INDEX NUMBER THEORY AND MEASUREMENT ECONOMICS

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April 4, 2016.

Chapter 16. The Treatment of Seasonal Commodities

1. Introduction

Most of the material on the treatment of seasonal commodities that will be presented in this Chapter is covered in Chapter 22 of the *Consumer Price Index Manual*. However, there is some new material in section 5 below.

It should be noted that all of the methods that will be suggested in this chapter to deal with seasonal commodities assume that the statistical agency is able to collect expenditure information on these seasonal commodities by month.[2] This expenditure information may be collected on a delayed basis through household expenditure surveys or possibly by obtaining detailed price and quantity data from retailers on a current basis (as is the case in the Netherlands for the groceries component of their national CPI).

Seasonal commodities are commodities which are either: (a) not available in the marketplace during certain seasons of the year or (b) are available throughout the year but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[3] A commodity that satisfies (a) is termed a *strongly seasonal commodity* whereas a commodity which satisfies (b) will be called a *weakly seasonal commodity*.[4]

Strongly seasonal commodities offer the biggest challenge to traditional bilateral index number theory, which assumes that all commodities are present in both periods being compared. Obviously, it is not possible to compare the price of a strongly seasonal commodity in a month where it is present in the marketplace with a nonexistent price for the commodity in a month where it is not available at all. However, even weakly seasonal commodities offer challenges to traditional index number theory since an increase in the

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[1] This Chapter was prepared for the Preconference Training Sessions on Price Measurement held on August 23-24, 2014, sponsored by the 33nd General Conference of the International Association for Research in Income and Wealth held in Rotterdam, Netherlands, August 24-30, 2014.

[2] Hardly any statistical agencies have monthly expenditure surveys and so the methods suggested in this chapter are not feasible. (Israel does have monthly consumer expenditure survey information). However, an increasing number of agencies are collecting weekly scanner data from retailers which have detailed price and quantity information on sales by individual product. Thus in the future, it will be possible to implement the methods suggested in this chapter for at least parts of the CPI for a country.

[3] This classification of seasonal commodities corresponds to Balk's narrow and wide sense seasonal commodities; see Balk (1980a; 7) (1980b; 110) (1980c; 68) (1981). Diewert (1998; 457) used the terms type 1 and type 2 seasonality.

[4] Mitchell (1927) noted that seasonal variations in prices and quantities are basically due to (i) fluctuations in the weather and (ii) customs.

index in a given month may simply be due to a seasonal increase in prices rather than a "true" increase in underlying inflation for the reference population.

In order to deal with seasonal commodities, the *Consumer Price Index Manual* offers *four types of index*:

- Year over year monthly indexes;
- Year over year annual indexes;
- Rolling year annual indexes and
- Month to month (chained) indexes.

In the following four sections, each type of index will be defined and (briefly) discussed. Section 6 will conclude that the first three types of index are conceptually sound but the fourth type of index suffers from a *chain drift problem* and hence is not a suitable index for statistical agencies to consider producing. The following Chapter 17 will offer an alternative month to month index that has superior properties.

Before proceeding to the technical definitions of the various indexes, it is necessary to discuss the notation that will be used and the interpretation of the variables. The algebra below will assume that the statistical agency has information on the monthly prices and quantities for the N commodities that enter the scope of the index. However, not all commodities will be present in each month. Denote the set of commodities n which are present in the marketplace during month m of any year as S(m).[5] Denote the price of commodity n in month m of year y as $p_n^{y,m}$ the corresponding quantity and expenditure share as $q_n^{y,m}$ and $s_n^{y,m} \equiv p_n^{y,m}q_n^{y,m}/\sum_{k\in S(m)} p_k^{y,m}q_k^{y,m}$ for y = 0,1, m = 1,2,...,12 and $n\in S(m)$.[6]

In the following four sections, various index number formulae will be defined using the above notation. However, the resulting indexes could refer to several situations:

- N is the total number of separate items that are to be distinguished in the overall consumer price index; i.e., the underlying assumption here is that we have complete price and quantity information on the universe of expenditures for the reference population.

[5] We are making the simplifying assumption that the set of strongly seasonal commodities remains the same as the years change. In practice, this assumption does not always hold; see Diewert, Finkel and Artsev (2009; 63). In order to deal with this situation where a price is present in one year for a month but not in the same month for the other year, a *price* for the commodity should be imputed for the month when the commodity is not available. Basically, the imputed price should be based on price movements of products that are thought to be comparable to the given seasonal product; see Diewert, Finkel and Artsev (2009; 61) for one such imputation method. For more systematic discussions of imputation methods, see Alterman, Diewert and Feenstra (1999) and Feenstra and Diewert (2001). The corresponding imputed quantity and monthly expenditure share for the missing price should be set equal to zero.

[6] The summation $\sum_{k\in S(m)} p_k^{y,m}q_k^{y,m}$ means that we sum expenditures in month m of year y over products k that are actually present in month m; i.e., strongly seasonal products that are not present in month m are excluded in this sum.

- N refers to the number of items in one particular *stratum* of the overall consumer price index. Standard index number theory is also applicable in this situation.
- N refers to the number of strata in the consumer price index and in this case, the prices $p_n^{y,m}$ are in fact, elementary indexes, and the corresponding $s_n^{y.m}$ are expenditure shares on the nth elementary category or stratum.[7]

Obviously, application of the first interpretation of the indexes is unrealistic; the statistical agency will typically not have access to true microeconomic data at the finest level of aggregation. However, application of the second interpretation of the indexes is quite possible; the existence of scanner data sets has led to the possibility of computing say true Fisher indexes for some strata of the CPI.[8] The third interpretation of the indexes is of course directly relevant to most statistical agencies. In what follows, the discussion of the indexes will use the first interpretation but the reader should keep in mind the more useful third interpretation.

2. Year over Year Monthly Indexes

For over a century,[9] it has been recognized that making year over year comparisons[10] of prices in the same month provides the simplest method for making comparisons that are free from the contaminating effects of seasonal fluctuations.

We will take the Fisher index as the "best" functional form for making bilateral comparisons. In the present context, the 12 *month over month Laspeyres*, *Paasche* and *Fisher* indexes, P_L, P_P and P_F, comparing the prices in year 1 for month m to those in year 0 for month m are defined as follows, using the notation in the previous section:

$$(1)\ P_L(p^{0,m},p^{1,m},s^{0,m}) \equiv \textstyle\sum_{n\in S(m)} (p_n^{1,m}/p_n^{0,m})s_n^{0,m}\ ; \qquad m= 1,...,12;$$
$$(2)\ P_P(p^{0,m},p^{1,m},s^{1,m}) \equiv [\textstyle\sum_{n\in S(m)} (p_n^{1,m}/p_n^{0,m})^{-1}s_n^{1,m}]^{-1}\ ; \qquad m= 1,...,12;$$
$$(3)\ P_F(p^{0,m},p^{1,m}, s^{0,m},s^{1,m}) \equiv [P_L(p^{0,m},p^{1,m},s^{0,m})P_P(p^{0,m},p^{1,m},s^{1,m})]^{1/2}\ ; \qquad m = 1,...,12$$

where $p^{y,m}$ is the vector of prices for commodities that are present in month m of year y and $s^{y,m}$ is the corresponding expenditure vector for y = 0,1 and m = 1,...,12. In the *Consumer Price Index Manual*,[11] it was noted that approximate month over month Laspeyres, Paasche and Fisher indexes could be defined, using information that is generally available to the statistical agency, by replacing the monthly expenditure shares $s_n^{0,m}$ and $s_n^{1,m}$ by the available monthly expenditure shares for the current base year $s_n^{b,m}$

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[7] In this case, the corresponding quantities are defined implicitly as $q_n^{y,m} \equiv s_n^{y,m}/p_n^{y,m}$ for $n\in S(m)$. A useful assumption that can ensure that the indexes constructed under this framework are true indexes (e.g., true Fisher indexes that are based on microeconomic data at the finest level of aggregation) is that within each stratum, prices of the products within the stratum vary proportionally over time.

[8] See Ivancic, Diewert and Fox (2011) and de Haan and van der Grient (2011) for applications of this type. We will discuss what can be done when scanner data are available in more detail in the following Chapter.

[9] See Jevons (1884;3), Flux (1921; 199) and Yule (1921; 199).

[10] In the seasonal price index context, this type of index corresponds to Bean and Stine's (1924; 31) Type D index.

[11] See the ILO (2004; 397).

for m = 1,...,12. Thus the 12 *approximate month over month Laspeyres*, *Paasche* and *Fisher* indexes, P_{AL}, P_{AP} and P_{AF}, are defined as follows:

$$(4)\ P_{AL}(p^{0,m},p^{1,m},s^{b,m}) \equiv \sum_{n\in S(m)} (p_n^{1,m}/p_n^{0,m})s_n^{b,m}\ ; \qquad m= 1,...,12;$$
$$(5)\ P_{AP}(p^{0,m},p^{1,m},s^{b,m}) \equiv [\sum_{n\in S(m)} (p_n^{1,m}/p_n^{0,m})^{-1}s_n^{b,m}]^{-1}\ ; \qquad m= 1,...,12;$$
$$(6)\ P_{AF}(p^{0,m},p^{1,m},s^{b,m}) \equiv [P_L(p^{0,m},p^{1,m},s^{b,m})P_P(p^{0,m},p^{1,m},s^{b,m})]^{1/2}\ ; \qquad m = 1,...,12$$

where $s^{b,m}$ is the vector expenditure shares for month m in the base year b.[12] The approximate Fisher year over year monthly indexes defined by (6) will provide adequate approximations to their true Fisher counterparts defined by (3) if the monthly expenditure shares for the base year b are not too different from their current year 0 and 1 counterparts.[13] Hence, it will be useful to construct the true Fisher indexes on a delayed basis in order to check the adequacy of the approximate Fisher indexes defined by (6).[14]

The year over year monthly approximate Fisher indexes defined by (6) will normally have a certain amount of upward bias, since these indexes cannot reflect long term substitution of consumers towards commodities that are became relatively cheaper over time. This reinforces the case for computing true year over year monthly Fisher indexes defined by (3) on a delayed basis so that this substitution bias can be estimated.

In the following section, it is shown how the various year over year monthly indexes defined in this section can be aggregated into an annual index.

3. Annual Year over Year Indexes

Mudgett (1955) in the consumer price context and Stone (1956) in the producer price context independently suggested a very effective way of dealing with seasonal commodities in the context of constructing an annual index:

"The basic index is a yearly index and as a price or quantity index is of the same sort as those about which books and pamphlets have been written in quantity over the years." Bruce D. Mudgett (1955; 97).

"The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities." Richard Stone (1956; 74-75).

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[12] Thus this vector excludes the strongly seasonal commodities that are not present in month m.
[13] We will evaluate how useful this approximate methodology is in Chapter 18.
[14] For the modified Turvey (1979) data set in the *Consumer Price Index Manual*, the approximate Fisher indexes were quite close to the true Fisher indexes; see the ILO (2004; 398-400). However, for the Israeli data set on fresh fruits, the correspondence between the true and approximate Fishers was not close for a large number of observations; i.e., for 16 out of 60 observations, the difference exceeded 5%: see Diewert, Finkel and Artsev (2009; 59-60). The Israeli example shows the importance of computing the true Fishers on a delayed basis when the true expenditure information becomes available.

Thus the basic idea is that each commodity in each month should be treated as a distinct commodity in an annual index. Thus the price of commodity n in January of year 1 is compared to the price of the same commodity in January of year 0 and so on.

Using the notation introduced in the previous section, the *Laspeyres, Paasche and Fisher annual indexes* comparing the prices of year 0 with those of year 1 can be defined as follows:

$$(7)\ P_L(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};q^{0,1},...,q^{0,12}) \equiv \sum_{m=1}^{12} \sum_{n\in S(m)} p_n^{1,m}q_n^{0,m} / \sum_{m=1}^{M} \sum_{n\in S(m)} p_n^{0,m}q_n^{0,m}\ ;$$

$$(8)\ P_P(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};q^{1,1},...,q^{1,12}) \equiv \sum_{m=1}^{12} \sum_{n\in S(m)} p_n^{1,m}q_n^{1,m} / \sum_{m=1}^{M} \sum_{n\in S(m)} p_n^{0,m}q_n^{1,m}\ ;$$

$$(9)\ P_F(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};q^{0,1},...,q^{0,12};q^{1,1},...,q^{1,12}) \equiv [P_L(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};q^{0,1},...,q^{0,12})P_P(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};q^{1,1},...,q^{1,12})]^{1/2}.$$

The above formulae can be rewritten in terms of price relatives and monthly expenditure shares as follows:

$$\begin{aligned}(10)\ \ & P_L(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};\sigma^{0,1}s^{0,1},...,\sigma^{0,12}s^{0,12}) \\ & \equiv \sum_{m=1}^{12} \sum_{n\in S(m)} \sigma^{0,m}s_n^{0,m}\,(p_n^{1,m}/p_n^{0,m}) \\ & = \sum_{m=1}^{12} \sigma^{0,m}\, P_L(p^{0,m},p^{1,m},s^{0,m})\ ;\end{aligned}$$

$$\begin{aligned}(11)\ \ & P_P(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};\sigma^{1,1}s^{1,1},...,\sigma^{1,12}s^{1,12}) \\ & \equiv [\sum_{m=1}^{12} \sum_{n\in S(m)} \sigma^{1,m}s_n^{1,m}\,(p_n^{1,m}/p_n^{0,m})^{-1}]^{-1} \\ & = [\sum_{m=1}^{12} \sigma^{1,m} \sum_{n\in S(m)} s_n^{1,m}\,(p_n^{1,m}/p_n^{0,m})^{-1}]^{-1} \\ & = [\sum_{m=1}^{12} \sigma^{1,m}\,[P_P(p^{0,m},p^{1,m},s^{1,m})]^{-1}]^{-1}\ ;\end{aligned}$$

$$\begin{aligned}(12)\ \ & P_F(p^{0,1},...,p^{0,12};p^{1,1},...,p^{1,12};\sigma^{0,1}s^{0,1},...,\sigma^{0,12}s^{0,12};\sigma^{1,1}s^{1,1},...,\sigma^{1.12}s^{1,12}) \\ & \equiv [\sum_{m=1}^{12} \sum_{n\in S(m)} \sigma^{0,m}s_n^{0,m}\,(p_n^{1,m}/p_n^{0,m})\{\sum_{m=1}^{12} \sum_{n\in S(m)} \sigma^{1,m}s_n^{1,m}\,(p_n^{1,m}/p_n^{0,m})^{-1}\}^{-1}]^{1/2} \\ & = [\{\sum_{m=1}^{12} \sigma^{0,m}\, P_L(p^{0,m},p^{1,m},s^{0,m})\}\{\sum_{m=1}^{12} \sigma^{1.m}\,[P_P(p^{0,m},p^{1,m},s^{1,m})]^{-1}\}^{-1}]^{1/2}\end{aligned}$$

where *the expenditure share for month m in year t* is defined as:

$$\begin{aligned}(13)\ \ & \sigma^{t,m} \equiv \sum_{n\in S(m)} p_n^{t,m}q_n^{t,m} / \sum_{i=1}^{12} \sum_{j\in S(i)} p_j^{t,i}q_j^{t,i}\ ; \qquad m = 1,2,...,12\ ;\ t = 0,1 \\ & = p^{t,m}\cdot q_n^{t,m} / \sum_{i=1}^{12} p^{t,i}\cdot q^{t,i}\end{aligned}$$

and the year over year monthly Laspeyres and Paasche price indexes $P_L(p^{0,m},p^{1,m},s^{0,m})$ and $P_P(p^{0,m},p^{1,m},s^{1,m})$ were defined by (1) and (2) respectively. As usual, the annual Fisher index P_F defined by (12) is the geometric mean of the Laspeyres and Paasche indexes, P_L and P_P, defined by (10) and (11). The last equations in (10) and (11) show that the annual Laspeyres index can be written as a *weighted arithmetic average* of the year over year monthly Laspeyres indexes $P_L(p^{0,m},p^{1,m},s^{0,m})$, where the weights are the monthly expenditure shares $\sigma^{0,m}$ in year 0, and the annual Paasche index can be written as a *weighted harmonic average* of the year over year monthly Paasche indexes

$P_P(p^{0,m},p^{1,m},s^{1,m})$, where the weights are the monthly expenditure shares $\sigma^{1,m}$ in year 1. Hence once the year over year monthly indexes defined in the previous subsection have been numerically calculated, it is easy to calculate the corresponding annual indexes.

The approach to computing annual indexes outlined in this subsection, which essentially involves taking monthly expenditure share weighted averages of the 12 year over year monthly indexes, should be contrasted with the usual approach to the construction of annual indexes that simply takes the arithmetic mean of the 12 monthly indexes. The problem with the latter approach is that months where expenditures are below the average (e.g., February) are given the same weight in the unweighted annual average as months where expenditures are above the average.[15]

As in section 2, *Approximate Laspeyres, Paasche and Fisher counterparts* to the true indexes defined by (10)-(12) can be defined by these same equations, except that the monthly expenditure shares, $\sigma^{0,m}$ and $\sigma^{1,m}$, are replaced by the corresponding base year expenditure shares, $\sigma^{b,m}$, for m = 1,...,12 and the within the month expenditure shares, $s_n^{0,m}$ and $s_n^{1,m}$, are replaced by the corresponding within the month base year expenditure shares, $s_n^{b,m}$ for m = 1,...,12 and $n \in S(m)$.

For the artificial data set in the *Consumer Price Index Manual*, the approximate annual Fisher index provided a very close approximation to the true Fisher indexes. This result needs further validation[16] but it should be noted that the approximate Fisher index can be computed using the same information set that is normally available to statistical agencies.

4. Rolling Year Annual Indexes

In the previous subsection, the price and quantity data pertaining to the 12 months of a calendar year were compared to the 12 months of a base calendar year. However, there is no need to restrict attention to calendar year comparisons: any 12 consecutive months of price and quantity data could be compared to the price and quantity data of the base year, provided that the January data in the noncalendar year is compared to the January data of the base year, the February data of the noncalendar year is compared to the February data of the base year, …, and the December data of the noncalendar year is compared to the December data of the base year.[17] Alterman, Diewert and Feenstra (1999; 70) called the resulting indexes *rolling year* or *moving year* indexes.[18]

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[15] In addition, the Mudgett-Stone approach to the construction of annual indexes can be given a strong justification from the viewpoint of the economic approach to index number theory; see Diewert (1999).
[16] The empirical results presented in Chapter 18 indicate that the approximate indexes are subject to some upward substitution bias.
[17] Diewert (1983) suggested this type of comparison and termed the resulting index a “split year” comparison.
[18] Crump (1924; 185) and Mendershausen (1937; 245) respectively used these terms in the context of various seasonal adjustment procedures. The term “rolling year” seems to be well established in the business literature in the UK.

In order to theoretically justify the rolling year indexes from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1999a; 56-61).

Of course, approximate counterpart to rolling year Laspeyres, Paasche and Fisher indexes can be obtained by replacing the various expenditure shares in the current year by their counterparts in the base year. The details and an example based on the Turvey data can be found in pages 402-406 of the *Consumer Price Index Manual*. The bottom line is that the true rolling year indexes were very smooth and free of seasonal influences and the approximate rolling year Fisher index approximated its true counterpart very well.[19]

Basically, the rolling year indexes are generally quite smooth and free from seasonal fluctuations. Each rolling year index can be viewed as a *seasonally adjusted annual consumer price index* that compares the data of the 12 consecutive months that end with the current month with the corresponding price and quantity data of the 12 months in the base year. Thus rolling year indexes offer statistical agencies an *objective* and *reproducible* method of seasonal adjustment that can compete with existing time series methods of seasonal adjustment.[20] Rolling year indexes are probably the most reliable indexes that can be constructed when there is substantial seasonality in prices.

There are two main difficulties with the use of rolling year indexes:

- The measure of inflation generated by the current index value is for a rolling year that is centered around the month that took place six months ago and
- It is not a valid measure of month to month inflation.

Thus in the following subsection, month to month inflation indexes are considered.

5. Maximum Overlap Month to Month Price Indexes

A possible method for dealing with seasonal commodities in the context of picking a target index for a month to month CPI is the following one:[21]

- Determine the set of commodities that are present in the marketplace in both months of the comparison.

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[19] Similar results were obtained using the Israeli data; see Diewert, Artsev and Finkel (2009; 61).

[20] For discussions on the merits of econometric or time series methods versus index number methods of seasonal adjustment, see Diewert (1999a; 61-68) and Alterman, Diewert and Feenstra (1999; 78-110). The basic problem with time series methods of seasonal adjustment is that the target seasonally adjusted index is very difficult to specify in an unambiguous way; i.e., there are an infinite number of possible target indexes. For example, it is impossible to identify a temporary increase in inflation within a year from a changing seasonal factor. Hence different econometricians will tend to generate different seasonally adjusted series, leading to a lack of reproducibility.

[21] For more on the economic approach and the assumptions on consumer preferences that can justify month to month maximum overlap indexes, see Diewert (1999a; 51-56).

- For this maximum overlap set of commodities, calculate one of the three indexes recommended in previous sections; i.e., calculate the Fisher, Walsh or Törnqvist Theil index.

Thus the bilateral index number formula is applied only to the subset of commodities that are present in both periods.[22]

The question now arises: should the comparison month and the base month be adjacent months (thus leading to chained indexes) or should the base month be fixed (leading to fixed base indexes)? In the *Consumer Price Index Manual*, a preference was expressed for chained indexes over fixed base indexes for the following two reasons:[23]

- The set of seasonal commodities which are present in the marketplace during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed base month (like January of a base year). Hence the comparisons made using chained indexes will be more comprehensive and hence more accurate than those made using a fixed base.
- In many economies, on average 2 or 3 percent of price quotes disappear each month due to the introduction of new commodities and the disappearance of older ones. This rapid sample attrition means that fixed base indexes rapidly become unrepresentative and hence it seems preferable to use chained indexes which can more closely follow marketplace developments.

Thus in the present section, we will follow the advice in the Manual and define the relevant Laspeyres, Paasche and Fisher maximum overlap month to month chain links. Some new notation is required. Let there be N commodities that are available in at least one month of the current year and let $p_n^{y,m}$ and $q_n^{y,m}$ denote the price and quantity of commodity n that is in the marketplace in month m of year y (if the commodity is unavailable, define $p_n^{y,m}$ and $q_n^{y,m}$ to be 0). Let $p^{y,m} \equiv [p_1^{y,m},p_2^{y,m},...,p_N^{y,m}]$ and $q^{y,m} \equiv [q_1^{y,m},q_2^{y,m},...,q_N^{y,m}]$ be the month m and year y price and quantity vectors respectively. Let S(y,m) be the *set of commodities that is present in month m of year y and the following month*. Then the *maximum overlap Laspeyres, Paasche and Fisher indexes* going from month m of year t to the following month can be defined as follows:[24]

$$(14)\ P_L(p^{y,m},p^{y,m+1},q^{t,m},S(y,m)) \equiv \sum_{n\in S(y,m)} p_n^{y,m+1}q_n^{y,m}/\sum_{n\in S(y,m)} p_n^{y,m}q_n^{y,m};\quad m = 1,2,...11;$$

$$(15)\ P_P(p^{y,m},p^{y,m+1},q^{y,m+1},S(y,m)) \equiv \sum_{n\in S(y,m)} p_n^{y,m+1}q_n^{y,m+1}/\sum_{n\in S(y,m)} p_n^{y,m}q_n^{y,m+1};\quad m = 1,2,...11;$$

$$(16)\ P_F(p^{y,m},p^{y,m+1},q^{y,m},q^{y,m+1},S(y,m))$$

[22] Keynes (1930; 95) called this the highest common factor method for making bilateral index number comparisons. Of course, this target index drops those strongly seasonal commodities that are not present in the marketplace during one of the two months being compared. Thus the index number comparison is not completely comprehensive. Mudgett (1951; 46) called the "error" in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the "homogeneity error".

[23] See the ILO (2004; 407-411).

[24] The formulae are slightly different for the indexes that go from December to January of the following year. In order to simplify the exposition, these formulae are left for the reader.

$$\equiv [P_L(p^{y,m},p^{y,m+1},q^{y,m},S(y,m))P_P(p^{y,m},p^{y,m+1},q^{y,m+1},S(y,m))]^{1/2}\ ; \qquad m = 1,2,...,11.$$

Note that P_L, P_P and P_F depend on the two (complete) price and quantity vectors pertaining to months m and m+1 of year y, $p^{y,m},p^{y,m+1},q^{y,m},q^{y,m+1}$, but they also depend on the set S(y,m), which is the set of commodities that are present in both months. Thus the commodity indices n that are in the summations on the right hand sides of (14) and (15) include indices n that correspond to commodities that are present in *both* months, which is the meaning of $n\in S(y,m)$; i.e., n belongs to the set S(y,m).

We will not convert formulae (14) and (15) into expenditure share form; the details on how this can be done are in the *Consumer Price Index Manual*.[25]

The performance of maximum overlap chained indexes has not been satisfactory; indexes based on this methodology tend to suffer from a *chain drift problem*; i.e., indexes constructed using this methodology tend to exhibit a downward bias.[26]

An objective method to test for the existence of chain drift in a price index $P(p^1,p^2,q^1,q^2)$ is the following *multiperiod identity test*,[27] which was initially proposed by Walsh (1901, 401):

$$(17)\ P(p^1,p^2,q^1,q^2)\ P(p^2,p^3,q^2,q^3)\ P(p^3,p^1,q^3,q^1) = 1.$$

$P(p^1,p^2,q^1,q^2)$ and $P(p^2,p^3,q^2,q^3)$ are price indexes between periods 1 and 2, and then 2 and 3, respectively, where p^t and q^t are the price and quantity vectors pertaining to periods t for t = 1, 2, 3. Their product gives the chained price index between periods 1 and 3. Note that there is a final link in the chain in (17), $P(p^3,p^1,q^3,q^1)$, which is a price index between periods 3 and 4, where the period 4 price and quantity data are the same as the period 1 data. The price index formula P will not suffer from *chain drift* or *chain link bias* if the product of all of these factors equals 1.

To see the relevance of the test defined by (17), suppose that in year y, month 1 and month 4 have exactly the same price and quantity data so that $p^{y,1} = p^{y,4}$ and $q^{y,1} = q^{y,4}$. Under these conditions, we would like the consumer price index chain links defined by (14)-(16) to be such that the month 1 and month 4 index values to be identical. Thus if the Fisher index is being used, in order to achieve this identity of index values for months 1 and 4 in year y, we require the following equality:

$$(18)\ P_F(p^{y,1},p^{y,2},q^{y,1},q^{y,2},S(y,1))P_F(p^{y,2},p^{y,3},q^{y,2},q^{y,3},S(y,2))\ P_F(p^{y,3},p^{y,1},q^{y,3},q^{y,1},S(y,3)) = 1.$$

The equality in (18) will not hold in general and thus chained maximum overlap month to month indexes will be subject to chain drift.[28]

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[25] See the ILO (2004; 408).
[26] For documentation of this phenomenon, see the ILO (2004; 409), Ivancic, Diewert and Fox (2011) and de Haan and van der Grient (2011).
[27] Diewert (1993; 40) gave the test this name. Our exposition of how to define chain drift follows that of Ivancic, Diewert and Fox (2011; 26).

What causes the downward drift of chained superlative indexes? A large part of the downward drift can be attributed to the problem of sales. A small numerical example will illustrate the problem.

Suppose that we are given the following price and quantity data for 2 commodities for 4 periods:

Table 1: Price and Quantity Data for Two Commodities

Period t	p_1^t	p_2^t	q_1^t	q_2^t
1	1.0	1.0	10	100
2	0.5	1.0	5000	100
3	1.0	1.0	1	100
4	1.0	1.0	10	100.

The first commodity is subject to periodic sales (in period 2), when the price drops to one half of its normal level of 1. In period 1, the "normal" off sale demand for commodity 1 is equal to 10 units. In period 2, the sale takes place and demand explodes to 5000 units. In period 3, the commodity is off sale and the price is back to 1 but most shoppers have stocked up in the previous period so demand falls to only 1 unit. Finally in period 4, the commodity is off sale but we are back to the "normal" demand of 10 units. Commodity 2 is an unexciting commodity: its price is 1 in all periods and the quantity sold is 100 units in each period. Note that the only thing that has happened going from period 3 to 4 is that the demand for commodity one has picked up from 1 unit to 10 units. Also note that the period 4 data are exactly equal to the period 1 data so if Walsh's test is satisfied, the product of the period to period chain links should equal one.

In Table 2, the fixed base Fisher, Laspeyres and Paasche price indexes, P_F, P_L and P_P and as expected, they behave well in period 4, returning to the period 1 level of 1. The chained Fisher, Törnqvist, Laspeyres and Paasche price indexes, P_{FCH}, P_{TCH}, P_{LCH} and P_{PCH} are listed. Obviously, the chained Laspeyres and Paasche indexes have chain link bias that is very large but what is interesting is that the chained Fisher has a 2% downward bias and the chained Törnqvist has a close to 3% downward bias.

Table 2: Fixed Base and Chained Fisher, Törnqvist, Laspeyres and Paasche Indexes

Period	P_F	P_L	P_P	P_{FCH}	P_{TCH}	P_{LCH}	P_{PCH}
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	0.69759	0.95455	0.50980	0.69759	0.69437	0.95455	0.50980
3	1.00000	1.00000	1.00000	0.97944	0.97232	1.87238	0.51234
4	1.00000	1.00000	1.00000	0.97944	0.97232	1.87238	0.51234

[28] The term "chain drift" seems to be due to Frisch: "The divergency which exists between a chain index and the corresponding direct index (when the latter does not satisfy the circular test) will often take the form of a systematic drifting." Ragnar Frisch (1936; 8).

If the above data were monthly, and they repeated themselves 3 times over the year, the overall chain link bias would build up to the 6 to 8% range for the two chained superlative indexes, P_{FCH}, P_{TCH}, which is fairly large.

The sales problem can be explained as follows: when commodity one comes off sale and goes back to its regular price in period 3, the corresponding quantity does not return to the level it had in period 1: the period 3 demand is only 1 unit whereas the period 1 demand for commodity 1 was 10 units. It is only in period 4 that demand for commodity one recovers to the period 1 level. However, since prices are the same in periods 3 and 4, all of the chain links show no change (even though quantities are changing) and this is what causes the difficulties. If demand for commodity one in period 3 had immediately recovered to its "normal" period 1 level of 10, then there would be no problem with chain drift.

The problem of chain drift does not always lead to a downward bias in chained superlative indexes. Feenstra and Shapiro (2003) provide an empirical example where the chain drift bias goes in the opposite direction and they explain why this result occurred:

"The fixed base Törnqvist does not equal the chained Törnqvist in general, and for our sample of weekly tuna data, we find that the differences between these two indexes are rather large: the chained Törnqvist has a pronounced upward bias for most regions of the United States. The reason for this is that periods of low price (i.e., sales) attract high purchases only when they are accompanied by advertising, and this tends to occur in the final weeks of a sale. Thus, the initial price decline, when the sale starts, does not receive as much weight in the cumulative index as the final price *increase* when the sale ends. The demand behavior that leads to this upward bias of the chained Törnqvist—with higher purchases at the end of a sale—means that consumers are very likely purchasing goods for inventory accumulation. The only theoretically correct index to use in this type of situation is a fixed base index, as demonstrated in section 5.3." Robert Feenstra and Matthew Shapiro (2003, 125).

Thus Feenstra and Shapiro suggested that a solution to the problems generated by big fluctuations in prices and quantities due to sales is to move from chained indexes to fixed base indexes. In the following Chapter, this suggestion will be discussed in more detail.

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