Kevin J. Fox Interview of W. Erwin Diewert

by W.E. Diewert and K.J. Fox June 1, 2016.

Erwin Diewert,
Discussion Paper 16-02,
Vancouver School of Economics,
University of British Columbia,
Vancouver, B.C.,
Canada, V6N 1Z1.
Email: erwin.diewert@ubc.ca

Kevin J. Fox,
School of Economics,
University of New South Wales,
Sydney 2052,
Australia.
Email : K.Fox@unsw.edu.au

Abstract

In this paper, Fox interviews Diewert on his academic career. His contributions to the development of flexible functional forms, superlative index numbers, the difference approach to index number theory, the measurement of waste, the user cost of capital, the measurement of Total Factor Productivity, the use of generalized concavity in economics, the measurement of financial sector outputs and inputs and the comparative statics of maximizing behavior are covered. His work on the development of international manuals on the Consumer Price Index, the Producer Price Index, on Property Prices and on the International Comparison of Prices is also noted. His pioneering work on the nonparametric of preferences and of technologies is also mentioned.

Key Words

Flexible functional forms, superlative index numbers, measurement of waste, user cost of capital, Total Factor Productivity measurement, generalized concavity, the Consumer Price Index, the Producer Price Index, Property Price Indexes, nonparametric estimation of preferences and technology, International Comparisons.

Journal of Economic Literature Classification Numbers

C23, C43, C51, C61, D12, D24, D90, E01, E31, E41, F11, H21, I31, O47, O51, O53.

INTERVIEW of
Professor W. Erwin Diewert†

Interviewed by Kevin J. Fox*

21 May 2016

Erwin Diewert has an exceptionally distinguished career as one of the world's most respected economists. His remarkable publication record speaks for itself. He is in his fifth decade of publishing in the leading journals of the economics profession, continuing to make significant influential contributions. While he has made his mark on an uncommonly diverse range of fields in economics, he is perhaps best known for his contributions to duality theory, index number theory, user cost of capital, functional form specification, international comparisons, international trade and revealed preference theory.

Besides his major impact on the academic literature, he also has valuable engagement with national statistical offices and organisations such as the IMF and World Bank, in particular through contributing significantly to a series of manuals that are used to guide statistical agency practice in both developed and developing countries.

He has received many prestigious honours, including the following:

- Distinguished Fellow, American Economic Association
- Distinguished Fellow, Canadian Economics Association
- Fellow, Econometric Society
- Member, Royal Society of Canada
- Fellow, Academy of the Social Sciences in Australia
- Fellow, Society for Economic Measurement
- Research Associate, National Bureau of Economic Research

Many concepts that are now standard in economics derive from his contributions. His influence is pervasive. A few examples are sufficient to illustrate this: he named, generalised and popularised Shephard's Lemma (Journal of Political Economy, 1971), he defined "superlative" index numbers, he defined the class of "flexible" functional forms (which include the popular translog and AIDS functional forms), he introduced (with Caves and Christensen) the Malmquist productivity index, and developed and promoted the concept of the user cost of capital.

---

† Vancouver School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1Z1 and the School of Economics, UNSW Australia, Sydney, erwin.diewert@ubc.ca
* School of Economics & Centre for Applied Economic Research, UNSW Australia, Sydney 2052, K.Fox@unsw.edu.au, Tel: +612 9385 3320.

Acknowledgement: This interview came about due to the encouragement of Peter C.B. Phillips.

Shephard's Lemma is covered in standard microeconomics text books, the theoretical foundations he provided for index numbers led major statistical agencies to change their index formulae (at both aggregate and elementary levels), flexible functional forms are used in a huge variety of empirical contexts in academia and policy circles, the Malmquist productivity index is used extensively in literatures on efficiency and productivity analysis, as well as in operations research and management science, and the user cost of capital is used by leading statistical agencies in calculating capital services for official productivity statistics.

The interview covers many of these contributions. While much of his work has been highly technical, his ability to write and otherwise communicate clearly, as I believe is evidenced by his expositions in this interview, has assisted in much of his work having a broad impact beyond the academic literature.

I will make brief comments on two other aspects of his professional life which may not be obvious from the interview. First, as a co-author, he is generous, enthusiastic, patient and much fun to work with. He enjoys interacting and discussing ideas, with a notable preference for doing so in convivial surroundings. There is a distinct lack of ego in these interactions, with the focus always on the quality of the ideas and points being made. Second, he is a dedicated educator; he never shirks teaching responsibilities, prepares and updates extensive class notes, and is a considerate, engaged and inspirational thesis advisor. He often emphasizes the importance of educating future generations of public and professional economists, as well as academic researchers.

After decades of remarkable and innovative work, he continues to energetically conduct research on diverse topics, engage with government and international agencies, and undertake teaching and supervisory commitments. He modestly claims that his goal has simply been to be a useful member of society. I hope that, at least in terms of his research contributions, this interview adequately communicates the extent to which he has most certainly achieved this admirable goal.

How did you come to be interested in economics?

It is a bit of a long story. In high school (1955-1959 at King Edward High School in Vancouver), I was very interested in history and had in mind either becoming a High School teacher or an archaeologist. Fortunately, I had a very good math teacher in grades 10 and 11 (Mr. Brown) who taught us Euclidean geometry and at this point, I saw the beauty and power of mathematics to solve problems. I took quite a few science courses in high school and of course, having a strong math background helped me to get quite good marks in science subjects. In our final year Provincial high school exams, I got the third highest mark in physics in the Province and so I went to the University of British Columbia in 1959 and entered the honours physics degree program. I soon learned that physics was a very mathematical subject; the teachers were using math concepts that we had not yet studied in my companion math courses! Thus in my second year at UBC, I dropped out of the physics program and went into the honours math program, with the idea that once I learned more math, I could go back to physics. However, a close friend of mine, Ronald Boyes, was in my carpool and as we drove out to the University, he talked about an introductory economics course that he was taking. It sounded interesting and so I sat in one of his classes and found the material quite fascinating. In fact, there seemed to be some sort of mathematical structure to economics. In the following two years, as I continued on with my math degree, I took all of my outside courses in economics. In addition to the usual micro and macro courses, I took advanced micro theory and industrial organization from Milton Moore, development economics from Ibrihim Poroy, international finance from Gordon Munroe, econometrics from Gideon Rosenbluth and mathematical economics from Gideon and Rodrigo Restrepo. (It turns out that Rodrigo was a student of Samuel Karlin and I adopted the mathematical notation used by Karlin and Restrepo for the rest of my life). I graduated with an honours math degree from UBC in 1963 but I really did not know what to do with my life at that point. I decided to postpone any serious decision about what direction to take by enrolling for a Master's degree in mathematics at UBC as I still enjoyed learning about the different branches of mathematics. I should mention that in my MA thesis ("Analysis of Variance Estimators for the Seasonal Adjustment of Economic Time Series") I tried to devise a method for determining whether the seasonal adjustment factors were additive or multiplicative to the trend. This is a difficult topic and I returned to it periodically over the years. In any case, I continued to take outside units in economics during my MA year in 1963-4. Milton Moore was very influential at this point; he urged me to apply for graduate school in economics and so I applied to the University of California at Berkeley and did my PhD in economics there during the years 1964-68. That was the start of my career as an economist. Looking back, I was very fortunate in having many great math teachers during my early years.

Who influenced you during your student days at Berkeley?

Again, I was lucky enough to have many great teachers during my years at Berkeley, including Gerard Debreu (mathematical economics), Sidney Winter and Daniel McFadden (microeconomics), Amartya Sen (macroeconomics), Richard van Slyke (linear programming), Olvi Mangasarian (nonlinear programming) and Roger Wets (stochastic programming) in Industrial Engineering and Edward Barankin (stochastic processes) and Erich Lehmann (hypothesis testing and nonparametric methods). Lehmann had a unique teaching style: every class, he would give us a couple of problems to solve and hand in at the beginning of the next class. I very much liked this approach (there was no need to cram for exams; one learned the material as the course progressed) and so I eventually adopted his style in my own classes.

The two most influential teachers I had at Berkeley were Dale Jorgenson and Dan McFadden. Dale was very active in the Econometrics Workshop at Berkeley and I enrolled in this course during my first year. Somehow Dale took an interest in my education. We would have meetings in his office every month or so. Initially, we talked about capital theory and applied general equilibrium models that could be estimated econometrically. He would direct me to various articles and books to read (e.g., he noted that Walras had developed user cost theory way back in 1874). Then at the next meeting, we would discuss what I had read. I was super impressed with Dale's ability to pick up the conversation one month later exactly at the same point where the last conversation left off. Dale also spoke very quickly. I was a bit of a country bumpkin and found it difficult initially to keep up with him but after a while, I got better at following his arguments. In any case, Dale has had a profound influence on me. Basically, Dale uses economic theory and econometric methods to solve important applied economics problems. Thus Dale has made important contributions to capital theory, applied general equilibrium modeling, consumer theory, production theory and tax policy to name a few. Throughout my career, I have tried to follow in his footsteps.

Dan McFadden has a similar thrust to his research and he too profoundly influenced me. During the summer of 1967, I worked in Ottawa for the Department of Manpower and Immigration on the problem of the demand for different types of labour for an industry. The approach that was being used at that time was very simple and was called the *manpower* (nowadays we would say *personpower*) *requirements approach* which can be explained as follows. Let y be industry output during some period of time, let a_i be the input output coefficient for the ith type of labour and let x_i be total labour demand for the ith type of labour. Then $x_i = a_i y$ for $i = 1,...,N$ where there are N types of labour. I had taken production theory from McFadden and Winter at Berkeley and we learned about factor substitution in their course. Thus I thought that the above approach should be generalized to allow for factor substitution and so I proposed that the ith labour demand function should have the following form: $x_i(y,p) = \Sigma_{j=1}^{N} a_{ij}(p_j/p_i)y$ for $i = 1,...,N$ where $p_1,...,p_N$ are prices or wage rates for the N types of labour and the a_{ij} are parameters to be estimated. Thus during the summer of 1967, I estimated a small model of the type indicated above and in the fall of 1967, I presented my results to the Econometrics Workshop at Berkeley. Dan McFadden was in the audience and commented on my presentation as follows: "But Erwin, your demand system is not integrable." I thought to myself (or maybe I actually said it in the seminar), "What the heck is integrability?" Needless to say, after the seminar, Dan explained the concept. When we weight the ith demand function by p_i and then sum the resulting $p_i x_i(y,p)$ over i, we obtain the producer's cost function, $C(y,p) = \Sigma_{i=1}^{N} p_i x_i(y,p)$. So far, so good. But now comes the integrability requirement: it turns out that if C(y,p) is differentiable with respect to its input price components, then consistency of the model requires that the first order partial derivative of C(y,p) with respect to the ith input price p_i must equal the ith input demand function; i.e., the following equations must hold: $\partial C(y,p)/\partial p_i = x_i(y,p)$ for $i = 1,...,N$. This is Shephard's Lemma. It should be noted that Hicks (1946; 331) established this result in the consumer context but his starting point was the consumer's utility function whereas Shephard (1953; 11) established his result starting with the cost function. When we carry out this exercise for my proposed demand system, we end up with $C(y,p) = \Sigma_{i=1}\Sigma_{j=1}^{N} a_{ij}p_jy$. When we differentiate this cost function with respect to p_j, we obtain $x_j^* = \Sigma_{i=1} a_{ij}y$ for j = 1,...,N which is not equal to my original jth demand function, $x_j(y,p)$. Dan directed me to Ronald Shephard's (1953) book and to his recent Berkeley working paper, McFadden (1966), to read up on this problem. It is then that I discovered duality theory; if producers or consumers behaved as price takers, then their technologies and preferences (with some regularity conditions) could be perfectly described by dual cost or profit functions (for producers) and dual expenditure or indirect utility functions (for consumers). A large portion of my early research revolved around duality theory and its applications. But back to my cost function problem. I had to accept that Dan's criticism of my suggested demand system was valid so I was a bit disappointed that my simple system was not going to be the answer to getting factor substitution into producer demand systems. However, one day as I was sitting in a class and my mind wandered, I thought: what if I insert a square root sign into my suggested demand system so that the ith demand function becomes: $x_i(y,p) = \Sigma_{j=1}^{N} a_{ij}(p_j/p_i)^{1/2}y$ for i = 1,...,N. If we impose the symmetry conditions, $a_{ij} = a_{ji}$ for all $i \neq j$, then this new demand system passes the integrability test and moreover, I was able to show that the resulting cost function was a *flexible functional form*; i.e., it could approximate an arbitrary differentiable cost function that was dual to a constant returns to scale convex technology to the second order around any point. These results became a part of my Ph.D thesis (McFadden became my thesis advisor) and led to the Generalized Leontief Cost function and my first published paper, Diewert (1971). Thus it can be seen that I owe a lot to Dan McFadden.

How would you describe the content of your Ph.D thesis?

The title of my thesis was "Functional Form in the Theory of Production and Consumer Demand". Basically, what I was trying to do is to come up with new methods for deriving systems of consumer demand and producer supply and demand functions which were consistent with optimizing behavior on the part of consumers and producers and where the unknown parameters which characterize preferences and technology could be estimated using basically linear regression techniques. At the same time, I wanted the preferences or production functions to be able to provide second order approximations to arbitrary twice continuously differentiable preferences or technologies; i.e., I wanted the functional forms to be *flexible*. Thus my thesis came up with flexible functional forms for single output production technologies, for a multiple output but single input technologies and for general multiple output and input technologies. These papers were later published as Diewert (1971) (1973) (1974a).

Flexible functional forms, such as the translog, are now commonly used in empirical work, but more restrictive Cobb-Douglas and Constant Elasticity of

Substitution (CES) functional forms are still the "workhorses" for many economists. From your perspective, what is the importance of the flexibility concept?

If a functional form can provide a second order approximation to a utility or production function or to its dual cost function, then the resulting consumer or input demand functions can provide a first order approximation to arbitrary demand systems and the resulting pattern of demand elasticities can be completely arbitrary, consistent with the restrictions imposed by cost minimizing behavior. The problem with traditional functional forms like the Cobb-Douglas or CES is that the elasticities of demand that these functions generate are severely restricted a priori. Thus if these functions are used for policy purposes, there is a good chance that the results will be seriously biased due to the inflexibility of these functional forms.

It does not seem to be all that difficult to find flexible functional forms; why not just use say a quadratic function of the inputs in the context of a one output, many input technology?

The problem with letting the production function be say $y = a_0 + a \cdot x + (1/2)x \cdot Ax$ where y is output, x is an input vector a_0 is a scalar parameter, a is an N dimensional vector of parameters and A is a symmetric N by N matrix of parameters is that in many applications, we want the production function to be homogeneous of degree one in the inputs. If we impose this requirement on the quadratic production function, the resulting function becomes the linear function, $y = a \cdot x$, which can only provide a first order approximation to an arbitrary twice continuously linearly homogeneous production function. The total cost function $C(y,p)$ which is dual to a linearly homogeneous production function has the form $C(y,p) = c(p)y$ where y is the output that is produced, p is a vector of input prices and $c(p)$ is the unit cost function; i.e., it gives the minimum cost of producing one unit of output given that the producer faces the input price vector p. Now the unit cost function $c(p)$ must be homogeneous of degree one and if we assume that $c(p)$ is quadratic function, say $c(p) = b_0 + b \cdot p + (1/2)p \cdot Bp$ where b_0 is a scalar parameter, b is an N dimensional vector of parameters and B is a symmetric N by N matrix of parameters, then in order for $c(p)$ to be linearly homogeneous, we must have $c(p) = b \cdot p$, which again can only provide a first order approximation to an arbitrary unit cost function. Thus the problem of finding flexible functional forms is complicated by the linear homogeneity problem.

What attracted you to duality theory? Specifically, what is the advantage of using duality theory to generate systems of derived demand and supply functions?

Suppose we start with a flexible functional form for a single output production function and generate the system of input demand functions that correspond to the given functional form by solving the associated constrained cost minimization problem. The resulting demand functions are typically highly nonlinear in the unknown parameters and in some cases, it is not even possible to find explicit expressions for the demand functions. Thus econometric estimation of such demand systems is not straightforward. Contrast this direct approach to the generation of input demand functions with the dual approach, which starts with a strategically chosen functional form for the dual unit cost function. Use Shephard's Lemma to generate the ith input demand function by partially differentiating the cost function with respect to the ith input price. We get $x_i(y,p) = \partial C(y,p)/\partial p_i = y\partial c(p)/\partial p_i$ for i = 1,...,N. If we choose the functional form for the unit cost function to be a quadratic form in the square roots of input prices as in the Generalized Leontief Cost function mentioned earlier, we get a system of input demand functions that are *linear* in the unknown parameters that characterize the unit cost function. This facilitates econometric estimation. The cross equation symmetry conditions can either be imposed or one can test for their validity. McFadden (1966; 13) basically noted this advantage of duality theory (as a simple way of obtaining derived demand functions) but I think my contribution was to work out specific examples of how his idea could be implemented with functional forms which were also flexible (and linear or almost linear in the unknown parameters). Diewert (1974b) (1993a) were survey papers on these applications of duality theory to producer and consumer theory. The counterpart to Shephard's Lemma in the multiple output and input case was first worked out by Hotelling (1932; 597) and applications of Hotelling's Lemma to generate systems of derived input demand and output supply equations using flexible functional forms for variable profit functions can also be found in Diewert (1973) (1974b) and McFadden (1978).

Is it true that you introduced the terms flexible functional form, Shephard's Lemma and Hotelling's Lemma?

Yes. It is always nice to introduce terms into the economics literature which catch on.

Who were the students at Berkeley with whom you interacted?

I had a great peer group at Berkeley. The economics students whom I talked to the most while there were Michael Denny, Melvyn Fuss and Lawrence (Larry) Lau and to a lesser extent, Charles (Chuck) Hulten and Laurits (Lau) Christensen. I continued to see these folks in later years and even eventually collaborated with Hulten (on price measurement) and Christensen (on index number theory and the measurement of TFP). All of these students did their thesis work with either Jorgenson or McFadden and their theses were on either capital theory or applications of duality theory or both. Larry Lau played a big role in the development of the translog functional form. After I had come up with the Generalized Leontief cost function, Dale realized that rather than taking a quadratic form in square roots of prices to form a unit cost function (or a quadratic form in square roots of input quantities in order to form a constant returns to scale production function), one could take a quadratic form in the logarithms of prices and set the resulting functional form equal to the logarithm of the unit cost function. Similarly, one could take a quadratic form in the logarithms of input quantities and set the resulting functional form equal to the logarithm of the production function.

Dale showed me the resulting translog functional forms in one of our monthly meetings. I was skeptical about the functional form for the unit cost function and pointed out that the unit cost function had to be a linearly homogeneous function in input prices and I did not see how these restrictions could be imposed on his translog functional form. However, Larry Lau figured out exactly the restrictions on the parameters of the translog functional form that would ensure that it was linearly homogeneous, and moreover it turned out that these restrictions did not destroy the flexibility of the resulting functional form; see Christensen, Jorgenson and Lau (1971). This was a very useful accomplishment: the translog functional form is one of the most frequently estimated flexible functional forms in the applied economics literature. However, there is a problem with both the translog and Generalized Leontief cost functions that arises when these functions are econometrically estimated. The problem is that these estimated cost functions should be concave functions in their input prices in order to be consistent with cost minimizing behavior, at least over the ranges for the input prices in the sample. Experience has shown that in most cases, the estimated cost functions are not concave in input prices over the entire sample. This is a problem with existing flexible functional forms that bothered me for many years. In the 1980s, I finally came up with a satisfactory solution to this problem which we can discuss later.

Let's go back to your last year at Berkeley. How did things go for you on the job market?

I went to the annual American Economic Association meetings in January of 1968. At the time, I did not have a complete thesis yet but I had written up my Generalized Leontief paper and my thesis supervisor thought that I was ready to go on the job market. I remember interviewing for jobs at the Commerce Department at the University of British Columbia (UBC) and at the Economics Departments at the Universities of Western Ontario, Chicago and MIT. I was surprised to get job offers from all four departments. I wanted to go back to UBC but I did not quite see how my research interests in basic measurement problems would be relevant in a business school setting. Western Ontario was dominated by Chicago and MIT. Paul Samuelson and Robert Solow, two of my economic heroes, were at MIT and I was not sure that I was quite good enough to be a fellow professor with those giants of the profession so I was leaning towards going to Chicago, where I thought I might fit in better. I did not know at the time that there were many giants at Chicago as well! I remember meeting Zvi Griliches at my seminar there and I knew something about him since the classic paper by Jorgenson and Griliches (1967) on measuring Total Factor Productivity (TFP) growth had just appeared. Hirofumi Uzawa (1964) was also listed on the faculty at the time and he had written a great paper on the duality between cost and production functions. But at the time, I did not know too much about the other faculty members at Chicago. In any case, there was another complication on the horizon. I was planning to get married to my wife, Virginia, during the summer of 1968. She was a dentist who worked for Vancouver General Hospital in Vancouver, but she was interested in getting a Master's degree in Orthodontics. Al Harberger, who was a Professor at Chicago at the time helped us out by putting in a good word with a Dentistry Professor (Hal Perry) he knew at Northwestern University and so it transpired that Virginia was admitted to the Master's program at Northwestern. So this cinched the deal; we went to Chicago in the fall of 1968. I had not finished my thesis at that point but I managed to write it up during the academic year 1968-69 and I got my Ph.D from Berkeley (signed by Ronald Reagan, who was then the Governor of California) in 1969.

What was it like teaching at the University of Chicago?

It was a great experience. I taught a graduate course in Mathematical Economics and I had some excellent students, including William Barnett, Vernon Henderson, Rachel McCulloch, Mike Mussa, and Douglas Purvis. Doug also influenced my teaching style. He noticed that I had very detailed written notes which I dutifully transcribed onto the blackboard so he suggested: why not just distribute these notes to the class? I thought that this was a pretty reasonable request so ever since then, when I teach a course, I give the students a copy of my lecture notes. This actually increases productivity since the students do not have to waste time copying down the notes and I can just breeze through the notes, emphasizing the important points.

Who were the faculty members at Chicago that you interacted with?

The senior faculty members that I interacted most with were Zvi Griliches, Marc Nerlove and Arnold Harberger. Nerlove was very interested in cost function estimation and Griliches was interested in all aspects of economic measurement. He had a profound influence on me that was similar to the influence of Jorgenson and McFadden. Arnold Harberger and I talked about methods for measuring economic welfare and the fundamentals of cost benefit analysis. These conversations stayed with me for a long time and eventually led to a number of papers on the measurement of individual and social welfare (Diewert (1976a), (1985a) (1992a)) and on cost benefit analysis (Diewert (1983a)). Harry Johnson was at the University of Chicago at the time but we somehow did not talk much until my second year there and then he started to give me some papers to referee for the Journal of Political Economy. I attended the econometrics workshop regularly and that was always interesting. The senior faculty attending the workshop were Griliches, Nerlove, Hans Theil and Arnold Zellner. Theil and Zellner did not get along very well: they used to attack each other (and their students) if they presented at the workshop. But it was more or less normal that there were vigorous discussions at Chicago seminar presentations. It took me a while to adjust to this somewhat confrontational style. I remember giving a presentation of a chapter out of my Ph.D thesis on estimating a Generalized Leontief cost function using US aggregate data. Zvi Griliches was not impressed. He told me after the seminar that there were too many parameters in the function and the data did not support the estimation of so many parameters. I was totally crushed by this negative assessment of what I thought was a great idea; i.e., the estimation of a flexible functional form using US data. But after a while, I dismissed his criticism: after all, how could official US data be unreliable? I will come back to this point later in this interview when we discuss my work in the 1980s.

Other senior faculty members at the University of Chicago who I talked to occasionally were Robert Fogel and Robert Mundell. I also shared an office for one year with J. Richard Zecher (a monetary theorist) and Stanley Fischer (who is now a very famous central banker) so I got to know these younger faculty members quite well. Other junior faculty members that I interacted with were Robert Gordon and Dierdre McCloskey. Robert Gordon has done a great deal of research on price measurement, bias in the CPI and the measurement of Total Factor Productivity (TFP) so our paths have crossed frequently over the years.

I understand that you were asked to referee a paper by Sydney Afriat for the Journal of Political Economy, which eventually led to an influential publication on your part. Can you tell us about this?

Afriat's paper that I was asked to referee was a very interesting and innovative one and it was eventually published as Afriat (1973). The problem was that the paper referred to an earlier published paper, Afriat (1967). This earlier paper showed how a finite set of price and quantity data pertaining to a household could be tested to see whether the data were consistent with utility maximizing behavior. If Afriat's test passed, then he showed how the household's preferences could be represented by a concave utility function and he showed how to construct this function. His approach was entirely nonparametric; i.e., it was not necessary to make parametric assumptions about the functional form of the utility function. Thus Afriat (1967) was a very fundamental paper. But it was extremely difficult to read and this meant that the 1973 paper, which drew heavily on the 1967 paper, was virtually unreadable. I remember spending two weeks trying to figure out what was going on with the two Afriat papers and I finally succeeded. I figured out how to represent Afriat's 1967 testing procedure by setting up a simple linear programming problem involving the observed data. If the optimized objective function for this program turned out to be zero, then the data were consistent with utility maximizing behavior and the concave utility function which rationalizes the data could readily be constructed. I also noted that it was necessary to add a couple of restrictions on the class of utility functions to ensure that Afriat's Theorem would be true (the preferences had to be continuous from above and be subject to a local nonsatiation assumption—Afriat did not place any restrictions on the utility function) and I provided a much simpler proof of his result. I wrote all of this up in a very detailed referee report and asked the author (Afriat) to make use of this material to make his new paper more readable. Sydney refused to make any changes so the JPE rejected his paper. A couple of years later, I thought about Sydney's 1967 paper and how it was too bad that hardly anyone understood his test and so I decided to dig up my old referee report and I turned it into a paper, "Afriat and Revealed Preference Theory" which was eventually published; see Diewert (1973). In this paper, I also figured out how to extend Afriat's test (which applied to general utility functions) to a test for the consistency of homothetic utility maximization. As people came to appreciate Afriat's results, there was a great flowering of papers in this area, starting with Varian (1982) who developed a more efficient method for checking the Afriat conditions. Over the years, I have returned to this nonparametric consumer theory area from time to time: see Diewert and Parkan (1985) (on testing for separability) and Diewert (2012a) (on extensions to choice under uncertainty). It turns out that Afriat's publications stimulated my research in a couple of other areas as we shall see.

Why did you leave the University of Chicago in 1970? It has been quoted that, in response to your decision to return to Canada, Arnold Harberger said "Erwin has a great production function but a lousy utility function."

I really enjoyed the intellectual atmosphere at Chicago but unfortunately, I found it hard to adjust to the climate (hot in the summer and very cold in the winter) having lived on the West Coast of North America all my life. At the time, Illinois power plants burned coal that had very high sulfur content and this affected my health somewhat. When my wife's 18 month Master's degree program at Northwestern ended, I looked to go back to Vancouver and I managed to get a job at the UBC Economics Department, starting in September of 1970.

I believe that you had a few months before taking up the job at UBC. What did you do with your time?

I left Chicago at the end of March 1970 and commenced at UBC in the fall of 1970. Zvi Griliches was only at the University of Chicago during my first academic year at Chicago, 1968-1969. In the following year, he left Chicago to take up an appointment at Harvard; he felt that it was time for a change. I was very fortunate that he took an interest in my research during our year together at Chicago, perhaps because we were both interested in fundamental measurement problems. In any case, when I quit Chicago at the end of March in 1970, he invited me to visit Harvard until the fall of 1970 when I started teaching at UBC. Zvi arranged office space for me; I had a large closet that was attached to Giora Hanoch's office; Giora was another visitor that Zvi supported. I gave a talk on my referee report on the Afriat paper while at Harvard and Giora attended my seminar and realized that the same nonparametric approach could be applied to production theory. Thus Giora wrote up a paper on the topic which was eventually published as Hanoch and Rothschild (1972). Independently, Sydney Afriat also realized that his nonparametric consumer theory approach could be adapted to production theory and he also published a nonparametric production function paper around this time, Afriat (1972a). A couple of Ph.D students of mine, Celik Parkan and Nimfa Mendoza, made further contributions to the nonparametric approach to production theory which eventually led to a couple of published papers, Diewert and Parkan (1983) and Diewert and Mendoza (2007). Hal Varian, another McFadden Berkeley student, also made important contributions to this nonparametric approach to production theory; see Varian (1984).

What research topics did you work on when you arrived at UBC?

Initially, a lot of my time was devoted to publishing the discussion papers that I produced while at Berkeley and Chicago. It was amazing that I was hired as an Associate Professor in 1970 and I did not have a single published paper until 1971, but I had quite a few papers in the pipeline. I also embarked on some new research projects in the early 1970s.

I should explain a bit of background before I describe the first of these research projects. I regard economics as the study of choice under constraint. We have two main constrained maximization problems that we use to model the economy: (i) consumers maximizing utility subject to a budget constraint and (ii) producers maximizing profits subject to their production function or more generally, their technology constraints. These two constrained maximization problems generate household and producer demand and supply equations which interact to produce equilibrium prices. Governments enter the picture by introducing tax wedges and using tax revenues to produce various goods and services as well as monetary transfers to certain households.

Hicks (1946) and Samuelson (1947) were the pioneers in establishing the mathematical properties of these derived demand and supply functions and more generally in working out the implications of maximizing behavior. The two sides of the market are brought together in the study of general equilibrium theory. But the temporary equilibrium theory of Hicks (1946; 126-127) where producers and consumers only form expectations about future prices is a much more realistic framework than the pure futures equilibrium which was dismissed by Hicks (1946; 140) as being a poor approximation to reality. Thus in the early 1970s, I wrote a couple of papers which attempted to provide somewhat practical models of the temporary equilibrium that could perhaps be econometrically implemented and then used for policy purposes.

The first paper was Diewert (1974c) which looked at the consumer's intertemporal utility maximization problem with a wealth constraint and showed how duality theory could be used to simplify the econometric estimation of nonseparable intertemporal preferences. The second paper was Diewert (1977) which laid out a more complete temporary equilibrium model and highlighted the importance of a financial sector which rented out durable inputs that were owned by the household sector to the producers with one period production functions. This was my first attempt to deal with the accounting problems that arise when producers use durable inputs in their one period production functions. Both of these papers derived the user cost of capital in a much simpler way than had been done in the current literature; i.e., following Walras and Hicks, the user cost of capital can be set equal to the beginning of the period cost of purchase of a durable input (or commodity) minus the discounted price that the used capital good (or commodity) could sell for in the marketplace at the end of the accounting period where the discount rate is the economic agent's cost of financial capital.

What else did you work on in the 1970s?

One minor (but interesting) paper was "A Note on Aggregation and Elasticities of Substitution", Diewert (1974d). This note explains why elasticities of substitution in single output production function studies tend to be small in magnitude if the number of inputs is small but these elasticities tend to grow in magnitude as there are more inputs in the model. The reason for this is as follows: if there are only two inputs, the two inputs must be substitutes so the elasticity of substitution must be positive (or zero). But as we disaggregate, complementarity becomes more common; i.e., negative elasticities of substitution occur and become more frequent. If we aggregate the N input model into a two input model, the positive and negative elasticities largely cancel each other out, leading to a relatively small aggregate elasticity of substitution between the two aggregate inputs.

A much more substantial paper was "Optimal Tax Perturbations", Diewert (1978a). This paper was intended to be a supplement to the optimal tax literature which was developed around this time. The basic idea is that governments cannot typically make large changes to tax rates so that approaching an optimal situation is going to be a gradual process. Thus the paper set up a (static) general equilibrium model with many consumers and producers, used duality theory to simplify the demand and supply equations and looked for optimal directions of tax change using a given social welfare function. I specialized the model at the end of the paper to derive the usual optimal tax results that were in the literature at the time but I also introduced a method for determining whether a Pareto improving direction of tax change could be implemented. This method relied on Motzkin's Theorem of the Alternative which I learned in my course in nonlinear programming at Berkeley taught by Mangasarian (1969). This technique proved to be very useful and I used it in a number of joint papers with Alan Woodland who was a colleague of mine during the 1970s and with Arja Turunen who was a Ph.D student of mine in the 1980s; see Diewert, Turunen-Red and Woodland (1989) (1991).

I also continued my research on application of flexible functional forms during this period. Ernst Berndt was a colleague at UBC during the 1970s and we wrote a paper together, along with my first Ph.D. thesis student, Masako Darrough, which integrated income distribution information with the estimation of a translog demand system; see Berndt, Darrough and Diewert (1977).

You are very well known for your influential research on index numbers, yet there has been no mention of this work so far. When and how did you get into index number research?

During my time at Chicago, I started to get interested in index number theory. I realized that it would not be possible to estimate flexible functional forms if the number of commodities in the model was large (a flexible functional form requires approximately $N^2/2$ parameters where N is the number of commodities in the model). So then the question arises: how exactly should we aggregate the number of commodities into a manageable number so that a complete matrix of price elasticities of demand (or supply) could be estimated?

Around 1972, I started reading papers on index number theory. I found two papers, by Robert Pollak (1983) (the discussion paper version of this paper was published by the Bureau of Labor Statistics in 1971) and Sydney Afriat (1972b) in particular, which were very interesting: they related functional forms for a consumer's utility function to functional forms for bilateral index number formulae, like the Laspeyres, Paasche and Fisher indexes. In particular, Afriat (1972b; 45) noted a result first derived by the Russian mathematician, Buscheguennce (Byushgens (1925)), that if a consumer maximized a homogeneous quadratic utility function, say $u(q) = (q \cdot Aq)^{1/2}$ where A is a symmetric matrix of parameters, for two time periods facing the commodity price vectors p^0 and p^1 with solution vectors q^0 and x^1, then the utility ratio $u(q^1)/u(q^0)$ was *exactly* equal to the Fisher (1922) ideal quantity index (the geometric mean of the Laspeyres and Paasche quantity indexes) and the true cost of living index (equal to the ratio of the dual unit cost functions $c(p^1)/c(p^0)$ where $c(p) = (p \cdot A^{-1}p)^{1/2}$ if A has full rank) was *exactly* equal to the Fisher ideal price index (the geometric mean of the Laspeyres and Paasche price indexes).

This seemed to me to be a very important result since I was able to prove that the homogeneous quadratic function was a *flexible functional form* (in the class of twice continuously differentiable linearly homogeneous functions) and hence could approximate any differentiable homothetic preference function to the second order. Using the Byushgens result, it is possible to construct aggregate prices and quantities that are consistent with utility maximizing (or cost minimizing) behavior and will closely approximate the "truth" without having to undertake any econometric estimation. Moreover, although the underlying aggregator function had to be linearly homogeneous to apply the Byushgens result, the number of commodities in the aggregate could be arbitrarily large. Thus there was a connection between flexible functional forms and certain index number formulae.

How were you able to find this remarkable paper by Byushgens?

During the 1970s and 1980s, Larry Lau, my old classmate from Berkeley, was able to invite me to visit Stanford during the summer to participate in the Mathematical Economics Workshop run by Mordecai Kurz. Thus I had access to Stanford's Hoover Library which had a huge collection of post-World War I Russian journals and books. Fortunately, during my undergrad years at UBC, I took three years of Russian and so I was able to browse through the library until I located Byushgens (1925). But during my browsing, I also discovered a great paper by Konüs and Byushgens (1926) which had not been noticed in the literature.

This paper not only had the Byushgens (1925) result but it also had detailed proofs of the result. Moreover, it had other exact index number results in it; for example, they showed that the share weighted Jevons quantity index (using base period expenditure shares) was exact for a Cobb-Douglas utility function. I was able to modify the proofs in the 1926 paper to cover additional classes of preferences such as the quadratic mean of order r and translog aggregator functions (and their dual unit cost functions) and find exact index number formulae which corresponded to these flexible functional forms. I also clarified the conditions that needed to be imposed on the matrix of coefficients A in the homogeneous quadratic utility function, $u(q) \equiv (q \cdot Aq)^{1/2}$. (The symmetric matrix A needed to have a positive eigenvalue with a strictly positive eigenvector and the remaining eigenvalues of A had to be zero or negative). Diewert and Hill (2012) developed additional material on regularity conditions for the Fisher aggregator functions.

I gave a seminar on my results at Stanford in 1973 and the paper was eventually published as Diewert (1976b);it was rejected by five major economics journals but Dennis Aigner was brave enough to publish it in the *Journal of Econometrics*. In addition to deriving results for the case of linearly homogeneous aggregator functions (utility or production functions), I also showed that the Törnqvist[1] price index was exact for a general translog (nonhomothetic) cost function and provided a similar exactness result for nonhomothetic translog distance function; see Diewert (1976b; 122-124). It seems that not many researchers know about these nonhomothetic translog exactness results. I also made my first attempt to measure Total Factor Productivity (TFP) growth using exact index numbers and making translog assumptions about the technology; see section 3 of Diewert (1976b). I was attempting to justify the Divisia measures of TFP growth that were used by Jorgenson and Griliches (1967) (1972). My results on TFP measurement in this paper were not entirely satisfactory; I made some separability assumptions between inputs and outputs and so the results were not general enough. Later, Diewert and Morrison (1986) and Diewert and Fox (2010) made less restrictive assumptions and derived much more satisfactory exact index number measures of TFP growth. It turns out that the translog functional form is well suited to exact index number applications.

The term "superlative index" is now commonly used to describe indexes such as the Fisher ideal and Törnqvist indexes. Did you introduce this term into the economics literature?

Not quite but I gave the term a more precise meaning. Irving Fisher (1922) in his famous book on the axiomatic approach to index number theory introduced the term "superlative index". However, he did not really give a proper definition for this term; he undertook an empirical comparison of several hundred index number formula using a US data set on the prices and quantities of raw materials during World War I. He classified an index number formula as being "superlative" if it was

[1] A typical reference for this index is Törnqvist (1936). However, as noted in Diewert (2005b, endnote 20), the formula does not appear explicitly in this reference. It does appear in Törnqvist and Törnqvist (1937: 18) where it is termed the "geometric ideal." This index first appeared as Formula 123 in Fisher (1922: 473). Fisher (1922: 265) listed it as one of his best 29 formulae. Persons (1928: 21–22) recommended it as one of nine indexes which satisfied his test approach.

numerically close to his Fisher ideal index. In my 1976 index number paper, I termed an index number formula to be "superlative" if it was *exact* for either a linearly homogeneous aggregator function (or its dual unit cost function) where either the aggregator function (or the unit cost function) was a *flexible functional form* in the class of linearly homogeneous functions. Thus the Fisher ideal index and the Törnqvist indexes are both superlative indexes. The term superlative indexes has caught on and statistical agencies are increasingly using superlative indexes as their target indexes instead of Paasche and Laspeyres indexes.

I believe you wrote an additional paper with further related results following this.

Yes, Diewert (1978b) was a follow up paper on my 1976b paper. There were a number of interesting results in this paper. First, I was able to show that all known superlative index number formulae approximated each other numerically to accuracy of a second order Taylor series approximation if the derivatives were evaluated at a point where the two price vectors were equal and the two quantity vectors being compared were equal. Second, I considered the two stage aggregation of superlative indexes and compared the two stage index with its single stage superlative counterpart. Again, I was able to show that the two stage index approximated its single stage counterpart to the second order around an equal price and equal quantity point. Thus superlative indexes, while not exactly consistent in aggregation, were approximately consistent in aggregation. Finally, I did a numerical example using annual Canadian national accounts data and showed that chaining the indexes reduced the spread between the Paasche and Laspeyres formulae and also reduced the spread between commonly used superlative indexes. From this exercise, I concluded that using chained indexes was probably more appropriate than using fixed base indexes, at least for annual data (but the same conclusion does not necessarily hold for sub annual data). I should add that Diewert (2002a) casts some light on the class of functional forms for the aggregator function that will lead to superlative index number formulae.

These papers seem to have been influential in changing the practice of some national statistical offices.

Yes, I believe that they helped provide a justification for the US Bureau of Economic Analysis to move away from the fixed price indexes they were using to measure GDP growth prior to 1996 and to implement the chained Fisher index methodology that they are currently using.

What were some of the other areas of economic research during the 1970s?

I was still working on the Hicks and Samuelson research agenda which attempted to determine the empirical implications of (competitive) optimizing behavior. The first paper along these lines was Diewert and Woodland (1977) followed by Diewert (1977b) (1981a) (1981b) (1983b) (1984a) (1985b) and Diewert and Lewis (1982). Most of these papers made use of duality theory. There was also a related paper, Blackorby and Diewert (1979), where we showed that a local second order approximation to a utility function also provided a local second order approximation to its dual expenditure function. Charles Blackorby was another colleague of mine at UBC and we spent a lot of time over beers (with Chris Archibald, David Donaldson and William Schworm) discussing the finer points of separability, duality theory and the measurement of social welfare.

You also followed up your initial research efforts on the user cost of capital in the latter half of the 1970s.

Yes. Around that time, I started to attend the meetings of the National Bureau of Economic Research (NBER) and in particular, the meetings of the Conference for Research in Income and Wealth (CRIW). Dan Usher organized a CRIW conference on the measurement of capital held in Toronto in 1976 and I contributed a paper on the problems associated with the measurement and aggregation of capital: Diewert (1980). Around this time, Dale Jorgenson and Zvi Griliches (1972) got into a controversy with Edward Denison on how exactly should capital services be aggregated. Denison suggested that capital stock components should be aggregated into an aggregate real capital stock and then the price of capital services used by a firm, industry or economy could be obtained residually by dividing Gross Operating Surplus of the production unit by the appropriate real capital stock. Jorgenson and Griliches on the other hand argued for the construction of user costs for each individual capital stock component and then they defined the aggregate flow of capital services by aggregating up the individual capital stock service flows. They demonstrated that the method of aggregation matters empirically. I sided with Jorgenson and Griliches in this dispute.

But I also got into a bit of a dispute with Dale that has persisted to the present day. The user cost of a particular capital stock component used in production comprises of the sum of interest rate, depreciation rate and tax rate terms less the *expected or actual ex post* rate of asset price appreciation over the accounting period times the beginning of the period asset price. Jorgenson has always maintained that the *actual ex post rate* of asset price inflation is the appropriate term to insert into the user cost formula whereas I maintained that the *expected rate* of asset price inflation should be used. Using expected rates of inflation has the drawback that expected rates are uncertain whereas actual ex post rates are definite. However, using ex post rates for assets that are subject to large price fluctuations will lead to user costs which have large fluctuations and in the case of land assets, the use of ex post inflation rates will generally lead to *negative user costs*, which does not make a great deal of sense. In the following decades, I returned to the topic of capital measurement repeatedly; see for example Diewert and Lawrence (2000), Diewert (2005a) (2009a) (2010a) and Diewert and Fox (2016b).

Let us turn to your research interests in the early 1980s. One of these seems to be the measurement of waste.

The early pioneers in this area were Hotelling, Allais, Boiteux and Debreu. Debreu (1951; 285) distinguished three types of waste in an economy: (i) underemployment of existing resources (i.e., unemployment), (ii) technical inefficiency and (iii) inefficiency due to the imperfection of economic organization (i.e., different economic agents face different prices for the same resource due to say tax distortions). But how exactly can we measure these types of waste quantitatively? I tried to answer this question in a series of papers; see Diewert (1981c) (1983c) (1984b) (1985a) (1985b).

Perhaps the best paper in this group was the 1983c paper, which assumed that we could observe a small open economy with tax or monopolistic distortions. Now freeze all households in the economy at their observed tax distorted equilibrium levels of utility and then set up a constrained maximization problem where a central planner tries to maximize the extra amount of foreign exchange the economy could produce, using existing technologies and ensuring that each household gets their initial tax distorted levels of utility. This extra amount of foreign exchange is always nonnegative and serves to measure the amount of waste in the economy. I went on to provide a second order approximation to this measure of waste, which of course depends on the magnitude of the distortions and various elasticities of substitution in production and consumption. This methodology avoids the problems associated with deriving “reasonable” weights for a social welfare function, a topic I tried to address in Diewert (1985a).

Diewert (1985b) is also of some interest. In this paper, I showed that deadweight losses due to tax distortions were generally much higher in magnitude in an intertemporal economy where there are many periods than the corresponding deadweight losses in a one period economy. This is due to the fact that investments in an intertemporal economy will generally flow into protected sectors of the economy and this leads to a *magnification* of the static deadweight losses.

I made another contribution to the measurement of inefficiency in the production sector of an economy with Raymond Kopp. Farrell (1957) showed how technical and allocative inefficiency in production could be measured if one had estimates of the best practice technology production possibility sets in hand. In Kopp and Diewert (1982), we showed how Farrell’s methodology could be applied if instead of a direct measure of the efficient production possibilities set, only an indirect representation was available in the form of a best practice cost function. This paper is widely cited so it served a useful purpose.

Another research interest in the early 1980s seems to have been the study of generalized concavity.

This was a fun area for me. Obviously, concavity and quasiconcavity arise naturally in economics and so I was well aware of the importance of generalizations of concavity to economics. Mangasarian introduced me to the concept of pseudoconcavity in the differentiable case while I was a student at Berkeley.

Pseudoconcave functions have the property that the first order necessary conditions for maximizing a differentiable function are also sufficient for a maximum and so these functions are also useful in economic applications. In any case, in the early 1980s, I came into contact with a couple of Israeli industrial engineers, Mordecai Avriel and Israel Zang, and we interacted to produce the paper, "Nine Kinds of Quasiconcavity and Concavity"; see Diewert, Avriel and Zang (1981).

I went on to produce two more papers on this topic, Diewert (1981d) (1981e). The second paper is a nice one for me; in it, I was able to prove a Generalized Mean Value Theorem without making any differentiability assumptions. This is probably my one and only theorem in the mathematics literature! In this paper, I also generalized the concept of pseudoconcavity to the nondifferentiable case. In any case, I joined up with Avriel, Zang, and another industrial engineer from Germany, Siegfried Schaible, to produce the book, *Generalized Concavity*, which was published in 1988. What is remarkable is that in 2010, the Society for Industrial and Applied Mathematics reprinted this book as Volume 63 in their series, *Classics in Applied Mathematics*; see Avriel, Diewert, Schaible and Zang (2010).

In addition to your work on the measurement of waste and on generalized concavity, you seemed to continue your work on index number theory.

Yes, I had four papers on index number theory appear in 1981-1982 and two of these papers turned out to be quite influential.

The first paper was Diewert (1981f), which gave a comprehensive review of the economic approach to index number theory. This paper drew heavily on the earlier work by Konüs and Byushgens (1926), Pollak (1983) (which appeared in 1971) and my work on exact and superlative indexes that was published in the 1970s.

The second paper was joint work with Robert Allen, who was an economic historian at UBC during this period. He was attempting to measure the productivity of US steel mills between 1889 and 1909, where productivity growth is measured as an output index divided by an input index. Bob collected data on the prices and quantities of outputs produced and inputs used by US steel mills over this period but the question was: should we form price indexes for outputs and inputs using our favorite price index formula and then calculate quantity indexes residually by deflating output and input values by their corresponding price indexes, or should we form quantity indexes for outputs and inputs *directly* using our favorite quantity index formula? Using the direct strategy means that the price indexes are calculated residually by deflating output and input values by their corresponding quantity indexes.

Allen and Diewert (1981; 433) showed that the choice of aggregation strategy mattered empirically for their data set: the direct Törnqvist output *quantity* index for their data set finished up 5% higher than the corresponding implicit quantity index that used the Törnqvist *price* index as the starting point for output aggregation. How should one choose between these aggregation strategies? There was some guidance from the theoretical literature on aggregation: Leontief's (1936; 54-57) Aggregation Theorem suggested that the direct aggregation approach was appropriate if all quantities in the aggregate varied in strict proportion over time while Hicks' (1946; 312-313) Composite Commodity Theorem suggested that the indirect aggregation approach was appropriate if all prices varied in strict proportion. Moreover, if prices varied proportionally over time, then all "reasonable" price index formulae will generate the factors of price proportionality as the price indexes (and hence the corresponding implicit quantity indexes will all coincide). If quantities varied proportionally over time, then all "reasonable" quantity index formulae will generate the factors of quantity proportionality as the quantity indexes. Thus if the variation in prices is more proportional than the variation in quantities, then Allen and Diewert thought it best to aggregate prices first and generate the corresponding quantities residually and if the variation in quantities is more proportional than the variation in prices, it is best to aggregate quantities directly.

But how can we decide whether prices vary more proportionally than quantities? Allen and Diewert (1981; 433) suggested the following procedure for the case of two observations: regress $\log(p_{1n}/p_{0n})$ on a constant for $n = 1,...,N$ and let $D(p_0,p_1)$ be the sum of squared residuals for the resulting regression where $p_t \equiv (p_{t1},...,p_{tN})$ for $t = 0,1$. Note that $D(p_0,p_1) \geq 0$. If $D(p_0,p_1) = 0$, then it can be seen that p_0 is proportional to p_1. If $D(p_0,p_1) > 0$, then the magnitude of $D(p_0,p_1)$ serves to measure how nonproportional the two price vectors are. Similarly, regress $\log(q_{1n}/q_{0n})$ on a constant for $n = 1,...,N$ and let $D(q_0,q_1)$ be the sum of squared residuals for the resulting regression where $q_t \equiv (q_{t1},...,q_{tN})$ for $t = 0,1$. If $D(p_0,p_1) < D(q_0,q_1)$, then Allen and Diewert defined prices to be more proportional than quantities. They found that output prices were in fact much more proportional than output quantities in their data set so they used the indirect method for forming their output aggregates. Many years later, I returned to the problems associated with measuring the degree of proportionality (or the amount of similarity) between two positive vectors of the same dimension. It turns out that these questions play an important role in making comparisons of prices and quantities across countries; see Diewert (2009b). I also note that Hicks developed his aggregation theorem in the context of consumer theory. I looked at its application in a production theory context in Diewert (1978c).

The third and fourth papers on index number theory that I helped write during the early 1980s were joint work with Laurits (Lau) Christensen and Douglas Caves. Christensen, after graduation from Berkeley, was an Assistant Professor at the University of Wisconsin in Madison. Caves was a Ph.D student of Christensen who ended up as a founding member of Christensen Associates, Lau's consulting company. Caves and Christensen were very active in the field of productivity measurement, mostly in the context of regulated industries, where TFP growth plays a big role in the regulatory process. In any case, Lau invited me out to give a seminar at Madison in 1980 and this led me to collaborate with them to produce two papers.

In Caves, Christensen and Diewert (1982a), we defined output and input indexes for very general multiple output, multiple input technologies using Malmquist (1953) Shephard (1953) distance functions. A feature of these definitions was that these output and input indexes did not depend on output or input prices and thus these definitions appealed to industrial engineers and operations researchers. However, in order to calculate these indexes without knowledge of prices one had to know the underlying technology, information which is usually not available. In this paper, we showed that if the output distance functions for a production unit could be represented by certain translog functional forms for two time periods and the economic agent engaged in revenue maximizing behavior, then a certain Malmquist output index was exactly equal to the Törnqvist output quantity index. Similarly, we showed that if the input distance functions for a production unit could be represented by certain translog functional forms for two time periods and the economic agent engaged in cost minimizing behavior, then a certain Malmquist input index was exactly equal to the Törnqvist input quantity index. These results are fine. The paper also provided a definition of productivity growth using Malmquist indexes and Theorem 3 in Caves, Christensen and Diewert (1982a; 1404) attempted to derive and exact index number formula for a measure of TFP growth, assuming that the two technology sets can be described by translog output distance functions.

In order to derive this result, we assumed (competitive) revenue maximizing behavior conditional on input quantities and (competitive) cost minimizing behavior conditional on output quantities. These assumptions are satisfactory provided that the underlying technology sets are subject to either constant returns or decreasing returns to scale (and producers take prices as given). But it is well known that competitive revenue maximizing behavior is not consistent with increasing returns. Moreover, our definition of productivity growth was really a definition of technical progress; i.e., of a shift in the technology set over time or space. Thus the paper was not entirely satisfactory on the topic of measuring TFP growth when there are increasing returns to scale. However, Diewert and Fox (2010; 89) provided a theoretically sound relationship between the Törnqvist output and input indexes and measures of technical progress and returns to scale (in the increasing returns to scale case) by introducing monopolistic markups into the translog distance function framework used by Caves, Christensen and Diewert. A specialization of the Diewert and Fox results to the competitive case with constant returns to scale led to the measure of TFP growth used by Jorgenson and Griliches (1967). In a related paper, Diewert and Fox (2008; 177-178) used translog cost functions (instead of distance functions), monopolistic markups and exact index number techniques to derive relatively simple relationships between output and input growth rates, returns to scale and measures of technical progress. I should note here that Hall (1988) noted that traditional index number measures of TFP growth were not valid if producers had market power, since in this case output prices are not equal to the corresponding marginal costs. Morrison and Diewert (1990; 273-275), Diewert and Lawrence (2005) and Lawrence and Diewert (2006) followed up on this insight of Hall's and provided various methods for estimating monopolistic markups in the context of productivity measurement studies.

My second paper with Caves and Christensen was my first paper on making multilateral index number comparisons; i.e., comparisons of output, input and productivity across many production units located across space (and time). It was natural for Caves and Christensen to be interested in this area of research because they were heavily involved in consulting activities, attempting to measure the relative productivity of production units in regulated industries. Later on, I also got involved in this regulatory area.

To understand the contribution, we need a bit of background information on multilateral index number theory. Making index number comparisons across space is generally trickier than making comparisons of the same production unit over time. There are two main methods used to make comparisons across time: (i) using a bilateral index number formula with a fixed base or (ii) by using the chain system. In the first method, we pick say the first observation in the sample to act as the base period and then use our favorite bilateral index number formula (say the Fisher or Törnqvist) to compare every other observation in the sample with the base period observation. In the second method, we start out by using our bilateral index number formula to compare the second period to the first. Then instead of comparing the third period directly with the first period as in the fixed base system, we use the bilateral index number formula to compare period 3 with period 2. Then we use this link index to update our price or quantity level for period 2 by multiplying the period 2 level by the index that links period 3 to 2.

To illustrate how the two methods work, consider the case of 3 observations; i.e., we have 3 N dimensional price vectors, say p^1, p^2 and p^3, and the corresponding quantity vectors are q^1,q^2 and q^3. Examples of bilateral price indexes, comparing the prices of period 2 with those of period 1, are the Laspeyres, Paasche and Fisher indexes, defined as follows: $P_L(p^1,p^2,q^1,q^2) \equiv p^2\cdot q^1/p^1\cdot q^1$; $P_P(p^1,p^2,q^1,q^2) \equiv p^2\cdot q^2/p^1\cdot q^2$ and $P_F(p^1,p^2,q^1,q^2) \equiv [P_L(p^1,p^2,q^1,q^2)P_P(p^1,p^2,q^1,q^2)]^{1/2}$. The logarithm of the Törnqvist price index, P_T, is defined as $\ln P_T(p^1,p^2,q^1,q^2) \equiv \Sigma_{n=1}^N (1/2)(s_{1n}+s_{2n})\ln(p_{2n}/p_{1n})$ where $p^t \equiv [p_{t1},...,p_{tN}]$ for t = 1,2 and s_{tn} is the expenditure share on commodity n during period t. Denote a generic bilateral price index that compares the prices of period 2 to those of period 1 by $P(p^1,p^2,q^1,q^2)$. The *fixed base sequence* of aggregate price levels for periods 1-3 is as follows: 1, $P(p^1,p^2,q^1,q^2)$, $P(p^1,p^3,q^1,q^3)$. The corresponding sequence of *chained* aggregate price levels is: 1, $P(p^1,p^2,q^1,q^2)$, $P(p^1,p^2,q^1,q^2)P(p^2,p^3,q^2,q^3)$. Pick one of these two sequences of prices and label the resulting price levels as P^1, P^2 and P^3. Then the corresponding aggregate quantity levels are defined as $Q^1 \equiv p^1\cdot q^1/P^1$, $Q^2 \equiv p^2\cdot q^2/P^2$ and $Q^3 \equiv p^3\cdot q^3/P^3$ so that the corresponding quantity aggregates are obtained by deflating the period-by-period value aggregates by our preferred price levels.

When making annual index number comparisons across time for the same production unit, it is generally best to use the chain system rather than fixed base indexes because, with the smooth trends in prices and quantities, chaining will reduce the spread between the Paasche and Laspeyres indexes (and also reduce the spread between alternative superlative formulae using the same data set). Thus chaining will probably get us closer to the "truth". However, when constructing indexes to compare production units across space, there is no natural ordering of the data to determine which observations have the most similar structure of prices and quantities (similar structures lead to the most accurate index number comparisons). We could take one country as the base country and then construct fixed base indexes of output and input across the production units in our sample using our favorite bilateral index number formula. However, it turns out that the resulting indexes are not invariant to the choice of the base country. Gini (1931) provided a simple solution to this lack of invariance problem: he constructed fixed base Fisher indexes using each country in turn as the base country and then he took the geometric mean of these base country specific sequences of indexes. His method for making international price or quantity comparisons is known as the GEKS (Gini, Eltetö, Köves and Szulc) method.

Caves, Christensen and Diewert (1982b) showed how Gini's methodology could be used to construct indexes of output, input and productivity for cross sectional or panel data sets, which consisted of prices and quantities for the inputs used and outputs produced for the production units. But instead of using the Fisher formula to do the aggregation of outputs and inputs, they drew on the results of Caves, Christensen and Diewert (1982a) and used Törnqvist output and input indexes as their bilateral index number formula. The major advantage of this adaptation of Gini's approach is that we were able to give a strong production theory justification for the use of the Törnqvist formula, drawing on the results in Caves, Christensen and Diewert (1982a) for the case of constant returns to scale technologies where there were no inconsistent results. Although the methodology in Caves, Christensen and Diewert (1982b) has been widely used, there are two problems with it that are sometimes troublesome: (i) all output and input quantities have to be positive for every observation in the panel and (ii) the methodology cannot be used to make comparisons of value added (or GDP) across production units; only gross outputs can be compared using this methodology. The problem is that the CCD methodology relies on output and input distance functions and output distance functions do not exist in general if there are negative outputs (i.e., intermediate inputs) in the output aggregate. This second difficulty has recently been addressed by Inklaar and Diewert (2016) where instead of making functional form assumptions on the output distance function, we followed the approach taken by Diewert and Morrison (1986) and made functional form assumptions on the value added or GDP functions that characterize the production units. This new approach allowed us to have value added or GDP as the output concept and to make TFP comparisons across production units.

Around this time, you also got involved with giving advice to Statistics Canada. How did this come about?

Martin Wilk became the Chief Statistician of Statistics Canada at the end of 1980. He was a professional statistician and he had some definite ideas on how to improve Statistics Canada. He decided that his staff should interact more with the public and the academic community and so he organized a conference on price measurement that was held in Ottawa in November, 1982. Since I had written a fair number of papers on index number theory by that time, I was asked to help organize the conference and I and Claude Montmarquette (a Ph.D student of mine) edited the conference proceedings which appeared in 1983. Another improvement that was initiated by Martin Wilk was to set up technical advisory committees for the different divisions of Statistics Canada. The members of these committees were academics, business economists, knowledgeable users and staff members from other statistical agencies. This was a good idea and it has been copied widely elsewhere (in the United States and Australia for example). I was the chair of the first of these Statistics Canada advisory committees, the Prices Advisory Committee, starting in 1983 and continuing to the present. Wilk's decision to make me chair of the Prices Advisory Committee had some important implications for me later in life, as we shall see. Martin retired as Chief Statistician in 1985 but he continued to serve on various advisory committees such as the Services and the Science and Technology Committees, where I was also a member. He had very strong views on almost everything and we had some vigorous discussions about many issues. But he was basically a nice guy and I enjoyed arguing with him over the years (and I think he also enjoyed our discussions).

Was the 1982 Statistics Canada Conference on Price Measurement a success?

I think that it was a big success. We were able to publish Robert Pollak's classic 1971 paper on the CPI in the volume, Dale Jorgenson and Daniel Slesnick had a classic paper on social cost-of-living indexes and Wolfgang Eichhorn and Joachim Voeller had a classic paper on the axiomatic or test approach to index number theory. I should mention here that Wolfgang played an important part in keeping alive research in the area of index number theory; he had a series of conferences on index number theory in the 1970s and 1980s that were held in Karlsruhe, Germany. I was able to attend all of these conferences except the first one. They were great conferences! During one of these conferences, I developed the axiomatic approach to making multilateral index number comparisons; see Diewert (1988) (1999b).

Did any of your own research appear in the Statistics Canada Conference volume?

Yes, I published three papers in this volume, Diewert (1983d) (1983e) and (1983f). The first paper was on cost-of-living indexes and the problems associated with aggregating them. The second paper broke some new ground; it was my first paper that attempted to measure the effects of changes in the terms of trade. It used the properties of GDP functions and was a precursor to my research with Cathy Morrison. The final paper was on the problems that seasonal commodities cause for consumer price indexes. In particular, *strongly seasonal commodities* (commodities which are not available in all seasons of the year) cause major problems. Price index number theory relies on the *matching principle*; i.e., we attempt to match the prices of commodities in one period with exactly the same commodities in another period. Now suppose there are 4 seasons and 4 commodities but commodity i is only available in the marketplace in season i for i = 1,2,3,4. It can be seen that it is impossible to construct a quarterly price index under these conditions; no matches are possible as we progress through the seasons in a year. This is obviously an extreme example, but you can see that strongly seasonal commodities are going to cause us grief if we want to construct quarterly or monthly price indexes. What can be done under these circumstances? Obviously, we could construct an annual index: simply compare the quarterly price in this year with the corresponding quarterly price in a base year, do this for each quarter, and we can construct annual indexes in the usual way. This method for dealing with strongly seasonal commodities in the context of an annual index was suggested by Mudgett (1955) and Stone (1956). I suggested a modification of this annual index and that was to construct a rolling year annual index; i.e., take the data for the last 4 consecutive quarters and compare these data with the corresponding seasons in a base year. This gives rise to Rolling Year Mudgett Stone indexes. These indexes can deal satisfactorily with strongly seasonal commodities but they have the drawback that they only give longer run trend estimates of inflation and hence they are not suitable measures of quarter-to-quarter (or month-to-month) inflation. An implication of this paper is that the existence of strongly seasonal commodities means that more than one CPI should be constructed in order to measure short and long term inflation.

I should note that my interest in this topic was stimulated by a paper by Ralph Turvey (1979) who constructed an artificial data set (with strongly seasonal commodities) and sent it to every major statistical agency in the world and asked them to use their normal seasonal adjustment procedures to construct a monthly index for his data set. Needless to say, the indexes which were sent to him in reply were all over the map! I maintain that Rolling Year Mudgett Stone indexes are suitable *target indexes* for central banks that have inflation targets but I have to acknowledge that this idea has not caught on! This is an area of research that I keep returning to since it is so difficult to provide completely satisfactory solutions to the problem of strongly seasonal commodities. For my later work in this area, see Diewert (1998a) (1999a) (2014a), Alterman, Diewert and Feenstra (1999), Armknecht and Diewert (2009), Feenstra and Diewert (2001), Diewert, Alterman and Feenstra (2011), Diewert, Armknecht and Nakamura (2011) and Diewert, Finkel and Artsev (2011). The last paper pointed out a problem with the Stone Mudgett rolling year indexes: namely, that empirically, fresh fruits and vegetables did not always appear in the marketplace in the same months of successive years. This is a problem that still has not been resolved. I should note that Robert Feenstra is another student of mine; he did his undergrad honours thesis under my supervision at UBC.

What else caused you to realize that official data were not always reliable?

On June 10, 1985, while I was visiting Stanford, I read an article in the San Francisco Chronicle on the growth of sales of those huge earth satellite dishes that were becoming popular around that time. Over the 5 years 1980-1985, the article gave the US sales of these dishes which were (in thousands), 4, 20, 60, 225 and 450. The article also gave the average prices of a dish for those years which were (in thousands of dollars), 40, 20, 10, 5 and 2. When I saw these figures, I thought to myself: I wonder if earth satellite dishes are in the US CPI? Of course, the answer was no and then I thought: I wonder how many other new goods either did not enter the CPI basket of products at all or until the price of the new product had dropped dramatically?

It was then that I realized that it was not an easy matter to calculate price indexes or to measure real output accurately. I recalled the comment by Zvi Griliches on my flexible functional form paper that I gave at the University of Chicago in 1969 (that the data did not support the estimation of all of the parameters in a flexible functional form). I realized that Zvi was right and from that time on, I devoted most of my research effort to improving economic measurement. I gave a talk on the new goods problem at Zvi's NBER Productivity Workshop on July 14, 1986 based on the Chronicle article. I reviewed the early history of the new goods problem in Diewert (1993b; 59-63). Following Hicks' (1940; 114) analysis of the new good problem, I produced several additional papers on this topic; see Diewert (1980; 498-503), (1987; 779), (1996) (1998b; 51-54). As a side note on the NBER Productivity Workshop, Zvi Griliches started this workshop in 1978 at Stanford but he soon moved it to the NBER in Cambridge Massachusetts. He ran this workshop until his untimely death in 1999 and then Ernst Berndt (a former colleague of mine at UBC) took over and ran it until 2011.

This workshop brought together academics who were interested in economic measurement as well as economic statisticians from North America and Europe. It was a tremendously successful workshop. Initially, there was a certain lack of communication between the official statisticians and the academics attending the workshop: the statisticians were reluctant to confess that some of their estimates were really not all that good and the academics did not realize the constraints that the statisticians faced. But over the years, this distrust completely vanished and thanks, largely to the efforts of Griliches and Berndt, both sides have worked together to solve practical (but difficult) measurement problems. I attended virtually all of the Productivity Workshops of this group and learned a lot.

It seems that in the middle to late 1980s your attention once again turned to the issue of finding flexible functional forms with nice properties.

Yes; as I mentioned earlier, a problem with existing flexible functional forms for say a cost or expenditure function was that the estimated functional form was usually not locally concave in prices at the data points in the sample. This is a major problem since an implication of cost minimizing behavior is that the estimated cost or expenditure function be concave in prices. It was possible to impose concavity on the Generalized Leontief and translog cost functions but the methods suggested in the literature for doing this were not satisfactory. Concavity could be imposed globally on the Generalized Leontief cost function by squaring the off diagonal coefficients in the matrix of parameters that characterize this functional form's unit cost function, but this method had the drawback of forcing all inputs to be substitutes so that it ruled out complementary inputs a priori and thus destroyed the flexibility of the functional form. Similarly, concavity could be imposed on the translog unit cost function at a single point but experience showed that this generally did not lead to global concavity and thus again, the flexibility of the functional form was destroyed. Terrence Wales and I addressed these problems in a series of papers.

In Diewert and Wales (1985), we suggested $c(p) \equiv b \cdot p + (1/2)(p \cdot Bp/\alpha \cdot p)$ as a functional form for a unit cost function where the vector of parameters b is strictly positive, α is a nonnegative, nonzero vector of coefficients that is determined by the investigator (usually chosen to be a vector of ones or to equal the sample average of the input quantity vectors in the production context) and B is an N by N symmetric negative semidefinite matrix of parameters that satisfies N constraints of the form $Bp^* = 0_N$ where 0_N is a vector of zeros and p^* is an arbitrary strictly positive vector (usually chosen to be the first input price vector in the sample). It turns out that this is a parsimonious flexible functional form for a unit cost function so it has the minimum number of free parameters to be a flexible functional form. It can be seen that if B is a matrix of zeros, then this functional form collapses to a Leontief unit cost function. We showed that if the estimated B matrix was negative semidefinite, then the functional form for the unit cost function was globally concave. But what happens if the estimated B matrix is not negative semidefinite? Then we are back to the same problem we had with the existing flexible functional forms; i.e., they do not satisfy the required curvature conditions. But Diewert and Wales solved this problem: if the estimated B matrix is not negative semidefinite, replace it with $-AA^T$ where A is an N by N lower triangular matrix and A^T is the transpose of A. It can be verified that $B = -AA^T$ is automatically negative semidefinite and we also showed that any negative semidefinite matrix could be represented in this form. We had a parsimonious flexible functional form which would automatically be globally concave in input prices. We called this functional form the *normalized quadratic* unit cost function.

We generalized the results in this paper in various directions over the next decade. In Diewert and Wales (1988), we introduced the concept of *semi-flexible* functional forms. As I mentioned earlier, in models where N is large, it becomes impossible to estimate flexible functional form in the time series context because the number of unknown parameters is roughly $N^2/2$ which may be larger than the available number of degrees of freedom. But we suggested that it is not necessary to estimate all of the parameters in the lower triangular A matrix: one could start by setting $B = -a^1a^{1T}$ where a^1 is an arbitrary N dimensional vector of parameters. Then in the next nonlinear regression, set $B = -a^1a^{1T} - a^2a^{2T}$ where a^2 is another N dimensional vector of parameters where the first component is set equal to 0. And so on. At some stage, we stop the procedure of adding more columns to the A matrix, either because we are getting low on degrees of freedom or because at some stage k of the algorithm, the estimated a^k vector turns out to be a vector of zeros. At this stage, we know that we have fitted the most negative semidefinite matrix for B that is consistent with our data. This is a very useful paper because it shows that parsimonious flexible functional form techniques can be applied to models where the number of commodities is quite large.

In Diewert and Wales (1992) (1993), we generalized the basic normalized quadratic functional form to situations where either the technology or preferences are nonhomothetic by splining the b vector. Finally, in Diewert and Wales (1995a), we showed how the normalized quadratic functional form could be used to test for separability. In Diewert (1992a), I showed that there was a bilateral index number formula that was exact for the normalized quadratic unit cost function and hence this index was another example of a superlative index.

Did you work with Terry Wales on any other topics?

We did one more joint paper, Diewert and Wales (1995b). There are many nonparametric methods for smoothing a time series. Each method has a smoothing parameter, which will trade off smoothness of the smoothed series with the fit of the smooth. The smoothing parameter which we suggested was the number of regions of concavity and convexity for the smooth; i.e., the smoothing parameter is the maximum number of wiggles that are allowed for the smoothed version of the original series. If one is interested in long run trends, then we would only want a few changes of curvature. If one wanted to look at business cycles, then we would allow for more changes of curvature. It turns out that to solve this problem, one needs to solve a lot of quadratic programming problems so we worked out an efficient algorithm for doing this. We did some applications of the method and it seemed to work very well. I really liked this paper, but referees did not like it at all. They wanted an asymptotic theory for the model, which I thought was rather ridiculous. In any case, we eventually succeeded in publishing it in a volume. Perhaps in the future, econometricians may rediscover the method and popularize it!

You have mentioned some joint work with Cathy Morrison, a UBC PhD student of yours. Perhaps you could expand on this?

I regard Diewert and Morrison (1986) as a very important paper on how to measure Total Factor Productivity in a production theory context. Total Factor Productivity growth between two points in time for a production unit is generally measured as an output quantity index divided by the corresponding input quantity index. But this definition is not immediately connected to production theory and leaves open exactly how to choose the bilateral index number formula that measures aggregate output and input growth for the production unit. Thus Caves, Christensen and Diewert (1982a) (1982b) addressed this problem in a satisfactory manner for constant returns to scale technologies using translog distance functions to describe the technologies at the two points in time and then finding an appropriate exact index number formula to describe productivity growth. However, as I mentioned earlier, this methodology cannot be applied to situations where the output aggregate includes intermediate inputs. In particular, if the output aggregate is the value added for an industry or the GDP for an economy, the CCD methodology cannot be applied since the output distance function is not well defined in this situation.

Diewert and Morrison solved this problem by making general translog assumptions on the value added or GDP functions for the production units at the two points in time (instead of making general translog assumptions on the distance functions). The end result is that we provided a strong justification, based on production theory, for measuring TFP growth as the implicit Törnqvist output index divided by the (direct) Törnqvist input index. Moreover, we showed how the overall Törnqvist input index could be decomposed exactly into a product of terms involving the input growth of each individual input and we developed a similar decomposition of the Törnqvist output price index into the product of terms involving the rate of price change for each individual price in the output aggregate. Diewert and Morrison (1986; 668) went on to apply this price change analysis to determine the effects on the GDP of an economy of changes in export and import prices; i.e., we attempted to determine the "welfare" effects on an economy of changes in the terms of trade.

We realized that an increase in export prices or a decrease in import prices, holding all else constant, should lead to an increase in "welfare" that is similar to a productivity improvement. We attempted to develop a framework that would measure both effects in a consistent production theory context. However, the analysis was not completely convincing and a satisfactory resolution of this problem did not emerge until two decades later as we shall see. I should note that Ulrich Kohli (1990) independently worked out the Diewert and Morrison translog measure of TFP growth but he took our analysis one step further: he rearranged the TFP growth measure that we both derived and he obtained a decomposition of nominal GDP growth of the production unit between the two periods into a product of explanatory factors, including output price inflation, productivity growth and real input growth. This is a very useful decomposition for macro economists. Kohli (1978) also was one of first economists to assume that all exports and imports flowed through the production sector of an economy. Using this assumption, a large portion of trade theory could be analyzed in a production theory context instead of in a much more complicated general equilibrium context. Diewert and Morrison used this idea and I have used it many times in other papers; e.g., see Diewert (1983c) (1983e). Finally, I should mention that Ulrich was my second Ph.D student.

Turning now to the 1990s, did you address any new measurement problems?

Yes, I ventured into a new area of research for me and that is to come up with tractable functional forms for consumer preferences over states of nature that are uncertain. I was inspired by a paper by Blackorby, Donaldson and Davidson (1977) where they derived the class of preferences that are implied by the expected utility theorem by using separability arguments. In Diewert (1993c; 401-433), I extended their separability approach to more general classes of preferences that could be characterized by implicitly separable preferences over states of nature. I applied this more general model to problems in modeling insurance, gambling and investing. I followed up this paper with a more specific model that could be applied to insurance and gambling, Diewert (1995b). Another Ph.D student of mine, Kam Yu (2009) generalized my nonexpected utility gambling model to an actual lottery and his generalization worked well. I think that a lot more could be done with this approach to modeling choice under uncertainty.

I believe that you first became acquainted with Marshall Reinsdorf in the early 1990s?

I first met Marshall on January 7, 1993 when I discussed his paper, "Seller Substitution Bias in the US Consumer Price Index" at the Anaheim meetings of the American Economic Association. In this paper, Marshall compared US CPI components for gasoline and food products with corresponding average price series compiled by the Bureau of Labor Statistics (BLS) over the years 1980-1992. He found that the BLS CPI component series grew somewhere between 1.1% and 1.4% per year faster than their corresponding average price series over this period. These results indicated the possibility of a considerable amount of upward bias in the official US CPI, which is used for a wide variety of indexation purposes. In my discussion of the paper, I remarked that "this paper is the measurement paper of the decade". A revised version of this paper was eventually published as Reinsdorf (1998). However, a discussion paper version of the paper emerged in 1994 and this paper had a very large influence on the measurement community. It started a detailed study of how CPIs were constructed in practice and led to the Boskin Commission Report on bias in the US CPI in 1996; see Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1996). The Boskin Report had implications around the world.

Marshall attributed his results to problems with the BLS procedures for aggregating specific price quotes at the first stage of aggregation where information on quantities is not available. A bilateral price index that does not make use of quantity information is called an elementary index. After discussing Marshall's paper in Anaheim, I got interested in elementary indexes and I soon wrote a discussion paper on the topic in January of 1995 which was published as Diewert (1995b). This was actually quite a good paper (my estimates for the amount of bias in the CPI were listed in this paper and were similar to those of the Boskin commission plus I introduced the axiomatic approach to elementary indexes). I followed up this paper with a couple of papers on bias in the CPI, Diewert (1996) (1998b). In the first paper, I basically defended the Boskin Commission's estimates of the upward bias in the US CPI as being in the range 1.3 to 1.7% per year and in the second paper, I tried to provide an analytic framework for the numerical estimation of the various biases that might exist in consumer price indexes.

To sum up: Marshall's paper led to substantial changes in how CPIs are constructed around the world. I would also like to praise Paul Armknecht, who was in charge of the Prices Division of the BLS at the time Marshall produced his research. Paul was brave enough to allow Marshall to present his results at the Anaheim meetings even though he knew there would be some substantial fallout from this decision. My discussion of Marshall's paper in 1993 led to a friendship with him and a nice joint paper; see Diewert, Reinsdorf and Ehemann (2002).

The Ottawa Group on Price Indices got started around this time. How did you manage to become a founding member of this Group?

This is an interesting story. The United Nations Economic Commission for Europe and the International Labour Organization team up every two years and hold the Meeting of the Group of Experts on Consumer Price Indices in Geneva in May. These experts were members of national and international statistical agencies. During the 1994 meeting of this Group, three participants got together after a day of listening to country reports for a beer or two and they lamented the fact that the meetings did not have a very high research component at that time. These participants were Paul Armknecht (head of the Prices Division at the BLS), Bert Balk (a very well-known index number theorist at the Dutch Central Bureau of Statistics) and Bohdan Schultz (head of price research at Statistics Canada and a founding father for the GEKS method for making international comparisons). Around this time, UN City Groups were forming. These City Groups were set up by national and international statistical agencies to have periodic meetings which would present research papers on an area of economic statistics. The idea was that national statistical agencies face similar measurement problems but have limited research resources, and these City Group meetings offer an efficient way of transmitting advances in economic measurement across a wider audience. Essentially, the research efforts of individual statistical agencies could be pooled by these meetings. The first of these UN sponsored City Groups was the Voorburg Group on Services Statistics which met in Voorburg for the first time in 1987. In any case, Armknecht, Balk and Schultz thought that it would be a good idea to form a UN City Group on Price Indices. Bohdan went back to Statistics Canada after the Geneva meeting and convinced Jacob Ryten, the Deputy Chief Statistician, to form the Ottawa Group (more formally, the UN International Working Group on Price Indices). The first meeting of this Group was held in Ottawa, October 31-November 4, 1994. Because I was the Chair of the Statistics Canada Prices Advisory Committee, I was invited to this inaugural meeting. I was able to circulate Diewert (1995b) at this meeting and I believe this paper had some influence on John Astin, who attended the meeting and was the father of Eurostat's Harmonized Index of Consumer Prices (HICP). My paper looked at the properties of elementary indexes, which are price indexes between two periods which are functions of only the price vectors, say p^0 and p^1, for the same N commodities for periods 0 and 1. Examples of elementary price indexes are the arithmetic and geometric averages of the price ratios, known as the Carli index, $P_C(p^0,p^1) \equiv \Sigma_{n=1}^N (1/N)(p_n^1/p_n^0)$, and the Jevons index, $P_J(p^0,p^1) \equiv [\Pi_{n=1}^N (p_n^1/p_n^0)]^{1/N}$. An important property for an elementary index is the time reversal test, which is $P(p^0,p^1)P(p^1,p^0) = 1$ where P is a generic elementary index. This test says if prices in period 2 revert back to the prices of period 0, then the product of the price indexes going from period 0 to 1 and then from 1 back to 0 should equal unity; i.e., the price index should end up where it started from under these conditions. The Jevons index satisfies this test but the Carli index does not and worse, it has a built in upward bias; i.e., $P_C(p^0,p^1)P_C(p^1,p^0) \geq 1$ with a strict inequality unless p^1 is proportional to p^0. Diewert (1995b) pointed out this fact, but it did not originate with me. Fisher (1922; 66) seems to have been the first to establish the upward bias of the Carli index and he made the following observations on its use by statistical agencies:

"In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose." Irving Fisher (1922; 29-30).

Unfortunately, many statistical agencies did not heed Fisher's warning and some agencies still continue to use the Carli index in constructing their CPIs. But fortunately, John Astin did heed Fisher's warning and he outlawed the use of the Carli formula as an input into the HICP. As an aside, in Diewert (2002b), I was somewhat critical of the HICP for basically two reasons: (i) the scope of the index did not fit into the framework of the System of National Accounts (it was in between a household CPI and a producer price index for consumption expenditures) and (ii) it tried to outlaw the use of any kind of imputation. But imputation is required in a CPI for at least two reasons: (i) quality change is pervasive and adjusting for it requires imputations and (ii) an imputation for the flow of services from consumer durables is required in order to measure total consumption services for households. The second problem is a serious one for measuring the service flow from Owner Occupied Housing (OOH). The HICP did not really know how to deal with OOH so it simply excluded it from the index. Unfortunately, this destroys the comparability of the HICP across countries which have differing degrees of home ownership.

The Ottawa Group had its 14th meeting in Tokyo in 2015 and I have had the privilege of attending every meeting. Since 2007, the Ottawa Group has met every second year, alternating with meetings of the ILO/UNECE Experts on Consumer Price Indices in Geneva (which I also attend). These two Groups have been tremendously influential in transmitting improvements in the measurement of prices across a much wider audience. Very few academics attend these meetings so I am honoured to be able to participate in these meetings and be accepted as a useful contributor. The decision of Martin Wilk to form a Prices Advisory Committee for Statistics Canada had far reaching consequences for my research.

Evidently, your participation in these groups led to your participation in a series of international price measurement manuals.

Yes, the ILO and UNECE realized that it was time to revise the existing international price index manuals in the light of new developments. The members of the Ottawa Group and other statistical agency experts were called upon to help draft new manuals on price Indexes; see ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004a) (2004b) (2009). The *Consumer Price Index Manual: Theory and Practice* was the first of these Manuals (edited by Peter Hill) followed by the corresponding *Producer Price Index Manual* (edited by Paul Armknecht) and the *Export and Import Price Index Manual* (edited by Mick Silver). I wrote a large proportion of the theoretical parts of these manuals, with the help of the editors and Bert Balk, David Fenwick, Carsten Hansen and others.

In the CPI Manual, I showed that the four major approaches to index number theory (basket approaches, axiomatic or test approaches, stochastic approaches and economic approaches) led to the same three bilateral index number formulae: the Fisher ideal index, the Törnqvist and the Walsh (1901) index. I have defined the first two formulae earlier and the definition of the Walsh bilateral price index is $P_W(p^0,p^1,q^0,q^1) \equiv \sum_{n=1}^N p_n^1(q_n^0q_n^1)^{1/2}/\sum_{j=1}^N p_j^0(q_j^0q_j^1)^{1/2}$. The fixed basket approach provides a strong justification for the use of the Fisher and Walsh formulae. The axiomatic approach leads to the Fisher index as being "best" while a stochastic approach leads to the Törnqvist as being "best". Finally, the economic approach provides a strong justification for the use of all three indexes. The material that I wrote up in the manuals on these four approaches to index number theory drew on my earlier published work: see Diewert (1997) on the basket approach, Diewert (1992b) on the axiomatic approach, Diewert (2011a) on the stochastic approach (this paper appeared in discussion paper form in 1995) and Diewert (1976b) (2001a) on the economic approach.

At the first meeting of the Ottawa Group in 1994, there was a major split among the participants. The participants from North America tended to favour the economic approach while the participants from Europe tended to favour the other approaches. But based on Diewert (1978b), I argued in the manuals that it did not matter so much which approach was chosen: each approach leads to the same three indexes which will numerically approximate each other to the second order around an equal price and quantity point. Eventually, this point of view was accepted and in later meetings of the Ottawa Group, we stopped arguing about which approach to index number theory should be chosen and focused on other measurement problems.

You made the above point in all three manuals, so you obviously thought there was a need to reiterate this in the different contexts of your theoretical chapters.

Yes, there was. But there was some new material that was developed in the *Producer Price Index Manual* that is important: basically, I showed that the traditional method for forming real estimates for outputs and intermediate inputs in an input-output framework did not lead to accurate estimates. The traditional method uses the *same* price index to deflate an entire row of value estimates for commodities produced or used by industries for a particular commodity class. But each commodity in the Input-Output (I-O) tables is actually an aggregate of hundreds if not thousands of individual products and the transactions in a given commodity class across two industries will have micro product weights that are specific to the bilateral transactions in that commodity class for the two industries under consideration. Thus the construction of accurate real I-O tables requires more information on individual transactions than will be available to statistical agencies. The lesson here is that published real I-O tables are inherently unreliable using traditional methodology and hence industry productivity estimates that rely on these tables should be regarded with some caution. Diewert (2006b) addressed additional problems with Input-Output tables that are due to the treatment of tax and transportation margins. I also developed some new material on the problems caused by transfer prices in the *Export and Import Price Index Manual*. The difficulties associated with the estimation of transfer prices is a growing problem as globalization proceeds; see also Diewert (1985c) on these transfer pricing issues.

You also started to do some consulting in Australia during the mid-1990s.

Yes, Denis Lawrence was a former Ph.D student of mine, and he really trained me to be a reasonably effective consultant. Starting in 1994, I would visit Denis in Canberra for a month or so every summer for many years and we would work on various consulting problems in three areas: (i) measuring the marginal excess burdens of taxes using flexible functional form techniques; (ii) measuring the productivity of countries using superlative index number techniques and (iii) measuring the productivity of regulated firms as an aid to more effective regulation. Published papers on excess burdens include Diewert and Lawrence (1994) (1996) (1998) (2002) and Diewert, Lawrence and Thompson (1998). The 2002 paper introduced an important generalization of the normalized quadratic functional form: we showed how this functional form could be adapted so that it would be flexible at two data points instead of just at a single data point. Published papers on measuring the TFP of countries and firms include Diewert and Lawrence (1999), (2000), (2005) (2006) and Lawrence, Diewert and Fox (2006). Published papers on measurement problems in the regulatory context include Lawrence and Diewert (2006), Lawrence, Diewert and Fox (2006) and Diewert, Lawrence and Fallon (2009). I also wrote a number of papers with you starting from this period in three main areas: (i) the theory of regulation (Diewert and Fox 2000); (ii) measurement errors as an explanation for the productivity slowdown that occurred around the world starting in the mid-1970s (Diewert and Fox 1999a, 2001) and (iii) measuring productivity (Diewert and Fox 1999b, 2008, 2010, 2011, 2014a, 2014b, 2015).

In addition to joining the Ottawa Group on Prices Measurement in 1994, you also joined another similar UN Group in 1997, namely the Canberra Group on Capital Measurement.

Yes, there were actually two separate Canberra Groups on capital measurement. Canberra I (officially the Expert Group on Capital Stock Statistics) had meetings over the years 1997-1999 and Canberra II (the Expert Group on the Measurement of

Nonfinancial Assets) ran from 2003-2007. I attended all of the meetings of these two groups and I wrote a number of research papers as a result of my interactions with these Groups; see for example, Diewert and Schreyer (2008) on capital theory. These areas of research can be grouped into five main areas.

The first main area was concerned with the *measurement of depreciation*. My interest in this area actually started many years ago: in Diewert (1977; 117) (1980; 473), I developed a very general model of depreciation where the amount of depreciation that occurred over a time period depended on the intensity of use of the asset and other inputs which could offset depreciation of the asset. I later realized that my model was pretty closely related to an accounting framework which was proposed by Hicks (1961) and the accountants Edwards and Bell (1961). These authors worked out a very nice framework for placing durable inputs into a single period production function model: at the beginning of an accounting period, the production unit has available to it a vector of durable capital inputs, which it values at beginning of the period opportunity costs and it combines these durable inputs with a vector of variable (nondurable) inputs to produce a vector of outputs and a vector of end-of-the-period (depreciated) durable inputs (which are to be valued at expected or actual market opportunity cost values). The variable outputs and inputs and the vector of end of period capital stocks are to be discounted at one plus the production unit's cost of capital. This one period model of production is a very sensible one and enables us to deal with variable depreciation rates in a very satisfactory manner. I spelled out the implications for the measurement of depreciation and the construction of user costs using this framework in Diewert (2010a).

The second capital measurement problem that I addressed as a result of working with the Canberra Groups was how to measure *inventory change* and the user cost of inventories in the Hicks, Edwards and Bell accounting framework; see Diewert and Smith (1994) and Diewert (2005b). In the latter paper, I noted that index number theory breaks down if the value aggregate can change sign between periods and this is clearly the case if the value aggregate is inventory change (or net exports). If we take inventory stocks at the beginning and end of the accounting period, these stocks can be deflated using normal index number theory and real inventory change can be measured as the difference between the deflated stocks. I note that the Bureau of Economic Analysis now uses this method for defining real inventory change.

The third capital measurement problem that I addressed during this period was how to measure the contribution of *R&D investments* in production. The Canberra Group II recommended introducing R&D investments as a productive asset into the international System of National Accounts. Prior to the 2008 version of the SNA, investments in R&D were immediately expensed which is clearly inappropriate since the benefits of a successful R&D project persist for many periods. However, it is also inappropriate to treat cumulated investment expenditures on an R&D project in the same manner as we treat investments in machinery and equipment and structures. The capital stocks that correspond to these durable inputs yield capital services which are inputs into a production function and can be varied over time. However, a successful R&D project generates a recipe or blueprint for either making a new commodity or for providing a more efficient method for making an old commodity. This R&D "input" cannot be varied over time; i.e., R&D capital stocks are very different animals from traditional reproducible capital stocks and require a different accounting framework.

Basically, a successful R&D project has a cost: the cumulated investment expenditures associated with it. Going forward, this cost is a fixed cost. The benefits of the project are the discounted stream of excess cash flows that the successful blueprint generates. Depreciation of the R&D asset in an accounting period is basically the loss of the excess cash flow that the project generated. This is very difficult to measure and so rather than tackle the difficult measurement issues associated with the fixed cost nature of R&D investments, national income accountants have simply made more or less arbitrary assumptions about R&D depreciation rates and treated R&D stocks in exactly the same manner as they would treat investments in trucks. Ning Huang (another former student of mine) and myself developed a rather complicated method for estimating R&D depreciation rates and more importantly, worked out how the System of National Accounts would have to be restructured to deal with R&D capital stocks in a more satisfactory manner; see Diewert and Huang (2011a) (2011b).

My fourth area of research into capital measurement is associated with the third area above and that is determining the implications for the measurement of capital services and depreciation when we have *sunk costs*; see Diewert (2009c) for the implications of machinery sunk costs and Diewert and Fox (2016a) for the implications of sunk costs in land and structures.

A fifth contribution to capital theory that I made around this time was to provide a solution to the *negative user cost problem*. The usual user cost of land is a financial opportunity cost of using the land for an accounting period; i.e., it is equal to the beginning of the period value of the land less the discounted expected (or actual) value of the land at the end of the period. Using actual end of period values of land almost always lead to negative user costs for at least some years over the longer run and even using expected end of period land prices leads to at least occasional negative user costs for land. This user cost for land is a financial cost of postponing selling the land for one period. But there is another way of valuing the cost of using the land for the accounting period under consideration and that is what one could rent the services of the land for during the period. This opportunity cost valuation for the services of land during the accounting period will always be nonnegative and typically will be positive. Thus in Diewert (2011a), I suggested the *opportunity cost approach* to the valuation of land services, which sets the value of land services to the *maximum* of the rental price and the user cost of land. The discussion paper version of this paper appeared in 2007. The resulting valuation for the services of owned land will always be nonnegative. This approach was followed up by Diewert, Nakamura and Nakamura (2009) and Diewert and Nakamura (2011).

In 1999, you started attending the meetings of a new group that focused on economic measurement problems.

Yes, this was the Economic Measurement Group annual workshop, usually held in December which started at the University of New South Wales in 1999 under your leadership and grew from an initial attendance of 6 people to well over 100 experts from all over the world. I have attended 14 of the 15 meetings of this group. One of the most influential papers to come out of these workshops was Ivancic, Diewert and Fox (2011), which introduced the Rolling Year GEKS method for addressing the chain drift problem which arises if scanner data and a chained superlative index are used in the construction of a CPI.

The problem of *chain drift* is caused by sales of a product: during a sale, the price drops to say half of its regular price but the quantity sold on sale can grow by a factor of 1000. When the sale is over and the product goes back to its regular price, the volume sold is typically less than the volume sold just before the sale (due to the fact that consumers have stocked up on the product) and this causes chained superlative indexes to register a level that is somewhat below the pre-sale level, which leads to a downward bias in the index over time. Jan de Haan (2008) brought this problem to our attention during the 2008 Economic Measurement Workshop and this paper led to the published papers on the chain drift problem by de Haan and van der Grient (2011) and Ivancic, Diewert and Fox (2011). In the latter paper, drawing on the earlier work of Balk (1981), we suggested a solution to this problem which drew on the GEKS method described above for making international comparisons between countries. Our suggested solution to the chain drift problem was to treat 13 consecutive months of price and quantity data on a class of consumer expenditures as if they corresponded to the data for 13 countries and use the GEKS method to construct a sequence of price levels for the expenditure category for the 13 consecutive months. It turns out that the resulting indexes are not subject to the chain drift problem. However, a new problem arises: what happens when new price and quantity data for the next month becomes available? Our suggested solution to the problem was to drop the data pertaining to the first month in our initial string of 13 months and add the new month’s data to the remaining 12 months of data, compute new GEKS indexes for the new sample of 13 months and use the rate of change in the new GEKS indexes for the last two months to update the index level for the 13th month in our original sample. We called the resulting method for constructing price indexes over time the *Rolling Year GEKS* method.

In the 2009 discussion paper version of Ivancic, Diewert and Fox (2011), we also suggested adapting another method for making international comparisons, the Rolling Year Weighted Time Product Dummy (RYTPD) Method, to address the chain drift problem; the unweighted version of this method for making international comparisons is due to Summers (1973) and the weighted version is due to Rao (2005) and Diewert (2004) (2005c). We found that the Rolling Year GEKS and the RYWTPD methods gave much the same results in an empirical example. I expect that these new methods for dealing with the chain drift problem will be incorporated into the construction of Consumer Price Indexes in the years to come. The EMG Workshops presented many other influential measurement papers over the years.

It seems that there is a general capacity to adapt to the time series context index number methods that were originally suggested for making comparisons across countries .

Yes. For example, Diewert and Fox (2011) adapted multilateral index number theory to deal with the problems associated with measuring the contribution of entering and exiting firms to aggregate productivity growth.

You have also worked on the problems associated with quantifying the benefits of a favourable change in a country's terms of trade.

I started thinking about this problem in the 1980s in Diewert (1983e) and Diewert and Morrison (1986) but the real breakthrough came in 2005 when I did a research paper on this topic for the Bureau of Economic Analysis, Diewert (2005d). The first basic idea in this paper runs as follows. I followed the example of Kohli (1978) and assumed that exports are produced by the production sector of the economy and that all imports into the economy are either used by the production sector or have some value added to them by transportation and retailing inputs. The production sector of an economy produces nominal GDP over a time period t and this value of output is distributed to the primary inputs used in production. Let the nominal GDP function for the economy in period t be $g^t(p^t,x^t) \equiv \max_y\{p^t \cdot y: (y,x^t) \in S^t\}$ where y is a net output vector (if a component of y is an imported commodity, this quantity component has a minus sign attached to it), x^t is the positive vector of primary inputs used by the production sector during period t, p^t is the vector of positive prices of output prices and import prices that faces the economy during period t and S^t is the period t production possibilities set for the economy.

At this point, we can use the methodology developed by Diewert and Morrison (1986) and Kohli (1990) to decompose the growth of GDP over two periods (and hence decompose the growth in nominal income generated by the production sector over the two periods) into a product of three effects: (i) growth of net output prices; (ii) growth of primary inputs and (iii) technical progress (i.e., expansion of the production possibilities set). The growth effects on nominal income of changes in import and export prices can be precisely measured using the Diewert-Morrison-Kohli (DMK) methodology. Instead of using nominal prices p^t for period t, we could deflate these prices by an appropriate consumer price index for period t and then we could use the same DMK methodology to get a decomposition of the real income generated by the production sector over the two periods into changes in real net output prices, growth of primary inputs and technical progress and these effects can be precisely measured. This was the first main idea in Diewert (2005d).

The second main idea was related to my work on the user cost of capital. The usual user cost of capital contains a depreciation term and this is quite appropriate if we are working in a GDP framework. But gross product cannot be consumed; depreciation should be deducted from the value of gross output if we want to have an estimate of (sustainable) income that could be spent on consumption. Thus in the second half of Diewert (2005d), I advocated moving to a net product framework. It is easy to do this: simply decompose the user cost of capital into the sum of two terms: one involving depreciation and the other consisting of the remaining terms. Now take the part of capital services involving depreciation out of the primary input category and treat this expenditure as an additional intermediate input expenditure (like imports) and subtract it from gross product to form net product; see Diewert (2010a) for a justification of this procedure. Now apply the same DMK decomposition as before with our new classification of (net) outputs and primary inputs. We applied this new decomposition to Japanese, Australian, Canadian and US data; see Diewert, Mizobuchi and Nomura (2005), Diewert and Lawrence (2006), Diewert and Yu (2012) and Diewert (2014b) respectively. I note that Hide Mizobuchi, Denis Lawrence and Emily Yu were all Ph.D students of mine.

In addition to your work on "regular" index number theory, you have also published papers on the difference approach to index number theory; can you explain this approach?

If we take the test or axiomatic approach to regular index number theory, a fundamental equation is the one which sets the value ratio for the aggregate over periods 0 and 1, say $p^1 \cdot q^1/p^0 \cdot q^0$, equal to the product of the price index $P(p^0,p^0,q^1,q^0)$ times the quantity index $Q(p^0,p^0,q^1,q^0)$ so we have $p^1 \cdot q^1/p^0 \cdot q^0 = P(p^0,p^0,q^1,q^0)Q(p^0,p^0,q^1,q^0)$. This approach to index number theory can be described as the *ratio approach* to index number theory; i.e., the price index function is interpreted as an aggregate price ratio, P^1/P^0, using this approach. The *difference approach* to index number theory sets the difference in the value aggregate over the two periods, $p^1 \cdot q^1 - p^0 \cdot q^0$, equal to the sum of the *indicator of price change* $\Delta P(p^0,p^0,q^1,q^0)$ plus the *indicator of quantity change* $\Delta Q(p^0,p^0,q^1,q^0)$ so we have $p^1 \cdot q^1 - p^0 \cdot q^0 = \Delta P(p^0,p^0,q^1,q^0) + \Delta Q(p^0,p^0,q^1,q^0)$. In this approach, the function $\Delta P(p^0,p^0,q^1,q^0)$ is interpreted as an aggregate price difference, $P^1 - P^0$. Using this approach, we impose reasonable tests or properties on the function $\Delta P(p^0,p^0,q^1,q^0)$ so that we can determine a "best" functional form that satisfies these properties. Note that once $\Delta P(p^0,p^0,q^1,q^0)$ is determined, $\Delta Q(p^0,p^0,q^1,q^0)$ is also determined as $p^1 \cdot q^1 - p^0 \cdot q^0 - \Delta P(p^0,p^0,q^1,q^0)$. Examples of indicators of price change are the Laspeyres indicator, $\Delta P_L \equiv q^0 \cdot (p^1 - p^0)$, the Paasche indicator, $\Delta P_P \equiv q^1 \cdot (p^1 - p^0)$, and the Bennet (1920) indicator, $\Delta P_B \equiv (1/2)(q^1 + q^0) \cdot (p^1 - p^0)$. Note that the Laspeyres and Paasche indicators of aggregate price change are counterparts to the Laspeyres and Paasche price indexes that occur in the ratio approach to index number theory. In Diewert (2005e), I developed the test approach to determining the "best" functional form for the indicator of price change and found that the Bennet indicator was "best"; i.e., it was the difference approach counterpart to the Fisher ideal price index which was "best" using the ratio approach to index number theory.

In addition to the above test approach to index number theory using differences, there is also an economic approach. I will first outline the economic approach to index number theory using ratios. Let C(u,p) be the cost or expenditure function that is dual to a consumer's utility function, f(q). Then Konüs (1939) defined the consumer's *true cost of living index* between periods 0 and 1 that corresponds to the reference utility level u = f(q) as the ratio $C(u,p^1)/C(u,p^0)$ where p^t is the vector of prices that the consumer faces in period t for t = 0,1. Konüs singled out two special cases of this family of price indexes by setting u equal to $f(q^0)$ and to $f(q^1)$ where q^t is the consumer's observed consumption vector for period t for t = 0,1. This leads to the two price indexes, $C(u^0,p^1)/C(u^0,p^0)$ and $C(u^1,p^1)/C(u^1,p^0)$. There is a corresponding economic approach to quantity indexes that also uses the cost function. Allen (1949) defined a family of quantity indexes as $C(f(q^1),p)/C(f(q^0),p)$ where p is a positive vector of reference prices. It is natural to set p equal to p^0 or p^1, leading to the two special cases of this family of quantity indexes, $C(f(q^1),p^0)/C(f(q^0),p^0)$ and $C(f(q^1),p^1)/C(f(q^0),p^1)$. As an aside, I applied this theoretical framework for price and quantity indexes to find exact and superlative price and quantity indexes for the case of nonhomothetic preferences in Diewert (2009d). This is the essence of the economic approach to index number theory, using ratios. But the ratios in this theory can be replaced by differences. Samuelson (1974) realized that C(f(q),p) was a valid cardinalization of the consumer's utility function for any reference price vector p and thus $C(f(q^1),p) - C(f(q^0),p)$ is a valid cardinal measure of the difference in the consumer's utility between periods 0 and 1. Again, it is useful to set the reference price vector p equal to p^0 or p^1. Thus Hicks (1942; 127-128) defined the *equivalent and compensating variations* as the differences $C(f(q^1),p^0) - C(f(q^0),p^0)$ and $C(f(q^1),p^1) - C(f(q^0),p^1)$. Hicks (1946; 331-332) also defined the following measures of price change (or income compensation changes), $C(f(q^0),p^1) - C(f(q^0),p^0)$ and $C(f(q^1),p^1) - C(f(q^1),p^0)$. It can be seen that these measures are difference counterparts to the corresponding Konüs true cost of living indexes. In Diewert (1992a) and Diewert and Mizobuchi (2009), we looked for flexible functional forms for the utility function f such that we could identify these Hicksian measures of price and utility change using observed data on prices and quantities for the two periods under consideration; i.e., we looked for superlative indicators of price and quantity change.

In recent years you have provided advice to the World Bank on their International Comparison Program (ICP). What is this program concerned with?

The World Bank has taken the lead in a worldwide partnership to collect comparable price and expenditure data on the components of GDP for most of the countries in the world. National price indexes cannot be compared across countries because they have different units of measurement. Thus the ICP collects prices of individual commodities in national currencies but in the same physical units of measurement that are comparable across countries. These comparable prices are aggregated into national price indexes called PPPs (Purchasing Power Parities). The PPPs are then used to deflate the components of a country's GDP into aggregate quantities (or volumes) that are (in principle) comparable across countries. Thus the ICP enables us to compare real GDP (and components of GDP) across countries. The ICP is a fundamental building block for the Penn World Tables which are widely used to compare real output and consumption across countries over time. The actual collection of the individual commodity prices has historically only taken place once every 5 to 10 years; the last two rounds of price collection took place in 2005 and 2011. The World Bank has set up a Technical Advisory Group (TAG) to provide methodological guidance for the construction of the PPPs. Because I had written papers on the properties of multilateral price indexes during the 1980s and 1990s (see Diewert (1988) (1999b)), I was invited to join the TAG for the 2005 round and initially, I was the Chair of the TAG for the 2011 round of the ICP. It proved to be too difficult to be the Chair since I was so far away from Washington D.C. and so I resigned as the Chair and was replaced by Paul McCarthy and Fred Vogel as joint Chairs. One very interesting methodological problem that we faced for the 2005 and 2011 ICP rounds was how to adapt existing multilateral index number theory (which treated each country in a perfectly symmetric manner) to deal with situations where a subgroup of countries (e.g., European Union Countries) first undertook a comparison of real GDP for all countries in the subgroup and then entered into a worldwide comparison of GDP by country with the restriction that the worldwide comparison be consistent with the subgroup comparison. This problem is of practical importance since some EU subsidy programs depend on the relative magnitude of the real GDPs of countries within the EU. Thus the EU wanted the European and worldwide relative estimates of GDP for EU countries to be consistent. I developed methodologies to deal with this consistency in aggregation problem as well as other ICP problems; see Diewert (2004) (2010b) (2010c) (2013a) (2013b). It was a pleasure to work with the members of the TAG; in particular, I would like to express my gratitude to Bettina Aten, Angus Deaton, Bert Balk, Yuri Dikhanov, Alan Heston, Robert Hill, Francette Koechlin, Paul McCarthy, Prasada Rao, Sergey Sergeev, Mick Silver, Marcel Timmer, Kim Zieschang and Fred Vogel for helpful discussions and comments over the years. I should add that I was not always persuasive in my recommendations to this group: I was (and still am) very much in favour of using Robert Hill's (2001) (2004) (2009) spatial linking method for making international comparisons of real output across countries but the TAG favoured using GEKS as the primary method.

It seems that another area of research that you have contributed to in recent years is how to quality adjust prices using hedonic regression techniques.

A hedonic regression is a regression of prices of a product at a point in time on the quantities of its price determining characteristics. But there are many different ways that this basic methodology can be adapted to construct constant quality price indexes over time. I initially did not want to get involved in this area; I thought that I would leave the methodological problems in this area for experts in the area like

Jack Triplett and Mick Silver to solve. However, I eventually got involved. In Diewert (2003a), I tried to look at a hedonic regression from the viewpoint of traditional consumer theory and find restrictions on preferences which would justify the usual hedonic regression approach. In Diewert (2003b), I took a somewhat systematic look at many of the unresolved issues in this area. In Diewert (2006a), I looked at the axiomatic properties of a two period time dummy hedonic regression model. In Diewert, Heravi and Silver (2009), we did a detailed comparison of the time dummy and hedonic imputation methods for running hedonic regressions. Jan de Haan (2010) did an excellent follow up paper which cast more light on these issues. You will recall our earlier discussion on the Rolling Year Weighted Time Product Dummy method for dealing with the chain drift problem (Ivancic, Diewert and Fox, 2009). This method was successfully married to hedonic regression methods for quality adjustment in a brilliant paper by de Haan and Krsinich (2014).

In recent years, you and your coauthors have looked at a particular application of hedonics, namely its application to the construction of property price indexes.

In the last 5 or 6 years, I tried to address the problems associated with the lack of information in the national accounts of most countries on the price and quantity of land in the economy. This information on land is needed in order to estimate the Total Factor Productivity of industries and economies as well as to estimate rates of return on assets in the economy. There are also problems with our estimates of depreciation rates on structures in the national accounts. To address these problems, I suggested a simple hedonic regression model which I called the *builder's model*; see Diewert (2011a). When I was a university student many years ago, I used to work for my father, Ewald Diewert, where we built a house in Vancouver each summer. The value of the newly built house was equal to the cost of the land plot that the structure sits on plus the cost of construction. This simple observation can be turned into a hedonic regression model. Let V_{tn} be the value of the new house n in a local area during time period t, let S_{tn} be the floor space area of the structure and let L_{tn} be the area of the land plot. Then we could postulate that the value of the house is equal to cost of the land plus the cost of the structure plus an error term ε_{tn}; i.e., we have $V_{tn} = \alpha_t L_{tn} + \beta_t S_{tn} + \varepsilon_{tn}$ where α_t is the price of land per unit area and β_t is the construction cost per unit of floor space area in period t. Note that we are assuming that the price of land and the building cost are constant over houses in the local area during period t. This regression model can be generalized to the case where "used" houses are sold. Let A(t,n) be the age of house n in period t (in years) and suppose that the used house n was sold in period t at price V_{tn}. Then the new hedonic regression model is $V_{tn} = \alpha_t L_{tn} + \beta_t (1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}$ where δ is the annual geometric depreciation rate for the structure. Thus we now have a nonlinear regression model where the land prices α_t, the new structure construction cost prices β_t and the geometric depreciation rate δ are the unknown parameters to be estimated. I worked with the Dutch statisticians Jan de Haan and Rens Hendriks to implement this model using data on house sales from the small Dutch town of "A". However, when we implemented the model, we found that there was substantial multicollinearity between L_{tn} and S_{tn}; i.e., large land plots came with larger structures. Thus although the resulting overall price index was reasonable, the estimated α_t and β_t were unreasonably volatile; i.e., they tended to move in opposite directions over time. We tried various methods to overcome this multicollinearity problem and we eventually set the period t structure price β_t equal to a single unknown quality adjustment parameter β times an official construction cost index, say p_t. This cured the multicollinearity problem and generated a reasonable land price series α_t for the town of "A"; see Diewert, de Haan and Hendriks (2011) (2015). Chihiro Shimizu and I generalized the above very simple builder's model to encompass additional housing characteristics in Diewert and Shimizu (2015a). The builder's model is also described in some detail in a Handbook that Eurostat commissioned to act as a guide for statistical agencies to construct house price indexes; see de Haan and Diewert (2011). For an adaptation of the builder's model to the sales of condominium units, see Diewert and Shimizu (2015b). The adaptation is not straightforward since the height of the building, and the height of the unit that is sold, affect the value of the unit and there are also problems associated with determining the unit's share of common space in the building. We also adapted the builder's model to the commercial property context; see Diewert and Shimizu (2016). This paper points out a limitation of using the builder's model to determine structure depreciation. In addition to the structure depreciation rate that is estimated by the builder's model, there is an additional source of depreciation that the model does not pick up: namely *demolition depreciation*. This is the depreciation which results when a building is demolished before it reaches the end of its planned lifetime. Diewert and Shimizu (2016) found that the "regular" depreciation rate that we determined applying our hedonic regression model to Tokyo commercial properties was about 2% per year but demolition depreciation added an additional 2% per year to determine an overall (geometric) commercial property depreciation rate of approximately 4% per year. Finally, for broader discussions on how to organize the price and quantity data for commercial properties in the System of National Accounts, see Diewert and Shimizu (2015c), Diewert and Fox (2016a) and Diewert, Fox and Shimizu (2016).

Since the turn of the century, you seem to have addressed a number of problems associated with productivity measurement. Could you elaborate on your efforts in this area?

I would first like to thank Alice Nakamura for her help on property price indexes but also on the measurement of TFP; e.g., see Diewert and Nakamura (1999) (2003). Alice also organized a series of very successful workshops in Ottawa with the help of the Bank of Canada on productivity topics during the last 15 years. During this time period, I wrote a number of papers on productivity topics. Several of these papers complained a bit about the slow progress that national statistical agencies have made in improving our estimates of national and industry TFP growth; see Diewert (2001b) (2008a) (2008b). The biggest problem is the lack of information on land used in production. Another problem is that the System of National accounts does not present enough detail on the incidence of taxes which makes it difficult to construct accurate user costs and to construct sectoral prices that producers face.

Here is a list of other productivity issues that I (and my coauthors) have attempted to address in recent years. (i) In the Nonmarket Sector of an economy, there are no prices to value the outputs produced by this sector and so national income accountants have measured output growth by the corresponding input growth. Using this methodology, TFP growth is automatically zero. In Diewert (2011b) (2012b), I suggested several methods for dealing with this problem in a more satisfactory manner. (ii) If estimates of an industry's best practice technology are available, then following Balk (2003), the TFP growth of a firm in that industry can be decomposed into a product of explanatory factors such as increases in the firm's technical efficiency, returns to scale and technical progress. In Diewert (2014c), I provided a similar decomposition under the assumption that an estimate of the industry's best practice cost function is available. (iii) Following the example of Tulkens (1993), in Diewert and Fox (2014a), we provided a general framework for the decomposition of productivity growth into explanatory factors making weaker assumptions on the reference best practice technology; i.e., we assumed that the reference technology satisfied free disposability rather than the usual convexity assumption. In Diewert and Fox (2014b), we showed how our model could be implemented by setting the reference technology equal to the free disposal hull of the observations in a sample of observations. (iv) In Diewert (2015a), I showed how economy wide TFP growth (or Labour Productivity Growth) could be decomposed into industry contribution factors. (v) In the productivity literature, it has long been known that estimated TFP growth using gross output as the output concept is much smaller than estimated TFP growth using value added as the output concept. In Diewert (2015b), I showed the exact relationship between these two ways of measuring TFP growth if Laspeyres, Paasche or Fisher indexes are used to aggregate outputs and inputs. (vi) Finally, Diewert and Wei (2016) suggested that geometric depreciation is not an appropriate depreciation model for computers, rather one hoss shay depreciation (or a more general model of depreciation) is more appropriate. We illustrated the difference between our suggested model and the geometric model using Australian data.

Another research area that you have looked at recently is the measurement of financial services in the System of National Accounts.

Yes, this is a very difficult area where there is little agreement in the literature on how to proceed. Diewert, Fixler and Zieschang (2012) took a user cost approach to the problems associated with measuring the outputs and inputs of banks and illustrated how alternative treatments of inputs and outputs led to different measures of bank output using US data on the banking sector. In Diewert (2014d), I used a similar user cost approach to financial transactions to provide a framework for integrating a firm's financial transactions with its "real" production decisions. There are still many issues to be resolved in this area.

Any final words?

I would like to thank all of my coauthors over the years (including the ones that we did not discuss in this interview): it has been a pleasure working with you over the years! I would also like to thank my wife, Virginia Diewert, for her support over all the years that we have been married.

References

Afriat, S.N. (1967), "The Construction of Utility Functions from Finite Expenditure Data", *International Economic Review* 8, 67-77.

Afriat, S.N. (1972a), "Efficiency Estimation of Production Functions", *International Economic Review* 13, 568-598.

Afriat, S.N. (1972b), "The Theory of International Comparisons of Real Income and Prices", pp. 13-69 in *International Comparisons of Prices and Output*, D.J. Daly (ed.), Studies in Income and Wealth, Volume 37, New York: National Bureau of Economic Research.

Afriat, S.N. (1973), "On a System of Inequalities in Demand Analysis: An Extension of the Classical Method", *International Economic Review* 14, 460-472.

Allen, R.G.D. (1949), "The Economic Theory of Index Numbers", *Economica* 16, 197-203.

Allen, R.C. and W.E. Diewert (1981), "Direct Versus Implicit Superlative Index Number Formulae", *The Review of Economics and Statistics* 63:3, 430-435.

Alterman, W.F., W.E. Diewert and R.C. Feenstra (1999), *International Trade Price Indexes and Seasonal Commodities*, Washington D.C.: Bureau of Labor Statistics.

Armknecht, P. and W.E. Diewert (2009), "Treatment of Seasonal Products", pp. 546-580 in the *Export and Import Price Index Manual, Theory and Practice*, Washington D.C.: International Monetary Fund.

Avriel M., W.E. Diewert, S. Schaible and I. Zang (2010), *Generalized Concavity*, SIAM Classics in Applied Mathematics, Vol. 63, Philadelphia: Society for Industrial and Applied Mathematics.

Balk, B.M. (1981), "A Simple Method for Constructing Price Indices for Seasonal Commodities", *Statistische Hefte* 22:1, 1-8.

Balk, B.M. (2003), "The Residual: On Monitoring and Benchmarking Firms, Industries and Economies with Respect to Productivity", *Journal of Productivity Analysis* 20, 5-47.

Bennet, T.L. (1920), "The Theory of Measurement of Changes in Cost of Living", *Journal of the Royal Statistics Society* 83, 455-462.

Berndt, E.R., M.N. Darrough and W.E. Diewert (1977), "Flexible Functional Forms and Expenditure Distributions: An Application to Canadian Consumer Demand Functions", *International Economic Review* 18, 651-675.

Blackorby, C., R. Davidson and D. Donaldson (1977), "A Homiletic Exposition of the Expected Utility Hypothesis", *Economica* 44, 351-358.

Blackorby, C. and W.E. Diewert (1979), "Expenditure Functions, Local Duality, and Second Order Approximations", *Econometrica* 47, 579-601.

Boskin, M.J. (Chair), E.R. Dullberger, R.J. Gordon, Z. Griliches and D.W. Jorgenson (1996), *Final Report of the Commission to Study the Consumer Price Index*, U.S. Senate, Committee on Finance, Washington DC: US Government Printing Office.

Byushgens, S.S. (1925), "Ob odnom klasse giperpoverkhnostey: po povodu 'idealnova indeksa' Irving Fisher a pokupatelnoi sili deneg", Mathematischeskii Sbornik 32, 625-631.

Caves, D.W., L.R. Christensen and W.E. Diewert (1982a), "The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity", *Econometrica* 50, 1393-1414.

Caves, D.W., L.R. Christensen and W.E. Diewert (1982b), "Multilateral Comparisons of Output, Input and Productivity using Superlative Index Numbers", *Economic Journal* 96, 659-679.

Christensen, L.R., D.W. Jorgenson and L. Lau (1971), "Conjugate Duality and the Transcendental Logarithmic Production Function", *Econometrica* 39, 255-256.

Debreu, G. (1951), "The Coefficient of Resource Utilization", *Econometrica* 19, 273-292.

de Haan, J. (2008), "Reducing Drift in Chained Superlative Price Indexes for Highly Disaggregated Data", paper presented at the EMG Workshop 2008, December 10-12, Crowne Plaza Hotel, Coogee Beach, Sydney.

de Haan, J. (2010), "Hedonic Price Indexes: A Comparison of Imputation, Time Dummy and 'Re-pricing' Methods'', *Journal of Economics and Statistics* (*Jahrbücher fur Nationalökonomie und Statistik)* 230, 772-791.

de Haan, J. and W.E. Diewert (eds.) (2011), *Residential Property Price Indices Handbook*, Luxembourg: Eurostat.

de Haan, J. and H.A. van der Grient (2011), "Eliminating Chain drift in Price Indexes Based on Scanner Data", *Journal of Econometrics* 161, 36-46.

de Haan, J. and F. Krsinich (2014), "Scanner Data and the Treatment of Quality Change in Nonrevisable Price Indexes," *Journal of Business and Economic Statistics*, 32:3, 341-358.

Diewert, W.E. (1971), "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function", *Journal of Political Economy* 79, 481-507.

Diewert, W.E. (1973), "Functional Forms for Profit and Transformation Functions", *Journal of Economic Theory* 6, 284-316.

Diewert, W.E. (1974a), "Functional Forms for Revenue and Factor Requirements Functions", *International Economic Review* 15, 119-130.

Diewert, W.E. (1974b), "Applications of Duality Theory", pp. 106-171 in *Frontiers of Quantitative Economics*, Volume 2, M.D. Intriligator and D.A. Kendrick (eds.), Amsterdam: North-Holland.

Diewert, W.E. (1974c), "Intertemporal Consumer Theory and the Demand for Durables", *Econometrica* 42:3, 497-516.

Diewert, W.E. (1974d), "A Note on Aggregation and Elasticities of Substitution", *Canadian Journal of Economics* 7, 12-20.

Diewert, W.E. (1976a), "Harberger's Welfare Indicator and Revealed Preference Theory", *American Economic Review* 66:1, 143-152.

Diewert, W.E. (1976b), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 114-145.

Diewert, W.E. (1977a), "Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium", pp. 73-126 in *Equilibrium and Disequilibrium in Economic Theory*, E. Schwödiauer (ed.), Dordrecht: Reidel Publishing.

Diewert, W.E. (1977b), "Generalized Slutsky Conditions for Aggregate Consumer Demand Functions", *Journal of Economic Theory* 15:2, 353-362.

Diewert, W.E. (1978a), "Optimal Tax Perturbations", *Journal of Public Economics* 10, 138-177.

Diewert, W.E. (1978b), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Diewert, W.E. (1978c), "Hicks' Aggregation Theorem and the Existence of a Real Value Added Function", pp. 17-51, Vol. 2, *in Production Economics: A Dual Approach to Theory and Applications*, M. Fuss and D. McFadden, editors, North-Holland, Amsterdam.

Diewert, W.E. (1980), "Aggregation Problems in the Measurement of Capital", pp. 433-528 in *The Measurement of Capital*, D. Usher (ed.), Chicago: The University of Chicago Press.

Diewert, W.E. (1981a), "The Elasticity of Derived Net Supply and a Generalized Le Chatelier Principle", *Review of Economic Studies* 48, 63-80.

Diewert, W.E. (1981b), "The Comparative Statics of Industry Long Run Equilibrium", *Canadian Journal of Economics* 14, 78-92.

Diewert, W.E. (1981c), "The Measurement of Deadweight Loss Revisited", *Econometrica* 49, 1225-1244.

Diewert, W.E. (1981d), "Generalized Concavity and Economics", pp.511-541 in *Generalized Concavity in Optimization and Economics*, S. Schaible and W.T. Ziemba (eds.), New York: Academic Press.

Diewert, W.E. (1981e), "Alternative Characterizations of Six Kinds of Quasiconcavity in the Nondifferentiable Case with Applications to Nonsmooth Programming", pp. 51-93 in in *Generalized Concavity in Optimization and Economics*, S. Schaible and W.T. Ziemba (eds.), New York: Academic Press.

Diewert (1981f) "The Economic Theory of Index Numbers: A Survey", pp. 163-208 in *Essays in the Theory and Measurement of Consumer Behaviour in Honour of Sir Richard Stone*, edited by A. Deaton, Cambridge: Cambridge University Press.

Diewert, W.E. (1983a), "Cost-Benefit Analysis and Project Evaluation: A Comparison of Alternative Approaches", *Journal of Public Economics* 22:3, 265-302.

Diewert, W.E. (1983b), "A Fundamental Matrix Equation of Production Theory with Applications to the Theory of International Trade", pp. 181-194 in *Methods of Operations Research*, Vol. 46, P. Stahly (ed.), Konigstein: Verlag Anton Hain.

Diewert, W.E. (1983c), "The Measurement of Waste Within the Production Sector of an Open Economy", *Scandinavian Journal of Economics* 85:2,159-179.

Diewert, W.E. (1983d), "The Theory of the Cost-of-Living Index and the Measurement of Welfare Change", pp. 163-233 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada.

Diewert, W.E. (1983e), "The Theory of the Output Price Index and the Measurement of Real Output Change", pp. 1049-1113 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada.

Diewert, W.E. (1983f), "The Treatment of Seasonality in a Cost of Living Index", pp. 1019-1045 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada.

Diewert, W.E. (1984a), "Sensitivity Analysis in Economics", *Computers and Operations Research* 11:2, 141-156.

Diewert, W.E. (1984b), "The Measurement of Deadweight Loss in an Open Economy", *Economica* 51:1, 23-42.

Diewert, W.E. (1985a), "Measurement of Waste and Welfare in Applied General Equilibrium Models", pp. 42-102 in *New Developments in Applied General Equilibrium Analysis*, J. Piggot and J. Whalley, (eds.), Cambridge: Cambridge University Press.

Diewert, W.E. (1985b), "A Dynamic Approach to the Measurement of Waste in an Open Economy", *Journal of International Economics* 19, 213-240.

Diewert, W.E. (1985b), "Measurement of Waste and Welfare in Applied General Equilibrium Models", pp. 42-102 in *New Developments in Applied General Equilibrium Analysis*, J. Piggot and J. Whalley, (eds.), Cambridge: Cambridge University Press.

Diewert, W.E. (1985c), "Transfer Pricing and Economic Efficiency", pp 47-81 in *Multinationals and Transfer Pricing*, A.M. Rugman and L. Eden (eds.), London: Croom Helm, London.

Diewert, W.E. (1987), "Index Numbers", pp. 767-780 in *The New Palgrave A Dictionary of Economics*, Vol. 2, J. Eatwell, M. Milgate and P. Newman (eds.), London: The Macmillan Press.

Diewert, W.E. (1988), "Test Approaches to International Comparisons", pp. 67-86 in *Measurement in Economics: Theory and Applications of Economic Indices*, W. Eichhorn (ed.), Heidelberg: Physica-Verlag.

Diewert, W.E. (1992a), "Exact and Superlative Welfare Change Indicators", *Economic Inquiry* 30, 565-582.

Diewert, W.E. (1992b), "Fisher Ideal Output, Input and Productivity Indexes Revisited", *Journal of Productivity Analysis* 3, 211-248.

Diewert, W.E. (1993a), "Duality Approaches to Microeconomic Theory", pp. 105-175 in *Essays in Index Number Theory*, Volume 1, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland.

Diewert, W.E. (1993b), "The Early History of Price Index Research", pp. 33-65 in *Essays in Index Number Theory*, Volume 1, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland.

Diewert, W.E. (1993c), "Symmetric Means and Choice under Uncertainty"', pp. 355-433 in *Essays in Index Number Theory*, Volume 1, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North-Holland.

Diewert, W.E. (1995a), "Functional Form Problems in Modeling Insurance and Gambling", *The Geneva Papers on Risk and Insurance Theory* 20, 135-150.

Diewert, W.E. (1995b), "Axiomatic and Economic Approaches to Elementary Price Indexes", *Proceedings of the International Conference on Price Indices*, October 31-November 2, 1994, L.M. Ducharme (ed.), First Meeting of the International Working Group on Price Indices, Ottawa: Statistics Canada.

Diewert, W.E. (1996), "Comment on CPI Biases", *Business Economics*, Vol. 31:2, 30-35.

Diewert, W.E. (1997), "Commentary on Mathew D. Shapiro and David W. Wilcox: Alternative Strategies for Aggregating Prices in the CPI", *The Federal Reserve Bank of St. Louis Review*, Vol. 79:3, 127-137.

Diewert, W.E. (1998), "Index Number Issues in the Consumer Price Index", *The Journal of Economic Perspectives*12:1, 47-58.

Diewert, W.E. (1998), "High Inflation, Seasonal Commodities and Annual Index Numbers", *Macroeconomic Dynamics* 2, 456-471.

Diewert, W.E. (1999a), "Index Number Approaches to Seasonal Adjustment", *Macroeconomic Dynamics* 3, 1-21.

Diewert, W.E. (1999b), "Axiomatic and Economic Approaches to International Comparisons", pp. 13-87 in *International and Interarea Comparisons of Income, Output and Prices*, A. Heston and R.E. Lipsey (eds.), Studies in Income and Wealth, Volume 61, Chicago: The University of Chicago Press.

Diewert, W.E. (2001a), "The Consumer Price Index and Index Number Purpose", *The Journal of Economic and Social Measurement* 27, 167-248.

Diewert, W.E. (2001), "Which (Old) Ideas on Productivity Measurement are Ready to Use?" pp. 85-101 in *New Developments in Productivity Analysis*, C.R. Hulten, E.R. Dean and M.J. Harper (eds.), Chicago: University of Chicago Press.

Diewert, W.E. (2002a), "The Quadratic Approximation Lemma and Decompositions of Superlative Indexes", *Journal of Economic and Social Measurement* 28, 63-88.

Diewert, W.E. (2002b), "Harmonized Indexes of Consumer Prices: Their Conceptual Foundations", *Swiss Journal of Economics and Statistics* 138, 547-637.

Diewert, W.E. (2003a), "Hedonic Regressions: A Consumer Theory Approach", pp.317-348 in *Scanner Data and Price Indexes*, R.C. Feenstra and M.D. Shapiro (eds.), Chicago: University of Chicago Press.

Diewert, W.E. (2003b), "Hedonic Regressions: A Review of Some Unresolved Issues", Paper presented at the Seventh Meeting of the Ottawa Group, Paris, France, May 27-29.

Diewert, W.E. (2004), "On the Stochastic Approach to Linking the Regions in the ICP", Department of Economics, Discussion Paper 04-16, University of British Columbia, Vancouver, B.C., Canada, V6T 1Z1.

Diewert, W.E. (2005a), "Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes and Interest Rates", pp. 479-542 in Measuring *Capital in the New Economy*, C. Corrado, J. Haltiwanger and D. Sichel (eds.), Chicago: University of Chicago Press.

Diewert, W.E. (2005b), "On Measuring Inventory Change in Current and Constant Dollars", Discussion Paper 05-12, Department of Economics, University of British Columbia, Vancouver, B.C., Canada.

Diewert, W.E. (2005c), "Weighted Country Product Dummy Variable Regressions and Index Number Formulae", *Review of Income and Wealth* 51:4, 561-571.

Diewert, W.E. (2005d), "Welfare, Productivity and Changes in the Terms of Trade", Report prepared for the Bureau of Economic Analysis, Washington D.C., November 22, 2005.

Diewert, W.E. (2005e), "Index Number Theory Using Differences Rather Than Ratios", *American Journal of Economics and Sociology* 64:1, 311-360.

Diewert, W.E. (2006a), "Adjacent Period Dummy Variable Hedonic Regressions and Bilateral Index Number Theory", Annales d'Économie et de Statistique, No. 79/80, 759-786.

Diewert, W.E. (2006b), "Comment", pp. 287-307 in *A New Architecture for the U.S. National Accounts*, D.W. Jorgenson, J.S. Landefeld and W.D. Nordhaus (eds.), Studies in Income and Wealth, Volume 66, Chicago: University of Chicago Press.

Diewert, W.E. (2008a), "What Is To Be Done For Better Productivity Measurement", *International Productivity Monitor*, No. 16 (Spring), 40-52.

Diewert, W.E. (2008b), "OECD Workshop on Productivity Analysis and Measurement: Conclusions and Future Directions", pp. 11-36 in *Proceedings From the OECD Workshops on Productivity Measurement and Analysis*, Paris: OECD.

Diewert, W.E. (2009a), "Durables and Owner Occupied Housing in a Consumer Price Index", pp. 445-506 in *Price Index Concepts and Measurement*, NBER, Studies in Income and Wealth Vol. 70, W.E. Diewert, J. Greenlees and C.R. Hulten (eds.), Chicago: University of Chicago Press.

Diewert, W.E. (2009b), "Similarity Indexes and Criteria for Spatial Linking", Chapter 8 in *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, D.S. Prasada Rao (ed.), Cheltenham, UK: Edward Elgar, (2009), pp. 183-216.

Diewert, W.E. (2009c), "The Aggregation of Capital over Vintages in a Model of Embodied Technical Progress", *Journal of Productivity Analysis* 32:1, 1-19.

Diewert, W.E. (2009d), "Cost of Living Indexes and Exact Index Numbers", pp. 207-246 in *Quantifying Consumer Preferences*, edited by Daniel Slottje in the Contributions to Economic Analysis Series, United Kingdom: Emerald Group Publishing.

Diewert, W.E. (2010a), "User Costs versus Waiting Services and Depreciation in a Model of Production", *Journal of Economics and Statistics* 230/6, 759-771.

Diewert, W.E. (2010b), "New Methodological Developments for the International Comparison Program", *Review of Income and Wealth* 56, S11-31.

Diewert, W.E. (2010c), "Understanding PPPs and PPP-based National Accounts", *American Economic Journal: Macroeconomics* 2, 36-45.

Diewert, W.E. (2011a), "The Paris OECD-IMF Workshop on Real Estate Prices: Conclusions and Future Directions", pp. 87-116 in *Price and Productivity Measurement: Volume 1—Housing*, W. Erwin Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), Victoria, Canada: Trafford Press.

Diewert, W.E. (2011b), "Measuring Productivity in the Public Sector: Some Conceptual Problems", *Journal of Productivity Analysis* 36, 177-191.

Diewert, W.E. (2012a), "Afriat's Theorem and Some Extensions to Choice Under Uncertainty", *Economic Journal* 122, 305–331.

Diewert, W.E. (2012b), "The Measurement of Productivity in the Nonmarket Sector", *Journal of Productivity Analysis* 37, 217-229.

Diewert, W.E. (2012c), "On the Stochastic Approach to Index Numbers", pp. 235-262 in *Price and Productivity Measurement: Volume 6, Index Number Theory*, W.E. Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), Trafford Press.

Diewert, W.E. (2013a), "Methods of Aggregation above the Basic Heading Level within Regions", pp. 121-167 in *Measuring the Real Size of the World Economy: The Framework, Methodology and Results of the International Comparison Program—ICP*, Washington D.C.: The World Bank.

Diewert, W.E. (2013b), "Methods of Aggregation above the Basic Heading Level: Linking the Regions", pp. 169-196 in *Measuring the Real Size of the World Economy: The Framework, Methodology and Results of the International Comparison Program—ICP*, Washington D.C.: The World Bank.

Diewert, W.E. (2014a), "An Empirical Illustration of Index Construction using Israeli Data on Vegetables", Discussion Paper 14-04, Vancouver School of Economics, University of British Columbia, Vancouver, B.C., Canada.

Diewert, W.E. (2014b), "US TFP Growth and the Contribution of Changes in Export and Import Prices to Real Income Growth", *Journal of Productivity Analysis* 41, 19-39.

Diewert, W.E. (2014c), "Decompositions of Profitability Change using Cost Functions, *Journal of Econometrics* 183, 58-66.

Diewert, W.E. (2014d), "The Treatment of Financial Transactions in the SNA: A User Cost Approach", *Eurostat Review of National Accounts and Macroeconomic Indicators* 1, 73-89.

Diewert, W.E. (2015a), "Decompositions of Productivity Growth into Sectoral Effects", *Journal of Productivity Analysis* 43, 367-387.

Diewert, W.E. (2015b), "Reconciling Gross Output TFP Growth with Value Added TFP Growth", *International Productivity Monitor*, Number 29 (Fall), 60-67.

Diewert, W.E., W.F. Alterman and R.C. Feenstra (2011), "Time Series Versus Index Number Methods of Seasonal Adjustment," pp. 29-52 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (2009), *Price and Productivity Measurement: Volume 2: Seasonality*, Trafford Press.

Diewert, W.E., P.A. Armknecht and A.O. Nakamura (2011), "Methods for Dealing With Seasonal Products In Price Indexes," chapter 2 in Diewert, W.E., B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (2009), *Price and Productivity Measurement: Volume 2: Seasonality*, Trafford Press, 5-28.

Diewert, W.E., M. Avriel and I. Zang (1981), "Nine Kinds of Quasiconcavity and Concavity", *Journal of Economic Theory* 25:3, 397-420.

Diewert, W.E., Y. Finkel and Y. Artsev (2011), "Empirical Evidence on the Treatment of Seasonal Products: The Israeli Experience", pp. 53-78 in *Price and Productivity Measurement: Volume 2: Seasonality,* W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (eds.), Victoria, Canada: Trafford Press.

Diewert, W.E., J. de Haan and R. Hendriks (2011), "The Decomposition of a House Price Index into Land and Structures Components: A Hedonic Regression Approach", *The Valuation Journal*, 6:1, 58-106.

Diewert, W.E., J. de Haan and R. Hendriks (2015), "Hedonic Regressions and the Decomposition of a House Price Index into Land and Structure Components", *Econometric Reviews*, 34:1-2, 106-126.

Diewert, W.E., D. Fixler and K. Zieschang (2012), "Problems with the Measurement of Banking Services in a National Accounting Framework", Discussion Paper 12-03, Department of Economics, University of British Columbia, Vancouver, B.C., Canada.

Diewert, W.E. and K.J. Fox (1999a), "Can Measurement Error Explain the Productivity Paradox?", *Canadian Journal of Economics* 32:2, 251-280.

Diewert, W.E. and K.J. Fox (1999b), "Is the Asia Pacific Region Different? Technical Progress Bias and Price Estimates for 18 OECD Countries, 1960-1992", pp. 125-144 in *Economic Efficiency and Productivity Growth in the Asia Pacific Region*, T-T. Fu, C. Huang and C.A.K. Lovell (eds.), Cheltenham: Edgar Elgar.

Diewert, W.E. and K.J. Fox (2000), "Incentive Indexes for Regulated Industries", *Journal of Regulatory Economics* 17:1, 5-24.

Diewert, W.E. and K.J. Fox (2001), "The Productivity Paradox and Mismeasurement of Economic Activity", pp. 175-197 in *Monetary Policy in a World Knowledge-based Growth, Quality Change, and Uncertain Measurement*, K. Okina and T. Inoue (eds.), New York: MacMillan.

Diewert, W.E. and K.J. Fox (2008), "On the Estimation of Returns to Scale, Technical Progress and Monopolistic Markups", *Journal of Econometrics* 145, 174-193.

Diewert, W.E. and K.J. Fox (2010), "Malmquist and Törnqvist Productivity Indexes: Returns to Scale and Technical Progress with Imperfect Competition", *Journal of Economics* 101, 73-95.

Diewert, W.E. and K.J. Fox (2011), "On Measuring the Contribution of Entering and Exiting Firms to Aggregate Productivity Growth", pp. 41-66 in *Price and Productivity Measurement, Volume 6, Index Number Theory,* W.E. Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), Trafford Press.

Diewert, W.E. and K.J. Fox (2014a), "Decomposing Bjurek Productivity Indexes into Explanatory Factors", Discussion Paper 14-07, Vancouver School of Economics, University of British Columbia, Vancouver, B.C., Canada.

Diewert, W.E. and K.J. Fox (2014b), "Reference Technology Sets, Free Disposal Hulls and Productivity Decompositions", *Economics Letters* 122, 238–242.

Diewert, W.E and K.J. Fox (2015), "Output Growth and Inflation Across Space and Time", UNSW Business School Research Paper No. 2015 ECON 4, University of New South Wales, Sydney 2052 Australia.

Diewert, W.E and K.J. Fox (2016a), "Sunk Costs and the Measurement of Commercial Property Depreciation", *Canadian Journal of Economics*, forthcoming.

Diewert, W.E and K.J. Fox (2016b), "Alternative User Costs, Rates of Return and TFP Growth Rates for the US Nonfinancial Corporate and Noncorporate Business Sectors: 1960-2014", Discussion Paper 16-03, Vancouver School of Economics, The University of British Columbia, Vancouver, Canada, V6T 1L4.

Diewert, W.E., K.J. Fox and C. Shimizu (2016), "Commercial Property Price Indexes and the System of National Accounts", *The Journal of Economic Surveys*, forthcoming.

Diewert, W.E., S. Heravi and M. Silver (2009), "Hedonic Imputation versus Time Dummy Indexes", pp. 161-202 in *Price Index Concepts and Measurement*, W.E. Diewert, J. Greenlees and C.R. Hulten (eds.), Chicago: University of Chicago Press.

Diewert, W.E. and R.J. Hill (2012), "Alternative Approaches to Index Number Theory", pp. 263-278 in *Price and Productivity Measurement: Volume 6, Index Number Theory*, W.E. Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), Trafford Press.

Diewert, W.E. and N. Huang (2011a), "Estimation of R&D Depreciation Rates: A Suggested Methodology and Preliminary Application", *Canadian Journal of Economics*, 44:2, 387-412.

Diewert, W.E. and N. Huang (2011b), "Capitalizing R&D Expenditures", *Macroeconomic Dynamics*15:4, 537-564.

Diewert, W.E. and D. Lawrence (1994), "New Zealand's Excess Burden of Tax", *Agenda: A Journal of Policy Analysis and Reform*, 2:1,27-34.

Diewert, W.E. and D. Lawrence (1996), "The Deadweight Costs of Taxation in New Zealand", *Canadian Journal of Economics* 29, S658-S673.

Diewert, W.E. and D. Lawrence (1998), "The High Costs of Capital Taxation in Australia", *Agenda: A Journal of Policy Analysis and Reform* 5:1, 355-361.

Diewert, W.E. and D. Lawrence (1999), "Measuring New Zealand's Productivity", New Zealand Treasury Working Paper 99/5, Wellington: NZ Treasury.

Diewert, W.E. and D. Lawrence (2000), "Progress in Measuring the Price and Quantity of Capital", pp.273-326 in *Econometrics Volume 2: Econometrics and the Cost of Capital: Essays in Honor of Dale W. Jorgenson*, Lawrence J. Lau (ed.), Cambridge: The MIT Press (2000).

Diewert, W.E. and D. Lawrence (2002), "The Deadweight Costs of Capital Taxation in Australia", pp. 103-167 in *Efficiency in the Public Sector*, K.J. Fox (ed.), Boston: Kluwer Academic Publishers.

Diewert, W.E. and D. Lawrence (2005), *Estimating Aggregate Productivity Growth for Australia*, Department of Communications, Information Technology and the Arts, Canberra: Australian Government.

Diewert, W.E. and D. Lawrence (2006), *Measuring the Contributions of Productivity and Terms of Trade to Australia's Economic Welfare*, Consultancy Report to the Productivity Commission, Australian Government, Canberra, March.

Diewert, W.E., D. Lawrence and J. Fallon (2009), *The Theory of Network Regulation in the Presence of Sunk Costs*, Report prepared by Economic Insights Pty Ltd for the New Zealand Commerce Commission. www.economicinsights.com.au

Diewert, W.E., D. Lawrence and F. Thompson (1998), "The Marginal Costs of Taxation and Regulation", pp. 135-171 in *Handbook of Public Finance*, F. Thompson and M.T. Green (eds.), New York: Marcel Dekker.

Diewert, W.E. and T.R. Lewis (1982), "The Comparative Dynamics of Efficient Programs of Capital Accumulation and Resource Depletion", pp. 301-325 in *Economic Theory of Natural Resources*, W. Eichhorn, R. Henn, K. Neumann, R.W. Shephard (eds.), Wien: Physica-Verlag.

Diewert, W.E. and N.F. Mendoza (2007), "The Le Chatelier Principle in Data Envelopment Analysis", pp. 63-82 in *Aggregation, Efficiency, and Measurement*, Rolf Färe, Shawna Grosskopf and Daniel Primont (eds.), New York: Springer.

Diewert, W.E. and H. Mizobuchi (2009), "Exact and Superlative Price and Quantity Indicators", *Macroeconomic Dynamics*, 13 (Supplement 2), 335-380.

Diewert, W.E., H. Mizobuchi and K. Nomura (2005), "On Measuring Japan's Productivity, 1955-2003", Discussion Paper 05-22, Department of Economics, University of British Columbia, Vancouver, B.C., Canada.

Diewert, W.E. and C.J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade", *Economic Journal* 96, 659-679.

Diewert, W.E. and A.O. Nakamura (1999), "Benchmarking and the Measurement of Best Practice Efficiency: An Electricity Generation Application", *Canadian Journal of Economics* 32:2, 570-588.

Diewert, W.E. and A.O. Nakamura (2003), "Index Number Concepts, Measures and Decompositions of Productivity Growth", *Journal of Productivity Analysis*, 19, 127-159.

Diewert, W.E. and A.O. Nakamura (2011), "Accounting for Housing in a CPI", (with A.O. Nakamura), pp. 7-32 in *Price and Productivity Measurement: Volume 1—Housing*, W. Erwin Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), Victoria, Canada.

Diewert, W.E., A.O. Nakamura and L. Nakamura (2009), "The Housing Bubble and a New Approach to Accounting for Housing in a CPI", *Journal of Housing Economics* 18:3, 156-171.

Diewert, W.E. and C. Parkan (1983), “Linear Programming Tests of Regularity Conditions for Production Functions”, pp. 131-158 in *Quantitative Studies on Production and Prices*, W. Eichhorn, R. Henn, K. Neumann and R.W. Shephard (eds.), Würzburg: Physica-Verlag.

Diewert, W.E. and C. Parkan (1985), “Tests for the Consistency of Consumer Data”, *Journal of Econometrics* 30, 127-147.

Diewert, W.E., M.B. Reinsdorf and C. Ehemann (2002), “Additive Decompositions for Fisher, Törnqvist and Geometric Mean Indexes”, *Journal of Economic and Social Measurement* 28, 51-61.

Diewert, W.E. and P. Schreyer (2008), “Capital Measurement”, in *The New Palgrave Dictionary of Economics*, Second Edition, S.N. Durlauf and L.E. Blume (eds.), London: Palgrave Macmillan.

Diewert, W.E. and C. Shimizu (2015a), “Residential Property Price Indices for Tokyo”, *Macroeconomic Dynamics* 19, 1659–1714.

Diewert, W.E. and C. Shimizu (2015b), “Hedonic Regression Models for Tokyo Condominium Sales”, Discussion Paper 15-07, School of Economics, The University of British Columbia, Vancouver, Canada, V6T 1Z1.

Diewert, W.E. and C. Shimizu (2015c), “A Conceptual Framework for Commercial Property Price Indexes”, *Journal of Statistical Science and Application* 3:9-10, 131-152.

Diewert, W.E. and C. Shimizu (2016), “Alternative Approaches to Commercial Property Price Indexes for Tokyo”, *Review of Income and Wealth*, forthcoming.

Diewert, W.E. and A.M. Smith (1994), “Productivity Measurement for a Distribution Firm”, *The Journal of Productivity Analysis* 5, 335-347.

Diewert, W.E., A.H. Turunen-Red and A.D. Woodland (1989), “Productivity and Pareto Improving Changes in Taxes and Tariffs”, *Review of Economic Studies* 56, 199-216.

Diewert, W.E., A.H. Turunen-Red and A.D. Woodland (1991), “Tariff Reform in a Small Open Multi-Household Economy with Domestic Distortions and Nontraded Goods”, *International Economic Review* 32, 937-957.

Diewert, W.E. and T.J. Wales (1985), “Flexible Functional Forms and Global Curvature Conditions”, *Econometrica* 55, 43-68.

Diewert, W.E. and T.J. Wales (1988), “A Normalized Quadratic Semiflexible Functional Form”, *Journal of Econometrics* 37, 327-342.

Diewert, W.E. and T.J. Wales (1992), “Quadratic Spline Models For Producer’s Supply and Demand Functions”, *International Economic Review* 33:3, 705-722.

Diewert, W.E. and T.J. Wales (1993), “Linear and Quadratic Spline Models For Consumer Demand Functions”, *Canadian Journal of Economics* 26, 77-106.

Diewert, W.E. and T.J. Wales (1995a), “Flexible Functional Forms and Tests of Homogeneous Separability”, *Journal of Econometrics* 67, 259-302.

Diewert, W.E. and T.J. Wales (1995b), “A ‘New’ Approach to the Smoothing Problem”, pp. 104-144 in *Money, Measurement and Computation*, Michael T. Belongia and Jane M. Binner (eds.), New York: MacMillan.

Diewert, W.E. and H. Wei (2016), "Getting Rental Prices Right for Computers", *Review of Income and Wealth*, forthcoming.

Diewert, W.E. and A.D. Woodland (1977), "Frank Knight's Theorem in Linear Programming Revisited", *Econometrica* 45, 375-398.

Diewert, W.E. and E. Yu (2012), "New Estimates of Real Income and Multifactor Productivity Growth for the Canadian Business Sector, 1961-2011", *International Productivity Monitor*, No. 24, (Fall), 27-48.

Edwards, E.O. and P.W. Bell (1961), *The Theory and Measurement of Business Income*, Berkeley: University of California Press.

Farrell, M.J. (1957), "The Measurement of Production Efficiency", *Journal of the Royal Statistical Society*, Series A, 120, 253-278.

Feenstra, R.C. and W.E. Diewert (2001), "Imputation and Price Indexes: Theory and Evidence from the International Price Program", Bureau of Labor Statistics Working Paper 335, U.S. Department of Labor, Washington D.C.

Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.

Gini, C. (1931), "On the Circular Test of Index Numbers", *Metron* 9:9, 3-24.

Hall, R.E. (1988), "The Relation Between Price and Marginal Cost in US Industry", *Journal of Political Economy* 96, 921-947.

Hanoch, G. and M. Rothschild (1972), "Testing the Assumptions of Production Theory: A Nonparametric Approach", *Journal of Political Economy* 80, 256-275.

Hicks, J.R. (1940), "The Valuation of the Social Income", *Economica* 7, 105-140.

Hicks, J.R. (1942), "Consumers' Surplus and Index Numbers", *The Review of Economic Studies* 9, 126-137.

Hicks, J.R. (1946), *Value and Capital*, Second Edition, Oxford: Clarendon Press.

Hicks, J.R. (1961), "The Measurement of Capital in Relation to the Measurement of Other Economic Aggregates", pp. 18-31 in *The Theory of Capital*, F.A. Lutz and D.C. Hague (eds.), London: Macmillan.

Hill, R.J. (1997), "A Taxonomy of Multilateral Methods for Making International Comparisons of Prices and Quantities", *Review of Income and Wealth* 43(1), 49-69.

Hill, R.J. (1999a), "Comparing Price Levels across Countries Using Minimum Spanning Trees", *The Review of Economics and Statistics* 81, 135-142.

Hill, R.J. (1999b), "International Comparisons using Spanning Trees", pp. 109-120 in *International and Interarea Comparisons of Income, Output and Prices*, A. Heston and R.E. Lipsey (eds.), Studies in Income and Wealth Volume 61, NBER, Chicago: The University of Chicago Press.

Hill, R.J. (2001), "Measuring Inflation and Growth Using Spanning Trees", *International Economic Review* 42, 167-185.

Hill, R.J. (2004), "Constructing Price Indexes Across Space and Time: The Case of the European Union", *American Economic Review* 94, 1379-1410.

Hill, R.J. (2009), "Comparing Per Capita Income Levels Across Countries Using Spanning Trees: Robustness, Prior Restrictions, Hybrids and Hierarchies", pp. 217-244 in *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, D.S. Prasada Rao (ed.), Cheltenham UK: Edward Elgar.

Hotelling, H. (1932), “Edgeworth’s Taxation Paradox and the Nature of Demand and Supply Functions”, *Journal of Political Economy* 40, 577-616.

Inklaar, R. and W.E. Diewert (2016), “Measuring Industry Productivity and Cross Country Convergence”, *Journal of Econometrics*, forthcoming.

Ivancic, L., W.E. Diewert and K.J. Fox (2009), “Scanner Data, Time Aggregation and the Construction of Price Indexes”, Discussion Paper 09-09, Department of Economics, University of British Columbia, Vancouver, Canada.

Ivancic, L., W.E. Diewert and K.J. Fox (2011), “Scanner Data, Time Aggregation and the Construction of Price Indexes”, *Journal of Econometrics* 161, 24-35.

Jorgenson, D.W. and Z. Griliches (1967), The Explanation of Productivity Change”, *The Review of Economic Studies* 34, 249-283.

Jorgenson, D.W. and Z. Griliches (1972), “Issues in Growth Accounting: A Reply to Edward F. Denison”, *Survey of Current Business* 52:4, Part II (May), 65-94.

Kohli, U.R.J. (1978), “A Gross National Product Function and the Derived Demand for Imports and Supply of Exports”, *Canadian Journal of Economics* 11, 167-182.

Kohli, U. (1990), “Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates”, *Journal of Economic and Social Measurement* 16, 125-136.

Konüs, A.A. and S.S. Byushgens (1926), “K probleme pokupatelnoi cili deneg”, *Voprosi Konyunkturi* 2, 151-172.

Konüs, A.A. (1939), “The Problem of the True Index of the Cost of Living”, *Econometrica* 7, 10-29.

Kopp, R.J and W.E. Diewert (1982), “The Decomposition of Frontier Cost Function Deviations into Measures of Technical and Allocative Efficiency”, *Journal of Econometrics* 19, 319-331.

Lawrence, D. and W.E. Diewert (2006), “Regulating Electricity Networks: The ABC of Setting X in New Zealand”, pp. 207-241 in *Performance Measurement and Regulation of Network Utilities*, T. Coelli and D. Lawrence (eds.), Cheltenham: Edward Elgar Publishing.

Lawrence, D., W.E. Diewert and K.J. Fox (2006), “The Contributions of Productivity, Price Changes and Firm Size to Profitability”, *Journal of Productivity Analysis* 26, 1-13.

Malmquist, S. (1953), “Index Numbers and Indifference Surfaces”, *Trabajos de Estastistica* 4, 209-242.

Mangasarian, O.L. (1969), *Nonlinear Programming*, New York: McGraw-Hill.

McFadden, D. (1966), “Cost, Revenue and Profit Functions: A Cursory Review”, IBER Working Paper No. 86, University of California, Berkeley.

McFadden, D. (1978), “Cost, Revenue and Profit Functions”, pp. 3-109 in *Production Economics: A Dual Approach*, Volume 1, M. Fuss and D. McFadden (eds.), Amsterdam: North-Holland.

Morrison, C.J. and W.E. Diewert (1990), “New Techniques in the Measurement of Multifactor Productivity”, *The Journal of Productivity Analysis* 1, 265-285.

Mudgett, B.D. (1955), “The Measurement of Seasonal Movements in Price and Quantity Indexes”, *Journal of the American Statistical Association* 50, 93-98.

Persons, W. M. (1928). The Construction of Index Numbers. Cambridge, MA: Riverside Press.

Pollak, R.A. (1969), “Conditional Demand Functions and Consumption Theory”, *Quarterly Journal of Economics* 83, 60-78.

Pollak, R.A. (1983), “The Theory of the Cost-of-Living Index”, pp. 87-161 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada.

Rao, D.S.P. (2005), “On the Equivalence of Weighted Country Product (CPD) Method and the Rao System for Multilateral Price Comparisons”, *Review of Income and Wealth* 4, 571–580.

Reinsdorf, M.B. (1998), “Formula Bias and Within-Stratum Substitution Bias in the U.S. CPI”, *The Review of Economics and Statistics* 80:2, 175-187.

Samuelson, P.A. (1947), *Foundations of Economic Analysis*, Cambridge, MA: Harvard University Press.

Samuelson, P.A. (1974), “Complementarity—An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory”, *The Journal of Economic Literature* 12, 1255-1289.

Shephard, R.W. (1953), *Cost and Production Functions*, Princeton N.J.: Princeton University Press.

Summers, R. (1973), “International Comparisons with Incomplete Data”, *Review of Income and Wealth* 29:1, 1-16.

Theil, H. (1967), *Economics and Information Theory*, Amsterdam: North-Holland.

Törnqvist, L. (1936), “The Bank of Finland’s Consumption Price Index” *Bank of Finland Monthly Bulletin*, No. 10, 1-8.

Törnqvist, L. and E. Törnqvist (1937), Vilket är förhållandet mellan finska markens och svenska kronans köpkraft?”, *Ekonomiska Samfundets Tidskrift* 39, 1-39 reprinted as pp. 121-160 in *Collected Scientific Papers of Leo Törnqvist*, Helsinki: The Research Institute of the Finnish Economy, 1981.

Tulkens, H. (1993), “On FDH Efficiency Analysis: Some Methodological Issues and Application to Retail Banking, Courts, and Urban Transit”, *Journal of Productivity Analysis* 4, 183–210.

Turvey, R. (1979), “The Treatment of Seasonal Items in Consumer Price Indices”, *Bulletin of Labour Statistics*, Fourth Quarter, International Labour Office, Geneva, 13-33.

Uzawa, H. (1964), “Duality Principles in the Theory of Cost and Production”, *International Economic Review* 5, 291-299.

Varian, H.R. (1982), “The Nonparametric Approach to Demand Analysis”, *Econometrica* 50, 945-973.

Varian, H.R. (1984), “The Nonparametric Approach to Production Analysis”, *Econometrica* 52, 579-597.

Walsh, C.M. (1901), *The Measurement of General Exchange Value*, New York: Macmillan and Co.