-\frac{1}{2}(1+\sigma)(\hat{y}_t + \hat{y}_t^*) - \phi \hat{y}_t^* - \psi_{2t} k \phi - \frac{1}{2}(\psi_{2t} - \psi_{1t}) k (1+\sigma) - \frac{1}{2} \psi_{4t}(1+\sigma) + \frac{1}{2} \psi_{3t}(1-\sigma) = 0, \quad (A2)\]

\[-\frac{\theta \pi_t}{k} - \psi_{1t} k = 0, \quad (A3)\]

\[-\frac{\theta \pi_t^*}{k} - \psi_{2t} k = 0, \quad (A4)\]

\[-\psi_{3t} + \gamma_{1t} = 0, \quad (A5)\]

\[-\psi_{4t} + \gamma_{2t} = 0. \quad (A6)\]

Combining (A1)-(A6) with (26)-(29), and imposing the condition that all expected future variables are expressed as \(\mu\) times their current values, gives (32) and (33) where:

\[\Omega = -\frac{(-\beta \mu + \beta \mu^2 - \mu k \phi - \mu - \mu k + 1)}{((\phi + 1)(-\theta k \phi - \theta k + \beta \mu - 1))} > 0. \quad (A8)\]

Note that when \(\sigma = (\ <, >\ )\ 1, \Omega_\sigma = (\ <, >\ ) \Omega.\]
References


Figure 1. Natural interest rates

\[ \lambda \]

\[ \tilde{\lambda} \]

\[ \tilde{r} \] - \[ \tilde{r}^{*} \]
Figure 2. Natural interest rates and optimal policy

\[
\lambda^H \quad \bar{\lambda} \quad \tilde{r} \quad \tilde{r}^* \quad \max \left\{ 0, \tilde{r}^* - \frac{\Omega - \Omega}{\Omega + \tilde{r}} \right\}
\]
Figure 3. Exchange rate response
\[ r^* = \max \{0, \bar{r}^* - \frac{\Omega_{\sigma} - \Omega}{\Omega_{\sigma} + \Omega} \} \]