

## APPLIED ECONOMICS

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### Chapter 7: The Measurement of Capital

#### 1. Introduction

“Capital (I am not the first to discover) is a very large subject, with many aspects; wherever one starts, it is hard to bring more than a few of them into view. It is just as if one were making pictures of a building; though it is the same building, it looks quite different from different angles.” John Hicks (1973; v).

“Perhaps a more realistic motive for reading earlier writers is not to rediscover forgotten truths, but to gain a perspective of how present day ideas have evolved and, perhaps, by reading the original statements of important ideas, to see them more vividly and understand them more clearly.” Geoffrey Whittington (1980; 240).

When a firm buys a durable capital input, it is not appropriate to charge the entire purchase price to the period when the input was purchased. If this is done, then profits will be unreasonably *understated* in the period of purchase and *overstated* in subsequent periods when the durable input continues to contribute to production. Rather than charging the entire purchase price of the asset to the first period of use (and charging nothing for the subsequent periods of use), it would be more appropriate to charge a *rental price* or a *user cost* for the asset for each period that it is used. If there are market rental prices for the asset (and for the used asset as it ages), then these market rental prices could be used to price the services of the asset in each period. But frequently, such market rental prices do not exist and so other techniques must be used to price the period by period services of the asset. We will indicate some of these alternative techniques in this chapter.<sup>1</sup>

In section 2, we discuss some of the problems that occur when an economy is experiencing very high inflation. Under these conditions, it will be necessary for the national price statistician to shorten the accounting period (or give up price measurement altogether). We also discuss some problems relating to the beginning, middle and end of the period.

In section 3, we present the basic equations relating stocks and flows of capital assuming that data on the prices of vintages of a homogeneous capital good are available. This framework is not applicable under all circumstances<sup>2</sup> but it is a framework that will allow us to disentangle the effects of general price change, asset specific price change and depreciation.

Section 4 continues the theoretical framework that was introduced in section 3. We show how information on vintage asset prices, vintage rental prices and vintage depreciation

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<sup>1</sup> The material in this chapter draws on Diewert and Lawrence (2000) and Diewert (2001) (2004a) (2004b) (2005a) (2005b).

<sup>2</sup> Most notably, our framework cannot deal with unique or one of a kind assets, which by definition, do not have vintages.

rates are all equivalent under certain assumptions; i.e., knowledge of any one of these three sequences or profiles is sufficient to determine the other two.

Section 5 discusses alternative sets of assumptions on nominal interest rates and anticipated asset price changes. We specify three different sets of assumptions that could be used in empirical implementations of the suggested methods.

Section 6 discusses the problems involved in aggregating over vintages of capital, both in forming capital stocks and capital services. Instead of the usual perpetual inventory method for aggregating over vintages, which assumes perfectly substitutable vintages of the same stock, we suggest the use of a superlative index number formula to do the aggregation.

Sections 7-10 show how the general algebra presented in sections 3 and 4 can be adapted to deal with four specific models of depreciation. The four models considered are the one hoss shay model, the geometric model of depreciation, straight line depreciation and the linear efficiency decline model.

Finally, inventory stocks are a type of capital input to production but the present System of National Accounts treatment of inventories and inventory change is somewhat confusing. Thus in Appendix 1, a theoretical framework that provides a unified treatment for measuring inventory change and the user cost of inventories is explained.<sup>3</sup> Appendix 2 gives some background information on the origins of the theoretical framework used in Appendix 1.

## **2. Inflation, the Length of the Accounting Period and the Measurement of Economic Activity**

Our goal in this chapter is twofold: (1) to measure the price and quantity of the *stock* of reproducible capital held by a production unit (an establishment, a firm, an industry or an entire economy) at a *point* in time and (2) to measure the price and quantity of the *flow* of reproducible capital services utilized by a production unit over a *period* of time. In particular, we want to extend the procedures for measuring capital stocks and flows to cover situations where there is general price level change or *inflation*. In this section, we shall review some of the general measurement problems that arise when inflation is high.

When capital flows are measured, the normal period of time is either a year or a quarter. Under conditions of high inflation, the aggregation of homogeneous commodity flows within a quarter or a year is complicated by the fact that the within period transactions are valued at very different prices. The recent national income accounting literature explains the problem as follows:

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<sup>3</sup> This methodology is based on Diewert and Smith (1994) and Diewert (2004b; 36). The accounting methodology can also be found in Diewert (2005b; 21-23), Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).

“Conventional index number theory is mostly concerned with comparisons between *points* of time whereas, in national accounts, price and quantity comparisons have to be made between discrete *periods* of time. Significant changes in price and quantity flows may occur not only between different periods but also within a single accounting period, especially one as long as a year. Indeed, the central problem of accounting under high inflation is that prices are much higher at the end of the accounting period than at the beginning.” Peter Hill (1996; 11).

“The underlying problem is not a traditional index number problem. It stems from the use of current value data as inputs into the calculation of indirect price or quantity measures under high inflation. Current accounts permit identical quantities of the same homogeneous product to be valued at very different prices during the course of the same year. Implicitly, quantities sold at higher prices later in the year are treated as if they were superior qualities when they are not.” Peter Hill (1996; 12).

“Under high inflation, the monetary value of flows of goods and services at different points of time within the same accounting period are not commensurate with each other because the unit of currency used as the *numeraire* is not stable. Adding together different quantities of the same good valued at different prices is equivalent, from a scientific point of view, to using different units of measurement for different sets of observations on the same variable. In the case of physical data, however, it is rather more obvious that adding quantities measured in grams to quantities measured in ounces is a futile procedure.” Peter Hill (1996; 32).

“Before the preparation of the 1993 SNA, issues connected with high or significant inflation had not been dealt with at all in international recommendations concerning national accounts. Uneasiness especially with the recording of nominal interest had been often expressed, for instance in Europe and North America at the time of two digit inflation and above all in countries, like in Latin America, experiencing high or hyper inflation. In relation with the latter situations, uneasiness extended to the whole set of accounts, because, due to the significant rate of inflation within each year, annual accounts in current values could no longer be deemed homogeneous as regards the level of prices in each year. They combine intra-annual flows that are valued at very different prices and are not, strictly speaking, additive. The effect of the intra-annual change in the general price level can be neglected for the sake of simplicity only when the rate of inflation is low. When it is high, the meaning of annual accounts in current values becomes fuzzy.” André Vanoli (1998).

“When inflation is high, the aggregation of flows from different periods becomes very much a case of ‘adding apples and bananas’— the flows at the end of the period will carry a much greater weight than the flows at the beginning of the period, so that the change on average will reflect development at the end of the period disproportionately. Annual national accounts at current prices become virtually meaningless and computation of national accounts at constant prices becomes very problematic.” Ezra Hadar and Soli Peleg (1998; 2).

Of course, concern over the effects of general price level change has a much longer history in the general cost accounting literature; see Baxter (1984), Tweedie and Whittington (1984) and Whittington (1992) for example.<sup>4</sup>

We now discuss in more detail the accounting problems caused by high inflation that are referred to in the above quotations. The basic problem is this: all discrete time economic theories and most of index number theory assumes that all of the transactions of a production unit in a homogeneous commodity within the accounting period can be

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<sup>4</sup> The inflation accounting literature extends back to Middleditch: “Today’s dollar is, then, a totally different unit from the dollar of 1897. As the general price level fluctuates, the dollar is bound to become a unit of different magnitude. To mix these units is like mixing inches and centimeters or measuring a field with a rubber tape-line.” Livingston Middleditch (1918; 114-115).

represented by a *single price* and a *single quantity*. It is natural to let the single quantity be the *sum* of the quantities sold (in the case of an output) or the *sum* of the quantities purchased (in the case of an input). But then, if we want the single price times the single quantity to equal the *value* of transactions for the commodity in the period, the single price must equal the value of transactions divided by the sum of quantities purchased or sold; i.e., the single price must equal a *unit value*.<sup>5</sup> But when there is substantial inflation within the accounting period, unit values give a much higher weight to transactions that occur near the end of the period compared to transactions that occurred near the beginning; it is as if the end of period transactions are being *artificially quality adjusted* to be more valuable than the beginning of the period transactions.

The obvious solution to this artificial implicit weighting problem is to choose the accounting period to be small enough so that the general inflation within the period is small enough to be ignored. This is precisely the solution suggested by the index number theorist Fisher<sup>6</sup> and the measurement economist Hicks: the length of the accounting period should be the Hicksian “week”:

“I shall define a week as that period of time during which variations in price can be neglected.” John R. Hicks (1946; 122).

Thus it seems that there is a simple solution to the problem of constructing meaningful accounting period prices and quantities for homogeneous commodities when there is high inflation: simply shorten the accounting period!

Hill (1996) however noted that there are at least three classes of problems associated with the above solution:

“In order to keep these issues in perspective, it is useful to summarise the problems created by continually shortening the accounting period.

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<sup>5</sup> The early index number theorists Walsh (1901; 96) (1921; 88), Fisher (1922; 318) and Davies (1924; 96) all suggested unit values as the prices that should be inserted into a bilateral index number formula. Walsh nicely sums up the case for unit values as follows: “Some nice questions arise as to whether only what is consumed in the country, or only what is produced in it, or both together are to be counted; and also there are difficulties as to the single price quotation that is to be given at each period to each commodity, since this, too, must be an average. Throughout the country during the period a commodity is not sold at one price, nor even at one wholesale price in its principle market. Various quantities of it are sold at different prices, and the full value is obtained by adding all the sums spent (at the same stage in its advance towards the consumer), and the average price is found by dividing the total sum (or the full value) by the total quantities.” Correa Moylan Walsh (1921; 88).

<sup>6</sup> “Essentially the same problem enters, however, whenever, as is usually the case, the data for prices and quantities with which we start are averages instead of being the original market quotations. Throughout this book, ‘the price’ of any commodity or ‘the quantity’ of it for any one year was assumed given. But what is such a price or quantity? Sometimes it is a single quotation for January 1 or July 1, but usually it is an average of several quotations scattered through the year. The question arises: On what principle should this average be constructed? The *practical* answer is *any* kind of average since, ordinarily, the variations during a year, so far, at least, as prices are concerned, are too little to make any perceptible difference in the result, whatever kind of average is used. Otherwise, there would be ground for subdividing the year into quarters or months until we reach a small enough period to be considered practically a point.” Irving Fisher (1922; 318).

1. The compilation of accounts for shorter time periods requires more information about the times at which various transactions take place. Enquiries may have to be conducted more frequently thereby creating additional costs for the data collectors. More burdens are also placed on the respondents supplying the information. In many cases, they may be unable to supply the necessary information because their own internal records and accounts do not permit them to do so, especially when they traditionally report their accounts for longer time periods, such as a year.
2. As production is a process which can extend over a considerable period of time, its measurement becomes progressively more difficult the shorter the accounting period. The problem is not confined to agriculture or forestry where many production processes take a year or more. The production of large fixed assets such as large ships, bridges, power stations, dams or the like can extend over several years. The output produced over shorter periods of time then has to be measured on the basis of work in progress completed each period. ...
3. Because many transactions, especially large transactions, are not completed within the day, there are typically many receivables and payables outstanding at any given moment of time. They assume greater importance in relation to the flows as the accounting period is reduced. This makes it more difficult to reconcile the values of different flows in the accounts, especially if the two parties to the transaction perceive it as taking place at different times from each other and do not record it in the same way required by the system. ... Peter Hill (1996; 34-35).

Thus shortening the accounting period leads to increased costs for the statistical agency and the businesses being surveyed. Moreover, firm accounting is geared to years and quarters and it may not be possible for production units to provide complete accounting information for periods shorter than a quarter. As the accounting period becomes shorter, it is less likely that production, shipment, billing and payment for the same commodity will all coincide within the accounting period. Also as the accounting period becomes shorter, work in progress will tend to become ever more important relative to final sales, creating difficult valuation problems.<sup>7</sup> Put another way, more and more inputs will shift from being intermediate inputs (inputs that are used up within the accounting period) to being durable inputs (inputs whose contribution to production extends over more than one period). In addition to these difficulties, there are others. For example, as the accounting period becomes shorter, transactions tend to become more erratic and sporadic. Many goods will not be sold in a supermarket in a particular day or week. Normal index number theory breaks down under these conditions: it is difficult to compare a positive amount of a good sold in one period with a zero amount sold in the next period.

A related difficulty is that many commodities are produced or demanded on a *seasonal* basis. If the accounting period is a year, then there are no seasonal commodity difficulties but as we shorten the period from a year, we will run into the problem of seasonal fluctuations in prices and quantities. In many cases, a seasonal commodity will not be available in all seasons and we again run into the problem of comparing positive values with zero values in the periods when the commodity is out of season. Even if the seasonal commodity does not disappear, the application of standard index number theory is not straightforward.<sup>8</sup>

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<sup>7</sup> There are very few price indexes for work in progress! This is to be expected since there are very few transactions involving partially completed products.

<sup>8</sup> Hadar and Peleg (1998; 5) comment on the importance of seasonal adjustment procedures in the context of high inflation: "As a by-product of the emphasis on quarterly estimates at constant prices the seasonal adjustment got large attention and many resources were spent to improve the adjustment." Diewert (1996)

Nevertheless, even in the face of the above difficulties, it seems that the only possible solution to the artificial implicit weighting problem that is generated by high inflation is to shorten the accounting period so that normal index number theory can be applied in order to construct meaningful economic aggregates.<sup>9</sup>

In addition to the above general problems associated with economic measurement of flow variables under conditions of high inflation, there are some additional problems associated with the measurement of capital. These additional problems are associated with the stock and flow aspects of capital. We will conclude this section by explaining these problems.

Given an accounting period of some predetermined length, we can associate with it at least three separate points in time:

- The beginning of the accounting period;
- The middle of the accounting period; and
- The end of the accounting period.

In interpreting the national accounts or the accounts of a business unit, we generally think of all flow variables as being concentrated in the middle of the period. If we follow this convention in the context of high inflation, then we require one (nominal) interest rate to index the value of money or financial capital going from the beginning of the period to the middle of the period and we require another (nominal) interest rate to index the value of money going from the middle of the period to the end of the period. Given these two interest rates, we could construct *centered user costs of capital* for each type of reproducible capital, which would be the appropriate flow variables that would match up with the other flow variables in the production accounts of the production unit. However, in order to reduce notational complexity, we do not construct centered user costs in what follows. Instead, for each type of asset, we construct either a *beginning of the period user cost* (which measures the cost of using the asset for the period under consideration from the perspective of the price level prevailing at the beginning of the period) or an *end of the period user cost* (which measures the cost of using the asset for the period under consideration from the perspective of the price level prevailing at the end of the period). Of course, armed with a knowledge of the appropriate half period interest rates, it is easy to convert these “bookend” user costs into centered user costs.

In the following section, we explain the fundamental equations relating stocks and flows of capital.

### 3. The Fundamental Equations Relating Stocks and Flows of Capital

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(1998) (1999) reviews possible approaches to the problems involved in treating seasonal commodities (and suggests solutions) when there is high inflation.

<sup>9</sup> Our discussion in the previous paragraph indicates that this cannot be done if the economy is experiencing a hyperinflation. Thus meaningful economic measurement becomes impossible under very high inflation. This is a hidden cost of inflation that is not discussed very much in the literature on the costs of inflation.

Before we begin with our algebra, it seems appropriate to explain why accounting for the contribution of capital to production is more difficult than accounting for the contributions of labour or materials. The main problem is that when a reproducible capital input is purchased for use by a production unit at the beginning of an accounting period, we cannot simply charge the entire purchase cost to the period of purchase. Since the benefits of using the capital asset extend over more than one period, the initial purchase cost must be distributed somehow over the useful life of the asset. This is *the fundamental problem of accounting*.<sup>10</sup> Hulten (1990) explains the consequences for accountants of the durability of capital as follows:

“Durability means that a capital good is productive for two or more time periods, and this, in turn, implies that a distinction must be made between the value of using or renting capital in any year and the value of owning the capital asset. This distinction would not necessarily lead to a measurement problem if the capital services used in any given year were paid for in that year; that is, if all capital were rented. In this case, transactions in the rental market would fix the price and quantity of capital in each time period, much as data on the price and quantity of labor services are derived from labor market transactions. But, unfortunately, much capital is utilized by its owner and the transfer of capital services between owner and user results in an implicit rent typically not observed by the statistician. Market data are thus inadequate for the task of directly estimating the price and quantity of capital services, and this has led to the development of indirect procedures for inferring the quantity of capital, like the perpetual inventory method, or to the acceptance of flawed measures, like book value.” Charles R. Hulten (1990; 120-121).

The value of an asset at the beginning of an accounting period is equal to the discounted stream of future rental payments that the asset is expected to yield. Thus the *stock value* of the asset is equal to the discounted future *service flows*<sup>11</sup> that the asset is expected to yield in future periods. Let the price of a new capital input purchased at the beginning of

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<sup>10</sup> “The difficulty of imputing expenses to individual sales or even to the gross earnings of the accounting period, the month or year, is an ever present problem for the accountant in the periodic determination of enterprise income. The longer the period for which the income is to be determined, the smaller the relative amount of error. Absolute accuracy can be attained only when the venture is completed and the enterprise terminated.” William T Crandell (1935; 388-389).

“Early enterprises and partners working in the main in isolated trading ventures, needed only an irregular determination of profit. But before the business corporation had been very long in operation it was evident that it needed to be treated as a continuing enterprise. For example, calculating dividends by separate voyages was found impractical in the East India Company by 1660. Profit calculation therefore became a matter of periodic estimates in place of the known results of completed ventures.” A.C. Littleton (1933; 270).

“The third convention is that of the annual accounting period. It is this convention which is responsible for most of the difficult accounting problems. Without this convention, accounting would be a simple matter of recording completed and fully realized transactions: an act of primitive simplicity.” Stephen Gilman (1939; 26).

“All the problems of income measurement are the result of our desire to attribute income to arbitrarily determined short periods of time. Everything comes right in the end; but by then it is too late to matter.” David Solomons (1961; 378). Note that these authors do not mention the additional complications that are due to the fact that future revenues and costs must be discounted to yield values that are equivalent to present dollars.

<sup>11</sup> Walras (1954) (first edition published in 1874) was one of the earliest economists to state that capital stocks are demanded because of the future flow of services that they render. Although he was perhaps the first economist to formally derive a user cost formula as we shall see, he did not work out the explicit discounting formula that Böhm-Bawerk (1891; 342) was able to derive.

period  $t$  be  $P_0^t$ . In a noninflationary environment, it can be assumed that the (potentially observable) sequence of (cross sectional) vintage rental prices prevailing at the beginning of period  $t$  can be expected to prevail in future periods. Thus in this no general inflation case, there is no need to have a separate notation for future expected rental prices for a new asset as it ages. However, in an inflationary environment, it is necessary to distinguish between the observable rental prices for the asset at different ages at the beginning of period  $t$  and future *expected* rental prices for assets of various ages.<sup>12</sup> Thus let  $f_0^t$  be the (observable) rental price of a new asset at the beginning of period  $t$ , let  $f_1^t$  be the (observable) rental price of a one period old asset at the beginning of period  $t$ , let  $f_2^t$  be the (observable) rental price of a 2 period old asset at the beginning of period  $t$ , etc. Then *the fundamental equation* relating the *stock value of a new asset* at the beginning of period  $t$ ,  $P_0^t$ , to the sequence of *cross sectional rental prices for assets of age  $n$*  prevailing at the beginning of period  $t$ ,  $\{f_n^t : n = 0, 1, 2, \dots\}$  is<sup>13</sup>:

$$(1) P_0^t = f_0^t + [(1+i_1^t)/(1+r_1^t)] f_1^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)] f_2^t + \dots$$

In the above equation,<sup>14</sup>  $1+i_1^t$  is the *rental price escalation factor* that is *expected* to apply to a one period old asset going from the beginning of period  $t$  to the end of period  $t$  (or equivalently, to the beginning of period  $t+1$ ),  $(1+i_1^t)(1+i_2^t)$  is *the rental price escalation factor* that is *expected* to apply to a 2 period old asset going from the beginning of period  $t$  to the beginning of period  $t+2$ , etc. Thus the  $i_n^t$  are *expected rates of price change for used assets of varying ages  $n$*  that are formed at the beginning of period  $t$ . The term  $1+r_1^t$  is the discount factor that makes a dollar received at the beginning of period  $t$  equivalent to a dollar received at the beginning of period  $t+1$ , the term  $(1+r_1^t)(1+r_2^t)$  is the discount factor that makes a dollar received at the beginning of period  $t$  equivalent to a dollar received at the beginning of period  $t+2$ , etc. Thus the  $r_n^t$  are one period *nominal interest rates* that represent the *term structure of interest rates* at the beginning of period  $t$ .<sup>15</sup>

<sup>12</sup> Note that these future expected rental prices are not generally observable due to the lack of futures markets for these future period rentals of the assets of varying ages.

<sup>13</sup> The sequence of (cross sectional) vintage rental prices  $\{f_n^t\}$  is called the *age-efficiency profile* of the asset.

<sup>14</sup> It should be noted that Irving Fisher (1897; 365) seemed to be well aware of the complexities that are imbedded in equation (1): "There is not space here to discuss the theory in greater detail, nor to apply it to economic problems. A full treatment would take account of the various standards in which income is or may be expressed, of the case in which the rates of interest at different dates and for different periods does not remain constant, of the fact that the services of capital which are discounted in its value are only *expected* services, not those which actually materialise, and of the consequent discrepancy between income anticipated and income realised, of the propriety or impropriety of including man himself as a species of income-bearing capital, and so on."

<sup>15</sup> Peter Hill has noted a major problem with the use of equation (1) as the starting point of our discussion: namely, *unique assets* will by definition not have used versions of the same asset in the marketplace during the current period and so the cross sectional rental prices  $f_n^t$  for assets of age  $n$  in period  $t$  will not exist for these assets! In this case, the  $f_n^t$  should be interpreted as expected future rentals that the unique asset is expected to generate at today's prices. The  $(1+i_n^t)$  terms then summarize expectations about the amount of asset specific price change that is expected to take place. This reinterpretation of equation (1) is more fundamental but we chose not to make it our starting point because it does not lead to a completely objective method for national statisticians to form reproducible estimates of these future rental payments. However, in many situations (e.g., the valuation of a new movie), the statistician will be forced to attempt to implement Hill's (2000) more general model.

We now generalize equation (1) to relate the *stock value of an n period old asset* at the beginning of period t,  $P_n^t$ , to the sequence of *cross sectional vintage rental prices* prevailing at the beginning of period t,  $\{f_n^t\}$ ; thus for  $n = 0, 1, 2, \dots$ , we assume:

$$(2) P_n^t = f_n^t + [(1+i_1^t)/(1+r_1^t)] f_{n+1}^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)] f_{n+2}^t + \dots$$

Thus older assets discount fewer terms in the above sum; i.e., as n increases by one, we have one less term on the right hand side of (2). However, note that we are applying the same price escalation factors  $(1+i_1^t)$ ,  $(1+i_1^t)(1+i_2^t)$ , ..., to escalate the cross sectional rental prices prevailing at the beginning of period t,  $f_1^t, f_2^t, \dots$ , and to form estimates of future expected rental prices for each vintage of the capital stock that is in use at the beginning of period t.

The rental prices prevailing at the beginning of period t for assets of various ages,  $f_0^t, f_1^t, \dots$  are potentially observable.<sup>16</sup> These cross section rental prices reflect the relative efficiency of the various vintages of the capital good that are still in use at the beginning of period t. It is assumed that these rentals are paid (explicitly or implicitly) by the users at the beginning of period t. Note that the sequence of asset stock prices for various ages at the beginning of period t,  $P_0^t, P_1^t, \dots$  is not affected by general inflation provided that the general inflation affects the expected asset rates of price change  $i_n^t$  and the nominal interest rates  $r_n^t$  in a proportional manner. We will return to this point later.

The physical productivity characteristics of a unit of capital of each age are determined by the sequence of cross sectional rental prices. Thus a brand new asset is characterized by the vector of current rental prices for assets of various ages,  $f_0^t, f_1^t, f_2^t, \dots$ , which are interpreted as “physical” contributions to output that the new asset is expected to yield during the current period t (this is  $f_0^t$ ), the next period (this is  $f_1^t$ ), and so on. An asset which is one period old at the start of period t is characterized by the vector  $f_1^t, f_2^t, \dots$ , etc.<sup>17</sup>

We have not explained how the expected rental price rates of price change  $i_n^t$  are to be estimated. We shall deal with this problem in section 5 below. However, it should be noted that there is no guarantee that our expectations about the future course of rental prices are correct.

At this point, we make some simplifying assumptions about the expected rates of rental price change for future periods  $i_n^t$  and the interest rates  $r_n^t$ . We assume that these anticipated specific price change escalation factors at the beginning of each period t are all equal; i.e., we assume:

$$(3) i_n^t = i^t; \quad n = 1, 2, \dots$$

<sup>16</sup> This is the main reason that we use this escalation of cross sectional rental prices approach to capital measurement rather than the more fundamental discounted future expected rentals approach advocated by Hill.

<sup>17</sup> Triplet (1996; 97) used this characterization for capital assets of various vintages.

We also assume that the term structure of (nominal) interest rates at the beginning of each period  $t$  is constant; i.e., we assume:

$$(4) r_n^t = r^t; \quad n = 1, 2, \dots$$

However, note that as the period  $t$  changes,  $r^t$  and  $i^t$  can change.

Using assumptions (3) and (4), we can rewrite the system of equations (2), which relate the sequence or profile of *stock prices* of age  $n$  at the beginning of period  $t$   $\{P_n^t\}$  to the sequence or profile of (cross sectional) *rental prices* for assets of age  $n$  at the beginning of period  $t$   $\{f_n^t\}$ , as follows:

$$(5) \begin{aligned} P_0^t &= f_0^t + [(1+i^t)/(1+r^t)] f_1^t + [(1+i^t)/(1+r^t)]^2 f_2^t + [(1+i^t)/(1+r^t)]^3 f_3^t + \dots \\ P_1^t &= f_1^t + [(1+i^t)/(1+r^t)] f_2^t + [(1+i^t)/(1+r^t)]^2 f_3^t + [(1+i^t)/(1+r^t)]^3 f_4^t + \dots \\ P_2^t &= f_2^t + [(1+i^t)/(1+r^t)] f_3^t + [(1+i^t)/(1+r^t)]^2 f_4^t + [(1+i^t)/(1+r^t)]^3 f_5^t + \dots \\ &\dots \\ P_n^t &= f_n^t + [(1+i^t)/(1+r^t)] f_{n+1}^t + [(1+i^t)/(1+r^t)]^2 f_{n+2}^t + [(1+i^t)/(1+r^t)]^3 f_{n+3}^t + \dots \end{aligned}$$

On the left hand side of equations (5), we have the sequence of period  $t$  asset prices by age starting with the price of a new asset,  $P_0^t$ , moving to the price of an asset that is one period old at the start of period  $t$ ,  $P_1^t$ , then moving to the price of an asset that is 2 periods old at the start of period  $t$ ,  $P_2^t$ , and so on. On the right hand side of equations (5), the first term in each equation is a member of the sequence of rental prices by age of asset that prevails in the market (if such markets exist) at the beginning of period  $t$ . Thus  $f_0^t$  is the rent for a new asset,  $f_1^t$  is the rent for an asset that is one period old at the beginning of period  $t$ ,  $f_2^t$  is the rent for an asset that is 2 periods old, and so on. This sequence of current market rental prices for the assets of various vintages is then extrapolated out into the future using the anticipated price escalation rates  $(1+i^t)$ ,  $(1+i^t)^2$ ,  $(1+i^t)^3$ , etc. and then these future expected rentals are discounted back to the beginning of period  $t$  using the nominal discount factors  $(1+r^t)$ ,  $(1+r^t)^2$ ,  $(1+r^t)^3$ , etc. Note that given the period  $t$  expected asset inflation rate  $i^t$  and the period  $t$  nominal discount rate  $r^t$ , we can go from the (cross sectional) sequence of vintage rental prices  $\{f_n^t\}$  to the (cross sectional) sequence of vintage asset prices  $\{P_n^t\}$  using equations (5). We shall show below how this procedure can be reversed; i.e., we shall show how given the sequence of cross sectional asset prices, we can construct estimates for the sequence of cross sectional rental prices.

It seems that Böhm-Bawerk was the first economist to use the above method for relating the future service flows of a durable input to its stock price:

“If the services of the durable good be exhausted in a short space of time, the individual services, provided that they are of the same quality— which, for simplicity’s sake, we assume— are, as a rule, equal in value, and the value of the material good itself is obtained by multiplying the value of one service by the number of services of which the good is capable. But in the case of many durable goods, such as ships, machinery, furniture, land, the services rendered extend over long periods, and the result is that the later services cannot be rendered, or at least cannot be rendered in a normal economic way, before a long time has expired. As a consequence, the value of the more distant material services suffers the same fate as the value of future goods. A material service, which, technically, is exactly the same as a service of this year, but which cannot be rendered before next year, is worth a little less than this year’s service; another similar

service, but obtainable only after two years, is, again, a little less valuable, and so on; the values of the remote services decreasing with the remoteness of the period at which they can be rendered. Say that this year's service is worth 100, then next year's service— assuming a difference of 5 % per annum— is worth in today's valuation only 95.23; the third year's service is worth only 90.70; the fourth year's service, 86.38; the fifth, sixth and seventh year's services, respectively, worth 82.27, 78.35, 74.62 of present money. The value of the durable good in this case is not found by multiplying the value of the current service by the total number of services, but is represented by a sum of services decreasing in value." Eugen von Böhm-Bawerk (1891; 342).

Böhm-Bawerk (1891; 342) considered a special case of (5) where all service flows  $f_n$  were equal to 100 for  $n = 0, 1, \dots, 6$  and equal to 0 thereafter, where the asset inflation rate was expected to be 0 and where the interest rate  $r$  was equal to .05 or 5 %.<sup>18</sup> This is a special case of what has come to be known as the *one hoss shay model* and we shall consider it in more detail in section 7.

Note that equations (5) can be rewritten as follows:<sup>19</sup>

$$\begin{aligned} (6) \quad P_0^t &= f_0^t + [(1+i^t)/(1+r^t)] P_1^t \quad ; \\ P_1^t &= f_1^t + [(1+i^t)/(1+r^t)] P_2^t \quad ; \\ P_2^t &= f_2^t + [(1+i^t)/(1+r^t)] P_3^t \quad ; \\ &\dots \\ P_n^t &= f_n^t + [(1+i^t)/(1+r^t)] P_{n+1}^t \quad ; \dots \end{aligned}$$

The first equation in (6) says that the value of a new asset at the start of period  $t$ ,  $P_0^t$ , is equal to the rental that the asset can earn in period  $t$ ,  $f_0^t$ ,<sup>20</sup> plus the expected asset value of the capital good at the end of period  $t$ ,  $(1+i^t) P_1^t$ , but this expected asset value must be divided by the discount factor,  $(1+r^t)$ , in order to convert this future value into an equivalent beginning of period  $t$  value.<sup>21</sup>

Now it is straightforward to solve equations (6) for the sequence of period  $t$  cross sectional rental prices,  $\{f_n^t\}$ , in terms of the cross sectional asset prices,  $\{P_n^t\}$ :

$$\begin{aligned} (7) \quad f_0^t &= P_0^t - [(1+i^t)/(1+r^t)] P_1^t = (1+r^t)^{-1} [P_0^t (1+r^t) - (1+i^t) P_1^t] \\ f_1^t &= P_1^t - [(1+i^t)/(1+r^t)] P_2^t = (1+r^t)^{-1} [P_1^t (1+r^t) - (1+i^t) P_2^t] \\ f_2^t &= P_2^t - [(1+i^t)/(1+r^t)] P_3^t = (1+r^t)^{-1} [P_2^t (1+r^t) - (1+i^t) P_3^t] \\ &\dots \end{aligned}$$

<sup>18</sup> Böhm-Bawerk (1891; 343) went on and constructed the sequence of vintage asset prices using his special case of equations (5).

<sup>19</sup> Christensen and Jorgenson (1969; 302) do this for the geometric depreciation model except that they assume that the rental is paid at the end of the period rather than the beginning. Variants of the system of equations (6) were derived by Christensen and Jorgenson (1973), Jorgenson (1989; 10), Hulten (1990; 128) and Diewert and Lawrence (2000; 276). Irving Fisher (1908; 32-33) also derived these equations in words.

<sup>20</sup> Note that we are implicitly assuming that the rental is paid to the owner at the beginning of period  $t$ .

<sup>21</sup> Another way of interpreting say the first equation in (6) runs as follows: the purchase cost of a new asset  $P_0^t$  less the rental  $f_0^t$  (which is paid immediately at the beginning of period  $t$ ) can be regarded as an investment, which must earn the going rate of return  $r^t$ . Thus we must have  $[P_0^t - f_0^t](1+r^t) = (1+i^t)P_1^t$  which is the (expected) value of the asset at the end of period  $t$ . This line of reasoning can be traced back to Walras (1954; 267).

$$f_n^t = P_n^t - [(1+i^t)/(1+r^t)] P_{n+1}^t = (1+r^t)^{-1} [P_n^t (1+r^t) - (1+i^t) P_{n+1}^t]; \dots$$

Thus equations (5) allow us to go from the sequence of rental prices by age  $n$   $\{f_n^t\}$  to the sequence of asset prices by age  $n$   $\{P_n^t\}$  while equations (7) allow us to reverse the process.

Equations (7) can be derived from elementary economic considerations. Consider the first equation in (7). Think of a production unit as purchasing a unit of the new capital asset at the beginning of period  $t$  at a cost of  $P_0^t$  and then using the asset throughout period  $t$ . However, at the end of period  $t$ , the producer will have a depreciated asset that is expected to be worth  $(1+i^t) P_1^t$ . Since this offset to the initial cost of the asset will only be received at the end of period  $t$ , it must be divided by  $(1+r^t)$  to express the benefit in terms of beginning of period  $t$  dollars. Thus the expected net cost of *using* the new asset for period  $t$ <sup>22</sup> is  $P_0^t - [(1+i^t)/(1+r^t)] P_1^t$ .

The above equations assume that the actual or implicit period  $t$  rental payments  $f_n^t$  for assets of different ages  $n$  are made at the *beginning* of period  $t$ . It is sometimes convenient to assume that the rental payments are made at the *end* of each accounting period. Thus we define the *end of period  $t$  rental price or user cost* for an asset that is  $n$  periods old at the beginning of period  $t$ ,  $u_n^t$ , in terms of the corresponding *beginning of period  $t$  rental price*  $f_n^t$  as follows:

$$(8) u_n^t \equiv (1+r^t) f_n^t ; n = 0, 1, 2, \dots$$

Thus if the rental payment is made at the end of the period instead of the beginning, then the beginning of the period rental  $f_n^t$  must be escalated by the interest rate factor  $(1+r^t)$  in order to obtain the end of the period user cost  $u_n^t$ <sup>23</sup>.

Using equations (8) and the second set of equations in (7), it can readily be shown that the sequence of end of period  $t$  user costs  $\{u_n^t\}$  can be defined in terms of the period  $t$  sequence of asset prices by age  $\{P_n^t\}$  as follows:

$$(9) \begin{aligned} u_0^t &= P_0^t (1+r^t) - (1+i^t) P_1^t \\ u_1^t &= P_1^t (1+r^t) - (1+i^t) P_2^t \\ u_2^t &= P_2^t (1+r^t) - (1+i^t) P_3^t \\ &\dots \\ u_n^t &= P_n^t (1+r^t) - (1+i^t) P_{n+1}^t ; \dots \end{aligned}$$

<sup>22</sup> This explains why the rental prices  $f_n^t$  are sometimes called *user costs*. This derivation of a user cost was used by Diewert (1974; 504), (1980; 472-473), (1992a; 194) and by Hulten (1996; 155).

<sup>23</sup> It is interesting that Böhm-Bawerk (1891; 343) carefully distinguished between rental payments made at the beginning or end of a period: "These figures are based on the assumption that the whole year's utility is obtained all at once, and, indeed, obtained in anticipation at the beginning of the year; e.g., by hiring the good at a year's interest of 100 payable on each 1<sup>st</sup> January. If, on the other hand, the year's use can only be had at the end of the year, a valuation undertaken at the beginning of the year will show figures not inconsiderably lower. ... That the figures should alter according as the date of the valuation stands nearer or farther from the date of obtaining the utility, is an entirely natural thing, and one quite familiar in financial life."

Equations (9) can also be given a direct economic interpretation. Consider the following explanation for the user cost for a new asset,  $u_0^t$ . At the end of period  $t$ , the business unit expects to have an asset worth  $(1+i^t) P_1^t$ . Offsetting this benefit is the beginning of the period asset purchase cost,  $P_0^t$ . However, in addition to this cost, the business must charge itself either the explicit interest cost that occurs if money is borrowed to purchase the asset or the implicit opportunity cost of the equity capital that is tied up in the purchase. Thus offsetting the end of the period benefit  $(1+i^t) P_1^t$  is the initial purchase cost and opportunity interest cost of the asset purchase,  $P_0^t (1+r^t)$ , leading to a end of period  $t$  net cost of  $P_0^t (1+r^t) - (1+i^t) P_1^t$  or  $u_0^t$ .

It is interesting to note that in both the accounting and financial management literature of the past century, there was a reluctance to treat the opportunity cost of *equity capital* tied up in capital inputs as a genuine cost of production.<sup>24</sup> However, more recently, there is an acceptance of an imputed interest charge for equity capital as a genuine cost of production.<sup>25</sup>

In the following section, we will relate the asset price profiles  $\{P_n^t\}$  and the user cost profiles  $\{u_n^t\}$  to *depreciation profiles*. However, before turning to the subject of depreciation, it is important to stress that the analysis presented in this section is based on a number of restrictive assumptions, particularly on future price expectations. Moreover, we have not explained how these asset price expectations are formed and we have not explained how the period  $t$  nominal interest rate is to be estimated (we will address these topics in section 5 below). We have not explained what should be done if the sequence of second hand asset prices  $\{P_n^t\}$  is not available and the sequences of vintage rental prices or user costs,  $\{f_n^t\}$  or  $\{u_n^t\}$ , are also not available (we will address this problem in later sections as well). We have also assumed that asset values and user costs are independent of how intensively the assets are used. Finally, we have not modeled uncertainty (about future prices and the useful lives of assets) and attitudes towards risk on the part of producers. Thus the analysis presented in this chapter is only a start on the difficult problems associated with measuring capital input.

#### 4. Cross Sectional Depreciation Profiles

Recall that in the previous section,  $P_n^t$  was defined to be the price of an asset that was  $n$  periods old at the beginning of period  $t$ . Generally, the decline in asset value as we go from one age to the next older age is called *depreciation*. More precisely, we define the *cross sectional depreciation*  $D_n^t$ <sup>26</sup> of an asset that is  $n$  periods old at the beginning of period  $t$  as

<sup>24</sup> This literature is reviewed in Diewert and Fox (1999; 271-274).

<sup>25</sup> Stern Stewart & Co. has popularized the idea of charging for the opportunity cost of equity capital and has called the resulting income concept, EVA, Economic Value Added.

<sup>26</sup> This terminology is due to Hill (1999) who used this terminology to distinguish the decline in second hand asset values due to aging (cross sectional depreciation) from the decline in an asset value over a period of time (time series depreciation). Triplett (1996; 98-99) uses the cross sectional definition of depreciation and shows that it is equal to the concept of capital consumption in the national accounts but he does this under the assumption of no expected real asset inflation.

$$(10) D_n^t \equiv P_n^t - P_{n+1}^t ; n = 0, 1, 2, \dots$$

Thus  $D_n^t$  is the value of an asset that is  $n$  periods old at the beginning of period  $t$ ,  $P_n^t$ , minus the value of an asset that is  $n+1$  periods old at the beginning of period  $t$ ,  $P_{n+1}^t$ .<sup>27</sup>

Obviously, given the sequence of period  $t$  used asset prices  $\{P_n^t\}$ , we can use equations (10) to determine the period  $t$  sequence of declines in asset values by age  $n$ ,  $\{D_n^t\}$ . Conversely, given the period  $t$  cross sectional depreciation sequence or *profile*,  $\{D_n^t\}$ , we can determine the period  $t$  asset prices by age by adding up amounts of depreciation:

$$(11) \begin{aligned} P_0^t &= D_0^t + D_1^t + D_2^t + \dots \\ P_1^t &= D_1^t + D_2^t + D_3^t + \dots \\ &\dots \\ P_n^t &= D_n^t + D_{n+1}^t + D_{n+2}^t + \dots \end{aligned}$$

Rather than working with first differences of asset prices by age, it is more convenient to reparameterize the pattern of cross sectional depreciation by defining the *period  $t$  depreciation rate*  $\delta_n^t$  for an asset that is  $n$  periods old at the start of period  $t$  as follows:

$$(12) \delta_n^t \equiv 1 - [P_{n+1}^t / P_n^t] = D_n^t / P_n^t ; n = 0, 1, 2, \dots$$

In the above definitions, we require  $n$  to be such that  $P_n^t$  is positive.<sup>28</sup>

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<sup>27</sup> Of course, the objections to the use of second hand market data to determine depreciation rates are very old: “We readily agree that where a market is sufficiently large, generally accessible, and continuous over time, it serves to coordinate a large number of subjective estimates and thus may impart a moment of (social) objectivity to value relations based on prices forced on it. But it can hardly be said that the second-hand market for industrial equipment, which would be the proper place for the determination of the value of capital goods which have been in use, satisfies these requirements, and that its valuations are superior to intra-enterprise valuation.” L.M. Lachmann (1941; 376-377). “Criticism has also been voiced about the viability of used asset market price data as an indicator of in use asset values. One argument, drawing on the Akerlof Lemons Model, is that assets resold in second hand markets are not representative of the underlying population of assets, because only poorer quality units are sold when used. Others express concerns about the thinness of resale markets, believing that it is sporadic in nature and is dominated by dealers who under-bid.” Charles R. Hulten and Frank C. Wykoff (1996; 17-18).

<sup>28</sup> This definition of depreciation dates back to Hicks (1939) at least and was used extensively by Hulten and Wykoff (1981a) (1981b), Diewert (1974; 504) and Hulten (1990; 128) (1996; 155): “If there is a perfect second hand market for the goods in question, so that a market value can be assessed for them with precision, corresponding to each particular degree of wear, then the value-loss due to consumption can be exactly measured...” John R. Hicks (1939; 176). Current cost accountants have also advocated the use of second hand market data (when available) to calculate “objective” depreciation rates: “But as a practical matter the quantification and valuation of asset services used is not a simple matter and we must fall back on estimated patterns as a basis for current cost as well as historic cost depreciation. For those fixed assets which have active second hand markets the problem is not overly difficult. A pattern of service values can be obtained at any time by comparing the market values of different ages or degrees of use. The differences so obtained, when related to the value of a new asset, yield the proportions of asset value which are normally used up or foregone in the various stages of asset life.” Edgar O. Edwards and Philip W. Bell (1961; 175).

Obviously, given the sequence of period  $t$  asset prices by age  $n$ ,  $\{P_n^t\}$ , we can use equations (12) to determine the period  $t$  sequence of *cross sectional depreciation rates* by age,  $\{\delta_n^t\}$ . Conversely, given the cross sectional sequence of period  $t$  depreciation rates,  $\{\delta_n^t\}$ , as well as the price of a new asset in period  $t$ ,  $P_0^t$ , we can determine the period  $t$  asset prices by age as follows:

$$(13) \begin{aligned} P_1^t &= (1 - \delta_0^t) P_0^t \\ P_2^t &= (1 - \delta_0^t)(1 - \delta_1^t) P_0^t \\ &\dots \\ P_n^t &= (1 - \delta_0^t)(1 - \delta_1^t)\dots(1 - \delta_{n-1}^t) P_0^t; \dots \end{aligned}$$

The interpretation of equations (13) is straightforward. At the beginning of period  $t$ , a new capital good is worth  $P_0^t$ . An asset of the same type but which is one period older at the beginning of period  $t$  is less valuable by the amount of depreciation  $\delta_0^t P_0^t$  and hence is worth  $(1 - \delta_0^t) P_0^t$ , which is equal to  $P_1^t$ . An asset which is two periods old at the beginning of period  $t$  is less valuable than a one period old asset by the amount of depreciation  $\delta_1^t P_1^t$  and hence is worth  $P_2^t = (1 - \delta_1^t) P_1^t$  which is equal to  $(1 - \delta_1^t)(1 - \delta_0^t) P_0^t$  using the first equation in (13) and so on. Suppose  $L - 1$  is the first integer which is such that  $\delta_{L-1}^t$  is equal to one. Then  $P_n^t$  equals zero for all  $n \geq L$ ; i.e., at the end of  $L$  periods of use, the asset no longer has a positive rental value. If  $L = 1$ , then a new asset of this type delivers all of its services in the first period of use and the asset is in fact a nondurable asset.

Now substitute equations (12) into equations (9) in order to obtain the following formulae for the sequence of the *end of the period user costs* by age  $n$   $\{u_n^t\}$  in terms of the price of a new asset at the beginning of period  $t$ ,  $P_0^t$ , and the sequence of cross sectional depreciation rates,  $\{\delta_n^t\}$ :

$$(14) \begin{aligned} u_0^t &= [(1+r^t) - (1+i^t)(1 - \delta_0^t)] P_0^t \\ u_1^t &= (1 - \delta_0^t)[(1+r^t) - (1+i^t)(1 - \delta_1^t)] P_0^t \\ &\dots \\ u_n^t &= (1 - \delta_0^t)\dots(1 - \delta_{n-1}^t)[(1+r^t) - (1+i^t)(1 - \delta_n^t)] P_0^t; \dots \end{aligned}$$

Thus given  $P_0^t$  (the beginning of period  $t$  price of a new asset),  $i^t$  (the new asset inflation rate that is expected at the beginning of period  $t$ ),  $r^t$  (the one period nominal interest rate that the business unit faces at the beginning of period  $t$ ) and given the sequence of cross sectional depreciation rates by age prevailing at the beginning of period  $t$  (the  $\delta_n^t$ ), then we can use equations (14) to calculate the sequence of end of the period user costs for period  $t$ , the  $u_n^t$ . Of course, given the  $u_n^t$ , we can use equations (8) to calculate the beginning of the period user costs (the  $f_n^t$ ) and then use the  $f_n^t$  to calculate the sequence of asset prices  $P_n^t$  using equations (5) and finally, given the  $P_n^t$ , we can use equations (12) in order to calculate the sequence of depreciation rates, the  $\delta_n^t$ . Thus *given any one of these sequences or profiles, all of the other sequences are completely determined*. This means

that assumptions about depreciation rates, the pattern of user costs by age or the pattern of asset prices by age *cannot be made independently of each other*.<sup>29</sup>

It is useful to look more closely at the first equation in (14), which expresses the user cost or rental price of a new asset at the beginning of period  $t$ ,  $u_0^t$ , in terms of the depreciation rate  $\delta_0^t$ , the one period nominal interest rate  $r^t$ , the new asset inflation rate  $i^t$  that is expected to prevail at the beginning of period  $t$  and the beginning of period  $t$  price for a new asset,  $P_0^t$ :

$$(15) u_0^t = [(1+r^t) - (1+i^t)(1 - \delta_0^t)] P_0^t = [r^t - i^t + (1+i^t)\delta_0^t] P_0^t.$$

Thus the user cost of a new asset  $u_0^t$  that is purchased at the beginning of period  $t$  (and the actual or imputed rental payment is made at the end of the period) is equal to  $r^t - i^t$  (a nominal interest rate minus an asset inflation rate which can be loosely interpreted<sup>30</sup> as a *real interest rate*) times the initial asset cost  $P_0^t$  plus  $(1+i^t)\delta_0^t P_0^t$  which is *depreciation* on the asset at beginning of the period prices,  $\delta_1^t P_0^t$ , times *the inflation escalation factor*,  $(1+i^t)$ .<sup>31</sup> If we further assume that the expected asset inflation rate is 0, then (15) further simplifies to:

$$(16) u_0^t = [r^t + \delta_0^t] P_0^t.$$

Under these assumptions, the user cost of a new asset is equal to the interest rate plus the depreciation rate times the initial purchase price.<sup>32</sup> This is essentially the user cost formula that was obtained by Walras in 1874:

“Let  $P$  be the price of a capital good. Let  $p$  be its gross income, that is, the price of its service inclusive of both the depreciation charge and the insurance premium. Let  $\mu P$  be the portion of this income representing the depreciation charge and  $\nu P$  the portion representing the insurance premium. What remains of the gross income after both charges have been deducted,  $\pi = p - (\mu+\nu)P$ , is the *net* income.

<sup>29</sup> This point was first made explicitly by Jorgenson and Griliches (1967; 257): “An almost universal conceptual error in the measurement of capital input is to confuse the aggregation of capital stock with the aggregation of capital service.” See also Jorgenson and Griliches (1972; 81-87). Much of the above algebra for switching from one method of representing vintage capital inputs to another was first developed by Christensen and Jorgenson (1969; 302-305) (1973) for the geometrically declining depreciation model. The general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989), Hulten (1990; 127-129) (1996; 152-160) and Diewert and Lawrence (2000).

<sup>30</sup> We will provide a more precise definition of a real interest rate later.

<sup>31</sup> This formula was obtained by Christensen and Jorgenson (1969; 302) for the geometric model of depreciation but it is valid for any depreciation model. Griliches (1963; 120) also came very close to deriving this formula in words: “In a perfectly competitive world the annual rent of a machine would equal the marginal product of its services. The rent itself would be determined by the interest costs on the investment, the deterioration in the future productivity of the machine due to current use, and the expected change in the price of the machine (obsolescence).”

<sup>32</sup> Using equations (13) and (14) and the assumption that the asset inflation rate  $i^t = 0$ , it can be shown that the user cost of an asset that is  $n$  periods old at the start of period  $t$  can be written as  $u_n^t = (r^t + \delta_n^t)P_n^t$  where  $P_n^t$  is the beginning of period  $t$  second hand market price for the asset.

We are now able to explain the differences in gross incomes derived from various capital goods having the same value, or conversely, the differences in values of various capital goods yielding the same gross incomes. It is, however, readily seen that the values of capital goods are rigorously proportional to their net incomes. At least this would have to be so under certain normal and ideal conditions when the market for capital goods is in equilibrium. Under equilibrium conditions the ratio  $[p - (\mu + \nu)P]/P$ , or the rate of net income, is the same for all capital goods. Let  $i$  be this common ratio. When we determine  $i$ , we also determine the prices of all landed capital, personal capital and capital goods proper by virtue of the equation  $p - (\mu + \nu)P = iP$  or  $P = p/[i + \mu + \nu]$ ." Léon Walras (1954; 268-269).

However, the basic idea that a durable input should be charged a period price that is equal to a depreciation term plus a term that would cover the cost of financial capital goes back much further<sup>33</sup>. For example, consider the following quotation from Babbage:

"Machines are, in some trades, let out to hire, and a certain sum is paid for their use, in the manner of rent. This is the case amongst the frame-work knitters: and Mr. Hensen, in speaking of the rate of payment for the use of their frames, states, that the proprietor receives such a rent that, besides paying the full interest for his capital, he clears the value of his frame in nine years. When the rapidity with which improvements succeed each other is considered, this rent does not appear exorbitant. Some of these frames have been worked for thirteen years with little or no repair." Charles Babbage (1835; 287).

Babbage did not proceed further with the user cost idea. Walras seems to have been the first economist who formalized the idea of a user cost into a mathematical formula. However, the early industrial engineering literature also independently came up with the user cost idea; Church described how the use of a machine should be charged as follows:

"No sophistry is needed to assume that these charges are in the nature of rents, for it might easily happen that in a certain building a number of separate little shops were established, each containing one machine, all making some particular part or working on some particular operation of the same class of goods, but each shop occupied, not by a wage earner, but by an independent mechanic, who rented his space, power and machinery, and sold the finished product to the lessor. Now in such a case, what would be the shop charges of these mechanics? Clearly they would comprise as their chief if not their only item, just the rent paid. And this rent would be made up of: (1) Interest. (2) Depreciation. (3) Insurance. (4) Profit on the capital involved in the building, machine and power-transmitting and generating plant. There would also most probably be a separate charge for power according to the quantity consumed.

Exclude the item of profit, which is not included in the case of a shop charge, and we find that we have approached most closely to the new plan of reducing any shop into its constituent production centres. No one would pretend that there was any insuperable difficulty involved in fixing a just rent for little shops let out in this plan." A. Hamilton Church (1901; 907-908).

"A production centre is, of course, either a mechanic, or a bench at which a hand craftsman works. Each of these is in the position of a little shop carrying on one little special industry, paying rent for the floor space occupied, interest for the capital involved, depreciation for the wear and tear, and so on, *quite independently of what may be paid by other production centres* in the same shop." A. Hamilton Church (1901; 734).

Church was well aware of the importance of determining the "right" rate to be charged for the use of a machine in a multiproduct enterprise. This information is required not only to price products appropriately but to determine whether an enterprise should make

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<sup>33</sup> Solomons (1968; 9-17) indicates that interest was regarded as a cost for a durable input in much of the nineteenth century accounting literature. The influential book by Garcke and Fells (1893) changed this.

or purchase a particular commodity. Babbage and Canning were also aware of the importance of determining the right machine rate charge:<sup>34</sup>

“The great competition introduced by machinery, and the application of the principle of the subdivision of labour, render it necessary for each producer to be continually on the watch, to discover improved methods by which the cost of the article he manufactures may be reduced; and, with this view, it is of great importance to know the precise expense of every process, as well as of the wear and tear of machinery which is due to it.” Charles Babbage (1835; 203).

“The question of ‘adequate’ rates of depreciation, in the sense that they will ultimately adjust the valuations to the realities, is often discussed as though it had no effect upon ultimate profit at all. Of some modes of valuing, it is said that they tend to overvalue some assets and to undervalue others, but the aggregate of book values found is nearly right. If the management pay no attention at all to the unit costs implied in such valuations, no harm is done. But if the cost accountant gives effect to these individually bad valuations through a machine-rate burden charge, and if the selling policy has regard for apparent unit profits, the valuation may lead to the worst rather than to the best possible policy.” John B. Canning (1929; 259-260).

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<sup>34</sup> Under moderate inflation, the difficulties with traditional cost accounting based on historical cost and no proper allowance for the opportunity of capital, the proper pricing of products becomes very difficult. Diewert and Fox (1999; 271-274) argued that this factor contributed to the great productivity slowdown that started around 1973 and persisted to the early 1990’s. The traditional method of cost accounting can be traced back to a book first published in 1887 by the English accountants, Garcke and Fells, who suggested allocating the “indirect costs” of producing a good proportionally to the amount of labour and materials costs used to make the item: “In some establishments the direct expenditures in wages and materials only is considered to constitute the cost; and no attempt is made to allocate to the various working or stock orders any portion of the indirect expenses. Under this system the difference between the sum of the wages and materials expended on the articles and their selling price constitutes the gross profit, which is carried in the aggregate to the credit of profit and loss, the indirect factory expenses already referred to, together with the establishment expenses and depreciation, being particularised on the debit side of that account. This method has certainly simplicity in its favour, but a more efficient check upon the indirect expenses would be obtained by establishing a relation between them and the direct expenses. This may be done by distributing all the indirect expenses, such as wages of foremen, rent of factory, fuel, lighting, heating, and cleaning, etc. (but not the salaries of clerks, office rent, stationery and other establishment charges to be referred to later), over the various jobs, as a percentage, either upon the wages expended upon the jobs respectively, or upon the cost of both wages and materials.” Emile Garcke and John Manger Fells (1893; 70-71). Compare this rather crude approach to cost accounting to the masterful analysis of Church! Garcke and Fells endorsed the idea that depreciation was an admissible item of cost that should be allocated in proportion to the prime cost (i.e., labour and materials cost) of manufacturing an article but they explicitly ruled out interest as a cost: “The item of Depreciation may, for the purpose of taking out the cost, simply be included in the category of the indirect expenses of the factory, and be distributed over the various enterprises in the same way as those expenses may be allocated; or it may be dealt with separately and more correctly in the manner already alluded to and hereafter to be fully described. The establishment expenses and interest on capital should not, however, in any case form part of the cost of production. There is no advantage in distributing these items over the various transactions or articles produced. They do not vary proportionately with the volume of business. ... The establishment charges are, in the aggregate, more or less constant, while the manufacturing costs fluctuate with the cost of labour and the price of material. To distribute the charges over the articles manufactured would, therefore, have the effect of disproportionately reducing the cost of production with every increase, and the reverse with every diminution, of business. Such a result is greatly to be depreciated, as tending to neither economy of management nor to accuracy in estimating for contracts. The principles of a business can always judge what percentage of gross profit upon cost is necessary to cover fixed establishment charges and interest on capital.” Emile Garcke and John Manger Fells (1893; 72-73). The aversion of accountants to include interest as a cost can be traced back to this quotation.

The above equations relating asset prices by age  $P_n^t$ , beginning of the period user costs by age  $f_n^t$ , end of the period user costs by age  $u_n^t$  and the (cross sectional) depreciation rates by age  $\delta_n^t$  are the fundamental ones that we will specialize in subsequent sections in order to measure both wealth capital stocks and capital services under conditions of inflation. In the following section, we shall consider several options that could be used in order to determine empirically the interest rates  $r^t$  and the asset inflation rates  $i^t$ .

## 5. The Empirical Determination of Interest Rates and Asset Inflation Rates

We consider initially three broad approaches<sup>35</sup> to the determination of the nominal interest rate  $r^t$  that is to be used to discount future period value flows by the business units in the aggregate under consideration:

- Use the ex post rate of return that will just make the sum of the user costs exhaust the gross operating surplus of the production sectors in the aggregate under consideration.
- Use an aggregate of nominal interest rates that the production sectors in the aggregate might be facing at the beginning of each period.
- Take a fixed real interest rate and add to it actual ex post consumer price inflation or anticipated consumer price inflation.

The first approach was used for the entire private production sector of the economy by Jorgenson and Griliches (1967; 267) and for various sectors of the economy by Christensen and Jorgenson (1969; 307). It is also widely used by statistical agencies. It has the advantage that the value of output for the sector will exactly equal the value of input in a consistent accounting framework. It has the disadvantages that it is subject to measurement error and it is an *ex post rate of return* which may not reflect the economic conditions facing producers at the beginning of the period.

The second approach suffers from aggregation problems. There are many interest rates in an economy at the beginning of an accounting period and the problem of finding the “right” aggregate of these rates is not a trivial one.

The third approach works as follows. Let the consumer price index for the economy at the beginning of period  $t$  be  $c^t$  say. Then the ex post general consumer inflation rate for period  $t$  is  $\rho^t$  defined as:

$$(17) \quad 1 + \rho^t \equiv c^{t+1}/c^t .$$

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<sup>35</sup> Other methods for determining the appropriate interest rates that should be inserted into user cost formulae are discussed by Harper, Berndt and Wood (1989) and in Chapter 5 of Schreyer (2001). Harper, Berndt and Wood (1989) evaluate empirically 5 alternative rental price formulae using geometric depreciation but making different assumptions about the interest rate and the treatment of asset price change. They show that the choice of formula matters.

Let the production units under consideration face the real interest rate  $r^*$ . Then by the Fisher (1896) effect, the relevant nominal interest rate that the producers face should be approximately equal to  $r^t$  defined as follows:

$$(18) r^t \equiv (1+r^*)(1+\rho^t) - 1.$$

The Australian Bureau of Statistics assumes that producers face a real interest rate of 4 %. This is consistent with long run observed economy wide real rates of return for most OECD countries which fall in the 2 to 5 per cent range. Thus following the example of the ABS, we could choose the real rate of return to be 4 % per annum; i.e., we assume that the nominal rate  $r^t$  is defined by (18) with the real rate defined by

$$(19) r^* \equiv .04$$

assuming that the accounting period chosen is a year.<sup>36</sup>

We turn now to the determination of the asset inflation rates, the  $i^t$ , which appear in most of the formulae derived in the preceding sections of this chapter. There are three broad approaches that can be used in this context:

- Use actual ex post asset inflation rates over each period.
- Assume that each asset inflation rate is equal to the general inflation rate for each period.
- Estimate anticipated asset inflation rates for each period.

The problem with the first alternative is that ex post asset inflation rates tend to be very volatile and including actual ex post asset inflation rates in the user cost formula will tend to lead to very volatile user costs or even negative user costs. If our intention is to construct user costs that approximate market rental rates for the assets under consideration, then it usually will not be appropriate to use ex post asset inflation rates in the user cost formula.

When we assume that each asset inflation rate is equal to the general inflation rate  $\rho^t$  defined by (17), the equations presented earlier simplify. Thus if we replace  $1+i^t$  by  $1+\rho^t$  and  $1+r^t$  by  $(1+r^*)(1+\rho^t)$ , equations (5), which relate the vintage asset prices  $P_n^t$  to the vintage rental prices  $f_n^t$ , become:

$$(20) \begin{aligned} P_0^t &= f_0^t + [1/(1+r^*)] f_1^t + [1/(1+r^*)]^2 f_2^t + [1/(1+r^*)]^3 f_3^t + \dots \\ P_1^t &= f_1^t + [1/(1+r^*)] f_2^t + [1/(1+r^*)]^2 f_3^t + [1/(1+r^*)]^3 f_4^t + \dots \\ &\dots \\ P_n^t &= f_n^t + [1/(1+r^*)] f_{n+1}^t + [1/(1+r^*)]^2 f_{n+2}^t + [1/(1+r^*)]^3 f_{n+3}^t + \dots \end{aligned}$$

Note that only the constant real interest rate  $r^*$  appears in these equations.

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<sup>36</sup> If we are in a high inflation situation so that the accounting period becomes a quarter or a month, then  $r^*$  must be chosen to be appropriately smaller.

If we replace  $1+i^t$  by  $1+\rho^t$  and  $1+r^t$  by  $(1+r^*)(1+\rho^t)$ , equations (14), which relate *the end of period vintage user costs*  $u_n^t$  to the vintage depreciation rates  $\delta_n^t$ , become:

$$\begin{aligned}
 (21) \quad u_0^t &= (1+\rho^t)[(1+r^*) - (1 - \delta_0^t)] P_0^t &&= (1+\rho^t)[r^* + \delta_0^t] P_0^t \\
 u_1^t &= (1+\rho^t)(1 - \delta_0^t)[(1+r^*) - (1 - \delta_1^t)] P_0^t &&= (1+\rho^t)(1 - \delta_0^t)[r^* + \delta_1^t] P_0^t \\
 &\dots && \\
 u_n^t &= (1+\rho^t)(1 - \delta_0^t) \dots (1 - \delta_{n-1}^t)[(1+r^*) - (1 - \delta_n^t)] P_0^t &&= (1+\rho^t)(1 - \delta_0^t) \dots (1 - \delta_{n-1}^t) [r^* + \delta_n^t] P_0^t.
 \end{aligned}$$

Now use equations (8) and  $1+r^t = (1+r^*)(1+\rho^t)$  and substitute into (21) to obtain the following equations, which relate the *beginning of period vintage user costs*  $f_n^t$  to the vintage depreciation rates  $\delta_n^t$ :

$$\begin{aligned}
 (22) \quad f_0^t &= (1+r^*)^{-1}[r^* + \delta_0^t] P_0^t \\
 f_1^t &= (1+r^*)^{-1}(1 - \delta_0^t)[r^* + \delta_1^t] P_0^t \\
 &\dots && \\
 f_n^t &= (1+r^*)^{-1}(1 - \delta_0^t) \dots (1 - \delta_{n-1}^t) [r^* + \delta_n^t] P_0^t.
 \end{aligned}$$

Note that only the constant real interest rate  $r^*$  appears in equations (22) but equations (21) also have the general inflation rate  $(1+\rho^t)$  as a multiplicative factor.

As mentioned above, in our third class of assumptions about asset inflation rates, we want to estimate *anticipated inflation rates* and use these estimates as our  $i^t$  in the various formulae we have exhibited. Unfortunately, there are any number of forecasting methods that could be used to estimate the anticipated inflation asset inflation rates. Alternatively, one could take a somewhat different approach than the pure forecasting one: one could simply *smooth* the observed ex post inflation rates and use these smoothed rates as our estimates of anticipated asset inflation rates.<sup>37</sup> However, there are a wide variety of smoothing methods and so again, we run into the *lack of reproducibility* problem.

To summarize our discussion on choosing interest rates and asset inflation rates to go into a user cost formula: there are many plausible alternatives and economists and statisticians have not been able to agree on “best” alternatives to use in practice.

In the next section, we turn our attention to the problem of aggregating across vintages of the same capital good.

## 6. Aggregation over Vintages of a Capital Good

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<sup>37</sup> Unfortunately, different analysts may choose different smoothing methods so there may be a problem of a lack of reproducibility in our estimating procedures. Harper, Berndt and Wood (1989; 351) note that the use of time series techniques to smooth ex post asset inflation rates and the use of such estimates as anticipated price change dates back to Epstein (1977).

In previous sections, we have discussed the beginning of period  $t$  *stock price*  $P_n^t$  of an asset that is  $n$  periods old and the corresponding beginning and end of period *user costs*,  $f_n^t$  and  $u_n^t$ . The stock prices are relevant for the construction of *real wealth measures* of capital and the user costs are relevant for the construction of *capital services measures*. We now address the problems involved in obtaining quantity series that will match up with these prices.

Let the period  $t-1$  investment in a homogeneous asset for the sector of the economy under consideration be  $I^{t-1}$ . We assume that the starting capital stock for a new unit of capital stock at the beginning of period  $t$  is  $K_0^t$  and this stock is equal to the new investment in the asset in the previous period; i.e., we assume:

$$(23) K_0^t \equiv I^{t-1}.$$

Essentially, we are assuming that the length of the period is short enough so that we can neglect any contribution of investment to current production; a new capital good becomes productive only in the period immediately following its construction. In a similar manner, we assume that the capital stock available of an asset that is  $n$  periods old at the start of period  $t$  is  $K_n^t$  and this stock is equal to the gross investment in this asset class during period  $t-n-1$ ; i.e., we assume:

$$(24) K_n^t \equiv I^{t-n-1}; \quad n = 0, 1, 2, \dots$$

Given these definitions, the value of the capital stock in the given asset class for the sector of the economy under consideration (*the wealth capital stock*) at the start of period  $t$  is

$$(25) W^t \equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \dots \\ = P_0^t I^{t-1} + P_1^t I^{t-2} + P_2^t I^{t-3} + \dots \quad \text{using (24).}$$

Turning now to the capital services quantity, we assume that the quantity of services that an asset of a particular vintage at a point in time is proportional (or more precisely, is equal) to the corresponding stock. Thus we assume that the quantity of services provided in period  $t$  by a unit of the capital stock that is  $n$  periods old at the start of period  $t$  is  $K_n^t$  defined by (24) above. Given these definitions, the value of capital services for all vintages of asset in the given asset class for the sector of the economy under consideration (*the productive services capital stock*) during period  $t$  using the end of period user costs  $u_n^t$  defined by equations (8) above is

$$(26) S^t \equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \dots \\ = u_0^t I^{t-1} + u_1^t I^{t-2} + u_2^t I^{t-3} + \dots \quad \text{using (24).}$$

Now we are faced with the problem of decomposing the value aggregates  $W^t$  and  $S^t$  defined by (25) and (26) into separate price and quantity components. If we assume that each new unit of capital lasts only a finite number of periods,  $L$  say, then we can solve

this value decomposition problem using normal index number theory. Thus define the period  $t$  vintage stock price and quantity vectors,  $P^t$  and  $K^t$  respectively, as follows:

$$(27) P^t \equiv [P_0^t, P_1^t, \dots, P_{L-1}^t] ; K^t \equiv [K_0^t, K_1^t, \dots, K_{L-1}^t] = [I^{t-1}, I^{t-2}, \dots, I^{t-L-1}] ; t = 0, 1, \dots, T.$$

Fixed base or chain indexes may be used to decompose value ratios into price change and quantity change components. In empirical work involving annual data, it is preferable to use the chain principle.<sup>38</sup> Thus the value of the capital stock in period  $t$ ,  $W^t$ , relative to its value in the preceding period,  $W^{t-1}$ , has the following index number decomposition:

$$(28) W^t / W^{t-1} = P(P^{t-1}, P^t, K^{t-1}, K^t) Q(P^{t-1}, P^t, K^{t-1}, K^t) ; t = 1, 2, \dots, T$$

where  $P$  and  $Q$  are *bilateral price and quantity indexes* respectively.

In a similar manner, we define the period  $t$  vintage end of the period user cost price and quantity vectors,  $u^t$  and  $K^t$  respectively, as follows:

$$(29) u^t \equiv [u_0^t, u_1^t, \dots, u_{L-1}^t] ; K^t \equiv [K_0^t, K_1^t, \dots, K_{L-1}^t] = [I^{t-1}, I^{t-2}, \dots, I^{t-L-1}] ; t = 0, 1, \dots, T.$$

We ask that the value of capital services in period  $t$ ,  $S^t$ , relative to its value in the preceding period,  $S^{t-1}$ , has the following index number decomposition:

$$(30) S^t / S^{t-1} = P(u^{t-1}, u^t, K^{t-1}, K^t) Q(u^{t-1}, u^t, K^{t-1}, K^t) ; t = 1, 2, \dots, T$$

where again  $P$  and  $Q$  are *bilateral price and quantity indexes* respectively.

There is now the problem of choosing the functional form for either the price index  $P$  or the quantity index  $Q$ .<sup>39</sup> For most empirical work, we recommend the *Fisher (1922) ideal price and quantity indexes*. These indexes appear to be “best” from the axiomatic viewpoint<sup>40</sup> and can also be given strong economic justifications.<sup>41</sup>

It should be noted that our use of an index number formula to aggregate over ages both stocks and services is more general than the usual aggregation over age procedures, which essentially assume that the different age of the same capital good are perfectly substitutable so that linear aggregation techniques can be used.<sup>42</sup> However, as we shall see in subsequent sections, the more general method of aggregation suggested here frequently reduces to the traditional linear method of aggregation provided that the prices by age all move in strict proportion over time.

<sup>38</sup> Given smoothly trending price and quantity data, the use of chain indexes will tend to reduce the differences between Paasche and Laspeyres indexes compared to the corresponding fixed base indexes and so chain indexes are generally preferred; see Diewert (1978; 895) for a discussion.

<sup>39</sup> Obviously, given one of these functional forms, we may use (28) to determine the other.

<sup>40</sup> See Diewert (1992b; 214-223).

<sup>41</sup> See Diewert (1976; 129-134).

<sup>42</sup> This more general form of aggregation was first suggested by Diewert and Lawrence (2000). For descriptions of the more traditional linear method of aggregation, see Jorgenson (1989; 4) or Hulten (1990; 121-127) (1996; 152-165).

Many researchers and statistical agencies relax the assumption that an asset lasts only a fixed number of periods,  $L$  say, and make assumptions about the distribution of retirements around the average service life,  $L$ . Usually, this extra degree of generality will not make much difference. However, the simultaneous retirement assumption can readily be relaxed (at the cost of much additional computational complexity) using the following methodology developed by Hulten:

“We have thus far taken the date of retirement  $T$  to be the same for all assets in a given cohort (all assets put in place in a given year). However, there is no reason for this to be true, and the theory is readily extended to allow for different retirement dates. A given cohort can be broken into components, or subcohorts, according to date of retirement and a separate  $T$  assigned to each. Each subcohort can then be characterized by its own efficiency sequence, which depends among other things on the subcohort’s useful life  $T_i$ .” Charles R. Hulten (1990; 125).

We now have all of the pieces that are required in order to decompose the capital stock of an asset class and the corresponding capital services into price and quantity components. However, in order to construct price and quantity components for capital services, we need information on the relative efficiencies  $f_n^t$  of the various ages of the capital input or equivalently, we need information on cross sectional depreciation rates by age  $\delta_n^t$  in order to use (30) above. The problem is that frequently, we do not have accurate information on either of these series and so instead, what is often done is that a standard asset life  $L$  is *assumed* and we make additional *assumptions* on the either the pattern of efficiencies or depreciation rates by age. Thus in a sense, this strategy follows the same somewhat mechanical strategy that was used by the early cost accountants:<sup>43</sup>

“The function of depreciation is recognized by most accountants as the provision of a means for spreading equitably the cost of comparatively long lived assets. Thus if a building will be of use during twenty years of operations, its cost should be recognized as operating expense, not of the first year, nor the last, but of all twenty years. Various methods may be proper in so allocating cost. The method used, however, is unimportant in this connection. The important matter is that at the time of abandonment, the cost of the asset shall as nearly as possible have been charged off as expense, under some systematic method.” M.B. Daniels (1933; 303).

However, our suggested mechanical strategy is more complex than that used by early accountants in that we translate assumptions about the pattern of cross section depreciation rates into implications for the pattern of rental prices and asset prices by age, taking into account the complications induced by discounting and expected future asset price changes.

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<sup>43</sup> Canning (1929; 204) criticized this strategy as follows: “The interminable argument that has been carried on by the text writers and others about the relative merits of the many formulas for measuring depreciation has failed, not only to produce the real merits of the several methods, but, more significantly, it has failed to produce a rational set of criteria of excellence whereby to test the aptness of any formula for any sub-class of fixed assets.”

In the following sections, we will consider 4 different sets of assumptions and show how the resulting aggregate capital stocks and services could be constructed.<sup>44</sup>

## 7. The One Hoss Shay Model of Efficiency and Depreciation

In section 3 above, we noted that Böhm-Bawerk (1891; 342) postulated that an asset would yield a constant level of services throughout its useful life of  $L$  years and then collapse in a heap to yield no services thereafter. This has come to be known as the one hoss shay or light bulb model of depreciation. Hulten noted that this pattern of relative efficiencies has the most intuitive appeal:

“Of these patterns, the one hoss shay pattern commands the most intuitive appeal. Casual experience with commonly used assets suggests that most assets have pretty much the same level of efficiency regardless of their age—a one year old chair does the same job as a 20 year old chair, and so on.” Charles R. Hulten (1990; 124).

Thus the basic assumptions of this model are that the period  $t$  efficiencies and hence cross sectional rental prices  $f_n^t$  are all equal to say  $f^t$  for ages  $n$  that are less than  $L$  periods old and for older ages of the asset, the efficiencies fall to zero. Thus we have:

$$(31) \begin{aligned} f_n^t &= f^t && \text{for } n = 0, 1, 2, \dots, L-1; \\ &= 0 && \text{for } n = L, L+1, L+2, \dots \end{aligned}$$

Now substitute (31) into the first equation in (5) and get the following formula<sup>45</sup> for the rental price  $f^t$  in terms of the price of a new asset at the beginning of year  $t$ ,  $P_0^t$ :

$$(32) f^t = P_0^t / [1 + (\gamma^t) + (\gamma^t)^2 + \dots + (\gamma^t)^{L-1}]$$

where the period  $t$  discount factor  $\gamma^t$  is defined in terms of the period  $t$  nominal interest rate  $r^t$  and the period  $t$  expected asset inflation rate  $i^t$  as follows:

$$(33) \gamma^t \equiv (1 + i^t) / (1 + r^t).$$

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<sup>44</sup> For an actual empirical example using Canadian data, see Diewert (2001) or (2004a). Diewert constructed machinery and equipment and structures capital stocks for Canada. Average lives for these two asset classes varies greatly across countries. One source of information about asset lives is the OECD (1993) where average service lives for various asset classes were reported for 14 or so OECD countries. For machinery and equipment (excluding vehicles) used in manufacturing activities, the average life ranged from 11 years for Japan to 26 years for the United Kingdom. For vehicles, the average service lives ranged from 2 years for passenger cars in Sweden to 14 years in Iceland and for road freight vehicles, the average life ranged from 3 years in Sweden to 14 years in Iceland. For buildings, the average service lives ranged from 15 years (for petroleum and gas buildings in the US) to 80 years for railway buildings in Sweden. Faced with this wide range of possible lives, Diewert followed the example of Angus Madison (1993) and assumed an average service life of 14 years for machinery and equipment and 39 years for nonresidential structures.

<sup>45</sup> This formula simplifies to  $P_0^t [1 - (\gamma^t)^L] / [1 - \gamma^t]$  provided that  $\gamma^t$  is less than 1 in magnitude. This last restriction does not always hold empirically, since for some years,  $i^t$  could exceed  $r^t$ . However, (32) is still valid under these conditions.

Now that the period  $t$  rental price  $f^t$  for an unretired asset has been determined, substitute equations (31) into equations (5) and determine the sequence of period  $t$  asset prices by age,  $P_n^t$ :

$$(34) \begin{aligned} P_n^t &= f^t [1 + (\gamma^t) + (\gamma^t)^2 + \dots + (\gamma^t)^{L-1-n}] & \text{for } n = 0, 1, 2, \dots, L-1 \\ &= 0 & \text{for } n = L, L+1, L+2, \dots \end{aligned}$$

Finally, use equations (8) to determine the end of period  $t$  rental prices,  $u_n^t$ , in terms of the corresponding beginning of period  $t$  rental prices,  $f_n^t$ :

$$(35) u_n^t = (1 + r^t) f_n^t ; \quad n = 0, 1, 2, \dots$$

Given the asset prices by age defined by (34), we can use equations (12) above to determine the corresponding cross sectional depreciation rates  $\delta_n^t$ .

We turn now to our second model of depreciation and efficiency.

## 8. The Declining Balance or Geometric Depreciation Model

The declining balance method of depreciation dates back to Matheson (1910; 55) at least.<sup>46</sup> In terms of the algebra presented in section 4 above, the method is very simple: all of the cross sectional vintage depreciation rates  $\delta_n^t$  defined by (12) are assumed to be equal to the same rate  $\delta$ , where  $\delta$  a positive number less than one; i.e., we have for all time periods  $t$ :

$$(36) \delta_n^t = \delta ; \quad n = 0, 1, 2, \dots .$$

Substitution of (36) into (14) leads to the following formula for the sequence of period  $t$  vintage user costs:

$$(37) \begin{aligned} u_n^t &= (1 - \delta)^{n-1} [(1+r^t) - (1+i^t)(1 - \delta)] P_0^t ; & n = 0, 1, 2, \dots \\ &= (1 - \delta)^{n-1} u_0^t & ; \quad n = 1, 2, \dots . \end{aligned}$$

The second set of equations in (37) says *that all of the vintage user costs are proportional to the user cost for a new asset*. This proportionality means that we do not have to use an index number formula to aggregate over vintages to form a capital services aggregate. To see this, using (37), the period  $t$  services aggregate  $S^t$  defined earlier by (26) can be rewritten as follows:

$$(38) \begin{aligned} S^t &\equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \dots \\ &= u_0^t [K_0^t + (1 - \delta) K_1^t + (1 - \delta)^2 K_2^t + \dots] \end{aligned}$$

<sup>46</sup> Matheson (1910; 91) used the term “diminishing value” to describe the method. Hotelling (1925; 350) used the term “the reducing balance method” while Canning (1929; 276) used the term the “declining balance formula”.

$$= u_0^t K_A^t$$

where the *period t capital aggregate*  $K_A^t$  is defined as

$$(39) K_A^t \equiv K_0^t + (1 - \delta) K_1^t + (1 - \delta)^2 K_2^t + \dots$$

If the depreciation rate  $\delta$  and the vintage capital stocks are known, then  $K_A^t$  can readily be calculated using (39). Then using the last line of (38), we see that the value of capital services (over all vintages),  $S^t$ , decomposes into the price term  $u_0^t$  times the quantity term  $K_A^t$ . Hence, it is not necessary to use an index number formula to aggregate over vintages using this depreciation model.

A similar simplification occurs when calculating the wealth stock using this depreciation model. Substitution of (36) into (13) leads to the following formula for the sequence of period t vintage asset prices:

$$(40) P_n^t = (1 - \delta)^{n-1} P_0^t; \quad n = 1, 2, \dots$$

Equations (40) say *that all of the vintage asset prices are proportional to the price of a new asset*. This proportionality means that again, we do not have to use an index number formula to aggregate over vintages to form a capital stock aggregate. To see this, using (40), the period t wealth aggregate  $W^t$  defined earlier by (25) can be rewritten as follows:

$$\begin{aligned} (41) W^t &\equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \dots \\ &= P_0^t [K_0^t + (1 - \delta) K_1^t + (1 - \delta)^2 K_2^t + \dots] \\ &= P_0^t K_A^t \end{aligned}$$

where  $K_A^t$  was defined by (39). Thus  $K_A^t$  can serve as both a capital stock aggregate or a flow of services aggregate, which is a major advantage of this model.<sup>47</sup>

There is a further simplification of the model which is useful in applications. If we compare equation (39) for period t+1 and period t, we see that the following formula results using equations (39):

$$(42) K_A^{t+1} \equiv K_0^{t+1} + (1 - \delta) K_A^t.$$

Thus the period t+1 aggregate capital stock,  $K_A^{t+1}$ , is equal to the investment in new assets that took place in period t, which is  $K_0^{t+1}$ , plus  $1 - \delta$  times the period t aggregate capital stock,  $K_A^t$ . This means that given a starting value for the capital stock, we can readily update it just using the depreciation rate  $\delta$  and the new investment in the asset during the prior period.

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<sup>47</sup> This advantage of the model has been pointed out by Jorgenson (1989) (1996b) and his coworkers. Its early application dates back to Jorgenson and Griliches (1967) and Christensen and Jorgenson (1969) (1973).

We now need to address the problem of determining the depreciation rate  $\delta$  for a particular asset class. Matheson was perhaps the first engineer to address this problem. On the basis of his experience, he simply postulated some approximate rates that could be applied:

“In most [brick or stone] factories an average of 3 per cent for buildings will generally be found appropriate, if due attention is paid to repairs. Such a rate will bring down a value of £ 1000 to £ 400 in thirty years.” Ewing Matheson (1910; 69).

“Buildings of wood or iron would require a higher rate, ranging from 5 to 10 per cent, according to the design and solidity of the buildings, the climate, the care and the regularity of the painting, and according also, to the usage they are subjected to.” Ewing Matheson (1910; 69).

“Contractors’ locomotives working on imperfect railroads soon wear out, and a rate of 20 per cent is generally required, bringing down the value of an engine costing £ 1000 to £ 328 in five years.” Ewing Matheson (1910; 86).

“In engineering factories, where the work is of a moderate kind which does not strain the machines severely, and where the hours of working do not average more than fifty per week, 5 per cent written off each year from the diminishing value will generally suffice for the wear-and-tear of machinery, cranes and fixed plant of all kinds, if steam engines and boilers be excluded.” Ewing Matheson (1910; 82).

“The high speed of the new turbo generators introduced since 1900, and their very exact fitting, render them liable to certain risks from variations in temperature and other causes. Several changes in regard to speed and methods of blading have occurred since their first introduction and if these generators are taken separately, only after some longer experience has been acquired can it be said that a depreciation rate of 10 per cent on the diminishing value will be too much for maintaining a book-figure appropriate to their condition. Such a rate will reduce £ 1000 to £ 349 in ten years.” Ewing Matheson (1910; 91).

How did Matheson arrive at his estimated depreciation rates? He gave some general guidance as follows:

“The main factors in arriving at a fair rate of depreciation are:

1. The Original value.
2. The probable working Life.
3. The Ultimate value when worn out or superceded.

Therefore, in deciding upon an appropriate rate of depreciation which will in a term of years provide for the estimated loss, it is not the original value or cost which has to be so provided for, but that cost less the ultimate or scrap value.” Ewing Matheson (1910; 76).

The algebra corresponding to Matheson’s method for determining  $\delta$  was explicitly described by the accountant Canning (1929; 276). Let the initial value of the asset be  $V_0$  and let its scrap value  $n$  years later be  $V_n$ . Then  $V_0$ ,  $V_n$  and the depreciation rate  $\delta$  are related by the following equation:

$$(43) V_n = (1 - \delta)^n V_0.$$

Canning goes on to explain that  $1 - \delta$  may be determined by solving the following equation:

$$(44) \log(1 - \delta) = [\log V_n - \log V_0]/n.$$

It is clear that Matheson used this framework to determine depreciation rates even though he did not lay out formally the above straightforward algebra.

However, Canning had a very valid criticism of the above method:

“This method can be summarily rejected for a reason quite independent of the deficiencies of formulas 1 and 2 above [(43) and (44) above]. Overwhelming weight is given to  $V_n$  in determining book values. ... Thus the least important constant in reality is given the greatest effect in the formula.” John B. Canning (1929; 276).

Thus Canning pointed out that the scrap value,  $V_n$ , which is not determined very accurately from an a priori point of view, is the tail that is wagging the dog; i.e., this poorly determined value plays a crucial role in the determination of the depreciation rate.<sup>48</sup>

An effective response to Canning’s criticism of the declining balance method of depreciation did not emerge until relatively recently when Hall (1971), Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b) used an entire array of used asset prices at point in time in order to determine the geometric depreciation rate which best matched up with the data.<sup>49</sup> Another theoretical possibility would be to use information on vintage rental prices in order to deduce the depreciation rate.<sup>50</sup> Hulten and Wykoff summarize their experience in estimating depreciation rates from used asset prices by concluding that the assumption of geometric or declining balance depreciation described their data relatively well:

“We have used the approach to study the depreciation patterns of a variety of fixed business assets in the United States (e.g., machine tools, construction equipment, autos and trucks, office equipment, office buildings, factories, warehouses, and other buildings). The straight line and concave patterns [i.e., one hoss

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<sup>48</sup> “There may be cases in which the formula fits the facts, but ... the chance of its being a formula of close fit is remote indeed. Its chief usefulness seems to be to furnish drill in the use of logarithms for students in accounting.” John B. Canning (1929; 277).

<sup>49</sup> Jorgenson (1996a) has a nice review of most of the empirical studies of depreciation. It should be noted that Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1996; 22) showed that equation (44) must be adjusted to correct for the early retirement of assets. The accountant Schmalenbach (1959; 91) (the first German edition was published in 1919) also noticed this problem: “The mistake should not be made, however, of drawing conclusions about useful life from those veteran machines which are to be seen in most businesses. Those which one sees are but the rare survivors; the many dead have long lain buried. This can be the source of serious errors.”

<sup>50</sup> This possibility is mentioned by Hulten and Wykoff (1996; 15): “In other words, if there were active rental markets for capital services as there are for labor services, the observed prices could be used to estimate the marginal products. And the rest of the framework would follow from these estimates. But, again, there is bad news: most capital is owner utilized, like much of the stock of single family houses. This means that owners of capital, in effect, rent it to themselves, leaving no data track for the analyst to observe.”

shay patterns] are strongly rejected ; geometric is also rejected, but the estimated patterns are extremely close to (though steeper than) the geometric form, even for structures. Although it is rejected statistically, the geometric pattern is far closer than either of the other two candidates. This leads us to accept the geometric pattern as a reasonable approximation for broad groups of assets, and to extend our results to assets for which no resale markets exist by imputing depreciation rates based on an assumption relating the rate of geometric decline to the useful lives of assets.” Charles C. Hulten and Frank C. Wykoff (1996; 16).

This brings us to our next problem: how can assumptions about asset lives in years be converted into geometric depreciation rates? In particular, how should we convert an estimated asset life of 39 years for structures and 14 years for machinery and equipment into comparable geometric rates?

One possible method for converting an average asset life,  $L$  periods say, into a comparable geometric depreciation rate is to argue as follows. Suppose that the straight line model of depreciation is the correct one (see Problem 1 below for a description of this model) and the asset under consideration has a useful life of  $L$  periods. Suppose further that investment in this type of asset is constant over time at one unit per period and asset prices are constant over time. Under these conditions, the long run equilibrium capital stock for this asset would be<sup>51</sup>:

$$(45) 1 + [(L-1)/L] + [(L-2)/L] + \dots + [2/L] + [1/L] = L(L+1)/2L = (L+1)/2.$$

Under the same conditions, the long run equilibrium geometric depreciation capital stock would be equal to the following sum:

$$(46) 1 + (1-\delta) + (1-\delta)^2 + \dots = 1/[1-(1-\delta)] = 1/\delta.$$

Now find the depreciation rate  $\delta$  which will make the two capital stocks equal; i.e., equate (45) to (46) and solve for  $\delta$ . The resulting  $\delta$  is:

$$(47) \delta = 2/(L+1).$$

Obviously, there are a number of problematical assumptions that were made in order to derive the depreciation rate  $\delta$  that corresponds to the length of life  $L$ <sup>52</sup> but (47) gives us at least a definite method of conversion from one model to the other.

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<sup>51</sup> The linear depreciation model implies that the vintage asset prices are proportional. Hence Hicks' Aggregation Theorem will imply that the capital aggregate will be the simple sum on the right hand side of (45).

<sup>52</sup> The two assumptions that are the least justified are: (1) the assumption that the straight line depreciation model is the correct model to do the conversion and (2) the assumption that investment has been constant back to minus infinity. Hulten and Wykoff (1996; 16) made the following suggestions for converting an  $L$  into a  $\delta$ : “Information is available on the average service life,  $L$ , from several sources. The rate of depreciation for non-marketed assets can be estimated using a two step procedure based on the ‘declining balance’ formula  $\delta = X/L$ . Under the ‘double declining balance’ formula,  $X = 2$ . The value of  $X$  can be estimated using the formula  $X = \delta L$  for those assets for which these estimates are available. In the Hulten-Wykoff studies, the average value for of  $X$  for producer’s durable equipment was found to be 1.65 (later revised to 1.86). For nonresidential structures,  $X$  was found to be 0.91. Once  $X$  is fixed,  $\delta$  follows for other assets whose average service life is available.”

If we assume that the average length of life for nonresidential construction is  $L$  equal to 39 years, applying the conversion formula (47) implies that the corresponding  $\delta$  equals .05; similarly, an assumed life of 14 years for machinery and equipment translates into a  $\delta$  equal to a 13.1/3% geometric depreciation rate for this asset class.

There is one remaining problem associated with the geometric depreciation model: how do we obtain a starting value for a geometric capital stock?

One method for dealing with this problem is due to Kohli<sup>53</sup> and it works as follows. Suppose we have collected investment data on a particular asset class for a number of periods, starting at period 0. Let the period  $t$  (real) investment be  $I^t$  and suppose that one plus the rate of growth of investment going from period  $t-1$  to  $t$  is

$$(48) 1+g^t \equiv I^t/I^{t-1} \text{ for } t = 1, 2, \dots, T.$$

Calculate the geometric average of these sample rates of growth  $g$  as follows:

$$(49) 1+g \equiv \{\prod_{t=1}^T (1+g^t)\}^{1/T}.$$

If investments in this asset class had been growing in the past at the average sample rate of growth  $g$  and the geometric depreciation rate is  $\delta$ , then the geometric capital stock at the start of period 0,  $K^0$ , would be equal to:

$$\begin{aligned} (50) K^0 &= I^0(1+g)^{-1} \{1 + [(1-\delta)/(1+g)] + [(1-\delta)/(1+g)]^2 + [(1-\delta)/(1+g)]^3 + \dots\} \\ &= I^0(1+g)^{-1} / (1 - [(1-\delta)/(1+g)]) \\ &= I^0 / [(1+g) - (1-\delta)] \\ &= I^0 / [g + \delta]. \end{aligned}$$

This completes our discussion of the geometric depreciation model for capital. Given its simplicity, it is our recommended model.

## 9. The Straight Line Method of Depreciation

The *straight line method of depreciation* is very simple in a world without price change: one simply makes an estimate of the most probable length of life for a new asset,  $L$  periods say, and then the original purchase price  $P_0^t$  is divided by  $L$  to yield as estimate of period by period depreciation for the next  $L$  periods. In a way, this is the simplest possible model of depreciation, just as the one hoss shay model was the simplest possible model of efficiency decline. We now set out the equations which describe the straight line model of depreciation in the general case when the anticipated asset inflation rate  $i^t$  is nonzero. Assuming that the asset has a life of  $L$  periods and that the cross sectional amounts of depreciation  $D_n^t \equiv P_n^t - P_{n+1}^t$  defined by (10) above are all equal for the assets

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<sup>53</sup> This method for obtaining a starting value for the geometric capital stock is due to Griliches (1980; 427) and Kohli (1982); see also Fox and Kohli (1998).

in use, then it can be seen that the beginning of period  $t$  vintage asset prices  $P_n^t$  will decline linearly for  $L$  periods and then remain at zero; i.e., the  $P_n^t$  will satisfy the following restrictions:

$$(51) \begin{aligned} P_n^t &= P_0^t [L - n]/L & n &= 0, 1, 2, \dots, L \\ &= 0 & n &= L+1, L+2, \dots \end{aligned}$$

### Problems

1. Recall definition (12) above, which defined the cross sectional depreciation rate for an asset that is  $n$  periods old at the beginning of period  $t$ ,  $\delta_n^t$ .

(a) Using (51) and the  $n$ th equation in (13), show that:

$$(i) (1 - \delta_0^t)(1 - \delta_1^t) \dots (1 - \delta_{n-1}^t) = P_n^t / P_0^t = 1 - (n/L) \quad \text{for } n = 1, 2, \dots, L.$$

(b) Using (i) for  $n$  and  $n+1$ , show that

$$(ii) (1 - \delta_n^t) = [L - (n+1)]/[L - n] \quad n = 0, 1, 2, \dots, L - 1.$$

(c) Now substitute (i) and (ii) into the general user cost formula (14) in order to obtain a formula for the *period  $t$  end of the period straight line user costs*,  $u_n^t$ , for  $n = 0, 1, 2, \dots, L-1$ .<sup>54</sup>

Comments: Equations (51) give us the sequence of vintage asset prices that are required to calculate the wealth capital stock while your answer to part (c) gives us the vintage user costs that are required to calculate capital services for the asset.

(d) Suppose that the nominal interest rate  $r^t$  and the nominal asset inflation rate  $i^t$  are both zero. Calculate the sequence of asset prices  $P_n^t$  and user costs for  $n = 0, 1, 2, \dots, L$  under these conditions.

2. Consider the one hoss shay model of depreciation.

(a) Suppose that the nominal interest rate  $r^t$  and the nominal asset inflation rate  $i^t$  are both zero. Calculate the sequence of asset prices  $P_n^t$  and user costs for  $n = 0, 1, 2, \dots, L$  for the one hoss shay model under these conditions.

(b) Compare the answers you obtained in part (a) of this question with the answer you obtained in part (d) of problem 1. Are there any relationships between your answers?

We conclude this section with some information on the early history of the straight line method for depreciation.

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<sup>54</sup> The user costs for  $n = L, L+1, L+2, \dots$  are all zero.

The accountant Canning summarized the straight line depreciation model as follows:

“*Straight Line Formula* ... In general, only two primary estimates are required to be made, viz., scrap value at the end of  $n$  periods and the numerical value of  $n$ . ... Obviously the number of periods of contemplated use of an asset can seldom be intelligently estimated without reference to the anticipated conditions of use. If the formula is to be respectable at all, the value of  $n$  must be the most probable number of periods that will yield the most economical use.” John B. Canning (1929; 265-266).

The following quotations indicate that the use of straight line depreciation dates back to the 1800’s at least:

“Sometimes an equal installment is written off every year from the original value of the plant; sometimes each machine or item of plant is considered separately; but it is more usual to write off a percentage, not of the original value, but from the balance of the plant account of the preceding year.” Ewing Matheson (1910; 55).

“In some instances the amount charged to revenue account for depreciation is a fixed sum, or an arbitrary percentage on the book value.” Emile Garcke and John Manger Fells (1893; 98).

The last two quotations indicate that the declining balance or geometric depreciation model (to be considered in the next section) also dates back to the 1800’s as a popular method for calculating depreciation.

## 10. The Linear Efficiency Decline Model

Recall that our first class of models (the one loss shay models) assumed that the efficiency (or cross section user cost) of the asset remained constant over the useful life of the asset. In our third class of models (the straight line depreciation models), we assumed that the cross sectional depreciation of the asset declined at a linear rate. In our second class of models (the geometric depreciation models), we assumed that cross section depreciation declined at a geometric rate. Comparing the third class with the second class of models, it can be seen that geometric depreciation is more *accelerated* than straight line depreciation; i.e., depreciation is relatively large for new vintages compared to older ones. In this section, we will consider another class of models that gives rise to an accelerated pattern of depreciation: the class of models that exhibit *a linear decline in efficiency*.

It is relatively easy to develop the mathematics of this model. Let  $f_0^t$  be the period  $t$  rental price for an asset that is new at the beginning of period  $t$ . If the useful life of the asset is  $L$  years and the efficiency decline is linear, then the sequence of period  $t$  cross sectional user costs  $f_n^t$  is defined as follows:

$$(52) \begin{aligned} f_n^t &\equiv f_0^t [L - n]/L ; & n &= 0, 1, 2, \dots, L - 1 ; \\ &\equiv 0 & ; & n &= L, L+1, L+2, \dots \end{aligned}$$

### Problems

3. (a) Substitute (52) into the first equation in (5) and obtain a formula for the rental price  $f_0^t$  in terms of the price of a new asset at the beginning of year  $t$ ,  $P_0^t$ .

(b) Substitute the formula for  $f_0^t$  that you obtained in part (a) above into (5) and obtain the sequence of period  $t$  vintage asset prices,  $P_n^t$ , for  $n = 0, 1, 2, \dots, L-1$  (the  $P_n^t$  will be 0 for  $n = L, L+1, L+2, \dots$ ).

4. Assume that the nominal interest rate  $r^t$  and the nominal asset inflation rate  $i^t$  are both zero. (a) Using your answer in part 3 (b) above, calculate:

$$(i) D_n^t \equiv P_n^t - P_{n+1}^t \quad \text{for } n = 0, 1, 2, \dots, L.$$

Comment: Formula (i) should show that when  $r^t = i^t = 0$ , *depreciation declines at a linear rate* for the linear efficiency decline model. When depreciation declines at a linear rate, the resulting formula for depreciation is called *the sum of the year digits formula*.<sup>55</sup> Thus just as the one hoss shay and straight line depreciation models coincide when  $r^t = i^t = 0$ , so too do the linear efficiency decline and sum of the digits depreciation models coincide.

### Appendix 1: A Theoretical Treatment of Inventory Change

A theoretical framework is needed to measure the contribution of the change inventory stock over a period to production. It is also necessary to work out the user cost of the beginning of the period stock of inventories. A framework to answer these questions is outlined, taken from Diewert and Smith (1994).

First consider the theory for a single inventory stock item. Consider a firm that perhaps produces a noninventory output during period  $t$ ,  $Y^t$ , uses a noninventory input  $X^t$ , sells the amount  $S^t$  of an inventory item during period  $t$  and makes purchases of the inventory item during period  $t$  in the amount  $B^t$ . Suppose that the average prices during period  $t$  of  $Y^t$ ,  $X^t$ ,  $S^t$  and  $B^t$  are  $P_Y^t$ ,  $P_X^t$ ,  $P_S^t$  and  $P_B^t$  respectively. Then neglecting balance sheet items, the firm's period  $t$  *cash flow* is:<sup>56</sup>

$$(A1) CF^t \equiv P_Y^t Y^t - P_X^t X^t + P_S^t S^t - P_B^t B^t.$$

Let the firm's beginning of period  $t$  stock of inventory be  $K^{t-1}$  and let its end of period stock of inventory be  $K^t$ . These inventory stocks are valued at the balance sheet prices prevailing at the beginning and end of period  $t$ ,  $P_K^{t-1}$  and  $P_K^t$  respectively. Note that all 4 prices involving inventory items,  $P_S^t$ ,  $P_B^t$ ,  $P_K^{t-1}$  and  $P_K^t$  can be different.

<sup>55</sup> Canning (1929; 277) describes the method in some detail so it was already in common use by that time.

<sup>56</sup> Note that this framework is flexible enough to allow the firm to either purchase or produce internally inventory items. Note also that firm purchases of inventory items from other domestic firms would appear in the national accounts as intermediate input purchases and purchases from foreign suppliers would appear as imports. On the other hand, sales of inventory items by the firm to domestic producers, households or foreigners would appear in the national accounts as gross outputs, final household consumption or exports respectively.

The firm's period  $t$  *economic income* or *net profit* is defined as its cash flow plus the value of its end of period  $t$  stock of inventory items less  $(1+r^t)$  times the value of its beginning of period  $t$  stock of inventory items:<sup>57</sup>

$$(A2) EI^t \equiv CF^t + P_K^t K^t - (1+r^t) P_K^{t-1} K^{t-1}$$

where  $r^t$  is the nominal cost of capital that the firm faces at the beginning of period  $t$ . Thus in definition (A2), it is assumed that the firm has to borrow financial capital or raise equity capital at the cost  $r^t$  in order to finance its initial holdings of inventory items. This cost could be real (in the case of a firm whose initial capital is funded by bonds) or it could be an opportunity cost (in the case of a firm entirely funded by equity capital).

The end of period stock of inventory is related to the beginning of the period stock by the following equation:

$$(A3) K^t = K^{t-1} + B^t - S^t - U^t$$

where  $U^t$  denotes inventory items that are lost, spoiled, damaged or are used internally by the firm. In the case of livestock inventories, there is a natural growth rate of inventories over the period so equation (A3) is replaced by:

$$(A4) K^t = K^{t-1} + B^t - S^t + G^t$$

where  $G^t$  denotes the natural growth of the stock over period  $t$ .<sup>58</sup>

Define the *change in inventory stocks* over period  $t$  as:

$$(A5) \Delta K^t \equiv K^t - K^{t-1} .$$

Using (A5), both (A3) and (A4) can be written as:

$$(A6) K^t = K^{t-1} + \Delta K^t .$$

Now substitute (A6) into the definition of economic income (A2) and the following expression is obtained:

$$(A7) EI^t \equiv CF^t + P_K^t [K^{t-1} + \Delta K^t] - (1+r^t) P_K^{t-1} K^{t-1} \\ = CF^t + P_K^t \Delta K^t - [r^t P_K^{t-1} - (P_K^t - P_K^{t-1}) ]K^{t-1} .$$

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<sup>57</sup> In this definition of economic income, we are assuming that current flow variables are realized at the end of the accounting period and we are "discounting" the beginning of the period values of the inventory items in place at the beginning of the period to the end of the period. The resulting user costs will be end of the period user costs.

<sup>58</sup> If the firm is constructing inventory items either for direct sale or as an intermediate step in its production processes, then these produced additions to the stock would be included in the term  $G^t$ .

Thus *economic income is equal to cash flow plus the value of the change in inventory (valued at end of period balance sheet prices) minus the user cost of inventories times the starting stocks of inventories* where this *period t user cost* is defined as

$$(A8) P_U^t \equiv r^t P_K^{t-1} - (P_K^t - P_K^{t-1}).$$

Note that the above algebra works for both livestock and ordinary inventory items.

There can be two versions of the user cost:

- An ex post version where the *actual* end of period balance sheet price of inventories is used or
- An ex ante version where at the beginning of period t, we estimate a *predicted value* for the end of period balance sheet price.

For the production accounts in the SNA, the ex ante version is the appropriate version, which means the national income accountant has some leeway in forming estimates of the end of period balance sheet price for the inventory item. Looking at (A7), it is important to note that the change in inventories that occurred over period t,  $\Delta K^t$ , should be valued at the end of period t price for the inventory item,  $P_K^t$ .<sup>59</sup>

If the firm is using or selling many inventory items, say J items, then equation (A7) becomes:

$$(A9) EI^t \equiv CF^t + \sum_{j=1}^J P_{K_j}^t \Delta K_j^t - \sum_{j=1}^J [r^t P_{K_j}^{t-1} - (P_{K_j}^t - P_{K_j}^{t-1})] K_j^{t-1}$$

where the notation is obvious. The terms involving the value of the change in inventories over the period are the following ones:

$$(A10) \sum_{j=1}^J P_{K_j}^t \Delta K_j^t = \sum_{j=1}^J P_{K_j}^t [K_j^t - K_j^{t-1}]$$

$$(A11) \quad = \sum_{j=1}^J P_{K_j}^t K_j^t - \sum_{j=1}^J P_{K_j}^t K_j^{t-1}.$$

Looking at (A10), it would appear that normal index number theory could be applied to the sum of terms in the value aggregate on the right hand side, with prices defined as the end of period t balance sheet prices  $P_{K_j}^t$  and corresponding quantities defined as the inventory changes  $K_j^t - K_j^{t-1}$  over period t. However, this value aggregate is not necessarily of one sign over time: it could be positive, negative or zero. *Normal index number theory breaks down for value aggregates that can be either positive or negative over time.*<sup>60</sup> Thus index number theory should not be applied to the value aggregate on

<sup>59</sup> However, the current SNA methodology requires that inventory change over the production period be evaluated at the average prices of the period. This requirement could be accommodated in our framework by replacing the end of period price of the inventory item,  $P_K^t$ , by an appropriate average inventory price for period t. If this is done, and if the actual end of period price of the inventory item is used for balance sheet purposes, then a reconciliation entry will be required in the Revaluation Accounts.

<sup>60</sup> To see why this breakdown occurs, consider a situation where the value aggregate just happens to be zero in the base period. Laspeyres price and quantity indexes will be undefined under these circumstances and

the right hand side of (A10). Instead, it is recommended that index number theory be applied *separately* to the two value aggregates on the right hand side of (A11).<sup>61</sup> Thus  $\sum_{j=1}^J P_{Kj}^t K_j^t$  should be decomposed (using normal index number theory) into  $P_{KE}^t K_E^t$  where  $P_{KE}^t$  is the scalar end of period t aggregate price of inventories and  $K_E^t$  is the corresponding end of period t aggregate stock and  $\sum_{j=1}^J P_{Kj}^t K_j^{t-1}$  should be decomposed into  $P_{KB}^t K_B^t$  where  $P_{KB}^t$  is the scalar beginning of period t aggregate price of inventories and  $K_B^t$  is the corresponding beginning of period t aggregate stock. Then in place of the current single aggregate for inventory change that is reported in the current System of National Accounts, it is recommended that *inventory change be treated in a manner that is symmetric to the treatment of aggregate exports and imports in the accounts*; i.e., the end of period aggregates  $P_{KE}^t$  and  $K_E^t$  (the counterparts to the aggregate price of exports and the aggregate quantity of exports) and the beginning of period aggregates  $P_{KB}^t$  and  $K_B^t$  would be reported separately just as exports and imports are reported separately in the current SNA.

There is another treatment of inventory change that could be used by statistical agencies that is much more straightforward. The definition of economic income, (A2) above, can be rewritten as follows:

$$(A12) EI^t \equiv CF^t + P_K^t K^t - P_K^{t-1} K^{t-1} - r^t P_K^{t-1} K^{t-1}.$$

Using (A12), *the value of inventory change for period t* is simply defined as the end of period t value of the stock,  $VK^t$ , less the beginning of period t value of the stock,  $VK^{t-1}$ :

$$(A13) VK^t - VK^{t-1} = P_K^t K^t - P_K^{t-1} K^{t-1}.$$

Using this decomposition of economic income, the user cost value aggregate is defined as the last term on the right hand side of (A12) and so *the new user cost of inventories* is:

$$(A14) P_U^{t*} \equiv r^t P_K^{t-1}.$$

The new user cost of inventories,  $P_U^{t*}$  defined by (A14), can be compared to the initial user cost of inventories,  $P_U^t$  defined by (A8), and the new value of inventory change defined by (A13) can be compared to the earlier expression for the value of inventory change defined by (A11). Both the old and the new decomposition of economic income are theoretically valid. However, note that a nominal interest rate  $r^t$  appears in (A14) whereas a type of real interest rate appeared in (A8). Hence for a country experiencing high inflation, the new user cost of inventories will be higher than the old user cost and similarly, the new value of inventory change defined by (A13) will be higher than the old

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nonsensical numbers will be obtained if the value aggregate is very close to zero in the base period. However, if the Laspeyres, Paasche or Fisher formula is used in forming a larger aggregate that is bounded well away from zero, then the right hand side of (A10) can be used when forming this larger aggregate and the same results will be obtained as using the right hand side of (A11) in forming the larger aggregate.

<sup>61</sup> This solution to the aggregation problem was suggested by Diewert (2004b; 36).

value of inventory change defined by (A11).<sup>62</sup> Thus nominal GDP will tend to be higher using the new decomposition compared to the initial one and it will be substantially higher under conditions of high inflation.

There are advantages and disadvantages of using the second decomposition of economic income compared to the first:

- The *main advantage* of the second decomposition is that it is much more straightforward and will be easier to explain to users. Also, it is much easier to reconcile quarterly changes in inventories to annual changes using the second decomposition.
- The *main disadvantage* of the second decomposition is that the resulting user cost of inventories is *different* from the user cost formula for reproducible capital and so an awkward asymmetry would be introduced into the SNA if a user cost approach to reproducible capital were introduced.<sup>63</sup>

Both decompositions of economic income involve a difference in two value aggregates where the sign of the difference cannot be bounded away from zero. Hence for both decompositions, it is recommended that the beginning and end of period values be separately deflated and shown as two items in the real accounts in a manner that is analogous to the present treatment of exports less imports.

## Appendix 2: The Underlying Model of Production

In this Appendix, the motivation for the model of production that was defined by equations (A1) and (A2) in the previous Appendix is explained. This model of is based on a well established model of production that is used both by economists and thoughtful accountants as the following two quotations will show:

“We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output” John R. Hicks (1961; 23).

“The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period

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<sup>62</sup> If the initial decomposition of economic income is used, then the beginning of the period inventory stocks are valued at the higher end of period prices but since this value aggregate is given a minus sign, this will reduce nominal GDP.

<sup>63</sup> The ex ante user cost for a reproducible capital asset contains an anticipated asset inflation rate in it similar to (A8), which offsets the nominal interest rate term. The ex ante user cost concept should be close to an actual rental or leasing price for the asset since it based on the same considerations that an owner would consider in setting a rental price. Hence, it seems desirable to have the user cost of inventories aligned with the user cost of reproducible capital.

specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity.” Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with “flow” inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period “flow” outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).<sup>64</sup> The model could be viewed as an Austrian model of production in honour of the Austrian economist Böhm-Bawerk (1891) who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.

Now relate this theoretical model of production to equations (A1) and (A2) in the previous Appendix. All of the “flow” inputs that are purchased during the period and all of the “flow” outputs that are sold during the period are the inputs and outputs that appear in the definition of cash flow in definition (A1). These are the flow inputs and outputs that are very familiar to national income accountants. But this is not the end of the story: the firm inherits an endowment of assets at the beginning of the production period and at the end of the period, the firm will have the net profit or loss that has occurred due to its sales of outputs and its purchases of inputs during the period. As well, *it will have a stock of assets that it can use when it starts production in the following period.* Hence it seems clear that just focusing on the flow transactions that occur within the production period will not give a complete picture of the firm’s productive activities. National income accountants are aware of this when they make allowance for “work in progress”; i.e., production that takes place during the period but without any visible sales because it takes multiple periods to produce a saleable unit. Hence, to get a complete picture of the firm’s production over the course of a period, it is necessary to add the value of the closing stock of assets less the beginning of the period stock of assets to the cash flow that accrued to the firm from its sales and purchases of market goods and services during the accounting period. Using the notation explained in the previous appendix, this leads to the following definition of the firm’s period  $t$  *gross income* or *gross profit*, defined as its cash flow plus the value of its end of period  $t$  stock of inventory items less the value of its beginning of period  $t$  stock of inventory items:

$$(A15) GI^t \equiv CF^t + P_K^t K^t - P_K^{t-1} K^{t-1}.$$

The gross income or profit approach does not explicitly recognize interest as a cost of production. However, in order to induce investors in the firm to hold the starting stocks of capital items for productive purposes (instead of immediately selling them), it is necessary to pay interest. Thus it is necessary to subtract interest times the beginning of

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<sup>64</sup> For more on this model of production and additional references to the literature, see the Appendices in Diewert (1977) (1980).

the period value of the capital stock from gross income in order to get the economic income or net profit  $EI^t$  defined earlier by (A2), which is repeated here for convenience:

$$(A16) EI^t \equiv CF^t + P_K^t K^t - (1+r^t) P_K^{t-1} K^{t-1}.$$

There are two versions of economic income that could be considered for national income accounting purposes:

- An *ex post* version that uses the *actual* end of period  $t$  price as the price  $P_K^t$  in (A16) or
- An *ex ante* version that uses an *anticipated* end of period  $t$  price as the price  $P_K^t$  in (A16).

Diewert (1980; 476) and Hill and Hill (2003) endorsed the *ex ante* version for most purposes, since it will tend to be smoother than the *ex post* version and it will generally be closer to a rental or leasing price for the asset.

However, there are several practical measurement issues that will make it difficult to implement the *ex ante* version of net income:<sup>65</sup>

- There may be difficulties in estimating the beginning of the period values of the various stocks held by firms since by definition, these stocks are being held (and not sold immediately) and so there are no unambiguous market prices to value these stocks.
- There may be difficulties in determining the right opportunity cost of financial capital  $r^t$ .
- It will be difficult to provide reproducible estimates of the anticipated end of period prices for the capital stocks being held by firms.

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<sup>65</sup> For noninventory assets, there will be difficulties in determining appropriate depreciation rates as well as the difficulties listed below.

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