consumer model. Over the 20 year period individual welfare has increased at a trend rate of only 0.4 per cent per annum. After an initial improvement in welfare between 1972 and 1975 of nearly 10 per cent, welfare levels then fell and were at levels slightly below their 1972 level between 1981 and 1983. Despite a steady increase after 1983, individual welfare levels were only 15 per cent better in 1991 than they were in 1972.

The individual consumption of goods and services index also shown in Figure 3.7 indicates that much of the recent improvement in individual welfare comes from increases in leisure. By the end of the period individual consumption levels were only slightly above what they had been 20 years earlier.

**Figure 3.7: Individual Welfare and Consumption Indexes**

![Graph showing Individual Welfare and Consumption Indexes]

*Source: Derived from Swan Consultants (Canberra) New Zealand database.*

The profitability of the private production sector is reflected in the real rates of return presented in Figure 3.8. The before-tax real rate of return averaged 2.1 per cent for the 20 year period. The highest before-tax real rate of return achieved was 5.0 per cent in 1974. However, the following year the real return plummeted to 0.6 per cent and stayed at low levels through until 1984. Since then the before-tax real return has remained in the range of 2 to 3 per cent. The post-tax real return, the return which drives investment decisions, has been far from healthy over most of the period. After starting at reasonable levels, the post-tax real return has been very low or negative for most of the period. It exceeded 3 per cent in only 1973 and 1974 before dropping to -0.9 per cent in 1975. The post-tax return remained negative through until 1983 with the exception of 1979. An increasing tax rate since 1984 has kept the post-tax real return at very low, albeit positive, levels despite the modest recovery in the before-tax return in recent years.
The real after-tax rate of return for most western countries has been found to be in the range of 3 to 5 per cent (Robbins and Robbins 1992). The average real after-tax rate of return for the United States was found to be 3.3 per cent for the period from 1954 to 1990. The average real after-tax rate of return observed for New Zealand, however, over the 20 years to 1991 was only 0.7 per cent. Clearly, the after-tax profitability of the private sector has not been good and a serious re-examination of the taxation of capital in New Zealand is warranted if New Zealand is to become attractive as a place to invest.

3.4 The New Zealand Tax System

The extensive reforms to the New Zealand tax system introduced over the last decade have been guided by three broad principles (Toder and Himes 1992):

- the imposition of a broader-based income tax with flatter rates;
- more reliance on indirect taxes; and,
- a reduction of tax-created and welfare payment poverty traps.

In reforming the income tax, the government cut the top personal income marginal tax rate from 66 to 33 per cent. The current tax structure has only two rates - 24 per cent for income up to around $31,000 and 33 per cent for income over this amount. A number of tax credits for family support and tax rebates typically introduce a tax free income amount for most taxpayers. A comprehensive fringe benefits tax was introduced to capture in-kind payments to employees. Corporate tax rates are now set equal to the top personal rate at 33 per cent and full imputation is allowed on dividends paid to shareholders to eliminate double taxation of dividends.
Withholding taxes have been introduced on interest and dividends at rates of 24 and 33 per cent, respectively. However, some forms of income remain tax-exempt, most notably capital gains except for those people classified as asset traders.

The simplification of the income tax system and reduction in marginal tax rates was facilitated by the introduction of a single-rate comprehensive value-added tax, known as the Goods and Services Tax (GST), which replaced the mish-mash of wholesale taxes existing beforehand. The GST applies at a uniform rate of 12.5 per cent and is generally regarded as one of the most ‘pure’ of its kind in the world with exports and financial transactions being about the only exemptions.

Quotas on imports which were prevalent during the late 1970s and early 1980s have been replaced with tariffs which are generally being phased down. Tariffs on many items remain high, however, by OECD standards.

Export incentives, accelerated depreciation and investment allowances, and other business investment tax incentives were generally eliminated. The reduction in the relative size of tax wedges on business investment and their absolute magnitude between 1984 and 1990 has been demonstrated by Rich (1991).

An area where the tax reforms have been less successful is the objective of reducing adverse effects on the incentive to work. While the decline in the top marginal tax rate has substantially increased the incentives to work for high income earners, effective marginal tax rates affecting labour supply decisions have not declined for lower income earners. The way social security benefits are provided has served to aggravate problems in this area.

Recent attention in the tax reform process has focused on the interface of the tax system with the rest of the world. A number of withholding taxes on international transactions have reduced the attractiveness of New Zealand as a place to invest although ad hoc processes such as the ‘Approved Issuers’ Levy’ have been put in place to remove major impediments. A recent tax treaty with Australia has removed obstacles to New Zealand companies repatriating profits from Australian investments as tax paid on profits in Australia will now be recognised by New Zealand. Previously a withholding tax of 33 per cent had to be paid on all profits repatriated to New Zealand, regardless of whether foreign tax had already been paid or not.

In spite of the impressive gains made in reforming the New Zealand tax system, some major problems remain. The most striking of these has been the rapid growth of taxation revenue as a percentage of Gross Domestic Product (GDP). As shown in Figure 3.9, New Zealand now has a higher share of taxation in GDP than Germany, the United Kingdom, Australia, Japan and the
United States. In 1966 New Zealand’s tax share of GDP at 24.7 per cent was only 1.5 percentage points higher than Australia’s and 1.2 percentage points less than that of the United States. By 1991 New Zealand’s GDP tax share of 38.2 per cent was 7.4 percentage points higher than Australia’s and 8.3 percentage points higher than that of the United States. The increase in New Zealand’s tax take was around double that for Australia and the United States.

Figure 3.9: **Tax Revenue as a Percentage of GDP — Selected OECD Countries**

![Graph showing tax revenue as a percentage of GDP for selected OECD countries from 1966 to 1990.](image)

*Source: OECD Revenue Statistics.*

In 1991 New Zealand ranked tenth out of the 24 OECD countries in terms of the tax share of GDP (Figure 3.10). However, all those with higher tax shares are high tax European countries. In 1991 income and profits taxes accounted for 58 per cent of New Zealand’s tax revenue. Although less than the OECD average of 62 per cent, this figure was still higher than that for Australia and the United Kingdom indicating that income taxes still play a leading role in New Zealand. Goods and services taxes accounted for one third of New Zealand tax revenue in 1991, 3.5 percentage points more than the OECD average. Countries where goods and services taxes played a larger part in the tax base included Norway, Finland and Ireland. Property taxes accounted for around 6 per cent of New Zealand tax collections in 1991 compared to the OECD average of 5 per cent. The remaining 2 per cent of revenue was accounted for by payroll taxes.

The changing composition of New Zealand tax revenue is shown in Figure 3.11. Over the last decade the share of income and profits taxes has fallen from around 70 per cent to 58 per cent while the importance of value-added and sales taxes has more than doubled from 10 to 22 per cent reflecting the introduction of the GST in 1986. Other taxes have remained relatively small contributors to total revenue with the importance of import duties declining and that of excise taxes fluctuating somewhat.
Figure 3.10: Composition of 1991 Tax Revenue as a Percentage of GDP — OECD Countries

Source: OECD Revenue Statistics.

Figure 3.11: Composition of New Zealand Revenue

Source: OECD Revenue Statistics.
In calculating the deadweight losses caused by taxation it is necessary to know the size of the ‘wedges’ taxes impose between the price paid by the consumer or user and the price received by the producer or supplier. These tax wedges or tax rates on producer prices were estimated for New Zealand using taxation statistics from the International Monetary Fund’s Government Finance Statistics and the OECD’s Revenue Statistics. Before this information can be used to calculate the size of the tax wedges, however, it has to be allocated to the various factors of production and commodities. The process by which this was done is explained in detail in Appendix A. The six principal tax rates derived are those applying to labour, capital, general consumption (excluding housing and transport), motor vehicles, imports and housing property.

Tax rates on labour and capital income are presented in Figure 3.12. Labour income is defined to be the value of wages and salaries paid plus a return to the self-employed to cover the opportunity cost of their time. Capital income is calculated as the profit the private sector earns from its production activities and is defined as the value of its outputs (consumption goods, investment goods, exports and sales to government) less the value of variable inputs (imports and labour including the opportunity cost of the self-employed). In allocating tax payments to the two factors of production the main task is allocating the large payments by individuals between labour and capital. To do this, use was made of information supplied by The Treasury on details of source deduction tax payments and the income base for the residual ‘other persons’ payments category.

**Figure 3.12: Tax Rates on Labour and Capital Income**

The tax rate on labour income has increased throughout the last two decades from a rate of 20 per cent in 1972 to around 32 per cent in 1991. However, most of this increase occurred in the period between 1972 and 1983 with increases since 1983 being relatively minor. Capital tax
rates on profit have fluctuated more widely due to the residual nature of profits as defined. After starting at levels similar to the labour tax rate, capital tax rates quickly increased to a very high level of 46 per cent in 1975 before generally falling back to a low point of around 15 per cent in 1984. Since then capital tax rates have again increased steadily to finish at levels similar to those applying to labour income.

Figure 3.13: Capital Tax Rate on Assets

As noted earlier, real rates of return in New Zealand have not been healthy since the mid-1970s. Consequently, to obtain a more accurate representation of capital tax rates it is necessary to look at capital tax payments relative to the value of assets. It is this tax rate which drives investment decisions. From Figure 3.13 it can be seen that capital tax rates on assets fell from an early high of 1.7 per cent in 1974 to a low of 0.9 per cent in 1984 before increasing steadily to a very high rate of 2.1 per cent 1990. This illustrates that once the variability of profit and low rates of return are netted out in looking at the more stable and more important capital tax rate on assets series, changes to the New Zealand tax system since 1974 have fallen heavily on capital with tax rates more than doubling. In the same period labour tax rates increased by only one sixteenth. However, recent reforms have been aimed at easing some of the high capital tax rates, particularly by changing some withholding tax arrangements. The capital tax rate on assets fell from 2.1 per cent in 1990 to 1.8 per cent in 1991.

The increasing importance of indirect taxes is again illustrated in Figure 3.14 where the tax rate in terms of producer’s prices on general consumption (excluding housing and transport) can be seen to have increased from around 11 per cent in 1972 to 32 per cent in 1991. Most of this increase has occurred since 1986 with the introduction of the Goods and Services Tax.
The producer price tax rate for an important consumer item, motor vehicles, can be seen to have changed dramatically over the last two decades. After starting at high levels in excess of 50 per cent in 1972 and increasing to a massive 83 per cent in 1978, the tax rate was halved in 1980 and has since been reduced further to end up at 13 per cent in 1991.

Finally, import duty rates and housing property tax rates are presented in Figure 3.15. The average import duty rate declined between 1972 and 1981 from 5.1 to 3.1 per cent. However, this marked a time when greater use was being made of import quotas to protect domestic industry. With the reforms progressively implemented from the early 1980s quotas were replaced initially by tariffs and then phased down. Consequently, average import duty rates again increased from 1981 to 1988 to peak at 5.7 per cent before falling away sharply after 1988 as the economy was opened up to international competition. The average import duty rate in 1991 was 2.6 per cent.
Housing property tax rates have increased steadily throughout the last two decades from 0.6 per cent in 1972 to 1.1 per cent in 1991. This is in line with experience in most OECD countries with the move to higher levels of cost recovery and user-pays pricing for services provided by local government.

3.5 Government Expenditure

While reform of the New Zealand tax system over the last decade has been impressive, it represents only one side of the government budget. Some beneficial reforms have also been made to government expenditure. However, major problem areas remain. The analysis of government expenditure and the fiscal position is complicated by the range of accounting conventions which have been adopted over the years and the tendency in recent years to remove some items from the government balance sheet as the public sector has been restructured and state owned enterprises have been privatised and corporatised (New Zealand Business Roundtable 1990). The composition of government expenditure expressed as a proportion of GDP using consistent data supplied by The Treasury is presented in Figure 3.16.

The most notable areas of reform in government expenditure have been the reduced expenditure on industry development and reduced debt servicing expenditure. Industry development expenditure fell from a peak of 5.4 per cent of GDP in 1984 to 1.2 per cent of GDP in 1992 as assistance levels to industry were reduced and the economy was exposed more to international competition. Debt servicing expenditure steadily increased from 3.9 per cent of GDP in 1980 to 8.0 per cent in 1988. It has since fallen back to 5.4 per cent although, as noted above, care should be exercised in interpreting debt servicing figures with recent rearrangements within the public sector.

Figure 3.16: Composition of Government Expenditure as a Percentage of GDP

![Graph showing government expenditure as a percentage of GDP](image)

*Source: The Treasury.*

MARGINAL COSTS OF TAXATION IN NEW ZEALAND
The largest single item of government expenditure remains social services and the proportion of GDP accounted for by this item has increased steadily until recently. Government expenditure on social services accounted for 11 per cent of GDP in 1980 and increased sharply between 1987 and 1991 to peak at 14.8 per cent before falling back to 13.7 per cent in 1992. Of major concern though is not only the volume of expenditure on social services but also the form of the expenditure and its effects on incentives. Generous benefits are paid to low income earners and these allow other income to be earned up to a threshold level from which time benefits are phased out as more non-benefit income is received. This leads to high effective marginal tax rates for most low income earners. In many cases the effective marginal tax rate is around 100 per cent up to incomes of $20,000. An example of an effective marginal tax rate schedule for a welfare recipient is presented in Figure 3.17. Clearly, when effective marginal tax rates are so high there is little reason for recipients to seek employment and contribute more to supporting themselves. This will have an adverse impact on economic performance as potential output from these individuals is forgone and bad demonstration effects reduce the incentives for those in the workforce to work harder.

**Figure 3.17: Effective Marginal Tax Rates for Sole Parent with Three Children Receiving Domestic Purposes Benefit and Family Support**

 throughout most of the 1980s there has also been a continuing imbalance between government expenditure and taxation revenue. Treasury figures for the central government in Figure 3.18 indicate that net expenditure has exceeded total taxation revenue for all years since 1980 except for 1990. This has been a significant contributing factor to the growth in New Zealand’s foreign debt. In 1992 total net foreign debt stood at around 80 per cent of GDP. Net public debt stood at around 55 per cent of GDP in 1992-93 (Richardson 1992). This points to the need for
greater discipline in restraining government expenditure so that debt levels can be reduced, paving the way for a sustainable reduction in taxation levels.

Figure 3.18: The Gap between Central Government Net Expenditure and Taxation

Source: The Treasury.
4. ALTERNATIVE MODELS OF MARGINAL EXCESS BURDEN

4.1 Introduction

Recall that in Chapter 2 above, we gave a diagrammatic and algebraic exposition of the partial equilibrium approach to measuring the marginal excess burden of a tax and expenditure increase.\(^1\) This literature on excess burdens makes the following major point: since additional government expenditures have to be financed by raising taxes and since tax wedges generally distort choices of consumers and producers away from an efficient allocation of resources, the additional loss of efficiency due to the raising of a tax rate should be added to the monetary costs of the additional government spending. Thus a government project should earn a rate of return that is sufficiently high to cover the additional excess burden that is created by raising taxes.

Browning (1987) noted that a wide variety of estimates for the welfare costs of additional government spending have been obtained due to uncertainty over the magnitude of various elasticities of supply and demand. However, Stuart (1984), Ballard (1990) and Fullerton (1991) have all noted that another important source of differences in estimates of marginal excess burdens is due to differences in assumptions. Thus in this chapter we shall lay out our assumptions in some detail in the context of a highly simplified general equilibrium model of an economy. We shall make two sets of assumptions which lead to two alternative concepts of the marginal excess burden of a tax increase. We shall use the second concept in our model of the New Zealand economy that will be explained in Chapters 5, 6 and 7 below.

In Section 4.2 below, we consider an idealised planned economy where an optimal allocation of resources can be attained without tax instruments. This model is not presented for its realism, but to introduce our assumptions on consumers and producers and to illustrate what a first best allocation of resources looks like.

In Section 4.3, we introduce taxes and a decentralised market economy. Our first concept of marginal excess burden is introduced here.

In Section 4.4, we introduce our second concept of marginal excess burden which follows the example of Kay and Keen (1988) and uses a variant of Debreu's (1951) (1954) coefficient of

---
resource utilisation to measure the excess burden of a tax increase.\textsuperscript{2} In this second concept, as a tax rate increases, consumers are given an offsetting transfer which keeps them at the same level of real income. Thus, our second concept of marginal excess burden can be viewed as a rigorous general equilibrium specification of the original Harberger (1964) – Browning (1976) marginal excess burden measure. Section 4.5 concludes with a nonmathematical summary of this chapter.

4.2 The Optimal Allocation of Resources

We consider a very simple model of a closed economy. There are three goods in the economy: (i) a consumption good $C$; (ii) labour $L$ or leisure $h = H - L$ (where $H$ is total hours potentially available for work in the period under consideration) and (iii) a fixed factor $K$ (an aggregate of land and capital). There are three sectors in the economy: (i) a household sector that demands consumer goods and supplies labour; (ii) a private production sector that produces a composite good that is consumed both by consumers and the government and uses labour $L_P$ as an input and (iii) a government sector that consumes goods $G$ and uses the amount of labour $L_G$ to produce general government services.

The technology of the private production sector can be represented by a production function $f$ where

\begin{equation}
C + G = f(L_P, K).
\end{equation}

$C + G$ is the total output produced given that $L_P$ units of labour are used.

The preferences of the household sector are represented by the utility function $U$. The utility level achieved \( u \) depends on consumption $C$ and leisure $h$ where

\begin{equation}
U = U(C, h);
\end{equation}

\begin{equation}
h = H - L_P - L_G,
\end{equation}

so that total labour supply is $L = L_P + L_G$.

The maximum level of welfare that is achievable in this economy can be obtained by maximising utility, $U(C, H - L_P - L_G)$, subject to the production function constraint, $C + G = F(L_P, K)$, with respect to consumption $C$ and privately utilised labour supply $L_P$. The government requirements for goods $G$ and labour $L_G$ are held fixed. Upon substituting the

\textsuperscript{2} Debreu's work was preceded by that of Allais (1943) (1977). The loss measures of Allais and Debreu were put in a unified framework by Diewert (1983) (1984).
production function constraint into the utility function, our welfare maximisation problem reduces to:

\[
\max_{L_p} \ U \left( f(L_p, K) - G, H - L_G - L_p \right).
\]

The first order necessary condition for solving (4) is:

\[
U_C f_L + U_h(-1) = 0.
\]

Thus at the optimal solution, we will have

\[
f_L \left( L_p, K \right) = \frac{U_h \left( C^*, h^* \right)}{U_C \left( C^*, h^* \right)}.
\]

Equation (6) implies that the slope of the production function will equal the slope of the consumer's indifference curve at the optimum.

The geometry of problem (4) is illustrated by Figure 4.1.

**Figure 4.1: An optimal situation**

The distance \( OA \) is equal to \( LG \), the government's labour requirements. The total output that can be produced by the economy is the curve \( AB \), the production function constraint. The distance \( FE = AC \) is the government's goods requirement \( G \). The curve \( CD \) is \( AB \) shifted down by \( G \).
The indifference curve $II$ is the highest one that is tangent to $CD$ and thus the consumer’s equilibrium point is $E$ and the producer’s equilibrium point is at $F$. Note that the slope of the line tangent to $F$ is equal to the slope of the line tangent to $E$; this is the geometry behind condition (6) above.

Of course, in real life economies, the government expenditures on goods, $G$, and on labour, $LG$, must be financed by taxes or user charges. Thus in the following Section, we introduce taxes into the above model.

### 4.3 Marginal excess burdens: A first approach

In order to model a tax distorted equilibrium, we need to introduce the following tax rates: $t_1$ is the rate of taxation on consumption goods, $t_2$ is the rate of taxation on labour and $t_3$ is the rate of taxation on the fixed factor. We denote the producer price for the consumption good by $p_1$ and the producer price for labour by $p_2$. The consumer prices for these two goods are $p_1(1 + t_1)$ and $p_2(1 - t_2)$ respectively.

In order to obtain the equations which characterise a tax distorted equilibrium, it is convenient to use duality theory. Thus we assume that the expenditure function dual to the utility function $U(C, h)$ is $e(p_1(1 + t_1), p_2(1 - t_2), u)$ and the profit function dual to the production function $f(L_p, K)$ is $\pi(p_1, p_2, K) = \pi(p_1, p_2)$ where we have dropped $K$ since it is held fixed.

The equations which define a tax distorted equilibrium are (7) – (10) below.

\[
\begin{align*}
(7) & \quad e_1[p_1(1 + t_1), p_2(1 - t_2), u] = \pi_1(p_1, p_2) - G \\
(8) & \quad e_2[p_1(1 + t_1), p_2(1 - t_2), u] = \pi_2(p_1, p_2) + H - LG \\
(9) & \quad e[p_1(1 + t_1), p_2(1 - t_2), u] = (1 - t_3)\pi(p_1, p_2) + p_2(1 - t_2)H \\
(10) & \quad t_1p_1e_1[p_1(1 + t_1), p_2(1 - t_2), u] + t_2p_2[H - e_2[p_1(1 + t_1), p_2(1 - t_2), u]] + t_3\pi(p_1, p_2) = p_1G + p_2LG
\end{align*}
\]

Equations (7) and (8) are the demand equals supply equations for goods and labour, respectively; (9) is the household budget constraint and (10) is the government’s budget constraint. Differentiation of a function with respect to its $i$th variable is denoted by a subscript $i$.

---

3 For expositions of duality theory, see Diewert (1974) (1993; Ch. 6) or Varian (1984; Ch. 1 and 3).
As is usual in general equilibrium theory, not all four equations (7) — (10) are independent. Hence we drop equation (10). We also require a normalisation on prices. We choose the producer price of labour \( p_2 \) to be the numeraire good and hence, we have

\[
(11) \quad p_2 = 1.
\]

Equations (7) — (10) can now be regarded as 3 simultaneous equations in 6 unknowns: \( u \) (the level of household utility), \( G \) (the level of government expenditure), \( p_1 \) (the price of the consumer good), \( t_1 \) (the tax rate on consumer goods), \( t_2 \) (the tax rate on labour) and \( t_3 \) (the tax rate on capital).

We regard \( u, G \) and \( p_1 \) as endogenous variables and the tax rates \( t_1, t_2 \) and \( t_3 \) as exogenous variables. Thus equations (7) — (9) determine the functions \( u(t_1, t_2, t_3) \), \( G(t_1, t_2, t_3) \) and \( p_1(t_1, t_2, t_3) \).

In order to determine the overall effects of a tax increase, we have to aggregate the consumer’s change in utility with the change in government real expenditures. One way of aggregating utility \( u \) and real government expenditures on goods \( G \) is by assuming that an overall social welfare function that is additive in the two components exists.\(^4\) Thus, define the following money metric\(^5\) welfare indicator \( W \) as follows:

\[
(12) \quad W(u, G, P_1, P_2) = e\left(P_1, P_2, u\right) + P_1 G + P_2 L_G
\]

where \( P_1 = (1 + t_1) p_1 \) and \( P_2 = (1 - t_2) p_2 \) are reference consumer prices for the consumption good and labour (or leisure), respectively. Thus, we measure the overall welfare of the representative consumer by private (per capita) expenditures on goods and leisure, \( e\left(P_1, P_2, u\right) \) plus (per capita) government expenditures on goods, \( P_1 G \), plus (per capita) government expenditures on labour, \( P_2 L_G \), where all expenditures are evaluated at the reference prices \( P_1 \) and \( P_2 \). As utility \( u \) and government expenditures \( G \) are changed due to the change in the tax rates \( t_1, t_2 \) or \( t_3 \), we hold the reference prices \( P_1 \) and \( P_2 \) constant in (12).

To determine welfare as a function of the tax rates \( t_1, t_2 \) and \( t_3 \), we simply substitute our solution functions \( u(t_1, t_2, t_3) \) and \( G(t_1, t_2, t_3) \) to the system of equations (7) — (9) into (12) to obtain the welfare function \( W^*(t_1, t_2, t_3) \):

\[
(13) \quad W^*(t_1, t_2, t_3) = e\left(P_1, P_2, u(t_1, t_2, t_3)\right) + P_1 G(t_1, t_2, t_3) + P_2 L_G.
\]

\(^4\) This type of assumption was used by Atkinson and Stern (1974; 174).

\(^5\) The term money metric scaling is due to Samuelson (1974; 1262).
In order to calculate the marginal excess burden of a tax increase in \( t_i \), \( EB_i \), we simply calculate minus the change in welfare due to the increase in \( t_i \) and divide by the change in real government spending on goods; i.e., define \( EB_i \) as follows for \( i=1,2,3 \):

\[
(14) \quad EB_i = -\left[ \frac{\partial W^*}{\partial t_i} \right] / \left[ \frac{\partial G(t_1, t_2, t_3)}{\partial t_i} \right]
\]

\[
(15) \quad = -\left[ e_u(P_1, P_2, u)u_i(t_1, t_2, t_3) + P_1 G_1(t_1, t_2, t_3) \right] / G_i(t_1, t_2, t_3)
\]

where \( e_u \) denotes the partial derivative of \( e(P_1, P_2, u) \) with respect to \( u \), \( G_i \) denotes the partial derivative of \( G(t_1, t_2, t_3) \) with respect to \( t_i \) and \( u_i \) denotes the partial derivative of \( u(t_1, t_2, t_3) \) with respect to \( t_i \) for \( i=1,2,3 \). Note that \( EB_2 \) defined by (14) is a general equilibrium counterpart to the partial equilibrium excess burden measure \( MEB(t_i) \) defined earlier by equation (3) in Section 2.1 above.

The marginal excess burden measures, \( EB_i \) defined by (14), can be illustrated by means of Figure 4.2. The initial producer equilibrium point is at the point \( F \) on the private sector production possibilities frontier \( AB \). The government's labour requirements are \( OA \) and the government's initial goods requirements \( G_1 \) are \( FE \). The initial consumer equilibrium is at the point \( E \) which is on the indifference curve \( I_1I_1 \) which corresponds to the initial utility level \( u_1 \).

**Figure 4.2: Marginal excess burden**
The tax rate $t_1$ on the consumer good is increased. This shifts the consumer's budget line down and the new consumer equilibrium is at the point $K$ on the indifference curve $I_2I_2$ which corresponds to the lower utility level $u_2'$. The new producer equilibrium is at the point $J$ and the government's new consumption of goods $G_2$ is the distance $JK$. Measuring utility in terms of the consumption good, it can be seen that the decrease in utility $u_1' - u_2'$ is equal to the distance $EH$. From the diagram, the distance $JK$ is less than the distance $FH$. This corresponds to the inequality $G_2 < G_1 + u_1' - u_2'$ or $u_2' + G_2 < u_1' + G_1$. Thus, as the tax rate on consumption goods $t_1$ increases, government spending $G$ increases but utility $u$ decreases and overall welfare measured in units of the consumer good, $u+G$, decreases. This welfare decrease divided by the government expenditure increase corresponds to the marginal excess burden measure $EB_1$ defined by (14).

In the remainder of this Section, we shall compute the partial derivatives which occur in the right hand side of (15), which defines the excess burden measures $EB_i$. Unfortunately, the computations are rather long and complex and the reader who is not interested in the details is advised to skip to the end of this Section.

In order to calculate the partial derivatives $u_i(t_1, t_2, t_3)$ and $G_i(t_1, t_2, t_3)$, it is necessary to totally differentiate equations (7) — (9) above with respect to the endogenous variables $G$, $u$ and $P_i$ and the exogenous tax variables $t_1$, $t_2$ and $t_3$. Using matrix notation, the resulting system of equations is:

$$
\begin{bmatrix}
1, & e_{1u}, & e_{11}(1 + t_1) - \pi_{11} & dG_i \\
0, & e_{2u}, & e_{21}(1 + t_1) - \pi_{21} & du \\
0, & e_u, & e_1(1 + t_1) - (1 - t_3)\pi_1 & dp_1
\end{bmatrix}
= 
\begin{bmatrix}
-e_{11}p_1, & e_{12}p_2, & 0 & dt_1 \\
-e_{21}p_1, & e_{22}p_2, & 0 & dt_2 \\
-e_1p_1, & -[H - e_2]p_2, & -\pi & dt_3
\end{bmatrix}
$$

where $e_i \equiv \partial e(P_1, P_2, u) / \partial P_i$ and $e_{ij} \equiv \partial^2 e(P_1, P_2, u) / \partial P_i \partial P_j$ are the first and second order partial derivatives of the expenditure function $e(P_1, P_2, u)$ with respect to the consumer prices $P_i$ and $P_j$ for $i, j = 1, 2$; $e_{1u} \equiv \partial^2 e(P_1, P_2, u) / \partial P_i \partial u$ is the cross partial derivative with respect to $P_i$ and $u$ for $i = 1, 2$ and $\pi_i \equiv \partial \pi(P_1, P_2) / \partial P_i$ and $\pi_{ij} \equiv \partial^2 \pi(P_1, P_2) / \partial P_i \partial P_j$ are
first and second order partial derivatives of the producer's profit function \( \pi(P_1, P_2) \) with respect to the producer prices \( P_i \) and \( P_j \) for \( i, j = 1, 2 \).

From duality theory, the first derivatives of the profit and expenditure functions with respect to prices are equal to net supply functions and (Hicksian) consumer demand functions, respectively. Thus, using the notation developed in Section 4.2 above, we have:

\[
(17) \quad e_1 = C; \quad e_2 = L; \quad \pi_1 = C + G; \quad \pi_2 = -L_p.
\]

Moreover, the second order derivatives \( e_{iu}, e_{ij}, \) and \( \pi_{ij} \) satisfy the following restrictions:

\[
(18) \quad P_1 e_{iu}(P_1, P_2, u) + P_2 e_{2u}(P_1, P_2, u) = e_u(P_1, P_2, u);
\]
\[
(19) \quad P_1 e_{11}(P_1, P_2, u) + P_2 e_{21}(P_1, P_2, u) = 0;
\]
\[
(20) \quad P_1 e_{12}(P_1, P_2, u) + P_2 e_{22}(P_1, P_2, u) = 0;
\]
\[
(21) \quad p_1 \pi_{12}(P_1, P_2) + p_2 \pi_{21}(P_1, P_2) = 0;
\]
\[
(22) \quad p_1 \pi_{12}(P_1, P_2) + p_2 \pi_{22}(P_1, P_2) = 0.
\]

The \( e_{ij} \) also satisfy the Hicks (1946) Samuelson (1947) symmetry conditions

\[
(23) \quad e_{12} = e_{21}
\]

and the \( \pi_{ij} \) also satisfy Hotelling's (1932) symmetry conditions:

\[
(24) \quad \pi_{12} = \pi_{21}.
\]

We also have the following sign restrictions on the second order partial derivatives of \( e \) and \( \Pi \):\(^7\)

\[
(25) \quad e_{11} \leq 0, \quad e_{22} \leq 0 \quad \text{and} \quad e_{12} = e_{21} \geq 0;
\]
\[
(26) \quad \pi_{11} \geq 0, \quad \pi_{22} \geq 0 \quad \text{and} \quad \pi_{12} = \pi_{21} \leq 0.
\]

Moreover, if any one of the inequalities in (25) is strict, then they are all strict and if any one of the inequalities in (26) is strict, then they are all strict.

The derivative formula that is obtained by simply inverting the matrix on the left hand side of (16) is very difficult to interpret. Thus we have found it more instructive to calculate the

\[^6\] See Diewert (1974; 112 and 137) (1993; 131 and 166) for example.

\[^7\] For (25), see Hicks (1946; 311), Samuelson (1947; 64-69) or Diewert (1993; 149). For (26), see Hicks (1946; 321), Samuelson (1953; 10) or Diewert (1974; 143-45).
derivatives of \( u(t_1, t_2, t_3) \) and \( G(t_1, t_2, t_3) \) that appear in (15) in a two step procedure. In the first step, take \( P_1 \) times the first equation in (16) and add it to \( P_2 \) times the second equation in (16). Using (18) - (21), we obtain the following equation:

\[
P_1 dG + e_u du - \left[ P_1 \pi_{11} - P_2 \pi_{21} \right] dp_1 = 0 dt_1 + 0 dt_2 + 0 dt_3.
\]

Recall that the \( P_i \) (the prices that consumers face) are related to the \( p_i \) (the prices that producers face) by the following equations:

\[
P_1 = p_1 (1 + t_1); \quad P_2 = p_2 (1 - t_2)
\]

where \( t_1 \) is the consumption tax rate and \( t_2 \) is the labour tax rate. Using (28), we have:

\[
P_1 \pi_{11} + P_2 \pi_{21} = p_1 (1 + t_1) \pi_{11} + p_2 (1 - t_2) \pi_{21}
\]

\[
= p_1 t_1 \pi_{11} - p_2 t_2 \pi_{21} \quad \text{using (21)}
\]

\[
= (t_1 + t_2) p_1 \pi_{11} \quad \text{using (21) again.}
\]

Substitution of (29) into (27) yields for \( i = 1, 2, 3 \):

\[
P_i \partial G / \partial t_i + e_u \partial u / \partial t_i = (t_1 + t_2) p_1 \pi_{11} dp_1 / \partial t_i.
\]

Note that the left hand side of (30) is (minus) the numerator of the excess burden measure \( EB_i \) defined by (15) above.

We can also use the last equation in (16) in order to eliminate the terms \( \partial u / \partial t_i \) from equations (30). Using also equations (17), we obtain the following expressions for \( \partial G / \partial t_i \):

\[
P_1 \partial G / \partial t_1 = p_1 C + \left[ (t_1 + t_3) C - (1 - t_3) G + (t_1 + t_2) p_1 \pi_{11} \right] dp_1 / \partial t_1;
\]

\[
P_1 \partial G / \partial t_2 = p_2 L + \left[ (t_1 + t_3) C - (1 - t_3) G + (t_1 + t_2) p_1 \pi_{11} \right] dp_1 / \partial t_2;
\]

\[
P_1 \partial G / \partial t_3 = \pi + \left[ (t_1 + t_3) C - (1 - t_3) G + (t_1 + t_2) p_1 \pi_{11} \right] dp_1 / \partial t_3.
\]

For small tax rates \( t_i \) and small government expenditures on goods \( G \), the first terms on the right hand sides of (31), (32), and (33) which are large and positive, will dominate the remaining terms and hence \( G \) will increase as a tax rate \( t_i \) increases; i.e., the derivatives \( \partial G / \partial t_i \) will have the expected positive signs. Thus the denominator derivatives, \( G_i = \partial G(t_1, t_2, t_3) / \partial t_i \) in (15), will tend to be positive. However, the signs of the numerator derivatives, \( -[e_u \partial u / \partial t_i + P_1 \partial G / \partial t_i] \) in (15), will be equal to the sign of
\[-(t_1 + t_2)p_1 \pi_{11} \partial p_1 / \partial t_1\] in view of (30). Since \(t_1, t_2, p_1\) and \(\pi_{11}\) will all generally be positive, we see that the sign of \(EB_i\) will generally be opposite to the sign of \(\partial p_1 / \partial t_1\). Thus in order to calculate the excess burden measures \(EB_i\), we need only use the last two equations in (16) to calculate the derivatives \(\partial p_1 / \partial t_i\) for \(i = 1, 2, 3\). If increasing a tax rate \(t_i\) decreases the producer price of output (relative to the wage rate which we are holding constant), then \(EB_i\) will have the expected positive sign; i.e., increasing \(t_1\) to finance additional government expenditures will lead to a decrease in the producer price of output which in turn will lead to a drop in output and utility. The decrease in utility will be sufficiently large to outweigh the increase in government consumption of goods and overall welfare will drop.

Using the last two equations in (16) to calculate the response of the output price \(p_1\) to changes in the tax rates \(t_i\) leads to the following equations:

(34) \[
\partial p_1(t_1, t_2, t_3) / \partial t_1 = \left[e_u e_{21} - Ce_{2u}\right] p_1 / D;
\]

(35) \[
\partial p_1(t_1, t_2, t_3) / \partial t_2 = -\left[e_u e_{22} + Le_{2u}\right] p_2 / D;
\]

(36) \[
\partial p_1(t_1, t_2, t_3) / \partial t_3 = -e_{2u} \pi / D
\]

where total labour supply is \(L = L_p + L_g\) and \(D\) is defined as:

(37) \[D = e_{2u} \left[C(1 + t_1) - (1 - t_3)(C + G)\right] - e_u \left[e_{21}(1 + t_1) - \pi_{21}\right].\]

It is interesting to note the key role played by \(e_{2u} = \partial^2 e(P, P_2, u) / \partial P_2 \partial u\), which is the response of leisure demand to an increase in real income \(u\). If \(e_{2u} = 0\) and the strict inequalities hold in (25) and (26), then \(D < 0\), \(\partial p_1 / \partial t_1 < 0\), \(\partial p_1 / \partial t_2 < 0\) and \(\partial p_1 / \partial t_3 = 0\). Thus in this case, overall welfare will decrease as we increase the tax rate on consumption \(t_1\) and the tax rate on labour earnings \(t_2\) and it will remain unchanged as we increase the tax rate \(t_3\) on the fixed (in this model) factor capital. However, if \(e_{2u}\) is sufficiently large and positive and \(D\) remains negative, then we can obtain the rather anomalous results \(\partial p_1 / \partial t_1 > 0\), \(\partial p_1 / \partial t_2 > 0\) and \(\partial p_1 / \partial t_3 > 0\) which means that overall welfare will increase as we increase \(t_1\), \(t_2\) and \(t_3\). Assuming also that \(\partial G / \partial t_i\) is positive for each \(i\), we find under the above conditions that the marginal excess burden measures \(EB_i\) defined by (14) become negative so that there is an excess benefit instead of an excess burden associated with the tax increase. An intuitive explanation for this rather anomalous result can be

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8 D is the determinant of the two by two submatrix involving \(du\) and \(dp_1\) of the matrix on the left hand side of (16).
made as follows: the tax increase leads to a fall in real income or utility, the fact that $e_{2u}$ is large leads to a large drop in the demand for leisure which in turn leads to a large increase in the supply of labour which leads to a large enough increase in output such that the increase in $G$ outweighs the decline in $u$.\(^9\)

The analysis in this Section can be summarised as follows: the marginal excess burden measures $EB_i$ associated with an increase in the tax rate $t_i$ were defined by (15). The derivatives in the numerators of (15) can be expressed in terms of initial tax rates $t_i$, the initial allocation of resources (C, h, L and G), the responses of (Hicksian) consumer demands to changes in prices $e_{ij}$, and the responses of producer net supply functions to changes in prices $\pi_{ij}$, using equations (30) and (35) — (37). Similar formulae for the derivatives of (15) can be obtained using (31) — (33) and (34) — (37). The resulting formulae for the marginal excess burdens are rather complex to say the least.

In addition to complexity, there is another major difficulty with the above approach to measuring excess burdens. The difficulty is that our method for measuring total welfare $W$ defined by (12) by summing together consumer expenditures on private goods with expenditures by governments on goods is rather arbitrary. The problem becomes even more acute in a many consumer context since there is no universally accepted metric for aggregating changes in private utility with changes in government expenditures in order to obtain a measure of overall welfare change.\(^10\) Thus, in the next section we pursue an approach to measuring the excess burdens due to tax increases which avoids this measurement problem.

### 4.4 Marginal excess burdens: A second approach

Our second approach to measuring the excess burden of a tax increase is based on the approach to efficiency measurement pioneered by Allais (1943) (1977) and Debreu (1951) (1954).\(^11\) In order to avoid the problem of adding together a utility change with a change in public good production, we hold each consumer’s utility constant as a tax rate is increased.

In the context of our representative consumer model described in Section 4.3 above, utility $u$ is held constant by adding a transfer payment $T$ to the consumer’s income. The endogenous

---

\(^9\) Our intuitive explanation for the existence of marginal excess benefits follows that of Fullerton (1991; 305). For some values of the parameters in their applied general equilibrium models, Hansson and Stuart (1985; 333) and Ballard (1990; 269) found negative marginal excess burdens or positive excess benefits.

\(^10\) This point is made rather effectively in Kay and Keen (1988; 268).

variables in our simple general equilibrium model become $G$ (government expenditures on goods), $T$ (the transfer) and $p_1$ (the producer price of output). The numeraire good is again labour and the producer price of labour $p_2$ is held constant. The exogenous variables are again $t_1, t_2$ and $t_3$, the tax rates on consumption, labour earnings and profits, respectively. The new system of equations which describes our model is given by (7) and (8) (the demand equals supply equations for goods and labour) and equations (38) and (39) below:

\begin{align}
(38) & \quad e\left[p_1(1 + t_1), p_2(1 - t_2), u\right] = (1 + t_3) \Pi(p_1, p_2) + p_2(1 - t_2)H + T; \\
(39) & \quad t_1p_1e\left[p_1(1 + t_1), p_2(1 - t_2), u\right] + t_2p_2e\left[p_1(1 + t_1), p_2(1 - t_2), u\right] + t_3\pi(p_1, p_2) = p_1G + p_2L_G + T.
\end{align}

Equation (38) is the consumer’s budget constraint and (39) is the government budget constraint.

As is usual in general equilibrium theory, the four equations (7), (8), (38) and (39) are dependent. We drop (39) and use the remaining equations to solve for $G$, $T$, and $p_1$ as functions of the tax rates $t_1, t_2$ and $t_3$. The partial derivatives of these solution functions can be obtained by totally differentiating (7), (8) and (38) with respect to $G$, $T$, $p_1$, $t_1$, $t_2$ and $t_3$. Using matrix notation, the resulting equations may be written as follows:

\begin{align}
(40) & \quad \begin{bmatrix} 1, & 0, & e_{11}(1 + t_1) - \pi_{11} \\
0, & 0, & e_{21}(1 + t_1) - \pi_{21} \\
0, & -1, & C(1 + t_1) - (1 - t_3)(C + G) \end{bmatrix} \begin{bmatrix} dG \\
dT \\
dp_1 \end{bmatrix} \\
& \quad = \begin{bmatrix} -e_{11}p_1, & e_{12}p_2, & 0 \\
-e_{21}p_1, & e_{22}p_2, & 0 \\
-Cp_1, & -Lp_2, & -\pi \end{bmatrix} \begin{bmatrix} dt_1 \\
dt_2 \\
dt_3 \end{bmatrix}
\end{align}

where we have also used (17) in evaluating the derivatives in (40).

We turn now to the problem of defining marginal excess burdens in our present model. Since utility remains constant, any benefits that an increase in taxation might generate are equal to the change in government purchases of goods $G$, valued at the initial consumer price of goods $P_1$. Thus our indicator of overall welfare in the present model is simply

\begin{align}
(41) & \quad W(t_1, t_2, t_3) = P_1G(t_1, t_2, t_3)
\end{align}
where \( G(t_1, t_2, t_3) \) (along with \( T(t_1, t_2, t_3) \), \( p_1(t_1, t_2, t_3) \)) are the functions obtained by solving (7), (8) and (38). From Equation (39), we see that the government revenue raised, \( R \), is equal to government expenditures on goods, \( p_1 G \), and labour, \( p_2 L_G \), plus government transfers to consumers, \( T \). Thus we can define tax revenue as a function of the tax rates \( t_1, t_2 \) and \( t_3 \) as follows:

\[
R(t_1, t_2, t_3) = p_1(t_1, t_2, t_3)G(t_1, t_2, t_3) + p_2L_G + T(t_1, t_2, t_3).
\]

Our general equilibrium measure of the marginal excess burden associated with increasing the tax rate \( t_i \), \( MEB_i \), can now be defined as (minus) the rate of change in welfare defined by (41) divided by the rate of change in revenue defined by (42) with respect to \( t_i \); i.e., for \( i = 1, 2, 3 \):

\[
MEB_i = -\left[ \frac{\partial W(t_1, t_2, t_3)}{\partial t_i} / \frac{\partial R(t_1, t_2, t_3)}{\partial t_i} \right] = -\frac{W_i(t_1, t_2, t_3)}{R_i(t_1, t_2, t_3)}
\]

\[
= -\frac{p_1 G_i(t_1, t_2, t_3)}{[p_1(t_1, t_2, t_3)G(t_1, t_2, t_3)} + T_i(t_1, t_2, t_3)]
\]

where \( W_i, R_i, G_i, p_1 \) and \( T_i \) are the partial derivatives of the functions \( W(t_1, t_2, t_3) \), \( T(t_1, t_2, t_3) \) with respect to \( t_i \) for \( i = 1, 2, 3 \). There will be an excess burden associated with increasing \( t_i \) if \( MEB_i \) is positive, an excess benefit if \( MEB_i \) is negative.

The derivatives \( G_i(t_1, t_2, t_3) \) that appear in (44) can be obtained by inverting the matrix on the left hand side of (40) but we shall follow the two step procedure that was used in the previous section in order to calculate these derivatives. Taking \( P_1 \) times the first equation in (40) and \( P_2 \) times the second equation in (40) (and using (18) — (20)) yields the following equation:

\[
P_1 dG + 0dT = \left[ P_1 \pi_{11} + P_2 \pi_{21} \right] dp_1 + 0dt_1 + 0dt_2 + 0dt_3.
\]

Substitution of (21) into (45) yields the following equations:

\[
P_1 G_i(t_1, t_2, t_3) = (t_1 + t_2)p_1 \pi_{11} p_i(t_1, t_2, t_3); \quad i = 1, 2, 3.
\]

Using the second equation in (40), it is very easy to solve for the partial derivatives \( p_1(t_1, t_2, t_3) \) of \( \partial p_1(t_1, t_2, t_3) / \partial t_i \):

\[
\frac{\partial p_1(t_1, t_2, t_3)}{\partial t_i} = \left[ e_{21} \left( 1 + t_1 \right) - \pi_{21} \right]^{-1} e_{21} p_1 \leq 0;
\]

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\[ \frac{\partial p_1(t_1, t_2, t_3)}{\partial t_2} = \left[ e_{21}(1 + t_1) - \pi_{21} \right]^{-1} e_{22} p_2 \leq 0; \]
\[ \frac{\partial p_1(t_1, t_2, t_3)}{\partial t_3} = 0 \]

where the inequalities in (47) and (48) follow from (25) and (26). Thus, an increase in \( t_1 \) or \( t_2 \) cannot increase the producer price of output. Substituting (46) and (49) yields:
\[ MEB_3 = 0; \]
i.e., the Allais-Debreu excess burden of a tax increase on profits is zero. This result is to be expected in the context of our model, which assumes that capital is a fixed factor which is not affected by a tax on its use. However, this result does not extend to a dynamic model where reproducible capital is endogenously determined. Thus the result (50) should not be used in the design of a real life tax system.

In order to obtain explicit expressions for \( MEB_1 \) and \( MEB_2 \) in terms of tax rates and various supply and demand elasticities, we need to calculate the transfer derivatives \( T_1(t_1, t_2, t_3) \) and \( T_2(t_1, t_2, t_3) \). This can readily be done using the third equation in (40). The resulting partial derivatives are:
\[ T_1(t_1, t_2, t_3) = p_1 C + \left[ (t_1 + t_3)C - (1 - t_3)G \right] \frac{\partial p_1(t_1, t_2, t_3)}{\partial t_1}; \]
\[ T_2(t_1, t_2, t_3) = p_2 L + \left[ (t_1 + t_3)C - (1 - t_3)G \right] \frac{\partial p_1(t_1, t_2, t_3)}{\partial t_2}. \]

Finally, (46) — (48), (51) and (52) may be substituted into (44) in order to obtain formulae for the Allais-Debreu excess burden measures \( MEB_1 \) and \( MEB_2 \) in terms of the initial allocation of resources, the initial tax rates and the responses of net demand and supply functions to changes in prices (the \( e_{ij} \) and \( \pi_{ij} \)). The resulting formulae are too complex to be exhibited here. However, we can present results for an approximation to the general formulae (44) which will be accurate for small tax rates \( t_i \) and small government expenditures on goods \( G \). Instead of calculating the full general equilibrium effects on government revenues of an increase in \( t_1 \) or

\[ \text{We need at least one of the inequalities } e_{21} \geq 0 \text{ and } \pi_{21} \leq 0 \text{ to be strict.} \]

\[ \text{Compare the unambiguous results (47) — (49) with the indeterminate results (34) — (36) that were obtained in the previous section.} \]

\[ \text{Theoretical and empirical research indicates that the efficiency costs of taxing the return to capital can be quite high; see Ballard, Shoven and Whalley (1985), Jorgenson and Yun (1986a) (1986b) (1990) (1991) and Dievert (1988; 23).} \]
we approximate the revenue increase by the first order rate of increase that ignore price effects. The resulting approximate derivatives are:

\[
R_1(t_1, t_2, t_3) \approx p_1 C;
\]

\[
R_2(t_1, t_2, t_3) \approx p_2 L.
\]

Note that the right hand sides of (53) and (54) are the initial tax bases for \(t_1\) and \(t_2\), respectively, the consumption and labour tax bases. Substitution of (46) — (48) and (53) — (54) into (44) leads to the following approximate Allais-Debreu excess burden measures, \(MEB_i^*\):

\[
MEB_1^* = (t_1 + t_2) \pi_{11} \pi_{21} e_{21} p_1 / \left[ e_{21} (1 + t_1) - \pi_{21} \right] C \geq 0;
\]

\[
MEB_2^* = -(t_1 + t_2) \pi_{11} \pi_{22} e_{22} p_1 / \left[ e_{21} (1 + t_1) - \pi_{21} \right] L \geq 0
\]

where the inequalities in (55) and (56) follow from the restrictions on the net demand derivatives \(e_{ij}\) given by (25) and the restrictions on the net supply derivatives \(\pi_{ij}\) given by (26).

For ease of interpretation, (55) and (56) can be expressed in terms of elasticities of net demand and supply rather than in terms of the derivatives \(e_{ij}\) and \(\pi_{ij}\). Define the cross elasticity of demand for consumption with respect to leisure as:

\[
\eta_{12} \equiv e_{12} p_2 / C \geq 0
\]

and minus the cross elasticity of supply of output with respect to labour as:

\[
\sigma_{12} \equiv -\pi_{12} p_2 / Y \geq 0
\]

where output \(Y = C + G\) and the inequalities in (57) and (58) follow from the restrictions (25) and (26). The elasticity \(\sigma_{12}(\eta_{12})\) is a measure of substitutability in production (consumption): the bigger \(\sigma_{12}(\eta_{12})\) is, the more substitutability there is in production (consumption). Substitution of (57) and (58) into (55) and (56) leads to the following expressions for the

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15 Terms involving the price derivatives \(P_i(t_1, t_2, t_3)\) are ignored in evaluating the revenue derivatives

\(R_i(t_1, t_2, t_3)\), where (46) is used to calculate \(G_i(t_1, t_2, t_3)\) in terms of \(P_i(t_1, t_2, t_3)\) and (51) and (52) are used to calculate the derivatives \(T_i(t_1, t_2, t_3)\) in terms of \(P_i(t_1, t_2, t_3)\).
approximate Allais-Debreu marginal excess burdens due to an increase in consumption and labour taxation respectively.\textsuperscript{16}

\begin{equation}
M_{EB1}^* = (t_1 + t_2) \sigma_{12} \eta_{12} / \left[ \eta_{12} s_C (1 + t_1) + \sigma_{12} (1 - t_2) \right] \geq 0; \tag{59}
\end{equation}

\begin{equation}
M_{EB2}^* = (t_1 + t_2) \sigma_{12} \eta_{12} s_C / s_L \left[ \eta_{12} s_C (1 + t_1) + \sigma_{12} (1 - t_2) \right] \geq 0 \tag{60}
\end{equation}

where $s_C = P_1 C / P_1 Y$ and $s_L = P_2 L / P_1 Y$ are the consumption and labour shares of output valued at consumer prices. A comparison of (59) and (60) shows that:

\begin{equation}
M_{EB2}^* = \left( s_C / s_L \right) M_{EB1}^*. \tag{61}
\end{equation}

Thus, if the consumption share of output, $s_C$, is greater than labour's share of output (valued at consumer prices), then the (approximate) marginal excess burden associated with raising labour taxes will exceed the burden associated with raising consumption taxes.

Examination of (59) and (60) shows that for normal parameter values, (approximate) marginal excess burdens will increase as the tax rates $t_1$ on consumption and $t_2$ on labour earnings increase and as substitutability in consumption and production increase (i.e., as $\eta_{12}$ and $\sigma_{12}$ increase).\textsuperscript{17} Note also that $M_{EB1}^*$ and $M_{EB2}^*$ will equal zero if either $\sigma_{12}$ or $\eta_{12}$ equals zero. Hence to get positive excess burdens in our simple general equilibrium model, we must have strict substitutability in both production and consumption. This is similar to the situation which occurred in the partial equilibrium model developed in Section 2.1 above.\textsuperscript{18} Finally, note that the excess burdens defined by (59) and (60) are approximately proportional to $t_1 + t_2$, the sum of the tax rates on consumption and labour earnings.

The geometry of the numerators of the marginal excess burden measures defined by (43) can be illustrated using Figure 4.2 again. Assume that the initial producer equilibrium is at the point $F$ and the initial consumer equilibrium is at the point $H$. The initial government consumption of goods is the distance $FH$. If the tax rate on consumption $t_1$ or the tax rate on labour $t_2$ is increased, then the consumer price line becomes less steeply sloped than the producer price line and thus producers move down the production possibilities set $AB$ to the point $J$ and consumers

\textsuperscript{16} We also used (20) — (24) in deriving (59) — (60).

\textsuperscript{17} These theoretical results are broadly consistent with the results obtained in the applied general equilibrium models of Stuart (1984; 360) and Ballard, Shoven and Whalley (1985; 128).

\textsuperscript{18} Compare the partial equilibrium formula for the marginal excess burden of a labour tax given by (10) in Section 2.1 with (60) in the present Section.
move along the indifference curve $I_2I_2$ to the point $K$. The new government consumption is $JK$ which is less than the initial government consumption $FH$.

4.5 Summary of the Chapter

The existing elasticity measures of the marginal excess burden of a tax increase have been based on simple partial equilibrium models. Our goal in this chapter has been to develop excess burden measures that are valid in a general equilibrium context.

Our first general equilibrium approach assumed that society's objective function (or social welfare function) was equal to a constant dollar sum of private household consumption and leisure plus the constant dollar sum of government expenditures on goods and services. However, the resulting measure proved to be too complex and moreover, it seemed to be a bit arbitrary: why should the benefits of government expenditures be exactly additive to the consumer's constant dollar consumption of goods and leisure?

Thus, in our second approach to measuring marginal excess burdens developed in section 4.4, we held consumers' money metric utility over private goods constant as we raised taxes to finance increased government expenditures. We used the increased taxes to increase government expenditures on goods while holding government expenditures on labour constant. Since private utility is held constant and government expenditures on labour are held constant, our measure of social welfare became government expenditures on goods, at constant reference prices: see (41) in section 4.4.

The intuition behind our second model of marginal excess burden can be explained as follows. A tax rate is increased and the increased revenues are initially used to increase the outputs of the government sector. However, the increased tax wedge causes increased deadweight loss and, in particular, a decrease in private utility for consumers. To restore consumers to their pre tax increase levels of private utility, the government provides consumers with a tax transfer. It turns out that this tax transfer more than exhausts the increase in revenue that the initial tax increase created. Thus a government project that is financed by the initial tax increase should be valued by consumers by enough of a premium to overcome the effects of the increased loss of efficiency that is generated by the initial tax increase. This premium rate is our estimated marginal deadweight loss. We do not want to imply that government investments should not go ahead, but they should be sufficiently valuable to society that they can overcome the tax induced increase in deadweight loss. We are simply trying to provide approximate estimates of the required excess premium rate that government projects should earn.
In Chapter 7 below, we will develop a more realistic model of marginal excess burden using the Allais-Debreu approach explained here. In the next two chapters, we turn our attention to the empirical specification of producer and consumer models for the New Zealand economy.
5. A MODEL OF PRODUCER BEHAVIOUR FOR NEW ZEALAND

5.1 The theoretical model

Most empirical work on calculating marginal excess burdens is subject to some severe limitations: either the underlying theoretical model used is a partial equilibrium model or an applied general equilibrium model is used which uses rather restrictive functional forms for producer's production functions and consumer's preference functions. Finally, the elasticity estimates that are used in these models are often taken from empirical studies pertaining to other countries.

In this Chapter, we use the data pertaining to the New Zealand economy that is developed in Appendix A below in order to estimate a system of private producer supply and demand equations. Flexible functional form techniques are used: i.e., the functional form we use to model the technology does not impose unwarranted a priori restrictions on elasticities of substitution between the outputs and inputs. In the present Section, we lay out the details of the model and in the following Section we present our empirical results.

The inputs and outputs in the New Zealand economy were aggregated into two classes of goods: (i) those that are variable during the course of a year and (ii) those that are fixed. There were five variable goods: (1) motor vehicle output; (2) general consumption and investment (including government consumption of goods); (3) exports of goods and services; (4) imports of goods and services and (5) labour input into the market sector (including the self employed but excluding general government employment). The two fixed factors or stocks were: (1) all non-land capital (non-residential structures, other construction, machinery and equipment and inventory stocks) and (2) the stock of land. General government holdings of these stocks were not included.

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19 Comparing (59) and (60) in Chapter 4 with formula (10) in Chapter 2 shows that the general equilibrium estimates for marginal excess burdens can be quite different from the partial equilibrium estimates.

20 The three most commonly used functional forms in applied general equilibrium theory are the Cobb-Douglas, constant elasticity of substitution and Leontief (no substitution) functional forms. If the number of goods in the model is greater than two, each of the above functional forms imposes a severe a priori restriction on elasticities of substitution; e.g., strict complementarity is ruled out; see Diewert (1985a). For an excellent survey of applied general equilibrium modelling, see Shoven and Whalley (1984).

21 The applied general equilibrium models of Jorgenson and Yun (1986a) (1986b) (1990) (1991) are not subject to the above criticisms.
Our treatment of export, imports and consumption is not completely conventional but it has appeared in the literature in the past 15 years: see the articles by Kohli (1978) (1993) and Lawrence (1989). Consumption, exports and imports are all regarded as separate goods in our model. Of course, some goods are both consumed and exported. However, the transportation, marketing and storage of these "identical" goods will serve to make them different; eg. export margins will generally be different from domestic margins for the same good. Moreover, we are dealing with aggregates of thousands of goods in each of the categories, "exports" and "consumption". The relative proportions of each micro good in these aggregates are different and hence the price indexes for each of these aggregates will be quite different, even if each aggregate is composed of a different mix of exactly the same goods. The only practical way to deal with these aggregation difficulties is to treat "export" and "consumption" as separate goods. If they are in fact virtually the same, then this fact will show up as extremely high substitutability between the two goods in our econometric work. "Imports" and "consumption" are also treated as distinct goods: virtually all imports will have domestic inputs added to them in terms of transportation, storage, packaging, wholesaling and retailing inputs. As a matter of national income accounting conventions, imports do not simply disappear: after domestic value added has been added to them, they reappear as components of consumption.

We also treat New Zealand as a small country, which means that we assume that the prices of New Zealand's export and imports are set on foreign markets and treated as exogenous. We also hold the trade balance constant as we vary taxes in our simulation exercises. This is a completely conventional treatment of trade in applied general equilibrium models. Alternative treatments are possible: a world demand curve for New Zealand's exports and a world supply curve for New Zealand's imports could be estimated. However, this would entail a major modeling effort — an effort which was beyond our limited resources.

In what follows, \( x = (x_1, x_2, x_3, x_4, x_5) \) denotes a vector of variable net outputs for the New Zealand economy\(^{22}\), \( p = (p_1, p_2, p_3, p_4, p_5) \) is the corresponding positive vector of variable input and output prices that producers face, \( s = (s_1, s_2) \) is a vector of stocks that are available to producers at the beginning of the year under consideration and \( w = (w_1, w_2) \) is a vector of ex post rental prices associated with the stocks.

The technology is represented by a GNP function (or variable profit function), \( \pi(p, s, t) \), defined as follows:

\(^{22}\) The components \( x_4 \) and \( x_5 \) are indexed with minus signs since imports and labour are inputs into the private production sector. We follow Kohli's (1978) treatment of international trade where all imports flow through the private production sector.
(1) \[ \pi(p, s, t) \equiv \max_x \left\{ p \cdot x : (x, s) \text{ belongs to } S^t \right\} \]

where \( S^t \) is the private sector technology set in period \( t \).\(^{23}\) Note that if \( x_i \) is an input, then \( x_i \) is negative.

Estimating equations can be obtained by differentiating the profit function with respect to the prices \( p_i \) and stocks \( s_j \).\(^{24}\)

(2) \[ x_i(p, s, t) = \frac{\partial \pi(p, s, t)}{\partial p_i}, i = 1, 2, 3, 4, 5; \]

(3) \[ w_j(p, s, t) = \frac{\partial \pi(p, s, t)}{\partial s_j}, j = 1, 2. \]

The functional form for \( \pi \) that we chose was a variation of the normalised quadratic functional form,\(^{25}\) since this functional form allows us to impose the appropriate curvature conditions without destroying its flexibility properties. Using matrix notation, the function can be defined as follows:

(4) \[ \pi(p, s, t) \equiv p \cdot Cs + p \cdot ch \cdot st + p \cdot gd \cdot st + \left( \frac{1}{2} \right) p \cdot Ap h \cdot s / p \cdot g - \left( \frac{1}{2} \right) s \cdot Bsp \cdot g / h \cdot s \]

where the vectors \( g \equiv \left( g_1, g_2, g_3, g_4, g_5\right) \) and \( h \equiv \left( h_1, h_2\right) \) were chosen a priori to be the absolute values of the sample means of the observed \( x^t \equiv \left( x_1^t, \ldots, x_5^t\right) \) and \( w^t \equiv \left( w_1^t, \ldots, w_5^t\right) \), normalised so that:

(5) \[ p^* \cdot g = 1; s^* \cdot h = 1 \]

where the \( p^* \) and \( s^* \) were fixed vectors.\(^{26}\) The variable \( t \) which appears in (4) is a scalar time variable which serves as a proxy for technological change. The parameter vectors \( c \) and \( d \) and the parameter matrices \( A = \left[ a_{ij}\right], B = \left[ b_{ij}\right] \) and \( C = \left[ c_{ij}\right] \) are to be estimated, subject to some restrictions. In order to identify the components of \( c \) and \( d \), we imposed the following linear restriction:

(6) \[ d \cdot s^* = 0. \]

\(^{23}\) Notation: \( p \cdot x = \sum_{i=1}^{5} p_i x_i \) is the inner product of the vectors \( p \) and \( x \).

\(^{24}\) See Diewert (1974; 137 and 140) (1993; 166 and 168).


\(^{26}\) We chose \( p^* \) and \( s^* \) to be vectors of ones.
In order for $\pi(p, s, t)$ to be a well behaved profit function (convex in $p$ and concave in $s$), we set $A$ and $B$ to be the following products:

$$A = U^T U; B = V^T V$$

where $U^T$ denotes the transpose of the matrix $U$, and $U$ and $V$ are upper triangular matrices satisfying the following restrictions:

$$U p^* = 0_5; V s^* = 0_2$$

where $0_5$ and $0_2$ are vectors of zeros of dimension 5 and 2, respectively.

Differentiating the profit function (4) with respect to the components of $p$ leads to the following system of 5 estimating equations:

$$x = Cs + c h \cdot s t + g d \cdot s t + Ap h \cdot s / p \cdot g$$

$$-\left(\frac{1}{2}\right)p \cdot Ap h \cdot s g / (p \cdot g)^2 - \left(\frac{1}{2}\right)s \cdot Bs g / h \cdot s.$$

A vector of ex post rental prices $w$ for the stocks can be obtained by differentiating $\pi$ defined by (4) with respect to the components of the fixed stock vector $s$. The following two additional estimating equations are obtained:

$$w = \frac{C^T p + h c \cdot p t + d g \cdot p t + \left(\frac{1}{2}\right)p \cdot Ap h}{p \cdot g}$$

$$-Bs p \cdot g / h \cdot s + \left(\frac{1}{2}\right)s \cdot Bs p \cdot g h / (h \cdot s)^2.$$

For simplicity, we omitted the superscript $t$ from $x, p, w$ and $s$ in equations (9) and (10).

The ex post rental prices $w^t$ were constructed so that in each period $t$, variable profits were distributed to the two fixed factors so that the following adding up equations were satisfied:

$$p^t \cdot s^t = w^t \cdot s^t, \ t = 0, 1, 2, \ldots, 19.$$

Thus, equations (9) cannot be statistically independent from equations (10); i.e., we must drop at least one of the 7 estimating equations in (9) and (10). In order to obtain estimates that were invariant to the equation dropped, we pre-multiplied both sides of the $i$th equation in (9) by $p^t \cdot g \cdot s^t \cdot h$ for $i = 1, 2, 3, 4, 5$ (call the resulting dependent variables $y_i^t$) and we pre-multiplied both sides of the $j$th equation in (10) by $s^t \cdot p^t \cdot g \cdot s^t \cdot h$ for $j = 1, 2$ (call the resulting dependent variables $z_j^t$). These transformations of the estimating equations (9) and

---

27 See Diewert and Wales (1987; 52-53) for further explanation.
(10) reduced heteroskedasticity and in view of (11), the resulting dependent variables satisfied the following restrictions:

\( \sum_{i=1}^{5} y_i^t - \sum_{j=1}^{2} z_j^t = 0, \quad t = 0, 1, 2, \ldots, 19. \)

We appended normally distributed errors \( u_i^t \) and \( v_j^t \) to the transformed estimating equations (9) and (10) with means 0 and variance-covariance matrix \( \Sigma \). In view of the restrictions (12) on the dependent variables, the errors must satisfy the following restrictions:

\( \sum_{i=1}^{5} v_i^t - 1_2 \cdot s^t = 0 \quad \text{for} \quad t = 0, 1, \ldots, 19. \)

where \( v^t = (v_1^t, \ldots, v_5^t) \) and \( s^t = (s_1^t, s_2^t) \). Thus \( \Sigma \) must be a singular variance-covariance matrix satisfying

\[ \begin{bmatrix} 1_5^T, -1_2^T \end{bmatrix} \Sigma = \begin{bmatrix} 0_5^T, 0_2^T \end{bmatrix}. \]

With the above statistical specification of the errors, non-linear maximum likelihood programs such as SHAZAM (see White (1978)) can be used to estimate the unknown parameters which appear in (4), after dropping any one of the transformed estimating equations (9) and (10). The resulting estimates will be invariant to the equation dropped. Our treatment here is completely consistent with standard techniques that are used in the theory of demand, where the data are also subject to an adding up constraint.

\[ \text{(14)} \]

5.2 Empirical results for the production model

Producer prices \( p_i^t \) and quantities \( x_i^t \) for the five variable outputs and inputs were constructed for the New Zealand economy for the 20 years ending 31 March, 1972 to 31 March, 1991. Corresponding data for the two stocks \( s_j^t \) and their ex post rental prices were also constructed. The details of our data construction procedures can be found in Appendix A with the specific data used in our producer regressions being presented in Tables A25 and A26.

The non-linear regression program in SHAZAM was used to estimate the unknown coefficients appearing in (4). The five (transformed) equations in (9) and the first (transformed) equation in (10) were used as estimating equations. As mentioned in the previous Section, our estimates are invariant to the equation deleted. The \( R^2 \) between the observed and predicted variables for the 6 equations were: 0.608, 0.813, 0.725, 0.438, 0.935 and 0.629. The \( R^2 \) appear to be rather low for time series results but recall that we have transformed each variable which

28 See Diwerts and Wales (1994) for more discussion on this point.
appears on the left hand sides of (9) and (10) so that the resulting variables are approximately constant. Hence our fits are actually quite satisfactory.

The parameter estimates from our non-linear regression are reported below in Table 5.1. Full computer print-outs of the regression models including a complete range of diagnostic statistics are available from the authors on request.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>c11</td>
<td>0.3980</td>
<td>0.91</td>
</tr>
<tr>
<td>c12</td>
<td>0.9190</td>
<td>2.08</td>
</tr>
<tr>
<td>c21</td>
<td>31.8170</td>
<td>15.04</td>
</tr>
<tr>
<td>c22</td>
<td>18.0660</td>
<td>8.99</td>
</tr>
<tr>
<td>c31</td>
<td>8.8370</td>
<td>11.17</td>
</tr>
<tr>
<td>c32</td>
<td>5.9410</td>
<td>7.45</td>
</tr>
<tr>
<td>c41</td>
<td>-10.1160</td>
<td>-14.15</td>
</tr>
<tr>
<td>c42</td>
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<td>-11.19</td>
</tr>
<tr>
<td>c51</td>
<td>-19.4380</td>
<td>-18.85</td>
</tr>
<tr>
<td>c52</td>
<td>-14.1010</td>
<td>-13.68</td>
</tr>
<tr>
<td>c6</td>
<td>0.0090</td>
<td>0.09</td>
</tr>
<tr>
<td>c7</td>
<td>-0.2209</td>
<td>-1.06</td>
</tr>
<tr>
<td>c8</td>
<td>0.3234</td>
<td>6.00</td>
</tr>
<tr>
<td>c9</td>
<td>-0.0652</td>
<td>-0.89</td>
</tr>
<tr>
<td>c10</td>
<td>0.4018</td>
<td>4.03</td>
</tr>
<tr>
<td>d1</td>
<td>-0.1722</td>
<td>-4.27</td>
</tr>
<tr>
<td>u12</td>
<td>0.7050</td>
<td>0.21</td>
</tr>
<tr>
<td>u13</td>
<td>0.3700</td>
<td>0.33</td>
</tr>
<tr>
<td>u14</td>
<td>-1.9140</td>
<td>-1.35</td>
</tr>
<tr>
<td>u15</td>
<td>0.5540</td>
<td>0.27</td>
</tr>
<tr>
<td>u23</td>
<td>-0.5340</td>
<td>-0.41</td>
</tr>
<tr>
<td>u24</td>
<td>1.0780</td>
<td>0.49</td>
</tr>
<tr>
<td>u25</td>
<td>2.7810</td>
<td>2.38</td>
</tr>
<tr>
<td>u34</td>
<td>-0.5280</td>
<td>-0.40</td>
</tr>
<tr>
<td>u35</td>
<td>1.9600</td>
<td>1.30</td>
</tr>
<tr>
<td>u45</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>v12</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Estimates of the main diagonal elements $u_{ii}$ of the $U = \left[ u_{ij} \right]$ matrix can be obtained using the following equations (which are based on the first set of restrictions in (8)):

\[
(15) \quad u_{ii} = -\sum_{j=i+1}^{5} u_{ij}, \quad i = 1, 2, 3, 4;
\]

\[
(16) \quad u_{55} = 0.
\]

Similarly, the second set of restrictions in (8) can be used in order to obtain estimates of the main diagonal elements $v_{ii}$ of the $V = \left[ v_{ij} \right]$ matrix:
\[ v_{11} = -v_{12}; \quad v_{22} = 0. \]

Finally, the restriction (6) can be used to obtain an estimate for \( d_2 \):
\[ d_2 = -d_1. \]

From Table 5.1, it can be deduced that the variable goods substitution matrix \( A = U^T U \) has rank 3 instead of the maximum possible rank 4 and the stock substitution matrix \( B = V^T V \) has rank 0 instead of the maximum possible rank 1. With the exception of \( u_{25} \), the statistical significance of the components of these substitution matrices is generally low. However, the components of the \( C \) matrix and the components of the \( c \) and \( d \) vectors are generally highly significant.

A measure of the technical progress that took place during year \( t \) can be obtained by differentiating the profit function \( \pi(p, s, t) \) with respect to \( t \) and dividing by \( \pi(p, s, t) \) evaluated at \( p = p^t \) and \( s = s^t \). The resulting measure of technical progress turned out to be 0.061 for 1972 and trended upward to 0.106 for 1991, averaging 8.2 per cent per year for the twenty years in our sample. Since variable profits are only about one quarter of the total returns to labour and capital, the average rate of 8.2 per cent translates into an average total factor productivity improvement of about 2 per cent per year. The positive and statistically significant parameter estimates for \( c_3 \) and \( c_5 \) indicate that the technical progress was mainly export augmenting and labour saving.

Since the fitted net output of variable good \( i \) in period \( t \), \( \tilde{x}_i^t \), can be obtained by differentiating \( \pi(p^t, s^t, t) \) with respect to \( p_i \), the \( j \)th price elasticity of net supply for good \( i \) in period \( t \) can be defined as
\[ \sigma_{ij}^t = \left( \frac{\partial \tilde{x}_i^t}{\partial p_i} \right) \frac{\partial^2 \pi(p^t, s^t, t)}{\partial p_i \partial p_j}; \quad i, j = 1, \ldots, 5. \]

The sample means of the net supply elasticities are listed in Table 5.2.

From viewing Table 5.2, it can be seen that with the exception of the price elasticity of demand for labour (which averaged -0.47), the elasticities were rather small in magnitude. However, there were some interesting trends in the annual elasticity estimates: \( \sigma_{11}^t \) (the own price elasticity of supply for motor vehicles) trended upwards from 0.06 in 1972 to 0.12 in 1991; \( \sigma_{22}^t \) (the own price elasticity of supply for general output) trended up from 0.23 to 0.36; \( \sigma_{33}^t \) (the own price elasticity of supply for exports) stayed approximately constant at 0.16; \( \sigma_{44}^t \) (the

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\[ 29 \] Recall equation (2) above; thus we have \( \tilde{x}_i^t = \partial \pi(p^t, s^t, t) / \partial p_i \) for \( i = 1, 2, 3, 4, 5. \)
Table 5.2: Average price elasticities of net supply $\sigma_{ij}$

<table>
<thead>
<tr>
<th>Change in quantity of:</th>
<th>Motor vehicles</th>
<th>Consumption &amp; investment</th>
<th>Exports</th>
<th>Imports</th>
<th>Labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor vehicles</td>
<td>0.09</td>
<td>0.19</td>
<td>0.10</td>
<td>-0.52</td>
<td>0.13</td>
</tr>
<tr>
<td>Consumption and invest</td>
<td>0.01</td>
<td>0.28</td>
<td>0.06</td>
<td>-0.11</td>
<td>-0.24</td>
</tr>
<tr>
<td>Exports</td>
<td>0.01</td>
<td>0.15</td>
<td>0.16</td>
<td>-0.03</td>
<td>-0.29</td>
</tr>
<tr>
<td>Imports</td>
<td>0.04</td>
<td>0.25</td>
<td>0.03</td>
<td>-0.30</td>
<td>-0.02</td>
</tr>
<tr>
<td>Labour</td>
<td>-0.01</td>
<td>0.33</td>
<td>0.17</td>
<td>-0.02</td>
<td>-0.47</td>
</tr>
</tbody>
</table>

own price elasticity of demand for imports) stayed approximately constant at -0.30 and then trended to -0.24 during the last 5 years and $\sigma_{55}'$ (the own price elasticity of demand for labour) trended up in magnitude from -0.35 in 1971 to -0.71 in 1991.\textsuperscript{30} From our discussion in Section 4.4 above, the increasing magnitudes of $\sigma_{22}'$ and $\sigma_{55}'$ suggests that the excess burden of increased government spending in New Zealand will probably be increasing over time. As we shall see in Chapter 7 below, this expectation of increasing excess burdens turns out to be true.

We turn now to the specification of our consumer model for New Zealand.

\textsuperscript{30} We have found this same upward trend in the magnitude of the price elasticity of demand for labour for most OECD countries in similar production models. This suggests that increases in wage rates have led to greater rates of unemployment in these countries.
6. A MODEL OF CONSUMER BEHAVIOUR FOR NEW ZEALAND

6.1 The consumer model

In this Chapter, we use the data for New Zealand explained in Appendix A below to estimate a system of consumer demand and labour supply equations. As in the previous Chapter, we do not want to restrict a priori elasticities of substitution so we again use flexible functional form techniques. In the present Section we present our theoretical model and in the following Section we present empirical results.

We assume that a representative consumer has preferences defined over 4 current period goods: (1) the consumption of the services of the current stock of motor vehicles; (2) general consumption (excluding motor vehicles and housing); (3) the consumption of the services of the current stock of housing; and (4) the consumption of leisure.

The economy’s total consumption of the above 4 goods was divided by the adult, working age population, aged 15-64 inclusive. Each working age adult was given a time endowment of 2000 hours per year. Per capita leisure $h$ was defined as $2000-L$ where $L$ is per capita hours of work supplied during the year under consideration. The resulting (rental) prices $p_i^r$ and per capita quantities $x_i^r$ are listed in Table A24 of Appendix A.

The consumer’s preferences can be represented by the expenditure function, $e(u, p)$, which is dual to the utility function, $u = f(x)$, where $p$ and $x$ are price and quantity vectors pertaining to consumer expenditure categories.\(^{31}\) As in the previous Chapter, we again use a normalised quadratic functional form,\(^ {32}\) since curvature conditions can be imposed on this functional form without destroying its flexibility. The functional form used in this Chapter is defined as follows:

\[
e(u, p) = \begin{cases} 
  a \cdot p + b \cdot pu + \left(\frac{1}{2}\right)p \cdot Cu / p \cdot g & \text{for } u \leq u^* \\
  a \cdot p + b \cdot pu^* + c \cdot p(u - u^*) + \left(\frac{1}{2}\right)p \cdot Cu / p \cdot g & \text{for } u > u^* 
\end{cases}
\]

\(^{31}\) For expositions of the use of duality theory in modelling consumer preferences, see Diewert (1974; 120-133) (1993; 148-154).

where \( g = (g_1, g_2, g_3, g_4) \) is a predetermined parameter vector; \( u^* \) is a predetermined level of utility; \( a = (a_1, a_2, a_3, a_4) \), \( b = (b_1, b_2, b_3, b_4) \) and \( c = (c_1, c_2, c_3, c_4) \) are parameter vectors to be estimated and \( C = \begin{bmatrix} c_{ij} \end{bmatrix} \) is a symmetric parameter matrix to be determined. The parameter vectors \( a, b \) and \( c \) satisfy the following restrictions:

\[
a \cdot p^* = 0; \quad b \cdot p^* = 1; \quad c \cdot p^* = 1
\]

where \( p^* \) is a predetermined price vector.\(^{33}\) The parameter matrix \( C \) satisfies the following restrictions:

\[
C = -U^TU
\]

where \( U = \begin{bmatrix} u_{ij} \end{bmatrix} \) is an upper triangular matrix which satisfies the following restrictions:

\[
Up^* = 0_4.
\]

The restrictions (2) — (4) impose money metric scaling\(^{34}\) on the utility function; i.e., utility change can be measured in terms of income or expenditure change at the reference prices \( p^* \).

The \( i \)th Hicksian demand function, \( x_i(u, p) \) can be obtained by differentiating the expenditure function with respect to the \( i \)th consumer price, \( p_i; \) i.e., we have:

\[
x_i(u, p) = \partial e(u, p) / \partial p_i, \quad i = 1, 2, 3, 4.
\]

The Hicksian demand functions defined by (5) have (unobservable) utility as an independent variable. We obtain an analytic expression for utility in period \( t, u^t \), by setting the expenditure function evaluated at period \( t \) utility, \( u^t \), and period \( t \) prices, \( p^t = (p_1^t, p_2^t, p_3^t, p_4^t) \), equal to period \( t \) expenditures on the 4 goods, \( Y^t \). We then solve the resulting equation \( e(u^t, p^t) = Y^t \) for \( u^t = g(Y^t, p^t) \). The function \( g \) is the consumer's indirect utility function and it is substituted into the equations (5) in order to obtain the following system of estimating equations:

\[
x^t_i = \partial e \left( g(Y^t, p^t), p^t \right) / \partial p_i, \quad i = 1, 2, 3, 4.
\]

To reduce heteroskedasticity, we multiply both sides of the \( i \)th equation in (6) by \( p_i / Y^t \), which transforms (6) into a system of expenditure share equations. Since these shares sum to

\[^{33}\] We chose \( p^* = 1_4 \), a vector of ones.

\[^{34}\] The term money metric scaling is due to Samuelson (1974) but the concept may be found in Hicks (1946).
unity in each period, we must drop one of those share equations when estimating the parameters using a non-linear regression package. We dropped the last equation, but as was the case in the previous Chapter, our estimates are invariant to the equation dropped.

Examination of definition (1) above shows that the term \( b \cdot pu \) changes into the terms \( b \cdot pu^* + c \cdot p(u - u^*) \) as \( u \) passes through the break point \( u^* \). Thus, income elasticities of demand can change arbitrarily as \( u \) passes through \( u^* \). We have not yet discussed how this break point \( u^* \) was chosen, a task which we now undertake.

It is known that constructing a chain Fisher (1922) ideal quantity index using the per capita price and quantity data tabled in Appendix A will give a close approximation to the period \( t \) indirect utility, \( u^t = g(Y^t, p^t) \). For more evidence to support this assertion, see Table 6.1 which lists \( u^t \) estimates in column 2 (based on our parameter estimates of the expenditure function \( e(u, p) \) defined by (1) — (4) above) and lists the chain Fisher quantity indexes in column 3. The units are in thousands of 1972 New Zealand dollars. As can be seen from examining Table 6.1, the index number estimates of per capita real consumption (including the

<table>
<thead>
<tr>
<th>Year</th>
<th>Utility estimate</th>
<th>Quantity index estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>3.085</td>
<td>3.085</td>
</tr>
<tr>
<td>1973</td>
<td>3.219</td>
<td>3.225</td>
</tr>
<tr>
<td>1974</td>
<td>3.319</td>
<td>3.327</td>
</tr>
<tr>
<td>1975</td>
<td>3.368</td>
<td>3.374</td>
</tr>
<tr>
<td>1976</td>
<td>3.303</td>
<td>3.305</td>
</tr>
<tr>
<td>1977</td>
<td>3.238</td>
<td>3.239</td>
</tr>
<tr>
<td>1978</td>
<td>3.142</td>
<td>3.139</td>
</tr>
<tr>
<td>1979</td>
<td>3.115</td>
<td>3.113</td>
</tr>
<tr>
<td>1980</td>
<td>3.120</td>
<td>3.117</td>
</tr>
<tr>
<td>1981</td>
<td>3.078</td>
<td>3.072</td>
</tr>
<tr>
<td>1982</td>
<td>3.066</td>
<td>3.061</td>
</tr>
<tr>
<td>1983</td>
<td>3.066</td>
<td>3.058</td>
</tr>
<tr>
<td>1984</td>
<td>3.166</td>
<td>3.156</td>
</tr>
<tr>
<td>1985</td>
<td>3.239</td>
<td>3.227</td>
</tr>
<tr>
<td>1986</td>
<td>3.269</td>
<td>3.255</td>
</tr>
<tr>
<td>1987</td>
<td>3.342</td>
<td>3.328</td>
</tr>
<tr>
<td>1988</td>
<td>3.410</td>
<td>3.398</td>
</tr>
<tr>
<td>1989</td>
<td>3.489</td>
<td>3.485</td>
</tr>
<tr>
<td>1990</td>
<td>3.541</td>
<td>3.543</td>
</tr>
<tr>
<td>1991</td>
<td>3.551</td>
<td>3.554</td>
</tr>
</tbody>
</table>

35 See Diewert and Wales (1993; 101). To obtain the close correspondence, the quantity index must be set equal to expenditures in the base period, which is the period which has prices equal to the reference prices \( p^* \). In our case, the base period was the first period.
consumption of leisure) coincide with the econometric estimates of indirect utility to three significant figures. Note that real consumption trends up from 1972 to 1975 ($3,085 to $3,374 in constant 1972 New Zealand dollars), trends down until 1983 (hitting a low of $3,058) and then trends up to 1991, ending up at $3,554.

The information in the last column of Table 6.1 (which can be computed without econometric estimation) allows us to choose the break point $u^*$. We set $u^* = 3.39$, which means that only the last 4 observations will be in the utility region $u > u^*$. With this choice of $u^*$, the system of estimating equations defined by (6) can be determined by differentiating the expenditure function defined by (1). The first 16 observations in our sample are assumed to be in the region $u \leq u^*$ and the last 4 observations are assumed to be in the regions $u > u^*$. This completes the theoretical specification of our model.

6.2 Empirical results for the consumer model

The model based on the transformed equations (6) (where the last equation was dropped since the expenditure shares sum to one in each period) was run using the non-linear regression program in SHAZAM. Autocorrelation proved to be a problem, so the model was rerun using the AUTO option.\(^{36}\) The resulting parameter estimates are listed Table 6.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>0.0175</td>
<td>0.24</td>
</tr>
<tr>
<td>a2</td>
<td>0.3363</td>
<td>0.65</td>
</tr>
<tr>
<td>a3</td>
<td>0.2305</td>
<td>8.85</td>
</tr>
<tr>
<td>b1</td>
<td>0.0387</td>
<td>1.65</td>
</tr>
<tr>
<td>b2</td>
<td>0.5119</td>
<td>3.12</td>
</tr>
<tr>
<td>b3</td>
<td>-0.0074</td>
<td>-0.91</td>
</tr>
<tr>
<td>c1</td>
<td>0.0641</td>
<td>1.36</td>
</tr>
<tr>
<td>c2</td>
<td>-0.7405</td>
<td>-3.03</td>
</tr>
<tr>
<td>c3</td>
<td>0.0672</td>
<td>5.15</td>
</tr>
<tr>
<td>u12</td>
<td>0.1416</td>
<td>1.33</td>
</tr>
<tr>
<td>u13</td>
<td>0.0276</td>
<td>3.39</td>
</tr>
<tr>
<td>u14</td>
<td>-0.0643</td>
<td>-0.66</td>
</tr>
<tr>
<td>u23</td>
<td>-0.0172</td>
<td>-1.96</td>
</tr>
<tr>
<td>u24</td>
<td>-0.4538</td>
<td>-6.38</td>
</tr>
<tr>
<td>u34</td>
<td>-0.0345</td>
<td>-3.90</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.6757</td>
<td>9.81</td>
</tr>
</tbody>
</table>

\(^{36}\) The same autocorrelation coefficient $\rho$ was estimated for each equation.
Estimates for $a_4$, $b_4$ and $c_4$ were obtained using the estimates listed in Table 6.2 and equations (2). Estimates for the diagonal elements $u_{ii}$ of the U matrix were obtained using equations (4). The parameters describing the consumer model were significantly different from zero for the most part. The $R^2$ between observed and predicted variables in the three estimating equations were 0.9227, 0.9273 and 0.9996, which was quite satisfactory considering that the dependent variables were shares.

Since the period $t$ fitted demand for commodity $i$, $\bar{x}_i^t$, is equal to the derivative of the estimated expenditure function with respect to the $i$th price evaluated at the period $t$ data, $\bar{u}_i^t = g \left( Y_i^t, p_i^t \right)$ and $p_i^t$ (i.e., we have $\bar{x}_i^t = \partial e(\bar{u}_i^t, p_i^t) / \partial p_i$, the Hicksian price elasticity of demand for consumer good $i$ with respect to price $j$ can be defined as

$$\eta_{ij}^t = \left( p_j^t / \bar{x}_i^t \right) \partial^2 e(\bar{u}_i^t, p_i^t) / \partial p_i \partial p_j; \quad i, j = 1, \ldots, 4.$$  

The sample means of the demand elasticities $\eta_{ij}^t$ are listed in Table 6.3.

**Table 6.3: Average compensated price elasticities of demand**

<table>
<thead>
<tr>
<th>Change in quantity of:</th>
<th>Motor vehicles</th>
<th>General consumption</th>
<th>Housing</th>
<th>Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor vehicles</td>
<td>-0.29</td>
<td>0.36</td>
<td>0.060</td>
<td>-0.14</td>
</tr>
<tr>
<td>General consumption</td>
<td>0.03</td>
<td>-0.41</td>
<td>0.005</td>
<td>0.38</td>
</tr>
<tr>
<td>Housing</td>
<td>0.05</td>
<td>0.05</td>
<td>-0.030</td>
<td>-0.07</td>
</tr>
<tr>
<td>Leisure</td>
<td>-0.02</td>
<td>0.82</td>
<td>-0.010</td>
<td>-0.79</td>
</tr>
</tbody>
</table>

From Table 6.3, it can be seen that the demand for housing (good 3) is quite inelastic: no price change significantly changes the demand for housing. The demand for motor vehicles (good 1) is also inelastic with respect to changes in the price of housing and the price of leisure (good 4) but motor vehicle demand is moderately substitutable with general consumption (good 2) since the average cross elasticity of demand for motor vehicles with respect to general consumption is 0.36. General consumption is quite substitutable with leisure since the two average cross elasticities are 0.38 and 0.82. The average price elasticity of demand for leisure, -0.79, is quite high in magnitude. This price elasticity of demand ranged between -0.64 in 1991 to -0.93 in 1976.

Hicksian real income elasticities of demand\(^\text{37}\) can be defined as follows:

$$\eta_{iu}^t = \left( \bar{u}_i^t / \bar{x}_i^t \right) \partial^2 e(\bar{u}_i^t, p_i^t) / \partial p_i \partial u; \quad i = 1, \ldots, 4.$$  

\(^{37}\) These elasticities are equal to ordinary income elasticities of demand.
The sample average income elasticities were: $\eta_{1u} = 0.99; \eta_{2u} = 0.42; \eta_{3u} = 0.14$ and $\eta_{4u} = 2.40$. These results are a bit unusual since it is generally thought that the income elasticity of demand for housing exceeds unity. However, over the first 16 observations in our sample, real income (or utility) changed very little so we cannot expect to obtain very accurate estimates for income elasticities of demand. The average $\eta_{ui}$ over the last 4 observations (when real incomes increased quite dramatically) were: $\eta_{1u} = 1.46; \eta_{2u} = 1.21; \eta_{3u} = 1.11$ and $\eta_{4u} = 5.13$. Thus, over the last four years in our sample, it appears that motor vehicles, housing and leisure all had income elasticities exceeding unity.

We turn now to a general equilibrium model of excess burdens for the New Zealand economy that utilises the consumer model described in this Chapter and the producer model described in the previous Chapter.
7. MARGINAL EXCESS BURDENS IN NEW ZEALAND

7.1 The theoretical model

In this Chapter, we bring together the producer model of Chapter 5 and the consumer model of Chapter 6 and construct a small (static) general equilibrium model for the New Zealand economy.

Let $C_i$ denote the per capita\(^{38}\) consumption of good $i$ for $i=1,2,3,4$. As in Chapter 6, consumer good 1 is the services of motor vehicles, good 2 is general consumption, good 3 is the services of the beginning of the period housing stock and consumer good 4 is the consumption of leisure. Denote the per capita stock of vehicles by $S_1$, the per capita stock of housing by $S_3$ and the per capita stock of time that is potentially available for labour supply during the year under consideration by $H$.\(^{39}\) The prices that consumers face for the four consumer goods are denoted by $P_1, P_2, P_3$ and $P_4$.\(^{40}\) These consumer data are listed in Table B.1 of Appendix B.

Let $Y_i$ denote the per capita net output of producer good $i$ for $i=1,\ldots,5$. As in Chapter 5, producer good 1 is new motor vehicles produced or imported during the year, good 2 is general consumption plus investment plus government consumption of goods, good 3 is exports of goods and services, good 4 is (minus) imports of goods and services during the year under consideration and good 5 is (minus) the demand for labour. The prices that the market sector of the economy faces for the five producer goods are denoted by $p_1, p_2(1+s_2), p_3(1+s_3), p_4(1+t_4)$ and $p_5$ where $s_2$ is the subsidy rate on general output, $s_3$ is the subsidy rate on exports and $t_4$ is the total indirect tax and tariff rate on imports.\(^{41}\)

\(^{38}\) The population variable is the adult population between ages 15 and 64 inclusive. All “per capita” variables have been formed by dividing by this population.

\(^{39}\) We chose $H$ to be 2000 hours. After converting to monetary units and producer prices, $H$ turned out to be 3.3098.

\(^{40}\) Note that we have changed our notation for the consumer prices and quantities compared to that used in Chapter 6.

\(^{41}\) We have changed the notation for the producer prices and quantities compared to that used in Chapter 5. Denote the producer prices used in this Chapter by $p_{i*}$ and the corresponding producer prices used in Chapter 5 and listed in Appendix A by $p_{i}$, $i=1,\ldots,5$. The relationships between the 5 sets of prices are as follows: $p_1 = p_{1*}$, $p_2(1+s_2) = p_{2*}$, $p_3(1+s_3) = p_{3*}$, $p_4(1+t_4) = p_{4*}$ and $p_5 = p_{5*}$. The subsidy rates $s_2$ and $s_3$ and the tariff rate $t_4$ are listed in Table B.3. The per capita fitted $Y_i$ used in this Chapter are listed in Table B.2 of Appendix B.
There are seven equations in our static general equilibrium model. They are listed below as:

(1) \[ C_1 = Y_1 + S_1; \]
(2) \[ C_2 = Y_2 - I - G; \]
(3) \[ C_3 = S_3; \]
(4) \[ B = p_3 Y_3 + p_4 Y_4; \]
(5) \[ C_4 = Y_5 - L_G + H; \]
(6) \[ E = (1 - t_6)\pi + p_5 (1 - t_5)H + p_1 S_1 + p_3 S_3 - p_1 [1 - (\delta_1 + r)] Y_1 - p_3 t_3 S_3 - B - p_2 I - p_2 D + T; \]
(7) \[ p_1 t_1 Y_1 + p_2 t_2 C_2 + p_3 t_3 C_3 - p_4 t_4 Y_4 + p_5 t_5 (H - C_4) + t_6 \pi \]
\[ - p_2 s_2 Y_2 - p_3 t_3 S_3 + p_2 D = p_2 G + p_5 L_G + T. \]

The tax rates \( t_1 \) to \( t_6 \) (on purchases of motor vehicles, general consumption, housing, imports, labour and profits, respectively) as well as the subsidy rates on general output \( s_2 \) and on exports \( s_3 \) are listed in Table B.3 of Appendix B.\(^{42}\) Estimates of real per capita government expenditures on goods \( G \) and on labour \( L_G \) may be found in Table B.4 of Appendix B along with some additional variables used in equations (1) to (7) above.

Before we present detailed interpretations of equations (1) to (7), we need to express the consumer prices \( P_1, P_2, \) and \( P_4 \) in terms of the producer prices \( p_i \):

(8) \[ P_1 = [t_1 + (r + \delta_1)]p_1; \]
(9) \[ P_2 = (1 + t_2)p_2; \]
(10) \[ P_4 = (1 - t_5)p_5; \]

\(^{42}\) We set \( s_2 = s_3 \); i.e., we assumed a common subsidy rate on all output due to a lack of more specific information. The housing tax rate \( t_3^* \) is the housing tax rate applied to the purchase price rather than the rental price. To calculate the rental price tax rate \( t_3 \) from \( t_3^* \), use \( t_3 = t_3^* / \left[ t_3^* + (r + \delta_3) \right] \) where \( r \) is the real after tax interest rate and \( \delta_3 = 0.015 \) is the housing depreciation rate. The motor vehicle depreciation rate used was \( \delta_1 = 0.1385 \).
where $\delta_1 = 0.1385$ is the depreciation rate for motor vehicles and $r$ is the after tax real rate of return. The right hand side of (8) is the rental price for motor vehicles assuming that the motor vehicle inflation rate is equal to the general inflation rate. The real rates of return for the New Zealand economy are listed in Table B.4 of Appendix B. The variables $E$ (per capita consumer expenditures) and $\pi$ (per capita gross profits) also appear in equations (6) and (7) above. These variables are defined in terms of per capita consumer purchases $C_i$ and per capita producer net outputs $Y_i$, as follows:

$$E \equiv \sum_{i=1}^{4} P_i C_i,$$

$$\pi \equiv p_1 Y_1 + p_2 (1 + s_2) Y_2 + p_3 (1 + s_3) Y_3 + p_4 (1 + t_4) Y_4 + p_5 Y_5.$$

Now we can provide explanations for each of the equations (1) — (7) in our static general equilibrium model. All quantity variables are expressed in per capita terms (or, more accurately, on a per working age population basis).

Equation (1) is the demand ($C_i$) equals supply equation for motor vehicles. Supply is made up of new additions ($Y_i$) plus the existing stock ($S_i$). Equation (2) is the demand ($C_2 + I + G$) equals supply ($Y_2$) equation for the general output of the economy. Equation (3) equates the demand for housing ($C_3$) to the stock available at the beginning of the period ($S_3$).$^{43}$ $B$ is the (exogenous) balance of trade for the New Zealand economy (in per capita terms) and in equation (4), $B$ is equated to the per capita value of exports ($p_3 Y_3$) minus the per capita value of imports ($p_4 Y_5$).$^{44}$ Thus $B$ is the balance of trade surplus (deficit if $B$ is negative) converted into New Zealand dollars.$^{45}$ In equation (5), the demand for leisure ($C_4$) is equated to the stock of economically relevant time in the period ($H$) minus time worked in the government sector ($-L_G$) minus time worked in the private sector ($Y_5$).$^{46}$ Equation (6) is the representative consumer’s budget constraint and equation (7) is the government’s budget constraint in per capita terms. The terms on the left hand side of (7) are per capita tax revenues on motor vehicles ($p_4 t_4 Y_4$), general consumption ($p_2 t_2 C_2$), housing ($p_3 t_3 C_3$), imports ($-p_4 t_4 Y_4$), labour earnings ($p_5 t_5 [H - C_4]$) and profits ($t_6 \pi$) less per capita subsidy expenditures on general output ($-p_2 s_2 Y_2$) and exports ($-p_3 s_3 Y_3$) plus the general government

---

$^{43}$ New additions to the stock of housing are included in investment $I$.

$^{44}$ Recall that $Y_4$ is indexed with a minus sign.

$^{45}$ Thus $p_3$ is the foreign export price and $p_4$ is the overseas import price converted into New Zealand dollars.

$^{46}$ Remember $Y_5$ is indexed with a minus sign since it is an input.
per capita budget deficit \(p_2D\).\(^{47}\) The terms on the right hand side of (7) are per capita government expenditures on goods \(p_2G\) and labour \(p_5L_G\) at producer prices plus the per capita transfer \(T\) that is required to keep consumers at their initial utility level as exogenous tax and subsidy rates are varied. \(T\) is always set equal to zero before the tax change takes place.

Given equations (1) — (5) and (7) and the identities (8) — (12), we can derive the consumer’s budget constraint (6) from the remaining equations. On the left hand side of (6), we have per capita expenditures on consumer goods including leisure, \((E)\). On the right hand side, we have the following sources of income: per capita after tax profits \(\left[1 - t_5\right]\pi\); the value of the consumer’s time endowment \(p_5\left[1 - t_5\right]H\); the value of consumer stocks of motor vehicles \(P_1S_1\) and housing \(P_3S_3\). Also appearing on the right hand side of (6) are some terms expressing the tax treatment of stocks (the terms \(-p_1\left[1 - \left(\delta_1 + r\right)\right]Y_1\) and \(-P_3t_3S_3\)) as well as some adjustments for the treatment of trade, investment and financing the deficit (the terms \(-B - p_2I - p_2D\)). The final term on the right hand side of the consumer’s budget constraint is \(T\), new transfer income from the government which compensates the consumer for any adverse changes in tax or subsidy rates.

Since the 7 equations (1) — (7) are dependent, we drop equation (7) in what follows. We replace the per capita consumption variables \(C_i\) by the price derivatives of the consumer’s expenditure function, \(\partial e[u, p_1\left(1 + \left(\delta_1 + r\right)\right), p_2\left(1 + t_2\right), p_3, p_5\left(1 - t_5\right)] / \partial p_i\) for \(i=1,2,3,4\) and we replace the per capita net output variables \(Y_i\) by the derivatives of the per capita profit function \(\pi\left[p_1, p_2\left(1 + s_2\right), p_3\left(1 + s_3\right), p_4\left(1 + t_4\right), p_5\right]\) with respect to its \(i\)th price variable for \(i=1,2,3,4,5\). The endogenous variables in the resulting system of 6 equations (1) — (6) are the following 6 variables: \(T, G, p_1, p_2, P_3\) and \(p_5\). The international prices for exports and imports, \(p_3\) and \(p_4\), are regarded as being exogenous to the model and these international prices act as numeraire prices for our model in each time period.

The utility level \(u\), the stock levels \(S_1\) and \(S_2\), investment \(I\), the balance of trade \(B\), the government’s labour requirements \(L_G\) and the real government deficit \(D\) were all regarded as exogenous variables.\(^{48}\) The tax and subsidy rate variables were the exogenous variables of interest. We totally differentiated equations (1) — (6) with respect to our 6 endogenous

\(^{47}\) \(D\) can be more accurately described as the real per capita deficit minus real per capita transfers from the government to consumers.

\(^{48}\) Our econometric models of consumer and producer behaviour were used to generate the per capita consumption and net output variables \(C_i^t\) and \(Y_i^t\). Then equations (1) — (5) and (7) were used to generate estimates for \(S_1^t, I^t, S_3^t, B^t, L_G^t\) and \(D^t\) respectively. All of these variables are listed in Appendix B.
variables and 7 of our exogenous tax and subsidy instruments. We did not differentiate with respect to \( t_3 \), the property tax rate, because the treatment of housing requires an intertemporal model. Various second derivatives of the expenditure function and the per capita profit function appeared in the resulting 6 simultaneous equations. We estimated these derivatives by converting our elasticity estimates discussed in chapters 5 and 6 above into derivatives. After inverting a 6 by 6 matrix for each period (using the matrix operations in the econometrics program SHAZAM), we were able to calculate the partial derivatives of \( G, T, p_2 \) and \( p_5 \) with respect to the \( t_i \) and \( s_j \).

We utilise the Allais-Debreu excess burden concept discussed in Section 4.4 above. Since utility is held constant, our indicator of overall welfare is simply the value of government consumption of goods \( G \) times the (constant) consumer price of general consumption \( P_2 \). Thus, define welfare as a function of the exogenous tax and subsidy rates as follows:

\[
W(t_1, t_2, t_4, t_5, t_6, s_2, s_3) = P_2 G(t_1, t_2, t_4, t_5, t_6, s_2, s_3).
\]

The left hand side of the government budget constraint is essentially net government revenues and the right hand side is essentially government expenditures. Since the right hand side of (7) has fewer terms than the left hand side, we define the net revenue function \( R \) as a function of the exogenous tax and subsidy instruments as follows:

\[
R(t_1, t_2, t_4, t_5, t_6, s_2, s_3) = P_2 G(t_1, \ldots, s_3) + P_5(t_1, \ldots, s_3)L + T(t_1, \ldots, s_3)
\]

where \( G(t_1, \ldots, s_3), T(t_1, \ldots, s_3), p_1(t_1, \ldots, s_3), p_2(t_1, \ldots, s_3), p_3(t_1, \ldots, s_3) \) and \( p_5(t_1, \ldots, s_3) \) are the solution functions to the system of simultaneous equations (1) — (6).

The Allais-Debreu general equilibrium measure of the marginal excess burden associated with increasing the tax rate \( t_i, MEB(t_i) \), is defined as in Section 4.4 as (minus) the rate of change in welfare defined by (13) divided by the rate of change in revenue defined by (14) with respect to \( t_i \); i.e., for \( i=1,2,4,5,6 \):

\[
MEB(t_i) = \left[ \frac{\partial W(t_1, \ldots, s_3)}{\partial t_i} \right] / \left[ \frac{\partial R(t_1, \ldots, s_3)}{\partial t_i} \right].
\]

Similar measures of marginal excess burden associated with decreasing the subsidy rate \( s_j \), can be defined as follows for \( j=2,3 \):

\[
MEB(s_j) = \left[ \frac{\partial W(t_1, \ldots, s_3)}{\partial s_j} \right] / \left[ \frac{\partial R(t_1, \ldots, s_3)}{\partial s_j} \right].
\]
7.2 Empirical results

The marginal excess burden measures defined by (15) and (16) were evaluated using the elasticities and data generated by our models of producer and consumer behaviour for New Zealand for the 20 years in our sample. The resulting marginal excess burdens are presented in Table 7.1.

Table 7.1: Marginal excess burdens for New Zealand

<table>
<thead>
<tr>
<th>Year</th>
<th>MEB(_1) Motor vehicles</th>
<th>General consumption</th>
<th>MEB(_4) Imports</th>
<th>MEB(_5) Labour</th>
<th>MEB(_2) General production</th>
<th>MEB(_3) Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>-0.0341</td>
<td>0.049</td>
<td>0.019</td>
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<td>0.037</td>
<td>0.183</td>
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<td>Average</td>
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<td>0.083</td>
<td>0.026</td>
<td>0.095</td>
<td>0.070</td>
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</table>

The marginal excess burden \( MEB(_6) \) associated with increasing the profits tax was zero in each period and hence was not listed in Table 7.1. This zero excess burden result is entirely due to our treatment of capital and investment: we assumed that investment was exogenously determined and not affected by capital taxation. In order to remove this assumption we would need to construct a complete intertemporal general equilibrium model and time constraints did not allow us to undertake this extension of our model. However, as was mentioned in Chapter 4, existing intertemporal general equilibrium models do find large excess burdens associated with the taxation of capital and so we would expect to also find large marginal excess burdens due to capital taxation.\(^49\)

From Table 7.1, we see that the marginal excess burden of financing increased government expenditures by increasing the tax rate $t_1$ on new motor vehicles is actually a marginal excess benefit which averaged 2.53 per cent over the sample period. This means that, on average, a government project financed by increased motor vehicle taxation could earn a real rate of return which was 2.53 per cent below the normal real rate of return and consumer overall welfare would remain unchanged. This anomalous result is due to the fact that motor vehicles appeared to be complementary to many goods, both in consumption and production.

The important marginal excess burdens are $MEB(t_2)$ and $MEB(t_5)$, those burdens associated with increasing consumption and labour taxation. Both of these excess burdens are quite significant: an average of 8.3 per cent for consumption taxation and an average of 9.5 per cent for labour taxation. Because we use a general equilibrium framework, our deadweight cost estimates apply year after year once a change in taxation has occurred. Consequently, if a government project is to be justified taking deadweight losses into account, it must provide a return each year which exceeds its direct cost (including a normal return) by at least the amount of the deadweight cost. This is equivalent to earning an ongoing real rate of return over and above the normal rate of return by at least the estimated percentage of deadweight costs. Thus, a government project financed by additional consumption (labour) taxation should have on average earned a real rate of return 8.3 per cent (9.5 per cent) above the normal real rate of return in order to overcome the adverse effects of increased taxation. These are very large excess rates of return since in most countries the after tax real rate of return is between 1 and 3 per cent.\footnote{We estimated the private sector's average real rate of return to be 0.6% over the 20 years in our sample.}

However, the sample average excess burdens do not tell the whole story. Examination of Table 7.1 and the tax rates listed in Table B.3 of Appendix B show that as tax rates in the New Zealand economy increased, marginal excess burdens have also tended to increase. Thus, the marginal excess burden associated with increased consumption (labour) taxation grew from 4.9 per cent (5.3 per cent) in 1972 to 13.7 per cent (18.3 per cent) in 1991. These are spectacular rates of increase.

It can be seen from Table 7.1 that the marginal excess burden associated with increasing the tax rate on international trade $t_4$ averages 2.6 per cent during the sample period. The annual $MEB(t_4)$ showed only a gradual upward trend. The general tendency for excess burdens to increase markedly over time was offset in this case by the reductions in trade taxes that took place over the 1970s and 1980s.