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WHY AN INFORMED PRINCIPAL MAY LEAVE RENTS TO AN AGENT*

BY PAUL BEAUDRY

This paper characterizes incentive contracts for the situation where a principal is privately informed about the technology governing an agency relationship. In contrast to a standard principal-agent relationship, it is shown that a principal who values effort highly will choose to induce effort by paying a high base wage and low bonus payments. Moreover, the equilibrium contract has the principal transferring rents to the agent even though contracting possibilities are unrestricted and both principal and agent are risk neutral. Consequently, the informed-principal framework is shown to provide a rational for the payment of efficiency wages.

1. INTRODUCTION

One of the insights of the moral-hazard literature is that a principal can generally choose an optimal incentive contract by first determining the proper state-contingent payments and then by determining the level of the base wage that extracts all the rents from the relationship (or equivalently by demanding the appropriate entrance fee). This literature also suggests that incentive schemes will vary in accordance with the principal’s valuation of the agent’s effort, with an agent bearing more risk when his effort is highly valuable. Therefore, as Carmichael (1984), (1990) has emphasized, agency theory can explain why similar workers are paid different wages but does not explain why an employer would choose to leave ex ante rents to an employee.

In this paper, an informed-principal approach to agency relationships is shown to substantially alter the previous characterization of optimal incentive contracts. The informed-principal problem relates to the situation in which a principal has private information relevant to the agency relationship at the time of negotiating the contract. This informational asymmetry seems justified in many employment situations since, at the time of hiring, an employer often knows more about the work place, future demand conditions, and promotion prospects than does a new recruit. Because a principal’s information can potentially affect the agent’s

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1 The author wishes to thank Russ Cooper, Larry Katz, Kevin Lang, Michel Poitevin, Joseph Stiglitz, Andy Weiss and especially an anonymous referee for useful comments. This research is partially supported by National Science Foundation grant SES 9209573.
2 See Grossman and Hart (1983) for a thorough analysis of the principal-agent problem and see Carmichael (1984) for a clear presentation of the argument that it is generally optimal for the principal to extract all rents from the agent.
3 The issue of whether it may be optimal for employers to choose employment contracts that leave rents to workers is central to the debate surrounding efficiency wages. For an overview of the efficiency-wage debate, see Carmichael (1990) and Lang and Kahn (1990).
incentives, an optimal contract must trade off the advantages of revealing information and providing proper incentives. In fact, the principal’s privileged information transforms the contracting problem into a game of incomplete information (i.e., a signaling game with multiple signals). If the agent finds it useful to know the principal’s private information before deciding his own actions, he may try to infer this information from the principal’s choice of contract. In this situation, the elements of the contract become signals of the principal’s private information.

The main question addressed in this paper is how a principal’s private information, regarding the technology governing an agency relationship, changes the characteristics of optimal incentive contracts. In particular, it is examined whether this informational asymmetry can lead the principal to transfer rents to the agent even when contracting possibilities are unrestricted and when both principal and agent are risk neutral. This latter question is of special interest since it is central to the debate surrounding the optimality of efficiency wages.⁴

The focus of the paper differs from previous papers on informed-principal problems by offering a detailed characterization of equilibrium contracts for a situation in which the informed-principal faces an agent moral-hazard problem. In comparison, Myerson’s (1983) analysis of the informed-principal problem focuses mainly on the interrelationship between different solution concepts. Its limited characterization of equilibrium mechanisms does not address whether agents receive ex ante rents. Maskin and Tirole’s (1989), (1992) analyses of the informed principal problem do offer detailed characterizations of equilibrium contracts but do not cover the situation addressed here. In particular, in both these latter papers the principal does not face any moral-hazard problem on the part of the agent.

The main result of the paper is that an agent’s uncertainty about the technology governing the relationship is likely to create a situation where a principal finds it profitable to induce effort by offering a low-bonus high-wage contract that leaves rents to the agent. The transfer of rents arises because, beyond choosing a contract that encourages effort by making the attainment of an objective desirable for the agent, the principal must also ensure that the attainment of the objective is believable by offering a credible signal. Consequently, by showing why a principal may choose to leave ex ante rents to the agent, this paper offers a simple rational for the payment of efficiency wages.

The remaining sections of the paper are structured as follows. Section 2 presents a stylized principal-agent problem. The optimal contract for the case where the principal does not have an informational advantage in characterized. Section 3 characterizes the principal’s choice of contract for the informed-principal problem. The main results on the terms of this contract and on the possibility of rent transfers are derived in this section. Section 4 briefly discusses a multiperiod extensions of the model and Section 5 contains concluding comments.

⁴ Starting with Weiss (1980), many papers have analyzed the opposite situation to the one analyzed here, that is, the situation where the worker has better information about productivity than the employer. In particular, see Guasch and Weiss (1980), Sappington (1983) and Mookherjee (1988).
2. THE PRINCIPAL-AGENT RELATIONSHIP

To clearly identify the contractual implications of an informed-principal relationship, it is helpful to begin by considering an agency relationship for a case where the principal does not have an informational advantage. Throughout the paper, the principal will be referred to as the employer and the agent as the worker, but other interpretations are obviously possible.

The employer and the potential worker are assumed to be risk neutral, with the worker's reservation utility level denoted by \( U_r \). The output resulting from employment is a random variable which can take on two values \( \phi_1 \) and \( \phi_2 \), where \( \phi_1 \geq \phi_2 \). The probability of attaining a high value of production, \( \phi_1 \), depends both on the characteristics of the job and on an action taken by the worker. There are evidently many ways in which jobs can differ. The situation of interest for analyzing agency problems is when jobs differ with respect to the value of the action taken by the worker. Let \( \beta \in [0, B] \) represent a technological characteristic of a job and let \( e \in [0, E] \) represent the agents action. The worker's action will be referred to as effort and is assumed not to be observable by the employer. The function \( \pi(e, \beta) \) will represent the probability of realizing \( \phi_1 \), given an effort of \( e \) by a worker in a job of type \( \beta \). The worker associates a monetary cost, \( g(e) \), with undertaking an effort equal to \( e \). Assumptions 1 and 2 impose conditions on the continuous functions \( \pi(\cdot) \) and \( g(\cdot) \) that ensure that the contracting problem can be characterized by first order conditions.

\[
\text{A1.} \quad \text{The agent's cost of effort } e \text{ is } g(e), \text{ with } g'(e) > 0, \ g''(e) > 0, \ g(0) = 0, \ g'(0) = 0 \text{ and } \lim_{e \to E} g'(e) = \infty. 
\]

\[
\text{A2.} \quad \text{The probability of realizing } \phi_1 \text{ given an effort } e \text{ and job characteristic } \beta \text{ is } \pi(e, \beta), \text{ with } \pi_\beta(e, \beta) > 0, \ \pi_{ee}(e, \beta) \leq 0, \ \pi(e, 0) = 0 \text{ and } \pi_e(e, \beta) > 0 \text{ for } \beta > 0. 
\]

Given this technology, the employer faces the problem of choosing a contract which is acceptable to the worker and which creates the right incentives. In this relationship, the form of the contract needs only to specify two parameters, for example, the wage paid in each of the two states of nature. Equivalently, but for convenience sake, the contract will be considered as specifying a base wage, \( w \), and a bonus payment, \( s \), paid when the high state of production is realized. Assuming the employer has the bargaining power and assuming that all characteristics of the job are common knowledge, the employer's problem can be represented by the following program:

\[
\max_{w, s, e} \pi(e, \beta)(\phi_1 - w - s) + (1 - \pi(e, \beta))(\phi_2 - w) \\
\text{s.t. (i) } e \in \arg \max_{e'} w + \pi(e', \beta)s - g(e') \\
\text{(ii) } w + \pi(e, \beta)s - g(e) \geq U_r. 
\]

The employer maximizes profits by choosing a contract \((w, s)\) and a level of effort that is incentive compatible and is acceptable to the worker (i.e. individually
rational). Proposition 1 characterizes the firm’s optimal contract. As is well known, the worker does not receive any rents in this situation. The bonus gives the worker the optimal incentives, and the base wage is used to extract all the rents. Note that this contract could alternatively be presented as a sell-out contract, that is, a contract where the worker simply buys the job from the employer at the price $\phi_2 - w^*(\beta)$.

**Proposition 1.** If the worker knows $\beta$, the principal extracts all the rents from the worker regardless of the characteristic of the job. The optimal contract for a job with characteristic $\beta$ is defined as follows:

$$s^*(\beta) = \phi_1 - \phi_2$$

$$w^*(\beta) = U_r - \pi(\bar{e}(\phi_1 - \phi_2, \beta), \beta)\phi_1 - \phi_2 + g(\bar{e}(\phi_1 - \phi_2, \beta))$$

where the function $\bar{e}(\phi_1 - \phi_2, \beta)$ is implicitly defined by

$$g'(\bar{e}) = (\phi_1 - \phi_2)\beta \pi_e(\bar{e}, \beta).$$

**Proof of Proposition 1.** All proofs are given in the Appendix.

The optimal contract described in Proposition 1 has the property that all jobs pay the same bonus payment and jobs in which effort is more valuable pay a lower base wage. The fact that a worker is ready to accept a lower base wage and the same bonus payment for jobs in which effort is more valuable obviously gives an employer incentives to misrepresent the characteristic of a job. For example, by misleading the worker, an employer with a low $\beta$ could manage to pay a lower base wage. Therefore, the simple characterization of incentive contracts described in Proposition 1 depends crucially on the worker knowing the characteristic of the job when being hired.

3. **THE INFORMED-PRINCIPAL PROBLEM**

This section compares the previous analysis to a situation in which the characteristic of a job is assumed to be the employer’s privileged information. For the sake of simplicity, the employer is assumed to have a job with either characteristic $\beta_H$ or $\beta_L$, where $\beta_H > \beta_L$. As a result of this asymmetry of information, the worker is no longer assumed to know the technology governing the relationship when being hired. The employment relationship must therefore be modeled as a game of incomplete information.

In an employment relation, the employer usually moves first by offering an employment contract followed by a response from the worker. This sequence of moves is maintained when analyzing the employment relation as a game: the employer offers a contract and the worker either accepts or rejects it. By adding an initial move to this game where nature chooses the characteristic of the job, the game becomes one of imperfect information (a signalling game with multiple signals). After being offered a contract, the worker forms beliefs about the
characteristic of the job. The worker then chooses to accept or reject the offer and chooses the level of effort that he will provide in this job.

This sequence of actions defines an extensive form game in which there are several Perfect Bayesian Equilibria. A Perfect Bayesian Equilibrium (PBE) requires that the employer and the worker act sequentially rational and that the worker’s belief be updated using Bayes rule when possible. The paper concentrates on characterizing the separating PBE equilibria of the game. Proposition 2 provides a qualitative assessment of the difference between the structure of payment offered in a $\beta_H$ type job versus that offered for employment in a $\beta_L$ type job in any separating equilibria. The justification for concentrating only on the separating equilibria of this game is that, as shown in Beaudry (1989), separation is the only equilibrium configuration which is robust to the application of the Intuitive Criterion (Cho and Kreps 1987).

**Proposition 2.** When an employer has private information about the marginal value of effort, in any separating PBE equilibrium, the equilibrium contract associated with the more productive job offers a higher base wage and a smaller bonus payment than the contract associated with the less productive job.

Proposition 2 demonstrates that, in any separating equilibria, a more productive job pays a higher base wage and provides a lower bonus payment than the less productive job. This characterization of equilibrium contracts is in dire contrast with that found in Proposition 1. Instead of having the employer who values effort highly offering an incentive scheme that forces the agent to bear more risk, the informational asymmetry about technology leads the more productive employer to choose an incentive scheme in which the agent bears less risk. In comparison to the standard principal-agency problem, the informed-principal approach therefore reverses the ordering between the importance of effort and the risk borne by the agent. It is also of interest to point out that the worker’s incentive towards effort in the $\beta_H$ type job is reduced in comparison with the symmetric-information situation. The employer does not restore this incentive to the first-best level, since being revealed as a more productive job is only valuable for an employer if the bonus is strictly less than $\phi_1 - \phi_2$. Consequently, the informed-principal paradigm provides an explanation for why an employer may want to maintain a residual claim in an employee’s production even though both the principal and the agent are risk neutral.

To understand why separation implies that the more productive job offers a contract with a higher base wage and a lower bonus payment, it is useful to consider each of these contractual elements in turn. First consider the bonus payment. If the

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5 Milgrom and Roberts (1986) examine the equilibrium properties of a signalling game with two signals for the case of product differentiation. Their analysis has certain similarities with the one presented here, although it is not a contracting problem and any possibility of rent transfers was ruled out by the formulation of their problem. Moreover, their focus was on identifying conditions under which two signals are used in equilibrium, while here the two signals are always used.

6 In Beaudry (1989), it is further shown that there is a unique separating equilibrium outcome that satisfies the Intuitive Criterion. Given that the propositions shown in this paper apply to all separating equilibria, they de facto apply to the unique PBE that satisfies the Intuitive Criterion.
bonus payment associated with the more productive job is higher than $\phi_1 - \phi_2$, then the employer is penalized each time the high state is realized. Since the high state of production is realized less often in the $\beta_L$ job, this penalty is less costly for an employer with this less productive job. Therefore, an employer with a $\beta_H$ type job cannot use a higher bonus payment to signal a high productivity job.

Now consider whether a more productive job can be identified by offering a contract which pays a bonus lower than $\phi_1 - \phi_2$ and a wage lower than $w^*(\beta_L)$. Again a $\beta_L$ type employer would always want to offer such a contract, since he would gain directly on the base wage and he would gain whenever the high state would arise. Therefore, in order to profitably separate, the employer with a more productive job must choose to increase the base wage above that paid in a $\beta_L$ type job, while paying a bonus lower than $\phi_1 - \phi_2$. The employer with a less productive job finds it unprofitable to mimic this behavior, since the compensating effect of the higher effort rendered by a worker who thinks he is employed in the more productive job is less valuable for him.

Proposition 2 exemplifies how the introduction of private information on the part of the employer can fundamentally change predictions regarding the pattern of payments in principal-agent relationships. In particular, the dichotomous roles of bonus payment versus base wage is loss. Recall that in the absence of asymmetric information on the part of the employer, the bonus payment is set to control effort and the base wage is used by the employer to extract all rents from the relationship. In contrast, in the informed-principal relationship the choice of base wage also effects incentives since it signals the type of job (and correspondingly the value of effort). This potential signalling role for the base wage indicates that the employer may not want to extract all the rents from the worker since it might conflict with the worker’s effort. It is this possibility of rent transfer that is addressed in Proposition 3. The proposition restricts attention to situations satisfying Assumption 3, that is, situations in which the employer would want to hire the worker even if the worker did not provide any discretionary effort. This additional assumption allows the statement of the proposition to be as simple as possible. In the absence of this assumption, the proposition would need to be stated in terms of the existence of a set (parameterized on $\beta_H$) such that, if $\beta_L$ and $\phi_2$ belong to the set, the job of type $\beta_H$ leaves rents to the worker.

A3. There are always potential gains from trade in the principal-agent relationship, that is, $\phi_2 > U_r$.

**Proposition 3.** For any $\beta_H$ there exists a $\beta^*(\beta_H) > 0$, such that if $\beta_L \leq \beta^*(\beta_H)$, all separating equilibria will be characterized by the employer with a job of type $\beta_H$ offering a contract that leaves rents to the worker.

Proposition 3 states that when a worker has “enough” uncertainty about the extent to which his effort affects the probability of receiving a bonus, an employer who values effort highly will choose a contract which transfers rents to the agent. In other words, if an employer with a job of type $\beta_H$ must distinguish himself from an employer of at least type $\beta^*(\beta_H)$, then the equilibrium contract transfers rents. In particular, if $\beta_L = 0$ then a $\beta_H$ job always transfers rents to the worker.
The intuition for why a large amount of uncertainty necessarily leads to a transfer of rents can most simply be explained by examining the tradeoffs associated with changing the bonus payment. Recall from Proposition 2 that separation always implies a higher base wage for a $\beta_H$ job than a $\beta_L$ job. Therefore, if the two types of jobs are very different, to simultaneously separate and extract all rents, a type $\beta_H$ employer must be offering a contract with a very low bonus. Otherwise the worker, knowing that this job pays the bonus more often, would prefer this job to his best alternative. But if the bonus is very low, the incentive effect of increasing the bonus must be profitable since it surely dominates the cost effect. Moreover, whenever it is profitable to increase the bonus, it is always more so for the job that values effort highest. Hence, a $\beta_H$ employer contemplating separation with complete rent extraction would prefer to increase both the wage and bonus in order to increase incentives while maintaining a credible signal.

In the case where the two jobs are quite similar, that is, when the worker's uncertainty about job characteristic is relatively small, it may be optimal for the employer with a $\beta_H$ job to extract all rents since he can do so while maintaining a relatively high bonus payment. However, it must be noted that Proposition 3 presents only a sufficient condition for rent transfers. Given Assumptions 1 through 3, it is not true that if $\beta_L > \beta^*(\beta_H)$, the worker in the $\beta_H$ job will not receive rents. In the case where $\beta_L > \beta^*(\beta_H)$, the previous argument in favor of increasing the bonus payment and thereby transferring rents in the $\beta_H$ job may or may not hold, depending on the particular shape of the function $\pi_e(e, \beta)$.

It is important to remember that the occurrence of rent transfers in an informed principal relationship does not rely on any restrictions on contracting possibilities. Neither $w$ nor $s$ have been restricted to be positive. A contract $(w, s)$ can easily be reinterpreted to include an entrance fee. To examine this possibility, let $x$ be the entrance fee, redefine the wage in the good state as $w - x + s$ and the wage in the bad state as $w - x$. It is obvious that the addition of an entrance fee does not change the effective space of contracts, and therefore does not affect the results. Similarly, adding the possibility for the worker to post a performance bond, which is redeemed only if the good state is realized, is also equivalent to renaming the variables.

Although the setup of this paper is very stylized, the intuition behind the rent transfer result may be quite general. As long as a principal wants to offer contingency payments in order to provide incentives, he is likely to need to convince the agent of the probability of each of these contingencies. In such a situation, the choice of contract will generally be a good instrument to signal information specific to the contingencies relevant to the particular relationship. A transfer of rents will emerge whenever the principal wants to make the attainment of a contingency desirable, by paying a high bonus, and simultaneously wants to make the attainment of the contingency believable, which will induce him to pay a high base wage.

4. EXTENSION TO A MULTIPERIOD SETTING

The previous section has shown that the introduction of ex ante private information on the part of the principal can fundamentally change the characteristic
of incentive contracts and, in particular, can explain why a principal may transfer rents to an agent as a means of providing incentives. Obviously, the economic relevance of such a result depends in part on its robustness with respect to modifications of the environment. One dimension of the analysis which is particularly questionable is the modelling of the informed principal’s problem as a one-shot game. Although such a formulation may be appropriate for some agency relationships, most employment relationships involve repeated interactions. Therefore, examining the robustness of the rent transfer result when an informed-principal relationship is repeated over time seems especially pertinent.

To examine the implications of repeated interaction, consider the following once repeated play of the informed-principal relationship. In the first period, the employer offers a wage-bonus contract, the worker either accepts and decides his level of effort or refuses the offer. The state of nature is then realized and the payments are made. In the second period, the employer again offers a wage-bonus contract which the worker can either accept or refuse. If the worker accepts the offer, he decides his level of effort, there is another realization of the state of nature, followed by payment. It is assumed that it is the same agent interacting with the principal in the two periods, however this assumption is absolutely unnecessary for the following result.

**Proposition 4.** Corresponding to any separating equilibria of the one-shot game where the worker receives rents when offered the equilibrium contract \((s_H, w_H)\), there exist an equilibrium of the twice repeated game in which the worker receives the offer \((s_H, w_H)\) in both periods when the job is of type \(\beta_H\), and therefore receives rents in both periods.

Proposition 4 shows that the possibility of rent transfers in an informed-principal framework does not rely on the relationship lasting only one period. In fact, the proof of the proposition consists of showing that equilibrium strategies of the one period game can be virtually repeated to form equilibrium strategies of the two period game.\(^7\) The crucial element behind this equilibrium is that the “better” principal must signal his information in each period. This behavior is supported by the belief that if the informed player does not signal again in the second stage, he is interpreted as being of type \(\beta_L\) even though he was thought to be of type \(\beta_H\) after the first stage of the game. Although such change in beliefs may at first seem awkward, most equilibrium concepts do not rule out such possibilities and, as discussed in Noldeke and van Damme (1990), there seems to be no compelling reason to do so.

In terms of our previous characterizations, Proposition 4 indicates that it may be optimal for an employer who values effort highly to repeatedly pay high wages and low bonuses as a means of encouraging effort. Moreover, such a contract offers the worker a level of utility in each period above his reservation level. This extension is especially relevant for the efficiency wage literature since it suggests that, when

\(^7\) In Beaudry (1989), it is further shown that these strategies also satisfy Cho’s (1987) extension of the Intuitive Criterion (once this criterion is suitably modified to apply to a situation with a continuous action space).
an employer is privately informed about the technology governing a relationship, he 
may want to pay above market wages for substantial periods of time. Therefore, an 
informed-principal approach to employment relationships provides an explanation 
of efficiency wages that is relevant for both short- and long-term relationships.\footnote{MacLeod and Malcomson (1989) offer an explanation of efficiency wages based on self-enforcing contracts. However, in contrast to the informed-principal approach, their explanation applies only to situations where the employment relationship potentially lasts for an infinite number of periods.}

5. CONCLUSIONS

The main thrust of the paper is that an informed-principal approach substantially 
alters standard predictions regarding the properties of incentive contract. In 
particular, the informed-principal approach predicts that a principal who values 
effort highly will offer a contract that pays a high base wage and a low bonus 
payment. Moreover, the informed-principal approach offers a simple explanation 
for why an employer may choose an incentive contract that offers ex ante rents to 
a worker even when contracting possibilities are unrestricted and both principal 
and agent are risk neutral. This transfer of rents was shown to arise because, when 
a principal is privately informed about the technology governing an agency 
relationship, the principal must make the attainment of any objective both desirable 
and believable for the agent. Since the informed-principal approach relies on agents 
being uncertain about the technology governing a new relationship, the theory 
developed here may be most relevant for understanding incentive schemes in 
emerging industries, or in industries in which technology is constantly evolving.

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APPENDIX

PROOF OF PROPOSITION 1. The employer’s program defined in (2.1) can be solved by 
the first order approach to principal-agent problems (the second order condition for 
the worker’s effort decision is always negative). Assuming that constraint (ii) holds with 
equality, the first order conditions defining the optimum are given by the following:

\[(\phi_1 - \phi_2)\beta \pi_e(e, \beta) = g'(e)\]
\[s \beta \pi_e(e, \beta) = g'(e)\]
\[w + \pi(e, \beta) s - g(e) = U_r,\]

It is clear that \(w^*(\beta)\) and \(s^*(\beta)\) solve these conditions. To verify that it is in fact 
optimal for the employer to make inequality (ii) bind (i.e., extract all rents), it is 
sufficient to show that the derivative of the associated value function with respect 
to \(U_r\) is always negative. This value function is

\[V(U_r) = \phi_2 - U_r + \pi(\bar{e}(\phi_1 - \phi_2, \beta), \beta)(\phi_1 - \phi_2) - g(\bar{e})\]

where \(\bar{e}(s, \beta)\) is implicitly defined by \(g'(\bar{e}) = s \beta \pi_e(\bar{e}, \beta)\).
PROOF OF PROPOSITION 2. The proof of this proposition requires showing that the contract \((w_H, s_H)\) offered by the high type in any separating equilibria is such that \(w_H > w_L = w^*(\beta_L)\) and \(s_H < s_L = s^*(\beta_L)\), where \((w_L, s_L)\) is the contract offered by the low type. This is done by first recognizing that separation always implies that \(w_L = w^*(\beta_L)\) and \(s_L = s^*(\beta_L)\), then by showing that \(s_H\) must be smaller than \(s^*(\beta_L) = \phi_1 - \phi_2\), and finally by showing that with \(s_H < s^*(\beta_L)\), \(w_H\) must be greater than \(w^*(\beta_L)\).

To see that separation implies \(w_L = w^*(\beta_L)\) and \(s_L = s^*(\beta_L)\), first note that an employer with a type \(L\) job can surely not do better than employ a worker with \(w_L = w^*(\beta_L)\) and \(s_L = s^*(\beta_L)\); otherwise, the worker would be receiving less than his reservation utility given that separation implies the worker knows the offer is for a type \(L\) job. Therefore, the only reason the employer would not offer \(w_L = w^*(\beta_L)\) and \(s_L = s^*(\beta_L)\) is that he expects it to be refused by the worker. However, there cannot exist a separating equilibrium where this offer is refused since (1) it is optimal for the worker to always accept (regardless of beliefs within the support) any offer \(c(\varepsilon) = (w^*(\beta_L) + \varepsilon, s^*(\beta_L))\) when \(\varepsilon > 0\), and (2) the employer’s best response is undefined if the worker only accepts offers \(c(\varepsilon)\) for which \(\varepsilon\) is strictly positive. Consequently, any equilibrium must have the worker accepting any offer \(c(\varepsilon)\) with \(\varepsilon \geq 0\), which implies that the offer \(w_L = w^*(\beta_L)\) and \(s_L = s^*(\beta_L)\) will be made and accepted in any separating equilibrium.

In order to prove that \(s_H\) must be smaller than \(s^*(\beta_L) = \phi_1 - \phi_2\), let us assume the contrary is true. First note that the two self-selection constraints plus the fact that the equilibrium is separating implies the following inequality:

\[
\{\pi(\bar{\varepsilon}(s_H, \beta_H), \beta_L) - \pi(\bar{\varepsilon}(s_H, \beta_H), \beta_H)\}(\phi_1 - \phi_2 - s_H) < 0.
\]

Since the first term is always negative, the second term must be positive which contradicts the assumption that \(s_H \geq \phi_1 - \phi_2\), therefore \(s_H\) must be smaller than \(\phi_1 - \phi_2\).

To see that \(w_H\) must be greater than \(w^*(\beta_L)\) when \(s_H < s^*(\beta_L)\), the self-selection constraint associated with type \(L\) can be rearranged as follows:

\[
w^*(\beta_L) - w_H \leq -\pi(\bar{\varepsilon}(s_H, \beta_H), \beta_H)(\phi_1 - \phi_2 - s_H) < 0.
\]

Since the right side of the above inequality is always negative, it directly implies that \(w_H > w^*(\beta_L)\). 

Q.E.D.

PROOF OF PROPOSITION 3. Assume the Proposition is false, then there must exist a sequence of \(\{\beta^k_L\}\) with \(\beta^k_L \to 0\) such that along the sequence, the equilibrium contract \((w_H^k, s_H^k)\) offered by the \(\beta_H\) job gives the worker a utility level equal to \(U_r\). Since the equilibrium is separating, we must have along the sequence

\[
\pi(\bar{\varepsilon}(s_H^k, \beta_H), \beta_H) s_H^k - g(\bar{\varepsilon}(s_H^k, \beta_H)) = U_r - w_H^k.
\]

But this leads to a contradiction. On the one hand, the optimality of the effort decision implies that the left side of the above equation is always positive. On the
other hand, the right side will converge to something smaller or equal to zero as \( \beta_L \rightarrow 0 \) (since \( w_t^{H_h} > w^*(\beta_L^t) \) by Proposition 2) and \( w^*(\beta_L^t) \) converges to \( U_r \). The statement that \( w^*(\beta_L^t) \) converges to \( U_r \) results from the fact that, given \( \pi(\epsilon, 0) = 0 \), the effort level associated with the less productive job converges to zero as \( \beta_L^t \rightarrow 0 \). 

\[ \text{Q.E.D.} \]

**Proof of Proposition 4.** The object of the proof is to show that it can be an equilibrium strategy for the employer to offer the same contract in both periods of the two period game as he would in the one-shot game. Let \((w_L, s_L)\) and \((w_H, s_H)\) denote separating equilibrium contract offers for the one-shot game. Consider the following strategies and beliefs, which are essentially the twice repeated version of strategies that uphold separating equilibria of the one period game.

The employer's strategy in period 1 is:

1. If the job is of type \( \beta_H \), offer the contract \((w_H, s_H)\).
2. If the job is of type \( \beta_L \), offer the contract \((w_L, s_L)\).

In period 2, regardless of history, play the same strategy as in period 1:

The worker's strategy in period 1 is:

1. If the contract offered is \((w_H, s_H)\), accept the offer and set \( e \) to satisfy \( g'(e) = s_H \beta_H \pi'(e \beta_H) \).
2. If the contract offered is \((w, s) \neq (w_H, s_L)\) but satisfies \( \pi(e, \beta_L)s + w - g(e) \geq U_r \) when \( e \) solves \( g'(e) = s \beta_L \pi'(e \beta_L) \), accept the offer and set \( e \) according to \( g'(e) = s \beta_L \pi'(e \beta_L) \).
3. Otherwise, refuse the offer.

In period 2, regardless of the history, play the same strategy as in period 1.

The workers beliefs in period 1 are:

1. If the contract offered is \((w_H, s_H)\), believe with probability one that the job is of type \( \beta_H \).
2. Otherwise believe with probability one the job is of type \( \beta_L \).

In period 2, regardless of the history, form beliefs exactly as in period 1.

In order to validate the statement of the proposition, it is necessary to verify whether the specified strategies form a PBE. Given that the second period strategies are those that support a separating equilibrium of the one-shot game, there cannot exist a deviation initiated in the second period that would upset the equilibrium. Therefore, if there exists a deviation that can break this equilibrium it must be initiated in the first period. However, the beliefs stated above make the history of play virtually irrelevant and hence make the decision with respect to deviating in the first period identical to that in the one-shot game, moreover it is clear that these beliefs satisfy Bayes Rule. Consequently, by the assumption that the contract offers \((w_L, s_L)\) and \((w_H, s_H)\) can be supported by a PBE equilibria of the one-shot game, they can be supported in both periods of the two period game.

\[ \text{Q.E.D.} \]
REFERENCES


