Preventing Self-fulfilling Debt Crises: The Role of Expectations*

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Abstract

This paper studies the role of expectations in determining the effectiveness of policy measures aimed at preventing expectations-driven sovereign debt crises. To endogenize expectations, I use global games techniques in a micro-founded model of debt crises. I show that adjustments in expectations act as an amplification mechanism magnifying the response of the economy to policy changes. I analyze how the role played by expectations changes when there is uncertainty as to whether the government will implement announced policies. I use these insights to analyze policies directed at preventing debt crises, such as an increase in taxes or a fiscal stimulus.

Key words: sovereign debt crises, global games, expectations, political uncertainty, taxes, fiscal stimulus

JEL codes: D82, D84, F34

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"[...] the assessment of the Governing Council is that we are in a situation now where you have large parts of the Euro Area in what we call a bad equilibrium, namely an equilibrium where you have self-fulfilling expectations. [...] So, there is a case for intervening, in a sense, to "break" these expectations."

Mario Draghi, Press Conference, Frankfurt am Main, September 6, 2012

1 Introduction

Sovereign debt crises are a recurrent phenomenon. After the turbulent 1980s and a series of defaults in the late 1990s and early 2000s, sovereign defaults once again became a hotly debated topic. Indeed, one of the most important issues troubling the global economy for the last several years has been the sovereign debt crisis in Europe. One of the leading views on the sovereign defaults, as exemplified by the above quote, is that they are the result of an interplay between poor economic fundamentals and self-fulfilling expectations. According to that view, governments default in part because of low confidence among market participants.[1]

The possibility of sovereign debt crises driven by self-fulfilling expectations has raised a number of important questions, such as: Is it possible for the government to implement policies that help to restore confidence, and hence avert a crisis driven by self-fulfilling expectations? Do effects of government policies depend on market participants' expectations? If so, is there a systematic dependence or does it vary by policy? And finally, how does the political uncertainty that typically surrounds the process of government policy adjustments affect the above conclusions?

Despite the importance of the above questions, there exists little theoretical work that analyzes how self-fulfilling expectations interact with government policies within a model of sovereign default. The goal of this paper is to fill this gap in the literature. In particular, I develop a two-period model of sovereign debt crisis in which self-fulfilling crises can arise. In contrast to the earlier literature that studied expectations-driven debt crises, expectations are determined endogenously in the model. As such, I dispense with the assumptions of exogenous sunspots that typically drive crises in such models. Instead, I use insights from the theory of global games that allows me not only to transform expectations into an equilibrium object but also to uniquely pin them down in equilibrium.

The uniqueness of expectations, and hence of equilibrium, allows me to explicitly analyze how government actions affect households' and lenders' expectations and the equilibrium outcomes. Therefore, I am able to analyze the role played by endogenous

[1]See Cole and Kehoe (1996), Conesa and Kehoe (2012), or De Grauwe and Ji (2013).

expectations in determining desirability and effectiveness of various government policies. The main finding of the paper is that endogenous expectations tend to amplify the impact of government policies on the probability of default. It follows that if the government wants to avoid default, it can use this effect to its own advantage by implementing policies that decrease the likelihood of default in the first place, since even a small policy change, when amplified by the adjustment in expectations, can have a large effect on the equilibrium probability of default. Moreover, I find that for small policy changes (that is, marginal changes in policy parameters as captured by relevant derivatives) the effect of expectations is the same regardless of the policy considered. Finally, I show that the amplifying effect of endogenous expectations is particularly strong when agents have precise information about the economy, but it becomes weaker as agents become less certain as to whether the proposed policies will be implemented.

The paper consists of two parts. In the first part, I develop a micro-founded model of self-fulfilling crisis, where a crisis arises as a result of an interplay between poor fundamentals, foreign lenders' pessimistic expectations, and domestic households' pessimistic expectations. As such, my framework emphasizes that expectations of domestic and foreign agents both matter for sustainability of debt — a point often ignored in the literature, which tends to focus on lenders. In order to deal with the issue of indeterminacy of equilibrium that tends to arise in models of expectations-driven crises, I assume that households and lenders do not know the average productivity in the economy, and instead only receive noisy signals about it (which transforms the model into a "global game"). I show that this small departure from common knowledge is enough to restore the uniqueness of equilibrium within the class of monotone equilibria.[2]

In the second part of the paper, I use the model to investigate the role played by the endogenous expectations in determining the effect of government policies on the probability of debt crises. I start by considering a situation where the government announces its policy in advance and the announced policy is always implemented. I find that a change in the probability of default implied by any policy adjustment is equal to the product of the "direct effect" (the initial effect of the policy change on the government's incentive to default holding households' and lenders' beliefs constant) and the "multiplier effect" (the change in the government's default decision implied

[2] Even though the model has a unique equilibrium outcome, a debt crisis is still driven by expectations in the following sense: There is a region of the fundamentals where both crisis and no crisis outcomes are consistent with fundamentals and whether a crisis occurs depends only on agents' expectations. If agents expect default, then a crisis occurs, while if they expect repayment, then the government will indeed repay the debt; in that sense a crisis is self-fulfilling (see Morris and Shin, 1998). The difference between this model and the typical models of self-fulfilling debt crises is that here expectations are uniquely determined.

by the adjustment in households' and lenders' expectations). I show that the direct effect determines whether a given policy decreases or increases the likelihood of a crisis, while the multiplier effect, which captures the role played by expectations, acts like an amplification mechanism that always magnifies the initial response of the economy. Moreover, for the case of small policy adjustments, the magnitude of the multiplier effect is the same regardless of the policies considered. Finally, I show that as households' and lenders' information becomes more precise the multiplier effect grows stronger, and in the limit the multiplier effect drives all of the adjustment in the probability of default. From this, it follows that if the government wants to avoid default, it can use expectations to its own advantage: Even a small policy change, when amplified by expectations can significantly decrease the likelihood of default. If, on the other hand, the government implements a "bad" policy, then the amplifying effect of expectations can make a crisis much more likely. These results emphasize the importance of taking into account endogeneity of expectations when deciding on future policies.

I also investigate how the above conclusions are changed when policy adjustments are unexpected or when the households and lenders are uncertain as to whether the government will implement a policy it has announced. I show that when a policy change is unexpected, then the multiplier effect is absent. Moreover, I find that in the limit as the noise in the households' and lenders' information vanishes, an unexpected adjustment in government policies has no impact on the economy. Finally, I show that under uncertainty about policy implementation, the change in the default probability is equal to the weighted average of the change in the probability of default when the policy adjustment is expected (and implemented) and when the policy change is unexpected. These results suggest that the amplification of policy effects due to endogenous expectations is smaller when there is a substantial uncertainty as to whether the government will implement its announced policies or not.

I use the above observations to analyze the impact of an adjustment in a tax rate and the impact of a fiscal stimulus on the probability of default. I find that an expected increase in a tax rate tends to decrease the probability of default as long as its distortive effect on investment is small. Also I find that fiscal stimulus decreases the probability of default only if its cost is smaller than the differential increase in the utility from higher government spending in repayment versus that in default. Finally, I investigate numerically the effects of an increase in taxes and fiscal stimulus on the probability of the default. The numerical analysis suggests that, for reasonable parameter values, an increase in taxes tends to decrease probability of default while a fiscal stimulus tends to increase it. In accordance with analytical results, the multiplier effect accounts for a large fraction of the overall change in the probability of default.

The paper also makes two contributions to the theoretical literature on global games. First, the main result, according to which a change in the policy can be decomposed into the direct effect and the multiplier effect, is general and applies to any regime-change global game model. Second, the paper also provides a simple method for computing comparative statics results when players are uncertain as to whether a parameter of the model will actually change. This method can be applied to any incomplete information game with strategic complementarities and is particularly useful when the model does not admit a closed-form solution.

The framework developed in the paper unifies two popular approaches to modeling self-fulfilling debt crises: the micro-funded general equilibrium approach of Cole and Kehoe (2000) and the game-theoretic approach of global games as in Corsetti et al. (2006) and Morris and Shin (2006). The key difference between my model and that of Cole and Kehoe (2000) lies in the information structure, which leads to a unique equilibrium in my model. The uniqueness of equilibrium is the result of applying insights from the global games literature as started by Carlson and Damme (1993) and Morris and Shin (1998). Corsetti et al. (2006) and Morris and Shin (2006) use global games in the context of sovereign default to study the effectiveness of IMF assistance in preventing a self-fulfilling debt crisis and the moral hazard such assistance creates.[3] In contrast to their framework, I use a micro-founded model which allows for a more extensive policy analysis. Moreover, I explicitly focus on the role played by the endogenous expectations on the policy outcomes.

Models of self-fulfilling crises have a long tradition in the literature on sovereign default, beginning with Sachs (1984) and Calvo (1988). Following the debt crisis in Europe, this literature has experienced a revival. Corsetti and Dedola (2011), Corsetti and Dedola (2013), and Aguiar et al. (2013) investigate how monetary policy can help to avoid a crisis. Lorenzoni and Werning (2013) focus on the role of the interest rate as the main driver of sovereign default. Finally, Cooper (2013) studies the role of debt guarantees as a way to avert a crisis within a federation of countries.

This paper is also related to the literature on sovereign debt in the spirit of Eaton and Gersovitz (1981), which is summarized well in Aguiar and Amador (2014) and Panizza et al. (2009). More recently, this line of research has focused on developing quantitative models of sovereign default that can account for the observed dynamics surrounding the default episodes. (See Aguiara and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), or Mendoza and Yue (2012), and references therein, for more on quantitative models of sovereign default.) Cuadra and Sapriza (2008) study quantitatively

[3] See also Zwart (2007) for the signaling effects of IMF policy choices in a global game model of sovereign debt crisis. Morris and Shin (2003) provide an excellent survey of the early global games literature.

the role of political uncertainty. Typically, this strand of literature assumes away the possibility of a belief-driven crisis.

A large body of work, motivated by the recent events in Europe, studies possible policy responses to the recession that accompanied the European debt crisis. Several papers use DSGE models to evaluate the effectiveness of various policies. For example, Eggertsson et al. (2014) study the effects of structural reforms, while Corsetti et al. (2013) investigate the effects of expansionary fiscal policy. Austerity is the main theme of an entire discussion volume on this topic (Corsetti (2012)). See also Gali (2014) and Lane (2013). My work complements these papers by providing an analysis of austerity and fiscal stimulus in an environment with a self-fulfilling debt crisis.

2 Model

There are two periods, $t = 1, 2$, and three types of agents: a continuum of identical households, a continuum of identical lenders, and the government. The economy is characterized by the average productivity level A, which is distributed according to a normal distribution with mean A_{-1} and standard deviation σ_A - that is $A \sim N\left(A_{-1}, \sigma_A^2\right)$. Here, A_{-1} denotes the past average productivity level in the economy, which all agents know. The current average level of productivity, A, is realized at the beginning of period 1 and is constant across the two periods, but it is initially unobserved by the agents. Instead, households and lenders receive private noisy signals about A; its value is revealed to everyone at the end of period 1.

2.1 Households

There is a continuum of identical households, indexed by $i \in [0, 1]$. Households are risk averse and have preferences given by

$$\sum_{t=1,2} \left[\log\left(c_t\right) + \log\left(g_t\right)\right],$$

where c_t is private consumption and g_t is government spending. Each household initially is endowed with the same amount of capital k_1, and has access to a production function:

$$y_t^i = \widetilde{Z} e^{A_i} f\left(k_t^i\right),$$

where $f(k) = k^\alpha$, $0 < \alpha < 1$. Here, A_i is a household-specific productivity level; $\widetilde{Z}$ is the aggregate productivity level, which depends on the government's default decision; and f is a production function that takes as inputs capital and, implicitly, inelastically supplied labor. The proceeds from production are the only source of income for the household and are taxed at a rate $\tau > 0$. Finally, capital is assumed to fully depreciate each period.[4]

Households receive their idiosyncratic productivity shocks A_i at the beginning of period $t = 1$. The idiosyncratic productivity is constant across time and given by

$$A_i = A + \varepsilon_i,$$

where ε_i is *i.i.d.* across households and is uniformly distributed on $[-\varepsilon, \varepsilon]$, $\varepsilon > 0$. Note that this implies that A is the average level of productivity in the economy, and that knowing A is equivalent to knowing the aggregate output. After the households observe their respective productivity realizations, household i makes its investment decision, that is it choose its capital stock, k_2^i, for period 2. Households make these choices before $\widetilde{Z}$ is determined (and before the actual production takes place). Thus, when making their investment decisions, households face uncertainty regarding their future income.[5] Households are committed to their investment decisions; they cannot adjust them later. The production takes place at the end of period 1, after $\widetilde{Z}$ is determined, at which point the households invest the amount chosen earlier and consume the rest of their income.

Households make no decisions in period 2. They simply use their capital to produce, and they consume all of their after-tax income.

2.2 The Government

The government is benevolent and maximizes households' utility. In each period t, it provides households with public consumption goods, g_t, and finances its expenditure by taxing households' income and (in period 1) by borrowing in the bond market. The government enters period 1 with a legacy debt, B_1, which is due later in this period, and it initially does not observe the average level of productivity in the economy, A.

At the beginning of period 1, the government announces an interest rate $r > 0$ at which it is willing to borrow in the bond market. Once the households and lenders make their choices, the government observes A and decides how much to borrow, B_2; whether to default or not, d_1; and how much of public goods to provide to households, g_1. In period 2, the government repays its debt B_2, if it did not default on it earlier, and provides g_2 to households. The government can default only in period 1, in which case it defaults on all of its debt.[6]

[4] The assumption that capital fully depreciates implies that the households' optimal investment choice is linear in e^{A_i}, which simplifies the subsequent analysis.

[5] While non-standard, this assumption captures two realistic features of an investment process. First, investment takes time and often requires prior planning. Second, investment decisions are made under uncertainty regarding future economic conditions (in this case, uncertainty about $\widetilde{Z}$).

[6] I allow for default in period 1 only, because of an inherent asymmetry between the two periods

Following the large literature on sovereign default, I assume that default is costly and associated with a drop in aggregate productivity (and, hence, in output) by a factor Z. In particular, when the government defaults, $\widetilde{Z}$ takes a value $Z < 1$, while $\widetilde{Z} = 1$ otherwise.[7]

There is also an additional cost of default: If the government issues a positive amount of debt at $t = 1$ (i.e., $B_2 > 0$) and then decides to default, it faces a further cost of default equal to ξB_2, $0 < \xi \leq 1$. I interpret ξB_2 as a "litigation cost" associated with the legal battles between bondholders and the government following a default.[8]

2.3 Lenders and the Bond Market

There is a continuum of identical, risk-neutral lenders, indexed by $j \in [0, 1]$, each with finite wealth $b > 0$. Lenders choose at $t = 1$ whether to participate in the bond market or invest in a risk-free asset. The net return on the risk-free asset is normalized to 0, while the return from participating in the bond market is endogenous and determined in equilibrium. Lenders do not observe the realization of the average productivity; instead, each lender j observes a private signal x_j about A where

$$x_j = A + v_j, \;\; v_j \sim N\left(0, \sigma_x^2\right),$$

with v_j being *i.i.d.* across lenders and independent of A.[9]

Only the government and lenders have access to the bond market. I assume that the government has all the market power in the bond market, and therefore, the government sets an interest rate r at which it is willing to borrow new funds. Taking r as given, lenders decide whether to supply their funds to the bond market, determining the total funds available in the bond market, S. The government then chooses its new borrowing, B_2, where $B_2 \in [0, S]$. After the government raises new funds, the bond market shuts down and lenders invest the funds not borrowed by the government in storage. For each unit of funds lent to the government, lender j receives a gross return of $1 + r$ in period $t = 2$ if the government repays its debt, and nothing otherwise.

in the model. Since period 2 is the last period of the model, it is hard to support repayment as an equilibrium outcome in that period — compared to period 1— because in period 2 the government faces much smaller costs of default and lacks the ability to roll over part of its debt.

[7] One can think of this drop in output as resulting from a disruption in credit markets following a sovereign default. See Mendoza and Yue (2012) for microfoundations of this channel.

[8] Following a default, creditors tend to file a substantial number of lawsuits against a defaulting government. For example, in the case of default by Argentina in 2001, there were over 140 lawsuits filed abroad, including 15 class action lawsuits, in addition to a large number of lawsuits filed in Argentine courts (Panizza et al. (2009)). I interpret ξB_2 as the costs to the government associated with these legal battles. For technical reasons behind this assumption, see Footnote 14 below.

[9] Moreover, the v_j's are independent of the ε_i's so that x_j does not provide information regarding households' idiosyncratic productivity shocks, and vice versa.

2.4 Timing

The timing of period 1 is summarized in Figure 1.

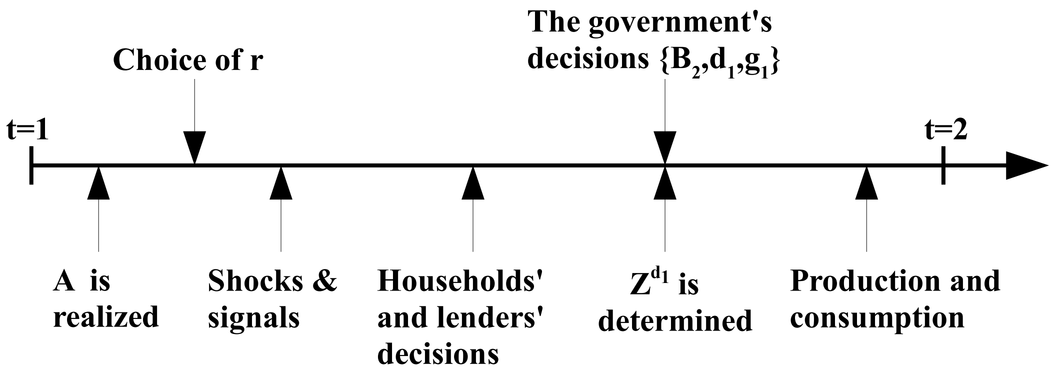


Figure 1: Timeline

At the beginning of period 1, nature draws the productivity level A, which is initially unobserved by the government as well as by the households and the lenders. Then, based only on the information contained in the prior belief, the government sets an interest rate r, at which it is willing to borrow from the lenders. Once r is announced, households receive their idiosyncratic productivity shocks and lenders observe their private noisy signals about A. Given their productivity shocks, households choose how much they want to invest, while lenders, using their private signals, decide whether to supply their funds in the market. At this point, the government learns the true A, and based on lenders' and households' decisions and the realization of A, it decides how much it will borrow today, B_2, whether to default or not, d_1, and how much of public goods to provide to households, g_1. Once the government borrows its desired amount, the bond market shuts down and the lenders' remaining funds are invested in the risk-free asset. Finally, at the end of the period, production, actual investment, and consumption take place and the average productivity level is revealed to all the agents.

Period 2 is much simpler. At the beginning of the period, production takes place. Then the government collects the taxes, provides public goods, g_2, and, if it did not default earlier, repays its remaining debt. Finally, households consume their after-tax output.

2.5 Discussion of Timing and Information Structure

The timing and the information structure of the model are chosen to satisfy three separate objectives: (1) to induce an information structure that is conducive to the existence of a unique equilibrium; (2) to render the government's problrm of choosing new borrowing, default, and spending decisions simple to solve; (3) to avoid any issue of learning or signaling in the model.

In order to achieve the first goal, following the large literature on global games, I assume that households and lenders do not observe the average productivity level, but rather the private noisy signals about A. This breaks common knowledge of the fundamentals among the agents, which is key to the uniqueness result.

To achieve the second goal, I let the government observe the average productivity, A, before making its new borrowing, default, and spending decisions in period 1. Under complete information, the government's optimal decisions can be solved, to a large extent, in a closed form. However, allowing the government to observe A creates a tension within the model. Since the government has an informational advantage, it can influence households' and lenders' decisions by signaling its private information through its choices. In order to avoid this complex to solve problem and achieve the third objective, I make two assumptions. First, I assume that the government sets the interest rate r at the beginning of period 1 based only on its prior belief, and thus its choice of r is uninformative about A. Second, I assume that households and lenders make their respective decisions simultaneously and before the government's choices of d_1, g_1, and B_2. From this it follows that households and lenders make their choices based only on the information contained in their prior beliefs and their respective signals.[10]

Allowing the government to set the interest rate has also an additional purpose in the model. It is well-known that prices tend to aggregate dispersed information efficiently (Grossman and Stiglitz, 1980). If in the model r were determined in a Walrasian market, then, in the absence of additional frictions or additional sources of uncertainty, the interest rate would perfectly reveal the underlying productivity. In order to prevent this one could inject additional sources of uncertainty, in which case the interest rate would act as a partially endogenous public signal. This would however complicate the model without altering the main conclusions of the paper.[11]

3 Equilibrium Analysis

An equilibrium in the model is defined as follows:

Definition 1 *An equilibrium is a set of government policy functions* $\{r, d_1, g_1, g_2, B_2\}$, *a profile of households' consumption and investment choices* $\{c_1, c_2, k_2\}_{i\in[0,1]}$, *a profile*

[10] Dropping these two assumptions would lead to a complex signaling game between the government and other agents, and, as shown by Angeletos et al. (2006) and Angeletos and Pavan (2013), could reintroduce multiplicity of equilibria into the model.

[11] The role of prices in global game models has been analyzed by Angeletos and Werning (2006) and Hellwig et al. (2006), among others.

of lenders' supply decisions $\{\beta\}_{j\in[0,1]}$, *and a supply function* S *such that the following hold:*

1. $\{r, d_1, g_1, g_2, B_2\}$ *solves the government's problems at* $t = 1,2$, *taking households' and lenders' decisions as given.*

2. *For every* i, $\{c_1^i, c_2^i, k_2^i\}$ *solves household* i*'s problems at* $t = 1,2$, *taking as given the other agents' decisions.*

3. *For every* j, β^j *solves lender* j*'s problem, taking as given the other agents' decisions.*

4. $S = \int_{j\in[0,1]} \beta^j dj$.

The above definition of an equilibrium is standard, and it requires that all the agents behave optimally in each subgame, taking as given the actions of the others. It also requires that the supply of funds in the bond market be consistent with lenders' supply decisions.

The equilibrium can be computed by backward induction, starting with period 2 and then moving to period 1. The key (and the most difficult step) is to solve simultaneously for the households' investment choices, the lenders' supply decisions, and the government's default decision. In what follows I will focus on equilibria in monotone strategies. This greatly simplifies the task of solving the model and renders the analysis more tractable.

3.1 Additional Assumptions

To simplify the analysis and ensure that the government problem is well-posed, I make the following assumptions.[12]

Assumption 1 *The legacy debt is large enough,* $B_1 > \overline{B}_1$ *for some threshold* $\overline{B}_1$.

Assumption 1 ensures that if the government decides to repay its legacy debt, it will find it optimal to borrow a positive amount. Otherwise, a debt crisis can only be a result of households' pessimistic expectations or poor fundamentals; in either of these two cases lenders stop playing any role in the model.

Assumption 2 *The wealth of the lenders is bounded above by* $\overline{b}$ *(i.e.,* $b < \overline{b}$*) and the "litigation costs" are large (i.e.,* $\xi \to 1$*).*

[12] For a discussion of these assumptions and the derivations of the respective bounds see Section D of the Appendix.

Assumption 2 is required to ensure that the government's incentive to default decreases as the supply of funds in the market increases, a property that substantially simplifies the analysis.

Assumption 3 *$Z > \underline{Z}$, that is, output cost of default is not too large.*

Assumption 3 implies that the government's optimal unconstrained borrowing, the amount it would like to borrow if it repays the debt, is monotone in A.

Given the above assumptions, I now analyze the equilibrium of the model. I compute the equilibrium using backward induction. Note that once the government makes its choices of B_2, d_1, g_1, no agent makes any decision and the equilibrium outcomes are determined. Therefore, I begin the analysis by describing the government's new borrowing, default, and spending decisions in period 1.

3.2 Period $t = 1$: The Government's Decisions

The government decides how much to borrow, whether or not to default, and how much to spend to maximize the households' utility, internalizing how each of these decisions affects consumption, aggregate productivity, and future tax revenues. The government makes these decisions after observing households' investment decisions, the supply of funds in the market, and the average level of productivity in the economy.

Let $\mathbf{k}_2 = \left\{k_2^i\right\}_{i\in[0,1]}$, and let $V_1^R(A, \mathbf{k}_2, S)$ be the value to the government of repaying its debt when the average productivity is equal to A, the households' investment profile is $\mathbf{k}_2$, and the supply of funds in the bond market is S. Then $V_1^R(A, \mathbf{k}_2, S)$ is given by

$$V_1^R(A, \mathbf{k}_2, S) = \max_{B_2\in[0,S]} \sum_{t=1,2} \left\{ \int_0^1 \left[\log\left(c_t^{i,R}\right) + \log\left(g_t^R\right)\right] di \right\}$$
$$s.t.\ g_1^R = \tau Y_1^R - B_1 + B_2$$
$$g_2^R = \tau Y_2^R - (1+r) B_2,$$

where g_t^R is the government spending in period t, Y_t^R is the aggregate output at time t if the government repays the debt. When the government decides to repay its debt, it chooses its new borrowing, B_2, to maximize households' utility subject to the available funds in the market, S, and its budget constraints.

Since at this point, S is already determined, it is possible that the optimal (unconstrained) amount the government would like to borrow, denoted by $B_2^{R,u}$, is larger than S; in this case, the government is constrained in the bond market and borrows $B_2 = S$. If, on the other hand, the supply of funds exceeds the government's unconstrained demand for funds, $B_2^{R,u}$, then the government sets $B_2 = B_2^{R,u}$. As shown below, the unconstrained demand for funds by the government, $B_2^{R,u}$, plays an important role in the lender's problem.

Let $V_1^D(A, \mathbf{k}_2, S)$ be the value associated with defaulting, that is,

$$V_1^D(A, \mathbf{k}_2, S) = \max_{B_2 \in [0,S]} \sum_{t=1,2} \left\{ \int_0^1 \left[\log\left(c_t^{i,D}\right) + \log\left(g_t^D\right) \right] di \right\}$$

$$s.t.\ g_1^D = \tau\left(ZY_1^R\right) + (1-\xi) B_2$$

$$g_2^D = \tau\left(ZY_2^R\right)$$

If the governments defaults, it borrows the maximum possible amount in the market (i.e., $B_2 = S$) and then repudiates all of its debt, and both of these actions tend to increase government spending in period 1. When $\xi \to 1$, this effect of borrowing as much as possible vanishes and the main benefit of default is an increase in the g_1 due to repudiation of the "legacy debt" B_1. The negative effect of defaulting is a drop in aggregate productivity by factor Z.[13]

When deciding whether or not to default, the government compares $V_1^R(A, \mathbf{k}_2, S)$ with $V_1^D(A, \mathbf{k}_2, S)$ and chooses to repay its debt if and only if the value associated with repaying is larger than the value associated with defaulting, that is, if and only if

$$\Delta V(A, \mathbf{k}_2, S) \equiv V_1^R(A, \mathbf{k}_2, S) - V_1^D(A, \mathbf{k}_2, S) \geq 0 \tag{1}$$

3.3 Default Decisions and the Fragility Region

For sufficiently low productivity levels, the government finds it optimal to default regardless of the households' and lenders' actions — when A is low, defaulting leads to an increase in government spending.[14] On the other hand, when the average level of productivity is high, the government always finds it optimal to repay the debt. Intuitively, for high A, defaulting not only leads to a drop in private consumption but also results in less government spending. Accordingly, for each interest rate r, there exist two thresholds, $\underline{A}(r)$ and $\overline{A}(r)$, such that the government always defaults if $A < \underline{A}(r)$ and never defaults if $A > \overline{A}(r)$.

For all $A \in \left[\underline{A}(r), \overline{A}(r)\right)$, the government's default decision depends on the households' and lenders' choices. If the lenders expect default, they invest all their funds in the risk-free asset. In this case, the government cannot roll over its debt, and hence

[13] From the definition of V_1^D, we can see that if $\xi < 1$, then the value of defaulting is an increasing function of S. When ξ is small, the value of defaulting increases with S at a faster rate than the value of repaying, and equilibrium in monotone strategies does not exist. The assumption that $\xi \to 1$ is needed to ensure the existence of equilibrium in monotone strategies while keeping default costly for the lenders.

[14] For sufficiently low A, we have $B_1 > (1-Z)\tau Y_1$, that is the fall in government revenues caused by the fall in the aggregate productivity is lower than the amount of debt that has to be repaid.

repaying B_1 becomes very costly in terms of the forgone utility from government spending. If, on the other hand, the households expect default, they decrease their investment, leading to a drop in the government's revenues (taxes) in the future. This translates into a drop in government expenditure in both periods (since the government smooths out the drop in its revenue across time) and leads to a higher cost of repaying the legacy debt. If $A \in \left[\underline{A}(r), \overline{A}(r)\right)$, these costs are large enough that in response to a shift in households' or lenders' expectations the government finds it optimal to default.

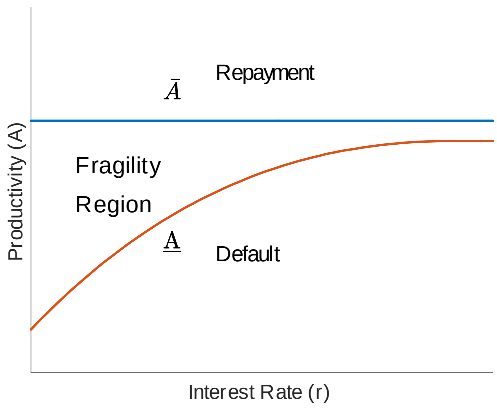


Figure 2: Fragility Region

Figure 2 depicts the fragility region $\left[\underline{A}(r), \overline{A}(r)\right)$. For all A in this region, the government's default decision depends on households' and lenders' expectations. If we were free to choose equilibrium beliefs, as in the case of complete-information models, both default and no default could be supported as equilibrium outcomes. However, given the assumed information structure, beliefs are not free objects, but rather are determined in equilibrium. As I show below, introducing uncertainty into the model allows me to pin down beliefs and prove the uniqueness of the equilibrium in monotone strategies.

3.4 Household's Problem

Consider household i with an idiosyncratic productivity shock A_i that must choose how much to invest. This household's problem can be written as

$$\max_{k_2} \ E\left[\sum_{t=1,2}\left[\log\left(c_t\right)+\log\left(g_t\right)\right]\middle| A_i,\boldsymbol{\sigma}\right]$$
$$s.t. \ c_1=(1-\tau)\,Z^{d_1(\boldsymbol{\sigma})}e^{A_i}f\left(k_1\right)-k_2$$
$$c_2=(1-\tau)\,Z^{d_1(\boldsymbol{\sigma})}e^{A_i}f\left(k_2\right)$$
$$\boldsymbol{\sigma}=\left\{\mathbf{k}_2,\boldsymbol{\beta},r,d_1,g_1,g_2,B_2\right\},$$

where $\boldsymbol{\sigma}$ is the strategy profile of all players and the expectations are taken over the government default decisions, $d_1(\boldsymbol{\sigma})$, as well as over the average level of productivity, A. Household i chooses k_2 to maximize its utility subject to the budget constraint, taking $\boldsymbol{\sigma}$ as given. Lemma 1 characterizes households' optimal investment when households believe that the government will always default if the average productivity is less than A^*.

Lemma 1 *Suppose that the government follows a monotone default strategy with threshold A^*. Then household i's optimal investment is given by*

$$k_2=(1-\tau)\,e^{A_i}f\left(k_1\right)\Lambda\left(A_i;\varepsilon,A^*\right),$$

where $\Lambda\left(A_i;\varepsilon,A^*\right)$ is increasing in the idiosyncratic productivity, A_i, and decreasing in the default threshold, A^*.[15]

3.5 Lender's Problem

Simultaneously with the households' investment choices, the lenders must decide whether to supply their funds to the bond market or to invest their funds in storage. Lenders base their decisions on the prior belief about A and their private signals, x_j. Let $\mathcal{R}(\boldsymbol{\sigma})$ be the government repayment set for a fixed strategy profile $\boldsymbol{\sigma}$. Then the expected payoff to lender j from supplying the funds to the bond market is given by

$$\int_{A\in\mathcal{R}(\boldsymbol{\sigma})}\left(1+r\min\left\{1,\frac{B_2^{R,u}\left(A;\boldsymbol{\sigma}\right)}{S\left(A;\boldsymbol{\beta}\right)}\right\}\right)f\left(A|x_j\right)dA,$$

where $f\left(A|x_j\right)$ is lender j's posterior belief about A, $B_2^{R,u}\left(A;\boldsymbol{\sigma}\right)$ is the unconstrained desired borrowing by the government in repayment, and $S\left(A;\boldsymbol{\beta}\right)$ is the supply function implied by the lenders' supply strategy profile $\boldsymbol{\beta}$. Finally, $\min\left\{1,B_2^{R,u}\left(A;\boldsymbol{\sigma}\right)/S\left(A;\boldsymbol{\beta}\right)\right\}$

[15] See Section A of the Appendix for the exact definition of $\Lambda\left(A_i;\varepsilon,A^*\right)$.

is the amount that lender j expects to lend to the government given that the average productivity level is A.[16] Lender j supplies his funds to the bond market if and only if the expected return from supplying the funds is higher than 1, the return from investing in storage.

As in the case of the household's problem, restricting attention to monotone strategies leads to a simple characterization of the lenders' optimal behavior.

Lemma 2 *Suppose that the government defaults if and only if $A < A^*$. Then an optimal strategy for each lender j is to supply the funds to the bond market if and only if he receives a signal $x_j \geq x^*$. Moreover, x^* is the unique solution to the equation*

$$\int_{A^*}^{\infty} \left(1 + r \min\left\{1, \frac{B_2^{R,u}(A;\boldsymbol{\sigma})}{S(A;\mathbf{x}^*)}\right\}\right) f(A|x^*)\, dA = 1,$$

where $S(A;\mathbf{x}^*)$ is the supply function when all lenders follow this strategy.

3.6 Equilibrium Default Threshold

Above I characterized the optimal behavior of each type of agent. This, in turn, allows me to prove the following proposition, which states that for any interest rate r there exists a unique equilibrium in monotone strategies.

Proposition 1 *There exist $\overline{\varepsilon} > 0$ and $\overline{\sigma}_x > 0$ such that for any interest rate r, any $\varepsilon \in (0, \overline{\varepsilon}]$, and any $\sigma_x \in (0, \overline{\sigma}_x]$, the model has a unique equilibrium in monotone strategies where the following hold:*

1. *The government defaults if and only if $A < A^*(r)$.*

2. *Each lender provides the funds if and only if $x_j \geq x^*(r)$.*

3. *Households' investment rules, k_2, are increasing in A_i.*

The proof of Proposition 1 builds on the insights and results of Athey (1996) and Morris and Shin (2003). The above result, however, is non-trivial for several reasons. First, difficulty comes from the fact that in the model, the global game is played by three different types of agents, each with its own preferences and choice sets. Second, the lenders' payoff function satisfies only a weak single-crossing condition, rather than

[16] For all $A \notin \mathcal{R}(\boldsymbol{\sigma})$, the government borrows all available funds in the market and then defaults, implying that in this case lender j earns nothing. If $A \in \mathcal{R}(\boldsymbol{\sigma})$, the government would like to borrow $B_2^{R,u}$.

global strategic complementarities, as in typical global games.[17] Finally, the regime-change condition (i.e., the condition that determines whether default will occur) arises endogenously from the government's optimal behavior — unlike in the typical global games literature, where it is exogenously imposed.

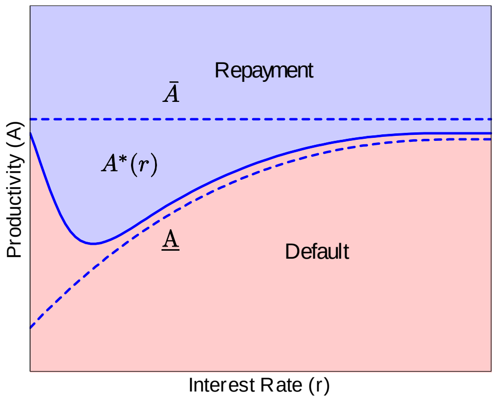


Figure 3: Default Threshold

Figure 3 depicts the equilibrium default threshold A^* as a function of the interest rate r. We see that $A^*(r)$ is a non-monotone function of r. To understand this, note that when the interest rate is low, few lenders supply their funds to the bond market. As a result, the government finds it optimal to default for most productivity values in the "fragility region". As r increases, the supply of funds increases since higher r compensates lenders for exposing themselves to default risk. At the same time, households' investment rules shift upwards since they anticipate that the government will choose to repay the debt for a larger set of productivity levels. This decreases the government's incentives to default and leads to a lower $A^*(r)$. A higher interest rate, however, increases the costs of rolling over the debt, discouraging the government from smoothing debt repayment over time. This tends to decrease the value of repaying debt to the government. For sufficiently high r, this negative effect dominates, implying that $A^*(r)$ becomes an increasing function of r.

It is important to stress that, while the default threshold is unique, the outcome of the model in the fragility region is driven fully by households' and lenders' expectations. For all productivity levels in the fragility region, both repayment and default could be supported as equilibrium outcomes if we had the freedom to choose the lenders' and

[17] Applying global games results in a complex environment in which payoff functions satisfy only the weak single-crossing condition, rather than global strategic complementarities, is not without cost. In particular, I need to restrict my attention to monotone strategies. Morris and Shin (2003) discuss why, in general, the single-crossing condition is not enough to prove uniqueness.

households' expectations. However, the households' and lenders' expectations are not free objects. An incomplete-information structure transforms beliefs into equilibrium objects and requires them to be sequentially rational and consistent with agents' strategy profiles. This imposes requirements on the beliefs that are not present in the complete-information game. The approach used in global games is thus to use an appropriate information structure that will lead to a unique equilibrium outcome.

3.7 Optimal Choice of r

It remains to characterize the government's optimal choice of interest rate, r. The government chooses the interest rate based on the current and past fundamentals of the economy, $\{B_1, k_1, A_{-1}\}$. The government also knows its future policy functions $\{d_1, g_1, g_2, B_2\}$ and realizes that it can affect consumption, investment, and the supply of funds through its choice of interest rate. To choose the optimal interest rate, the government solves the following problem:

$$W\left(A_{-1}, B_1, \mathbf{k}_1; \boldsymbol{\sigma}\right) = \max_r E\left[\sum_{t=1,2} \int_{i=0}^{1} \left[\log\left(c_t^i\right) + \log\left(g_t\right)\right] di \,\middle|\, A_{-1}\right]$$

$$s.t. \text{ policy functions } \{c_1, c_2, d_1, B_2, g_1, g_2\}$$

$$\text{lenders' and households' strategies } \{\boldsymbol{\beta}, \mathbf{k}_2\}.$$

When choosing the interest rate, the government faces the following trade-off: On the one hand, at least initially, a higher r tends to decrease the default threshold. On the other hand, a higher r increases the cost of borrowing at $t = 1$, making it more costly to roll over the maturing debt. Thus, the government weighs the positive effect of a lower default threshold against the increase in the borrowing costs.

The next lemma characterizes the interest rate in the limit when both the private information and the information content of the prior become very precise.

Lemma 3 *Let $\varepsilon, \sigma_x \to 0$. Then*

$$\lim_{\sigma_A \to 0} r^*\left(A_{-1}\right) = \begin{cases} 0 & \text{if } A_{-1} > A^*(0) \\ \widehat{r}\left(A_{-1}\right) & \text{if } A_{-1} \in \left[\underline{A}^*, A^*(0)\right] \end{cases}$$

where $\widehat{r}\left(A_{-1}\right) \equiv \min\{r : A^*(r) = A_{-1}\}$ and $\underline{A}^* = \min A^*(r)$. If $A < \underline{A}^*$, then the government always defaults, regardless of the interest rate.[18]

[18] For any σ_A, the equilibrium in the limit as $\varepsilon \to 0$ and $\sigma_x \to 0$ is well-defined. Therefore, one can ask the question, What happens as $\sigma_A \to 0$? Conceptually, this is the same as considering the limiting case, as the noise is vanishing in a standard global game.

In the limit, as the prior become perfectly informative, the trade-off described above takes a very simple form and the government simply chooses the lowest interest rate that allows it to avoid costly default. Thus, the optimal interest rate is a decreasing function of the fundamentals. If $A < \underline{A}^*$, that is, if A is smaller than the minimum default threshold that the government can induce by choosing the interest rate, then the government defaults regardless of the interest rate.

4 Preventing Self-fulfilling Debt Crises

In this section, I analyze under which conditions government policies can decrease the ex-ante probability of default (i.e., help to prevent a debt crisis). I assume that each policy change is announced in period 1 before the households and lenders make their decisions but after r is set, and that the government is committed to implementing the announced policies. The policy itself is, however, is not implemented until the end of that period. This allows me to focus on the role of beliefs while abstracting away from the effects of other forces. I relax these assumptions in the following sections. In Section 5, I analyze what happens if the policy adjustment is unexpected and if there is uncertainty as to whether the government will implement the announced policy, while in Section 6 I analyze the case when the policy announcement is made before the interest rate is set. Figure 4 depicts the timing for the policy adjustment considered in this section.

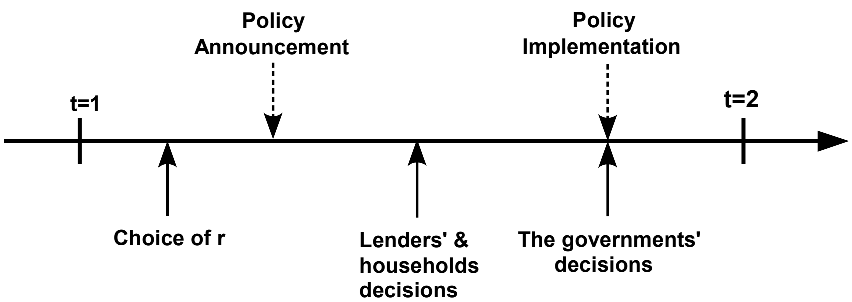


Figure 4: Timing of Policy Adjustments

In order to simplify analysis and make the problem more tractable, I make the following assumption:

Assumption 4 B_1 *is large enough so that for all* $A > \underline{A}(0)$ *the government's desired borrowing in repayment exceeds the supply of funds in the market.*[19]

[19] Recall from Section 3.3 that $\underline{A}(0)$ is the lower bound for the fragility region when $r = 0$. Thus, it

Assumption 4 simplifies the problem by eliminating the issue of competition between lenders in the bond market, in which case the lender's problem can be solved in closed form.[20]

4.1 Equilibrium Effects of Policy Adjustments

Before analyzing specific policies, it is useful to understand the equilibrium forces that are at play when the government adjusts its policy. For this purpose, consider an abstract policy adjustment, captured by a change in a parameter ψ.[21] We would like to understand how a change in ψ affects the ex-ante probability of default.

The ex-ante probability of default is equal to $\Pr(A < A^*)$ and hence is determined by the default threshold A^*. For a given interest rate r, the default threshold is determined by the following set of equations. The first equilibrium condition is the government's default equation, which says that at the threshold productivity level, A^*, the government is indifferent between repaying the debt and defaulting, that is,

$$\Delta V\left(A^*, \mathbf{k}_2, x^*; \psi\right) = 0.$$

The second equilibrium condition expresses the first-order conditions for optimal investment on the part of households. Given A_i (household i's productivity) and given A^{**} (households' and lenders' belief regarding the default threshold) thos condition determines household i's capital at $t = 2$ and can be written as

$$I\left(A_i, A^{**}, k_2^i; \psi\right) = 0.$$

The final equilibrium equation expresses the lenders' indifference conditions, which say that, given the lenders' belief regarding the default threshold, A^{**}, when lender j observes signal x^*, he is indifferent between supplying his funds to the bond market and investing in storage:

$$L\left(A^{**}, x^*; \psi\right) = 0.$$

Note that in equilibrium, the agents' belief about the threshold productivity at which the government will default have to be correct, and hence $A^{**} = A^*$. The above equations determine the effect of a change in a government policy on the default threshold, and hence on the ex-ante probability of default.

is the productivity level below which the government will always default, regardless of the interest rate and regardless of the households' and lenders' decisions.

[20] While Assumption 4 simplifies the comparative statics analysis, it does not affect its underlying logic. In particular, Proposition 2 holds in the same form regardless of whether we impose Assumption 4. For a more detailed discussion of the consequences of this assumption see Section D of the Appendix.

[21] For concreteness, one can think of this policy as an increase in taxes, in which case $\psi = \tau$.

The change in the default threshold implied by the adjustment in a policy parameter ψ is given by

$$\frac{dA^*}{d\psi} = \underbrace{\frac{1}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}} di}}_{\text{Multiplier effect}} \times \underbrace{\left(\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi} di\right)}_{\text{Direct effect}} \tag{2}$$

To understand the intuition behind the above expression, first keep households' and lenders' beliefs about A^* constant. Then a change in ψ affects the government's incentive to default, by changing the difference between the values of repaying and defaulting on the debt. This effect works through the government's indifference condition; I denote it by $\partial A^*/\partial\psi$, since it corresponds to the partial effect of a change in policy keeping strategies of households and lenders fixed. Moreover, the policy change potentially affects households' and lenders' decision problems, thereby leading households and lenders to adjust their strategies and in turn bringing about a further change in the government's incentive to default (these effects are captured by terms $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}$ and $\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}$, respectively). Thus, the "direct effect" is equal to the change in the default threshold, keeping households' and lenders' expectations fixed.

So far, I fixed agents' expectations regarding the default threshold. The households' and lenders' expectations, however, are not constant. In response to this initial change in the default threshold, the households and lenders adjust their expectations, and thus their strategies, which leads to a further change in A^*, inducing another round of adjustment in the households' and lenders' expectations and so on. The "multiplier effect" capture the change in default threshold driven by the adjustment in households' and lenders' expectations. Finally, note that the magnitude of the multiplier effect is the same regardless of the policy considered, as captured by a change in ψ.[22]

Proposition 2 *Let M denote the multiplier effect and D the direct effect.*

1. *For any $\varepsilon, \sigma_x > 0$, we have $M \in (1, \infty)$.*

2. *If either $\varepsilon \to 0$ or $\sigma_x \to 0$ (or both), then $M \to \infty$ and $D \to 0$, with $\lim_{\varepsilon,\sigma_x \to 0} (M \times D) \in \mathbb{R}$.*

3. *Suppose that $\varepsilon, \sigma_x \to 0$ and $\frac{\varepsilon}{\sigma_x} \to c$ where $c \in \overline{\mathbb{R}}_+$.*

   (a) *If $c = 0$, then in the limit the multiplier is driven only by adjustments in households' expectations.*

[22] The multiplier effect in the model is similar in spirit to the multiplier in Keynesian models (see for example Cooper and John (1988)). The difference is that here it operates through expectations and rather than directly through actions.

(b) If $c \in \mathbb{R}_{++}$, then in the limit the multiplier effect is driven by adjustments in both households' and lenders' expectations.

(c) If $c = \infty$, then in the limit the multiplier effect is driven only by adjustments in lenders' expectations.

Part (1) of the above Proposition establishes that the multiplier effect is always positive, and thus it always reinforces the initial effect of a policy change.[23] Thus, endogenous expectations tend to amplify the impact of government actions on the economy. If the government wants to avoid default, it can use this effect to its own advantage by implementing the right policies. Unfortunately, it also means that if the government wants to pursue other objectives and would like to implement a policy that tends to increase the probability of default, then adjustments in households' and lenders' expectations make such a policy more costly by magnifying its effect on the likelihood of default.

Part (2) of the Proposition implies that when lenders' and households' information is very precise, then the role played by the endogenous expectations is particularly strong. Thus, it becomes particularly important for the government to take into account the impact of its policies on expectations. In the limit, as the noise in the agents' signals vanishes, all of the adjustment in the probability of default is driven by the adjustment in expectations (which is captured by the multiplier effect). To understand this, note that in the model, agents face two types of uncertainty: (1) the fundamental uncertainty, that is the uncertainty about the underlying state of the economy, and (2) the strategic uncertainty that captures the uncertainty about the actions of others. The direct effect operates through the fundamental uncertainty, since a policy change is equivalent to a change in the underlying state of the economy.[24] On the other hand, the multiplier effect operates through the change in strategic uncertainty, since it captures the effect of the adjustment in the expectations regarding others' actions. As signals become more informative agents face less and less in the way of fundamental uncertainty, and hence the relative importance of the direct effect decreases while that of the multiplier effect increases. In the limit, as information becomes infinitely precise, the only type of uncertainty that agents face is the strategic uncertainty and, hence, all the adjustment in the default threshold is driven by the multiplier effect.[25]

[23] This result is driven by the presence of strategic complementarities in the model: An increase in default threshold always leads to adjustments in households' and lenders' strategies that make default even more attractive, and vice versa.

[24] Any change in a parameter results in a shift of the fragility region, and hence can be mapped into improvement (if the shift is down) or deterioration (if the shifts is up) in the state of the economy.

[25] As shown by Morris and Shin (2003), in global games agents face the highest degree of strategic uncertainty precisely in the limit as the noise vanishes. Thus, as agents' information becomes more

Finally, the last part of the Proposition shows that in the limit the relative contribution of the adjustment in households' and lenders' expectations depends on the relative informativeness of their signals. When households' information becomes precise at a faster rate than that of lenders, households face more strategic uncertainty than lenders, and hence they respond more strongly to changes in the expectations regarding A^* (the term $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}$ becomes larger than $\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}$). The opposite is true when it is lenders' information that becomes more informative at the faster rate. Thus, the informativeness of households' information relative to that of lenders determines whether it is households' or lenders' expectations that matter more.

Proposition 2 also implies that whether a change in policy increases or decreases the probability of default is determined by the "direct effect". Thus, to establish whether a given policy decreases or increases the likelihood of a debt crisis one can abstract from the issues of expectations and focus on how the policy affects the government incentive to default holding agents' expectations constant. However, expectations are key for quantifying the impact of the policy on the probability of default, and their role increases as the information available to economic agents becomes more precise. It follows that quantitative predictions of models in the spirit of Calvo (1988) or Cole and Kehoe (2000) may substantially underestimate the policy effects, since these models do not include the belief-driven multiplier effect.

4.2 Overview of Policies

Using the above insights, I now analyze specific government policies. I focus on two policy measures that received a lot of attention in the recent policy debates following the European debt crisis: (1) an increase in taxes and (2) a fiscal stimulus (financed with debt). Below, I describe how each of these policies is introduced into the model.

Increase in Taxes In the model, a rise in the tax rate is captured by an increase in τ, the fraction of output that the government takes away from households. For simplicity, I assume that once adjusted, τ is kept constant across periods and is the same regardless of whether the government defaults.

Fiscal Stimulus I model fiscal stimulus as an increase in the initial capital stock of each household from k_1 to $(1+s)\,k_1$ financed by the government, where s measures the size of the stimulus as a percentage of the initial capital stock. Thus, if the government

precise, not only does the fundamental uncertainty vanish but the strategic uncertainty tends to increase, further enhancing the relative strength of the multiplier effect.

decides to engage in a stimulus the total output of the economy will increase.[26] I do not explicitly model the government's financing decision. Instead, I assume that to finance a stimulus, the government issues additional debt at the end of the period preceding period 1. I consider separately the case where this additional debt matures at the end of period 1 together with B_1 (short-term debt financing with interest rate $r^{ST} \geq 0$) or in period 2 (long-term debt financing with interest rate $r^{LT} \geq 0$).

4.2.1 Increase in Taxes

As explained above, to understand the effect of an increase in the tax rate τ on the default threshold, it is enough to focus on its direct effects. A higher tax rate leads to a change in the government's incentives to repay debt equal to[27]

$$\underbrace{Y_1^{R,*}\left(u_{g_1}^R - u_{g_1}^D\right) + Y_2^R\left(u_{g_2}^R - u_{g_2}^D\right)}_{\text{Concavity effect}} + \underbrace{Y_1^R\left(1-Z\right)u_{g_1}^D + Y_2^R\left(1-Z\right)u_{g_2}^D}_{\text{Differential increase in tax revenues}}$$
$$\underbrace{-\frac{\alpha}{1-\tau}\tau Y_2^R\left(u_{g_2}^R - Zu_{g_2}^D\right)}_{\text{Investment distortion}}, \tag{3}$$

where $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utilities from government spending in period t in repayment and default, respectively, and is Y_t^R the total output of the economy in period t in repayment, all evaluated at the threshold productivity level A^*. If the expression in (3) is positive, then the government's incentive to repay its debt increases following an increase in τ.

The expression in (3) tells us that an increase in the tax rate affects the government's default incentives through three channels. First, a higher τ implies higher tax revenues. Since at A^*, government spending is lower in repayment than in default, the concavity of the utility function implies that a given increase in government spending leads to a

[26] This is a simple way to model a fiscal stimulus in the current framework. One should interpret the increase in k_1 not as an increase in physical capital owned by households but rather as an increase in government spending on public goods and services that enhance production (e.g., an increase in expenditure on infrastructure or on the maintenance of the rule of law). An alternative way to model stimulus would be to explicitly allow government spending to enter the production function, that is to write the household production function as $y_t^i = e^{A_i} f\left(k_t^i, h_t\right)$ where h_t captures explicitly the government expenditure that is important for production. However, the qualitative conclusions would remain unchanged.

[27] The expression in (3) corresponds to $\frac{\partial}{\partial \tau}\Delta V\left(A^*, k_2, x^*; \psi\right)$. The direct effect is equal to $\frac{\partial}{\partial \tau}\Delta V\left(A^*, k_2, x^*; \psi\right)$ divided by $-\frac{\partial}{\partial A^*}\Delta V\left(A^*, k_2, x^*; \psi\right) < 0$. In particular, the sum of the concavity effect and the differential increase in tax revenues divided by $-\frac{\partial}{\partial A^*}\Delta V\left(A^*, k_2, x^*; \psi\right)$ is equal to $\frac{\partial A^*}{\partial \psi}$, while the expression for investment distortion divided by $-\frac{\partial}{\partial A^*}\Delta V\left(A^*, k_2, x^*; \psi\right)$ corresponds to $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}$ in Equation (2).

greater increase in the value of repaying than in the value of defaulting, thus decreasing the government's default incentive (the "concavity effect"). Second, since the total output is higher in repayment, a given increase in the tax rate translates into a greater increase in tax revenues in repayment than in default, further decreasing the government's default incentives (the "differential increase in tax revenues"). The last term captures the negative effect of higher taxes on households' investment decisions, where $\alpha/(1-\tau)$ is the rate at which output decreases with higher taxes and $u_{g_2}^R - Zu_{g_2}^D$ measures how "painful" this decrease in spending is to households in repayment compared to default the "investment distortion")

When households' utility from the government spending is logarithmic, the concavity effect is equal to

$$\frac{1}{\tau}(B_1 - B_2)\, u_{g_1}^R + \frac{1}{\tau}(1+r)\, B_2 u_{g_2}^R - (1-Z)\, Y_1^R u_{g_1}^D - (1-Z)\, Y_2^R u_{g_2}^D$$

To understand this expression, note that government spending in repayment tends to be lower than in default, because of repayment of debt ($B_1 - B_2$ in period 1 and $(1+r)\, B_2$ in period 2). This tends to increase the marginal utility from the additional government spending in repayment, contributing positively to the concavity effect. This effect is captured by the first two terms in the above expression. In default, on the other hand, government revenues drop by $1 - Z$ in each period compared to the case when the government repays its debt. This contributes negatively to the concavity effect and is captured by the last two terms.

Note that the negative part of the concavity effect exactly offsets the positive effect of the differential increases in tax revenues as described in (3). Thus, the sum of the concavity effect and the differential increase in tax revenues is simply equal to $\frac{1}{\tau}(B_1 - B_2)\, u_{g_1}^R + \frac{1}{\tau}(1+r)\, B_2 u_{g_2}^R$. Finally, the investment distortion can be written as $\frac{\alpha\tau}{1-\tau}\frac{1}{\tau}(1+r)\, B_2 u_{g_2}^R$. Proposition 3 follows immediately from these observations.

Proposition 3 *Let* $\varsigma_\tau \equiv u_{g_1}^R [B_1 - B_2] + (1+r)\, B_2 u_{g_2}^R \left[1 - \frac{\alpha\tau}{1-\tau}\right]$,

1. *If* $\varsigma_\tau > 0$, *then an increase in taxes decreases the probability of default.*

2. *If* $\varsigma_\tau < 0$, *then an increase in taxes increases the probability of default.*

4.2.2 Fiscal Stimulus

Now consider the effect of a fiscal stimulus on the probability of default. A fiscal stimulus leads to a change in government's incentives to repay debt equal to

$$\underbrace{\tau\frac{\partial Y_1^R}{\partial s}\left(u_{g_1}^R - u_{g_1}^D\right) + \tau\frac{\partial Y_2^R}{\partial s}\left(u_{g_2}^R - u_{g_2}^D\right)}_{\text{Concavity effect}} + \underbrace{\left[\frac{\partial Y_1^R}{\partial s}u_{g_1}^D + \frac{\partial Y_2^R}{\partial s}u_{g_2}^D\right]\tau\left(1-Z\right)}_{\text{Differential increase in tax revenues}}$$
$$\underbrace{-u_{g_t}^R\left(1+r^{stim}\right)k_1}_{\text{Increase in debt}}, \tag{4}$$

where $r^{stim} \in \left\{r^{ST}, r^{LT}\right\}$ is the interest rate on the debt issued to finance the stimulus and where $u_{g_t}^R$, $u_{g_t}^D$ and Y_t^R are defined as in Section 4.2.1.

The expression in (4) tells us that a fiscal stimulus affects the government's default incentive through three channels: (1) the "\concavity effect; (2) a differential increase in government tax revenues in repayment and default (both of which were also present in the case of a tax increase); and (3) a negative effect due to an increase in the government's debt burden (equal to $u_{g_1}^R\left(1+r^{ST}\right)k_1$ if the stimulus is financed with short-term debt, or to $u_{g_2}^R\left(1+r^{ST}\right)k_1$ if financed with long-term debt).[28]

As in the case of an increase in taxes, one can rewrite the concavity effect as

$$\tau\frac{\partial Y_1^R}{\partial s}\left(B_1 - B_2\right)u_{g_1}^R + \tau\frac{\partial Y_2^R}{\partial s}\left(1+r\right)B_2 u_{g_2}^R - \tau\frac{\partial Y_1^R}{\partial s}\left(1-Z\right)u_{g_1}^D - \tau\frac{\partial Y_2^R}{\partial s}\left(1-Z\right)u_{g_2}^D,$$

and the intuition behind this expression is analogous. On the one hand, higher $(B_1 - B_2)$ and $(1+r)B_2$ mean more debt repayment if the government chooses to repay the debt, increasing the marginal utility from the additional government spending if the government chooses to repay its debt. On the other hand, a higher Z means lower tax revenues in default compared to the case of no default, increasing the marginal utility from the additional government spending in default. Since the marginal effect of the stimulus on output is αY_1 in period 1 and $\alpha^2 Y_2$ in period 2, on summing up the differential increase in tax revenues and the concavity effects, we obtain the following result.

Proposition 4 *Let* $\varsigma_{st} \equiv \alpha\left(B_1 - B_2\right)u_{g_1}^R + \alpha^2\left(1+r\right)B_2 u_{g_2}^R - \left(1+r^{ST}\right)k_1 u_{g_1}^R$

1. *If* $\varsigma_{st} > 0$*, then a stimulus decreases the probability of default.*

2. *If* $\varsigma_{st} > 0$*, then a stimulus increases the probability of default.*

When a stimulus is financed with long term debt the relevant condition becomes $\alpha\left(B_1 - B_2\right)u_{g_1}^R + \alpha^2 B_2 u_{g_2}^R > \left(1+r^{LT}\right)k_1 u_{g_2}^R$. Since in equilibrium the government

[28] The typical argument for the use of a fiscal stimulus is that, fiscal stimulus is beneficial because by expanding output, it actually lowers debt-to-GDP ratio and hence makes the debt sustainable. The expression in (4) reveals that, to the extent that the government maximizes households' welfare, this logic is faulty: whether the stimulus decreases or increases probability of default depends not on the absolute size of the "fiscal multiplier" but rather on the difference in the multiplier effect between the default case and the repayment case.

expenditure in period 1 is always lower than in period 2 (the government is unable to smooth repayment, on account of the limited supply of funds in the market), as long as r^{LT} is not significantly higher than r^{ST}, the cost of repaying the debt used to finance stimulus is incurred when it is less costly in terms of the forgone utility from the government spending. Hence, a fiscal stimulus financed with long-term debt is more likely to decrease the probability of default.

5 Political Uncertainty and Its Consequences

Above I considered a situation where a policy change was expected by both households and lenders. In this section, I investigate how the above results change if the households and lenders are uncertain as to whether the government will adjust its policies. The analysis is motivated by the observation that often there is strong disagreement among political parties and among policymakers regarding the political and economic desirability of given economic policies, thereby giving rise to a substantial political uncertainty. The recent experience of European countries during the European debt crisis has shown that such uncertainty may constitute an important factor in a debt crisis.[29]

I consider two cases. First, I investigate the model's predictions when a policy change is unexpected by the lenders and households. Then I analyze a situation where households and lenders expect that the government will adjust its policy with probability $p \in (0, 1)$. Here, p captures the magnitude of political uncertainty in the economy.

5.1 Unexpected Policy Adjustment

Proposition 5 *Suppose that a policy change is unexpected. Then*

$$\frac{dA^*}{d\psi} = \frac{\partial A^*}{\partial \psi}.$$

Moreover, $dA^*/d\psi \to 0$ *as* $\varepsilon, \sigma_x \to 0$.

[29] Political uncertainty played an important role in Greece, where after winning the unexpected early elections in January 2015 the Syriza-led coalition stopped implementation of reforms, only to suddenly change its mind six months later, but not until after pushing Greece to the verge of default ("Countdown to Grexit deadline day," *Financial Times*, July 8, 2015). One could argue that the issue of political uncertainty also played an important role in Italy. In response to the crisis, the Italian parliament formed a technocratic government, with Mario Monti as prime minister, to implement a package of structural reforms. This government had no direct connection to any major political party, however, and as such suffered from the lack of political support for some of its difficult measures. As a result, the government was less successful than expected in passing the reforms. The presence of political uncertainty in Italy was widely commented on in the press ("Time slips by for Monti's reform", *Financial Times*, May 29, 2012, and "Monti admits to losing support," *Financial Times*, June 7, 2012).

Proposition **??** tells us that when a policy change is unexpected the change in the default threshold is equal to the direct effect the policy has on the government's incentives to default. Since agents expect no policy adjustment, their strategies are unchanged, implying that the multiplier effect and the part of the direct effect that operates through households' and lenders' choices are absent. Since in the limit the change in the default threshold is always driven by the change in agents' expectations (see Proposition 2), in that case an unexpected policy change becomes completely ineffective.

The above proposition applied to the government policies considered above implies that, as long as $\varepsilon, \sigma_x > 0$ an unexpected increase in tax rate always leads to decrease in the probability of default. This is because the negative effect of higher taxes on households' investment choices is now absent (no investment distortion). On the other hand, a fiscal stimulus, if unexpected, leads to an expansion of output in period 1 only; households, not expecting an increase in their income, keep their investment unchanged. As a consequence, a fiscal stimulus is now more likely to lead a debt crisis than it was before.

5.2 Uncertainty about Reforms

To capture the political uncertainty surrounding economic reforms, I assume that agents expect the government to implement a given reform with probability $p \in (0,1)$. Thus, in contrast to the previous analysis, the policy change is implemented with probability p. Otherwise, the timing is the same as in Section 4.[30]

Let $dA^*/d\psi\,(p)$ denote the total change in the default threshold when the agents expect the policy to be implemented with probability p and the government does implement the announced policy. It can be shown that in this case we have

$$\frac{dA^*}{d\psi}(p) = p\frac{dA^*}{d\psi}(1) + (1-p)\frac{\partial A^*}{\partial \psi} \tag{5}$$

Thus, a change in the default threshold is a weighted average of the change in the default threshold when there is no uncertainty $(dA^*/d\psi\,(1))$ and when the policy change is unexpected $(\partial A^*/\partial\psi)$. Intuitively, when agents expect that the policy will be implemented with probability p, their response to the prospect of the policy adjustment is proportionately less than in the case of no political uncertainty. This results in an adjustment of the default threshold equal to $p\frac{dA^*}{d\psi}(1)$. On the other hand, with probability $1-p$ households and lenders do not expect the adjustment, in which case if the policy adjustment happens it is driven by the direct change in the government's default

[30] This timing aims to capture the uncertainty faced by firms and creditors, who often have to make investment decisions and decide whether to keep lending to the government or not, without knowing the final scope and shape of economic reforms.

incentive (and hence the adjustment in A^* is equal to the change in the default threshold when the policy adjustment is unexpected).

Proposition 1 establishes the effect of the political uncertainty on the effectiveness of the government's policy measures.

Proposition 6 *1.* $\frac{dA^*}{d\psi}(p) \to p\frac{dA^*}{d\psi}(1)$ *as* $\varepsilon, \sigma_x \to 0$. *Therefore, for sufficiently high* ε *and* σ_x *we have*

$$sgn\left(\frac{dA^*}{d\psi}(p)\right) = sgn\left(\frac{dA^*}{d\psi}(1)\right).$$

2. For any $\varepsilon, \sigma_x > 0$, *the level of uncertainty that minimizes* $\frac{dA^*}{d\psi}(p)$ *(i.e., maximizes the effectiveness of a policy* ψ*), denoted by* p^*, *is either* 0 *or* 1*:*

(a) $p^* = 0$ *if* $\frac{dA^*}{d\psi}(1) > \frac{\partial A^*}{\partial \psi}$.

(b) $p^* = 1$ *if* $\frac{dA^*}{d\psi}(1) < \frac{\partial A^*}{\partial \psi}$.

Proposition 1 follows immediately from Equation (5) and has two implications. First, to the extent that economic agents are well-informed about economic fundamentals, the political uncertainty will not change the desirability of the government's policies (i.e., whether they increase or decrease the probability of default), but only its quantitative impact (Part 1). Thus, if households and lenders have access to precise information, uncertainty about reform implementation makes policies that decrease probability of default less effective, but it also renders the policies that increase the probability of default less harmful. However, if agents' information about the fundamentals is not very precise, then political uncertainty can have a positive or a negative effect on the final outcome (Part 2).[31]

6 The Effect of the Interest Rate on Policy Adjustments[32]

When the policy announcement takes place before the interest rate is set, then the equilibrium interest rate will respond to the announced policy adjustment. In this case the change in the default threshold, $dA^*/d\psi$, is equal to a weighted sum of two terms. The first term is the effect familiar from the analysis in Section 4.1, that is the change in the default threshold holding the equilibrium interest rate constant. The second effect is a new one that captures the change in the default threshold implied by the adjustment in the equilibrium interest rate r^* when we allow the adjustments in households' and

[31]Note that in the above analysis, p should not be interpreted as the choice of policymakers but rather as a result of political bargaining within the government. Once p becomes a strategic choice of the government, households and lenders will take this into account when making their decisions, and hence the above analysis will not apply.

[32]For more details, see Section E of the Appendix.

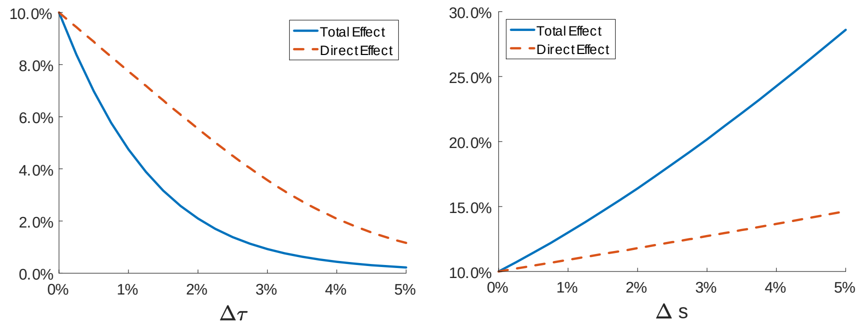
lenders' actions and beliefs to affect A^* only indirectly, through the impact they have on r^*

How does the adjustment in r^* alter the effectiveness of government policies considered in Section 4.2 and the importance of the expectation-driven multiplier effect? While it is difficult to answer this question analytically, intuition suggests that an adjustment in r^* tend to decrease the magnitude of the change in A^* implied by a change in ψ as long as the default threshold A^* is lower than the prior of the mean belief about A, A_{-1} (so that ex-ante probability of default is less than half). To understand this, note that a decrease in A^* decreases the benefit of a choosing a high r (since a lower A^* means that a further decrease in A^* driven by an adjustment in r translates into a smaller decrease in the probability of default) and increases the cost of a setting a high r (since a fall in A^* implies that the government has to incur the cost of a higher r for a larger set of productivity values). The opposite is true when A^* increases. As a result, the implied change in A^* is lower than if the policy change were announced after the interest rate is set. Regarding the second question, Proposition B establishes that the main result of Section 4.1 still holds: When households and lenders have very precise information, then the change in the default threshold is driven mainly by the multiplier effect (which now includes the change in r^* in response to the adjustment in households' and lenders beliefs').

Proposition 7 *Let M_{Total} denote the total multiplier effect, and D_{Total} denote the total direct effect in the model when the policy is announced before the interest r^* is set. If either $\varepsilon \to 0$ or $\sigma_x \to 0$ (or both) then $M_{Total} \to \infty$, $D_{Total} \to 0$ and $\lim_{\varepsilon,\sigma_x \to 0} (M_{Total} \times D_{Total}) \in \mathbb{R}$.*

7 Numerical Examples

In this section I investigate numerically the effects of adjustments in taxes and of a fiscal stimulus on the ex-ante probability of default when the parameters of the model are chosen so that the model resembles the GIIPS economies (i.e., Greece, Ireland, Italy, Portugal, and Spain) at the onset of the European debt crisis in 2008. I also report the contribution of the multiplier effect to the implied changes in the likelihood of default. While the above analysis indicates that the multiplier effect plays an important role in the model when policy adjustments are small (as approximated by the relevant derivatives), it is of interest to see whether these results generalize to large and discrete policy changes. Moreover, the above analysis was also conducted under the Assumption 4, which eliminated the competition effect. I dispense here with this assumption and consider a case where there is the competition in lending among the lenders.[33]



Panel A: Tax Increase   Panel B: Fiscal Stimulus

Figure 5: The effect of policy adjustment on the probability of default

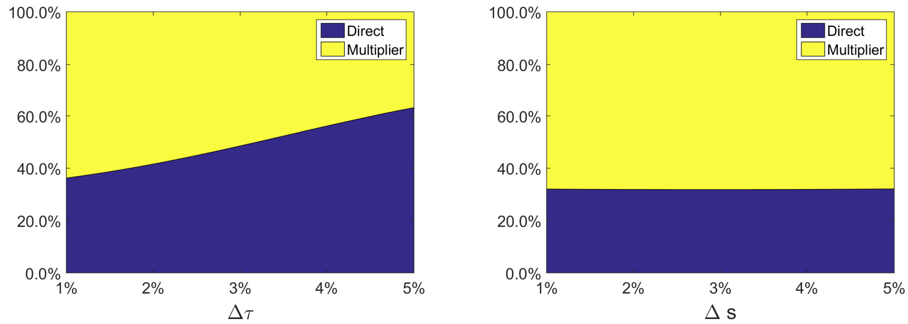
Figure 5 depicts a change in the probability of default implied by an increase in the tax rate (left panel) and by a fiscal stimulus financed with short-term debt (left panel).[34] We see that an increase in the tax rate tends to decrease the probability of default, as the positive effect of an increase in government tax revenues more than compensates for the negative effect of higher taxes on investment. The opposite is true for the fiscal stimulus (regardless of whether it is financed with short-term or long-term debt) as the cost of a stimulus (which is incurred only in repayment) is larger than its benefits.

Finally, one may wonder how important the multiplier effect is in the above calibration. From Figure 6 we see that, in accordance with Proposition 2, for small policy adjustments the multiplier effect plays an important role, explaining more than 60% of the overall change in the default threshold.[35] However, as the tax adjustment be-

[33] From the perspective of the policy analysis, the most important parameters are τ, the tax rate; Z, the output costs of default; k_1, the initial the capital stock; and α, the capital share of output, since these parameters determine directly the costs and benefits of both policies considered above. I set $\tau = 0.4$, the average ratio of governments' tax revenue to GDP in the Eurozone in 2011 as reported by Eurostat, and $Z = 0.92$, implying that in the case of a debt crisis, output declines by 8% (the observed output decline in Greece after it defaulted in 2010). I choose $k_1 = 0.66$ to match the average growth of net capital stock of 2% in the GIIPS economies in the run-up to the crisis (period 2004-2008), and $\alpha = 0.4$ (see Arpaia et al. (2009)). The information parameters are $\sigma_x = 1/20$, $\varepsilon = \sqrt{3}\sigma_x$, and $\sigma = 1/12$. Mean of prior, A_{-1}, is set to imply a 10% probability of default. The initial debt is $B_1 = 0.5$, and the total wealth of the lenders is four times the maturing debt, implying the ratio $b/B_1 = 4$, which is twice the average bid-to-cover ratio in the debt auctions in Germany and Italy as reported in Beetsma et al. (2013). The results are robust to alternative choices of parameters.

[34] The results for a fiscal stimulus financed with long-term debt are very similar.

[35] I consider the change in the default threshold rather than a change in the ex-ante probability of default, because the same absolute change in A^* results in a smaller contribution of the multiplier effect when the probability of default decreases than when the probability of default increases. This is because in the former case the multiplier effect pushes A^* away from the mean of the ex-ante distribution of productivity while in the latter case it moves A^* towards the mean.



Panel A: Tax Increase Panel B: Fiscal Stimulus

Figure 6: Relative Importance of the Direct and Multiplier Effect

comes larger, the relative importance of the multiplier effect decreases somewhat. This is because for a sufficiently large change in taxes, the initial default threshold lies in the region where, at the new tax rate, the government always repays the debt. As such, part of the adjustment in the default threshold A^* takes place in the "repayment region," where expectations play no role and where only the direct effect operates. As a tax rate increase being considered becomes larger, the larger part of the overall adjustment in A^* takes place in the repayment region increasing the overall contribution of the direct effect. This change in the relative importance of direct and multiplier effect is also present in the case of a fiscal stimulus if the fiscal stimulus is large enough.

8 Conclusions

In this paper, I develop a global game model of self-fulfilling sovereign debt crises and use it to understand how a policy adjustment affects the likelihood of a crisis. I show that any policy change has two effects on the probability of default: the direct effect that captures the initial impact of the policy adjustment on the government default incentives holding households' and lenders' expectations regarding default constant, and the multiplier effect that captures the change in the government's default decision implied by the adjustment in households' and lenders' expectations. I find that the direct effect determines whether the policy will lead to a decrease or an increase in the probability of default, while the multiplier effect is key for determining the strength of the equilibrium adjustment. Thus, I show that adjustment in households' and lenders' expectations act as an amplification mechanism always magnifying the initial response of the economy. I also consider how the above conclusions are altered if the households and lenders are uncertain as to whether the government will adjust its policies.

A word of caution is needed regarding the interpretation of the results. In this paper, I analyze a situation in which the government finds itself at a point where a debt crisis is possible. Indeed, the main question this paper addresses is how to avoid a debt crisis when one is likely in the very near future. For that purpose, the fact that the model presented herein is two-period is a minor issue. However, the fact that the model is not dynamic becomes key when trying to answer questions regarding medium-term policies. A question of particular importance is what the government should do to avoid facing another debt crisis in the future once a debt crisis has been averted in the present. This remains an important question for future research.

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Appendix
(For Online Publication)

This appendix contains the proofs of the results that have been stated in the paper and is divided into six sections. In Section A I solve the main model. This section contains the proofs of Lemma 1 and Lemma 2, the main uniqueness result (Proposition 1), and the proof of Lemma 3. Section B contains derivations of the direct and multiplier effects and the proofs of Propositions 2, 3, and 4 from the paper. Section C contains derivations of the total change in the default threshold when the agents expect the policy to be implemented with probability p, i.e., $dA^*/d\psi(p)$. In Section D I briefly discuss how the results would change if Assumption 4 was not imposed. Section E contains a discussion of the effect of an adjustment in the interest rate on the effects of policy changes while Section F contains several technical claims invoked in proofs throughout the Appendix.[36]

A Global Game model

A.1 Uniqueness Result

Proposition A *There exist $\overline{\varepsilon} > 0$ and $\overline{\sigma}_x > 0$ such that for all $\varepsilon \in (0, \overline{\varepsilon}]$ and all $\sigma_x \in (0, \overline{\sigma}_x]$ the model has a unique equilibrium in monotone strategies.*

To prove the above result, I first characterize the optimal households' and lenders' strategies in response to a monotone default strategy by the government. Then I show that in response to these households' and lenders' strategies the government indeed finds it optimal to follow a monotone default strategy. Finally, I show that there exists a unique fixed-point of this argument. Before proceeding any further I introduce notation that will be useful when analyzing the model.

Notation 4 *I will use the following notation throughout the Appendix:*

1. *A^* denotes the default threshold used by the government.*

2. *A^{**} denotes the default threshold expected by the households and lenders.*

[36] The solution to the complete information version of the model, and detailed derivations of the multiplier and direct effects when agents are uncertain whether announced policies will be implemented, can be found in the "Additional Results" document available on the author's website (http://economics.ubc.ca/faculty-and-staff/michal-szkup/).

A.1.1 Households

Suppose that households expect the government to repay its debt if and only if $A \geq A^{**}$. Household i's optimal investment then solves the household's problem specified in Section 3.4. Each household receives a productivity shock A_i, where $A_i = A + \varepsilon_i$ and $\varepsilon_i \in [-\varepsilon, \varepsilon]$.

If $A_i > A^{**} + \varepsilon$, then household i expects no default; in that case,

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \frac{\alpha}{1+\alpha}.$$ [37]

If household i receives productivity $A_i < A^{**} - \varepsilon$, then household i believes that the government will always default and

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \frac{\alpha Z}{1+\alpha}.$$

Finally, in the case when $A_i \in (A^{**} - \varepsilon, A^{**} + \varepsilon)$ the household is uncertain as to whether the government will default. In that case,

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \Lambda(A_i; \varepsilon, A^{**})$$

where

$$\begin{aligned}\Lambda(A_i; \varepsilon, A^{**}) \;=\; & \frac{\alpha(1+Z) + P(A^{**}|A_i) + Z(1 - P(A^{**}|A_i))}{2(1+\alpha)} \\ & - \frac{\sqrt{[\alpha(1+Z) + P(A^{**}|A_i) + Z(1-P(A^{**}|A_i))]^2 - 4\alpha Z(1+\alpha)}}{2(1+\alpha)}\end{aligned}$$

It is straightforward to show that $\Lambda(A_i; \varepsilon, A^{**})$ is increasing in A_i and decreasing in A^{**}. This establishes Lemma 1 in the paper.

Next, I perform a change of variables $\kappa = \frac{\varepsilon_i}{\varepsilon}$, where $\varepsilon_i \in [-\varepsilon, \varepsilon]$ so that $\kappa \in [-1, 1]$. This change of variables turns out to be useful for computing the output in the limiting case as $\varepsilon \to 0$, and in general, when analyzing the effect of changes in ε. Define

$$\Lambda(A + \kappa\varepsilon; \kappa, A^{**}) \equiv \begin{cases} \frac{\alpha}{(1+\alpha)} & \text{when } A_i = A + \kappa\varepsilon > A^{**} + \varepsilon \\ \Lambda(A_i; \varepsilon, A^{**}) & \text{when } A_i = A + \kappa\varepsilon \in (A^{**} - \varepsilon, A^{**} + \varepsilon) \\ \frac{\alpha Z}{(1+\alpha)} & \text{when } A_i = A + \kappa\varepsilon < A^{**} - \varepsilon \end{cases}$$

In what follows I will denote the optimal choice of capital as $k_2^*(A, \kappa, A^{**})$ to emphasize its dependence on A, κ and household's belief about the default threshold A^{**}.

[37] It is here that the assumption of full depreciation of households' capital simplifies the model. When the capital depreciates fully each period, the optimal choice of capital is linear. As we will see below, this will make the government's default condition near linear in e^A.

A.1.2 Lenders[38]

Denote by $p_x = 1/\sigma_x^2$ and $p_A = 1/\sigma_A^2$ the precisions of the lenders' private signals and the prior, respectively. As usual, it is more convenient to work with precisions rather than standard deviations or variances.

Let $u(1, A; x^{**}, A^{**})$ be the expected payoff to lender j from lending to the government when the average productivity is equal to A, the government uses a threshold strategy with cutoff A^{**}, and the other lenders use monotone strategies with cutoff x^{**}. Similarly, denote by $u(0, A; x^{**}, A^{**})$ the payoff to lender j from investing in the risk-free asset. Then

$$\begin{aligned} u(1, A; x^{**}, A^{**}) &= \begin{cases} 1 + r \min\left\{\frac{B_2^{R,u}(A)}{S(A;x^{**})}, 1\right\} & \text{if } A \geq A^{**} \\ 0 & \text{otherwise} \end{cases} \\ u(0, A; x^{**}, A^{**}) &= 1 \end{aligned}$$

Define $\Delta u(A; x^{**}, A^{**}) \equiv u(1, A; x^{**}, A^{**}) - u(0, A; x^{**}, A^{**})$.

It is immediate to see that for any pair (A^{**}, x^{**}), and regardless of the government's desired borrowing function $B_2^{R,u}$, the function $\Delta u(A; x^{**}, A^{**})$ satisfies a weak single crossing property in A.[39] Moreover, it is well-known that a family of normal density functions parameterized by x_j

$$\left\{(p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}}\right)\right\}_{x_j \in \mathbb{R}}$$

satisfies the strict monotone likelihood ratio (MLR) property, implying that $(p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}}\right)$ is strictly log-supermodular in (A, x_j) (see Athey, 1996). By Theorem 3.2 in Athey (1996),

$$\Delta U(x_j; x^*, A^{**}) \equiv \int_{A^{**}}^{\infty} \Delta u(A; x^{**}, A^{**}) (p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}}\right) dA$$

satisfies the strict single-crossing property in A^{**}. Thus, in response to monotone strategies by the government and the other lenders, lender j finds it optimal to follow a monotone strategy.

[38] In this section I make use of two results established in Athey (1996). The first of the results, Theorem 3.2 in Athey (1996), establishes that if g satisfies the weak single-crossing property, and if k is strictly log-supermodular and $k(s, \theta)$ has constant support in θ, then $G(\theta) \equiv \int_S g(s) k(s; \theta) ds$ satisfies the strict single-crossing property in θ. Theorem 3.4 in Athey (1996) extends this conclusion to the case where g also depends on θ under the additional assumption of piecewise continuity of g .

[39] A function $f(x)$, where $f : \mathbb{R} \to \mathbb{R}$, satisfies a weak single-crossing property in x if for all $x_H > x_L$, $f(x_L) > 0$ implies $f(x_H) \geq 0$.

Consider $\Delta U(x^*; x^*, A^{**})$, the expected utility difference from supplying the funds to the market versus not supplying them, evaluated at x^* and let $L(A^{**}, x^*) \equiv \Delta U(x^*; x^*, A^{**})$. I want to show that for each A^{**} there exists unique x^* such that $L(A^{**}, x^*) = 0$. First note that $\Delta u(A; x^*, A^{**})$ as defined above is increasing in x^*. This is because $S(A; x^*) = b\left(1 - \Phi\left(\frac{x^*-A}{p_x^{-1/2}}\right)\right)$ is decreasing in x^*. Moreover, for all $A \geq A^{**}$ $B_2^{R,u}(A)$ is differentiable in A and therefore $\Delta u(A; x^*, A^{**})$ is piecewise continuous. Second, note that $\Delta u(A; x^*, A^{**})(p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{(p_x+p_A)^{-1/2}}\right) \neq 0$, at least for all $A < A^{**}$. Then, by Theorem 3.4 in Athey (1996) it follows that $L(A^{**}, x^*)$ satisfies a strict single-crossing condition in x^*. This proves Lemma 2 in the text.

A.1.3 The Government's Monotone Default Strategy

Suppose that the households follow investment strategies as characterized above and the lenders use monotone strategies with a common threshold x^*. I show that $\Delta V(A, \mathbf{k}_2^*, S)$ is strictly increasing in A.

Define $\mathbf{k}_2^*(A, A^{**}) \equiv \{k_2(A, \kappa, A^{**})\}_{\kappa \in [-1,1]}$, that is, $\mathbf{k}_2^*(A)$ denotes the households' investment choices when the average productivity is equal to A and when all households expect that the default threshold is A^{**}. Note that if the lenders follow monotone strategies, then $S = b\left[1 - \Phi\left(\frac{x^*-A}{\sigma_x}\right)\right]$. Thus, with a slight abuse of notation I will write $\Delta V(A, \mathbf{k}_2^*(A, A^{**}), S)$ as $\Delta V(A; \mathbf{k}_2^*(A, A^{**}), x^*)$. Finally, let $B_2^{R,u}$ denote the government optimal unconstrained borrowing (see also Section 3.2 in the paper).

Using the definition of $\Delta V(A, \mathbf{k}_2^*(A, A^{**}), x^*)$, substituting for $\mathbf{k}_2^*(A)$ the expression found in Section $A.1.1$ and rearranging, we get

$$
\begin{aligned}
\Delta V(A, \mathbf{k}_2^*(A, A^{**}), x^*) &= \int_{-1}^{1} \frac{1}{2} \log\left(\frac{1 - \Lambda(A + \kappa\varepsilon, \kappa, A^{**})}{Z - \Lambda(A + \kappa\varepsilon, \kappa, A^{**})}\right) d\kappa + \log\left(\frac{\tau Y_1^R - B_1 + B_2^{R^*}}{\tau Z Y_1^R + (1-\xi) B_2^{D^*}}\right) \\
&\quad + \log\left(\frac{1}{Z}\right) + \log\left(\frac{\tau Y_2^R - (1+r) B_2^{R^*}}{Z \tau Y_2^R}\right),
\end{aligned}
$$

where

$$
B_2^{R^*} = \begin{cases} B_2^{R,u}(A) & \text{if } B_2^{R,u} \leq S(A, x^*) \\ S(A, x^*) & \text{if } B_2^{R,u} > S(A, x^*) \end{cases}.
$$

Differentiating with respect to A, simplifying, and taking the limit as $\xi \to 1$, we get

$$
\frac{\partial \Delta V(A; \mathbf{k}_2^*(A, A^{**}), x^*, A^*)}{\partial A} \geq \frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{(1+\alpha) B_2^{R^*}(1+r)}{\tau Y_2^R - (1+r) B_2^{R^*}}. \tag{6}
$$

where I used the observation that if $B_2^{R^*} = B_2^{R,u}(A)$, then by the optimality of the government borrowing choices the terms containing $\partial B_2^{R^*}/\partial A$ add up to 0, while otherwise their sum is strictly positive.

Add the above fractionson the right-hand side of 6. The resulting numerator can be written as

$$2(1+r)\left(B_2^{R^*}\right)^2 - B_2^{R^*}\left(\tau Y_2^R + 2(1+r)B_1 - (1+r)\tau Y_1^R\right) + B_1\tau Y_2^R.$$

This expression is quadratic in $B_2^{R^*}$. Let $B_2^{R^*,1}(A)$ and $B_2^{R^*,2}(A)$ be its two roots. Whether these roots are real or not depends on the parameters of the model. For all $A \in [\underline{A}, \overline{A}]$, define $\bar{b}(A) = \min\left\{B_2^{R^*,1}(A), B_2^{R^*,2}(A)\right\}$ if the roots are real, and $\bar{b}(A) = \infty$ if they are complex. Let $\bar{b} = \min_{A\in[\underline{A},\overline{A}]} \bar{b}(A)$. It follows that if $b < \bar{b}$ then the government's best response to monotone strategies is itself monotone. I assume that the lenders' wealth b satisfies this constraint (Assumption 3 in the paper).[40]

A.1.4 Uniqueness of Equilibrium

In light of the above results, to establish uniqueness it is enough to show that

$$\Delta V\left(A^*, \mathbf{k}_2^*(A^*, A^*), x^*(A^*)\right)$$

is monotone in A^*, where $\mathbf{k}_2^*(A^*) \equiv \{k_2(A^*, \kappa, A^*)\}_{\kappa\in[-1,1]}$ is a vector whose components are the individual households' investment strategies when the households have the correct expectations about the default threshold (i.e., $A^{**} = A^*$), and x^* is the common signal threshold used by the lenders when households and lenders expect the default threshold to be A^*. I denote the optimal lender's threshold by $x^*(A^*)$, to emphasize that it depends on A^*.

Fix $\eta > 0$, where η is a small positive number. Differentiating $\Delta V(A^*; \mathbf{k}_2^*(A^*), x^*(A^*))$ with respect to A^* and taking the limit as $\xi \to 1$ we get

$$\begin{aligned}\frac{d\Delta V}{dA^*} &= \int_{-1}^{1} \frac{-\frac{\partial\Lambda}{\partial A^*}[Z-\Lambda] + [1-\Lambda]Z\frac{\partial\Lambda}{\partial A^*}}{[1-\Lambda][Z-\Lambda]}d\kappa \\ &\quad + \frac{\frac{dB_2^{R^*}}{dA^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} - \frac{(1+r)\frac{\partial B_2^{R^*}}{\partial A^*}}{\tau Y_2^R - (1+r)B_2^{R^*}} \\ &\quad + \frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{(1+\Psi)(1+r)B_2^{R^*}}{\tau Y_2^R - (1+r)B_2^{R^*}},\end{aligned}$$

where

$$\Psi \equiv \frac{\int_{-1}^{1}\frac{1}{2}\frac{\partial}{\partial A^*}f\left(k_2\left(A^*+\kappa\varepsilon;\varepsilon,A^*\right)\right)d\kappa}{Y_2^R} \to \alpha \text{ as } \varepsilon \to 0.$$

Since

$$\lim_{\varepsilon\to 0}\frac{\partial \Pr\left(A^*|A^*+\kappa\varepsilon\right)}{\partial A^*} \to 0,$$

[40] One may wonder how restrictive this assumption is. The answer is that it depends on the parameters. However, numerical simulations suggest that unless α or Z is very close to 1 both roots are complex, which means that the bound can be made arbitrarily large (though it has to be finite). In particular, this is the case for the calibration used in the paper.

there exists $\overline{\varepsilon}$ such that for all $\varepsilon \in (0, \overline{\varepsilon}]$ we have

$$\int_{-1}^{1} \frac{-\frac{\partial \Lambda}{\partial A^*}[Z-\Lambda] + [1-\Lambda] Z \frac{\partial \Lambda}{\partial A^*}}{[1-\Lambda][Z-\Lambda]} d\kappa < \frac{\eta}{2}$$

Next, since $\frac{\partial S(A^*)}{\partial A^*} > -b \frac{p_A}{p_x^{1/2}} \frac{1}{\sqrt{2\pi}} \to 0$ as $p_x \to \infty$, it follows that there exists a large enough $\overline{p}_x$ such that for all $p_x > \overline{p}_x$ we have

$$\frac{\frac{dB_2^{R^*}}{dA^*}}{\left[\tau Y_1^R - B_1 + B_2^{R^*}\right]} - \frac{(1+r)\frac{\partial B_2^{R^*}}{\partial A^*}}{\left[\tau Y_2^R - (1+r) B_2^{R^*}\right]} > -\frac{\eta}{2}.$$ [41]

Finally, following the same argument as in Section $A.1.3$ one can show that there exists $\overline{\overline{b}}(\varepsilon)$ such that for all $b < \overline{\overline{b}}(\varepsilon)$ we have

$$\frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{(1+r) B_2^{R^*}}{\left[\tau Y_2^R - (1+r) B_2^{R^*}\right]} > \eta.$$

Therefore, for all ε with $0 < \varepsilon < \overline{\varepsilon}$ and all $p_x > \overline{p}_x$ we have

$$\frac{d\Delta V}{dA^*} > -\frac{\eta}{2} - \frac{\eta}{2} + \eta = 0$$

implying that there exists a unique default threshold A^* that satisfies all the equilibrium conditions.

The above analysis applies to a fixed value of A^*. However, since $A^* \in \left[\underline{A}, \overline{A}\right]$, which is a compact interval, there exists bounds $\overline{\varepsilon}$ and $\overline{p}_x$ which are independent of A^*, such that if $\varepsilon < \overline{\varepsilon}$ and $p_x < \overline{p}_x$, then $d\Delta V/dA^*$ is strictly positive for all $A^* \in \left[\underline{A}, \overline{A}\right]$. This completes the proof.

A.2 Optimal Interest Rate

Proof of Lemma 3. Let $W(r)$ denote the expected payoff to the government from choosing interest rate r. Consider first $A_{-1} > A^*(0)$, and fix $r > 0$.[42] I show that for sufficiently low σ_A there exists r' with $0 < r' < r$ such that $W(r') > W(r)$. To see this, consider any $r' \in (0, r)$ such that $A^*(r') > A^*(0)$.[43] If $A^*(r') \leq A^*(r)$, then the result follows immediately (for any $\sigma_A > 0$). Thus, assume that $A^*(r') > A^*(r)$. Then

$$\begin{aligned} W(r) - W(r') = & \int_{A^*(r')}^{\infty} \left[V_1^R(A, \mathbf{k}_2, S; r) - V_1^R(A, \mathbf{k}_2, S; r')\right] f(A|A_{-1})\, dA \\ & + \int_{A^*(r)}^{A^*(r')} \left[V_1^R(A, \mathbf{k}_2, S; r) - V_1^D(A, \mathbf{k}_2, S)\right] f(A|A_{-1})\, dA. \end{aligned}$$

[41] If $\partial B_2^{R^*}/\partial A^* = \partial B_2^{R,u}/\partial A^*$, then the sum of these terms is 0.

[42] Note that $A^*(0) = \max_{r \in [0,\overline{r}]} A^*(r)$.

[43] Such r' exists since $W(r)$ is increasing in r in the neighborhood of r_F.

Note that $V_1^R(A,\mathbf{k}_2,S;r)-V_1^R(A,\mathbf{k}_2,S;r')<0$ for all $A>A^*(r')$. Let C be a positive constant. Since $A_{-1}>A^*(0)\geq A^*(r')$, we have

$$\begin{aligned}
& \int_{A^*(r')}^{\infty}\left[V_1^R(A,\mathbf{k}_2,S;r)-V_1^R\left(A,\mathbf{k}_2,S;r'\right)\right]f(A|A_{-1})\,dA \\
< & \int_{A^*(r')}^{A_{-1}+C}\left[V_1^R(A,\mathbf{k}_2,S;r)-V_1^R\left(A,\mathbf{k}_2,S;r'\right)\right]f(A|A_{-1})\,dA \\
< & \max_{A\in[A^*(r),A_{-1}+C]}\left[V_1^R(A,\mathbf{k}_2,S;r)-V_1^R\left(A,\mathbf{k}_2,S;r'\right)\right]\left[F(A_{-1}+C|A_{-1})-F\left(A^*\left(r'\right)|A_{-1}\right)\right] \\
\to & \max_{A\in[A^*(r),A_{-1}+C]}\left[V_1^R(A,\mathbf{k}_2,S;r)-V_1^R\left(A,\mathbf{k}_2,S;r'\right)\right]<0 \quad \text{as } \sigma_A\to 0.
\end{aligned}$$

On the other hand, for all $A\in(A^*(r),A^*(r'))$, the government defaults if the interest rate is r' and repays its debt when the interest rate is r. Moreover, we know that $V_1^R(A,\mathbf{k}_2,S;r)-V^D(A)>0$ for all $A\in(A^*(r),A^*(r'))$. Therefore,

$$\begin{aligned}
& \int_{A^*(r)}^{A^*(r')}\left[V_1^R(A,\mathbf{k}_2,S;r)-V^D(A,\mathbf{k}_2,S)\right]f(A|A_{-1})\,dA \\
< & \left[\max_{A\in[A^*(r'),A^*(r)]}\left\{V_1^R(A,\mathbf{k}_2,S;r)-V^D(A)\right\}\right]\left[F\left(A^*\left(r'\right)|A_{-1}\right)-F(A^*(r)|A_{-1})\right] \\
\to & \ 0 \ \text{ as } \ \sigma_A\to 0.
\end{aligned}$$

Thus, $\lim_{\sigma_A\to 0}[W(r)-W(r')]<0$. Since r was arbitrary, the same argument holds for any $r>0$; thus, for all $A_{-1}>A^*(0)$ we must have $r^*(\sigma_A)\to 0$ as $\sigma_A\to 0$.

Using an analogous argument, one can establish that for each $A_{-1}\in\left[\min_{r\in[0,\bar{r}]}A^*(r),A^*(0)\right]$ the optimal interest rate is equal to r^* where $r^*=\min\{r:A^*(r)=A_{-1}\}$. ■

B Policy Analysis

Let ψ denote a parameter of the model (for concreteness, one can think of the tax rate, in which case $\psi=\tau$). Then, for given r^*, the equilibrium conditions can be written as

$$I\left(A^*+\kappa\varepsilon,A^{**},k_2^*(\kappa),\psi\right)=0,$$

which is the equilibrium condition for a households with productivity $A^*+\kappa\varepsilon$ and which determines the capital choice for a household with productivity shock $\kappa\varepsilon$;

$$L\left(A^{**},x^*,\psi\right)=0,$$

which is the equilibrium condition that describing the lenders' behavior and which determines x^*; and finally,

$$\Delta V\left(A^*,\{k_2^*(\kappa)\}_{\kappa\in[-1,1]},x^*,\psi\right)=0$$

which is the equilibrium condition that describes the government's default decision and determines A^*.[44]

Note that, for each $\kappa \in [-1,1]$, the equation $I(A^* + \kappa\varepsilon, A^*, k_2^*(\kappa), \psi) = 0$ specifies $k_2^*(\kappa)$ as a function of household's productivity $A^* + \kappa\varepsilon$, household's belief about the default threshold A^{**}, and the policy parameter ψ. for each $\kappa \in [-1,1]$. Similarly, the equation $L(A^*, x^*, \psi) = 0$ determines x^* as a function of the lenders' belief about the default threshold A^{**} and ψ. Without loss of generality, I assume that the households hold the same belief as the lenders in regard to the default threshold. In equilibrium, $A^{**} = A^*$, that is the households and lenders hold correct beliefs about the government's default decision. However, to derive the effect of a change in the households' and lenders' beliefs on the default threshold, we have to differentiate between the belief about the threshold held by the households and lenders and the actual default threshold, where the latter is defined as the level of productivity at which the government defaults.

B.1 The Effect of a Change in ψ on A^*

To compute the equilibrium change in A^* due to a change in ψ, I compute the total derivatives of the expressions on the both sides of equilibrium conditions and solve the resulting linear system of equations for $dA^*/d\psi$:

$$I_1(\kappa)\frac{dA^*}{d\psi} + I_2(\kappa)\frac{dA^{**}}{d\psi} + I_3(\kappa)\frac{dk_2^*(\kappa)}{d\psi} + I_4(\kappa) = 0 \tag{7}$$

$$L_1\frac{dA^{**}}{d\psi} + L_2\frac{dx^*}{d\psi} + L_3 = 0 \tag{8}$$

$$\Delta V_1\frac{dA^*}{d\psi} + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{dk_2^*(\kappa)}{d\psi}d\kappa + \Delta V_3\frac{dx^*}{d\psi} + \Delta V_4 = 0 \tag{9}$$

where I_n is the partial derivative of $I(A^* + \kappa\varepsilon, A^{**}, k_2^*(\kappa), \psi)$ with respect to its nth argument and similarly for L_n and ΔV_n. $dA^{**}/d\psi$ is the total change in agents' beliefs regarding the government default threshold implied by a change in ψ. In equilibrium, $dA^{**}/d\psi = dA^*/d\psi$, but for now it is important to keep the distinction between the two objects.

Solving for $dx^*/d\psi$ and $dk_2^*/d\psi$ using Equations (8) and (7) we get

$$\frac{dx^*}{d\psi} = -\frac{L_1}{L_2}\frac{dA^{**}}{d\psi} - \frac{L_3}{L_2}$$

$$\frac{dk_2^*(\kappa)}{d\psi} = -\frac{I_1(\kappa)}{I_3(\kappa)}\frac{dA^*}{d\psi} - \frac{I_2(\kappa)}{I_3(\kappa)}\frac{dA^{**}}{d\psi} - \frac{I_4(\kappa)}{I_3(\kappa)}$$

[44] Note that this condition implicitly assumes that the government's borrowing and spending decisions are optimal. In other words, $\Delta V = 0$ determines the productivity default threshold, given that the government behaves optimally in the case when it repays its debt as well as in the case when it chooses to default.

or, recognizing that $\partial x^*/\partial A^{**} = -L_1/L_2$, $\partial k_2^*(\kappa)/\partial A^* = -I_1(\kappa)/I_3(\kappa)$, $\partial k_2^*(\kappa)/\partial A^{**} = -I_2(\kappa)/I_3(\kappa)$, and $\partial k_2^*(\kappa)/\partial\psi = -I_4(\kappa)/I_3(\kappa)$:

$$\begin{aligned}\frac{dx^*}{d\psi} &= \frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial x^*}{\partial\psi} \\ \frac{dk_2^*(\kappa)}{d\psi} &= \frac{\partial k_2^*(\kappa)}{\partial A^*}\frac{dA^*}{d\psi} + \frac{\partial k_2^*(\kappa)}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial k_2^*(\kappa)}{\partial\psi}\end{aligned}$$

Substituting the above expressions into Equation (9) and rearranging, we get

$$\left[\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa\right]\frac{dA^*}{d\psi} = \tag{10}$$
$$-\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left[\frac{\partial k_2^*(\kappa)}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial k_2^*(\kappa)}{\partial\psi}\right]d\kappa - \Delta V_3\left[\frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial x^*}{\partial\psi}\right] - \Delta V_4,$$

where $\left[\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa\right]$ captures the effect of an increase in the productivity on the government's incentives to default.

At this point it is key to differentiate between a change in the households' investments due to a change in the households' strategies and a change in the households' investments due to merely a change in productivity holding households' strategies fixed. Recall that an individual household's investment strategy is a function that maps the individual productivity into an investment choice, that is it is a map $k_2^* : A_i \to \mathbb{R}$. Thus, a change in the household's strategy is defined as a shift in this mapping, that is a change in k_2^* for each A_i. On the other hand, holding household strategies constant, a change in A_i also affects household i's investments: It is simply a movement along the curve $k_2 : A_i \to \mathbb{R}$. Thus, the term $\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa$ captures the effect of a change in the productivity on the government's incentives to default holding households' and lenders' strategies constant.

Using the above observation, divide Equation (10) by $\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa$ to obtain

$$\begin{aligned}\frac{dA^*}{d\psi} = &\frac{-\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial\psi}d\kappa}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa} - \frac{\Delta V_3\frac{\partial x^*}{\partial\psi}}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}} - \frac{\Delta V_4}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}} \\ &- \frac{\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^{**}}d\kappa}{\Delta V_1 - \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}}\frac{dA^{**}}{d\psi} - \frac{\Delta V_3\frac{\partial x^*}{\partial A^{**}}}{\Delta V_1 - \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}}\frac{dA^{**}}{d\psi}\end{aligned}$$

The first three terms capture the direct effects of a change in ψ on the equilibrium strategies of the households', the lenders' and the government, respectively, holding households' and lenders' beliefs about the default threshold constant (i.e., holding A^{**} constant). The two remaining terms capture the effect of a change in ψ has on the the households' and lenders' beliefs. In particular, note that

$$\frac{\partial A^*}{\partial \psi} = -\frac{\Delta V_4}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa},$$

that is, the third term captures the partial effect of a change in ψ on the government's default incentives holding households' and lenders' strategies and beliefs constant. Similarly,

$$\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} = \frac{\Delta V_3}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*}}\frac{\partial x^*}{\partial A^{**}}$$

and, slightly abusing notation,

$$\int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa = -\int\limits_{-1}^{1} \frac{\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}},$$

where this term captures the effect of a change in the households' beliefs on the government's incentives to default. In a similar fashion,

$$\int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa = -\int\limits_{-1}^{1} \frac{\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa},$$

where $\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa$ captures the effect of a change in the households' strategies caused by a change in ψ holding the households' beliefs about the default threshold, A^{**}, constant.

Using the above notation, we obtain

$$\frac{dA^*}{d\psi} = \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \frac{\partial A^*}{\partial \psi} + \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa \frac{\partial A^{**}}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\frac{\partial A^{**}}{\partial \psi}$$

In equilibrium, $A^{**} = A^*$, and so it has to be the case that $\partial A^{**}/\partial \psi = dA^*/d\psi$. Thus, after rearranging,

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*} - \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa} \tag{11}$$

Finally, note that $\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa$ corresponds simply to $\int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial \psi} di$, while the term $\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa$ corresponds to $\int_0^1 \frac{1}{2}\frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial A^{**}} di$. Thus, we obtain

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial \psi} di}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*} - \int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial A^{**}} di},$$

which corresponds to Equation (2) in the paper.

B.2 Proofs of Propositions 2, 3 and 4

I first establish a preliminary result that will be useful in the proof of Proposition 2.

Lemma B.1 *Consider* $\left.\frac{\partial P(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}} / \left.\frac{\partial S}{\partial A^{**}}\right|_{A^{**}=A^{*}}$.

1. *If σ_x is fixed and $\varepsilon \to 0$, then*

$$\lim_{\varepsilon\to 0} \frac{\left.\frac{\partial P(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}}}{\left.\frac{\partial S}{\partial A^{**}}\right|_{A^{**}=A^{*}}} = +\infty$$

2. *If ε is fixed and $\sigma_x \to 0$, then*

$$\lim_{\sigma_x\to 0} \frac{\left.\frac{\partial P(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}}}{\left.\frac{\partial S}{\partial A^{**}}\right|_{A^{**}=A^{*}}} = 0$$

3. *If $\varepsilon \to 0$ and $\sigma_x = c\varepsilon^{\theta}$ for some $c, \theta \in \mathbb{R}_{++}$, then*

$$\lim_{\substack{\varepsilon\to 0\\ \sigma_x = c\varepsilon^{\theta}}} \frac{\left.\frac{\partial P(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}}}{\left.\frac{\partial S}{\partial A^{**}}\right|_{A^{**}=A^{*}}} = \begin{cases} 0 & \text{if } \theta \in (0,1) \\ \frac{-c}{\phi\left(\Phi^{-1}\left(\frac{1}{1+r}\right)\right)} & \text{if } \theta = 1 \\ -\infty & \text{if } \theta > 1 \end{cases}.$$

Proof. First, note that under Assumption 4 we have $\frac{B_2^{R,u}(A)}{S(A;x^{**})} = 1$, and hence it can be shown that

$$x^{*} = \frac{p_x + p_A}{p_x} A^{**} - \frac{p_A}{p_x} A_{-1} + \frac{\sqrt{p_x + p_A}}{p_x} \Phi^{-1}\left(\frac{1}{1+r}\right),$$

where $p_x = \frac{1}{\sigma_x^2}$ and $p_A = \frac{1}{\sigma_A^2}$. Therefore,

$$\left.\frac{\partial S}{\partial A^{**}}\right|_{A^{**}=A^{*}} = -b p_x^{1/2} \phi\left(\frac{x^{*} - A^{*}}{p_x^{-1/2}}\right) \frac{p_x + p_A}{p_x}.$$

Moreover,

$$\frac{\partial P\left(A^{**}|A^{*}+\kappa\varepsilon\right)}{\partial A^{**}} = \frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon - A_{-1}}{\sigma_A}\right) - \left(\frac{A-(1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)}$$

Thus,

$$\frac{\left.\frac{\partial P(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}}}{\left.\frac{\partial S}{\partial A^{**}}\right|_{A^{**}=A^{*}}} = \frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon - A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)}}{-b p_x^{1/2}\phi\left(\frac{x^{*}-A^{*}}{p_x^{-1/2}}\right)\frac{p_x+p_A}{p_x}}.$$

Taking the limit as $\varepsilon \to 0$, we see that the above expression tends to ∞. On the other hand, if ε is fixed and we take the limit as $\sigma_x \to 0$ (i.e., $p_x \to \infty$), then the above expression tends to 0 $\left(\text{since at } A^{**} = A^* \text{ we have } p_x^{1/2}(x^* - A^*) \to \Phi^{-1}\left(\frac{1}{1+r}\right)\right)$.

Finally, consider the case when $\varepsilon \to 0$, $\sigma_x \to 0$, and $\sigma_x = c\varepsilon^\theta$. Then

$$\lim_{\substack{\varepsilon,\sigma_x \to 0 \\ \sigma_x = c\varepsilon^\theta}} \frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+\varepsilon-A_{-1}}{\sigma_A}\right)-\left(\frac{A^*-\varepsilon-A_{-1}}{\sigma_A}\right)}}{-p_x^{1/2}\phi\left(\frac{x^*-A^*}{p_x^{-1/2}}\right)\frac{p_x+p_A}{p_x}} = \lim_{\varepsilon \to 0} \frac{c\varepsilon^\theta \frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{-\left[\Phi\left(\frac{A^*+\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-\varepsilon-A_{-1}}{\sigma_A}\right)\right]\phi\left(\frac{x^*-A^*}{p_x^{-1/2}}\right)\left[1+\frac{c^2\varepsilon^{2\theta}}{\sigma_A^2}\right]},$$

where I used the substitution $p_x = \frac{1}{c^2\varepsilon^{2\theta}}$ to eliminate p_x. As $\varepsilon \to 0$, both the numerator and denominator tend to 0. Thus, to compute the limit we can apply l'Hôpital's Rule. Differentiating the numerator with respect to εwe get

$$\theta c\varepsilon^{\theta-1}\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)$$

From this it follow that as $\varepsilon \to 0$ the numerator tends to ∞ if $0 < \theta < 1$, to $c\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)$ if $\theta = 1$ and to 0 if $\theta > 1$.

The derivative of the denominator with respect to ε is given by

$$\left[\frac{(1+\kappa)}{2\sigma_A}\phi\left(\frac{A^*+\varepsilon-A_{-1}}{\sigma_A}\right)+\frac{(1-\kappa)}{2\sigma_A}\phi\left(\frac{A^*-\varepsilon-A_{-1}}{\sigma_A}\right)\right]\phi\left(\frac{x^*-A^*}{p_x^{-1/2}}\right)\left[1+\frac{c^2\varepsilon^{2\theta}}{\sigma_A^2}\right]$$
$$+\left[\Phi\left(\frac{A^*+\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-\varepsilon-A_{-1}}{\sigma_A}\right)\right]\phi\left(\frac{x^*-A^*}{p_x^{-1/2}}\right)\frac{2\theta c^2\varepsilon^{2\theta-1}}{\sigma_A^2}$$

For any $\theta > 0$ and any $c > 0$, the above expression tends to

$$\frac{1}{2\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)\phi\left(\Phi^{-1}\left(\frac{1}{1+r}\right)\right) > 0,$$

which proves the result. ■

Proof of Proposition 2. **(1)** Recall from the proof of uniqueness that the government default condition, after taking into account the dual role of A^* as the average value of productivity in the economy and the default threshold, is strictly increasing in A^*. Thus,

$$\Delta V_1+\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa+\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left.\frac{\partial k_2^*(\kappa)}{\partial A^{**}}\right|_{A^{**}=A^*}d\kappa+\Delta V_3\left.\frac{\partial x^*}{\partial A^{**}}\right|_{A^{**}=A^*} > 0,$$

where the third and fourth terms capture the effect of a change in the households' and lenders' beliefs, respectively. Dividing the above expression by $\Delta V_1+\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa$ establishes the non-negativity of the multiplier effect.

Under Assumption 4 we have $\frac{B_2^{R,u}(A)}{S(A;x^{**})} = 1$ for all A, and hence it can be shown that $x^* = \frac{p_x+p_A}{p_x}A^{**} - \frac{p_A}{p_x}A_{-1} + \frac{\sqrt{p_x+p_A}}{p_x}\Phi^{-1}\left(\frac{1}{1+r}\right)$, implying that $\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} > 0$. Similarly, it is straightforward to show that $\partial k_2^*/\partial A^{**} < 0$. Since a higher investment by all households decreases the government's incentives to default ($\int_{-1}^{1} \frac{1}{2} \partial A^* / \partial k_2^* (\kappa)\, d\kappa < 0$), we have $\int_{-1}^{1} \frac{1}{2} \frac{\partial A^*}{\partial k_2^*(\kappa)} \frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa > 0$. It follows that the denominator of the multiplier effect is less than 1, so that the multiplier effect is greater than 1.

(2) I show first that $D \to 0$ as $\varepsilon \to 0$. Note that $\lim_{\varepsilon \to 0} \left. \frac{\partial \Pr(A^{**}|A^* + \kappa\varepsilon)}{\partial A^*} \right|_{A^{**}=A^*} = \infty$, and thus, $\lim_{\varepsilon \to 0} \left. \frac{\partial k_2^*(A^* + \kappa\varepsilon, \kappa, A^{**})}{\partial A^*} \right|_{A^{**}=A^*} = \infty$.[45] From this it follows that $\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa \to \infty$, that is the effect of higher productivity on government's default incentive tends to infinity. On the other hand, for any parameter of the model ψ both $\frac{\partial k_2}{\partial \psi}$ and $\frac{\partial S}{\partial \psi}$ exist, and thus are finite. Moreover, the only dependence of these derivatives on ε and σ_x is though $P(A^*|A_i)$ and $S(A, x^*)$. Since both $P(A^*|A_i)$ and $S(A, x^*)$ converge to a constant as the households' and lenders' information becomes infinitely precise, $\frac{\partial k_2}{\partial \psi}$ and $\frac{\partial S}{\partial \psi}$ are finite in the limit as $\varepsilon, \sigma_x \to 0$, implying that $\frac{\partial A^*}{\partial k_2^*} \frac{\partial k_2^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*} \frac{\partial x^*}{\partial \psi} \to 0$. Similarly, $\Delta V_4 = \partial \Delta V / \partial \psi$ is well-defined for each parameter of the model. By the same argument, we know that $\lim_{\varepsilon, \sigma_A \to 0} \Delta V_4$ is finite and, thus $\frac{\partial A^*}{\partial \psi} \to 0$. It follows that $D \to 0$ as $\varepsilon \to 0$.

Next, consider the case when $\sigma_x \to 0$. Note that ΔV_1 includes the effect of an increase in S holding x^* constant. In particular, at the default threshold we have $S = b\left(1 - \Phi\left(\frac{x^* - A^*}{\sigma_x}\right)\right)$. Now holding x^* constant, we have that $\partial S / \partial A^* = b\phi\left(\frac{x^* - A^*}{\sigma_x}\right)\frac{1}{\sigma_x}$. Then $\lim_{\sigma_x \to 0} \frac{\partial}{\partial A^*} \left. S(A^*, x^*) \right|_{A^{**}=A^*} = \infty$, so that $\lim_{\sigma_x \to 0} \left[\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa\right] \to \infty$. Thus, by the same argument as in the case of $\varepsilon \to 0$, we conclude $D \to 0$ as $\sigma_x \to 0$. Finally, the case when $\varepsilon, \sigma_x \to 0$ follows from the above observations.

Next, I show that the multiplier effect tends to infinity as $\varepsilon \to 0$ or $\sigma_x \to 0$ or both. Consider first the case when $\varepsilon \to 0$ and $\sigma_x > 0$ is fixed. Note that in this case (as argued above) $\lim_{\varepsilon \to 0} \frac{\partial A^*}{\partial x^*} \frac{\partial x^*}{\partial A^{**}} = 0$, since $\lim_{\varepsilon \to 0} \left[\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 \left. \frac{\partial k_2^*(\kappa)}{\partial A} \right|_{A=A^*} d\kappa\right] = \infty$. Next, consider the remaining term in the definition of the multiplier effect:

$$\frac{\partial A^*}{\partial k_2^*} \frac{\partial k_2^*}{\partial A^{**}} = - \frac{\int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \left. \frac{\partial k_2^*(A^* + \kappa\varepsilon, \kappa, A^{**})}{\partial A^{**}} \right|_{A^{**}=A^*} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(A^* + \kappa\varepsilon, \kappa, A^{**})}{\partial A^*} d\widehat{\kappa}}$$

Recall that

$$\frac{\partial k_2^*(A^* + \kappa\varepsilon, \kappa, A^{**})}{\partial A^{**}} = (1 - \tau) e^{A^* + \kappa\varepsilon} f(k_1) \frac{\partial P(A^{**}|A^* + \kappa\varepsilon)}{\partial A^{**}} C_\Lambda (A^* + \kappa\varepsilon, \kappa, A^{**}),$$

where

$$C_\Lambda (A^* + \kappa\varepsilon, \kappa, A^{**}) \equiv (1 - Z)\left(1 - \frac{\alpha(1 + Z) + P(A^{**}|A^* + \kappa\varepsilon)(1 - Z) + Z}{\sqrt{[\alpha(1 + Z) + P(A^{**}|A^* + \kappa\varepsilon)(1 - Z) + Z]^2 - 4\alpha Z(1 + Z)}}\right) < 0.$$

[45] For the proof of this statement and other statements regarding the limiting behavior of $P(A^{**}|A^* + \kappa\varepsilon)$, see Section F of this appendix.

Define

$$T_1(\kappa) \equiv \frac{1}{2} \frac{(1-Z) \frac{C_\Lambda(A^*+\kappa\varepsilon,\kappa,A^{**})}{\Lambda(A^*+\kappa\varepsilon,\kappa,A^{**})} k_2^*(A^*+\kappa\varepsilon,\kappa,A^*)}{(Z-\Lambda(A^*+\kappa\varepsilon,\kappa,A^{**}))(1-\Lambda(A^*+\kappa\varepsilon,\kappa,A^{**}))} < 0$$

$$T_2(\kappa) \equiv \frac{1}{2} \frac{\alpha(1+r)S}{Y_2(\tau Y_2-(1+r)S)} e^{A^*+\kappa\varepsilon} \frac{C_\Lambda(A^*+\kappa\varepsilon,\kappa,A^{**}) f(k_2^*(\kappa))}{\Lambda(A^*+\kappa\varepsilon,\kappa,A^{**})} < 0,$$

where

$$\frac{1}{2}\Delta V_2(\kappa) = [T_1(\kappa)+T_2(\kappa)] \left.\frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^{**}}\right|_{A^{**}=A^*} \frac{1}{\left.\frac{\partial P(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}},$$

so that we can concisely write

$$\int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa) \left.\frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^{**}}\right|_{A^{**}=A^*} d\kappa = \int_{-1}^{1} [T_1(\kappa)+T_2(\kappa)] \left.\frac{\partial P(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*} d\kappa$$

Recall that

$$\int_{-1}^{1} \frac{1}{2} \frac{\partial A^*}{\partial k_2(\kappa)} \frac{\partial k_2(\kappa)}{\partial A^{**}} d\kappa = \frac{\int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa) \left.\frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^{**}}\right|_{A^{**}=A^*} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa}$$

Now

$$\int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa = \int_{-1}^{1} [T_1(\kappa)+T_2(\kappa)] \left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^*}\right| d\kappa$$

and

$$\Delta V_1 = \frac{B_1-S}{\tau Y_1-B_1+S} + \frac{S(1+r)}{\tau Y_2-(1+r)S} + \frac{\partial S}{\partial A^*}\left[\frac{1}{\tau Y_1-B_1+S} + \frac{(1+r)}{\tau Y_2-(1+r)S}\right] \equiv T_3 > 0,$$

so that ΔV_1 captures the effect of an increase in A^* on the government's incentive to default through an increase in output (the first two terms) and through an increase in the supply of funds. Thus,

$$\frac{\partial A^*}{\partial k_2} \frac{\partial k_2}{\partial A^{**}} = -\frac{\int_{-1}^{1} [T_1(\kappa)+T_2(\kappa)] \left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*} d\kappa}{\int_{-1}^{1} [T_1(\widehat{\kappa})+T_2(\widehat{\kappa})] \left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^*}\right|_{A^{**}=A^*} d\widehat{\kappa} + T_3}$$

Moreover,

$$\lim_{\varepsilon\to 0} \frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}} = \infty \text{ and } \lim_{\varepsilon\to 0} \frac{\left.\frac{\partial \Pr(A^*|A+\widehat{\kappa}\varepsilon)}{\partial A}\right|_{A=A^*}}{\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^*}\right|_{A^{**}=A^*}} = -1$$

where both limits are computed in the proof of Claim 7 in Section F of this appendix. Given the above limits, it follows that

$$\begin{aligned}
\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}} &= -\frac{\int_{-1}^{1}\left[T_1(\kappa)+T_2(\kappa)\right]\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}d\kappa}{\int_{-1}^{1}\left[T_1(\widehat{\kappa})+T_2(\widehat{\kappa})\right]\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^{*}}\right|_{A^{**}=A^*}d\widehat{\kappa}+T_3} \\
&= -\int_{-1}^{1}\frac{\left[T_1(\kappa)+T_2(\kappa)\right]d\kappa}{\int_{-1}^{1}\left[T_1(\widehat{\kappa})+T_2(\widehat{\kappa})\right]\frac{\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^{*}}\right|_{A^{**}=A^*}}{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}}+\frac{T_3}{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}}d\widehat{\kappa}} \\
&\to -\int_{-1}^{1}\frac{\left[T_1(\kappa)+T_2(\kappa)\right]d\kappa}{\int_{-1}^{1}(-1)\left[T_1(\widehat{\kappa})+T_2(\widehat{\kappa})\right]d\widehat{\kappa}}=1
\end{aligned}$$

Therefore, $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}} \to 1$, implying that $M \to \infty$. The remaining cases follow by a similar argument.

Finally, to see that $\lim_{\varepsilon,\sigma_x} M \times D$ is finite, recall that

$$M \times D = \frac{\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\frac{\partial A^*}{\partial \psi}}{1-\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^*}-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*}}$$

To see that the above product is finite multiply both the numerator and denominator by $\Delta V_1+\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left.\frac{\partial k_2^*(\kappa)}{\partial A}\right|_{A=A^*}d\kappa$ and then take the limit of the resulting expression.

(3) Consider the limiting behavior of

$$\frac{\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^*}}{\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*}}=\frac{\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left.\frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^{**}}\right|_{A^{**}=A^*}d\kappa}{\Delta V_3\frac{\partial x^*}{\partial A^*}}$$

Recall that

$$\Delta V_3\frac{\partial x^*}{\partial A^*}=(-1)\left[\frac{1}{\tau Y_1-B_1+S}-\frac{(1+r)}{\tau Y_2-(1+r)S}\right]bp_x^{1/2}\phi\left(\frac{x^*-A^*}{p_x^{-1/2}}\right)\frac{p_x+p_A}{p_x}$$

and (from the proof of part (2) of this proposition) that

$$\left.\frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^{**}}\right|_{A^{**}=A^*}=\left[T_1(\kappa)+T_2(\kappa)\right]\left.\frac{\partial P(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}$$

I consider first the case when σ_x is fixed and $\varepsilon \to 0$. Then

$$\begin{aligned}
\frac{\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^*}}{\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*}} &= \lim_{\varepsilon\to 0}\frac{\int_{\kappa=-1}^{1}\left[T_1(\kappa)+T_2(\kappa)\right]\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}d\kappa}{(-1)\left[\frac{1}{\tau Y_1-B_1+S}-\frac{(1+r)}{\tau Y_2-(1+r)S}\right]bp_x^{1/2}\phi\left(\frac{x^*-A^*}{p_x^{-1/2}}\right)\frac{p_x+p_A}{p_x}} \\
&= \infty
\end{aligned}$$

since $T_1(\kappa)$ and $T_2(\kappa)$ converge to negative constants, $\lim_{\varepsilon\to 0}\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}=\infty$ (see Lemma B.1), and the denominator is always finite and negative.[46] An anal-

[46] That $\frac{1}{\tau Y_1-B_1+S}-\frac{(1+r)}{\tau Y_2-(1+r)S}>0$ follows from the fact that in equilibrium the government is unable to borrow the unconstrained optimal amount.

ogous argument establishes that when ε is fixed and $\sigma_x \to 0$ (i.e., $p_x \to \infty$), then $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^*} \Big/ \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*}$ tends to 0.

Finally, consider the case where both $\varepsilon \to 0$ and $\sigma_x \to 0$ and $\sigma_x = c\varepsilon^\theta$. Then,

$$\frac{\frac{\partial A^*}{\partial k_2^*}\frac{\partial k_2^*}{\partial A^{**}}}{\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}} = \lim_{\substack{\varepsilon,\sigma_x\to 0\\ \sigma_x = c\varepsilon^\theta}} \frac{\int_{\kappa=-1}^{1}\left[T_1(\kappa)+T_2(\kappa)\right] \left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*} d\kappa}{(-1)\left[\frac{1}{\tau Y_1 - B_1 + S} - \frac{(1+r)}{\tau Y_2-(1+r)S}\right] b p_x^{1/2}\phi\left(\frac{x^*(A^*)-A^*}{p_x^{-1/2}}\right)\frac{p_x+p_A}{p_x}}$$

The above limit is determined by the limiting behavior of

$$\frac{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}}{p_x^{1/2}\phi\left(\frac{x^*(A^*)-A^*}{p_x^{-1/2}}\right)\frac{p_x+p_A}{p_x}}$$

The result then follows from Lemma B.1.[47] ■

B.3 Policies

The default threshold is determined by the following condition:

$$\begin{aligned} 0 \;=\; \Delta V\left(A^*, \mathbf{k}_2^*, x^*\right) = & \int_{-1}^{1} \log\left(c_1^R\right) d\kappa + \log\left(\tau Y_1^R - B_1 + B_2^{R^*}\right) \\ & + \int_{-1}^{1} \log\left(c_2^R\right) d\kappa + \log\left(\tau Y_2^R - (1+r)\, B_2^{R^*}\right) \\ & - \int_{-1}^{1} \log\left(c_1^D\right) d\kappa - \log\left(\tau Z Y_1^R\right) \\ & - \int_{-1}^{1} \log\left(c_2^D\right) d\kappa - \log\left(\tau Z Y_2^R\right), \end{aligned} \tag{12}$$

where c_t^R and c_t^D are the consumption in period t in repayment and default, respectively, Y_t^R is the total output of the economy in period t, and $B_2^{R^*}$ is the equilibrium borrowing by the government, all evaluated at the threshold productivity A^* Before proceeding further, note that $\int \log\left(\frac{c_t^R}{c_t^D}\right)$ is independent of τ and k_1 for $t = 1, 2$, and thus policy change will affect the government's incentive to default only through its effect on government spending in repayment and in default[48]. Equation (12) plays a key role in establishing Propositions 3 and 4.[49]

B.3.1 Proof of Proposition 3

Differentiate $\Delta V\left(A^*, \mathbf{k}_2^*, x^*\right)$ with respect to τ to obtain

$$u_{g_1}^R Y_1^R + u_{g_2}^R Y_2^R + u_{g_2}^R \tau \frac{\partial Y_2^R}{\partial \tau} - u_{g_1}^D Z Y_1^R + u_{g_2}^D Y_2^R + u_{g_2}^D Z\tau \frac{\partial Y_2^R}{\partial \tau},$$

[47] That we can pass the limit through the integral sign follows from the observation that the integrand is bounded for $\theta \geq 1$. For the case of $\theta < 1$ the results follows by Fatou's lemma.

[48] This is because $c_2^D = Zc_2^R$, $c_1^D = Z(1-\tau)e^{A_i} f(k_1) - k_2$, $c_1^R = (1-\tau)e^{A_i} f(k_1) - k_2$, and k_2 is linear in $f(k_1)$ and τ.

[49] Equations (3) and (4) can be computed directly from Equation 12.

where $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utility from government spending in period t in repayment and default, respectively, and Y_1^R the total output of the economy in period t in repayment, all evaluated at A^*.[50] Given households' investment choices, $\partial Y_2^R / \partial \tau = \frac{\alpha}{1-\tau}$. Thus, rearranging the terms in the above expression, we obtain

$$\underbrace{Y_1^R (1-Z) u_{g_1^*}^D + Y_2^R (1-Z) u_{g_1^*}^D}_{\text{Differential increase in tax revenues}} + \underbrace{Y_1^R \left[u_{g_1^*}^R - u_{g_1^*}^D\right] + Y_2^R \left[u_{g_2^*}^R - u_{g_2^*}^D\right]}_{\text{Concavity effect}} - \underbrace{\frac{\partial Y_2^R}{\partial \tau} \tau \left[u_{g_2^*}^R - Z u_{g_2^*}^D\right]}_{\text{Investment distortion}}$$

which corresponds to the expression (3) in the paper.

Consider the concavity effect. Note that

$$\begin{aligned} Y_1^R \left[u_{g_1^*}^R - u_{g_1^*}^D\right] &= Y_1^R \left[\frac{1}{\tau Y_1^R - B_1 + B_2^{R^*}} - \frac{1}{\tau Z Y_1^R}\right] = Y_1^R \left[\frac{(Z-1)\tau Y_1^R + B_1 - B_2^{R^*}}{\tau Z Y_1^R \left(\tau Y_1^R - B_1 + B_2^{R^*}\right)}\right] \\ &= Y_1^R \left[\frac{(Z-1)\left(\tau Y_1^R - B_1 + B_2^{R^*}\right) + Z\left(B_1 - B_2^{R^*}\right)}{\tau Z Y_1^R \left(\tau Y_1^R - B_1 + B_2^{R^*}\right)}\right] \\ &= Y_1^R (Z-1) u_{g_1^*}^D + \frac{\left(B_1 - B_2^{R^*}\right)}{\tau\left(\tau Y_1^R - B_1 + B_2^{R^*}\right)} \\ &= Y_1^R (Z-1) u_{g_1^*}^D + \frac{1}{\tau}\left(B_1 - B_2^{R^*}\right) u_{g_1^*}^R \end{aligned}$$

Similarly, following analogous steps, one can show that

$$Y_2^R \left[u_{g_2^*}^R - u_{g_2^*}^D\right] = Y_2^R (Z-1) u_{g_2^*}^D + \frac{1}{\tau}(1+r) B_2^{R^*} u_{g_2^*}^R$$

Thus, the concavity effect can be written as

$$\frac{1}{\tau}\left(B_1 - B_2^{R^*}\right) u_{g_1^*}^R + \frac{1}{\tau}(1+r) B_2^{R^*} u_{g_2^*}^R - Y_1^R (1-Z) u_{g_1^*}^D - (1-Z) Y_2^R u_{g_2^*}^D$$

It then follows that the sum of the concavity effect and differential increase in tax revenues is simply

$$\frac{1}{\tau}\left(B_1 - B_2^{R^*}\right) u_{g_1^*}^R + \frac{1}{\tau}(1+r) B_2^{R^*} u_{g_2^*}^R$$

Finally, consider the investment distortion effect. Note that

$$\begin{aligned} \frac{\partial Y_2^R}{\partial \tau} \tau \left[u_{g_2^*}^R - Z u_{g_2^*}^D\right] &= \frac{\alpha \tau Y_2^R}{1-\tau} \left[\frac{1}{\left(\tau Y_2^R - (1+r) B_2^{R^*}\right)} - \frac{Z}{\tau Z Y_2^R}\right] \\ &= \frac{\alpha \tau Y_2^R}{1-\tau} \left[\frac{\tau Y_2^R - \tau Y_2^R + (1+r) B_2^{R^*}}{\tau Y_2^R \left(\tau Y_2^R - (1+r) B_2^{R^*}\right)}\right] \\ &= \frac{\alpha \tau}{1-\tau} \frac{1}{\tau} (1+r) B_2^{R^*} u_{g_2^*}^R \end{aligned}$$

[50] There is no effect of a change of τ on $B_2^{R^*}$, the equilibrium level of borrowing, since under Assumption 4, $B_2^{R^*} = S(A, x^*)$ and $\partial S(A, x^*)/\partial \tau = 0$. If Assumption 4 were relaxed there would be an additional term capturing the potential impact of a change in taxes on government borrowing in equilibrium (via the competition effect among lenders).

Proposition 3 then follows from summing the terms that capture the concavity effect, differential increase in tax revenues and the investment distortion (ς_τ defined in Proposition 3 is proportional to their sum).

B.3.2 Proof of Proposition 4

The proof of Proposition 4 is similar to the proof of Proposition 3. I consider only a stimulus financed with short-term debt. The case of a stimulus financed with long-term debt is analogous.

Note first that when the government engages in a fiscal stimulus financed with short-term debt that matures at the end of period 1, government spending in repayment in period 1 becomes $\tau Y_1^R - B_1 + B_2^{R^*} - \left(1 + r^{ST}\right) sk_1$, where sk_1 is the size of stimulus. The positive effect of such a stimulus is that it leads to expansion of output. Differentiating both sides of the government indifference condition with respect to s, we get

$$\tau \frac{\partial Y_1}{\partial s} u_{g_1}^R - \left(1 + r^{ST}\right) k_1 u_{g_1}^R + \tau \frac{\partial Y_2}{\partial s} u_{g_2}^R - \tau Z \frac{\partial Y_1}{\partial s} u_{g_1}^D - \tau Z \frac{\partial Y_2}{\partial s} u_{g_2}^D$$

Rearranging, we get

$$\underbrace{\frac{\partial Y_1}{\partial s} \tau (1 - Z) u_{g_1}^D + \frac{\partial Y_2}{\partial s} \tau (1 - Z) u_{g_2}^D}_{\text{Differential increase in tax revenues}} + \underbrace{\tau \frac{\partial Y_1}{\partial s} \left[u_{g_1}^R - u_{g_1}^D\right] + \tau \frac{\partial Y_2}{\partial s} \left[u_{g_2}^R - u_{g_2}^D\right]}_{\text{Concavity effect}} - \underbrace{\left(1 + r^{ST}\right) k_1 u_{g_1}^R}_{\text{Increase in debt}}$$

When the government engages in a stimulus, $k_2^i = (1 - \tau) e^{A_i} f\left(k_1 (1 + s)\right) \Lambda\left(A_i; \varepsilon, A^*\right)$, and thus

$$\frac{\partial Y_1}{\partial s} = \frac{\alpha}{1 + s} Y_1^R \quad \text{and} \quad \frac{\partial Y_2}{\partial s} = \frac{\alpha^2}{1 + s} Y_2^R$$

As in the case of a tax increase, the concavity effect of a stimulus can be written as

$$\tau \frac{\partial Y_1}{\partial s} \frac{1}{\tau Y_1} (B_1 - B_2) u_{g_1}^R + \tau \frac{\partial Y_2}{\partial s} \frac{1}{\tau Y_2} (1 + r) B_2 u_{g_2}^R - \frac{\partial Y_1}{\partial s} \tau (1 - Z) u_{g_1}^D - \frac{\partial Y_2}{\partial s} \tau (1 - Z) u_{g_2}^D$$

Thus, the sum of the concavity effect and the differential increase in tax revenues is equal to

$$\frac{\alpha}{1 + s} (B_1 - B_2) u_{g_1}^R + \frac{\alpha^2}{1 + s} (1 + r) B_2 u_{g_2}^R,$$

and thus a further stimulus, from the level s, leads to a decrease in the probability of default if and only if

$$\frac{\alpha}{1 + s} (B_1 - B_2) u_{g_1}^R + \frac{\alpha^2}{1 + s} (1 + r) B_2 u_{g_2}^R - \left(1 + r^{ST}\right) k_1 u_{g_1}^R > 0$$

In particular, if the government is considering engaging in a small stimulus (when the alternative is not to engage in stimulus at all, so that $s = 0$), then the condition becomes

$$\alpha (B_1 - B_2) u_{g_1}^R + \alpha^2 (1 + r) B_2 u_{g_2}^R - \left(1 + r^{ST}\right) k_1 u_{g_1}^R > 0$$

which is the condition reported in Proposition 4.

B.4 Discussion of Assumption 4

The above analysis was conducted under the following assumption:

Assumption 4 *B_1 is large enough so that for all $A > \underline{A}(0)$ the government's desired borrowing in repayment exceeds the supply of funds in the market.*

To determine a bound on B_1, which is assumed implicitly in Assumption 4, assume that interest rate r is less than $\widehat{r}$ for some arbitrarily high $\widehat{r}$. Recall that

$$B_2^{R,u} = \frac{(1+r)\,B_1 + \tau Y_2^R - (1+r)\,\tau Y_1^R}{2\,(1+r)}.$$

For a fixed $r < \widehat{r}$, a higher B_1 increases B_2, not only directly, but also indirectly by shifting the lower bound of the fragility region, $\underline{A}(r)$, upwards. For sufficiently high $\underline{A}(r)$, we have $Y_2^R \underline{A}(r) > Y_1^R \underline{A}(r)$. Moreover, $\partial Y_2^R / \partial A = (1+\alpha)\,Y_2^R$ and $\partial Y_1^R / \partial A = Y_1^R$, implying that once $\underline{A}(r)$ is high enough so that $Y_2^R \underline{A}(r) > Y_1^R \underline{A}(r)$, a further increase in $\underline{A}(r)$ leads to an increase in $\tau Y_2^R - (1+r)\,\tau Y_1^R$, and hence in the desired borrowing. It follows that for a fixed b and a fixed r, there exists a high enough B_1 such that $B_2^{R,u} > b$. Since $[0, \widehat{r}]$ is a compact interval there exists a high enough B_1, call it $\widehat{B}_1$, such that if $B_1 > \widehat{B}_1$ then $B_2^{R,u} > b$ for all $r \in [0, \widehat{r}]$.[51]

Assumption 4 simplifies the lender's problem. The difficult part of the lender's problem is the competition effect: Ceteris paribus, a higher supply of funds in the bond market decreases the lenders' expected return from lending. This effect, however, is not present when $B_2^{R,u} > b$, in which case there exists a closed-form solution for x^*. In particular, under Assumption 4, we have

$$x^* = \frac{p_x + p_A}{p_x} A^{**} - \frac{p_A}{p_x} A_{-1} + \frac{\sqrt{p_x + p_A}}{p_x} \Phi^{-1}\left(\frac{1}{1+r}\right).$$[52]

This in turn substantially simplifies the analysis presented in Sections 4 and 5 of the paper.[53] In Section F of this appendix, I discuss briefly how the result change if Assumption 4 is not imposed in Section F of this Appendix.

C Policy Adjustments under Uncertainty

In this section I derive the change in the default threshold when households and lenders are uncertain as to whether the policy change will be implemented and assign probability p to the government implementing the new policy. As in Section B of this appendix, I am

[51] Assuming that Z is high enough would have the same effect.

[52] Derivations of the threshold x^* when there is no competition effect are standard and can be found, for example, in Szkup and Trevino (2015).

[53] Assuming that lenders ignore the competition effect would have the same implications.

interested in understanding the effect of an announcement of a small policy change on the default threshold. To do so, I start by considering a situation where with probability $(1-p)$ the policy parameter takes value ψ (which I associate with the case when the policy change is not implemented) and with probability p the policy parameter takes value ψ' (which I associate with the new level of the policy parameter if the policy is implemented). I then compute the effect of a further change in ψ' and I impose the condition that initially $\psi' = \psi$. By following these steps, I obtain the effect of an announcement of a change in the policy parameter when such a change will take place with probability p.

Let A^* be the threshold if the policy parameter takes value ψ (i.e., the policy change is not implemented) and $A^{*\prime}$ the policy threshold when the policy parameter takes value ψ' (i.e., the policy change is implemented).[54] Then the equilibrium conditions can be written as

$$
\begin{aligned}
(1-p)\, I\left(A^* + \kappa\varepsilon, A^*, k_2^*(\kappa), \psi\right) + pI\left(A^* + \kappa\varepsilon, A^{*\prime}, k_2^*(\kappa), \psi'\right) &= 0 \qquad (13)\\
(1-p)\, I\left(A^{*\prime} + \kappa\varepsilon, A^*, k_2^{*\prime}(\kappa), \psi\right) + pI\left(A^{*\prime} + \kappa\varepsilon, A^{*\prime}, k_2^{*\prime}(\kappa), \psi'\right) &= 0 \qquad (14)\\
(1-p)\, L\left(A^*, x^*, \psi\right) + pL\left(A^{*\prime}, x^*, \psi'\right) &= 0 \qquad (15)\\
\Delta V\left(A^*; \{k_2^*(\kappa)\}_{\kappa\in[0,1]}, x^*, A^*, \psi\right) &= 0 \qquad (16)\\
\Delta V\left(A^{*\prime}; \{k_2^*(\kappa)\}_{\kappa\in[0,1]}, x^*, A^{*\prime}, \psi'\right) &= 0, \qquad (17)
\end{aligned}
$$

where $k_2^*(\kappa)$ denotes an individual household's equilibrium investment when that household's productivity is equal to $A^* + \kappa\varepsilon$, while $k_2^{*\prime}$ denotes the individual household's equilibrium investment when that household's productivity is equal $A^{*\prime} + \kappa\varepsilon$.

When households and lenders are uncertain whether an announced policy will be implemented there are additional equilibrium equations compared to the case considered in Section B of this appendix. This is because we need to determine the default threshold both when the policy in implemented and when it is not (the possibility of a policy change also affects the threshold even if in the end the policy is not implemented). In particular, to compute the equilibrium default threshold when the policy parameter takes value ψ, we need both the government's default condition and the household investment decisions evaluated both evaluated at ψ (Equations 13 and 16). Similarly, to compute the equilibrium default threshold when the policy parameter takes value ψ', we need both the government's default condition and the household investment conditions evaluated both evaluated at ψ' (Equations 14 and 17).

[54] For example, if the relevant policy parameter is a tax rate τ and the government contemplates increasing the tax rate to $\tau' > \tau$ then $\psi = \tau$ while $\psi' = \tau'$.

To compute the effect of a policy announcement when the policy is expected to be implemented with probability p, one can follow an approach similar to the one in Section B of this appendix, that is consider the total derivatives of both sides of all equilibrium condition with respect to ψ'. Solving the resulting system of equations for $dA^*/d\psi$ and $dA^{*\prime}/d\psi'$ and evaluating all derivatives at $\psi = \psi'$ (since we consider a small policy change from its initial level at ψ) yields the desired result.[55]

C.1 Proofs of Propositions 5 and 6

Proposition 5 follows immediately from the discussion in the paper and part (2) of Proposition 2. Proposition 6 follows directly from Equation (5) in the paper which states that $\frac{dA^*}{d\psi}(p) = p\frac{dA^*}{d\psi}(1) + (1-p)\frac{\partial A^*}{\partial \psi}$, and part (2) of Proposition 2.

D Discussion of Assumptions 1–4

D.1 Assumptions 1–3

To solve the model described in Section 2 of the paper, I imposed Assumptions 1–3 (Section 3.1 in the paper). Assumption 1, which states that $B_1 \geq \underline{B}_1$ is needed to make the problem interesting. It is straightforward to show that the unconstrained optimal borrowing by the government when the interest rate is $r = 0$ is given by

$$B_2^{R,u} = \frac{B_1 + \tau Y_2 - \tau Y_1}{2}$$

If B_1 is low, then the government might have no incentives to borrow in the fragility region (low B_1 means that the fragility region contains low values of productivity A, for which Y_2 tends to be substantially smaller than Y_1). But in this case lenders' expectations stop playing role in the model. By imposing an appropriate lower bound on B_1, I can ensure that the government will always want to borrow in the fragility region.[56]

Assumption 2 consists of two parts. First, it imposes a bound on the total wealth of the lenders. This is needed for two reasons. First of all, an individual lender's wealth has to be bounded, since (given the assumption of risk-neutrality) after receiving a good signal he always supplies all his funds to the market. Thus, if lenders had an infinite amount of funds, the government would always be able to borrow funds from the few agents that receive high signals.[57] Second, a bound on b is needed to ensure

[55] The detailed derivations can be found in the "Additional Results" document available on author's website (http://economics.ubc.ca/faculty-and-staff/michal-szkup/).

[56] The details of deriving a sufficient bound on B_1 can be in "Addtional Results" document on the author's website.

[57] The second reason is to ensure that the government incentives to default decrease as A increases. As shown in Section B of this appendix $\partial \Delta V/\partial A$ depends on B_2, the amount that the government can

that $\Delta V\left(A^{*},\mathbf{k}_{2}^{*},x^{*}\right)$. The details of establishing the bound on b can be found in sections A.1.3 and A.1.4 in this appendix. The role of the assumption that $\xi\rightarrow 1$ is discussed in Footnote 14 in the paper and in Section A.3 in the "Additional Results" document on the author's website. To reiterate here, I need a high ξ in order to ensure that the government's incentives to default satisfies single-crossing as the supply of funds in the market, S, increases. If that is not the case, then no equilibrium in monotone strageies exists. In the limit as $\xi\rightarrow 1$, government's incentive to default is monotonically decreasing in S. Such monotonicity, while probably not necessary, greatly simplifies the analysis.

Finally, Assumption 3 implies that $B_{2}^{R,u}$ is increasing in the fragility region. This simplifies the analysis of the lender's problem (when the stronger Assumption 4 is not imposed), and I use it to establish that x^{*} is increasing in A^{*}. Under Assumption 3, a lender who observes a higher signal not only believes that default is less likely but also that he will be able to lend more to the government. The details of the derivations of the bound on Z can be found in Section A.5 in the "Additional Results" document on the author's website. Numerical simulations suggest that this assumption is not crucial for the model to have a unique equilibrium in monotone strategies.

D.2 Policy Analysis without Assumption 4

Assumption 4 is useful, since it simplifies the lender's problem. However, one can obtain a similar decomposition of $dA^{*}/d\psi$ when Assumption 4 is not imposed.

Without Assumption 4, a change in households' investment strategies will affect the lenders' equilibrium behavior. This is becase the government's desired unconstrained borrowing, $B_{2}^{R,u}$, depends on Y_{2}, and a change in $B_{2}^{R,u}$ translate into a change in x^{*}. Thus, the lenders' indifference condition has to be written as

$$L\left(A^{**},x^{*},\psi,k_{2}\right)=0$$

rather than as $L\left(A^{**},x^{*},\psi\right)=0$. This is the only change compared to the case when Assumption 4 is imposed. Following the same steps, one can show that

$$\frac{dA^{*}}{d\psi}=\frac{\frac{\partial A^{*}}{\partial\psi}+\int_{-1}^{1}\frac{1}{2}\frac{\partial A^{*}}{\partial k_{2}^{*}(\kappa)}\frac{\partial k_{2}^{*}(\kappa)}{\partial\psi}d\kappa+\frac{\partial A^{*}}{\partial x^{*}}\left[\frac{\partial x^{*}}{\partial\psi}+\int_{-1}^{1}\frac{\partial x^{*}}{\partial k_{2}^{*}(\kappa)}\frac{\partial k_{2}^{*}(\kappa)}{\partial\psi}d\kappa\right]}{1-\int_{-1}^{1}\frac{1}{2}\frac{\partial A^{*}}{\partial k_{2}^{*}(\kappa)}\frac{\partial k_{2}^{*}(\kappa)}{\partial A^{**}}d\kappa-\frac{\partial A^{*}}{\partial x^{*}}\left[\frac{\partial x^{*}}{\partial A^{*}}+\int_{-1}^{1}\frac{\partial x^{*}}{\partial k_{2}^{*}(\kappa)}\frac{\partial k_{2}^{*}(\kappa)}{\partial A^{**}}d\kappa\right]}$$

borrow. A bound on b, and hence on B_{2}, ensures that $\partial\Delta V/\partial A>0$ for all A in the fragility region and for all possible choices of B_{2}, that is for all $B_{2}\in[0,b]$. As numerical simulations suggest, unless parameters are extreme (Z is close to 1 or α close to 1) this is not an issue. However, analytically this is hard to show and hence I take care of this issue by imposing appropriate bound on b.

Thus, compared to the case when Assumption 4 holds, there is an additional term in the expression for the direct effect, $\frac{\partial A^*}{\partial x^*}\int\limits_{-1}^{1}\frac{\partial x^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi}d\kappa$. This is because a change in ψ leads to an adjustment in the households' investment which affects the government's desired borrowing. Without Assumption 4 there is "competition effect", higher supply of funds to the bond market tends to mean less lending per lender, hence a change in the households' investment strategies leads to an adjustment in x^*. Similarly, the multiplier effect has an additional term equal to $\frac{\partial A^*}{\partial x^*}\int\limits_{-1}^{1}\frac{\partial x^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}}d\kappa$, since now a change in households' expectations affects the lenders' behavior through its impact on the government's desired borrowing.

There are two main reasons why in the paper I consider a case when Assumption 4 holds. First of all, Assumption 4 substantially simplifies the subsequent analysis. This is particularly true when considering effects of an increase in taxes and of a fiscal stimulus, or when deriving an expression for $dA^*/d\psi$, since $\int_{-1}^{1}\left[\partial x^*/\partial k_2^*(\kappa)\right]\left[\partial k_2^*(\kappa)/\partial\psi\right]$ is a complicated object and can be computed only implicitly. Second, numerical simulations suggests that the competition effect, which is assumed away when Assumption 4 is imposed, plays only a minor role when determining the desirability of a particular policy.

E The Effect of the Interest Rate on Policy Adjustments

Above I analyzed the case when the policy change takes place after the interest rate has been set, and thus the change in the policy and the resulting change in the default threshold A^* do not affect the interest rate r. In this section I analyze what happens when the policy change is announced before the government chooses the interest rate, in which case we have to take into account how a policy change affects the choice of interest rate and how this change in the interest rate affects the default threshold.

Recall that the government chooses the interest rate to maximize the ex-ante welfare. The optimal interest rate is then the solution to the first-order condition associated with this problem, which can be written as

$$R\left(A^*, k_2, x^*, \psi, r^*\right) = 0$$

Here, we recognize that r^* depends on the government's future decisions, households' investment choices, and lenders' supply decisions. The choice of r^* is also affected by the policy parameters, since ψ affects the gains and costs associated with a higher r.

Following the same approach as in Section $B.1$ of this appendix I find that the total effect of a change in policy ψ on the default threshold is given by

$$\frac{dA^*}{d\psi} = \frac{1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}{M_{Total}}\left[\frac{\frac{\partial A^*}{\partial \psi}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^1\frac{\partial A^*}{\partial k_2}\frac{\partial k_2^i}{\partial \psi}di}{1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right]$$
$$+\frac{1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}{M_{Total}}\left[\frac{\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi}+\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^1\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)}{1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right],$$

where M_{Total} is the (total) multiplier effect that is present in the model when r can adjust; is given by

$$M_{Total} = \frac{1}{1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di-\frac{\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)}{1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial r}}\left(\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}+\int_{i=0}^1\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right)}.$$ [58]

To understand the above expression, note first that $\left[1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the multiplier effect in the case when we hold the interest constant, and $\left[1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^1\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the multiplier effect in the case when the government's default decision is affected by the change in households beliefs through only an implied adjustment in the interest rate. Then the first term in the expression for $dA^*/d\psi$ captures the change in the default threshold implied by a change in the policy holding the interest rate constant (the expression in the square brackets) weighted by the relative importance of the "partial" multiplier effect (i.e., multiplier effect when r is kept constant as in Section 4 of the paper) compared to the total multiplier effect, M_{Total}. This effect is familiar from the earlier analysis. The second term captures the total change in the default threshold implied by the adjustment in r^*. Here, $\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi}+\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^1\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)$ captures the effect that an adjustment in ψ has on r^* (and hence on A^*) holding households' and lenders' expectations constant: A change in ψ leads to a change in r^*, which then affects A^*. This effect is then reinforced by the associated multiplier effect that results from the initial adjustment in r^* and is adjusted by the relative importance of its partial multiplier effect.

How does an adjustment in r^* alter the effectiveness of various government policies compared to the case when r^* is constant? While it is difficult to answer this question analytically, intuition suggests that an adjustment in r^* tends to decrease the magnitude of the change in A^* implied by ψ as long as the default threshold A^* is lower than the prior of the mean belief about A, A_{-1}.[59] To understand this, note that a decrease in A^* decreases the benefit of a higher r (since a lower A^* means that a further decrease in A^* driven by an adjustment in r translates into a lower decrease in the probability of default) and increases the cost of a higher r (since a fall in A^* implies that the

[58]The above expression can be derived by following the same steps as in Section B.1.

[59]This is also confirmed by numerical simulations.

government has to incur the cost of a higher r for a larger set of productivity values). The opposite is true when A^* increases. This suggests that a policy change that leads to a decrease in A^* is accompanied by a decrease r^*, which decreases the positive effect of the policy adjustment. On the other hand, a policy change that leads to an increase in A^* is accompanied by an increase in r^*, which tends to partially offset the negative effect that such a policy has on the probability of default.

Finally, one may wonder how an adjustment in r^* affects the relative importance of the direct and the multiplier effects. The next proposition states that the main result of Section 4.1 in the paper still holds.[60]

Proposition B *Let M_{Total} denote the total multiplier effect and D_{Total} denote the total direct effect in the model when the policy is announced before the interest r is set. If either $\varepsilon \to 0$ or $\sigma_x \to 0$ (or both) then $M_{Total} \to \infty$ and $D_{Total} \to 0$ with $\lim_{\varepsilon,\sigma_x \to 0} (M_{Total} \times D_{Total}) \in \mathbb{R}$.*

The discussion above suggests that in absolute terms the multiplier effect is lower when the policy is announced before the interest rate is set. However, it is still true that when households and lenders have very precise information about the current state of the economy, the change in the probability of default is driven mainly by the multiplier effect. The intuition for this result is exactly the same as before: In the limit when there is no fundamental uncertainty, households and lenders care only about the actions (and hence beliefs) of others. Thus, if an agent expects that the beliefs of others do not change, he will not change his action in response to a policy adjustment, implying that the direct effect will be equal to 0.

F Auxiliary Results

In this section I provide proofs of several results that have been invoked throughout this appendix. First, I show that $\partial x^*/\partial A^* < \frac{p_x+p_A}{p_x}$. Then I compute limits of several expressions as $\varepsilon, \sigma_x \to 0$ and which where used in the proof of Proposition 2.

Lemma 5 *The derivative of x^* with respect to A^* is bounded from above by $\frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}$.*

Proof. Applying the implicit function theorem to the lenders' indifference condition, we get

$$\frac{dx^*}{dA^*} = -\frac{(-1)\left(1 + r\min\left\{1, \frac{B_2^u(A^*)}{S(A^*,x^*)}\right\}\right) f(A^*|x^*)}{\frac{\partial}{\partial x^*}\left[\int\limits_{A^*}^{\infty}\left(1 + r\min\left\{1, \frac{B_2^u(A)}{S(A.x^*)}\right\}\right) f(A|x^*)\, dA\right]},$$

[60] The proof of Proposition B is analogous to the proof of Proposition 2 provided in Section B in this appendix.

where $f(A|x)$ is the conditional density of A given lender j observed signal $x_j = x^*$. Define $\mathcal{A}^u = \left\{A \geq A^* | B_2^{R,u}(A) < S(A)\right\}$ and $\mathcal{A}^c = \left\{A \geq A^* | B_2^{R,u}(A) \geq S(A)\right\}$, and note that $B_2^{R,u}(A)$ and $S(A)$ intersect at most finitely many times. Without loss of generality, I assume that $B_2^{R,u}(A)$ and $S(A)$ intersect at least once (otherwise, the result follows immediately). Then we can write $\mathcal{A}^u$ and $\mathcal{A}^c$ as $\mathcal{A}^u = \cup_{i=1}^{N_u}\left[A_{i_0}^u, A_{i_1}^u\right]$ and $\mathcal{A}^c = \cup_{i=1}^{N_c}\left(A_{i_0}^c, A_{i_1}^c\right)$, where $N_u, N_c \in \mathbb{N}$, $\left\{A_{i_0}^u\right\}_{i=1}^{N_u}$ are the values of the productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from above and $\left\{A_{i_1}^u\right\}_{i=1}^{N_u}$ are the values of productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from below.[61] With these definitions, we can write the above derivative as

$$\frac{dx^*}{dA^*} = \frac{\left(1 + r\min\left\{1, \frac{BB_2^{R,u}(A)}{S(A^*,x^*)}\right\}\right) f(A^*|x^*)}{\sum \frac{\partial}{\partial x^*}\int\limits_{A_{i_0}^c}^{A_{i_1}^c}(1+r) f(A|x^*)\,dA + \sum \frac{\partial}{\partial x^*}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\left\{1 + r\frac{B_2^{R,u}(A)}{S(A,x^*)} f(A|x^*)\right\} dA}$$

Consider the case where at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. Then the denominator becomes:

$$\sum_{i=1}^{N_u}\int\limits_{A_{i_0}^c}^{A_{i_1}^c}\frac{\partial}{\partial x^*}(1+r) f(A|x^*)\,dA + \sum_{i=1}^{N_c}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{1 + r\frac{B_2^{R,u}(A)}{S(A,x^*)} f(A|x^*)\right\} dA$$

$$= \int\limits_{A^*}^{\infty}\frac{\partial}{\partial x^*}(1+r) f(A|x^*)\,dA + \sum_{i=1}^{N_c}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{r\left(\frac{B_2^{R,u}(A)}{S(A,x^*)} - 1\right) f(A|x^*)\right\} dA$$

It remains to show that the second of the above terms is positive. Intuitively, that is what we expect, since a higher x^* makes high values of A more likely and $B_2^u(A)$ is increasing in A. The remainder of this proof is devoted to establishing it analytically.

The idea of the next few steps is to change differentiation with respect to x^* with the differentiation with respect to A. First, note that, since $f(A|x^*) = (p_A + p_x)^{1/2}\phi\left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{(p_A + p_x)^{-1/2}}\right)$, we have

$$\int\limits_{A^*}^{\infty}\frac{\partial}{\partial x^*}(1+r) f(A|x^*)\,dA = -\frac{p_x}{p_x + p_A}\int\limits_{A^*}^{\infty}\frac{\partial}{\partial A}(1+r) f(A|x^*)\,dA = \frac{p_x}{p_x + p_A}(1+r) f(A^*|x^*)$$

Next, let $H(A, x^*) = \left(\frac{B_2^u(A)}{S(A,x^*)} - 1\right) f(A|x^*)$. Then,

$$\frac{\partial}{\partial x^*}H(A,x^*) = -\frac{p_x}{p_x + p_A}\frac{\partial}{\partial A}H(A,x^*) + \frac{\partial B_2^{R,u}(A)}{\partial A}\frac{1}{S(A,x^*)} f(A|x^*) - \frac{p_A}{p_x + p_A}\frac{B_2^{R,u}(A)}{S(A,x^*)}\frac{\frac{\partial}{\partial x^*}S(A,x^*)}{S(A,x^*)},$$

[61] If at A^* we have $S(A,x^*) > B_2^{R,u}(A)$, then $A_{1_0}^u = A^*$, $A_{1_1}^u = A_{1_0}^c$, $A_{i_1}^c = A_{2_0}^u$, and so on. If at A^* we have $S(A,x^*) < B_2^{R,u}(A)$ then $A_{1_0}^c = A^*$, $A_{1_1}^c = A_{1_0}^u$, $A_{i_1}^u = A_{2_0}^c$, and so on.

where, since $\partial B_2^{R,u}(A)/\partial A > 0$ and $\frac{\partial}{\partial x^*} S(A, x^*) < 0$, the last two terms are strictly positive.[62] Moreover, note that for $i = 1, ..., N_c$ we have $H\left(A_{i_1}^u, x^*\right) = H\left(A_{i_0}^u, x^*\right) = 0$. Therefore,

$$\sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} \frac{\partial}{\partial x^*} \left\{ r \left(\frac{B_2^{R,u}(A)}{S(A, x^*)} - 1 \right) f(A|x^*) \right\} dA$$

$$> \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} -\frac{p_x}{p_x + p_A} \frac{\partial}{\partial A} H(A, x^*) \, dA$$

$$= -\frac{p_x}{p_x + p_A} \sum_{i=1}^{N_c} \left[H\left(A_{i_1}^u, x^*\right) - H\left(A_{i_0}^u, x^*\right) \right] = 0$$

This establishes the claim for the conclusion of the Lemma when at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. The case when $B_2^u(A^*) < S(A^*, x^*)$ is established in an analogous way. ■

The next claim has been used in Section $A.1.4$ to establish uniqueness of equilibrium in monotone strategies.

Claim 6 $\lim_{\varepsilon \to 0} \frac{\partial \Pr(A^*|A^* + \kappa\varepsilon)}{\partial A^*} = 0$

Proof. Note that

$$\frac{\partial \Pr(A^*|A^* + \kappa\varepsilon)}{\partial A^*}$$

$$= \frac{\frac{1}{\sigma_A}\phi\left(\frac{A^* - A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^* - (1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^* + (1+\kappa)\varepsilon - A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^* - (1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)} +$$

$$+ \frac{\left[\frac{1}{\sigma_A}\phi\left(\frac{A^* + (1+\kappa)\varepsilon - A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^* - (1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)\right]\left[\Phi\left(\frac{A^* - A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^* - (1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^* + (1+\kappa)\varepsilon - A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^* - (1-\kappa)\varepsilon - A_{-1}}{\sigma_A}\right)\right]^2}$$

Taking the limit as $\varepsilon \to 0$ and using l'Hôpital's rule one can show that the first term converges to $\frac{A^* - A_{-1}}{\sigma_A} \frac{(1-\kappa)}{2}$ while the second term converges to $-\frac{A^* - A_{-1}}{\sigma_A} \frac{(1-\kappa)}{2}$. It follows that $\lim_{\varepsilon \to 0} \frac{\partial \Pr(A^*|A^* + \kappa\varepsilon)}{\partial A^*} = 0$.

■

The next two claims have been used in the proof of Proposition 2.

Claim 7 $\lim_{\varepsilon \to 0} \left. \frac{\partial \Pr(A^{**}|A^* + \kappa\varepsilon)}{\partial A^{**}} \right|_{A^{**} = A^*} = \infty$ *and* $\lim_{\varepsilon \to 0} \frac{\left. \frac{\partial \Pr(A^{**}|A^* + \widehat{\kappa}\varepsilon)}{\partial A^*} \right|_{A^{**} = A^*}}{\left. \frac{\partial \Pr(A^{**}|A^* + \kappa\varepsilon)}{\partial A^{**}} \right|_{A^{**} = A^*}} = -1$

[62] The second and third terms "correct" for the fact that

$$\frac{\partial}{\partial x^*} H(A, x^*) \neq -\frac{p_x}{p_x + p_A} \frac{\partial}{\partial A} H(A, x^*)$$

Proof. If $A^{**} \in (A^* - (1-\kappa)\varepsilon, A^* + (1+\kappa)\varepsilon)$, then

$$\Pr(A^{**}|A^* + \kappa\varepsilon) = \frac{\Phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Differentiating with respect to A^{**}, we get

$$\frac{\partial \Pr(A^{**}|A^* + \kappa\varepsilon)}{\partial A^{**}} = \frac{\phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Taking the limit as $\varepsilon \to 0$ at $A^* = A^{**}$, we get

$$\lim_{\varepsilon\to 0} \frac{\partial \Pr(A^{**}|A^{**} + \kappa\varepsilon)}{\partial A^{**}} = \infty$$

Next, consider

$$\frac{\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^*}\right|_{A^{**}=A^*}}{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}} = -\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}} - \frac{\frac{\left[\Phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\frac{1}{\sigma_A}\phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]^2}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}}$$

Using l'Hôpital's rule, one can establish that

$$\lim_{\varepsilon\to 0} -\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}} = -1$$

Similarly, using l'Hôpital's rule, one can show that the second term converges to 0. ■

Claim 8 $\lim_{\sigma_x\to 0} \frac{\partial}{\partial A} S(A, x^*)|_{A=A^*} = \infty$

Proof. Note that

$$S(A, x^*) = b\left[1 - \Phi\left(\frac{x^* - A}{\sigma_x}\right)\right]$$

Taking the derivative with respect to A, we get

$$\frac{\partial S(A, x^*)}{\partial A} = \frac{1}{\sigma_x}\phi\left(\frac{x^* - A}{\sigma_x}\right).$$

Under Assumption 4, we have $x^* = \frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}A^* - \frac{\sigma_x^2}{\sigma_A^2}A_{-1} + \sigma_x^2\sqrt{\frac{1}{\sigma_x^2}+\frac{1}{\sigma_A^2}}\Phi^{-1}\left(\frac{1}{1+r}\right)$, and thus $\lim_{\sigma_x\to 0}\frac{x^*-A}{\sigma_x} = \phi\left(\Phi^{-1}\left(\frac{1}{1+r}\right)\right)$. The Claim follows immediately from this observation. ■

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