Developing Land and Structure Price Indexes for Ottawa Condominium Apartments

Kate Burnett-Isaacs, Ning Huang and W. Erwin Diewert,[1] November 21, 2016
Discussion Paper 16-09,
Vancouver School of Economics,
The University of British Columbia,
Vancouver, Canada, V6T 1L4.

Abstract

Measuring the service flow and the stock value of condominium apartments in Canada and decomposing these values into constant quality price and quantity components is important for many purposes. In addition, the System of National Accounts requires that these service flows and stock values for condos be decomposed into constant quality land and structure components. In Canada and most other countries, such a land and structure decomposition of condominium apartment sale prices does not currently exist. In this paper, we provide such a decomposition of condominium apartment sales in Ottawa for the period 1996-2009. Specific attention is paid to the roles of communal land and structure space, as well as building commercial space, on condominium apartment unit selling prices. Key findings include methods to allocate land and building space to a single condominium unit, identifying the characteristics that best explain condominium prices, and developing an average depreciation rate for condos for the 14 year time period.

Keywords

Condominium apartment price indexes, land and structure price indexes, hedonic regressions, net depreciation rates, System of National Accounts.

JEL Classification Numbers

C2, C23, C43, E31, R21.

---

[1] Kate Burnett-Isaacs: Producer Prices Division, Statistics Canada, Ottawa, Canada, K1A 0T6 (email:kate.burnett-isaacs@canada.ca); Ning Huang: Consumer Prices Division, Statistics Canada, Ottawa, Canada, K1A 0T6 (email: nings.huang@canada.ca) and W. Erwin Diewert: School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1Z1 and the School of Economics, University of New South Wales, Sydney, Australia (email: erwin.diewert@ubc.ca). The authors thank Heidi Ertl, Mary Beth Garneau, Gaétan Garneau and Clément Yélou at Statistics Canada and Price Measurement Advisory Committee members Jan de Haan, François Des Rosiers, Pierre Duguay, John Mallon and Mick Silver for their helpful discussions and comments. The authors also thank Amy Jiang, Simon Trottier and Ross Beck-MacNeil for their technical help on the data treatment. The views expressed in this paper are solely those of the authors and do not necessarily reflect those of Statistics Canada. The last author gratefully acknowledges the financial support of the SSHRC of Canada and the Australian Research Council (LP0884095, DP150100830).

1. Introduction

Over the last 15 years, the condominium apartment sector has been a growing component of the Canadian residential property market.[2] To accurately measure the economic activity in this sector, Statistics Canada is developing a New Condominium Apartment Price Index (NCAPI).[3] The use of the NCAPI in statistical programs, for example as a deflator in various components of Gross Domestic Product and as an input into the Consumer Price Index, require price indexes for the total (land and structure components), as well as separate price index series for land and structure. Data on separate land and structure values are difficult to come by resulting in a knowledge gap in condominium apartment information that the NCAPI currently cannot fill.

In order to decompose a condominium apartment unit price into separate land and structure components this paper looks to the Builder's Model developed by Diewert and Shimizu (2017).[4] This hedonic model suggests that the value of a condominium unit is a result of the sum of the value of the land and structure components. The structure component can be viewed as the depreciated cost to build the structure itself.[5] The land component measures the impact that location and neighbourhood amenities, in addition to land size, have on the total price of a condominium apartment unit.

However, there are three key considerations that must be taken when decomposing the price of a condominium unit into separate land and structure components. The most difficult consideration to model is how to allocate communal land to a single condominium unit. Secondly, condominium units share communal space with the rest of the building. These communal building areas include lobbies, hallways, party rooms, gyms, pools, parking lots, etc. that though shared, are paid for by each unit and so need to be properly modeled. Lastly, we extend Diewert and Shimizu's model to include commercial space found in condominium apartment buildings. These spaces provide amenities to the building, and the rent for these commercial enterprises could contribute to the condominium association's revenues. Therefore, the impact that commercial space has on the selling price of a condominium apartment unit needs to be fully investigated.

The purpose of this study is to develop a methodology to create separate land and structure price indexes that can be used in the calculation of the NCAPI and future resale condominium property price indexes. In this study, we focus on the Ottawa high-rise

---

[2] For more details on the increase in condominium apartment trends see Read-Hobman (2015) "Evolution of housing in Canada, 1957 to 2014" Canadian Megatrends. Statistics Canada, Catalogue no. 11-630-X.

[3] Based on the definition of National Household Survey conducted by Statistics Canada, condominium dwelling refers to a private residential complex in which dwellings are owned individually while land and common elements are held in joint ownership with others. See "Homeownership and Shelter Costs in Canada" Catalogue, No. 99-014-X2011002 for more details. This definition does not specify the type of structure. When we define a condominium apartment, in this paper and in the context of the NCAPI, it does not include single family homes or row houses that have condominium type ownership.

[4] See Diewert and Shimizu (2017) for more details on the builder's model for Tokyo Condominium Sales.

[5] See Davis and Palumbo (2008) for more explanation on using construction costs as a proxy for the structure component.

condominium apartment market from 1996 to 2009. The paper is broken down in the following manner: section 2 explains the data used in this study; section 3 introduces the Builder's Model and how it must adapt to incorporate land, communal building space and commercial space for condominium apartments; section 4 focuses on finding the main determinants of land prices; section 5 introduces structural variables to the Builder's Model; section 6 explores how to introduce commercial space into the Builder's Model; section 7 explains the land, structure and total property index series derived from our proposed hedonic model and section 8 concludes.

2. Data

The source of data for this study is a combination of a residential property price research dataset, City of Ottawa building characteristics data set and some internet data sources. This research dataset was developed for new and resale condominium apartment units for 55 quarters from Q1 1996 to Q3 2009.[6] High rise condos, which are defined as those condo buildings with five and more floors, are the focus of this study. This threshold was chosen because buildings with four or less floors are built similarly to single family houses, with higher wood content than high rise buildings that are built with more glass and concrete materials. The dataset contains unit characteristic variables such as number of bedrooms, bathrooms, heating fuel, floor covering, the story the unit is on and unit square footage; land characteristics such as location of the condominium building described by the Forward Sortation Area (FSA),[7] land size and excess land; and building structure characteristics include building size, building height, unit height, and total number of units in building.

The dataset required grooming due to misreporting as well as outlier detection and trimming of maximum and minimum values for unit living area, selling price, bedroom, bathrooms, and age variables. The final dataset includes observations with the following characteristics:

- Living area between 300 and 1500 square feet (sqft);
- Selling price between bottom 1percent and top 5 percent by year of sale;[8]
- 1 to 4 bedrooms;
- 1 to 3 bathrooms;
- Age < 50 years.[9]

[6] Q4 2009 data was available, but with few observations. The estimates produced for this quarter were not reliable.
[7] A forward sortation area (FSA) is a geographic unit based on the first three characters in a Canadian postal code.
[8] These values for trimming were chosen in order to normalize the distribution of selling price.
[9] The age restriction of 50 years was chosen because buildings older than this age will most likely have gone through a major renovation. Since we use age of the building to estimate depreciation, including buildings with major renovations would not provide accurate results.

Descriptive statistics for the main variables that will be used in our analysis are listed in table 1. It can be seen that, even after data grooming and range trimming, there is still great variation in variables such as selling prices, total residential building area and lot size of the condo buildings.

Table 1: Descriptive Statistics for Key Variables

	Freq	Mean	Std. Deviation	Maximum	Minimum	Mode
Price (dollars)	9977	162701.97	73500.38	458000	32000	
Unit Living Area (sqft)	9977	667.15	155.26	1495.64	300.06	
Lot Size (sqft)	9977	92373.90	65176.89	268021.13	2029.00	
Building Size (sqft)	9977	222910.35	108449.69	614823.18	15021	
Age (years)	9977	20.67	8.96	42	0	23
Height of Building (stories)	9977	16.32	6.73	32	5	12
Story of Unit	9977	8.44	5.87	28	1	3
Bedrooms (number)	9977	1.92	0.51	4	1	2
Bathrooms (number)	9977	1.52	0.51	3	1	2

3. The Builder's Model for Condominium Apartments

The Builder's Model is based on the expected cost of building a property, either a single family home or a condominium apartment unit. This model suggests that the selling price of a property that has a newly built structure on it is driven by the cost of producing said property.[10] Thus the hedonic form of the Builder's Model states that property price is equal to the quality adjusted cost of land per square foot (α_t) times the square footage of land (TL_{tn}) plus the quality adjusted structure cost per square foot (β_t) times the square footage of the structure (S_{tn}) for n=1,…N_t, where N is the number of observations, for a given time (t).The Builder's Model can be approximated by the following hedonic regression model with an error term (ε_{tn}) that is assumed to be normally distributed with a mean of zero and a constant variance:

$$(1)\ P_{tn} = \alpha_t TL_{tn} + \beta_t S_{tn} + \varepsilon_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t$$

The above model applies to new properties. To incorporate depreciation that occurs in older structures, which devalues the structure in the absence of renovations, the Builder's Model can use information on the age of the structure (A_{tn}) in order to estimate a net

[10]For more details on the Builder's Model application to single family homes, see the Handbook on Residential Property Price Indexes (2013).

geometric depreciation rate (δ_t) as the structure ages one period with the following formula:

$$(2)\ P_{tn} = \alpha_t TL_{tn} + \beta_t (1-\delta_t)^{A_{tn}} S_{tn} + \varepsilon_{tn}; \quad t = 1,\ldots,55;\ n = 1,\ldots,N_t$$

In trying to estimate equation 2, multicollinearity between the land and structure variables warrants the use of a construction cost index to proxy for the change in cost of building the structure.[11] Multicollinearity occurs when two or more independent variables are correlated with each other. This can cause estimates to be unstable and difficult to interpret with potentially incorrect signs or magnitudes.[12] In this study of condominium apartments, the price per square foot from the 2004 model of the Apartment Building Construction Price Index (ABCPI) is used to proxy the cost of building a condominium unit.[13] The use of this variable is based on the assumption that the movement of condominium apartment building costs approximate those for non-condominium apartment buildings. This notion is based on the grounds that increasingly, apartment buildings are being constructed with similar finishes as condos. This price per square foot is then indexed using the ABCPI to get an estimated cost per square foot of structure space (PS_t) for each quarter from Q1 1996 to Q3 2009. The resulting hedonic model is:

$$(3)\ P_{tn} = \alpha_t TL_{tn} + \beta PS_t (1-\delta_t)^{A_{tn}} S_{tn} + \varepsilon_{tn}; \quad t = 1,\ldots,55;\ n = 1,\ldots,N_t$$

To apply the Builder's Model to condominium apartment units, we need to make additional considerations that would not be found in the Model for say single family homes. The main considerations are how to address the roles of communal land and structure space, as well as building commercial space, on the selling price of a condominium apartment unit.

3.1 Allocating the Unit's Land Share: Method 1

In our dataset, the variable for unit area is used to estimate the structure component for the unit only. However, land size is given for the whole building and not the single unit. Therefore, land size must be allocated appropriately to a single condo unit. The preliminary assumption is that each unit in the building equally enjoys the whole land area, therefore the land should be divided equally by all units in the building (TU_{tn}):[14]

$$(4)\ L_{tn} = \left(\frac{1}{TU_n}\right) TL_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

In section 4, alternative land imputation methods are investigated.

[11] The Handbook on Residential Property Price Indexes (2013) expands on the multicollinearity problem found in the Builder's Model; see also Diewert, de Haan and Hendricks (2015).
[12] For more details on the issues that arise with multicollinearity in hedonic models, see Greene (2003).
[13] Thus our model will achieve consistency with Canadian System of Macroeconomic Accounts estimates for the value of new construction.
[14] This notion of equally shared land is presented in Diewert and Shimizu (2017).

3.2 Allocating the Unit's Share of Communal Space

A condominium unit shares communal space with other units in its building. This space and the amenities in it are accounted for, in part,[15] in the selling price of the unit. However, when it comes to estimating these spaces in the Builder's Model, the floor space of the condominium unit only covers privately owned space.

Explicit values of communal space are not available in our dataset, therefore we need to estimate the proportion of the building space that is communal. Consultations were conducted with the construction industry and an estimate for communal space was calculated using the apartment building specifications from the 2004 model used in the ABCPI. From these two sources, it was determined that about 20-30 percent of the building space is allocated to communal areas. We tested the sensitivity of this assumption and find that there was very little difference in the estimates of the Builder's Model using 20, 25 or 30 percent values for communal space. Therefore, the hedonic models and results that follow will use an estimate for communal space of 25 percent of the total building area.

The floor area of the unit represents privately owned structure space. To capture all structural space allocated to a condominium unit, including communal space, the privately owned space in our model must be blown up by a factor that represents communal space. Since we are using the estimate of 25 percent of the building as communal space, 75 percent of the building is private space and so can be estimated by the unit floor space. To include that extra 25 percent of communal space in our model, structural space is estimated by $(1/0.75)S_{tn}$ or $(1.33)S_{tn}$.

The amenities contained in this 25 percent communal space can differ between buildings and could be a factor affecting the price of a condominium unit. Tests were conducted to determine the impact that indoor and outdoor parking, fitness facility, party room and indoor pool had on the price of a condominium unit. Fitness room and party room had a negative impact on the Builder's Model and parking (indoor and outdoor) and indoor pool had a marginal, positive impact on the model.[16] For these reasons, communal amenities are not including in this study.

3.3 The Treatment of Commercial Space

In our data set, we found about 7 percent of the condominium apartment buildings contained commercial space, meaning a section of the building area is rented out to commercial enterprises. These spaces not only provide amenities to the building residents, but also pay rent to the condominium association or building owners, which in turn could be used to pay for such things as renovations. Therefore, the assumption is that

---

[15] Condo fees also contribute to paying for and maintaining these amenities.

[16] A small impact is defined as improving the Log Likelihood value by a small amount relative to the increase in the number of parameters.

commercial space is a benefit to condominium inhabitants and so must be somehow reflected in the selling price of condominium units.

The only information we have in our data set on commercial space is whether a business exists in the building or not. No information is available on the type of business or the amenities it provides to the building residents. Therefore we use a commercial space dummy variable in our Builder's Model to assess the impact that commercial space has on the sale price of a condominium unit. Commercial space might have an impact on both the land and structure components of a condominium unit price. For the land component, the commercial space can provide amenities to the building, however commercial spaces tend to be on busy street that may be unattractive to residents. On the structure side, rent from commercial spaces could be used to pay for structural upkeep. The type of impact commercial space has on our Builder's Model, along with the definition of the commercial space dummy variable, the associated model and the estimates are discussed in section 5.3.

3.4 A Preliminary Builder's Model

Including depreciation rate, land imputation and cost per square foot of structural space enhances the Builder's Model to better represent the features found in condominium apartment units. In order to get initial land value estimates, which will be discussed in section 4, the depreciation rate is set to 2 percent. This is the estimate used by the Canadian System of Macroeconomic Accounts and productivity analysis at Statistics Canada for all residential housing depreciation rates. Therefore, we update model (2) to include the unit's share of land (L_{tn} defined by (4)), the communal space blow-up factor (1.33), the price per square foot of structural floor space (PS_t) and the annual depreciation rate (0.02):

$$(5)\ P_{tn} = \alpha_t L_{tn} + (1.33)\beta PS_t(1-0.02)^{A_{tn}} S_{tn} + \varepsilon_{tn};\ t=1,\ldots,55;\ n=1,\ldots,N_t$$

The R^2 value for this model is 0.6751, which indicates there is room for improvement in the model. However, there are more concerning results of this model. Most α_t values, which are the estimates for average price of land per square foot, are negative. Negative prices cannot exist in this context. Also, the β coefficient estimate is 5.03. The β coefficient now represents a general quality adjustment to the structure area. Our assumption is that the model can account for almost all quality adjustment to the structure implying that β should be closer to a value of 1. Thus the very large estimate for β has led to α_t estimates which are too small to be credible.

4. The Determinants of Condominium Land Prices

To improve the results of the model defined by (5), we are going to assume β = 1 and focus on finding the main determinants of land prices. To do this, we set imputed land value to be the dependant variable of our hedonic model. We derive an estimate for land value (LV_{tn}) by subtracting our imputed structure value (SV_{tn}) from the total property selling price (P_{tn}):

$$(6)\ LV_{tn} = P_{tn} - SV_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

Our imputed structure value is approximated by:

$$(7)\ SV_{tn} = (1.33)PS_t(1 - 0.02)^{A_{tn}}S_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

The above estimates for land value will now be used as the dependent variable in the models that are estimated in this section. The baseline model we will use to begin our analysis is that land value can be modeled by the price of land per square foot (α_t) multiplied by land equally distributed per unit (L_{tn}):

$$(8)\ LV_{tn} = \alpha_t L_{tn} + \varepsilon_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

This model gives us a starting point to assess the impact of additional land characteristics on the goodness of fit of the proposed model. It has an R square value of −0.6891 and a log likelihood value (LL) of −127456. Given the nonlinear nature of this and subsequent models, goodness of fit will be determined by the combined improvement in the LL and the R-square values.

4.1 Introducing Postal Code Dummy Variables

The results of model (8) clearly suggest that there needs to be an improvement in how we model land prices. The price of any property is heavily impacted by location. To capture this relationship we use Forward Sortation Area dummy variables ($FSA_{tn,i}$) in our hedonic model. The Forward Sortation Area is identified by the first three digits of the Canadian postal code. These 22 dummy variables are defined as:

(9) $FSA_{tn,i}$ = 1 if observation n in period t is in Forward Sortation Area i;
= 0 otherwise.

By adding the Forward Sortation Area dummy variables to model (8) we can account for how the land prices change based on location:

$$(10)\ LV_{tn} = \alpha_t(\textstyle\sum_{i=1}^{22} \theta_i FSA_{tn,i})\, L_{tn} + \varepsilon_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t$$

where the land size by unit (L_{tn}) is defined as in equation (4). The 55 α_t parameters and 22 θ_i parameters cannot all be identified. Therefore, we normalize α_1= 1. With α_1= 1, the value of all other α_t estimate represent the percentage change in land value due to the change in time from period 1 to period *t*. This is the definition of a price index and so we can use the parameter estimates of land price to create our land price index. The R-square for this model is 0.0957 and the LL is −124339, which is a large 3117 improvement from model (8), validating our assumption that location has a significant impact on the land prices in Ottawa.

4.2 Alternative Land Value Imputation Methods

Prior models assumed that land was equally distributed to each condominium apartment unit. However, land could also be allocated to a single unit proportionally to the size of the unit or a combination of equal and proportional allocation.

Land can be allocated to a single unit proportionally to its size compared to the rest of the building. Like in the case of condo fees, where larger units pay higher fees and thus contribute more to funding communal spaces, the logic in this assumption is that the larger units should have a larger share of the land. Proportional land size (L_{tn}) is defined as:

$$(11)\ L_{tn} = \left(\frac{S_{tn}}{TS_{tn}}\right) TL_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots, N_t.$$

Replacing the L_{tn} variable in equations (10) to the proportional land variable defined in (11), our model becomes:

$$(12)\ LV_{tn} = \alpha_t (\textstyle\sum_{i=1}^{22} \theta_i FSA_{tn,i}) \left(\frac{S_{tn}}{TS_{tn}}\right) TL_{tn} + \varepsilon_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots, N_t.$$

However, the R-square and LL values from this model, –0.0938 and –125288 respectively, are worse than those of model (10).

Given that model (12) provides worse results than model (10) we need to find a different method to determine the land share of a single unit. An alternative method is to distribute the total land among the units in the building by a *weighted average* of the equal and the proportional allocation:

$$(13)\ L_{tn} = \left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right) + (1-\rho)\left(\frac{1}{TU_{tn}}\right)\right] TL_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

The ρ coefficient is estimated in model (14) below:

$$(14)\ LV_{tn} = \alpha_t (\textstyle\sum_{i=1}^{22} \theta_i FSA_{tn,i}) \left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right) + (1-\rho)\left(\frac{1}{TU_{tn}}\right)\right] TL_n + \varepsilon_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots, N_t.$$

The estimate of $\hat{\rho}$ is 0.2525 (t stat = 12.10), therefore placing a higher weight towards equally distributing the land to a single unit. This makes sense given the poor performance of proportionally distributing land alone, as was found in model (12).

The R-square of model (14) is 0.1021 and the LL is –124304, which is an improvement of 984 on model (12) and an improvement of 35 on model (10). Therefore, subsequent models of land value and total property price will use this weighted land imputation method.

4.3 Introducing the Height of the Unit as an Explanatory Variable

Our expectation is that a unit on a higher floor will have a better view than those on lower floors. The view can be thought as the vertical dimension of land. Therefore, the floor the unit is on, or the height of the unit (H_{tn}), impacts the price of land. The variable H_{tn} is added to model (14) as a continuous variable because it represented the response of the unit's price to a change in height of the unit in a more parsimonious way than using height dummy variables. Thus we add $(1+\gamma(H_{tn}-1))$ to model (14) and obtain (15):

$$(15)\; LV_{tn} = \alpha_t(\textstyle\sum_{i=1}^{22}\theta_i FSA_{tn,i})(1+\gamma(H_{tn}-1))\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)(\frac{1}{TU_{tn}})\right]TL_{tn} + \varepsilon_{tn};\qquad t = 1,\ldots,55;\ n = 1,\ldots, N_t.$$

Even though H_{tn} is a continuous variable, we still normalize the impact that the height of the unit has over the lowest floor observed in our data, in this case the first story. The predicted value of land price will not be affected by those observations corresponding to a unit sold on the first floor. For any unit on a floor above the first floor, the land price will increase by γ for each story. Our estimate for $\hat{\gamma}$ is 0.041494 (t stat = 29.51). Therefore the predicted land price of a condominium unit will increase by 4.15 percent for every story above the first floor. The R-square of this model is 0.1901 and the LL is −123789, an increase of 515 over model (14).

4.4 Introducing the Number of Units in the Building as an Explanatory Variable

In order to build a condominium apartment building land needs to be zoned for the type and size of building. A building with more units will cost more in zoning fees and builders will pass these extra costs on to consumers. To test the extent to which an extra unit impacts the sale price of a condominium unit, we introduce the total number of units (TU_{tn}) into model (15) in a similar fashion to the height of the unit (H_{tn}) as a continuous variable. Again we normalized the impact that an extra unit will have above the minimum number of units found in a building in our dataset. In this case, that minimum number of units is 9. We update model (15) with $(1+\omega(TU_{tn}-9))$, where ω represents the percentage change in land value due to an increase of one unit in the total number of units found in a building. If the building has 9 units in it, land value will be unaffected.

Our new hedonic model is as follows:

$$(16)\; LV_{tn} = \alpha_t(\textstyle\sum_{i=1}^{22}\theta_i FSA_{tn,i})(1+\gamma(H_{tn}-1))\,(1+\omega(TU_{tn}-9))\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)(\frac{1}{TU_{tn}})\right]TL_{tn}+\varepsilon_{tn};\qquad t = 1,\ldots,55;\ n = 1,\ldots, N_t.$$

The R-square value of model (16) is 0.2935 and the LL is −123108, which is an increase of 681 over model (15). The resulting estimate for $\hat{\omega}$ is 0.008927 (t stat = 34.67) indicating that one extra unit in the building will increase the value of land for a single condominium unit by 0.9 percent.

4.5 Introducing the Height of the Building as an Explanatory Variable

Certain neighbourhoods are zoned for tall buildings, such as downtown areas. These buildings are generally more expensive, but to what extent is that because these buildings are tall or because they are in downtown? In model (10) we accounted for location, so now we want to determine the impact building height has on land values. To measure this effect, we introduce four height of building dummy variables to model (16) based on the quartiles of total building height (TH_{tn}) found in our dataset. Group 1 is defined as containing observations for $TH_{tn} < 11$ stories; group 2 contains observations where $11 \leq TH_{tn} < 15$; group 3 contains observations where $15 \leq TH_{tn} < 22$ and group 4 contains observations where $TH_{tn} \geq 22$. The quartile groupings were chosen to ensure that there were enough observations for all dummy variables. The total building height dummy variable is defined as:

$$(17)\; TH_{tn,j} = 1 \text{ if observation n in period t is in total building height group j;}$$
$$= 0 \text{ otherwise.}$$

It is important to note that the height of the building does not change over time. Since our observations are observed for a given time *t*, we include the time subscript in our variable definition. The hedonic model including total height dummy variables is as follows:

$$(18)\; LV_{tn} = \alpha_t\left(\textstyle\sum_{i=1}^{22} \theta_i FSA_{tn,i}\right)(1+\gamma(H_{tn}-1))\left(1+\omega(TU_{tn}-9)\right)\left(\textstyle\sum_{j=1}^{4} \vartheta_j TH_{tn,j}\right)$$
$$\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}+\varepsilon_{tn}; \qquad t = 1,\ldots,55;\ n = 1,\ldots, N_t.$$

The four total building height parameters (ϑ_j), the 22 Forward Sortation Area dummy parameters (θ_i) and the 55 land price parameters in model (18) cannot be all identified, therefore we apply the following normalizations on these parameters:

$$(19)\; \alpha_1=1;\ \vartheta_1 = 1.$$

The R-square value for model (18) is 0.3608 and the LL is −122608, an increase of 500 over the LL of model (16). The estimated total building height parameters increase as the building height increases, suggesting that even accounting for location, building height increases land prices.[17]

4.6 Introducing Excess Land as an Explanatory Variable

The excess land surrounding a condominium building can incorporate many land characteristics that we cannot account for given our data. Excess land is measured as the

[17] See Table 2.

total land size minus the building footprint (total building area divided by number of floors in the building). If a building has a large amount of excess land it could mean this excess property contains amenities such as outdoor parking, outdoor pools, parks and pathways. We have few of such characteristic variables in our dataset and so excess land can account for some of these extra land features. We create four excess land dummy variables ($EL_{tn,h}$) based on the quartiles of excess land size found in our data: group 1 is made up of observations where $EL_{tn} < 22254$ square feet; group 2 contains observations where $22254 \le EL_{tn} < 76424$ square feet; group 3 contains observation where $76424 \le EL_{tn} < 124269$ and group 4 contains observations where $EL_{tn} \ge 124269$. The quartile ranges were chosen to ensure that there were enough observations for each grouping of excess land. The excess land dummy variables are created as follows:

$$(20)\; EL_{tn,m} = 1 \text{ if observation n in period t is in excess land group m;}$$
$$= 0 \text{ otherwise;}$$

In addition to the normalizations imposed for model (18), we set $\sigma_1 = 1$. We then add these $EL_{tn,m}$ dummy variables to model (18) to get the following model:

$$(21)\; LV_{tn} = \alpha_t\left(\sum_{i=1}^{22} \theta_i FSA_{tn,i}\right)(1+\gamma(H_{tn}-1))\big(1+\omega(TU_{tn}-9)\big)\left(\sum_{j=1}^{4} \vartheta_j TH_{tn,j}\right)$$
$$\left(\sum_{m=1}^{4} \sigma_m EL_{tn,m}\right)\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}+\varepsilon_{tn};$$
$$t = 1,\ldots,55;\; n = 1,\ldots, N_t$$

where α_1=1; $\vartheta_1 = 1; and\ \sigma_1 = 1$.

The R-square value of model (21) is 0.6244 and the LL is –119956, which is a 2652 improvement over the LL of model (18). Even though excess land has a significant impact, the results are not what we originally expected. Due to the amenities and potential view that more excess land could offer a condominium unit, we would assume that more excess land would increase the price of land. However, as one can see in Table 2, the estimated σ_m decreases as the excess land gets bigger, which is a little counterintuitive to our assumptions. This is the same result found in Diewert and Shimizu (2017). The significant increase in LL with the inclusion of excess land signifies that the presence of extra land is an important factor in determining the sales price of a condominium apartment. However, the decrease in the estimated $\hat{\sigma}_m$ suggests there might be decreasing returns to scale for excess land.

4.7 Estimates for the Determinants of Land Value

Table 2 displays the estimated coefficients and T statistics for model (21), where $\hat{\theta}_i$ are the parameter estimates for the Forward Sortation Area dummy variables, $\hat{\alpha}_t$ are the parameter estimates for land value for a condominium unit in period t, $\hat{\gamma}$ is the estimate for change in land value of a unit due to an increase in the floor that unit is on, $\hat{\omega}$ is the estimate for the change in land value for a unit due to an extra unit in the building, $\hat{\vartheta}_j$ is the parameter estimate for the total building height dummy variables, $\hat{\sigma}_{\mathrm{m}}$ is the parameter estimate for the excess land dummy variables and lastly, $\hat{\rho}$ is the land imputation weight estimate.

Table 2: Estimated Coefficients for Model (21).

Coefficient	Estimate	T Stat	Coefficient	Estimate	T Stat	Coefficient	Estimate	T Stat
$\hat{\theta}_1$	82.577	6.60	$\hat{\alpha}_9$	0.9787	7.14	$\hat{\alpha}_{37}$	2.3345	9.20
$\hat{\theta}_2$	100.13	9.18	$\hat{\alpha}_{10}$	0.8401	8.99	$\hat{\alpha}_{38}$	2.3806	9.25
$\hat{\theta}_3$	105.53	9.01	$\hat{\alpha}_{11}$	0.9262	7.08	$\hat{\alpha}_{39}$	2.346	9.19
$\hat{\theta}_4$	64.885	8.79	$\hat{\alpha}_{12}$	1.0039	5.69	$\hat{\alpha}_{40}$	2.4029	9.08
$\hat{\theta}_5$	90.337	9.18	$\hat{\alpha}_{13}$	1.0390	7.48	$\hat{\alpha}_{41}$	2.5236	9.22
$\hat{\theta}_6$	106.44	9.15	$\hat{\alpha}_{14}$	0.9371	9.04	$\hat{\alpha}_{42}$	2.5298	9.25
$\hat{\theta}_7$	126.13	9.13	$\hat{\alpha}_{15}$	1.0564	8.02	$\hat{\alpha}_{43}$	2.5063	9.24
$\hat{\theta}_8$	187.18	9.26	$\hat{\alpha}_{16}$	1.0927	7.13	$\hat{\alpha}_{44}$	2.4544	9.19
$\hat{\theta}_9$	159.79	9.23	$\hat{\alpha}_{17}$	1.1296	7.89	$\hat{\alpha}_{45}$	2.6228	9.25
$\hat{\theta}_{10}$	214.59	9.14	$\hat{\alpha}_{18}$	1.1815	8.56	$\hat{\alpha}_{46}$	2.6739	9.26
$\hat{\theta}_{11}$	81.964	8.79	$\hat{\alpha}_{19}$	1.2383	7.84	$\hat{\alpha}_{47}$	2.7100	9.25
$\hat{\theta}_{12}$	100.28	9.15	$\hat{\alpha}_{20}$	1.2014	9.08	$\hat{\alpha}_{48}$	2.7479	9.23
$\hat{\theta}_{13}$	143.35	9.22	$\hat{\alpha}_{21}$	1.4036	9.13	$\hat{\alpha}_{49}$	2.8755	9.25
$\hat{\theta}_{14}$	252.58	9.16	$\hat{\alpha}_{22}$	1.5865	8.94	α_{50}	2.9970	9.28
$\hat{\theta}_{15}$	151.36	9.11	$\hat{\alpha}_{23}$	1.5818	8.78	$\hat{\alpha}_{51}$	2.8974	9.26
$\hat{\theta}_{16}$	92.482	9.19	$\hat{\alpha}_{24}$	1.5576	8.97	$\hat{\alpha}_{52}$	3.0339	9.25
$\hat{\theta}_{17}$	90.672	9.05	$\hat{\alpha}_{25}$	1.8164	9.09	$\hat{\alpha}_{53}$	2.8931	9.26
$\hat{\theta}_{18}$	106.18	9.19	$\hat{\alpha}_{26}$	2.0317	9.08	$\hat{\alpha}_{54}$	3.0543	9.28
$\hat{\theta}_{19}$	140.15	9.12	$\hat{\alpha}_{27}$	1.9698	9.05	$\hat{\alpha}_{55}$	3.0327	9.28
$\hat{\theta}_{20}$	121.55	8.12	$\hat{\alpha}_{28}$	2.0104	8.96	$\hat{\gamma}$	0.0075	12.68
$\hat{\theta}_{21}$	198.69	9.26	$\hat{\alpha}_{29}$	2.1058	9.06	$\hat{\omega}$	0.0114	40.94
$\hat{\theta}_{22}$	138.82	7.66	$\hat{\alpha}_{30}$	1.9988	9.24	$\hat{\vartheta}_2$	1.1266	109.75
$\hat{\alpha}_2$	1.0684	7.51	$\hat{\alpha}_{31}$	2.2484	9.16	$\hat{\vartheta}_3$	1.3746	79.60
$\hat{\alpha}_3$	1.1455	7.50	$\hat{\alpha}_{32}$	2.1976	9.04	$\hat{\vartheta}_4$	1.5901	72.79
$\hat{\alpha}_4$	0.9073	5.79	$\hat{\alpha}_{33}$	2.3165	9.18	$\hat{\sigma}_2$	0.5568	130.21
$\hat{\alpha}_5$	0.8874	8.49	$\hat{\alpha}_{34}$	2.3362	9.22	$\hat{\sigma}_3$	0.3081	83.64
$\hat{\alpha}_6$	1.061	7.65	$\hat{\alpha}_{35}$	2.2695	9.17	$\hat{\sigma}_4$	0.1959	63.86
$\hat{\alpha}_7$	0.9965	7.42	$\hat{\alpha}_{36}$	2.3274	9.14	$\hat{\rho}$	0.5064	61.99
$\hat{\alpha}_8$	0.9713	6.73						

With the final determinants of land all included in model (21), our parameters previously estimated have changed. The parameter change of note is the land imputation weight ($\hat{\rho}$) has grown from 0.2525 in model (14) to 0.5064 (t stat = 61.99) in model (21). This means that the proportional to size land imputation method has a little over equal share of the imputed land value at 50.64 percent. We also see that height of the unit ($\hat{\gamma}$) has a lesser impact on land value at 0.75 percent per additional floor. On the other hand, one extra unit in the building has a larger impact on land values than originally found in model (16) at 1.14 percent.

Certain trends continue in model (21). The parameter estimates for building height ($\hat{\vartheta}_j$) increase as the height increases. We also still observe that excess land coefficient estimates ($\hat{\sigma}_m$) decrease with the size of excess land. Though this variable significantly improves our model, as depicted by the 2652 increase in LL with the inclusion of this variable, the resulting estimates suggest that that extra land is not a priority for consumers looking to purchase condominium apartment units.

5. Quality Adjustment Variables for the Structure Component of Condo Value

Now that we have determined the main characteristics that contribute to land prices, we can use these variables in a Builder's Model that includes both land and structure components, a net geometric depreciation rate (δ) for the entire 55 quarters of our data and where condominium unit property price is once again the dependent variable of our model:

$$(22)\ P_{tn} = \alpha_t\left(\textstyle\sum_{i=1}^{22}\theta_i FSA_{tn,i}\right)\left(1+\gamma(H_{tn}-1)\right)\left(1+\omega(TU_{tn}-9)\right)\left(\textstyle\sum_{j=1}^{4}\vartheta_j TH_{tn,j}\right)$$
$$\left(\textstyle\sum_{m=1}^{4}\sigma_m EL_{tn,m}\right)\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}+(1.33)PS_t(1-\delta)^{A_{tn}}S_{tn}+\varepsilon_{tn};$$
$$t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

The R-square value of model (22) is 0.7006 and an LL of −119934. The estimate for the depreciation rate ($\hat{\delta}$) is 0.011911 (t stat = 11.20), which is much lower than expected. Therefore we need to consider structural quality adjustment factors, such as the number of bedrooms and number of bathrooms, in our Builder's Model.

5.1 Introducing the Number of Bedrooms as an Explanatory Variable

Even after accounting for unit size, the number of bedrooms in a condominium unit can impact its selling price. The condominium units found in our data have between 1 to 4 bedrooms. We group our observations based the number of bedrooms found in the unit: group 1 contains observations with one bedroom; group 2 observations have 2 bedrooms and group 3 observations have 3 or 4 bedrooms. Three and four bedrooms were grouped together due to the small sample size of units with four bedrooms. We introduce a bedroom dummy variable ($BD_{tn,k}$) into model (22) based on the following definition:

$$(23)\ BD_{tn,k} = 1 \text{ if observation n in period t is in bedroom group k;}$$
$$= 0 \text{ otherwise.}$$

The hedonic model accounting for the impact of bedrooms on selling price is as follows:

$$(24)\ P_{tn} = \alpha_t\left(\textstyle\sum_{i=1}^{22}\theta_i FSA_{tn,i}\right)\left(1+\gamma(H_{tn}-1)\right)\left(1+\omega(TU_{tn}-9)\right)$$
$$\left(\textstyle\sum_{j=1}^{4}\vartheta_j TH_{tn,j}\right)\left(\textstyle\sum_{m=1}^{4}\sigma_m EL_{tn,m}\right)\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}$$
$$+(1.33)PS_t(1-\delta)^{A_{tn}}\left(\textstyle\sum_{k=1}^{3}\tau_k BD_{tn,k}\right)S_{tn}+\varepsilon_{tn};$$
$$t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

We apply the following normalization parameters to model (24):

$$(25)\ \alpha_1 = 1;\ \vartheta_1 = 1;\ \sigma_1 = 1;\ \tau_1 = 1.$$

The R-square value for model (24) is 0.7799 and the LL is −118398, which is a 1536 increase in the LL from model (22). This huge increase in LL indicates that the number of bedrooms significantly impacts the selling price of a condominium unit. The coefficient estimates are increasing with the number of bedrooms in the unit signifying that more bedrooms will increase the sale price of a condominium apartment unit.

5.2 Introducing the Number of Bathrooms as an Explanatory Variable

Bathrooms are key features in any residential property and condominium apartments units are no exception. To test the exact impact that bathrooms have on the selling price of condominium units, we introduce number of bathroom dummy variables as structure quality adjustment variables. The condominium units found in our data have between 1 and 3 bathrooms. We group our observations based the number of bathrooms found in the condominium unit: group 1 observations have one bathroom; group 2 observations have 2 bathrooms and group 3 observations have 3 bathrooms. We introduce a bathroom dummy variable ($BT_{tn,c}$) into model (24) based on the following definition:

$$(26)\ BT_{tn,c} = 1 \text{ if observation n in period t is in bathroom group c;}$$
$$= 0 \text{ otherwise.}$$

The hedonic model including bathrooms is as follows:

$$(27)\ P_{tn} = \alpha_t\left(\sum_{i=1}^{22}\theta_i FSA_{tn,i}\right)\left(1+\gamma(H_{tn}-1)\right)\left(1+\omega(TU_{tn}-9)\right)$$
$$\left(\sum_{j=1}^{4}\vartheta_j TH_{tn,j}\right)\left(\sum_{m=1}^{4}\sigma_m EL_{tn,m}\right)\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}$$
$$+(1.33)PS_t(1-\delta)^{A_{tn}}\left(\sum_{k=1}^{3}\tau_k BD_{tn,k}\right)\left(\sum_{c=1}^{3}\varphi_c BT_{tn,c}\right)S_{tn}+\varepsilon_{tn};$$
$$t = 1,\ldots,55;\ n = 1,\ldots,N_t.$$

As in previous models, we need to apply normalization parameters to model (27), because all parameters cannot be identified. The normalization conditions are:

$$(28)\ \alpha_1=1;\ \vartheta_1 = 1;\ \sigma_1 = 1;\ \tau_1 = 1;\ \varphi_1 = 1.$$

Model (27) has an R-square value of 0.7871 and a LL of −118232, which is an increase in LL of 166 from model (25). The positive and increasing values of $\hat{\varphi}_2$ and $\hat{\varphi}_3$, at 2.07544 (t-stat = 73.18) and 2.251693 (t-stat = 49.82) respectively, suggest that the more bathrooms found in a condominium unit, the higher it will sell for.

5.3 Additional Structure Characteristics as Explanatory Variables

Other structural characteristic variables were tested with model (27) such as hardwood floors in the unit, natural gas in the unit, number of appliances in the unit, on-suite bathrooms, dens, balconies, whether or not the unit was a new build and condo fees. Only balconies and the presence of natural gas in the unit significantly improved the model.[18] Both of these variables are grouped into two categories: group 1 are for those observations that have the structural characteristic and group 2 are those observations that do not have the structural characteristic in question. To include balconies and natural gas into our Builder's Model we introduce a balcony ($BC_{tn,y}$) and a natural gas ($NG_{tn,z}$) dummy variables to model (27) defined as:

(29) $BC_{tn,y}$ = 1 if observation n in period t is in group y;
= 0 otherwise.
(30) $NG_{tn,z}$ = 1 if observation n in period t is in group z;
= 0 otherwise.

Both variables were added individually and combined to model (27). Adding balconies only to model (27) resulted in an R-square value of 0.7991 and a LL −117944, which is an improvement of 288 over the LL of model (27). Adding natural gas to model (27) resulted in an R-square value of 0.7978 and a LL of −117976, an improvement of 256. Since both structural characteristics individually improved the model, we introduced balcony and natural gas dummy variables to model (27) as shown in model (31):

$$(31)\ P_{tn} = \alpha_t\left(\textstyle\sum_{i=1}^{22}\theta_i FSA_{tn,i}\right)\left(1+\gamma(H_{tn}-1)\right)\left(1+\omega(TU_{tn}-9)\right)$$
$$\left(\textstyle\sum_{j=1}^{4}\vartheta_j TH_{tn,j}\right)\left(\textstyle\sum_{m=1}^{4}\sigma_m EL_{tn,m}\right)\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}+(1.33)PS_t$$
$$(1-\delta)^{A_{tn}}\ \left(\textstyle\sum_{k=1}^{3}\tau_k BD_{tn,k}\right)\left(\textstyle\sum_{c=1}^{3}\varphi_c BT_{tn,c}\right)\left(\textstyle\sum_{y=1}^{2}\pi_y BC_{tn,y}\right)\left(\textstyle\sum_{z=1}^{2}\eta_z NG_{tn,z}\right)S_{tn}+\varepsilon_{tn}$$
t = 1,…,55; n = 1,…, N_t.

As in previous models, every parameter cannot be identified, so we apply the following normalization restrictions to model (31):

(32) α_1=1; $\vartheta_1 = 1$; $\sigma_1 = 1$; $\tau_1 = 1$; $\varphi_1 = 1$;.$\pi_1 = 1$; $\eta_1 = 1$;

Model (31) has an R-square value of 0.8066 and a LL −117753, which is an improvement in LL over model (27) by 479. The coefficient estimates for the balcony and natural gas dummy variables are 1.246339 (t-stat = 135.41) and 1.22537 (t-stat = 130.76), respectively. These values are consistent with our expectations. Balconies can increase the price of a condominium unit because it provides additional living space as well as an ideal "observation post" for enjoying the view. Furthermore, natural gas is considered to be an important and preferred means of heating homes in Ottawa and so would logically increase the price of a condominium unit.

[18] Significantly improve the model in this case refers to improving the model by at least 100 LL points. Though this is not a critical value when conducting a Log Likelihood test, this threshold was chosen because adding more variables to our hedonic non-linear model made it more difficult for the model to converge. Therefore, the threshold of 100 was chosen to balance convergence with variable choice.

Table 3 lists the estimated coefficients for model (31). The coefficient estimates that we have added from table 2 include the estimate for net geometric depreciation rate ($\hat{\delta}$), the estimates for the bedroom dummy variables ($\hat{\tau}_k$), bathroom dummy variables ($\hat{\varphi}_c$), balcony dummy variable ($\hat{\pi}_2$) and the natural gas dummy variable ($\hat{\eta}_2$).

Table 3: Estimated Coefficients for Model 31

Coefficient	Estimate	T Stat	Coefficient	Estimate	T Stat	Coefficient	Estimate	T Stat
$\hat{\theta}_1$	16.258	5.28	$\hat{\alpha}_{11}$	0.86955	4.36	$\hat{\alpha}_{42}$	3.58645	6.49
$\hat{\theta}_2$	33.765	6.41	$\hat{\alpha}_{12}$	0.93726	4.05	$\hat{\alpha}_{43}$	3.62191	6.49
$\hat{\theta}_3$	35.351	6.28	$\hat{\alpha}_{13}$	0.97869	4.93	$\hat{\alpha}_{44}$	3.45678	6.46
$\hat{\theta}_4$	19.918	6.26	$\hat{\alpha}_{14}$	0.92696	6.10	$\hat{\alpha}_{45}$	3.76463	6.49
$\hat{\theta}_5$	30.792	6.39	$\hat{\alpha}_{15}$	1.09312	5.40	$\hat{\alpha}_{46}$	3.84937	6.50
$\hat{\theta}_6$	42.539	6.42	$\hat{\alpha}_{16}$	1.11226	4.90	$\hat{\alpha}_{47}$	3.89612	6.49
$\hat{\theta}_7$	49.403	6.42	$\hat{\alpha}_{17}$	1.21807	5.89	$\hat{\alpha}_{48}$	3.99630	6.48
$\hat{\theta}_8$	70.314	6.45	$\hat{\alpha}_{18}$	1.27303	6.08	$\hat{\alpha}_{49}$	4.10612	6.49
$\hat{\theta}_9$	62.074	6.45	$\hat{\alpha}_{19}$	1.42360	5.63	α_{50}	4.27321	6.50
$\hat{\theta}_{10}$	72.632	6.41	$\hat{\alpha}_{20}$	1.41738	6.33	$\hat{\alpha}_{51}$	4.22797	6.49
$\hat{\theta}_{11}$	25.173	6.21	$\hat{\alpha}_{21}$	1.86172	6.39	$\hat{\alpha}_{52}$	4.32355	6.49
$\hat{\theta}_{12}$	33.736	6.36	$\hat{\alpha}_{22}$	2.07061	6.22	$\hat{\alpha}_{53}$	4.22331	6.50
$\hat{\theta}_{13}$	55.922	6.44	$\hat{\alpha}_{23}$	2.10633	6.24	$\hat{\alpha}_{54}$	4.44432	6.50
$\hat{\theta}_{14}$	86.726	6.37	$\hat{\alpha}_{24}$	2.09352	6.27	$\hat{\alpha}_{55}$	4.57094	6.50
$\hat{\theta}_{15}$	53.838	6.40	$\hat{\alpha}_{25}$	2.45374	6.42	$\hat{\gamma}$	0.00955	14.01
$\hat{\theta}_{16}$	32.867	6.41	$\hat{\alpha}_{26}$	2.84905	6.41	$\hat{\omega}$	0.01372	37.76
$\hat{\theta}_{17}$	31.739	6.34	$\hat{\alpha}_{27}$	2.84515	6.41	$\hat{\vartheta}_2$	1.086788	95.27
$\hat{\theta}_{18}$	39.399	6.41	$\hat{\alpha}_{28}$	2.83455	6.31	$\hat{\vartheta}_3$	1.363746	73.61
$\hat{\theta}_{19}$	48.077	6.37	$\hat{\alpha}_{29}$	3.02023	6.40	$\hat{\vartheta}_4$	1.491565	65.15
$\hat{\theta}_{20}$	39.617	6.06	$\hat{\alpha}_{30}$	3.00737	6.48	$\hat{\sigma}_2$	0.575134	121.74
$\hat{\theta}_{21}$	76.387	6.46	$\hat{\alpha}_{31}$	3.32204	6.45	$\hat{\sigma}_3$	0.339337	72.51
$\hat{\theta}_{22}$	47.586	6.19	$\hat{\alpha}_{32}$	3.29699	6.41	$\hat{\sigma}_4$	0.216779	58.50
$\hat{\alpha}_2$	0.9742	4.74	$\hat{\alpha}_{33}$	3.46498	6.47	$\hat{\rho}$	0.099938	6.010
$\hat{\alpha}_3$	1.0028	4.95	$\hat{\alpha}_{34}$	3.45011	6.49	$\hat{\delta}$	0.024139	46.20
$\hat{\alpha}_4$	0.75397	3.78	$\hat{\alpha}_{35}$	3.30218	6.46	$\hat{\tau}_2$	1.572196	79.79
$\hat{\alpha}_5$	0.84490	5.13	$\hat{\alpha}_{36}$	3.38356	6.44	$\hat{\tau}_3$	1.552917	55.79
$\hat{\alpha}_6$	0.97242	5.07	$\hat{\alpha}_{37}$	3.39190	6.48	$\hat{\varphi}_2$	1.360253	91.10
$\hat{\alpha}_7$	0.90406	5.14	$\hat{\alpha}_{38}$	3.50522	6.49	$\hat{\varphi}_3$	1.507086	30.90
$\hat{\alpha}_8$	0.88059	4.55	$\hat{\alpha}_{39}$	3.37858	6.47	$\hat{\pi}_2$	1.246309	135.41
$\hat{\alpha}_9$	0.87329	4.75	$\hat{\alpha}_{40}$	3.45818	6.42	$\hat{\eta}_2$	1.225306	130.76
$\hat{\alpha}_{10}$	0.86223	6.12	$\hat{\alpha}_{41}$	3.72105	6.48			

6. Commercial Space in the Builder's Model

Buildings of structural mix (with both residential and nonresidential) are becoming more and more prevalent in Canadian condominium development, especially on busy streets or close to a commercial center. We test whether the presence of commercial space in the building impacts the selling price of a condominium unit by introducing a commercial space dummy variable ($CM_{tn,p}$) to model (31). The observations are then split into group 1 that contains observations in buildings without commercial space and group 2 whose observations are in buildings that contain commercial space. The dummy variable $CM_{tn,p}$ is defined as:

(33) $CM_{tn,z}$ = 1 if observation n in period t is in group z;

= 0 otherwise.

Commercial space can impact both the land and structure components. It can help pinpoint whether a building is in a residential or more mixed neighbourhood, therefore affecting the value of land. The structural component of the Builder's Model may also be impacted by the presence of commercial space because the rent from these establishments could contribute to the upkeep of the condominium building itself. This can in turn, effect the communal space aspect of the structure component.

With commercial space potentially impacting both the land and structure components, multiple tests were conducted to determine the specific effect commercial space has on the sale price of a condominium unit. We tested the impact of adding $CM_{tn,z}$ on the land component and found that it had no impact on the LL. This might be because we have already identified enough factors that impact the land component and so commercial space does not have additional explanatory power. We tested the impact of adding $CM_{tn,z}$ on the structure component and found that it improved the LL by 9. Lastly we tested the impact of adding a commercial space dummy variable on both the land and structure components. Though adding two dummy variables (one to each component) improved the LL by 30, possible concerns of multicollinearity impacting the coefficient estimates made this option not viable. After this analysis we determined that commercial space should be added to the structure component only.

The hedonic model containing commercial space is as follows:

$$(34)\ P_{tn} = \alpha_t(\textstyle\sum_{i=1}^{22} \theta_i FSA_{tn,i})(1+\gamma(H_{tn}-1))(1+\omega(TU_{tn}-9))$$

$$(\textstyle\sum_{j=1}^{4} \vartheta_j TH_{tn,j})(\sum_{m=1}^{4} \sigma_m EL_{tn,m})\left[\rho\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\rho)\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn}+(1.33)PS_t(1-\delta)^{A_{tn}}(\sum_{k=1}^{3}\tau_k BD_{tn,k})(\sum_{c=1}^{3}\varphi_c BT_{tn,c})$$

$$(\textstyle\sum_{y=1}^{2}\pi_y BC_{tn,y})(\sum_{z=1}^{2}\eta_z NG_{tn,z})(\sum_{p=1}^{2}\epsilon_z CM_{tn,p})S_{tn}+\varepsilon_{tn};$$

t = 1,…,55; n = 1,…, N_t

We apply the following normalizing conditions:

(35) α_1=1; $\vartheta_1 = 1$; $\sigma_1 = 1$; $\tau_1 = 1$; $\varphi_1 = 1$; $\epsilon_1 = 1$.

By normalizing ϵ_1=1 we are assuming that the presence of commercial space will affect the sale price of condominium units.

Model (34) has an R-square value of 0.8071 and an LL of -117741. Although the commercial space dummy variables does not impact the overall model as much as other variables in this study, only improving the LL by 9, the value of $\hat{\epsilon}_2$ is statistically significant with a value of 1.0905 (t-stat = 98.58). A possible explanation for this low impact to the model is due to the small sample size of observations that are in buildings with commercial space at 7 percent. In light of this result, the heterogeneity of commercial properties comes into question. A dummy variable alone may not be able to capture their complexity and the specific relationship commercial spaces have with sale prices of condominium apartments. To improve our confidence in the relationship of commercial space and sales prices of condominium apartment units, more detailed variables on the nature of commercial space need to be obtained and analyzed in the Builder's Model.

Table 4 lists the estimated coefficients for model (34). The coefficient estimates that we have added from table 3 is the estimate for the commercial space dummy variables ($\hat{\epsilon}_2$).

Table 4: Estimated Coefficients for Model 34

Coefficient	Estimate	T Stat	Coefficient	Estimate	T Stat	Coefficient	Estimate	T Stat
$\hat{\theta}_1$	16.5407	5.31	$\hat{\alpha}_{11}$	0.86765	4.32	$\hat{\alpha}_{42}$	3.57289	6.51
$\hat{\theta}_2$	34.5989	6.43	$\hat{\alpha}_{12}$	0.94102	4.09	$\hat{\alpha}_{43}$	3.60157	6.50
$\hat{\theta}_3$	35.9295	6.31	$\hat{\alpha}_{13}$	0.98144	4.93	$\hat{\alpha}_{44}$	3.43389	6.48
$\hat{\theta}_4$	20.2695	6.28	$\hat{\alpha}_{14}$	0.93297	6.13	$\hat{\alpha}_{45}$	3.74870	6.51
$\hat{\theta}_5$	31.3068	6.41	$\hat{\alpha}_{15}$	1.09200	5.41	$\hat{\alpha}_{46}$	3.82822	6.51
$\hat{\theta}_6$	43.1959	6.44	$\hat{\alpha}_{16}$	1.11285	4.94	$\hat{\alpha}_{47}$	3.87449	6.51
$\hat{\theta}_7$	50.3563	6.43	$\hat{\alpha}_{17}$	1.21437	5.89	$\hat{\alpha}_{48}$	3.97929	6.50
$\hat{\theta}_8$	70.0657	6.47	$\hat{\alpha}_{18}$	1.27561	6.09	$\hat{\alpha}_{49}$	4.09038	6.51
$\hat{\theta}_9$	63.7759	6.47	$\hat{\alpha}_{19}$	1.41963	5.61	α_{50}	4.24081	6.52
$\hat{\theta}_{10}$	74.5892	6.43	$\hat{\alpha}_{20}$	1.41498	6.35	$\hat{\alpha}_{51}$	4.20546	6.51
$\hat{\theta}_{11}$	25.7993	6.23	$\hat{\alpha}_{21}$	1.85845	6.40	$\hat{\alpha}_{52}$	4.30984	6.51
$\hat{\theta}_{12}$	34.1538	6.38	$\hat{\alpha}_{22}$	2.06064	6.24	$\hat{\alpha}_{53}$	4.19862	6.51
$\hat{\theta}_{13}$	55.8896	6.45	$\hat{\alpha}_{23}$	2.10232	6.24	$\hat{\alpha}_{54}$	4.41157	6.52
$\hat{\theta}_{14}$	87.3253	6.40	$\hat{\alpha}_{24}$	2.08533	6.28	$\hat{\alpha}_{55}$	4.54487	6.52
$\hat{\theta}_{15}$	54.1484	6.41	$\hat{\alpha}_{25}$	2.43923	6.44	$\hat{\gamma}$	0.00956	14.01
$\hat{\theta}_{16}$	33.4139	6.43	$\hat{\alpha}_{26}$	2.83768	6.42	$\hat{\omega}$	0.01343	37.49
$\hat{\theta}_{17}$	32.3098	6.35	$\hat{\alpha}_{27}$	2.83073	6.43	$\hat{\vartheta}_2$	1.09264	94.59
$\hat{\theta}_{18}$	39.7728	6.43	$\hat{\alpha}_{28}$	2.82598	6.33	$\hat{\vartheta}_3$	1.36362	73.88
$\hat{\theta}_{19}$	48.8472	6.39	$\hat{\alpha}_{29}$	3.00444	6.41	$\hat{\vartheta}_4$	1.48300	65.00
$\hat{\theta}_{20}$	40.2860	6.09	$\hat{\alpha}_{30}$	2.99844	6.49	$\hat{\sigma}_2$	0.58198	119.84
$\hat{\theta}_{21}$	77.5894	6.48	$\hat{\alpha}_{31}$	3.31125	6.47	$\hat{\sigma}_3$	0.34388	72.80
$\hat{\theta}_{22}$	48.3734	6.20	$\hat{\alpha}_{32}$	3.28068	6.43	$\hat{\sigma}_4$	0.21976	58.85
$\hat{\alpha}_2$	0.98005	4.78	$\hat{\alpha}_{33}$	3.45632	6.49	$\hat{\rho}$	0.11181	6.86
$\hat{\alpha}_3$	1.01181	4.99	$\hat{\alpha}_{34}$	3.43630	6.51	$\hat{\delta}$	0.02380	45.35
$\hat{\alpha}_4$	0.75482	3.81	$\hat{\alpha}_{35}$	3.29321	6.48	$\hat{\tau}_2$	1.54271	79.66
$\hat{\alpha}_5$	0.84710	5.16	$\hat{\alpha}_{36}$	3.37163	6.46	$\hat{\tau}_3$	1.52337	55.87

$\hat{\alpha}_6$	0.97755	5.10	$\hat{\alpha}_{37}$	3.37552	6.49	$\hat{\varphi}_2$	1.36017	91.26
$\hat{\alpha}_7$	0.90986	5.17	$\hat{\alpha}_{38}$	3.49603	6.50	$\hat{\varphi}_3$	1.51513	31.52
$\hat{\alpha}_8$	0.88609	4.59	$\hat{\alpha}_{39}$	3.36760	6.49	$\hat{\pi}_2$	1.25073	137.77
$\hat{\alpha}_9$	0.87100	4.80	$\hat{\alpha}_{40}$	3.44073	6.44	$\hat{\eta}_2$	1.22725	130.67
$\hat{\alpha}_{10}$	0.86584	6.14	$\hat{\alpha}_{41}$	3.70367	6.49	$\hat{\varepsilon}_2$	1.09055	98.58

With our final model (34) that incorporates both land and structure components we see a drastic change in the land imputation weight ($\hat{\rho}$). When it was first estimated in model (14), $\hat{\rho}$ was 0.2525, meaning that equal distribution of land amongst the building units dominated the imputed land value per unit. In model (21) $\hat{\rho}$ was 0.5064 suggesting that proportional to unit size allocation shares the imputed land value. Now, in model (34), the weight has swung back in favour of equal distribution of land where $\hat{\rho}$ is 0.111809 (t stat = 6.86), implying that, in our final Builder's Model, 89 percent of the imputed land value is estimated by the equal distribution of land amongst condominium units in the same building.

From model (34) we also get an estimate for the net geometric depreciation rate ($\hat{\delta}$) for the entire Q1 1996 to Q3 2009 period. The estimated depreciation rates do change between models. It dropped slightly from 0.024139 in model (31) to 0.023803 (t stat = 45.35) in our final model. This depreciation rate estimate of 2.4 percent is slightly higher than the estimate used by Statistics Canada's Canadian System of Macroeconomic Accounts for deflating residential construction activity and for conducting productivity analysis.

However, it should be kept in mind that our estimated geometric depreciation rate of 2.4 percent per year may be subject to some downward bias for two reasons:

- Capital expenditures on maintaining and renovating the structure are not taken into account in our model so that we are estimating a net of capital expenditures depreciation rate rather than a gross depreciation rate.[19]

- Our model does not take into account the premature demolition of condominium buildings; i.e., we only observe sales of surviving structures. This could be a major source of bias.[20]

7. The Resulting Price Indexes

7.1 Constructing Land, Structure and Total Condominium Sales Price Indexes

[19] This is not a major problem since maintenance, repair and renovation expenditures are a separate stratum in the Canadian CPI and so these expenditures are taken into account in the System of National Accounts. It would probably be appropriate to capitalize some of these expenditures and depreciate them separately but this alternative treatment would not materially affect the accounts.

[20] This problem can be addressed if information on the age of buildings when they are demolished is available; see Diewert and Shimizu (2016) for the details on how to treat this problem. For commercial structures in Tokyo, they found that this demolition depreciation added an additional 2% per year to their estimated net depreciation rate that they obtained using the builder's model.

Now that we have estimates for land prices we can use them to construct land, structure and total property price indexes. Since α_t estimates represent the percentage change in land price due to the change in time from period 1 to period *t*, we can uses these estimates to create land price indexes (IL_t) as:

$$(36)\ IL_t = \alpha_t \times 100; \qquad t = 1,\ldots,55.$$

The structure price index is the change in price of structure (PS_t). Since we indexed the price per square foot of structure with the ABCPI to get an approximate value of structure price for each period t, our structure price index (IS_t) is implicitly estimated by the ABCPI based to Q1 1996=100:

$$(37)\ IS_t = \frac{PS_t}{PS_1} \times 100 = ABCPI_t; \qquad t = 1,\ldots,55.$$

We start with calculating our total property price index using a fixed base Laspeyres price index formula because this is the formula that will be used in the NCAPI of Statistics Canada. The weights we use are the value shares of land and structures. First we calculate the value of land (LV_{tn}) and structures (SV_{tn}) for t = 1,…,55 and n = 1,…,N_t:

$$(38)\ LV_{tn} = \hat{\alpha}_t\left(\textstyle\sum_{i=1}^{22}\hat{\theta}_i FSA_{tn,i}\right)(1+\hat{\gamma}(H_{tn}-1))\big(1+\hat{\omega}(TU_{tn}-9)\big)$$
$$\left(\textstyle\sum_{j=1}^{4}\hat{\vartheta}_j TH_{tn,j}\right)\left(\textstyle\sum_{m=1}^{4}\hat{\sigma}_m EL_{tn,m}\right)\left[\hat{\rho}\left(\frac{S_{tn}}{TS_{tn}}\right)+(1-\hat{\rho})\left(\frac{1}{TU_{tn}}\right)\right]TL_{tn};$$

$$(39)\ SV_{tn} = (1.33)PS_t(1-\hat{\delta})^{A_{tn}}\ \left(\textstyle\sum_{k=1}^{4}\hat{\tau}_k BD_{tn,k}\right)\left(\textstyle\sum_{c=1}^{3}\hat{\varphi}_c BT_{tn,c}\right)S_{tn}.$$

In order to get total land and structure values for sales in period t, we sum the predicted values from (38) and (39) to get:

$$(40)\ LV_t = \textstyle\sum_{n=1}^{N_t} LV_{tn}; \qquad t=1,\ldots,55.$$

$$(41)\ SV_t = \textstyle\sum_{n=1}^{N_t} SV_{tn} \qquad t=1,\ldots,55.$$

We define the total property value of condominium sales for period t, V_t, as the sum of the predicted values LV_t and SV_t:

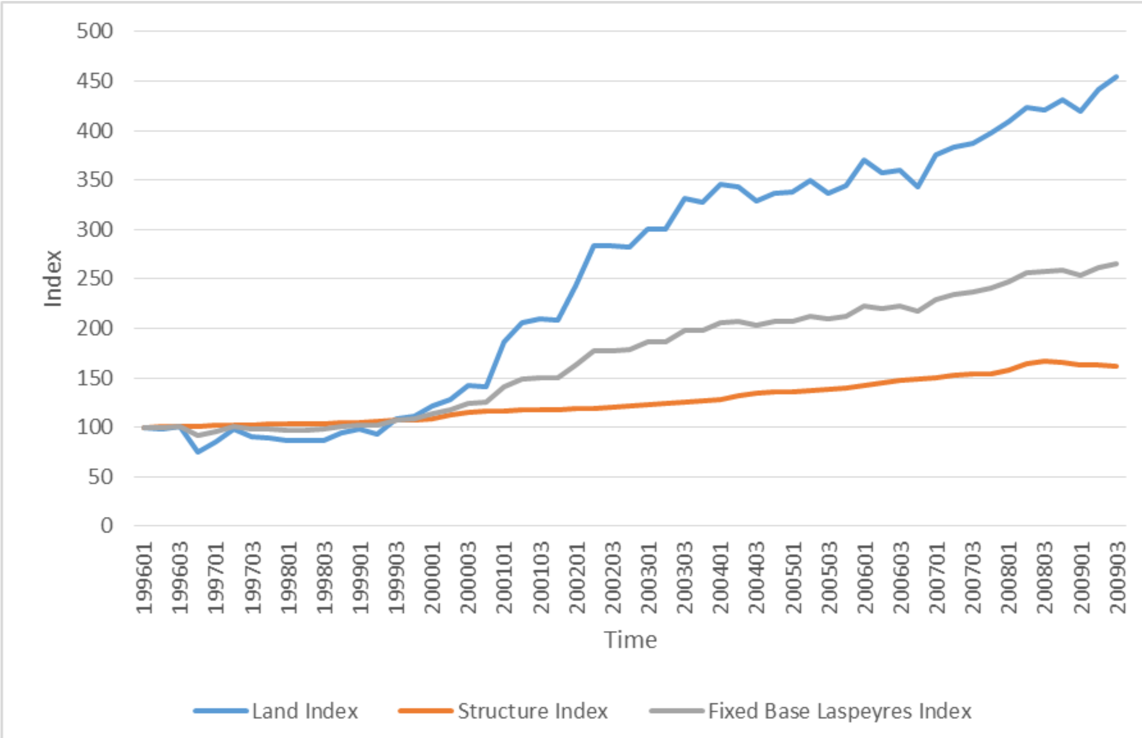
$$(42)\ V_t = LV_t + SV_t; \qquad t = 1,\ldots,55.$$

The fixed base Laspeyres index formula for period t can be written as follows:

$$(43)\ I_t = IL_t\left(\frac{LV_1}{V_1}\right) + IS_t\left(\frac{SV_1}{V_1}\right); \qquad t = 1,\ldots,55.$$

The land, structure and fixed base Laspeyres (or total property) price indexes for sales of condominium units are illustrated in Figure 1.

Figure 1: Land, Structure and Fixed Base Laspeyres Price Indexes

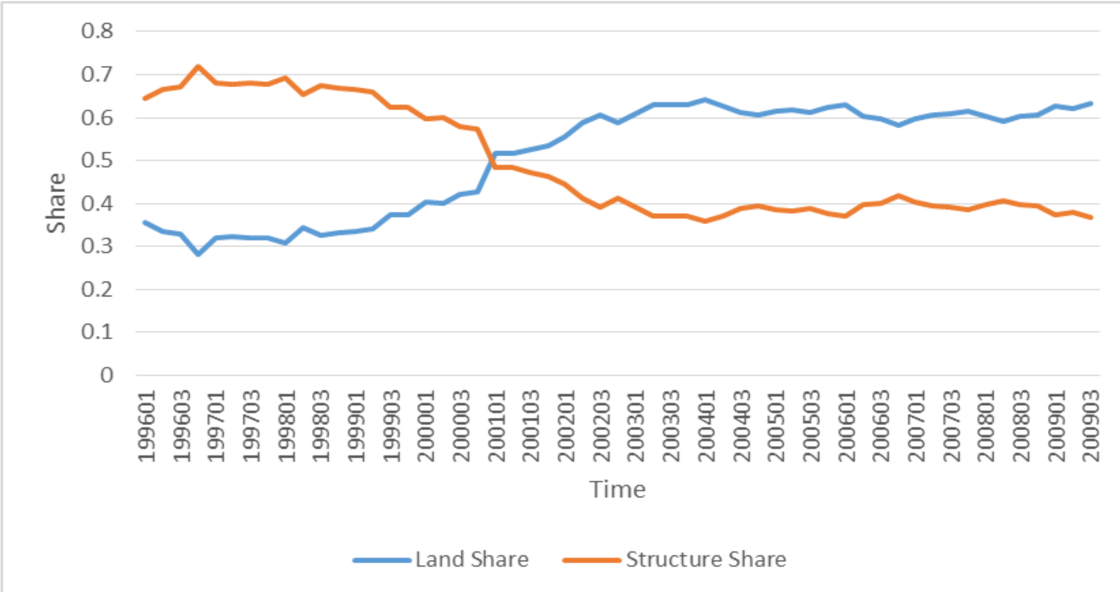


We can see that land prices have increased 4.56 fold between Q1 1996 and Q3 2009. From discussions with potential users of our land index including the Consumer Price Index, the Canadian System of Macroeconomic Accounts and other residential property price indexes produced at Statistics Canada these land results are deemed to be reasonable. This is the only condominium land price index of its kind in Canada, therefore, we cannot compare our results to any other land price index to legitimize these results. However, other condominium price indexes do exist that model the total property price of a unit. In section 7.3, we will compare our total property price index to other indexes that use different methods of calculation, but using the same data as is used in this section.

7.2 Land and Structure Value Shares and Alternative Total Property Price Indexes

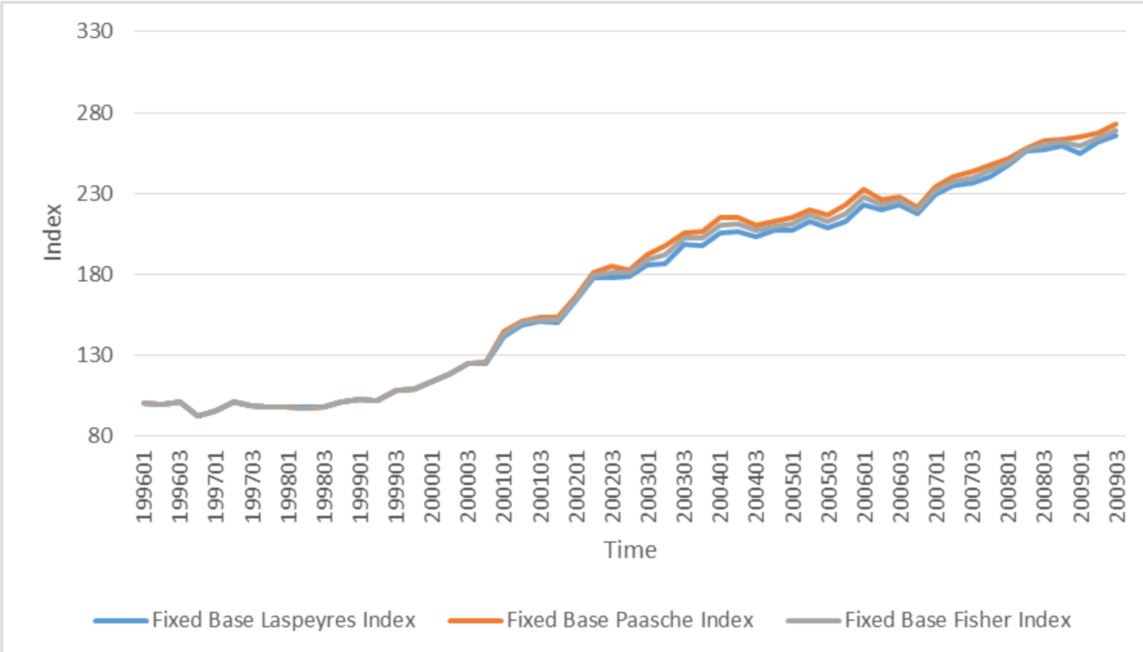
Before we go any further in comparing total property price indexes, we have to decide which formula we will use to calculate our hedonically imputed index. The fixed base Laspeyres index shown in figure 1 is misleading because over the 1996 to 2009 period, the land and structure value shares of condo sales change dramatically, as shown in Figure 2:

Figure 2: Land and Structure Value Shares over Time



We can see that at our base period, Q1 1996, the structure component has a 65 percent share of the total value. However, as of Q1 2001, land takes over the majority share. This means that if we were to calculate a fixed base Paasche or a Fisher Index, the total property price index will look quite different. Figure 3 illustrates the difference between the fixed base Laspeyres, Paasche and Fisher indexes calculated from our land and structure price and value estimates from model (34).

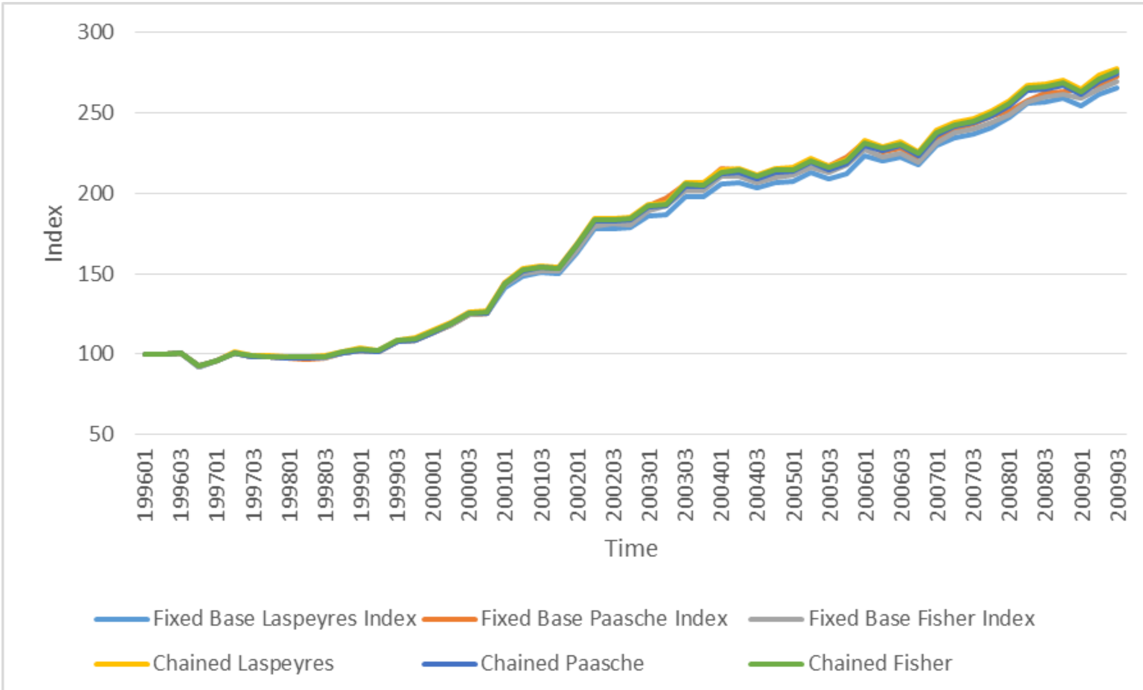
Figure 3: Fixed Base Laspeyres, Paasche and Fisher Total Property Price Indexes



Note that the fixed base Paasche and Fisher indexes are higher than the Laspeyres. This is counter intuitive to most cases where we see the Paasche and Fisher indexes are lower than the Laspeyres because of change in consumption patterns and weighting due to preferences towards cheaper goods. However, starting in 2001, the land value share is dominant, meaning that the land value, which exhibits much more growth than the structure value, gets a higher weight.

Due to this phenomenon in weighting patterns a chained Laspeyres, Paasche and Fisher would display different results than their fixed counterparts. Figure 4 illustrates the differences between the chained and fixed base Laspeyres, Paasche and Fisher indexes.

Figure 4: Chained versus Fixed Base Laspeyres, Paasche and Fisher Total Property Price Indexes



With weights in the chained Laspeyres being more timely, they are more representative for each comparison period, reflecting the changes in the land share over time. Then, we see, using the chained methodology, that the Laspeyres index is higher than the Paasche, which follows traditional index theory. Also, as predicted, the spread between the Laspeyres and Paasche is dramatically reduced, which is clearly shown in Figure 4. Thus, all the chained indexes more closely approximate each other than the fixed base indexes. However, we also need to point out, with some bounces in the land prices, the chained indexes could suffer a certain degree of chain drift.

7.3 Comparison with Other Total Property Indexes

As mentioned in section 7.1, we do not have any other official land price indexes that can be compared to our land price index. However, we can compare our fixed base and chained Fisher indexes from section 7.2 to total property indexes calculated by other methods, such as the hedonic method and the stratification method, using the same data.

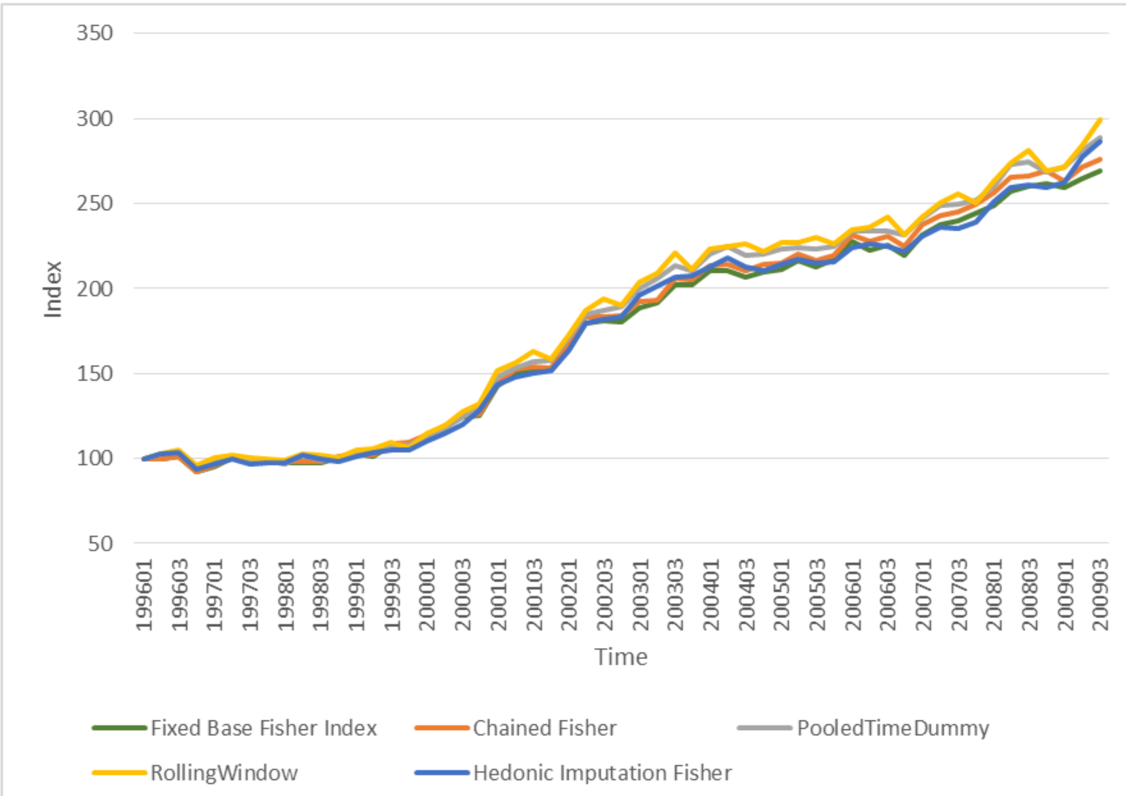
First we will compare our Fisher indexes to three hedonic indexes calculated by the following methods: the Pooled Time Dummy hedonic method, the Rolling Window Time Dummy hedonic method and the Hedonic Imputation approach.[21] Hedonic methods have

[21] For more details on hedonic methods to construct price indexes refer to *Handbook on Residential Property Prices Indices* by Eurostat (2013), page 51-55.

become a preferred method of constructing constant quality housing price indexes even though the data requirements are extensive and often expensive to obtain.[22]

The characteristics that we included in these hedonic models are the same as were used in the Builder's Model (34).[23] These characteristics reflect both quantitative and qualitative housing features that determine condominium prices. Figure 5 compares the different condo price indexes for Ottawa using the three alternative hedonic regression methods.

Figure 5: Our Total Property Fisher Indexes versus Alternative Hedonic Regression Base Indexes



The Pooled Time Dummy method[24] runs a single regression on both characteristics variables and time dummy variables. It is very simple to apply in practice. The price index can be obtained directly from the estimated regression equation. The dependant variable is the logarithm of the unit's selling price and the overall price index is obtained by taking the exponential of the time dummy coefficients.

[22] For more details on hedonic methods to construct price indexes refer to *Handbook on Residential Property Prices Indices* by Eurostat (2013) page 50-55.
[23] Commercial space is not included in the Rolling Window Time Dummy Variable and Hedonic Imputation methods. Since the sample size of commercial space is very small, we would have too many zero values in many time windows.
[24] See pages 51-52 on "Time dummy variable method" in the *Handbook on Residential Property Prices Indices* by Eurostat (2013).

A practical problem associated with the hedonic regression model is the reassessment of the parameters with more recent data available. The Rolling Window approach is a simple solution to this problem. The Rolling Window Time Dummy method[25] is similar to the Pooled Time Dummy method, with the difference that the Rolling Window Method runs a sequence of hedonic regressions for a fixed-window length, such as a year. This length of the window is determined when the model yields relatively robust estimates. We applied Rolling Window procedure with a length of 5 quarters. The advantage of this method over the Pooled Time Dummy method is that the Rolling Window method allows for gradual changes in consumer tastes or preferences over time.

In order to implement the Hedonic Imputation approach,[26] a separate hedonic regression is run using the data for each period.[27] In general, a set of fixed quantity of characteristics of a standard or matched model are chosen to impute the missing prices using the estimated coefficients from the hedonic regression model. Based on which time period the fixed characteristics belong to, the Laspeyres, Paasche and Fisher imputation indexes can be estimated. The chained Fisher index calculated by using the Hedonic Imputation method *(*labeled Hedonic Imputation Fisher*)* is shown in Figure 5. Comparing with the other two hedonic price indexes, we found that all three alternative hedonic indexes generally approximate each other fairly closely.

From figure 5 we can see that the fixed base Fisher index, calculated from our non-linear model (34) follows the same long term trend as the Pooled Time Dummy, Rolling Window and Hedonic Imputation models. The fixed base Fisher and the chained Fisher indexes exhibit a 1.77 and 1.84 fold increase, respectively, between Q1 1996 and Q3 2009. This is slightly less than the total growth of the three alternative hedonic models.[28] However, all four indexes do have similar quarterly movements with an average growth rate of 2 percent over the 14 year period.

The concern with using linear regression models such as the Pooled Time Dummy, Rolling Window and Hedonic Imputation models is that there can be multicollinearity between the variables causing misleading coefficient estimates, which are then used to calculate the indexes themselves.[29] Therefore, we want to compare our Fisher Index to three indexes using the following stratification methods: Mean Index and Median Index stratified by postal code and weighted by the sales in each quarter and the Median Index

---

[25] See page 94-95 on "Rolling window hedonic regressions" in the *Handbook on Residential Property Prices Indices* by Eurostat (2013).

[26] For more details on Characteristics prices method refer to *Handbook on Residential Property Prices Indices* by Eurostat (2013) page 53-55.

[27] See pages 62-64 in the *Handbook on Residential Property Prices Indices* by Eurostat (2013).

[28] Specifically, the fixed base Fisher index has a 177% increase, the chained Fisher index has a 184% increase, the Pooled Time Dummy Index has a 188.9% increase, the Rolling Window Index has a 199.5% increase and the Hedonic Imputation Method has a 186.7% increase. -

[29] See Diewert and Shimizu (2017). Of particular concern is the sign of the coefficient on the age variable in time dummy regression models that use the logarithm of the selling price as the dependent variable. If the sign of the age coefficient is positive instead of negative, then it is very likely that the overall price index generated by exponentiating the time dummy variables will have a downward bias.

using stratification method proposed by Prasad and Richards (2006).[30] These methods revolve around compiling a condominium price index using the mean or median price of each period. This methodology is simple and requires little information. However, this type of index has many disadvantages, such as it cannot fully account for quality change and the compositional change of the housing stock will affect the price indexes. Appropriate stratification can reduce bias caused by this compositional change.

Location is one of the natural stratification variables to use. We test the impact of using different fineness of classification scheme, such as district,[31] Ward and FSA, as the stratification indicator. Since condominium units are sold more frequently in certain areas and less frequently in the others, the alternative indexes exhibit different price change patterns in different locations, which indicates that keeping the homogeneity of each cell is very important for the accuracy of the index. However, when the stratification scheme is too fine, empty cells will occur for some periods. If the classification scheme is very coarse, we cannot sufficiently control the homogeneity of the cell. The stratified price series reported in the paper use FSA as the stratification variable. Figure 6 illustrates the comparison between our Fisher indexes and the Mean Index (Mean_FSA), Median Index (Med_FSA) and Median Index proposed by Prasad and Richards (Med(P&R)).

Figure 6: Fisher Indexes versus Stratified Indexes

[30] See Prasad and Richards (2006). They use essentially a two-layer stratification method. The first layer stratification is based on a minor geographic group and the second layer is based on the long term average price level of dwellings in those small regions. We slightly change this method by grouping the small regions based on the age of the condo units. The resulting index series approximates the indexes generated by our hedonic modeling.

[31] The district is the finest neighbourhood variable used in the data.

The period to period movements vary between the five indexes. However, the long term trends are similar across all five indexes. It can be seen that there are more fluctuations in the Median index stratified by FSA, especially after the first quarter of 2006, than those in the other four price series. This might be a result of using the sales as weight to aggregate index cross different FSA.[32]

8. Conclusion

The most important conclusion from this study is that we now have a method to create land price indexes for condominium units.[33] Condominium land and building characteristics data are difficult to find in Canada and attaining these data is a hurdle in putting the Builder's Model into practice. If the required information is obtained, we could apply this method to fill the missing gaps in the production of Statistics Canada's New Condominium Apartment Price Index and future residential property price indexes. Though we cannot fully determine the accuracy of our land index by comparing to other sources, because no such sources exist, the similarities between the Fisher indexes

[32]Although not shown in this paper, the unweighted Median Index stratified by FSA is smoother than the weighted one.

[33] Moreover, our estimated structure price index can be harmonized with current structure price indexes that are used in the System of National Accounts.

created from our Builder's Model and other hedonic and stratification methods, is promising for our proposed method of index calculation.

Through our modeling, we narrowed down the significant determinants of land prices to include location (determined by FSA), unit height, number of units in the building, building height and excess land. Our measurement of location by using FSA dummy variables is rather discrete. To improve our assessment of location, including neighbourhood characteristics, further research needs to be conducted and more detailed data need to be acquired.

We also identified structure quality adjustment variables such as the number of bedrooms, number of bathrooms, the presence of a balcony and natural gas heating in a condominium unit that impact price. Many other variables, such as dens, hardwood floors, condo fees and on-suite bathrooms were tested in our Builder's Model that appeared to have little impact. Comparing with the variables included in the model of Diewert and Shimizu (2017), we believe that the city characteristics will also have impact on determining the choice of variables added to the Builder's Model. For instance, due to the long winters in Ottawa, the means of heating is an important feature for determining the price of condominium units. All these findings could be helpful for designing a survey to effectively collect required information at a minimum cost.

A surprising conclusion from this study was the small impact that commercial space had on our hedonic model. Our analysis of adding a dummy variable to the land and structure components of the Builder's Model both separately and combined throws some light on how commercial space affects the condominium unit prices. The end result on including a commercial space dummy variable only on the structure component highlights the role that commercial space plays on total property prices.

Lastly, we determined a net geometric depreciation rate of 2.4 percent for the Q1 1996 to Q3 2009 period. This value is slightly larger than that currently used by the Canadian System of National Accounts.[34] This exercise highlights that not only can the Builder's Model provide a land price index for the National Accounts, but can also provide additional beneficial statistics for other parts of the System of National Accounts.

References

Davis, M., and M. Palumbo (2008),"The Price of Residential Land in Large US Cities", *Journal of Urban Economics* 63, 352-384.

Diewert, W.E., J. de Haan and R. Hendriks (2015), "Hedonic Regressions and the Decomposition of a House Price Index into Land and Structure Components", *Econometric Reviews*, 34:1-2, 106-126.

Diewert, W.E. and C. Shimizu (2016), "Alternative Approaches to Commercial Property Price Indexes for Tokyo", *Review of Income and Wealth*, forthcoming.

[34] However, as noted earlier, demolition depreciation is neglected in our model and so a geometric depreciation rate of 2.3% should be regarded as a lower bound on the overall depreciation rate.

Diewert, W.E. and C. Shimizu (2017), "Hedonic Regression Models for Tokyo Condominium Sales", *Regional Science and Urban Economics*, forthcoming.

Eurostat (2013), *Handbook on Residential Property Prices Indices (RPPIs)*, Luxembourg: Publications Office of the European Union.

Greene, W.H. (2003), *Econometric Analysis*, 5th Edition, Upper Saddle River: Pearson Education, Inc.

Prasad, N. and Richards, A. (2006), "Measuring Housing Price Growth: Using Stratification to Improve Median-Based Measures" Research Discussion Paper 2006-04, Reserve Bank of Australia.

Read-Hobman, T. (2015), *Evolution of Housing in Canada, 1957 to 2014: Canadian Megatrends*, Statistics Canada, Catalogue no. 11-630-X.

Situ, J. and S. LeVasseur (2013), *Homeownership and Shelter Costs in Canada*, Statistics Canada, Catalogue No. 99-014-X2011002.