Chapter 5

THE DEADWEIGHT COSTS OF CAPITAL TAXATION IN AUSTRALIA

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5.1 INTRODUCTION

Taxation reform is currently the focus of attention in Australia. The ramshackle wholesale sales tax was replaced on 1 July 2000 with a goods and services tax and the Review of Business Taxation has recommended extensive changes to some aspects of capital taxation. These include the reduction of the company tax rate from 36 per cent to 30 per cent and the removal of accelerated depreciation. However, many of these changes – particularly those affecting capital taxation – are being made in the absence of detailed quantification of the effects of both the old and new tax regimes.

This chapter takes the first steps towards creating a consistent analytical framework to analyse the efficiency costs of different taxes to the Australian economy. In particular, it provides estimates of the deadweight costs or marginal excess burden of capital taxation in Australia.

In recent years there has been a growing focus overseas on the costs of raising taxation revenue. Communities have come to realise that far from being free, government expenditure has to be financed sooner or later by increased taxation and that taxation imposes a number of costs on the economy. As well as the direct cost of the extra revenue and associated administrative and compliance costs, an important additional cost arises from the changes in behaviour induced by taxation. Taxes distort the
incentives to work, save and invest and the pattern of input use and production in the economy. These distortions impose costs on the economy by reallocating resources from their most productive uses to less productive ones. The losses created are known as deadweight costs or the excess burden of taxation. The deadweight cost of taxation is a measure of the value of the opportunities that are effectively lost when taxation diverts labour, land and capital from their best uses. By calculating the deadweight costs of taxation we can gauge the potential effects of taxation on the economy and society and work out the least costly combination of taxes. The size of deadweight costs is influenced by a range of factors but they are likely to be largest when the actions of producers and consumers are highly responsive to after-tax prices, when existing marginal tax rates are high and when savings are highly responsive to after-tax returns.

An earlier study by Dieuwert and Lawrence (1994) has done much to raise the awareness of New Zealand policy-makers and the general community to the deadweight costs of taxation in New Zealand. The key findings of the study were that the deadweight costs associated with labour taxation have increased from 5 per cent to over 18 per cent in the 20 years up to 1991. Over the same period the marginal excess burden of consumption taxation (all indirect taxes other than property taxes and import duties) has increased from 5 per cent to around 14 per cent.

However, while our first study made a number of advances in the measurement of deadweight costs, the estimates obtained are likely to be relatively conservative. By estimating a static model which treated investment as exogenous and capital as fixed each period we were not able to calculate the marginal excess burden of capital taxation. Other studies which have attempted to introduce dynamics and model capital accumulation decisions have shown that the marginal excess burden of capital taxation is generally higher than that for labour given capital's far greater mobility. This is especially likely to be the case for small open economies such as Australia and New Zealand trading in a world of ever-increasing capital mobility and globalisation.

There have been a few previous studies attempting to estimate marginal excess burdens for Australia, notably those of Findlay and Jones (1982), Han (1996) and Campbell and Bond (1997). However, these studies all use static models and concentrate on labour and commodity taxation. While capital tax deadweight losses are likely to be far higher, there have been few studies that have successfully quantified them due to the conceptual and implementation difficulties associated with building dynamic models. The work of Jorgenson and Yun (1991) in the United States is one important exception.

In this chapter we report the results of calculating dynamic deadweight losses for capital taxes in Australia based on an econometric model of the production sector. A brief outline of the chapter follows.

Section 5.2 provides an overview of our Australian database. Additional details are contained in our Data Appendix.

In section 5.3, we explain why taxing the return to capital can be expected to reduce the real output of an economy and create an efficiency loss in the context of a simple production function model.

In section 5.4, we discuss how the cost of a durable input should be allocated across the useful life of the input. This leads us into a discussion of the user cost of capital.

Sections 5.5 to 5.7 are more technical. These sections gradually relax some of the restrictive assumptions made in section 5.3. In particular, we need to generalize the model explained in section 5.3 to cover the case of many (noncapital) inputs and outputs and many capital inputs. We also need to extend the model to an open economy.

Section 5.8 introduces our econometric model which is based on relatively recent developments in the theory of flexible functional forms. However, in section 5.9, we discuss a technical problem with the functional form that is suggested in section 5.8: namely, it will tend to generate somewhat artificially trending elasticities in many data sets. Given the importance of getting accurate elasticity estimates for the computation of excess burdens, we address this problem in section 5.9. We suggest a new functional form that is completely flexible at two data points instead of the usual single data point.

Section 5.10 presents our empirical estimates for Australia estimated using data for the period 1967-1997.

Section 5.11 uses the elasticity estimates presented in section 5.10 and the theory of excess burden measurement developed in section 5.7 to present empirical estimates of the marginal excess burdens of capital taxation in Australia for the years 1967-1997. However, we regard our estimates as being preliminary: there is more work to be done both on developing better estimates of the allocation of taxes in Australia and in estimating more disaggregated econometric models.

Section 5.12 concludes.
5.2 KEY PERFORMANCE INDICATORS FROM THE AUSTRALIAN DATABASE

Before explaining the general approach used to calculate the capital tax deadweight losses, we will briefly outline some of the key performance indicators derived from our Australian database. The approach used to construct the database is generally similar to that used in our 1994 New Zealand study, with the exception of the treatment of capital and investment goods. The database runs for 31 years from 1966-67 to 1996-97 and the market sector producer model estimated contains 12 goods. These comprise 3 variable outputs (general private consumption; government consumption of goods and services; and exports), 2 variable inputs (imports; and labour), 3 investment goods (plant and equipment; non-residential and other construction; and inventories) and 4 capital stocks (plant and equipment; non-residential and other construction; inventories; and business and agricultural land). The construction of the database is described briefly in the appendix.

![Figure 5.1. Diewert-Lawrence Total Factor Productivity and ABS MFP Indexes](image)

The best summary measures of economic performance are total factor productivity and the economic rate of return. Total factor productivity (TFP) measures the amount of total outputs produced per unit of overall inputs. Technical change, improved management and the elimination of inefficient work practices can bring about improvements in TFP. The economic rate of return provides a measure of true profitability based on the current market value of assets.

The Australian economy’s market sector TFP for the 31-year period up to 1996-97 is presented in Figure 5.1 along with the corresponding Australian Bureau of Statistics (ABS) multifactor productivity (MFP) index. Over this period the market sector’s output grew at an average annual rate of 3.5 per cent while its inputs grew at an average rate of 2.0 per cent leading to an average annual TFP growth rate of 1.5 per cent. The ABS MFP series follows a similar trend over the period and produces the same average annual TFP growth rate. However, the indexes diverge in some years due to our more comprehensive TFP index covering a wider range of inputs, using different user costs for capital inputs and using producer prices to value all outputs and inputs.

![Figure 5.2. Nominal Rates of Return](image)
The profitability of the market production sector is reflected in the nominal rates of return presented in Figure 5.2. The before-tax nominal rate of return averaged 10.7 per cent for the 31-year period. The highest before-tax nominal rate of return achieved was 17.8 per cent in 1973. The lowest nominal before-tax rate of return was 7.5 per cent in 1997. The after-tax nominal rate of return averaged 9.3 per cent.

The weighted average real after-tax rate of return observed for Australia, after allowing for asset-specific rates of inflation, over the 31 years to 1997 was 4.2 per cent. This is consistent with the long-term real after-tax rate of return for most western countries which Robbins and Robbins (1992) found to lie in the range of 3 to 5 per cent.

![Figure 5.3. Labour and Capital Tax Rates](image)

In calculating the deadweight losses caused by taxation it is necessary to know the size of the 'wedges' taxes impose between the price paid by the consumer or user and the price received by the producer or supplier. Tax rates on labour and capital income are presented in Figure 5.3. Capital income is calculated as the profit the private sector earns from its production activities and is defined as the value of its outputs (consumption goods, sales to government, exports and investment goods) less the value of variable inputs (imports and labour).

![Figure 5.4. Capital Tax Rate on Assets](image)

The average tax rate on labour income increased over the 31-year period from a rate of 12.5 per cent in 1967 to 25.6 per cent in 1997. The labour tax rate peaked in 1989 at 26.4 per cent. Capital tax rates on profit have fluctuated more widely due to the residual nature of profits as defined. After starting at 27 per cent, capital tax rates progressively declined to a rate of 14.7 per cent in 1980. Since then capital tax rates have increased steadily to finish at a level of 53 per cent in 1997. This increase in the overall rate of capital tax can be attributed to the introduction of capital gains taxes, increasing reliance on transactions taxes and the progressive tightening of exemptions from the tax base.

To obtain a more accurate representation of capital tax rates it is necessary to look at capital tax payments relative to the value of assets. It is this tax rate which drives investment decisions. From Figure 5.4 it can be seen that capital tax rates started at 2.8 per cent in 1967 then rose to 3.2 per cent in 1974 before falling back to 2.1 per cent in 1983. Over the remainder of the period the capital tax rate on assets increased sharply to finish at 4 per cent in 1997. The more stable and more important capital tax rate on assets series confirms that changes to the Australian tax system since the mid 1980s have fallen relatively heavily on capital.
5.3 A SIMPLIFIED INTRODUCTION TO OUR METHODOLOGY

The construction and estimation of the dynamic models necessary to allow capital tax deadweight losses to be calculated is a notoriously complex process. In this section, we present a highly simplified summary of the full approach used in the study. The reader who is not interested in technical details can read this section in order to get the broad outline of our methodological approach and then skip the following technical sections and turn to the end of this chapter where we table our estimated marginal excess burdens.

There are many approaches to the determination of the efficiency costs of capital taxation. The approach we take is the following one. We assume that the private production sector of the economy uses inputs of capital, labour and imports to produce consumption goods, exports, government purchases of goods and services and investment goods. We assume that investment goods produced in the current year are added to the capital stock at the beginning of the following year. Domestic households and foreign investors require interest payments in order to induce them to supply financial capital to the production sector. We view the business income tax as falling on the return to capital and, thus, the rate of return that the private production sector must earn. The effect of the capital tax will be to reduce the equilibrium level of capital, investment and domestic net product. In an equilibrium situation, investment goods are produced so as to just offset depreciation and an optimal capital stock is one that maximises net output less interest payments subject to primary resource constraints. Capital taxation moves the economy away from this optimal situation. Our approach to capital taxation is based on that developed by Diewert (1981; 65–68) (1988; 19–23).

An empirical description of the production function (or the set of technologically feasible inputs and outputs for the private production sector) is required in order to evaluate the efficiency effects of varying levels of capital taxation. Broadly speaking, there are two approaches to the empirical estimation of production functions or technology sets: (i) the applied general equilibrium modelling approach pioneered by Shoven and Whalley (1972) (1984); and (ii) the econometric approach using flexible functional forms as used by Jorgenson and Yun (1991) and Diewert and Lawrence (1994) (1996). In the first approach, simplified production functions are estimated using the data pertaining to the economy for one period. In the second approach, first and second order parameters that characterise the technology are econometrically estimated using time series data for the economy under consideration. We use the second approach in our study since it will be empirically more accurate than the first approach.

In our econometric model, we have three variable capital stocks: equipment, structures and inventory stocks. Land is a fourth capital stock which is regarded as fixed. All other outputs and inputs are regarded as variable in our econometric work so their prices are taken as exogenous and the corresponding quantities are regarded as endogenous variables.

We illustrate the efficiency costs of taxing capital by considering a very simple model of a closed economy. We suppose that units of private sector reproducible capital are combined with factors that are held fixed during the short run to produce units of aggregate output that can be used for either consumption $C$ or investment $I$. Letting $L$ denote the number of units of labour and other factors that are fixed in the short run we have:

$$ Y = C + I = f(K, L) \tag{5.1} $$

where $f$ is the production function, $Y$ denotes output and $K$ denotes the beginning of the period capital stock. In this simple illustration we are assuming that units of the investment good $I$ are perfectly substitutable with units of the consumption good $C$. We also assume that investment goods produced during the current period are added to the reproducible capital stock at the beginning of the following period. Thus, investment goods can be viewed as intertemporal intermediate inputs into the private production sector: $I$ is produced this period so that it can be used as capital input next period and offset this period’s depreciation of the capital stock.

We assume that each unit of the capital stock has a physical decline in its efficiency over the period at the rate $\delta$; i.e. if $K$ units of the capital stock are in place at the beginning of the period, only $(1-\delta)K$ units are available for further use at the end of the current period.

We consider a steady state capital optimisation problem where investment is set equal to depreciation; i.e. we replace $I$ in (1) by $\delta K$ and maximise $C = f(K, L) - I = f(K, L) - \delta K$ with respect to $K$. Another way of viewing depreciation in this formulation is to regard it as a cost of production; i.e. the capital used at the beginning of the period, $K$, should be assessed a charge equal to the decline in value of the capital
stock due to deterioration and a shorter life. Another charge that should be assessed against the starting capital stock is the opportunity cost of capital; i.e. the interest cost which will be just sufficient to induce owners of the capital stock to hold the capital stock through the period. Thus, if the interest rate is \( r \), then the optimal long run capital stock \( K^* \) is the solution to the following maximisation problem:

\[
\max_K \left\{ f(K, L) - (r + \delta)K \right\}.
\]  
(5.2)

Since we are regarding \( L \) as fixed, write the production function \( f(K, L) \) as \( f(K) \). Then the first order necessary condition for \( K^* \) to solve (5.2) is:

\[
f'(K^*) = r + \delta
\]  
(5.3)

where \( f' \) denotes the first derivative of \( f \). We assume that the following second order sufficient condition is also satisfied:

\[
f''(K^*) < 0.
\]  
(5.4)

The geometry of the unconstrained maximisation problem (5.2) is illustrated in Figure 5.5 below. The curved line through the origin represents the production function constraint, \( C + I = f(K) \), while the straight line through the origin represents the depreciation and interest cost of capital. The difference between the two lines represents sustainable consumption (after interest payments) or surplus as a function of the beginning of the period capital stock \( K \). The maximum sustainable surplus \( S^0 \) is achieved at the capital stock \( K^0 \) where the slope of the production function equals the slope of the total capital cost function.

When capital is taxed, private producers will face the price \( r + \delta + \tau \) per unit of capital used, where \( \tau \) is the capital asset tax rate. Thus, instead of solving (5.2) in the long run, private producers will be induced to choose the capital stock \( K^* \) which solves:

\[
\max_K \left\{ f(K) - (r + \delta + \tau)K \right\}.
\]  
(5.5)

We may regard the \( K^* \) which solves (5.5) as a function of the asset tax rate \( \tau \); i.e. \( K^* = K(\tau) \). This solution to (5.5) will satisfy the following first order necessary condition:

\[
f'[K(\tau)] = r + \delta + \tau.
\]  
(5.6)

The fact that producers must pay capital taxes to the government increases the cost of using reproducible capital as an input and the resulting steady state capital stock \( K(\tau) \) is smaller than the optimal capital stock, \( K^0 = K(0) \), which solved (5.2). The tax distorted surplus, \( S^* = f(K^*) - (r + \delta)K^* \) is smaller than the optimal surplus \( S^0 = f(K^0) - (r + \delta)K^0 \) (see Figure 5.5).

Figure 5.5. Stylised Loss from Capital Taxation

Figure 5.5 illustrates qualitatively the effects of taxing reproducible capital — the higher the level of taxation, the lower will be the long run level of capital utilised and the corresponding surplus. In what follows, we indicate how a quantitative estimate of the decline in the sustainable surplus can be obtained.

First, differentiate equation (5.6) with respect to \( \tau \). We obtain the following equation for the change in capital due to a small increase in the tax rate, \( K'(\tau) \):
\[ K'(\tau) = 1 / f''[K(\tau)], \quad (5.7) \]

where \( f'' \) is the second derivative of the production function and will be negative under the usual assumptions on the production function. Now define producer surplus (or sustainable consumption after interest payments) as a function of the capital tax rate \( \tau \) as follows:

\[ S(\tau) = f[K(\tau)] - (r + \delta) \cdot K(\tau). \quad (5.8) \]

Differentiating (5.8) with respect to \( \tau \) and using (5.6) yields the following formula for the rate of change of surplus with respect to the level of capital taxation:

\[ S'(\tau) = [f''[K(\tau)] - (r + \delta)] \cdot K'(\tau) = \tau K'(\tau). \quad (5.9) \]

Evaluating (5.9) at \( \tau = 0 \) yields

\[ S'(0) = 0. \quad (5.10) \]

Differentiating (5.9) with respect to \( \tau \) and evaluating the resulting derivative at \( \tau = 0 \) yields

\[ S'(0) = K'(0) = 1 / f''[K(0)] < 0 \quad (5.11) \]

where the second equality in (5.11) follows from (5.7) evaluated at \( \tau = 0 \) and the inequality follows from \( K'' = K(0) \) and (5.4). We now use (5.10) and (5.11) to form the following second order Taylor series approximation to \( S(\tau) \):

\[ S(\tau) \approx S(0) + S'(0) \tau + \frac{1}{2} S''(0) \tau^2 \]
\[ = S(0) + \frac{1}{2} \tau^2 / f''[K(0)]. \quad (5.12) \]

Define \( L(\tau) \) as the loss of sustainable output as a fraction of optimal output \( Y(0) \):

\[ L(\tau) = [S(0) - S(\tau)] / Y(0). \quad (5.13) \]

Using (12), a second order approximation to \( L(\tau) \) is

\[ A(\tau) = -\frac{1}{2} \tau^2 / Y(0) \cdot f''[K(0)]. \quad (5.14) \]

We need to provide an economic interpretation for the second derivative of the production function, \( f''[K(0)] \), evaluated at the optimal capital stock \( K(0) \). The first derivative of the production function, \( f'[K(0)] \), is the optimal return to one unit of the capital stock or the rental price of capital, \( P_K = r + \delta \). Thus, the second derivative can be interpreted as the change in the rental price due to a small change in the use of capital, \( dp_K[K(0)] / dK \):

\[ dp_K[K(0)] / dK = f''[K(0)] = f''[K'] < 0 \quad (5.15) \]

where the inequality follows from (4). We convert \( dp_K / dK \) into a non-negative (inverse) elasticity of demand for capital \( e \) by changing the sign of \( f''[K(0)] \) and multiplying by \( K(0) / P_K[K(0)] = K(0) / f'[K(0)] \):

\[ e = -f''[K(0)] K(0) / f'[K(0)] = -f''[K(0)] K(0) / (r + \delta) \quad (5.16) \]

where the second equality in (5.16) follows from (5.3).

It is also useful to define the economy's optimal capital output ratio \( \gamma \) as the ratio of the optimal capital stock \( K(0) \) to the optimal gross output \( Y(0) = f[K(0)] \):

\[ \gamma = K(0) / Y(0). \quad (5.17) \]

Substitution of (5.16) and (5.17) into (5.14) yields the following formula for the approximate loss of producer surplus as a fraction of optimal output:

\[ A(\tau) = (1/2) \tau^2 \gamma / e(\tau + \delta) > 0. \quad (5.18) \]

For advanced industrial economies, a typical range for the capital tax rate \( \tau \) is 0.01 to 0.03 (ie one percent to three percent of the asset value of capital), for the capital output ratio \( \gamma \) is two to four, for the inverse elasticity of demand for capital \( e \) is 0.5 to 1.0, for the real after tax rate of return \( r \) is 0.01 to 0.05 and for the depreciation rate \( \delta \) is 0.04 to 0.08. If we substitute the midrange values for these parameters into the right hand side of (5.18), we find that the approximate output loss due to capital taxation at the rate \( \tau = 0.02 \) is \( A(0.02) = 0.0089 \) or 0.89 percent of gross domestic product.

Table 5.1 below indicates how the approximate loss of output \( A(\tau) \) varies as each parameter varies between the low and high values of its assumed range while letting the remaining parameters equal their midrange values.
Table 5.1. Percentage Loss of Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Midrange</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Values</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.05</td>
<td>2.00</td>
</tr>
<tr>
<td>% loss</td>
<td></td>
<td>0.89</td>
<td>0.22</td>
<td>2.00</td>
<td>1.14</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Note that the approximate loss of output increases as the square of the capital tax rate $\tau$ so if $\tau$ increases from 0.02 to 0.03 and the other parameters remain at their midpoint values, the loss of output due to capital taxation increases from 0.89% of GDP to two percent of GDP. These output losses persist year after year so that the present value of these annual output losses is substantial.

It should be emphasised that the above efficiency losses induced by the taxation of capital are entirely avoidable: equivalent amounts of revenue could be raised by taxing the final outputs of the private production sector or by taxing primary inputs. We note that the latter two forms of taxation do not involve a loss of productive efficiency for the economy whereas taxing an intermediate input like capital invariably involves a loss of productive efficiency.  

The efficiency losses listed above in Table 5.1 are likely to underestimate substantially the actual losses that are induced by capital taxation in an industrialised economy. The above model assumes only one capital stock with an average tax rate of $\tau$ which is applied to the asset value of reproducible capital. In actual economies, the system of business income taxation invariably taxes lightly some components of the capital stock and taxes other components very heavily. The efficiency losses associated with the differential taxation of each type of capital will grow approximately as the square of the tax rate. Thus, the large losses associated with the heavily taxed components will not be balanced by the small losses associated with the lightly taxed components and the total loss will be much larger than the loss obtained by applying an average tax rate to the total reproducible capital stock.$^2$

Another diagram may be helpful in illustrating the efficiency costs of capital taxation. Note that $K(\tau)$, the capital stock solution to equation (5.6), can be regarded as the long run demand for reproducible capital as a function of the tax rate $\tau$. Now rewrite equation (5.6) as follows:

$$ f'[K(r + \delta + \tau)] = r + \delta + \tau $$

(5.19)

ie the demand for capital $K(r + \delta + \tau)$ which solves (5.19) can be written as a function of the tax distorted rental price of capital, $r + \delta + \tau$. In Figure 5.6 below, the inverse of this demand for capital function is graphed as the curve DD. If there were no capital taxes, capital would be supplied to the private production sector at the rental price $r + \delta$ which would just cover the real interest and depreciation costs of using a unit of capital for the period. This horizontal supply of capital curve intersects the demand curve at the point $A$. The imposition of the capital tax $\tau$ shifts the supply of capital curve up and this tax distorted supply curve intersects the demand curve at $B$. Note that the equilibrium level of capital used has decreased from $K^e$ to $K^*$. 

![Figure 5.6. Alternative Representation of Capital Tax Loss](image)

Equation (5.19) can be integrated to obtain an expression for the gross change in output; i.e., the change in gross output produced due to capital taxation before deducting depreciation and interest costs; i.e. we have

$$ f(K^e) - f(K^*) = \int_{K^e}^{K^*} f'(K)K\,dK = \text{Area } BAK^eK^* $$

(5.20)
The efficiency cost of capital taxation is defined to be the net change in output after deducting depreciation and interest payments:

\[
S(0) - S(t) = \left[f(K^o) - (r + \delta)\right] - \left[f(K^*) - (r + \delta)K^*\right] \\
= f(K^o) - f(K^*) - (r + \delta)(K^o - K^*) \\
= \text{Area } BAK^*K^* - \text{Area } CAK^*K^* \\
= \text{Area } ABC. 
\]

Thus, the area of the shaded triangular region under the demand curve is a measure of the efficiency costs of capital taxation. This is a producer surplus measure of deadweight loss.

We now linearise the demand curve around the undistorted equilibrium point \( A \) and use the triangle \( AEF \) as an approximation to the exact deadweight loss \( ABC \). It can be verified that the absolute value of the slope of the linear approximation to DD at \( A \) is \( \varepsilon(r + \delta) / K^o \), where \( \varepsilon \) is the inverse elasticity of demand for capital. The vertical distance \( EF \) in Figure 5.6 is equal to \( \tau \) so the horizontal distance of the triangle \( AEF, AF \), will equal the vertical distance \( \tau \) divided by the slope \( \varepsilon(r + \delta) / K^o \). Thus,

\[
\text{Area } AEF = \left(1/2\right) \left[\frac{\varepsilon(r + \delta)}{K^o}\right] \tau \\
= \left(1/2\right) \tau^2 K^* / \varepsilon(r + \delta). 
\]

We note that for small tax rates \( \tau \), the approximate loss measure \( AEF \) should be quite close to the exact loss measure \( ABC \).

The approximate total efficiency loss or excess burden of capital taxation defined by (5.18) or (5.22) is not the most interesting number from the viewpoint of economic policy. A more interesting concept, initiated by Browning (1976) (1987), is the marginal excess burden (MEB) of capital taxation. This concept compares the increase in efficiency loss due to a small increase in the level of capital taxation to the increase in tax revenue that can be attributed to the tax increase.

In Figure 5.7 above, we have reproduced the approximate deadweight loss triangle \( AEF \) as in Figure 5.6 and this triangle corresponds to the efficiency loss when the capital tax rate is \( \tau \). We now increase the capital taxation rate by a small amount \( \Delta \tau \) and we note that the increase in efficiency loss is equal to the triangle \( EIJ \) plus the rectangle \( EFKJ \). The initial tax revenue is equal to the area of the rectangle \( EFNLM \) and the new tax revenue is equal to the area of the rectangle \( IKNL \). Thus, the change in tax revenue is equal to the area of \( UML \) minus the area of \( EFKJ \). This change in tax revenue (the incremental benefits of the tax increase) can be compared to the increased efficiency loss, \( EFKI \), (the incremental costs of the tax increase). The ratio of the latter to the former is termed the marginal excess burden of taxation. Note that if the initial level of taxation \( \tau \) is very high, then the incremental tax revenue can be negative; i.e. the induced reduction in the use of capital can outweigh the increased tax revenue per unit of capital used by private producers and the resulting marginal excess burden is negative.

We now provide an analytic formulation that corresponds to the marginal excess burden measure described by Figure 5.7. Equation (5.18) describes the approximate efficiency loss \( A(\tau) \) as a fraction of optimal
output \( Y(0) \). Differentiating this function with respect to \( \tau \) gives us the following formula for the marginal efficiency loss (as a fraction of \( Y(0) \)):

\[
A'(\tau) = \tau \gamma / \varepsilon (r + \delta) > 0. \tag{5.23}
\]

Define tax revenue as a function of the capital tax rate \( \tau \), \( T(\tau) \) as follows:

\[
T(\tau) = \tau K(\tau). \tag{5.24}
\]

Note that \( T(0) = 0 \) and the first and second order derivatives of \( T(\tau) \) evaluated at the no distortion point \( \tau = 0 \) are:

\[
T'(0) = K(0) \tag{5.25}
\]

\[
T''(0) = 2K'(0) \tag{5.26}
\]

Thus a second order Taylor series approximation to \( T(\tau) \) is

\[
T(\tau) \approx K(0)\tau + K'(0)\tau^2 \tag{5.27}
\]

Define the approximate benefit function \( B(\tau) \) as the right hand side of (5.27), divided by the optimal output \( Y(0) \):

\[
B(\tau) = \frac{[K(0)\tau + K'(0)\tau^2]}{Y(0)} = \frac{\gamma \tau + K'(0)\tau^2}{Y(0)} \tag{5.28}
\]

\[
= \frac{\gamma \tau - \gamma \tau^2}{\varepsilon (r + \delta)} \tag{5.28}
\]

Now differentiate (5.28) with respect to \( \tau \) which gives us a formula for the marginal benefit of increasing capital taxes \( B'(\tau) \) (as a fraction of optimal output \( Y(0) \)):

\[
B'(\tau) = \gamma [1 - 2\tau / \varepsilon (r + \delta)]. \tag{5.29}
\]

Finally, define the (approximate) marginal excess burden of capital taxation \( MEB(\tau) \) as the ratio of the marginal efficiency cost \( A'(\tau) \) defined by (5.23) to the marginal benefit \( B'(\tau) \) defined by (5.29):

\[
MEB(\tau) = A'(\tau) / B'(\tau) = \tau / [\varepsilon (r + \delta) - 2\tau]. \tag{5.30}
\]

Note that \( MEB(\tau) \) depends not only on the rate of capital taxation \( \tau \) but it also depends on the inverse elasticity of demand for capital \( \varepsilon \), the real interest rate \( r \) and the depreciation rate \( \delta \). However, \( MEB(\tau) \) does not depend on the capital output ratio \( \gamma \) in contrast to our earlier formula for the approximate total efficiency loss (as a fraction of optimal output), \( A(\tau) \), defined by (5.18).

In Table 5.2 below, we evaluate \( MEB(\tau) \) defined by (5.30) at our midrange estimates for the capital tax rate \( \tau = 0.02 \), the real interest rate \( r = 0.04 \), the depreciation rate \( \delta = 0.06 \) and the (inverse) elasticity of demand for capital \( \varepsilon = 0.75 \). We also table \( MEB(\tau) \) as each parameter varies between the low and high values of its assumed range, while letting the other parameter values equal their assumed midrange values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Values</th>
<th>( \tau )</th>
<th>( r )</th>
<th>( \delta )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEB</td>
<td>0.727</td>
<td>0.211</td>
<td>0.00</td>
<td>0.00</td>
<td>0.75</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>4.000</td>
<td>1.600</td>
<td>0.471</td>
<td>0.471</td>
<td>4.000</td>
<td>0.400</td>
</tr>
</tbody>
</table>

From Table 5.2, the marginal excess burden of capital taxation when \( \tau = 0.02, r = 0.03, \delta = 0.06 \) and \( \varepsilon = 0.75 \) is 72.7 percent. This means that if the government is contemplating financing a new recurring program expenditure by increasing capital taxation, then for each dollar of tax revenue spent on the program, its benefits should exceed 1.727 dollars; i.e., the loss of productive efficiency that is induced by a tax increase that yields an extra dollar of revenue is 72.7 cents. In contrast to the rather small numbers in Table 5.1, the numbers in Table 5.2 are rather large. For example, if the level of capital taxation increases from two percent to three percent, then the marginal excess burden increases from 72.7 percent to 400 percent; i.e., the marginal benefits that accrue to an incremental program that is financed by the increased level of capital taxation should exceed five dollars for each dollar of revenue raised. Of that five dollars, one dollar of benefits is required to make up for the one dollar of tax revenue that is diverted from private uses and the other four dollars of benefits are required to offset the loss of output that the increased level of capital taxation induces in the private production sector.
Table 5.2 indicates that the marginal excess burden of capital taxation is very sensitive to the parameter values that were inserted into formula (5.30). This is unfortunate, because it is difficult to determine \( \tau, r, \delta \) and \( \varepsilon \) with great precision in actual economies. Hence relatively small errors in these parameters can translate into relatively large errors in the associated excess burdens. However, our qualitative assessment of the numbers presented in Table 5.2 is that the marginal excess burdens generated by the taxation of reproducible capital are likely to be considerably larger than the marginal excess burdens generated by taxing consumption or labour.\(^3\) Our reason for this a priori expectation is that even though \( \tau \) is a relatively small fraction, it is a relatively large proportion of the undistorted rental price of capital \( r + \delta \) and hence has a relatively large effect on the allocation of resources.

The highly simplified model presented in this section assumed that there was no inflation in the economy and that the output good was identical to the capital good. In the following section, we relax these assumptions and look more closely at the problems involved in determining what the cost of using a unit of capital for a period is (as opposed to its purchase price).

### 5.4. THE USER COST OF CAPITAL

In the national accounts, interest (or more generally, the return to capital) is treated as a transfer (or as a distribution out of surplus) and not as a cost of production. Thus, the only cost associated with the use of reproducible capital in the national accounts is depreciation. The costs of using nonreproducible capital inputs like land are totally ignored in the national accounts. However, economic theory regards interest as a cost of production – it is the cost of inducing investors to defer consumption for the period under consideration. Thus, an appropriate cost of capital from the viewpoint of production theory and the measurement of deadweight costs is the user cost of capital which includes both interest and depreciation costs. This concept dates back at least a century to the economist Walras (1954; 269) and the industrial engineer Church (1901; 907-908). In more recent times, it was generalized to deal with the complications of the business income tax by Jorgenson (1996). We review this literature below.

We begin by deriving the user cost of capital in a world without the taxation of capital. We suppose that a firm purchases a capital asset (or durable input) such as a machine, a computer, a building, an inventory item or a plot of land at the beginning of an accounting period at the price \( P \). Since a durable asset by definition lasts longer than one period, the firm cannot simply charge the entire cost of the asset to the first accounting period: it must distribute the cost over the useful life of the asset. During the period, the asset declines in value according to the depreciation rate \( \delta \). So at the end of the accounting period, if there were no inflation during the period, the asset would be worth \( (1-\delta)P \). However, normally there will be some change in the price of the asset over the period. Let the inflation rate for a unit of the asset be denoted by \( \alpha \), so that the end of the period price for the depreciated asset will be \( (1-\delta)(1+\alpha)P \). Now we are ready to work out what the net cost of using the asset is for the first accounting period. The beginning of the period user cost of the asset, \( B \), is defined to be the asset’s purchase cost \( P \) minus the discounted end of period market value of the asset:

\[
B = P - (1-\delta)(1+\alpha)P(1+R) \tag{5.31}
\]

where \( R \) is the average cost of capital that the firm faces during the period; i.e., it is an average of the bond interest rate and the equity cost of capital that it faces at the beginning of the accounting period.\(^4\)

The above user cost of capital is the one that economists are most familiar with since they are used to working with discounted values. However, it is also possible to work with the end of the period user cost \( U \), which simply multiplies \( B \) by \( 1+R \):

\[
U = P(1+R) - (1-\delta)(1+\alpha)P \tag{5.32}
\]

\[
= [R - \alpha + \delta (1+\alpha)]P. \tag{5.33}
\]

Formula (5.32) for the end of period user cost of an asset should be intuitively appealing to accountants. As part of the cost of using the asset during the period, we need to charge not only the purchase price \( P \) of the asset, but also the direct bond interest costs associated with financing the purchase of the asset plus the opportunity cost of tying up equity capital in the asset (this is the cost \( RP \)). However, these costs are partially offset by the fact that at the end of the period, we have an asset that could be sold for the amount \( (1-\delta)(1+\alpha)P \). Thus, the net cost of using the asset during the
period (including the opportunity costs of the equity capital that is tied up in the purchase of the asset) is the right hand side of (32). Note that the end of the period value of the asset, \((1-\delta)(1+\alpha)P\), becomes next period's beginning of the period value of the asset.

Formula (5.33), which is simply a rearrangement of (5.32), also has a nice intuitive interpretation. It says that the user cost of an asset has an interest/opportunity cost of capital equal to \(RP\) less a capital gains component \(\alpha P\) plus a depreciation component that is indexed for asset inflation \(\delta (1+\alpha)P\). The first two components of this formula can be combined into the term \((R - \alpha)P\) which can be interpreted as a real interest rate term.

The user cost of capital plays a fundamental role in any economic approach to modeling producer behavior. It plays the role of a period specific price for a durable capital input and is analogous to a wage rate (as the price for a unit of labor) or an output price (as the price for a unit of output).

We now bring business income taxes into the picture. Unfortunately, the business income tax does not treat capital costs in the manner indicated above by formulae (5.32) or (5.33), which is an economic approach based on current opportunity costs. Most systems of business income taxation use the conventions of historical cost accounting to define period by period capital costs that are to be used in defining income for tax purposes. Thus, for tax purposes, an accounting user cost \(A\) for the asset described in the previous section might be defined by something like the following formula:

\[
A = (fR + d)P
\]  

(5.34)

where \(f\) is the fraction of interest and equity cost that is tax deductible (typically interest costs are deductible but equity opportunity costs are not so that \(f\) would depend on the firm's debt-equity ratio) and \(d\) is the depreciation rate for the asset that is prescribed by the tax code. The actual formula for \(A\) is typically a lot more complicated than the right hand side of (5.34) due to various incentives and exceptions that are invariably written into the tax code and due to the lack of indexation of depreciation allowances for inflation.

We continue to suppose the firm uses only one capital input, say the one described in the previous section. Suppose further that the firm purchases \(K\) units of this durable input at the beginning of the accounting period and has a cash flow (the value of outputs produced during the period minus the value of nondurable inputs used during the period) of \(CF\) during the period. Then the firm's profits before income taxes would be \(CF - UK\) where \(U\) is the user cost described by (5.32) or (5.33) above. The firm's profits after business income taxes, \(\pi\), are equal to before tax profits, \(CF - UK\), minus the business tax rate, \(t\), times taxable income as defined by the tax code, \(CF - AK\); i.e. we have the following definition for after business tax income:

\[
\pi = [CF - UK] - t[CF - AK]
\]

(5.35)

\[
= (1 - \delta)(CF - UK) + t[A - U]K
\]

(5.36)

rearranging terms

\[
= (1 - \delta)(CF - K[U + [t(1 - \delta)(U - A)]K]
\]

(5.37)

rearranging terms

\[
= (1 - \delta)(CF - W)K
\]

(5.38)

using (39) where the business tax wedge \(W\) is defined as:

\[
W = [t(1 - \delta)(U - A)]
\]

(5.39)

Note that if either the business income tax rate \(t\) equals 0 or if the no tax economic user cost \(U\) equals the tax accounting user cost \(A\), then the tax wedge \(W\) is 0 and the tax distorted user cost of capital \(U + W\) is equal to the undistorted user cost of capital \(U\).

The tax distorted user cost of capital \(U + W\) was first derived by Jorgenson (1963) and it is widely used by economists when they model the effects of the business income tax. In order for us to use it in our study, we would need a time series of business tax rates \(t\) and a time series of accounting user costs \(A\) for the various types of asset in our model.

However, the (business) tax distorted user cost of capital, \(U + W\), is not the end of the story when we want to calculate the deadweight cost of taxes on capital, because the above material neglects the fact that capital is not only taxed at the business level, it is also taxed at the personal level. In addition, there are various specific commodity taxes that fall on various capital stock components, like property taxes and sales taxes on purchases of machinery and equipment and structures.
Unfortunately, we do not have an accurate information base that would allow us to construct the personal and business tax wedges defined above. However, there is little evidence that businesses actually rearrange terms as was done in (5.35) to (5.38) above to obtain the Jorgenson tax adjusted user costs $U + W$. There is, however, some evidence that firms do use the user costs defined by (5.32) or (5.33). Typically, the business income tax is ignored entirely in cost allocation models and simply treated as a charge against earnings. In this case, a cost allocating business might treat the amount of business tax that it pays during the accounting period as a capital charge that should be spread evenly on its assets. Following this line of thought, turn to our one asset example again. Define the business asset tax rate $\tau$ as:

$$\tau = \frac{\text{business income taxes paid during the period}}{PK}$$  \hfill (5.40)

and the tax adjusted user cost becomes:

$$U' = U + \tau P$$

$$= [R - \alpha + \delta(1 + \alpha)]P$$ \hfill (5.41)

using (33). \hfill (5.42)

The user cost formula (5.42) is the user cost formula that we use in this study, except that we also include personal taxes on financial capital in definition (5.40). Thus we are assuming that business and personal taxes simply increase the nominal cost of capital, $R$.

From the viewpoint of real life firm accounting practices, the case for using the user cost formula (5.42) is just as strong as the case for using the Jorgensonian user cost formula $U + W$. Essentially, we have replaced the Jorgensonian tax wedge $W$ by the aggregate business and personal tax wedge $\tau P$.

The approach we adopt in this report to calculating the capital tax rate is to assume that the tax rate is the same across the various asset categories. It is calculated by taking the ratio of actual capital tax payments to the value of assets. This produces an estimate of the average capital tax rate for each year.

Using the average capital tax rate has the advantage of being based on relatively 'hard', observable data and, for most countries, it will provide a reasonable approximation of the capital tax burden faced by producers. However, an argument can also be mounted that deadweight loss studies should use estimates of the effective marginal tax rate (EMTR) producers face, (the Jorgensonian approach), as this will be a closer approximation to the rate producers respond to in making investment and other production decisions. While the use of EMTRs may be desirable, their construction is informationally very demanding. We hope to implement this approach in the future. In the meantime, we use the average tax rate on capital to proxy the tax burden producers face and to test our producer model.

To summarise: we assume that the undistorted user cost of capital for each of our three assets has the following form:

$$U = uP = [R - \alpha + \delta(1 + \alpha)]P$$ \hfill (5.43)

where $P$ is the beginning of the period asset price, $R$ is a tax free opportunity cost of capital, $\alpha$ is an anticipated asset inflation rate and $\delta$ is our assumed geometric or declining balance depreciation rate. The assumed depreciation rate for nonresidential structures was 3.48 per cent (ie $\delta_{NR} = 0.0348$), for machinery and equipment was 12.4 per cent (ie $\delta_{ME} = 0.124$) and for inventory stocks was zero (ie $\delta_{IS} = 0$).

To obtain the final tax distorted user cost for each asset, we need to add the total distortion wedge, $wP$, to the undistorted user costs defined by (43). These wedge terms $w$ for each asset are defined as follows:

$$w_{NR} \equiv \tau + \tau_{PNR}$$ \hfill (5.44)

$$w_{ME} \equiv \tau$$ \hfill (5.45)

$$w_{IS} \equiv \tau$$ \hfill (5.46)

where $\tau$ is the combined (asset) business and personal tax rate on capital and $\tau_{PNR}$ is the property tax rate on nonresidential capital. These tax rates are listed in Table 5.3 below along with $R$, the nominal pre-tax opportunity cost of capital, and the three (anticipated) asset inflation rates, $\alpha_{NR}$, $\alpha_{ME}$ and $\alpha_{IS}$.
### Table 5.3. Tax Rates and Asset Inflation Rates (percentages)

<table>
<thead>
<tr>
<th>Year</th>
<th>$\tau$</th>
<th>$\tau_{NR}$</th>
<th>$R$</th>
<th>$\alpha_{NR}$</th>
<th>$\alpha_{ME}$</th>
<th>$\alpha_{IS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>2.84</td>
<td>0.88</td>
<td>10.73</td>
<td>2.76</td>
<td>0.96</td>
<td>-0.13</td>
</tr>
<tr>
<td>1968</td>
<td>2.79</td>
<td>0.85</td>
<td>12.57</td>
<td>3.99</td>
<td>2.02</td>
<td>0.76</td>
</tr>
<tr>
<td>1969</td>
<td>2.70</td>
<td>0.80</td>
<td>13.38</td>
<td>5.23</td>
<td>3.09</td>
<td>1.73</td>
</tr>
<tr>
<td>1970</td>
<td>3.00</td>
<td>0.76</td>
<td>14.42</td>
<td>6.48</td>
<td>4.19</td>
<td>2.74</td>
</tr>
<tr>
<td>1971</td>
<td>2.86</td>
<td>0.73</td>
<td>14.00</td>
<td>7.71</td>
<td>5.32</td>
<td>3.77</td>
</tr>
<tr>
<td>1972</td>
<td>2.97</td>
<td>0.71</td>
<td>14.28</td>
<td>8.90</td>
<td>6.51</td>
<td>4.77</td>
</tr>
<tr>
<td>1973</td>
<td>2.94</td>
<td>0.67</td>
<td>17.77</td>
<td>9.47</td>
<td>7.71</td>
<td>6.03</td>
</tr>
<tr>
<td>1974</td>
<td>3.17</td>
<td>0.68</td>
<td>15.82</td>
<td>9.85</td>
<td>8.69</td>
<td>7.48</td>
</tr>
<tr>
<td>1975</td>
<td>2.99</td>
<td>0.78</td>
<td>13.76</td>
<td>10.27</td>
<td>9.46</td>
<td>8.87</td>
</tr>
<tr>
<td>1976</td>
<td>3.08</td>
<td>0.80</td>
<td>15.79</td>
<td>10.78</td>
<td>10.10</td>
<td>9.53</td>
</tr>
<tr>
<td>1977</td>
<td>2.95</td>
<td>0.77</td>
<td>14.64</td>
<td>11.07</td>
<td>10.64</td>
<td>9.51</td>
</tr>
<tr>
<td>1978</td>
<td>2.69</td>
<td>0.73</td>
<td>15.20</td>
<td>11.06</td>
<td>10.60</td>
<td>9.26</td>
</tr>
<tr>
<td>1979</td>
<td>2.27</td>
<td>0.70</td>
<td>15.77</td>
<td>10.97</td>
<td>9.78</td>
<td>8.85</td>
</tr>
<tr>
<td>1980</td>
<td>2.25</td>
<td>0.68</td>
<td>15.54</td>
<td>10.69</td>
<td>8.66</td>
<td>8.32</td>
</tr>
<tr>
<td>1981</td>
<td>2.60</td>
<td>0.66</td>
<td>14.90</td>
<td>10.43</td>
<td>7.40</td>
<td>7.69</td>
</tr>
<tr>
<td>1982</td>
<td>2.40</td>
<td>0.62</td>
<td>12.30</td>
<td>10.20</td>
<td>6.15</td>
<td>6.96</td>
</tr>
<tr>
<td>1983</td>
<td>2.09</td>
<td>0.62</td>
<td>12.56</td>
<td>9.82</td>
<td>5.33</td>
<td>6.14</td>
</tr>
<tr>
<td>1984</td>
<td>2.11</td>
<td>0.60</td>
<td>12.94</td>
<td>9.20</td>
<td>4.56</td>
<td>5.46</td>
</tr>
<tr>
<td>1985</td>
<td>2.40</td>
<td>0.59</td>
<td>12.28</td>
<td>8.41</td>
<td>3.77</td>
<td>4.93</td>
</tr>
<tr>
<td>1986</td>
<td>2.32</td>
<td>0.61</td>
<td>11.19</td>
<td>7.59</td>
<td>3.01</td>
<td>4.50</td>
</tr>
<tr>
<td>1987</td>
<td>2.57</td>
<td>0.61</td>
<td>11.05</td>
<td>6.72</td>
<td>2.37</td>
<td>4.05</td>
</tr>
<tr>
<td>1988</td>
<td>2.99</td>
<td>0.63</td>
<td>11.07</td>
<td>5.66</td>
<td>2.05</td>
<td>3.61</td>
</tr>
<tr>
<td>1989</td>
<td>3.06</td>
<td>0.64</td>
<td>10.55</td>
<td>4.52</td>
<td>1.90</td>
<td>3.14</td>
</tr>
<tr>
<td>1990</td>
<td>3.23</td>
<td>0.71</td>
<td>8.62</td>
<td>3.53</td>
<td>1.53</td>
<td>2.71</td>
</tr>
<tr>
<td>1991</td>
<td>3.37</td>
<td>0.78</td>
<td>7.88</td>
<td>2.71</td>
<td>0.81</td>
<td>2.35</td>
</tr>
<tr>
<td>1992</td>
<td>3.03</td>
<td>0.81</td>
<td>8.50</td>
<td>1.96</td>
<td>-0.19</td>
<td>2.03</td>
</tr>
<tr>
<td>1993</td>
<td>2.92</td>
<td>0.76</td>
<td>8.41</td>
<td>1.65</td>
<td>-1.72</td>
<td>1.88</td>
</tr>
<tr>
<td>1994</td>
<td>3.01</td>
<td>0.74</td>
<td>7.96</td>
<td>1.41</td>
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<td>1.72</td>
</tr>
<tr>
<td>1995</td>
<td>3.42</td>
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<td>7.48</td>
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<td>-4.73</td>
<td>1.58</td>
</tr>
<tr>
<td>1996</td>
<td>3.51</td>
<td>0.73</td>
<td>7.57</td>
<td>1.14</td>
<td>-6.33</td>
<td>1.48</td>
</tr>
<tr>
<td>1997</td>
<td>4.03</td>
<td>0.75</td>
<td>7.46</td>
<td>1.10</td>
<td>-8.01</td>
<td>1.41</td>
</tr>
<tr>
<td>Average</td>
<td>2.86</td>
<td>0.71</td>
<td>12.14</td>
<td>6.66</td>
<td>3.63</td>
<td>4.62</td>
</tr>
</tbody>
</table>

It can be seen that the combined personal and business tax rates on the return to capital are in the 2.1 per cent to 4.0 per cent range for our sample period, 1967-1997. Our estimated property tax rate which falls on the use of structures is in the 0.6 per cent to 0.9 per cent per year range.

The undistorted user costs for nonresidential structures, $u_{NR}$, machinery and equipment, $u_{ME}$, and for inventory stocks, $u_{IS}$ (in proportional form; see (5.43) above) are listed in Table 5.4 below, along with the tax wedges. The undistorted user costs and tax wedges are then graphed in Figures 5.8 and 5.9, respectively.
Finally, the beginning of the period asset prices (before commodity taxes), $P_{M}$, $P_{E}$ and $P_{I}$ are listed in Table 5.5 along with the corresponding beginning of the period capital stocks, $K_{M}$, $K_{E}$ and $K_{I}$ (in billions of 1967 Australian dollars). The capital stocks were constructed using the perpetual inventory method using the Australian Bureau of Statistics’ net capital stock estimates for 1967 as a starting point. The depreciation rate was then chosen so that our estimates for 1997 coincided with those of the ABS.

In the next 3 sections, we show how the simple excess burden model explained in section 5.2 above can be generalized to cover the case of many (noncapital) outputs and inputs and 3 types of reproducible capital (compared to the single reproducible capital stock model of section 5.2).
5.5 **EXCESS BURDENS IN A SINGLE CAPITAL MODEL USING CASH FLOW PROFIT FUNCTIONS**

The model presented in section 5.2 above was adequate to introduce the basic concepts involved in measuring the deadweight loss due to the taxation of capital. However, this model suffered from a number of defects including: (i) it was highly aggregated; (ii) the investment good was assumed to be perfectly substitutable with the consumption good and (iii) the economy was closed, i.e. there was no international trade in goods and services. In this section, we relax the above restrictions except we continue to assume that there is only one reproducible capital stock in the economy. In the following section, we shall deal with the multiple stock case.

In order to simplify our derivation of excess burden formulae, we make use of the producer's profit function. The profit function simply provides an alternative method for representing the production function, or more generally, the producer's production possibilities set. The use of the profit function not only facilitates the derivation of deadweight loss formulae but it is also very convenient from the viewpoint of the econometric estimation of production functions or technology sets.

We assume that there are \( M \) non-capital variable outputs and inputs that are produced and utilised in the private production sector. The positive prices that producers face in period \( t \) for these \( M \) variable commodities are denoted by \((p_1^t, \ldots, p_M^t) = p^t\). The corresponding variable outputs and inputs produced and used during period \( t \) are denoted by the quantity vector \( y^t = (y_1^t, \ldots, y_M^t) \). The list of outputs includes consumption goods and services, government purchases of goods and services from the private sector, an investment good that corresponds to the single reproducible capital stock in our model and exports. The list of variable inputs includes imports and labour. If commodity \( m \) is an input, then \( y_m^t \) has a negative sign. The price of one unit of the reproducible capital stock is \( p^t \) in period \( t \). The private business sector of the economy utilises the beginning of period \( t \) capital stock \( K^t \) and the fixed factor input \( F^t \). The period \( t \) set of feasible net output vectors \( y \), conditional on a beginning of the period capital stock \( K^t \) and fixed factor input \( F^t \) is denoted by the set \( S^t \). The private sector's period \( t \) cash flow profit function \( \pi^t \) is defined as follows:

\[
\pi^t(p^t, K^t, F^t) = \max_y \{ p^t \cdot y : (y, K^t, F^t) \in S^t \}
\]  

where \( p^t \cdot y \) denotes the inner product of the vectors \( p^t \) and \( y \). In words, \( \pi^t(p^t, K^t, F^t) \) is the maximum value added less the value of labour inputs that the private sector can produce given that producers face the prices \( p^t \) for these variable outputs and inputs and given that producers have the fixed stocks \( K^t \) and \( F^t \) of reproducible and non-reproducible capital available to them at the beginning of period \( t \). In other words, \( \pi^t(p^t, K^t, F^t) \) is the maximum cash flow that the economy can earn in period \( t \), given that it faces the price vector \( p^t \) for variable inputs and outputs and has the use of \( K^t \) units of reproducible capital and \( F^t \) units of fixed factors.

The counterpart to our earlier tax distorted profit maximisation problem (5.5) is now the following problem:

\[
\max_K \{ \pi^t(p^t, K, F^t) - (u^t + w^t) p^t K \} = \Pi^t(p^t, (u^t + w^t) p^t, F^t)
\]  

(5.48)

where \( u^t \) is the period \( t \) (deflated) undistorted user cost of capital defined in (5.43) in the previous section and \( w^t \) is the period \( t \) (deflated) total tax distortions wedge defined by one of (5.44)-(5.46) in the previous section. Note that the optimised objective function in (5.48) defines another profit function \( \Pi^t \), which we call the rents profit function. We will use the rents profit function in the next section. We assume that the observed period \( t \) capital stock \( K^t \) satisfies the first order necessary conditions for the unconstrained maximisation problem in (5.48):

\[
\partial \pi^t(p^t, K, F^t) / \partial K = (u^t + w^t) p^t.
\]  

(5.49)

Equation (5.49) is the counterpart to our old equation (5.6). For an arbitrary capital tax wedge \( w \), we denote the capital solution to (5.49), where \( w \) replaces \( w^t \), by \( K(w) \). Substituting \( K(w) \) into (5.49) and differentiating with respect to \( w \) yields the following equation for \( K'(w) \):

\[
K'(w) = \{ \partial^2 \pi^t(p^t, K^t, F^t) / \partial K^2 \}^{-1} \cdot P^t.
\]  

(5.50)

Recall the definition of the producer surplus function \( S(w) \), defined by (5.8) above. This function evaluated the outputs produced and the inputs used by the private sector at undistorted prices. Using the cash flow profit function and the capital demand function \( K(w) \), which solves (5.49) when the observed period \( t \) distortion term \( w' \) is replaced by an arbitrary distortion term \( w \), we redefine the surplus function for period \( t \), \( S(w) \) as follows:
\( S(w) = \pi'(p', K(w), F') - u'P'K(w) \). \tag{5.51}

Differentiating \( S(w) \) with respect to \( w \) yields the following equations:
\[
S'(w) = [\partial \pi'(p', K(w), F')/\partial K] K'(w) - u'P'K'(w)
\]
\( = [u' + w] P' K'(w) - u'P'K'(w) \)
\[
\text{using (5.49) with } w \text{ replacing } w'
\]
\( = w'P'K'(w) \).

Evaluating (5.52) at \( w = 0 \) and \( w = w' \) leads to the following equalities:
\[
S'(0) = 0; \tag{5.53}
\]
\[
S'(w') = w'P'K'(w') \tag{5.54}
\]
\[
= w'P'[\partial^2 \pi'(p', K', F')/\partial K^2]^{-1} P'
\]

using (50).

It is possible to approximate \( S(w') - S(0) \) by a second order Taylor series expansion, as we did in section 5.2 above. Of course, if \( S(w) \) is (locally) a quadratic function, this approximation will be (locally) exact. Another approximation for \( S(w') - S(0) \) that is exact if \( S(w) \) is quadratic is the following one:
\[
S(w) - S(0) \approx (1/2) [S'(w') + S'(0)] [w' - 0] \tag{5.55}
\]
\[
= (1/2) S'(w') w'
\]

using (5.53)
\[
= (1/2) (w')^2 P'[\partial^2 \pi'(p', K', F')/\partial K^2]^{-1} P'
\]

using (5.54).

Thus, we have the following quadratic approximation for \( S(0) - S(w') \):
\[
S(0) - S(w') \approx -(1/2) (w')^2 P'[\partial^2 \pi'(p', K', F')/\partial K^2]^{-1} P'
\] \tag{5.56}

Recall our earlier definition (5.13) of the loss of surplus due to the distortion wedge \( w^* \) as a fraction of the undistorted GDP, \( [S(0) - S(w^*)]/Y(0) \), and the second order approximation to this loss, \( A(w^*) \) defined by (5.14). We will now express the loss as a fraction of the tax distorted level of GDP, which we denote by \( Y(w') \) for period \( t \). Thus, using (5.56), our new second order approximation to the loss of output in period \( t \) is:
\[
A(w') = -(1/2)(w')^2 P'[\partial^2 \pi'(p', K', F')/\partial K^2]^{-1} P'/Y(w'). \tag{5.57}
\]

Given an econometrically estimated cash flow function \( \pi' \), we can readily calculate \( A(w') \) defined by (5.57).

We turn now to the problem of generalising our old marginal cost function defined earlier by (5.23). We now define the cost of the system of capital taxation function \( C(w) \) as the difference between the optimal value of output \( S(0) \) and the tax distorted value of output \( S(w') \):
\[
C(w') \equiv S(0) - S(w') \tag{5.58}
\]

where \( S(w) \) is now defined by (5.51) above. Differentiating (5.58) with respect to \( w \) and evaluating \( w \) at the period \( t \) tax distortion rate \( w' \) leads to the following period \( t \) marginal cost of the system of capital taxation:
\[
MC(w') = C'(w') \tag{5.59}
\]
\[
= -S'(w') \tag{5.58}
\]
\[
= -w'P'K'(w') \tag{5.54}
\]
\[
= -w'P'[\partial^2 \pi'(p', K', F')/\partial K^2]^{-1} P' \tag{5.60}
\]

Thus, (5.60) defines the marginal cost of increasing the period \( t \) distortion wedge \( w' \) by a small amount. We turn now to the problem of defining the corresponding marginal benefit function for our new model.

Recall equation (5.24) above, which expressed the total revenue \( T \) from all sources of capital taxation. The counterpart is now:
\[
T' = w'P'K'. \tag{5.61}
\]

We replace the period \( t \) distortion rate \( w' \) by a general distortion rate \( w \) and let \( K(w) \) be the solution to (5.49) when \( w' \) is replaced by \( w \). Making these substitutions into (5.61) leads to the following definition for the period \( t \) total tax revenues as a function of the distortion rate \( w \):
\[
T(w) = wP'K(w). \tag{5.62}
\]

We can now obtain the marginal increase in capital tax revenues by differentiating \( T(w) \) defined by (5.62) with respect to \( w \) and evaluating the resulting derivative at the observed period \( t \) distortion rate \( w' \). Thus, define the period \( t \) marginal benefit of an increase in the rate of capital taxation as:
\[
MB(w') = T'(w') \tag{5.63}
\]
\[
= P'K(w') + w'P'K'(w') \text{ differentiating (5.62)}
\]
\[
= P'K(w') + w'P'[\partial^2 \pi'(p', K', F')/\partial K^2]^{-1} P'
\]
using (5.54).

Thus, given an econometric estimate for the period $t$ cash flow function, $\pi'_t$, the right hand side of (5.64) can be evaluated using observable data. The first term on the right hand side of (5.54) will be positive and the second term will be negative. If there is a high degree of substitutability of capital for other inputs and outputs in period $t$, then the second term can make the overall marginal benefits of increasing capital taxes negative. In this case, the government will achieve both increased productive efficiency and higher tax revenues by reducing capital tax distortions.

The marginal excess burden of increasing the period $t$ capital tax distortion rate $w'$ by a small amount is simply the ratio of the marginal cost $MC(w')$ defined by (5.60) above divided by the marginal tax revenue $MB(w')$ defined by (5.64) above:

$$MEB(w') = MC(w')/MB(w') = -w'K'(w')/\{P'^tK(w') + w'P'K'(w')\}. \quad (5.65)$$

Thus, with the use of duality theory, it proved to be quite straightforward to generalise the very simple one output model of section 5.2 to an open economy with many outputs and inputs. However, the model presented in this section still only had a single reproducible capital input. Before we deal with the case of many capital inputs, we rework the analysis presented in this section using the pure rents profit function $\Pi'^t$ (see (5.48) above) in place of the cash flow profit function $\pi'$, since we used the former function in our econometric work.

5.6. EXCESS BURDENS IN A SINGLE CAPITAL MODEL USING PURE RENTS PROFIT FUNCTIONS

Recall that the period $t$ pure rents profit function $\Pi'(p'_t,(u'+w')P'_t,F'_t)$ was defined by (5.48) above. Recall also the capital demand function $K(w)$ that was the solution to the first order condition (5.49), where $w$ was replaced by a general distortion wedge $w$. It can be shown\(^4\) that the capital demand function $K(w)$ can be obtained directly from the pure rents profit function by differentiating $\Pi'(p'_t,(u'+w')P'_t,F'_t)$ with respect to the tax distorted user cost, which we denote by $V \equiv (u' + w)P'_t$:

$$K(w) = -\frac{\partial \Pi'(p'_t,(u'+w)P'_t,F'_t)}{\partial v}. \quad (5.66)$$

We can differentiate $K(w)$ with respect to $w$ and evaluate the resulting derivative at $w = w'$. We obtain the following counterpart to our old formula (5.50) in the previous section:

$$K'(w') = -[\frac{\partial^2 \Pi'(p'_t,(u'+w)P'_t,F'_t)}{\partial v^2}] P'. \quad (5.67)$$

Recall our old definition (5.51) of the surplus function $S(w)$ using the cash flow profit function $\pi'$. Using the pure rents profit function $\Pi'$, we can redefine $S(w)$ as follows:

$$S(w) = \Pi'(p'_t,(u'+w)P'_t,F'_t) + wP'K(w). \quad (5.68)$$

Differentiating $S(w)$ with respect to $w$ yields the following equations:

$$S'(w) = [\frac{\partial \Pi'(p'_t,(u'+w)P'_t,F'_t)}{\partial v}] P' + P'K(w) + wP'K'(w) \quad (5.69)$$

$$= -K(w)P' + P'K(w) + wP'K'(w) \quad \text{using (5.66)}$$

$$= wP'K'(w).$$

Evaluating (69) at $w = 0$ and $w = w'$ leads to the following equalities:

$$S'(0) = 0; \quad (5.70)$$

$$S'(w') = w'PK'(w') \quad (5.71)$$

$$= -w'P'[\frac{\partial^2 \Pi'(p'_t,(u'+w)P'_t,F'_t)}{\partial v^2}] P', \quad \text{using (67).} \quad (5.72)$$

Using (5.70) and (5.72) and making use of the quadratic approximation (5.55) again to approximate $S(0)$, we have the following quadratic approximation for $S(0) - S(w)$:

$$S(0) - S(w') = (1/2)(w')^2 P'[\frac{\partial^2 \Pi'(p'_t,(u'+w)P'_t,F'_t)}{\partial v^2}] P'. \quad (5.73)$$

Recall our earlier definition of the loss of surplus due to the distortion wedge $w$ as a fraction of undistorted GDP, $[S(0) - S(w)]/Y(0)$, and the second order approximation to this loss, $A(w)$ defined by (5.14). As in the previous section, we will again express the loss as a fraction of the tax distorted level of GDP, which we again denote by $Y(w')$ for period $t$. 


Thus, using (5.73), we obtain the counterpart to (5.57) in the previous section; i.e., our new second order approximation to the loss of output in period \( t \) is:

\[
A(w') = \left(1/2\right)(w')^2 P'[\partial^2 \Pi \quad \left(p', (u'+w)P', F'/\partial V^2\right) P'/Y(w').
\]

(5.74)

Given an econometrically estimated pure rents function \( \Pi' \), we can readily calculate \( A(w') \) defined by (5.74).

We turn now to the problem of generalising our old marginal cost function \( MC(w) \) defined earlier by (5.59). As in the previous section, we define the cost of the system of capital taxation function \( C(w) \) as the difference between the optimal value of output \( S(0) \) and the tax distorted value of output \( S(w') \):

\[
C(w') = S(0) - S(w')
\]

(5.75)

where \( S(w) \) is now defined by (5.68) above. Differentiating (5.75) with respect to \( w \) and evaluating \( w \) at the period \( t \) tax distortion rate \( w' \) leads to the following period \( t \) marginal cost of the system of capital taxation:

\[
MC(w') = C'(w')
\]

(5.76)

\[
= -S'(w') \quad \text{using (5.75)}
\]

\[
= -w'P'K'(w') \quad \text{using (5.71)}
\]

\[
= w'P'[\partial^2 \Pi \quad \left(p', (u'+w)P', F'/\partial V^2\right) P'\text{ using (5.72)}.
\]

(5.77)

Thus, (5.76) defines the marginal cost of increasing the period \( t \) distortion wedge \( w' \) by a small amount and (5.77) and (5.78) are formulae which can be used to evaluate this marginal cost. We turn now to the problem of defining the corresponding marginal benefit function for our new model.

Recall equations (5.24) and (5.61) above, which expressed the total revenue \( T \) from all sources of capital taxation in terms of the distortion wedge \( W = wP \). We can rewrite (5.24) for period \( t \) total capital tax revenue \( T' \) as a function of the period \( t \) distortion wedge \( w' \) as follows:

\[
T' = w'P'K'(w').
\]

(5.79)

We replace the period \( t \) distortion rate \( w' \) by a general distortion rate \( w \) and let \( K(w) \) be defined by (5.66). Making these substitutions into (5.79) leads to the following definition for the period \( t \) total tax revenues as a function of the distortion rate \( w \):

\[
T(w) = wP'K(w).
\]

(5.80)

Now differentiate \( T(w) \) defined by (5.80) with respect to \( w \) and evaluate the resulting derivative at the observed period \( t \) distortion rate \( w' \). This defines the period \( t \) marginal benefit of an increase in the rate of capital taxation as:

\[
MB(w') = T'(w')
\]

(5.81)

\[
= P'K(w')+w'P'K'(w') \quad \text{differentiating (5.80)}
\]

(5.82)

\[
= P'K(w')-w'P'[\partial^2 \Pi \quad \left(p', (u'+w)P', F'/\partial V^2\right) P' \quad \text{using (5.67)}.
\]

(5.83)

Thus, given an econometric estimate for the period \( t \) cash flow function, \( \Pi' \), the right hand side of (5.83) can be evaluated using observable data. As was the case with our earlier formulae (5.64), the first term on the right hand side of (5.83) will be positive and the second term will be negative. If there is a high degree of substitutability of capital for other inputs and outputs in period \( t \), then the second term can make the overall marginal benefits of increasing capital taxes negative. In this case, the government will achieve both increased productive efficiency and higher tax revenues by reducing capital tax distortions.

The marginal excess burden of increasing the period \( t \) capital tax distortion rate \( w' \) by a small amount is simply the ratio of the marginal cost \( MC(w) \) defined by (5.77) above divided by the marginal tax revenue \( MB(w') \) defined by (5.82) above:

\[
MEB(w') = \frac{MC(w')}{MB(w')} = -w'K'(w')/\left\{ P'K(w')+w'P'K'(w') \right\}.
\]

(5.84)

Note that our new formula for the \( MEB(w') \), (5.84), coincides with our formula for the marginal excess burden of an increase in the wedge \( W \) in the previous section, (5.65). However, in this section, we obtain an estimate of the capital demand derivative \( K'(w) \) using equation (5.67), which involves the second order partial derivative of the period \( t \) pure profits function \( \Pi' \) with respect to the tax distorted user cost \( V = (u+w)P \), whereas in the previous section, we obtained an estimate of the capital demand derivative \( K'(w') \) using equation (5.50), which involved the second order partial derivative of the period \( t \) cash flow function \( \pi' \) with respect to capital \( K \).
The profit function models presented in this section and the previous section had only a single reproducible capital input. In the next section, we relax this restriction.

5.7 A MULTIPLE CAPITAL STOCK MODEL

In this section, we assume that $K = (K_1, K_2, K_3)$ is now a three dimensional vector of reproducible capital stocks rather than being the scalar capital stock assumed in the previous sections of this chapter. We now adapt the analysis presented in the previous section to this multiple capital stock case.

The period $t$ cash flow profit function $\pi^t$ can still be defined by (5.47) above but we now interpret $K$ as a vector. Recall that the period $t$ pure rents profit function $\Pi^t(p^t, (u^t+w^t)P^t, F^t)$ was defined by (5.48) above. This same definition is still applicable with $K$ being interpreted as a vector but now $(u^t+w^t)P^t = [(u^t_1+w^t_1)P^t_1, (u^t_2+w^t_2)P^t_2, (u^t_3+w^t_3)P^t_3]$ is interpreted as a vector of period $t$ tax distorted user costs for the reproducible capital stocks. We define the period $t$ vector of stock prices for the reproducible capital stock components as $P^t = [P^t_1, P^t_2, P^t_3]$.

The vector of period $t$ undistorted (deflated) user costs of capital is defined as $u^t = [u^t_1, u^t_2, u^t_3]$ and the vector of deflated period $t$ distortion terms is defined as $w^t = [w^t_1, w^t_2, w^t_3]$. Each distortion term is defined as in equations (5.44)-(5.46) in section 5.3 above. The undistorted (deflated) user costs of capital, $u_n$, are defined as in section 5.3; i.e. we have:

$$u_n = R - c_n + \delta_n (1 + \alpha_0); \quad n = 1,2,3. \quad (5.85)$$

It can be shown that the vector of capital demand functions $K(w) = [K_1(w_1, w_2, w_3), K_2(w_1, w_2, w_3), K_3(w_1, w_2, w_3)]$ can be obtained directly from the pure rents profit function by differentiating $\Pi^t(p^t, (u^t+w^t)P^t, F^t)$ with respect to the components of the tax distorted user costs, which we denote by the vector $V = (u^t+w^t)P^t = [(u^t_1+w^t_1)P^t_1, (u^t_2+w^t_2)P^t_2, (u^t_3+w^t_3)P^t_3]$:

$$K(w) = - \nabla_V \Pi^t(p^t, (u^t+w^t)P^t, F^t) \quad (5.86)$$

where $\nabla_V \Pi^t(p^t, (u^t+w^t)P^t, F^t) = \nabla_V \Pi^t(p^t, V, F^t)$ denotes the vector of first order partial derivatives of $\Pi^t(p^t, V, F^t)$ with respect to the components of $V$; i.e. we have

$$\nabla_V \Pi^t(p^t, V, F^t) = [\partial \Pi^t(p^t, V, F^t)/\partial V_1, \partial \Pi^t(p^t, V, F^t)/\partial V_2, \partial \Pi^t(p^t, V, F^t)/\partial V_3].$$

We can differentiate the vector $K(w)$ defined by (5.86) with respect to the components of $w$ and evaluate the resulting three by three matrix of derivatives at $w = w^t$. We obtain the following counterpart to our old formula (5.67) in the previous section:

$$\nabla_w K(w') = - [\nabla^2_V V \Pi^t(p^t, (u^t+w^t)P^t, F^t)] \text{Diag}(P^t) \quad (5.88)$$

where $\nabla^2_V V \Pi^t$ denotes the three by three matrix of second order partial derivatives of $\Pi^t$ with respect to the three user costs and $\text{Diag}(P^t)$ is a three by three diagonal matrix with the elements of the three dimensional vector of period $t$ capital stock prices, $P^t = [P^t_1, P^t_2, P^t_3]$, running down the main diagonal.

Recall our old definition (5.68) of the surplus function $S(w)$. We can redefine the surplus function $S(w)$ using the cash flow profit function $\Pi^t$ as follows:

$$S(w) = \Pi^t(p^t, (u^t+w^t)P^t, F^t) + \sum_{i=1}^3 w_i P_i^t K_i(w). \quad (5.89)$$

Differentiating $S(w)$ with respect to the components of $w$ yields the following equations:

$$\partial S(w)/\partial w_n = [\partial \Pi^t(p^t, (u^t+w^t)P^t, F^t)/\partial V_n] P_i^t + P_i^t K_i(w) \quad (5.90)$$

$$+ \sum_{j=1}^3 w_j P_j^t \partial K_j(w)/\partial w_n$$

$$= - K_i(w) P_i^t + P_i^t K_i(w) + \sum_{j=1}^3 w_j P_j^t \partial K_j(w)/\partial w_n \quad \text{using (5.86)}$$

$$= \sum_{j=1}^3 w_j P_j^t \partial K_j(w)/\partial w_n \quad \text{for } n = 1,2,3.$$ Evaluating (5.90) at $w = 0$ and $w = w^t$ leads to the following equalities:

$$\partial S(0)/\partial w_n = 0 \quad \text{for } n = 1,2,3; \quad (5.91)$$

$$\partial S(w)/\partial w_n = \sum_{j=1}^3 w_j P_j^t \partial K_j(w)/\partial w_n \quad \text{for } n = 1,2,3 \quad (5.92)$$

$$= - \sum_{j=1}^3 w_j P_j^t [\partial^2 \Pi^t(p^t, (u^t+w^t)P^t, F^t)/\partial V_j \partial V_n] P_i^t. \quad \text{using (5.88).} \quad (5.93)$$
Using (5.91) and (5.93) and making use of Diewert's (1976; 118) quadratic approximation lemma, we have the following quadratic approximation for \( S(0) - S(w) \):

\[
S(0) - S(w) \approx \left( \frac{1}{2} \right) \sum_{j=1}^{3} \sum_{n=1}^{3} w_j' P_j' \left[ \partial^2 I'(p', (u' + w')P', F')/\partial Y_j \partial N_n \right] w_n P_n' \tag{5.94}
\]

As in the previous section, we will again express the loss (5.74) as a fraction of the tax distorted level of GDP, which we again denote by \( Y(w) \) for period \( t \). Thus, using (5.94), we obtain the following counterpart to (5.74) in the previous section; i.e. our new second order approximation to the loss of output in period \( t \) is:

\[
A(w) = \left( \frac{1}{2} \right) \sum_{j=1}^{3} \sum_{n=1}^{3} w_j' P_j' \left[ \partial^2 I'(p', (u' + w')P', F')/\partial Y_j \partial N_n \right] w_n P_n' / Y(w) \tag{5.95}
\]

Given an econometrically estimated pure rents function \( I'(p') \), we can readily calculate \( A(w) \) defined by (5.95).

We turn now to the problem of defining the marginal cost function. As in the previous section, we define the cost of the system of capital taxation tax function \( C(w) \) as the difference between the optimal value of output \( S(0) \) and the tax distorted value of output \( S(w) \):

\[
C(w) = S(0) - S(w) \tag{5.96}
\]

where \( S(w) \) is now defined by (5.89) above. Differentiating (5.96) with respect to the components of \( w \) and evaluating \( w \) at the period \( t \) tax distortion vector \( w^t \) leads to the following period \( t \) marginal costs with respect to a small increase in the \( nth \) deflated distortion wedge \( w_n^t \):

\[
MC_n(w^t) = \partial C(w^t)/\partial w_n^t \tag{for} n = 1, 2, 3 \tag{5.97}
\]

\[
= - \partial S(w^t)/\partial w_n^t \tag{using (96)} \]

\[
= - \sum_{j=1}^{3} w_j' P_j' \partial K_j(w^t)/\partial w_n^t \tag{using (92)} \tag{5.98}
\]

\[
= \sum_{j=1}^{3} w_j' P_j' \left[ \partial^2 I'(p', (u' + w')P', F')/\partial Y_j \partial N_n \right] P_n' \tag{using (103)} \tag{5.99}
\]

Thus, (5.97) defines the marginal cost of increasing the period \( t \) distortion wedge for capital input \( n, w_n^t \), by a small amount, and (5.98) and (5.99) are formulae which can be used to evaluate this marginal cost. However, now we encounter a difference in the multiple capital stock model of this section compared with the single capital stock model in the previous section. In the previous sections, we did not have to consider in detail the effects of changes in each tax policy parameter; all we had to know is whether the change in tax policy increased or decreased the single deflated wedge. Now we have to consider changes in each tax parameter separately. The changes in tax policy that we will consider in section 5.9 are:

- An increase in the business income tax rate \( \tau \) and
- An increase in the rate of property tax \( \tau_{pr} \).

To indicate how we can work out the marginal cost of each type of tax increase, we show how to do this for the first case above, an increase in the rate of business income taxation. Recall definitions (5.44) to (5.46) which defined each deflated wedge \( w_n \) as a function of all tax parameters. Now regard each \( w_n \) as a function of the business income tax rate, \( \tau \); i.e. we have:

\[
w_n(\tau) = g_n(\tau) \tag{for} n = 1, 2, 3 \tag{5.100}
\]

Now we define the cost of the business income tax function \( C(\tau) \) as the difference between the optimal value of output \( S(0) \) and the tax distorted value of output \( S(w) \), but where the wedges \( w_n^t \) are regarded as functions of \( \tau \):

\[
C(\tau) = S(0) - S[g_1(\tau), g_2(\tau), g_3(\tau)]. \tag{5.101}
\]

Now differentiate (5.101) with respect to \( \tau \) and evaluate the resulting derivatives at the observed period \( t \) tax rate \( \tau^t \). Using (5.101) leads to the following period \( t \) marginal cost of an increase in the business income tax rate:

\[
MC(\tau^t) = \partial C(\tau^t)/\partial \tau \tag{5.102}
\]

\[
= - \sum_{n=1}^{3} \left[ \partial S(w)/\partial w_n \right] \partial g_n(\tau)/\partial \tau \tag{using (100)} \]

\[
= - \sum_{n=1}^{3} \sum_{j=1}^{3} w_j P_j' \left[ \partial K_j(w)/\partial w_n \right] \partial g_n(\tau)/\partial \tau \tag{using (92)} \tag{5.103}
\]

\[
= \sum_{j=1}^{3} w_j P_j' \left[ \partial^2 I'(p', (u' + w')P', F')/\partial Y_j \partial N_n \right] P_n' \tag{using (103)} \tag{5.99}
\]
Equations (5.99) can be substituted into (5.103) in order to obtain a formula for $MC(\tau')$ that can be evaluated empirically, given an econometrically estimated pure rents function $J$. 

We turn now to the problem of defining the corresponding marginal benefit function for our new model. We continue to focus on changes in the business income tax rate. The treatment of changes in the property tax parameter is similar.

Recall equation (5.80) above, which expressed the total revenue $T$ from all sources of capital taxation in terms of the distortion wedge $W = wP$. The multiple capital stock generalisation of equation (5.90) in the previous section is (5.104) below; i.e., we can write total capital tax revenue $T'$ in period $t$ as a function of the distortion wedges $w' = [w_1', w_2', w_3']$ as follows:

$$T' = \sum_{n=1}^{3} w_n' P_n' K_n.$$

We replace the period $t$ vector of distortion rate $w'$ by a general vector of distortion rates $w$ and let $K(w)$ be defined by (5.86). Finally, we replace each wedge $w_n$ as a function of the business income tax rate $\tau$, as in equations (5.100) above. Making these substitutions into (5.104) leads to the following definition for the period $t$ total tax revenues as a function of the business income tax rate $\tau$:

$$T(\tau) = \sum_{n=1}^{3} w_n(\tau) P_n' K_n[w_1(\tau), w_2(\tau), w_3(\tau)].$$

We can obtain the marginal increase in capital tax revenues by differentiating $T(\tau)$ defined by (5.105) with respect to $\tau$ and evaluating the resulting derivative at the observed period $t$ tax rate $\tau'$. Thus, we define the period $t$ marginal benefit of an increase in the business income tax rate as:

$$MB(\tau') = T'(\tau')$$

$$= \sum_{n=1}^{3} \left[ \frac{\partial g_n(\tau')}{\partial \tau} \right] P_n' K_n[w']$$

$$+ \sum_{n=1}^{3} w_n' P_n' \left[ \sum_{j=1}^{3} \frac{\partial K_n(w')}{\partial w_j} \frac{\partial g_j(\tau')}{\partial \tau} \right]$$

$$- \sum_{n=1}^{3} \sum_{j=1}^{3} w_n' P_n' \left[ \frac{\partial^2 \Pi'(p', u'^t + w') P', F')}{\partial V_n \partial V_j} \right]$$

$$\frac{\partial g(\tau')}{\partial \tau}$$

where the last equality follows using (5.88).

Thus, given an econometric estimate for the period $t$ cash flow function, $\Pi'$, the right hand side of (5.106) can be evaluated using observable data. As was the case with our earlier formulae, (5.64) and (5.83), the first term on the right hand side of (5.106) will be positive but we can no longer guarantee that the second term will be negative. If there is a high degree of substitutability of capital for other inputs and outputs in period $t$, then as in the previous sections, the second term can make the overall marginal benefits of increasing capital taxes negative. In this case, the government will achieve both increased productive efficiency and higher tax revenues by reducing capital tax distortions.

The marginal excess burden of increasing the period $t$ business income tax rate $\tau'$ by a small amount is simply the ratio of the marginal cost $MC(\tau')$ defined by (5.103) above divided by the marginal tax revenue $MB(\tau')$ defined by (5.106) above:

$$MEB(\tau') = \frac{MC(\tau')}{MB(\tau')}.$$ 

The calculation of marginal excess burdens for the property tax parameter is similar.

We turn now to our econometric model.

### 5.8. The Producer Model

As we saw in the previous section, the key determinants of the size of the deadweight loss or loss of efficiency in the economy due to the taxation of capital are the size of the capital tax distortion wedges and the magnitudes of various elasticities of demand and supply for private sector producers. This section and the next one will focus on the empirical estimation of these producer elasticities.

In this chapter, we use the data pertaining to the Australian economy that is developed in the data Appendix below to estimate a system of private producer supply and demand equations. Flexible functional form techniques are used—i.e., the functional form we use to model the technology does not impose unwarranted a priori restrictions on elasticities of substitution between the outputs and inputs. In the present section, we lay out a preliminary version of our model.
In the next section, we note that there is a potential problem with our preliminary model: the elasticities that it generates may trend significantly over the sample period in a manner that is not warranted. Given the importance of determining accurate elasticities in order to calculate excess burdens, we discuss how this problem can be remedied.

When the number of commodities in an applied general equilibrium model is large, it becomes difficult or impossible to estimate flexible functional forms. When there are \(N+1\) commodities and \(T\) observations on prices and quantities for each commodity, there are \((N+1)T\) degrees of freedom available for econometric estimation and this number is an upper bound to the number of unknown parameters characterising technology that can be estimated. A bare bones basic flexible functional form for a production function (or the dual cost or profit functions) in the constant returns to scale case has \(N(N+1)/2\) unknown parameters. Hence, as soon as \(N\) (the number of commodities less one) is equal to or greater than \(2T\), it becomes impossible to estimate a flexible functional form using time series data.

The twelve commodities in our database are: (1) an aggregate of consumption (excluding housing), residential investment and motor vehicle additions; (2) government consumption of intermediate inputs plus investment; (3) exports; (4) imports; (5) labour input; (6) equipment investment; (7) nonresidential and other construction investment; (8) inventories investment; (9) stocks of machinery and equipment; (10) nonresidential and other construction stocks; (11) inventory stocks and (12) inputs of land and other fixed factors.

Flexibility is a desirable property for a functional form since an inflexible functional form will restrict elasticities of substitution between commodities in some arbitrary a priori fashion. A way of dealing with this inflexibility problem when the number of commodities is large relative to the number of observations was suggested by Diebert and Wales (1988) in the consumer theory context: instead of estimating a general \(N\) by \(N\) symmetric substitution matrix \(A\) of full rank, they restricted \(A\) to be a symmetric substitution matrix of rank \(J\) where \(J\) is smaller than \(N\). Diebert and Wales (1988) termed functional forms of this type semelflexible. In the present section, we shall adapt their technique to the producer context.

The technology of the private production sector could be described by a production, cost or variable profit function. In this study, we will describe technology by means of a pure rents or pure profits function of the type defined in the previous section.\(^{15}\)

Recall the definition of the period \(t\) cash flow profit function \(\pi^t\), (5.47) above, which we rewrite as (5.108) below:

\[
\pi^t(p^t, K^t, F^t) = \max_{y} \{ p^t \cdot y : (y, K^t, F^t) \in S^t \}
\]  
(5.108)

where \((p^t, \ldots, p^t) = p^t\) is the vector of positive prices that producers face in period \(t\) for the \(8\) noncapital variable inputs and outputs in our model and where \(p^t \cdot y\) denotes the inner product of the of the vectors \(p^t\) and \(y\). The corresponding variable inputs and outputs produced and used during period \(t\) are denoted by the quantity vector \(y^t = (y^t_1, \ldots, y^t_n)\). Recall that if commodity \(m\) is an input, then \(y^t_m\) has a negative sign. The private business sector of the economy utilises the beginning of period \(t\) capital stock vector \(K^t = (K^t_1, K^t_2, K^t_3)\) and the fixed factor input \(F^t\). The period \(t\) set of feasible net output vectors \(y\), conditional on a beginning of the period capital stock \(K^t\) and fixed factor input \(F^t\) is denoted by the set \(S^t\). In words, \(\pi^t(p^t, K^t, F^t)\) is the maximum value added less the value of labour inputs that the private sector can produce given that producers face the prices \(p^t\) for these variable inputs and outputs and given that producers have the vector of fixed stocks of reproducible capital \(K^t\) and the quantity \(F^t\) of non-reproducible capital available to them at the beginning of period \(t\).

As in the previous sections, we use the period \(t\) cash flow profit function \(\pi^t\) defined by (5.108) above in order to define the period \(t\) pure rent profit function \(\Pi^t\) as follows:

\[
\Pi^t(p^t, (u^t+w^t)P^t, F^t) = \max_{x} \{ \pi^t(p^t, K^t, F^t) - \sum_{s=1}^{3} (u^t_s + w^t_s)P^t_sK^t_s \}
\]  
(5.109)

where the price of one unit of the \(i\)th type of reproducible capital stock is \(P^t_s\) in period \(t\) and \(u^t = (u^t_1, u^t_2, u^t_3)\) is the vector of period \(t\) (deflated) undistorted user costs of capital defined by (5.85) in the previous section and \(w^t = (w^t_1, w^t_2, w^t_3)\) is the period \(t\) vector of (deflated) total tax distortion wedges defined by (5.44)-(5.46) in section 5.4.

In our econometric work, we hold the input of land and other fixed factors that are used by the Australian private production sector fixed throughout our sample period. Hence, in what follows, we will omit \(F^t\) from \(\Pi^t(p^t, (u^t+w^t)P^t, F^t)\). We will also absorb the three user costs of capital, \((u^t_1+w^t_1)P^t_1, (u^t_2+w^t_2)P^t_2\) and \((u^t_3+w^t_3)P^t_3\). Finally, the notation \(\Pi^t\) indicates that the pure rents profit function depends on the period \(t\) as well.
as on the price vector \( p = (p_1, p_2, \ldots, p_{11}) \). We will rewrite this dependence as \( \Pi(p, t) \).

Once a functional form for \( \Pi \) has been chosen, estimating equations can be obtained by differentiating the profit function with respect to the prices \( p_m \), see Diewert (1974a, 137 and 140), (1993; 166 and 168):

\[
y_m(p, t) = \frac{\partial \Pi(p, t)}{\partial p_m}; \quad m = 1, \ldots, 11. \tag{5.110}
\]

The functional form for the pure rents function \( \Pi \) that we chose was a variant of the normalised quadratic functional form, since this functional form allows us to impose the appropriate curvature conditions without destroying its flexibility properties. Using matrix notation, the function can be defined as follows:

\[
\Pi(p, t) = p^* \cdot \mathbf{b} + (t - 1) + (1/2) p^* \cdot A_p / p^* \cdot \mathbf{g}; \quad t = 1, 2, \ldots, 31 \tag{5.111}
\]

where \( \mathbf{b} = [b_1, \ldots, b_{11}] \) and \( \mathbf{d} = [d_1, \ldots, d_{11}] \) are parameter vectors to be estimated and \( A = [a_{mn}] \) is a 11 by 11 symmetric matrix of parameters to be estimated. The vector \( \mathbf{g} = [g_1, \ldots, g_{11}] \) is a vector of exogenously determined parameters. The components of \( \mathbf{g} \) were chosen to be the absolute values of the sample means of the observed net output vectors \( y^t = [y_1^t, \ldots, y_{11}^t] \) normalised so that:

\[
p^* \cdot \mathbf{g} = 1 \tag{5.112}
\]

where \( p^* \) was a fixed vector. The variable \( t \) which appears in (111) is a scalar time variable which serves as a proxy for technological change.

In order to ensure that \( \Pi(p, t) \) is a well behaved profit function (and hence is a convex function in its price variables \( p \)), we set \( A \) to equal the following product of two matrices, \( U \) and its transpose \( U^T \):

\[
A = U U^T \tag{5.113}
\]

where \( U \) is a lower triangular matrix and \( U^T \) is an upper triangular matrix which satisfies the following restrictions:

\[
U^T p^* = 0_{11} \tag{5.114}
\]

where \( 0_{11} \) is a vector of zeros of dimension 11.

Differentiating the profit function (5.111) with respect to the components of \( p \) leads to the following system of 11 estimating equations:

\[
y^t = b + d(t - 1) + Ap^t / p^* \cdot \mathbf{g} - (1/2) p^* \cdot Ap^t \cdot \mathbf{g} / (p^* \cdot \mathbf{g})^2 + e^t; \tag{5.115}
\]

where \( e^t = [e_1^t, \ldots, e_{11}^t] \) is a vector of independently distributed normal residuals where each of the residuals \( e_m^t \) has mean 0 and variance \( \sigma_m^2 \) for \( m = 1, \ldots, 11 \) and \( t = 1, \ldots, 31 \).

The vector of parameters \( d \) essentially adds a linear trend to each estimating equation in order to allow for the effects of technical progress in the Australian economy over our sample period.

Unfortunately, (5.113) and (5.115) did not represent our final model because there is a problem with the profit function defined by (5.111). The problem is that the elasticities of demand and supply derived from the profit function defined by (5.111) can have substantial trends built into them. This is a problem in the present context due to the importance of elasticities in determining marginal excess burdens. We deal with this problem in the following section.

5.9 THE PROBLEM OF TRENDING ELASTICITIES

If we differentiate the pure rents profit function defined by (5.111) above with respect to the \( n \)th component of the price vector \( p \), we obtain the following equation that describes the net supply of commodity \( m \) as a function of the price vector \( p \) in period \( t \):

\[
y_m(p, t) = b_m + d_m(t - 1) + \sum_{j=1}^{11} a_{mj} (p_j / p^g) - (1/2) g_m p A_p / (p^g)^2 \tag{5.116}
\]

Now differentiate (5.116) with respect to \( p_n \), the \( n \)th component of the price vector \( p \):

\[
\frac{\partial y_m(p, t)}{\partial p_n} = a_{mn} / p^g - \sum_{j=1}^{11} a_{mj} p_j g_m / (p^g)^2 \tag{5.117}
\]

Now turn (5.117) into the cross elasticity of net supply of commodity \( m \) with respect to a change in the price of commodity \( n \), \( e_{mn} \).
The last three terms on the right hand side of (5.118) will be zero when \( p = p^* \) and in our empirical work, these last three terms were typically small in magnitude. Thus, the key determinant of the magnitude of the elasticity \( e_{mn} \) will typically be the first term on the right hand side of (5.118), namely, \( a_{mn} \left( p_n / y_m p^* g \right) \). Of course, the parameter \( a_{mn} \) will be constant over time but the other terms, \( p_n \) (the price of commodity \( n \)), \( y_m \) (the net output of commodity \( m \)) and \( p^* g = \sum_{t=1}^{11} p_n g_n \) (a fixed basket price index of all 11 variable input and output prices) can all have substantial trends over our sample period. Thus, our chosen functional form has built in these possible trends in elasticities.

A solution to this problem is readily at hand but at a cost in terms of using up degrees of freedom. We have followed the example of most applied production function researchers and allowed technical progress to affect the constant terms in the system of net supply functions (5.116) but we left the substitution matrix \( A \) unchanged over time. To solve the problem of trending elasticities, all we have to do is allow \( A \) to change over time as well. Thus, in our empirical work, we set the \( A \) matrix in (5.116) above equal to weighted average of a matrix \( B \) (which characterises substitution possibilities in 1967) and a matrix \( C \) (which characterises substitution possibilities in 1998); i.e., we define \( A \) as follows in terms of \( B \) and \( C \) and the time variable \( t \):

\[
A' = \left(1 - [(t-1)/30]\right)B + [(t-1)/30]C; \quad t = 1,2,\ldots,31. \quad (5.119)
\]

Essentially, we now let technical progress affect not only the constant terms in (5.111) but we also allow it to affect substitution possibilities as well. Another way of viewing our new functional form is that we allow the functional form to be flexible at two points (the first sample point and the last) instead of the usual one point.

In order to impose the correct curvature conditions (globally), we need to set the 11 by 11 symmetric matrices \( B \) and \( C \) equal to the product of \( UU^T \) and \( VV^T \) respectively, where \( U \) and \( V \) are lower triangular matrices; i.e., we set:

\[
B = UU^T; \quad C = VV^T; \quad U \text{ and } V \text{ lower triangular.} \quad (5.120)
\]

We also impose the following normalisations on the matrices \( U \) and \( V \):

\[
U^T p^* = 0_1; \quad V^T p^* = 0_1. \quad (5.121)
\]

Now we are ready to describe our empirical results.

5.10 EMPIRICAL RESULTS FOR THE PRODUCTION MODEL

The unknown parameters which appear in (5.115) (where \( A \) is replaced by \( A' \) defined by (5.119) above) can in theory be estimated using nonlinear systems maximum likelihood estimation commands in econometric packages such as TSP or SHAZAM (see White (1978)). However, due to the very large number of parameters in our model, these econometric programs failed to converge. Thus, we decided to run all 11 of our estimating equations in (5.115) as one big nonlinear regression with only one variance parameter \( \sigma^2 \). However, in order to reduce heteroskedasticity, if \( y_m \) was the independent variable, we multiplied both sides of the estimating equation by the corresponding period \( t \) price for the \( m \)th variable input or output, \( p_m \), and divided both sides of the estimating equation by the period \( t \) private sector GDP. Thus, the estimating equations were turned into share of GDP equations. This approach proved to be quite successful using SHAZAM.

It should be noted that equations (5.115) are linear in the unknown vectors of parameters, \( b \) and \( d \), and linear in the unknown components of the matrices of parameters, \( B \) and \( C \). However, when we impose the correct curvature conditions on our estimated profit function by setting \( B = UU^T \) and \( C = VV^T \), the resulting estimating equations (5.115) turn out to be nonlinear in the components of the matrices \( U \) and \( V \). When we attempted to estimate the parameters in \( b \), \( d \), \( U \) and \( V \) by running one big regression, we found that it was difficult to achieve convergence if we attempted to estimate all of the parameters in an initial regression. Thus, we used the following strategy: (i) the parameters in the vectors \( b \) and \( d \) were estimated in an initial linear regression (with \( U \) and \( V \) being set equal to zero matrices initially); (ii) we ran nonlinear regressions, using equations (5.125), introducing one column of the \( U \) matrix and one column of the \( V \) matrix into our nonlinear regression; (iii) the final parameter values from
stage (ii) above were used as starting values in a new nonlinear regression where an additional column of \( U \) and \( V \) were added with starting values close to zero; (iv) step (iii) was repeated until all columns of the \( U \) and \( V \) matrices were entered into the big nonlinear regression, with at least one nonzero component. This algorithm lead us to introduce 5 columns of the \( U \) matrix and 5 columns of the \( V \) matrix. In view of the restrictions (5.121), this means that we should have 40 \( u_{mn} \) parameters and 40 \( v_{mn} \) parameters in our final regression. We also have 11 \( b_n \) and 11 \( d_m \) parameters or an additional 22 parameters to estimate. This means we have a total of 102 parameters to estimate with 11 times 31 or 341 degrees of freedom. This is a large number of parameters for the available degrees of freedom but most of them appear to be necessary to describe substitution elasticities for the Australian economy over our sample time period. Our estimated coefficients may be found in Table 5.6.

It should be noted that the elements in the last row of the lower triangular \( U \) matrix are defined in terms of the other elements of the \( U \) matrix as follows:

\[
u_{11,m} = -\sum_{j=1}^{m-1} u_{jm} ; \quad m = 1, \ldots, 5.
\]

(5.122)

Similarly, the elements in the last row of the lower triangular \( V \) matrix are defined in terms of the other elements of the \( V \) matrix as follows:

\[
u_{11,m} = -\sum_{j=1}^{m-1} v_{jm} ; \quad m = 1, \ldots, 5.
\]

(5.123)

Of the 80 price substitution coefficients \( w_{mn} \) and \( v_{mn} \) 26 had \( t \) statistics greater than two.

Recall definition (5.118) above, which defined the cross elasticity of net supply of commodity \( m \) with respect to a change in the price of commodity \( n, e_{mn} \). There are too many elasticities in the full 11 by 11 matrix of elasticities for us to list them all but we do list the own price elasticities of net supply \( e_{nn} \) in Table 5.7.
<table>
<thead>
<tr>
<th>Year</th>
<th>$e_{11}$</th>
<th>$e_{12}$</th>
<th>$e_{13}$</th>
<th>$e_{14}$</th>
<th>$e_{15}$</th>
<th>$e_{16}$</th>
<th>$e_{17}$</th>
<th>$e_{18}$</th>
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<th>$e_{11/11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>0.956</td>
<td>0.849</td>
<td>0.786</td>
<td>0.780</td>
<td>0.732</td>
<td>0.688</td>
<td>0.646</td>
<td>0.603</td>
<td>0.562</td>
<td>0.524</td>
<td>0.489</td>
</tr>
<tr>
<td>1968</td>
<td>0.767</td>
<td>0.569</td>
<td>0.522</td>
<td>0.476</td>
<td>0.433</td>
<td>0.397</td>
<td>0.362</td>
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<td>0.299</td>
<td>0.269</td>
<td>0.240</td>
</tr>
<tr>
<td>1969</td>
<td>0.863</td>
<td>0.677</td>
<td>0.652</td>
<td>0.625</td>
<td>0.596</td>
<td>0.569</td>
<td>0.543</td>
<td>0.516</td>
<td>0.490</td>
<td>0.465</td>
<td>0.443</td>
</tr>
<tr>
<td>1970</td>
<td>0.763</td>
<td>0.570</td>
<td>0.550</td>
<td>0.532</td>
<td>0.511</td>
<td>0.494</td>
<td>0.478</td>
<td>0.462</td>
<td>0.447</td>
<td>0.432</td>
<td>0.419</td>
</tr>
<tr>
<td>1971</td>
<td>0.763</td>
<td>0.664</td>
<td>0.639</td>
<td>0.618</td>
<td>0.597</td>
<td>0.578</td>
<td>0.558</td>
<td>0.538</td>
<td>0.519</td>
<td>0.499</td>
<td>0.480</td>
</tr>
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<td>1972</td>
<td>0.739</td>
<td>0.677</td>
<td>0.666</td>
<td>0.652</td>
<td>0.641</td>
<td>0.633</td>
<td>0.625</td>
<td>0.616</td>
<td>0.607</td>
<td>0.596</td>
<td>0.585</td>
</tr>
<tr>
<td>1973</td>
<td>0.665</td>
<td>0.700</td>
<td>0.690</td>
<td>0.685</td>
<td>0.677</td>
<td>0.662</td>
<td>0.653</td>
<td>0.648</td>
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<td>1974</td>
<td>0.531</td>
<td>0.610</td>
<td>0.609</td>
<td>0.609</td>
<td>0.608</td>
<td>0.606</td>
<td>0.604</td>
<td>0.602</td>
<td>0.600</td>
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<td>0.597</td>
</tr>
<tr>
<td>1975</td>
<td>0.604</td>
<td>0.680</td>
<td>0.679</td>
<td>0.676</td>
<td>0.674</td>
<td>0.671</td>
<td>0.669</td>
<td>0.666</td>
<td>0.663</td>
<td>0.660</td>
<td>0.658</td>
</tr>
</tbody>
</table>

* The variables are as follows: (1) an aggregate of consumption (excluding housing), residential investment and motor vehicle additions; (2) government consumption of intermediate inputs plus investment; (3) exports; (4) imports; (5) labour input; (6) equipment investment; (7) nonresidential and other construction investment; (8) inventories investment; (9) stocks of machinery and equipment; (10) nonresidential and other construction stocks; (11) inventory stocks and (12) inputs of land and other fixed factors.

It can be seen that many of the elasticities are fairly large, especially near the beginning of the sample period. This may be partially due to the rather large number of parameters in our model. However, it is likely that the large elasticities are simply due to the fact that we are using a very flexible functional form and we have disaggregated inputs and outputs to a greater degree than many previous econometric studies. From viewing Table 5.7, it can be seen that most of the elasticities trend downward in magnitude. Thus, the elasticity of consumption supply, $e_{11}$, trends down from about 0.9 at the beginning of the sample period, 1967, to finish at about 0.4 at the end of the sample period, 1997. Similar downward trends in magnitude can be found for the supply elasticity for government consumption $e_{12}$ (from approximately 6 to 1), for the export supply elasticity $e_{13}$ (from about 1 to 0.3), for the equipment supply elasticity $e_{14}$ (from about 1.8 to 1), for the machinery and equipment demand elasticity $e_{15}$ (from about 9 to 1), for the nonresidential and other construction demand elasticity $e_{16/10}$ (from about 1.8 to 0.7) and for the inventory input demand elasticity $e_{17/11}$ (from about 7 to 0.7).

However, the elasticity of supply for nonresidential and other construction investment $e_{17}$ has a strong upward trend from about 0.6 in 1967 to 4.5 in 1997. The import demand elasticity $e_{18}$ also exhibited a strong increase in magnitude from 0.7 in 1967 to 1.7 in 1987 and then it trended downward in magnitude to about 0.8 in 1997. The elasticity of demand for labour $e_{19}$ also trended upward in magnitude initially from 0.8 in 1967 to about 1 in 1975 and then it remained approximately constant for the remainder of the sample period. The elasticity of inventory net supply $e_{19}$ was large and erratic throughout the sample period (due to the small level of inventory net supply). Even though our elasticity estimates generally exhibited trends, with our new functional form, we can be reasonably certain that these elasticity trends are induced by the data rather than by the functional form.

Given the strong downward trends in the magnitudes of the capital input demand elasticities, we would expect on the basis of our back of the envelope model presented in section 5.3 above that the marginal excess burdens of capital taxation would decline over time. As we shall see in the next section, this proved to be the case.

Consistent with economic theory, we suspect that a more disaggregated model would yield bigger (in magnitude) elasticities. This is what we found in our studies of the New Zealand economy, moving from a highly aggregated model to one that is relatively disaggregated has increased the scope for substitution and, consequently, led to substantially larger elasticity estimates. Thus, it is likely that further disaggregation would lead to even higher elasticities of demand for capital and this would feed into higher estimates of deadweight losses and marginal excess burdens of capital taxation.

Our observation that disaggregation tends to lead to larger estimate of elasticities of supply and demand is one that has not been stressed in the
literature a great deal. However, given the importance of elasticity information for a wide variety of policy purposes, we believe that the point is an important one and deserves further research.22

We have now assembled all the necessary building blocks for the construction of marginal excess burdens for capital taxation. In the next section, we present our marginal excess burden estimates.

5.11 MARGINAL EXCESS BURDENS OF CAPITAL TAXATION FOR AUSTRALIA

As outlined in section 5.7 (see formula (5.95) above), we calculate a second order approximation to the total loss of output that results from the taxation of capital due to both the personal and business income tax, the property tax on structures and sales taxes on the purchases of durable capital equipment. This second order approach is the average of two first order approximations: one around the distorted equilibrium and one around the undistorted equilibrium. These total losses are reported as a fraction of business sector GDP in Table 5.8 and Figure 5.10 below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Loss</th>
<th>Year</th>
<th>Loss</th>
<th>Year</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>1.080</td>
<td>1978</td>
<td>0.810</td>
<td>1989</td>
<td>0.770</td>
</tr>
<tr>
<td>1968</td>
<td>0.950</td>
<td>1979</td>
<td>0.570</td>
<td>1990</td>
<td>0.850</td>
</tr>
<tr>
<td>1969</td>
<td>0.860</td>
<td>1980</td>
<td>0.500</td>
<td>1991</td>
<td>0.880</td>
</tr>
<tr>
<td>1970</td>
<td>0.970</td>
<td>1981</td>
<td>0.620</td>
<td>1992</td>
<td>0.690</td>
</tr>
<tr>
<td>1971</td>
<td>0.880</td>
<td>1982</td>
<td>0.550</td>
<td>1993</td>
<td>0.600</td>
</tr>
<tr>
<td>1972</td>
<td>0.950</td>
<td>1983</td>
<td>0.460</td>
<td>1994</td>
<td>0.620</td>
</tr>
<tr>
<td>1973</td>
<td>0.810</td>
<td>1984</td>
<td>0.430</td>
<td>1995</td>
<td>0.720</td>
</tr>
<tr>
<td>1974</td>
<td>0.860</td>
<td>1985</td>
<td>0.510</td>
<td>1996</td>
<td>0.740</td>
</tr>
<tr>
<td>1975</td>
<td>0.950</td>
<td>1986</td>
<td>0.520</td>
<td>1997</td>
<td>0.910</td>
</tr>
<tr>
<td>1976</td>
<td>1.060</td>
<td>1987</td>
<td>0.650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td>0.940</td>
<td>1988</td>
<td>0.810</td>
<td>Average</td>
<td>0.760</td>
</tr>
</tbody>
</table>

After starting from a relatively high proportion of around 1.08 per cent of GDP in 1967, the production loss from capital taxation progressively declines to 0.43 per cent of GDP in 1984 and then generally trends upward to finish at 0.91 per cent of GDP in 1997. We know that these productive efficiency losses grow roughly proportionally to the magnitude of elasticities and increase at a squared rate as the distortion wedges increase. From Table 5.4, it can be seen that the wedges were roughly U shaped over the entire sample period but that the wedges were about 30 to 40 per cent higher at the end of the sample period compared to the beginning. From Table 5.7, it can be seen that capital input demand elasticities started out at relatively high levels in 1967 and declined throughout the sample period. The interaction of these two effects led to the U shaped pattern of efficiency losses in Table 5.8.

As noted in sections 5.2 and 5.7 (see formula (5.107) above), the marginal excess burden of a tax parameter is the loss of output due to a marginal increase in the tax parameter divided by the increase in total tax revenue due to the same marginal change in the tax parameter. In Table 5.9 and Figure 5.11 below, we list the MEB for an increase in the business capital tax rate \( \tau \) and the MEB for an increase in the property tax on structures \( t_{PNR} \). In both cases, the increase in the tax rate increased tax revenues.

The Marginal Excess Burden of the business income tax started at 45 per cent in 1967 and declined rapidly to hit a low of 17 per cent in 1985. The MEB then fluctuated between 17 per cent and 25 per cent for the remainder of the sample period, finishing at 21.6 per cent in 1997. Generally declining (in magnitude) elasticities of demand for all three types of variable capital explain most of these fluctuations. The sample average of the MEB's for the business income tax was 26.15 per cent.

The MEB for the property tax on structures started at 27.8 per cent and increased rapidly to 64.7 per cent in 1976. The MEBs then declined to
26.8 per cent in 1984 (the corresponding total tax wedge \( w_{NR} \) was at its lowest point for the sample period, 2.71 per cent) and then the MEBs increased to 55.8 per cent in 1991 (the corresponding total tax wedge \( w_{NR} \) was at its highest point up to that time, 4.15 per cent). The MEBs for the property tax then generally trended down (driven by declining elasticities) to finish at 46.8 per cent in 1997. The sample average of the MEBs for the property tax on business structures was 40.2 per cent, which is considerably higher than the sample average MEB for the business income tax. This result is explained by the fact that the ratio of the distortion wedge \( w \) to the corresponding undistorted user cost \( u \) is much higher for structures than it is for machinery and equipment. We note that 40 per cent is roughly twice the marginal excess burden rate that we found for consumption and labour in New Zealand.23

<table>
<thead>
<tr>
<th>Year</th>
<th>Business Tax</th>
<th>Property Tax</th>
<th>Year</th>
<th>Business Tax</th>
<th>Property Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>45.39</td>
<td>27.81</td>
<td>1983</td>
<td>16.25</td>
<td>27.02</td>
</tr>
<tr>
<td>1968</td>
<td>39.77</td>
<td>27.49</td>
<td>1984</td>
<td>15.83</td>
<td>26.79</td>
</tr>
<tr>
<td>1969</td>
<td>37.21</td>
<td>30.18</td>
<td>1985</td>
<td>17.15</td>
<td>29.95</td>
</tr>
<tr>
<td>1970</td>
<td>40.09</td>
<td>33.75</td>
<td>1986</td>
<td>17.33</td>
<td>29.68</td>
</tr>
<tr>
<td>1971</td>
<td>34.76</td>
<td>36.25</td>
<td>1987</td>
<td>20.13</td>
<td>34.00</td>
</tr>
<tr>
<td>1972</td>
<td>37.14</td>
<td>40.51</td>
<td>1988</td>
<td>23.71</td>
<td>44.47</td>
</tr>
<tr>
<td>1973</td>
<td>32.66</td>
<td>39.91</td>
<td>1989</td>
<td>24.11</td>
<td>50.52</td>
</tr>
<tr>
<td>1974</td>
<td>32.00</td>
<td>42.72</td>
<td>1990</td>
<td>25.16</td>
<td>54.96</td>
</tr>
<tr>
<td>1975</td>
<td>34.35</td>
<td>62.44</td>
<td>1991</td>
<td>25.16</td>
<td>55.77</td>
</tr>
<tr>
<td>1976</td>
<td>35.60</td>
<td>64.68</td>
<td>1992</td>
<td>26.33</td>
<td>42.65</td>
</tr>
<tr>
<td>1977</td>
<td>31.09</td>
<td>53.10</td>
<td>1993</td>
<td>17.63</td>
<td>36.08</td>
</tr>
<tr>
<td>1978</td>
<td>27.79</td>
<td>44.35</td>
<td>1994</td>
<td>17.85</td>
<td>37.09</td>
</tr>
<tr>
<td>1979</td>
<td>21.54</td>
<td>34.36</td>
<td>1995</td>
<td>19.80</td>
<td>42.23</td>
</tr>
<tr>
<td>1980</td>
<td>19.31</td>
<td>32.03</td>
<td>1996</td>
<td>19.51</td>
<td>42.59</td>
</tr>
<tr>
<td>1982</td>
<td>18.88</td>
<td>31.75</td>
<td>Average</td>
<td>26.15</td>
<td>40.20</td>
</tr>
</tbody>
</table>

5.12.1 CONCLUSIONS

We conclude with the following observations:

- We should aim to reduce capital tax distortions (particularly disparities between different assets and producers) because they always involve a loss of productive efficiency. The loss of revenue has to be made up by taxing consumption or labour but with enough tax instruments at its disposal, the tax authority can always design a tax reform strategy that will increase overall welfare.
- Our estimates of the burdens of capital taxation are likely to be underestimated due to our use of average tax rates. The complexities of the tax code lead to a much more dispersed pattern of burdens than averages indicate and since the losses are approximately proportional to the squares of tax distortions, averaging tax distortions will lead to an underestimate of the true efficiency losses.
- Our estimates of the burdens of capital taxation are also likely to be underestimated due to the relatively high degree of aggregation in our model. There is theoretical and empirical evidence that elasticities of substitution increase in magnitude as we disaggregate over commodities.
These higher elasticity estimates would generate proportionately higher estimates of total and marginal efficiency losses.

It may be useful to spell out in more detail why it is inefficient to have a system of capital taxation that generates nonzero distortion wedges. The basic intuition behind the above algebra is explained rather well by Judd (1999):

“One general problem with this literature [on capital taxation] is the lack of economic intuition. ... In this paper, we ignore simple dynamic features such as the steady state behavior or long run elasticities, and instead put the zero long run tax results on more economically appealing foundations. To do this, we look to the commodity tax literature. Two results from that literature apply here; first, the optimality of uniform taxation with separable and sufficiently symmetric utility, and, second the prohibition of intermediate good taxation derived in Diamond and Mirrlees (1971). Our methods generalize previous work and tie the results to the commodity tax literature, a change which helps us understand why we often find that the average tax rate on capital income is zero in the optimal policy.” Kenneth L. Judd (1999; 2).

It is the second result from optimal tax theory, the prohibition against taxing intermediate inputs in production, that explains our results. Judd goes on to elaborate on this point:

“The second key principle we invoke is the Diamond-Mirrlees argument against the taxation of intermediate goods. This is relevant here since capital goods, physical and human, are intermediate goods. In fact, income taxation is equivalent to sales taxation of intermediate goods. This can be seen by noting, for example, that a 100 % sales tax on capital equipment is equivalent to a 50 % tax on the income from capital equipment. Since intermediate good taxation will generally put an economy on the interior of its production possibilities frontier, capital income taxation is likely to produce similar factor distortions, particularly if there are many capital goods. Therefore, an optimal tax structure would tax only final goods”. Kenneth L. Judd (1999; 5-6).

Thus, a reproducible capital stock component is both produced by the production sector (or imported at a fixed world price and, thus, is produced by an integrated world production sector) and used as an input in later periods; i.e. it is an intertemporal intermediate input. Hence, in order for an economy to achieve productive efficiency, it is necessary that all users and producers of an intermediate commodity face the same prices.24 However, the system of business income taxation causes users and producers of reproducible capital to face different (intertemporal) prices. Diewert (1988) made the same point as Judd:

“The other major thrust of this paper will be to indicate four major areas where our present tax system is inefficient. Thus in the second, third and fourth sections below, we discuss three different types of deadweight loss induced by our present system of business taxation. In the second section, we discuss the losses due to the fact that the tax system does not treat (nondurable) inputs and outputs in an even handed manner; that is, there are tariffs and sales taxes that fall within the business sector (the manufacturer’s sales tax) as well as various output subsidies. In the third section, we discuss the losses due to the uneven tax treatment of durable inputs, such as land, inventories and various types of capital.” W. Erwin Diewert (1988; 2).

Thus, Diewert noted that both intermediate input taxation and the taxation of reproducible capital inputs led to a loss of productive efficiency. The third type of deadweight loss that leads to a global loss of productive efficiency is transfer pricing. Diewert went on to characterize these three types of loss of productive efficiency as follows:

“The above three types of deadweight loss lead to both a loss of productive efficiency as well as a loss of overall efficiency defined earlier. A tax system is consistent with productive efficiency if the allocation of resources across the entire business sector is such that no output can be increased, holding other aggregate outputs and inputs fixed.” W. Erwin Diewert (1988; 2-3).

“In summary: in order to achieve productive efficiency, it is necessary that all producers in the economy face the same relative prices for their outputs and variable inputs.” W. Erwin Diewert (1988; 6).

Tax systems that lead to a loss of productive efficiency can always be redesigned so that the inefficiencies are eliminated, the same tax revenues are collected and the utilities of at least some households increase, provided that the government has a sufficient number of tax instruments at its disposal.

NOTES

* We gratefully acknowledge the financial support of the Social Sciences and Humanities Research Council of Canada and the helpful comments of an anonymous referee, Kevin Fox and Peter Robertson. The usual disclaimer applies.

1 See Diewert (1983a) (1983b) for the productive efficiency approach to tax policy. Although capital taxation cannot be justified on productivity or efficiency grounds, it still can be justified on equity grounds.
Reproducible capital stocks are stocks produced by the production sector. There are no efficiency losses associated with taxing capital stock components that are fixed (such as land).

Diewert and Lawrence (1994) (1996) found that the marginal excess burdens for labour and consumption taxes were in the 10 percent to 20 percent range for the New Zealand economy.

Note that is a nominal interest rate as opposed to the real interest rate in the previous section.

See Diewert and Fox (1999).

For structures, we also add the term \( \tau P \) to (42) where \( \tau \) is the property tax rate.

We used smoothed ex post asset inflation rates to proxy these anticipated asset inflation rates. We used the Lowess nonparametric smoothing option in SHAZAM with the smoothing parameter \( f = 0.2 \).

For additional material on profit functions and duality theory, see Diewert (1974a) (1993).

Thus our treatment of international trade follows that of Kohli (1978).

The capital demand function, \( K(w) \), and the surplus function, \( S(w) \), should actually be denoted as \( K'(w) \) and \( S'(w) \) to indicate that these functions depend on the period \( t \) price data and on the period \( t \) cash flow function \( f(t) \). The function \( f(t) \) depends on time \( t \) due to technical change.

See Diewert's (1976, 118) quadratic approximation lemma.

This is known as Hotelling's Lemma; see Diewert (1992, 166).

Note that we have changed our notation for the deflated wedges.

Since the pure rents profit function \( f(t) \) must be a convex function in its price arguments, the matrix of second order partial derivatives in (5.105) must be positive semidefinite. This means that the approximate loss defined by (5.105) must be nonnegative.

In this approach, we treat the user costs of our three types of reproducible capital as exogenous variables and the corresponding capital input demands are treated as endogenous variables. Thus, in our present econometric approach, the demand for capital is treated in a symmetric manner with the demand for labour. In contrast, in our earlier study of the New Zealand economy, Diewert and Lawrence (1994), we treated the stocks of reproducible capital as exogenous variables and the corresponding rental prices as endogenous variables. However, the use of this approach led to unrealistically large elasticities and excess burdens. We feel that our present approach is more appropriate in the context of determining the excess burdens of capital taxation, a topic that our earlier study did not address.


We chose \( x \) to be a vector of ones.

See Diewert and Wales (1987, 52-53) for further explanation.

Restrictions like (5.114) are required in order to identify the components of the \( b \) vector. Alternatively, restrictions (5.114) could be dropped but then the \( b \) vector would have to be dropped as well.

Adding an extra \( U \) or \( V \) column led to a negligible increase in the log likelihood after 4 columns were added so we stopped at 5 columns for the \( U \) and \( V \) matrices.

See Diewert (1974b).

One of the last serious discussions about the likely size of elasticities took place 50 years ago in the context of trade elasticities by Orcutt (1950) who argued that elasticities of import demand and export supply were likely to be larger than had been thought. We found that in our work on the New Zealand economy, trade elasticities in a 15 commodity model dropped substantially when we aggregated our two export commodities into a single export aggregate and when we aggregated our three import commodities into a single import aggregate.

22 See Diewert and Lawrence (1994).

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Han, S-H (1996), 'The Deadweight Costs of Taxes in Australia', Contributed paper to 25th Conference of Economists, Canberra.


Organisation for Economic Co-operation and Development (various years), Economic Outlook, Paris.


APPENDIX: DATA CONSTRUCTION

The construction of the database used in this study largely follows the approach outlined in detail in the appendices to Diewert and Lawrence (1994) (2000). The treatment of inventories follows that of Diewert and Smith (1994).

The principal data sources for this study are ABS and Organisation of Economic Cooperation and Development (OECD) data contained in Econdata (1996). A brief outline of the major variables in the model follows.
Variable Outputs and Inputs

General Private Consumption: This variable is based on the ABS private final consumption expenditure series. As in our 1994 study we exclude the housing and transport components of PFCE and replace them with investment in residential housing and sales of motor vehicles to consumers, respectively. General private consumption is then formed as the aggregate of these three components. Price deflators for the three components are sourced from the ABS National Accounts and the NIF model database.

Government Consumption of Goods and Services: Government final consumption expenditure is available from the ABS National Accounts. However, we need to deduct government expenditure on wages and salaries from this to form an estimate of government purchases from the market sector. A government wages and salaries series was formed from National Accounts and OECD Economic Outlook sources. The price index for this variable was taken as the implicit price deflator for GFCE.

Exports and Imports: The value of aggregate exports and imports and implicit price deflators were obtained from the National Accounts. Consistent with normal convention in estimating variable profit functions, the quantities of the variable inputs (imports and labour) were taken to be negative.

Labour: The value of market sector labour inputs is taken as the sum of wages, salaries and supplements for market sector employees plus a return to the self-employed. Market sector wages and salaries are obtained by subtracting non-market wages and salaries from the National Accounts total wages and salaries. Non-market wages and salaries are obtained from the National Accounts and the OECD Economic Outlook. The average wage rate is also taken from the OECD Economic Outlook and the self-employed are allocated a return equal to 75 per cent of the average private sector employee compensation.

Plant and Equipment, and Non-residential and Other Construction Investment: The values and prices of market sector plant and equipment investment and non-residential and other construction investment were obtained from the ABS Capital Stocks publication (catalogue number 5221.0). The investment output price variables were adjusted to allow for the asset-specific inflation rates and the post-tax rate of return.

Inventories Investment: The value of non-farm market sector inventories is formed from the National Accounts and the ABS National Balance Sheets (catalogue number 5241.0). The price is taken be the average of manufacturing articles and materials used in manufacturing. Livestock inventories are formed from Australian Bureau of Agricultural and Resource Economics (ABARE) and ABS data on the numbers and prices of sheep, cattle, pigs and horses. The value of the investment output variable for inventories is formed from the current period price of inventories, the next period quantity of inventories, the asset-specific inflation rate and the post-tax rate of return.

Capital Stocks

The values and prices of market sector plant and equipment, and non-residential and other construction stocks were constructed using the perpetual inventory method using the Australian Bureau of Statistics' net capital stock estimates for 1967 as a starting point. The depreciation rate was then chosen so that our estimates for 1997 coincided with those of the ABS. The value and price of the inventories stock was sourced from the ABS and ABARE sources listed above. The quantity of business and agricultural land was assumed to be fixed over the period and the value of the land stock was taken from the ABS National Balance Sheets backdated with information from Dippelsman and McDonald (1989) and Lawrence and McKay (1980) and updated using ABS information on capital city housing prices. User costs for the capital stocks are formed using nominal rates of return, asset-specific rates of inflation, property and capital tax rates and depreciation rates as outlined in section 5.4.

Taxes

Taxation data was compiled from a number of ABS sources, the NIF model database and the OECD Revenue Statistics. The main taxation categories formed were labour direct taxes, capital direct taxes, sales tax on motor vehicles, customs duties on imports, other indirect taxes, housing property taxes and business property taxes. Labour direct taxes were formed as the sum of net pay-as-you-earn taxes, payroll taxes (including fringe benefits) and a proportion of other individuals tax payments (based on the distribution of employees and self-employed). Capital direct taxes comprise enterprise taxes, non-resident and transactions taxes plus estate duties and the remainder of other individuals taxes. The producer model is estimated on data expressed entirely in producers' prices.