Is it Harmful to Allow Partial Cooperation?

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Abstract

In economics, politics and society, examples abound in economics, politics and society where agents can enter partial cooperation schemes, i.e., they can collude with a subset of agents. Several contributions devoted to specific settings have claimed that such partial cooperation actually worsens welfare compared to the no-cooperation situation. Our paper assesses this view by highlighting the forces that lead to such results. We find that the nature of strategic spillovers is central to determining whether partial cooperation is bad. Our propositions are then applied to various examples as industry wage bargaining or local public goods.

Keywords: Non-cooperative games; cooperation; wage bargaining; public goods

JEL classification: C72; E62; J5

I. Introduction

Partial cooperation qualifies a situation where each individual agent cooperates with a subset of other agents but not with everyone in the economy or society. Hence partial coalitions are formed: agents within coalitions cooperate but coalitions act non-cooperatively in regard to one another. Recently, a number of papers dealing with totally different issues have examined the effects of partial cooperation. The first example which comes to mind is the celebrated paper by Calmfors and Drifill (1988) on wage bargaining. In this study, Calmfors and Drifill claimed that wage bargaining at the industry level generates a lower level of employment than fully centralized “corporatist” bargaining or fully decentralized bargaining at the
firm level. They suggested that there exists a U-shaped relation between employment and the level of cooperation. Their view of the intermediate level of bargaining corresponds to what we call partial cooperation: trade unions collude within an industry to fix the wage rate to be applied by all firms within the industry, thus internalizing spillovers among firms of the same industry. But contrarily to what happens in “corporatist” fully centralized bargaining, industry-wide trade union coalitions do not cooperate among themselves, thus failing to internalize the spillovers among industries. Their result ran counter to intuition: one would expect partial cooperation to entail an outcome in-between those of centralization and decentralization, as agents would even partially internalize spillovers among themselves\(^1\). The Calmfors and Drifill paper generated other studies on the relationships between the level of bargaining and the wage/employment level; see e.g. Hoel (1991). In particular, some papers extended the model by allowing for openness of the economy; see Danthine and Hunt (1994), Sørrensen (1993), Rama (1994) and Corneo (1995). Lately, Cahuc (1995) and Cahuc and Zylberberg (1997) pointed to the signs of the various externalities among agents as the crucial factor for understanding the relationship between wages and the level of bargaining.

Another field of research where the notion of partial cooperation has been put to use is the issue of “regionalism” in the world economy. The current development of trading blocs with preferred economic links with “neighbours” could be contradictory to the intensification of free-trade agreements which corresponds to an attempt to form a worldwide coalition on trade rules. This issue is attracting attention from scholars who develop models implicitly based on partial cooperation, since a “regional” bloc can be viewed as an intermediate coalition; see Krugman (1991) and Bond and Syropoulos (1995, 1996). When regional blocs cover the entire world economy, this corresponds exactly to partial cooperation: countries cooperate within their blocs, but “regional” blocs do not cooperate when deciding on tariff matters. Other studies have used a set-up with partial cooperation. Sørrensen (1996) has recently studied the effects of fiscal cooperation among subsets of countries. Rotemberg and Woodford (1992) have examined the effect of collusion among firms belonging to the same industry on aggregate dynamics through variable mark-ups.

These references exemplify the widespread use of the idea of partial cooperation. But, of course, it can be applied to many other fields such as R&D consortia among firms (with or without public blessing), the cartellization of industries or the existence of clubs (sporting clubs, churches or military alliances) which compete for membership. The possibility of

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\(^1\)Actually, Calmfors and Drifill did not prove this analytically, as their model was too complex to be solved. They reached their conclusion through simulation exercises.
applying the notion of partial cooperation to a wealth of economic and social issues then raises a fairly general question: is partial cooperation necessarily bad, as could be deduced (admittedly by casually extending their reasoning) from the Calmfors and Drifill experiment? The various studies we have just quoted generally fail to address the normative issue raised by the possibility of partial cooperation and only aim to compare the level-of-action solution to a game based on partial cooperation with the no-cooperation and full-cooperation solutions; an exception is Sørensen (1996).

Here, we try to understand the positive and normative consequences of partial cooperation using a simple set-up. Our aim is to offer a more comprehensive view of the consequences of partial cooperation. From a positive point of view, we would like to understand how the equilibrium action(s) decided by agents is (are) altered by partial cooperation, compared to the outcomes of full cooperation (centralization) or no-cooperation (decentralization). From a normative point of view, we would like to know when do agents lose from partial cooperation. Our approach is to assess the impact of partial cooperation by means of fairly general reasoning. This issue is of importance, since there is hardly any economic act which an economic agent can perform without colluding explicitly or implicitly with others: in family life, in a firm, on a board of directors, in a trade union or in international forums, by trust or by act, by vote or by words, every aspect of our lives and socio-economic integration is enmeshed with partial cooperation.

The paper is organized as follows. In Section II, we set up a simple yet general symmetric game which permits us to distinguish three types of solutions: a fully decentralized non-cooperative equilibrium with no cooperation; an “intermediate” solution, corresponding to partial cooperation, where agents cooperate within (“natural”) groups; and a fully centralized solution where all agents cooperate. We then state rules allowing us to compare actions and welfares arising in these three equilibria. These rules depend on the various characteristics of spillovers between agents and are strikingly simple. Our set-up subsumes many of the specific models developed in the papers quoted above and our conclusions can readily be applied to these studies, suggesting a simple explanation of their results. In Section III, we offer examples of economic problems involving partial cooperation, and apply our results. Concluding comments follow.

II. The Benchmark Model

The Structure of the Game

We consider a game with groups of players. After introducing the actions and structure of the groups, we state the properties of the payoff functions. 

The players, the groups and the actions. We consider a symmetric game with \( K \times N \) players, dispatched in \( K \) groups; the size of each group is identical and equal to \( N \). A player \( i, i = 1, \ldots, N \) belonging to group \( k, k = 1, \ldots, K \), takes an action denoted by \( a_{ik} \in \mathbb{R} \). Any permutation of a vector of actions of players belonging to group \( k, k = 1, \ldots, K \), \((a_{1k}, \ldots, a_{Nk})\), is denoted by \( a^k \in \mathbb{R}^N \). Any permutation of a vector of actions of players different from \( i \) belonging to his own group \( k \), \((a_{1k}, \ldots, a_{i-1,k}, a_{i+1,k}, \ldots, a_{Nk})\), is denoted by \( a^{-k} \in \mathbb{R}^{N-1} \), and any permutation of a vector of actions of the players who do not belong to group \( k \), \((a^1, \ldots, a^{k-1}, a^{k+1}, \ldots, a^K)\), is denoted by \( a^{-k} \in \mathbb{R}^{(K-1)N} \).

The payoffs. The payoff to player \( i \) belonging to group \( k \) is defined by a twice continuously differentiable function \( V(a_{ik}, a^k_{-i}, a^{-k}) : \mathbb{R}^{NK} \rightarrow \mathbb{R} \). Given that \( a^k_{-i} \) and \( a^{-k} \) respectively denote any permutation of a given vector, the payoff function satisfies the following properties: \( V_l(a_{ik}, a^k_{-i}, a^{-k}) = V_{l'}(a_{ik}, a^k_{-i}, a^{-k}), (l, l') \in \{2, N\}^2 \) or \((l, l') \in \{N + 1, KN\}^2\), where a subscript for function \( V \) denotes a partial derivative. It is worth noting that such properties imply that the action of any player belonging to the same group as player \( ik \), but different from player \( ik \), has the same impact on the payoff to player \( ik \). Moreover, the action of any player who belongs to any group different from \( k \) also has an identical impact on the payoff function of player \( ik \). What this modelling suggests is that for each agent, the other agents can be divided in two: some \((N - 1)\) agents have a specific impact on her payoff, different from the impact of the other \((K - 1)N\) agents.

In this paper, we focus only on symmetric solutions. In this case, we write the payoff function of agent \( i \) belonging to group \( k \) in a situation where all the players belonging to his group take the same action \( a_k \) and all other players another action \( a_{-k} \) as follows: \( V(a_{ik}, a^k_{-i}, a^{-k}) = v(a_{ik}, a_k, a_{-k}) \), if \( a_{jk} = a_k, j = 1, \ldots, N, j \neq i, a_{jk'} = a_{-k}, k' = 1, \ldots, K, k' \neq k \) and \( j = 1, \ldots, N \). We define \( \sigma(a) \equiv v(a, a, a); \sigma(a) \) as the payoff to any agent when an action \( a \) is played by all agents.

The Three Situations

We want to study three different situations corresponding to three forms of cooperation and the corresponding symmetric solutions. The first is a decentralized no-cooperation game, where each player chooses his action non-cooperatively. Its equilibrium will correspond to the symmetric Nash equilibrium of the game. The second situation is an intermediate setting, where players of each group form a coalition. The outcome of the game will be the symmetric Nash equilibrium between the \( K \) coalitions, where the strategy of each coalition is a single action. The third is a symmetric
centralized setting, where a coalition chooses an action which maximizes the sum of the payoffs of all players.

The decentralized setting. In a decentralized game, the problem of player $i$ belonging to group $k$ is to maximize her payoff given the vector of actions of the other players. Restricting (w.l.o.g.) the analysis to identical actions taken by other players both within and outside group $k$, this problem can be written as follows:

$$\max_{a_{ik}} v(a_{ik}, a_k, a_{-k})$$

where $a_k$ denotes the action taken by the other members of group $k$ and $a_{-k}$ denotes the action taken by members of the groups other than $k$. In a symmetric equilibrium, the first-order condition yields the best reply function $a^*_k$:

$$v_1(a^*_k, a_k, a_{-k}) = 0.$$ 

Assumption 1. The first-order condition is necessary and sufficient to define a unique symmetric decentralized equilibrium, denoted by $a^D$, that is:

$$\sigma_1(a^D) = 0, \quad \text{where } \sigma_1(a) \equiv v_1(a, a, a), \quad a \in \mathbb{R}. \quad (1)$$

The partial cooperation setting. In the partial cooperation setting, the players choose cooperatively an action in each group, given the actions taken in the other groups. Then, restricting the analysis to identical actions taken by other players outside group $k$, the problem of the coalition in group $k$ can be written:

$$\max_{a_{ik}} \sum_{i=1}^{N} v(a_{ik}, a_k, a_{-k})$$

subject to: $a_{ik} = a_k, \ i = 1, \ldots, N$.

The first-order condition yields the best reply function $a^*_k$:

$$v_1(a^*_k, a^*_k, a_{-k}) + v_2(a^*_k, a^*_k, a_{-k}) = 0.$$ 

Assumption 2. The first-order condition is necessary and sufficient to define a unique symmetric intermediate equilibrium, denoted by $a^I$, that is:
\( \sigma_1(a^I) + \sigma_2(a^I) = 0, \) where \( \sigma_2(a) \equiv v_2(a, a, a), a \in \mathbb{R}. \) (2)

The centralized setting. In a centralized setting, there is a single coalition which maximizes the joint welfare of all players. In a symmetric equilibrium, the problem of the coalition is to choose an action which maximizes the sum of the payoffs of all players:

\[
\max_{a_{ik}} \sum_{k=1}^{K} \sum_{i=1}^{N} v(a_{ik}, a_k, a)
\]

subject to: \( a_{ik} = a_k = a, i = 1, \ldots, N; k = 1, \ldots, K. \)

Assumption 3. The first-order condition is necessary and sufficient to define a unique value of \( a \in \mathbb{R}, \) denoted by \( a^C, \) that is:

\[
\sigma_1(a^C) + \sigma_2(a^C) + \sigma_3(a^C) = 0, \quad \sigma_3(a) \equiv v_3(a, a, a), \quad a \in \mathbb{R}. \) (3)

Before presenting the analysis of the different equilibria, it is useful to introduce the following definitions:

Definition 1. The game exhibits positive (negative) spillovers within groups for the level of action \( a \in \mathbb{R} \) if and only if \( \sigma_2(a) > (<) 0. \)

Definition 2. The game exhibits positive (negative) spillovers between groups for the action \( a \in \mathbb{R} \) if and only if \( \sigma_3(a) > (<) 0. \)

These definitions deserve some comments. The concept of spillover used here follows Cooper and John (1988) and refers to the type of externalities implicit in the payoff structure. It has been extended to take into account the existence of groups, since the spillovers between and within groups may have different signs.

We now introduce the following properties for the various externalities:

Assumption 4. \( \sigma_2(a) \neq 0 \quad \sigma_3(a) \neq 0 \quad \sigma_2(a) + \sigma_3(a) \neq 0 \)

\( \forall a \in [a, \bar{a}], \quad a \equiv \inf(a^D, a^I, a^C), \quad \bar{a} \equiv \sup(a^D, a^I, a^C). \)

Assumption 4 implies that spillovers both within and between groups are never absent over an adequate interval, and that they do not cancel out,
implying that the net externality is never nil. In other words, the analysis is restricted to situations where a change in the strategy of any player will always affect the payoff of any other player in the neighbourhood of symmetric situations. It is worth noting that Assumption 4, together with the differentiability of the payoff function, implies that spillovers do not change sign over the interval $[a, \bar{a}]$.

Using the definitions and assumptions previously stated, we can now compare the different symmetric solutions.

Comparison of Solutions

Our aim is to compare the level of welfare obtained in the three solutions corresponding to the three different patterns of cooperation assumed between agents. More specifically, we would like to understand under which conditions the solution arising from partial cooperation is worse than the other two solutions. After presenting a lemma which allows us to compare the levels of action characterizing each solution, we offer two propositions, related to ranking welfares, first on a necessary condition, then on a sufficient condition.

**Lemma 1.** The ordering of the symmetric actions is related to the nature of spillovers as follows:

(i) $a^I > a^C$ \[ a^I > a^D \] \[ \iff \sigma_2(a) > 0 \text{ and } \sigma_3(a) < 0 \]

(ii) $a^I < a^C$ \[ a^I < a^D \] \[ \iff \sigma_2(a) < 0 \text{ and } \sigma_3(a) > 0 \]

(iii) $a^C < a^I < a^D$ \[ \iff \sigma_2(a) < 0 \text{ and } \sigma_3(a) < 0 \]

(iv) $a^C > a^I > a^D$ \[ \iff \sigma_2(a) > 0 \text{ and } \sigma_3(a) > 0 \]

**Proof:** See Appendix.

How should this Lemma be understood? Let us consider, as an example, the case where spillovers within groups are positive and spillovers between groups are negative. A given coalition in the intermediate game exploits the positive spillovers among its members and hence chooses an action that is larger than the symmetric non-cooperative action played in the fully decentralized game, given a set of actions taken by players outside this group. However, it plays a non-cooperative game with the coalitions formed by the
other groups in the intermediate game. Hence each coalition fails to take into consideration the negative externalities it generates on the other coalitions. In the case of full cooperation between agents, these externalities would have been taken into account, leading to an action smaller than the intermediate action.

The explanation is reversed when spillovers have the same sign. Suppose spillovers between as well as within groups are both positive (at the intermediate equilibrium). A given coalition in the intermediate game exploits the positive spillovers among its members and hence chooses an action that is larger than the symmetric non-cooperative action played in the fully decentralized game, given a set of actions taken by players outside this group. But now, even though a coalition fails to take into consideration the externalities it generates on other coalitions, these externalities are positive. Full cooperation between agents would have led to an even larger action than the intermediate one. Hence the result that the intermediate action is between the non-cooperative and the full-cooperative actions.

Turning now to the comparison of welfares and to the significance of partial cooperation, we first state a necessary condition:

**Proposition 1. (Necessary condition).** The intermediate equilibrium yields a lower payoff than the decentralized and the centralized equilibria only if the game exhibits spillovers within groups and between groups with opposite signs.

**Proof:** From the lemma, we know that \( a^C > a^I > a^D \) if and only if the game exhibits positive spillovers within groups and between groups. Then, using Assumption 3, one gets \( \sigma(a^C) > \sigma(a^I) > \sigma(a^D) \) if and only if the game exhibits positive spillovers within groups and between groups.

Similarly, \( a^C < a^I < a^D \) if and only if the game exhibits negative spillovers within groups and between groups. Then, using Assumption 3, one gets \( \sigma(a^C) > \sigma(a^I) > \sigma(a^D) \) if and only if the game exhibits negative spillovers within groups and between groups. ■

The understanding of Proposition 1 is rather straightforward. A structure of coalitions allows members from any group to internalize the spillovers within groups but not spillovers between groups. As long as these spillovers do not have an opposite effect on agents’ welfare, but work in the same way on welfare, partially integrating these spillovers represents an amelioration of welfare. A socially benevolent planner would push further in the same direction but would not reverse it. Hence, it is only in the case where spillovers act in opposite directions that partial cooperation may worsen agents’ welfare.
Next we state a sufficient condition for partial cooperation to yield the worst outcome of the three symmetric equilibria:

**Proposition 2.** (Sufficiency condition). The intermediate equilibrium yields a lower payoff than the centralized and the decentralized equilibria if \(|\sigma_2(a^C)| < |\sigma_3(a^C)|\) and the game exhibits spillovers within groups and between groups with opposite signs.

**Proof:** Let us first remark that, according to the first-order condition of the decentralized equilibrium \(\sigma_1(a^D) = 0\), the first-order condition of the centralized equilibrium, \(\sigma_1(a^C) = -(\sigma_2(a^C) + \sigma_3(a^C))\), and the fact that from Assumption 1, \(\partial \sigma_1(a^D)/\partial a < 0\), one gets, when \(|\sigma_2(a^C)| < |\sigma_3(a^C)|\): \(a^C > a^D\) if \(\sigma_3(a^C) > 0\), and \(a^C < a^D\) if \(\sigma_3(a^C) < 0\).

Then, assuming \(|\sigma_2(a^C)| < |\sigma_3(a^C)|\), \(\sigma_2(a^C) > 0\) and \(\sigma_3(a^C) < 0\) one gets, thanks to Lemma 1, \(a^C < a^D < a^I\). This implies, according to Assumption 3, that \(\sigma(a^C) > \sigma(a^D) > \sigma(a^I)\).

Assuming now \(|\sigma_2(a^C)| < |\sigma_3(a^C)|\), \(\sigma_2(a^C) < 0\) and \(\sigma_3(a^C) > 0\) one gets, thanks to Lemma 1, \(a^C > a^D > a^I\). This implies, according to Assumption 3, that \(\sigma(a^C) > \sigma(a^D) > \sigma(a^I)\). \(\blacksquare\)

To understand this proposition, suppose \(\sigma_2(a) > 0 > \sigma_3(a)\) and that \(|\sigma_2(a^C)| < |\sigma_3(a^C)|\). Granted that spillovers work in opposite directions, we know that from the socially benevolent planner’s point of view the net spillover effect is negative and, therefore, he chooses a cooperative action that is smaller than the non-cooperative one. This implies, according to Proposition 1, that the intermediate action is the largest of the three actions, the farthest from the cooperative one. Hence, given the concavity of the payoff function, it generates the third-best payoff, i.e., the worst one, as coalitions do not internalize the very bad global negative effect of an increase in the level of actions, and they only internalize the positive (not large enough) effect of partial cooperation which induces them to increase their level of actions, given the level of actions played by others.

The condition stated in Proposition 2 does not imply that the externality on a player A generated by a player B outside the “natural” group of A (assumed for ease of reasoning to be positive) is larger than the externality generated by another player C belonging to this group. Proposition 2 indicates that a sufficient condition for this to arise is that \(\sigma_2(a_I)\) and \(\sigma_3(a_I)\) be of opposite signs and that \(\sigma_3\) be greater in absolute value. At first glance this may appear rather unlikely to arise (almost pathological), since one would think that if \(\sigma_3\) is the stronger force, then our notion of a “natural” coalition seems awkward: why should we collude with players whose actions do not generate the largest spillovers? But this point is misleading. What is important to note is that \(\sigma_3\) is equal to the value of the spillovers \(V_3(a_I)\)

induced by a player who does not belong to the “natural” group of A, times the number \((K - 1)N\) of those players. Accordingly, \(\sigma_3\) can be large through the size effect even though the individual externality of an outside player is small, even negligible.

At this point, it is worth acknowledging two limitations of our results. First, the set-up is one of perfect symmetry, i.e., it is assumed that each group is of equal size, with identical individuals. The only difference between agents is that they belong to different coalitions, and each coalition has the same number of members. Hence the symmetry characteristics. The task of generalizing our results remains. Even though it may be of interest to relax our simplifying assumptions, we are confident that the results deduced from such exercises should be broadly consistent with our present results, at least qualitatively: partial cooperation is likely to worsen welfare when spillovers within and between coalitions differ in sign and when the aggregate effects of spillovers between coalitions more than offset the positive effects from internalizing spillovers within groups.\(^2\) Second, throughout the paper, we shy away from the issue of the stability of coalitions corresponding to partial cooperation. Instead, we focus exclusively on comparing three situations: decentralized decisions, partial cooperation of individuals within groups, and full cooperation. We chose this limited focus for three reasons. First, there are many economic examples that only require such comparisons. For instance, some forms of partial cooperation are enforced compulsorily with no (legal) possibility to defect. An example would be some legal enforcement of a given structure of wage bargaining, as in France. Second, we can implicitly assume the existence of incentive schemes or some repeated-game threatening arrangements which support an existing structure of partial coalitions. Moreover, as far as we are aware, there is still no general agreement on which approach to use in order to address the issue of endogenous coalition formation. Consequently, we take for granted the existence of partial cooperation, without explicitly studying its endogenous formation or stability.

**III. Economic Examples**

*Is Industry-wage Bargaining Necessarily Bad?*

*A tout seigneur, tout honneur ...* Let us first consider the case studied by Calmfors and Drifflf (1988): wage bargaining. Calmfors and Drifflf were the first to give prominence to what we call here “partial cooperation”. In an important and widely quoted paper, they claimed, using both empirical evidence and theoretical reasoning, that there is a U-shaped relationship

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\(^2\) Partial cooperation in asymmetric settings generates new issues such as “counterproductive” effects; see Rogoff (1985).
between the extent of centralization of wage bargaining and macroeconomic performance: an intermediary level of bargaining would lead to a worse outcome than a centralized or a decentralized setting. Using a classification scheme relying on two criteria (the level of coordination among employers and trade unions and the number of economywide representative organizations), they indicated that OECD data confirm their claim. Theoretically, they revealed a trade-off linked to the level of wage bargaining. A fully decentralized bargaining system leads bargaining parties, especially trade unions, to neglect the external effects of their decisions. A more centralized pattern of decisions would induce parties to better take into account spillovers generated by wage agreements. On the other hand, the market power of trade unions increases when the degree of centralization increases, which may lead to a wage level detrimental to employment. As such, both the fully centralized and the fully decentralized schemes have defects.

But then, so the argument goes, the intermediate level of bargaining, such as when bargaining takes place at the industry level (when wage agreements are negotiated by an industrywide trade union, covering every firm in a given industry), combines both defects. At the industry level, the coordination of decisions is incomplete: spillovers (especially on the general price index) are not fully internalized by bargaining parties. In the meantime, the market power of trade unions is notably increased, leading to a lower wage elasticity of the demand for labour in each industry, inducing trade unions to look for higher wages.

The theoretical models (and particularly the one proposed by Calmfors and Drifill) which support this claim rely on symmetric non-cooperative games. The case with industry bargaining is modelled as a partial cooperation game: coalitions form at the industry level, but industry coalitions do not cooperate. However, it has been insufficiently noted that Calmfors and Drifill make an important assumption: an industry encompasses firms producing (strongly) substitutable goods whereas goods produced in two different industries are weakly substitutable. This proves to be crucial for obtaining the hump-shaped relationship between the level of centralization and the level of wages, as becomes clear when applying our propositions to a wage bargaining set-up.

To see this, consider an economy with imperfect competition on the labour market. There are $NK$ firms, and $K$ industries. In each firm, there is a monopoly trade union, the preferences of which depend positively on the real wage obtained by employed workers and the level of employment in the firm. The game is played over real wages. A two-stage game takes place:

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3Calmfors and Drifill consider a more complex model where trade unions fix nominal wages and the price levels are endogenously determined. To keep the reasoning short and simple, we abstain from considering the effects of the wage bargaining level which goes through prices.
first, the union chooses the wage level to be applied, then the firm determines
the level of labour it requests. The demand for labour of the
ith firm in the
kth industry, which we denote \( l_{ik} \), is given by the following equation:

\[
 l_{ik} = L(w_{ik}, w_k, w_{-k})
\]

where \( w_{ik} \) denotes the real wage applied in firm \( ik \), \( w_k \) denotes the mean real
wage applied in other firms than \( i \) but belonging to the \( k \)th industry and \( w_{-k} \)
denotes the mean real wage applied in firms belonging to the other
industries. The demand for labour of firm \( ik \) depends not only (negatively)
on the real wage set by its counterpart union, but also on real wages set in
the other firms of the \( k \)th industry and on the wages set by firms in the other
industries as well, because the demands for goods are linked together as
goods may be complements or substitutes. The demand for labour of firm \( ik \)
is increasing in \( w_k \) if goods produced by firms within industry \( k \) are
substitutes: an increase in the real wages set in firms other than firm \( ik \)
increases their prices relative to the price charged by firm \( ik \) and it is induced
to increase its demand for labour. In contrast, it is decreasing in \( w_k \) if goods
produced by firms within the industry are complements: an increase in the
real wages set in firms other than firm \( ik \) increases their prices, which
implies a decrease in the quantities they are able to sell and hence in the
quantity firm \( ik \) itself is able to sell; it is induced to decrease its demand for
labour. Similar reasoning can be used to understand the effects of \( w_{-k} \) on
the demand for labour of firm \( ik \): it depends on the characteristics of the
goods produced in different industries (whether they are substitutes or
complements). To summarize, we may write:

\[
 L_1 = \frac{\partial L(w_{ik}, w_k, w_{-k})}{\partial w_{ik}} < 0; \\
 L_2 = \frac{\partial L(w_{ik}, w_k, w_{-k})}{\partial w_k} (>) 0 \text{ if goods produced in the } k \text{th} \\
    \text{industry are substitutes (complements); } \\
 L_3 = \frac{\partial L(w_{ik}, w_k, w_{-k})}{\partial w_{-k}} (>) 0 \text{ if goods produced by firms in different} \\
    \text{industries are substitutes (complements). Moreover, we assume that } \\
 L_1(w_{ik}, w_k, w_{-k}) + L_2(w_{ik}, w_k, w_{-k}) + L_3(w_{ik}, w_k, w_{-k}) < 0 \text{ in order to ensure that an increase in the} \\
 \text{wage level at symmetric equilibrium decreases employment.}
\]

The payoff function of the trade union in firm \( ik \) is:

\[
 U(w_{ik}, l_{ik}) = U(w_{ik}, L(w_{ik}, w_k, w_{-k}))
\]

where \( U \) is a concave function, twice differentiable and strictly increasing
with respect to its two arguments. It is then immediate to obtain the sign of
the spillovers, both within groups and between groups. The spillover within
groups is equal to: \( \partial U(w_{ik}, L(w_{ik}, w_k, w_{-k}))/\partial w_{ik} \equiv U_2 L_2 \) and is positive
(negative) if goods within industry \( k \) are substitutes (complements). The
spillover between groups is equal to: \( \partial U(w_{ik}, L(w_{ik}, w_k, w_{-k}))/\partial w_{-k} \equiv U_2 L_3 \) and is positive (negative) if goods produced in different industries
are substitutes (complements). Regarding the issue raised by Calmfors and Driffill, it is straightforward to apply our previous results to this simple setting. There is a hump-shaped relationship between the wage level and the degree of centralization when goods produced within an industry are substitutes and goods produced in different industries are complements. With any of the other three possibilities, such a phenomenon does not hold. Hence, a priori, this relationship has no special shape: every type of shape can be observed. Regarding the welfare issues, not addressed by Calmfors and Driffill, our propositions allow us to state that an (intermediate) industry level of wage bargaining does not necessarily generate the worst payoff to trade unions and their members.

Using a more complex model would only mitigate our results. In particular, by considering that trade unions set nominal wages, allowing for endogenization of the price level, Cahuc and Zylberberg (1997) have shown that the hump-shaped relationship exists when goods produced within an industry are strongly substitutable whereas goods produced in different industries are complements or weakly substitutable. This is indeed what was assumed by Calmfors and Driffill. But their result hinges on the same point: it is a possible asymmetry between goods which matters for assessing the relationship between the level of centralization of wage bargaining and the wage level. We should by no means think that it occurs in any case, or that it always generates the worst outcome in terms of employment.

Similarly, our general results can explain why some economists have argued that centralized bargaining (i.e., a national trade union sets the wages for every firm in every industry) may not be optimal when considering an open economy, or an economy in an economic zone with integrated goods markets; see Rama (1994). This extension means that a nationwide trade union no longer represents the fully centralized coalition but an intermediate coalition, leading to a scheme of partial cooperation. Again, Calmfors and Driffell’s results can be understood when pointing out the differences in consumer preferences between goods produced domestically and goods produced by other countries.

Will we Benefit from Fiscal Cooperation among Subsets of Countries?

The issue of international fiscal cooperation is under debate among macro-economists. It has been shown that cooperation does not necessarily represent an improvement in the sense that it generates a number of difficulties.

In a recent paper, Sørensen (1996) has addressed the issue of coordination of fiscal policies among subsets of countries. His main result is that, given the fact that global cooperation on fiscal policy is the first-best outcome, totally non-cooperative fiscal policies generate the second-best outcome, whereas partial coordination among subsets of countries (all countries
entering such agreements) is only third best. His set-up exactly fits the general framework we have just exposed and exploited. Hence our general propositions can be applied to Sørensen’s model and explain the reasons—in terms of externalities—for his result. They also suggest how it could be reversed.

Sørensen’s model derives from the standard monopolistic competition macromodel with imperfect competition on the labour market due to unions bargaining over wages. Its distinctive feature is that each differentiated good is produced by a number of countries. The government budget is balanced and public expenditures are equal to lump-sum taxes.

In (the representative firm of) each country, there is a union which is able to fix the wage rate, seeking to maximize the utility of the representative member. On the whole, the driving forces of the world economy are the public expenditures (denoted by $g_{ik}$) decided by national governments, and they generate externalities among countries by affecting relative prices of goods (terms of trade), hence welfare. Then, Sørensen considers and compares three equilibria corresponding to our three cases: (i) fully coordinated (among all countries) fiscal policy; (ii) coordination of fiscal policy among countries producing the same good; (iii) no coordination at all. He finds (p. 117) “that totally uncoordinated policies are too expansionary compared to the first-best (global coordination) and... that coordination among similar countries gives rise to an even more expansionary outcome. Hence, we may conclude that fiscal policy coordination among similar countries results in lower welfare than global coordination as well as no coordination at all”.

Given that Sørensen’s model perfectly fits our assumptions, we can envision his theoretical results above in terms of our propositions. There are $NK$ countries and $K$ differentiated goods, with each good produced by $N$ countries. The representative consumer in each country consumes the $K$ goods and her utility function is written as follows:

$$U_{ik} = K^{1/(1-\theta)} \left( \sum_{l=1}^{K} C_{ik,l}^{(\theta-1)/\theta} \right)^{\theta/(\theta-1)} - dl_{ik} + u(g_{ik}) \quad \forall i, \forall k \quad \theta > 1, \; d > 0$$

with $u(\cdot)$ concave. $C_{ik,l}$ is the consumption of good $l$ by the representative consumer of country $ik$, $l_{ik}$ is the quantity of labour used in country $ik$, and $g_{ik}$ is the public expenditure in country $ik$, for purchasing good $k$. The government budget is balanced and public expenditures are equal to lump-sum taxes $T_{ik}$. Using Sørensen’s resolution of the model, the indirect utility

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4This model has been developed by Dixon (1991) and others.
function of the representative agent \( ik \) can be written as a function of the various levels of public expenditures in the world economy:

\[
U_{ik} = v(g_{ik}, g_{k}, g_{-k}).
\]

The derivative \( v_2(g_{ik}, g_{k}, g_{-k}) \) is positive because an increase in public expenditures by countries producing the same good \( k \) increases the relative price of this good, which benefits to the representative agent in country \( ik \) through an income effect. The derivative \( v_3(g_{ik}, g_{k}, g_{-k}) \) is negative because an increase in public expenditures by countries producing another good than good \( k \) decreases the (relative) price of this good, which harms the representative agent in country \( ik \).

Given the opposite signs of the spillovers, this generates the ranking of actions. The ordering of actions comes from the concavity of \( u(\cdot) \). The explanations for these orderings in Sørensen’s model parallel our general propositions. The fact that:

\[
g^I > g^N > g^C
\]

(where \( g^I \), \( g^N \), \( g^C \) denote the equilibrium public expenditure levels in the intermediate game, the decentralized game and the centralized game, respectively) comes from spillovers within and between groups with opposite signs. Applying our analysis also enables us to explain the ranking of welfare obtained by Sørensen.

### Should we Encourage Local Cooperation on Local Public Goods in the Presence of Centralized Financing?

Consider an environment where there are \( K \) regions, each with two localities \((N = 2)\).\(^5\) For each locality, a public good can be provided at a level \( a_{ik} \geq 0 \), \( i = 1, 2 \) and \( k = 1, \ldots, K \). The direct benefits from a public good are assumed to accrue to the individuals in the specific locality of the public good as well as to the individuals in the particular region. In other words, local public goods are assumed to have regional externalities. We denote by \( U(a_{ik}, a_{jk}) \), \( i = 1, 2; j = 1, 2; i \neq j \), the utility from public goods obtained by individuals in locality \( i \) in region \( k \). We assume that \( U_1 > 0 \), \( U_2 > 0 \) and \( U_{12} < 0 \), so that public goods from different localities within a region are substitutes. Furthermore, in order to ensure that optimal levels of public goods are well defined, we assume that \( U(\cdot) \) is concave and that the limit of \( U_1(a, a) \) and \( U_2(a, a) \) is zero (or negative) as \( a \) goes to infinity.

\(^5\)This last assumption can easily be relaxed.
The cost of providing one unit of any local public good is equal to one (unit of available resource). In the fiscal environment we consider, a fraction \((1 - \alpha)\) of the cost of providing a public good is subsidized through general taxation, that is, each locality is taxed by an amount \(((1 - \alpha)/2K)\sum_{l=1}^{K} \sum_{j=1}^{2} a_{jl}\) for the general provision of public goods. The remaining costs are paid by the local authorities. The issue we want to address is whether, in this environment, it can be harmful to have localities coordinate regionally on the level of public goods as opposed to deciding on them completely independently at the local level. In order to see how the results of Section II can be applied in this case, we begin by making explicit the net payoffs to each locality. The net payoff to locality \(i\) in region \(k\) from the provision of the vector of public goods \(\{a_{ik}\}_{i=1,k=1}^{2,K}\) is given by:

\[
V(a_{ik}, a_{jk}, a) = \sum_{l=1}^{K} \sum_{j=1}^{2} a_{jl} (1 - \alpha) \frac{a_{ik} - (1 - \alpha) a_{jk}}{2K}
\]

In the above formulation it is assumed that a locality’s payoff incorporates the benefits derived directly from the public goods and is linearly decreasing in its total expenditure on public goods, including its payment of taxes. In the case of symmetric equilibria, the above payoff function simplifies to:

\[
u(a_{ik}, a_{jk}, a_k) = U(a_{ik}, a_{jk}) - \left(\alpha + \frac{(1 - \alpha)}{2K}\right) a_{ik} - (1 - \alpha) \frac{a_{jk}}{2K} - (1 - \alpha) a_k \frac{(K - 1)}{K}.
\]

We are now capable of using the propositions in Section II to make the following claim:

**Claim.** If there are sufficiently many regions \((K\) large enough) and if the local public good is sufficiently subsidized \((\alpha\) small enough), then it is welfare decreasing to allow localities within a region to coordinate on the level of public goods to be provided.

Proof: This problem satisfies the assumptions of Section II, therefore, by Proposition 2, it is sufficient to show that for sufficiently large $K$ and sufficiently small $\alpha$, $\sigma_2 < -\sigma_3$. Note that in the limit, when $K$ goes to infinity, $v_2(a_{ik}, a_{jk}, a_k) = U_2(a_{ik}, a_{jk})$, $v_3(a_{ik}, a_{jk}, a_k) = -(1 - \alpha)$ and the optimality condition for the regionally coordinated decision on $a$ is $U_1 + U_2 = 2\alpha$. Hence, if $\alpha < \frac{1}{3}$ as $K$ approaches infinity $\sigma_2$ will necessarily fall below $-\sigma_3$. ■

The intuition for this result is rather straightforward. When the number of regions is sufficiently large, a coalition of (two) localities only internalizes the benefits of public goods without adequately internalizing the costs, as the whole economy finances them. Since local public goods generate positive externalities within the group, a coalition of localities belonging to the same coalition is induced to favour more provision of public goods. If, moreover, the subsidization of local public goods is sufficiently strong, the overall level of public goods under coordinated decisions will tend to be excessively high. With $U(\cdot)$ concave, this implies that the welfare when countries collude in pairs will be smaller than when there is no coalition at all.

IV. Conclusion

We do not always lose by allowing partial cooperation. However, this does not mean that it should be encouraged at any cost. Using a simple yet general analytical framework, allowing us to consider and compare symmetric equilibria, we offered simple rules for comparing both actions and welfares that characterize the symmetric equilibrium relying on partial cooperation with the outcomes of games with full cooperation or no cooperation.

To obtain these rules, we developed a generic symmetric formal model with several features. Players are identical, insofar as they are characterized by the same payoff function. Each player in a game belongs to a “natural” group, characterized by the fact that spillovers between players within it differ from spillovers between agents belonging to different groups. We then defined partial cooperation by the fact that players cooperate within their “natural” group, but groups do not cooperate. We denominate this setting as the partial cooperation game.

In such a setting, partial cooperation turns out to be good, improving welfare as compared to the no-cooperation game, only if spillovers of any sort are of the same sign, when computed at the equilibrium of the partial cooperation game. If not, and if spillovers between groups on the aggregate, when estimated at the intermediate equilibrium, are more important in absolute value than the spillovers within groups, then partial cooperation is “bad”.

We also showed how these rules can be applied to many issues implying
partial cooperation, since many economic examples satisfy the assumptions of the generic model, such as industry-wage bargaining and local public goods.

On the whole, the message of this paper is that we should be cautious when offered the possibility to enter a scheme of partial cooperation, say, with our neighbours or closest kin – cautious, but not systematically opposed to such a prospect. Cooperation is an exercise in lucidity.

Appendix. Proof of Lemma 1

The proof of Lemma 1 is in three steps.

(1) First, we show that \( \sigma_1(a) + \sigma_2(a) + \sigma_3(a) \), \( \sigma_1(a) + \sigma_2(a) \) and \( \sigma_1(a) \) are decreasing functions of \( a \), respectively at \( a^C \), \( a^I \) and \( a^D \). Assumption 3 implies that \( \partial(\sigma_1(a^C) + \sigma_2(a^C) + \sigma_3(a^C))/\partial a < 0 \). The uniqueness of \( a^I \) (Assumption 2) together with Assumption 4 implies that \( \partial(\sigma_1(a^I) + \sigma_2(a^I))/\partial a > 0 \) is impossible because otherwise, there exists \( a \in [a, \bar{a}] \) such that \( \sigma_1(a) + \sigma_2(a) + \sigma_3(a) = \sigma_1(a) + \sigma_2(a) \), which is equivalent to \( \sigma_3(a) = 0 \), which is ruled out by Assumption 4 (see Figure A1). Therefore, we must have \( \partial(\sigma_1(a^I) + \sigma_2(a^I))/\partial a \leq 0 \). By similar reasoning, \( \partial\sigma_1(a^D)/\partial a > 0 \) is also impossible because, otherwise, there exists \( a \in [a, \bar{a}] \) such that \( \sigma_1(a) + \sigma_2(a) + \sigma_3(a) = \sigma_1(a) \), which is equivalent to \( \sigma_2(a) + \sigma_3(a) = 0 \), which is ruled out by Assumption 4. Therefore, \( \partial\sigma_1(a^D)/\partial a \equiv 0 \).

![Fig. A1.](image-url)
(2) Proof of (i) and (ii). We first compare $a^I$ and $a^D$ and then proceed to the comparison between $a^I$ and $a^C$.

**Comparison of $a^I$ and $a^D$**

(a) Consider the case where $\partial \sigma_1(a^D)/\partial a < 0$ and $(\sigma_1(a^I) + \sigma_2(a^I))/\partial a < 0$. The uniqueness of $a^D$ and $a^I$ (Assumptions 1 and 2) implies that $a^D > a^I$ iff $\sigma_2(a^I) < 0$, $a \in [a, \overline{a}]$.

(b) Consider the case where $\partial \sigma_1(a^D)/\partial a = 0$ and $(\sigma_1(a^I) + \sigma_2(a^I))/\partial a < 0$. Assume that $\sigma_2(a^I) < 0$. Suppose that $a^D < a^I$. Note that $\sigma_1(a^I)$ cannot be inverted U-shaped ($\sigma_1(a^I) < 0$, $a \neq a^D$) since this would imply $\sigma_2(a^I) > 0$ which is ruled out by assumption. But then, since $\sigma_1(a^I)$ is U-shaped, there exists $a \in [a^D, a^I]$ such that $\sigma_1(a^I) + \sigma_2(a^I) = \sigma_1(a^I)$, which is equivalent to $\sigma_2(a^I) = 0$, which is ruled out by Assumption 4. Thus, $a^D > a^I \iff \sigma_2(a^I) < 0$.

(c) Consider the case where $\partial \sigma_1(a^D)/\partial a < 0$ and $(\sigma_1(a^I) + \sigma_2(a^I))/\partial a = 0$. Assume that $\sigma_2(a^I) < 0$. Suppose that $a^D < a^I$. Note that $\sigma_1(a^I) + \sigma_2(a^I)$ cannot be U-shaped ($\sigma_1(a^I) + \sigma_2(a^I) < 0$, $a \neq a^I$) since this would imply $\sigma_2(a^D) > 0$ which is ruled out by assumption. But then, since $\sigma_1(a^I) + \sigma_2(a^I)$ is inverted U-shaped, there exists $a \in [a^D, a^I]$ such that $\sigma_1(a^I) + \sigma_2(a^I) = \sigma_1(a^I)$, which is equivalent to $\sigma_2(a^I) = 0$, which is ruled out by Assumption 4. Thus, $a^D > a^I \iff \sigma_2(a^I) < 0$.

(d) Consider the case where $\partial \sigma_1(a^D)/\partial a < 0$ and $(\sigma_1(a^I) + \sigma_2(a^I))/\partial a = 0$. First, note that $\sigma_1(a^I) + \sigma_2(a^I)$ cannot intersect between $a^D$ and $a^I$. This would imply that there exists $a \in [a, \overline{a}]$ such that $\sigma_2(a^I) = 0$, which is ruled out by Assumption 4. In other words, both functions cannot be simultaneously inverted U-shaped or U-shaped. First-order conditions using the uniqueness of $a^D$ and $a^I$, this implies that $\sigma_1(a^I) + \sigma_2(a^I)$ is non-negative, over $[a, \overline{a}]$ iff $\sigma_1(a^I)$ is non-negative over the same interval.

Assume that $\sigma_2(a^I) < 0$, implying that $\sigma_1(a^I) + \sigma_2(a^I)$ is inverted U-shaped (since $\sigma_1(a^I) + \sigma_2(a^I) = \sigma_2(a^I) < 0$) and $\sigma_1(a^I)$ is U-shaped. Suppose then that $a^D < a^I$. If $\sigma_3(a^I) < 0$, then $\sigma_2(a^I) + \sigma_3(a^I) < 0$ and $a^D > a^C$. But then, as $\sigma_1(a^I) + \sigma_2(a^I) + \sigma_3(a^I)$ is non-negative, there exists $a \in [a^D, a^I]$ such that $\sigma_1(a^I) + \sigma_2(a^I) = \sigma_1(a^I) + \sigma_2(a^I) + \sigma_3(a^I)$, which is equivalent to $\sigma_1(a^I) = 0$, which is ruled out by Assumption 4. If $\sigma_3(a^I) > 0$, then $\sigma_1(a^I) + \sigma_2(a^I) + \sigma_3(a^I)$ is positive, which means that $a^C > a^I$ (since $\sigma_1(a^I) + \sigma_2(a^I) + \sigma_3(a^I)$ is decreasing). As we suppose $a^I > a^D$, $\sigma_1(a^D) + \sigma_2(a^D) + \sigma_3(a^D)$ is positive. Hence there exists $a \in [a^D, a^I]$ such that $\sigma_1(a^I) = \sigma_2(a^I) + \sigma_3(a^I)$, implying $\sigma_2(a^I) + \sigma_3(a^I) = 0$, which is ruled out by Assumption 4.

Hence, whatever the sign of $\sigma_3(a^I)$, $a^D > a^I \iff \sigma_2(a^I) < 0$.

**Comparison of $a^I$ and $a^C$**

The first-order conditions imply that: $\sigma_1(a^C) + \sigma_2(a^C) + \sigma_3(a^C) = 0$ and $\sigma_1(a^C) + \sigma_2(a^C) = 0$. Moreover, the second-order conditions of the centralized solution imply that: $(\partial \sigma_1(a^C)/\partial a + \sigma_2(a^C) + \sigma_3(a^C) = 0)/\partial a = 0$.

(a) Consider the case where $\partial (\sigma_1(a^C) + \sigma_2(a^C))/\partial a < 0$. The uniqueness of $a^C$ and $a^I$ (Assumptions 2 and 3) implies that $a^C > a^I$ iff $\sigma_3(a^C) > 0$, $a \in [a^C, a^I]$. 

(b) Consider the case where $\partial (\sigma_1(a^l) + \sigma_2(a^l))/\partial a = 0$. Assume that $\sigma_3(a) < 0$. In this case, one has necessarily $\sigma_1(a^l) + \sigma_2(a^l) + \sigma_3(a^l) = \sigma_1(a^l) + \sigma_2(a^l)$, which implies $a^l > a^C$. In the reverse case, $a^l > a^C$ together with $\sigma_3(a) < 0$ implies $\sigma_1(a^l) + \sigma_2(a^l) + \sigma_3(a^l) < \sigma_1(a^l) + \sigma_2(a^l)$. Thus, $a^C < a^l \Rightarrow \sigma_3(a) < 0$. 

(3) Proof of (iii) and (iv).

(a) Suppose externalities to be positive: $\sigma_2(a) > 0$ and $\sigma_3(a) > 0$. We know from (i) and (ii) that $a^I$ takes an intermediate value between $a^D$ and $a^C$. From the first-order conditions for the centralized solutions and the sign of externalities, we get that: $\sigma_1(a^C) < 0$. If $\sigma_1(a)$ is monotone, it implies that $a^D < a^C$. If $\sigma_2(a)$ is not monotone and then inverted U-shaped, $a^C < a^D$ is impossible: it would imply that there exists $a \in [a^C, a^D]$ such that $\sigma_1(a^C) = \sigma_1(a^D)$, implying $\sigma_2(a) + \sigma_3(a) = 0$, which is ruled out by Assumption 4. Hence

$$\begin{align*}
\sigma_2(a) > 0 \\
\sigma_3(a) > 0
\end{align*} \Rightarrow a^D < a^l < a^C.
$$

The reverse implication is immediate when taking (i) and (ii) into consideration.

(b) Suppose externalities to be negative: $\sigma_2(a) < 0$ and $\sigma_3(a) < 0$. We know from (i) and (ii) that $a^I$ takes an intermediate value between $a^D$ and $a^C$. From the first-order conditions for the centralized solutions and the sign of externalities, we get that: $\sigma_1(a^C) > 0$. If $\sigma_1(a)$ is monotone, it implies that $a^D > a^C$. If $\sigma_2(a)$ is not monotone and then upward-sloping, $a^D < a^C$ is impossible: it would imply that there exists $a \in [a^C, a^D]$ such that $\sigma_1(a^C) = \sigma_1(a^D)$, implying $\sigma_2(a) + \sigma_3(a) = 0$, which is ruled out by Assumption 4. Hence

$$\begin{align*}
\sigma_2(a) < 0 \\
\sigma_3(a) < 0
\end{align*} \Rightarrow a^D > a^l > a^C.
$$

The reverse implication is immediate when taking (i) and (ii) into consideration.\textsuperscript{6}

References


\textsuperscript{6}The various cases we have just studied can be visualized by means of figures analogous to Figure A1.


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