Applications of Population Principles: A Note

David Donaldson and Krishna Pendakur

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Abstract

Because most principles for evaluating policies which result in population change require information about individual well-being as well as interpersonal comparisons, some way of assessing levels of well-being is essential to their use. In this paper, we look at one way of doing it using equivalent expenditures and equivalence scales. In certain circumstances, equivalent expenditures, which can be used to make interpersonal comparisons, can be estimated. This note examines this method in light of the demands of the critical-level generalized utilitarian family of principles. Properties of those principles are examined, and the problems of setting the critical level and of implementing priority for the less well off is included. In addition, the possibility of decisions based on theory and a few facts is investigated. The paper concludes with a discussion of population policy and the current world environmental crisis.

Most principles for evaluating policies which result in population change require some information about the existence and utility levels of those who ever live in the histories which corresponding to policy options, which we call alternative histories or alternative. Utilities are indexes of well-being, and in population ethics, they should be indexes of lifetime well-being. The utility estimates are hard to assess and, because lifetime utilities are needed, the problem is more difficult than the estimation of short-term well-being.

One way to estimate utilities, at least in short periods, is to use equivalent expenditures and equivalence scales. They make interpersonal comparisons of utilities, accounting for heterogeneity of household characteristics, and can be estimated from available data in some circumstances. Although short-term utilities are not adequate for population ethics, estimation of their short-term values is an important first step. This paper presents an outline of one such method (Section 4). It is not the only possible one, but it does work and, if a reference utility function is specified and estimated, an economy can be converted, for welfare purposes, into one with identical reference single individuals. Children are included; no one is left out.

In Section 1 and 2, we present several of the main principles and families of principles: classical utilitarianism, average utilitarianism, the critical-level generalized utilitarian

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1 Thanks to Charlie Blackorby, Walter Bossert, Toby Ord and the other participants in the workshop Population Ethics: Theory and Practice, University of Oxford, for suggestions and corrections.
(CLGU) family, and its subfamily, the critical-level utilitarian (CLU) principles. The CLU principles are utilitarian for same-number comparisons and the CLGU principles permit a social aversion to inequality, giving priority to the interests of people whose utility levels are low.

In Section 3, we discuss the principles themselves, focussing, in the main, on two properties: avoidance of the repugnant conclusion (Parfit [1984]), and independence of the existence of the dead (Blackorby, Bossert and Donaldson [2005]). The first of these rules out principles such as classical utilitarianism because, when utility levels are above neutrality, average utility can always be traded off against numbers of people.

A principle satisfies the repugnant conclusion if and only if, for every alternative in which all utilities are positive, there exists an alternative in which each person’s utility level is arbitrarily small but positive which is ranked as better, as long as the number if people is sufficiently large. We see avoidance of this property as important. It is avoided by all CLGU principles with positive critical levels.

Different principles require different information about those who ever live. Independence of the existence of the dead, or existence independence, allows knowledge of individuals whose lives are over in all alternatives to be ranked to be disregarded. If a principle has this property, alternatives are ranked in the same way whether the dead are included in or excluded from the calculation. Indeed, existence independence is usually implicitly assumed. For instance, Parfit [1984] discusses an example in which a couple can choose whether to become pregnant immediately or to wait for a while. In the discussion, no attention is paid to the past or, for that matter, to anyone who is unaffected by the decision. Average utilitarianism does not satisfy existence independence: in computing average utility, the existence and utility levels of Cleopatra and Socrates matter.

In addition, the problems of setting the critical-level parameter and, in the CLGU case, the problem of choosing a strictly concave function for the prioritarian case, in which social aversion to utility inequality is expressed, are discussed.

In Section 4, we outline the equivalent-expenditure method for obtaining and comparing utilities in economic environments. Although the analysis is short-run at present, there has been a little progress in moving towards lifetime utilities.

Sometimes it is possible to obtain qualitative results with a minimum of data. Several attempts are outlined in Section 5.

Section 6 asks the question “Are there too many people alive now?” and Section 7 concludes.

1. Principles consistent with same-number utilitarianism:

Welfarist population principles such as the classical-utilitarian and average-utilitarian principles use information about individual lifetime well-being to rank alternative histories of the world (or the universe if you like) according to their goodness. Lifetime utilities are needed to avoid confusing differences in length of life with population size. If, for example,
a public health project results in longer lives, the population does not change, but from a single period’s point of view, the population increases.

Individual utilities are indexes of lifetime well-being. We may reasonably expect them to be able to rank gains and losses so that the statement ‘Yuriko gains more in moving from $a$ to $b$ than from $c$ to $d$’ has meaning. This property is preserved if individual increasing linear (affine) transformations are applied to individual utilities. Utility functions of this type are often called cardinally measurable. Interpersonal comparisons of lifetime well-being narrow the set of possible utilities and, with cardinal measurability, comparisons at two utility levels are sufficient to provide interpersonal comparisons of utilities at all possible levels. Given that, a single set of utilities results, unique up to a common increasing linear transformation. In population ethics, it is common to normalize the individual utility number for neutrality to zero. Neutrality is the well-being achieved by a life without experiences. (For discussions, see Broome [1991, 2004] and Blackorby, Bossert and Donaldson [2005]). Thus, with the normalization, lives worth living are associated with positive utility levels, and lives not worth living with negative levels. A second normalization, such as a utility level of 100 may be chosen for an excellent life. With this addition, lifetime utilities are interpersonally comparable, unique, and numerically measurable.

Although it is possible to discount the interests of future people, the result is to put greater weight on the interests of those born sooner. Consequently, discounting favours the interests of the old. Discounting is therefore not impartial, and so fails to satisfy a basic ethical principle. If, for example, it is possible to extend a one of two individual’s lives by one year with the same addition to lifetime well-being for both, discounting demands that the the older person should receive the benefit, simply because of age. Individuals’ and households’ economic behaviour is consistent with discounting future benefits to household members and it can be argued that such preferences should count normatively. Our framework can accommodate such a move.

It may be objected that principles that do not discount future utilities may recommend actions that require substantial sacrifices from those currently alive with social discounting suggested as a remedy. The problem is that principles that describe the common good are distorted by importing the constraints of human nature into them. Consequently, we discuss the non-discounting case only.

Most population principles have an associated value function which ranks alternatives using the lifetime utilities of those who ever live. Equality of value in two alternatives expresses equal goodness and inequality of value implies that the alternative with higher value is better than the one with lower value.

Many population principles are utilitarian for same-number comparisons. Classical or total utilitarianism (CU) is one, and its value function is the simple sum of utilities of those who ever live. CU ranks alternative $x$ as at least as good as alternative $y$ if and only if the total utility of those alive in $x$ is at least as great as the corresponding total in $y$. In mathematical notation, $x$ is at least as good as $y$ if and only if

$$\sum_{i \in N(x)} U^i(x) \geq \sum_{i \in N(y)} U^i(y),$$

(1)
where $N(x)$ those who ever live in $x$ and $N(y)$ is those who ever live in $y$. A strict inequality implies that $x$ is better than $y$ and an equality implies that $x$ and $y$ are equally good.

The CU value function is equal to the number who ever live multiplied by their average utility. Writing $n(x)$ as the number who ever live in $x$, the value function for CU can be rewritten as $n(x) \left[ \sum_{i \in N(x)} U^i(x)/n(x) \right]$ which is population size multiplied by average utility.

The classical-utilitarian principle is illustrated in Figure 1.

![Figure 1: Classical Utilitarianism](image)

The curved lines are combinations of average utility and population size that are judged equally good by the classical principle. In addition, the horizontal axis, in which average utility is zero, is also a set of combinations of average utility and population size that are equally good. There are many more such lines of course. Although the drawn curves are continuous, population size must be a positive integer.\(^2\) Given classical utilitarianism, additions with positive utilities to a utility-unaffected population are desirable and additions with negative utilities are undesirable.

The value function for the average-utilitarian principle is just the average utility of all those who ever live, and $x$ is at least as good as $y$ if and only if the average utility of those

\(^2\) This is true of all the diagrams in the paper.
who ever live in $x$ is at least as great as the corresponding average utility in $y$. Equivalently, $x$ is at least as good as $y$ if and only if

$$\sum_{i \in N(x)} \frac{U^i(x)}{n(x)} \geq \sum_{i \in N(y)} \frac{U^i(y)}{n(y)}.$$  \tag{2}

The average-utilitarian principle is illustrated in Figure 2.

![Figure 2: Average Utilitarianism](image)

Each horizontal line is a ‘curve’ of equal goodness: combinations of average utility and population size judged equally good by average utilitarianism. AU requires knowledge of the numbers and utility levels of people who are long dead to compute the average.

An example of a family of principles which is utilitarian for same-number comparisons is the critical-level-utilitarian family. It uses a single fixed parameter called the critical level. Written as $\alpha$, it is normally chosen to be non-negative. The CLU principle with critical level $\alpha$ ranks $x$ as at least as good as $y$ if and only if the sum of individual utilities less the critical level ($\alpha$) of all those who ever live in $x$ is at least as great as the corresponding sum in $y$. In notation,

$$\sum_{i \in N(x)} [U^i(x) - \alpha] \geq \sum_{i \in N(y)} [U^i(y) - \alpha].$$  \tag{3}
or, equivalently,

\[
n(x) \left[ \frac{\sum_{i \in N(x)} U^i(x)}{n(x)} - \alpha \right] \geq n(y) \left[ \frac{\sum_{i \in N(y)} U^i(y)}{n(y)} - \alpha \right],
\]

where \( n(x) \) and \( n(y) \) is the number of people who ever live in \( x \) and \( y \). Thus, the CLU value function is equal to the number who ever live multiplied by their average utility less the critical level. Normally \( \alpha \) is non-negative.

CLU is illustrated in Figure 3.

\[\text{Figure 3: Critical-Level Utilitarianism}\]

The curves of equal goodness are similar to those of classical utilitarianism but shifted up by the amount equal to the critical level (\( \alpha \)).

CLU is a different principle for each choice of \( \alpha \). Additions to a utility-unaffected population are desirable if the added person’s utility level exceeds \( \alpha \), and undesirable if the added person’s utility is less than \( \alpha \). If the added person has a utility level equal to \( \alpha \), the expanded alternative is as good as the original. Critical level \( \alpha \) represents a floor on
the trade-off between average utility and numbers. CLU includes the classical-utilitarian principle ($\alpha = 0$).

Consider the following example in which the present generation of 100m people is threatened by an asteroid which will end human existence. The first alternative results from inaction and, in the second alternative, the present generation makes a significant sacrifice which diverts the asteroid allowing an additional 100m people. There is no generational overlap.

**Alternative a:** Average utility of the present generation of 100m is 80; there is no future generation.

**Alternative b:** Average utility of the present generation of 100m is 40; average utility of the future generation of 100m is 50.

Suppose critical level alpha is 30. Then the CLU value in millions for $a$ is 5000 and the CLU value for $b$ is 3000, so $a$ is ranked as better than $b$ for according to the CLU principle with a critical level of thirty. More generally, the CLU value for $a$ is $100(80 - \alpha) = 8000 - 100\alpha$ and the CLU value for $b$ is $100(40 - \alpha) + 100(50 - \alpha) = 9000 - 200\alpha$. If $\alpha$ is less than 10, $b$ is better; if $\alpha$ is greater then 10, it is not.

Confronted with this problem, economists might suggest comparing sums of discounted utilities in each alternative with an ethically appropriate social discount rate. This procedure commits the analyst to a particular ethical position that may be unintended. Imagine alternative $a'$ which duplicates $a$ but has a future generation with each person’s utility at zero. Whether future utilities are discounted or not, this procedure ranks $a$ and $a'$ as equally good. Consequently, adding people with utilities of zero to a utility-unaffected population results in an alternative that is equally good. Thus, when utilities are estimated, the level zero must be meaningful. Given a few basic properties, this property leads to the repugnant conclusion (Blackorby, Bossert, Donaldson and Fleurbaey [1998]).

Critical-level utilitarianism makes the judgement that adding people to a utility-unaffected population with utilities equal to the constant critical level ($\alpha$) is a matter of social indifference.

Estimates of the consequences of different policy options are often uncertain. The usual option is to attach probabilities to the uncertain ones. Although Harsanyi’s social-aggregation theorem, suitably corrected, can be invoked, both CLU and CLGU do well. A significant problem arises because people may live in some, but not all, states in some alternatives.

Another example is based on a suggestion by Berners-Lee and Clark [2013]. Suppose the countries of the world decide to cap the flow of fossil fuels from the earth. Two possibilities are considered: a low cap recommended by scientists, and a higher cap the politicians think feasible. There is some uncertainty: the high cap $H$ may be a success ($H^s$) or a failure ($H^f$) and the two possibilities are judged equally likely. (The numbers are unrealistic because, with more realism, climate action always wins.)

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3 For the five-part harmony of the social-aggregation theorem’s extension to a population environment, see Weymark [1991], Blackorby, Bossert and Donaldson [2002, 2005].
Alternative \( P \) (the present course): Average utility of the present generation of 7b is 50; average utility of future generations of 5b is 20.

Alternative \( H^s \) (success of the high cap): Average utility of the present generation of 7b is 45; average utility of future generations of 8b is 25.

Alternative \( H^f \) (failure of the high cap): Average utility of the present generation of 7b is 45; average utility of future generations of 5b is 20.

Alternative \( L \) (the low cap): Average utility of the present generation of 7b is 30; average utility of future generations of 20b is 35.

Suppose \( \alpha = 30 \). Then CLU values are 90 for \( P \), 65 for \( H^s \), 55 for \( H^f \), and 100 for \( L \). The extended version of CLU requires comparison of expected CLU values, which are 90 for \( P \), 60 for \( H \), and 100. In this case, therefore, the low cap is best.

More generally, CLU values for the four possibilities are \((450 - 12\alpha)\) for \( P \), \((515 - 15\alpha)\) for \( H^s \), \((415 - 12\alpha)\) for \( H^f \), and \((910 - 27\alpha)\) for \( L \). Expected CLU values, which are \((450 - 12\alpha)\) for \( P \), \((465 - 13.5\alpha)\) for \( H \), and \((910 - 27\alpha)\) for \( L \). It follows that \( H \) is better than \( P \) if and only if \( \alpha < 10 \). But, if \( \alpha > 10 \) \( H \) is not worth it. \( L \) is better than \( P \) if and only if \( \alpha < 30.7 \) and \( L \) is better than \( H \) if and only if \( \alpha < 29.2 \).

2. Principles that give priority to the interests of the less well off:

It has become popular in recent years to include the possibility of a social preference for utility equality, a property not possessed by utilitarianism.\(^4\) The critical-level generalized-utilitarian principles (CLGU), which include the CLU principles, allow for this. Each member of the family is distinguished by an increasing transformation \((g)\) and a critical level \((\alpha)\), and \( x \) as at least as good as \( y \) if and only if the total transformed utility of those who ever live in \( x \) is at least as great as the total transformed utility in \( y \). Consequently, the critical-level generalized-utilitarian principle with transformation \( g \) and critical level \( \alpha \) ranks \( x \) as at least as good as \( y \) if and only if the sum of \( g \)-transformed individual utilities less the \( g \)-transformed value of \( \alpha \) of those who ever live in \( x \) is at least as great as the corresponding sum in \( y \). In notation, \( x \) is at least as good as \( y \) if and only if

\[
\sum_{i \in N(x)} \left[ g(U_i(x)) - g(\alpha) \right] \geq \sum_{i \in N(y)} \left[ g(U_i(y)) - g(\alpha) \right].
\]

\( g \) is increasing and usually chosen so that \( g(0) = 0 \). If \( g \) is strictly concave, priority is given to the interests of people with low utilities. For that reason, such principles are sometimes called prioritarian (Parfit [1997]).

\(^4\) Utilitarianism, applied to a fixed population, ranks a benefit of equal worth (equal increase in utility) to a single person as equally good, no matter who receives the benefit. This property, together with the requirement that, when utilities are equal, more is better, characterizes utilitarianism [Blackorby, Bossert and Donaldson [2005; Theorem 4.9].
The critical-level generalized-utilitarian value function is equal to average transformed utility less the transformed value of the critical level (α) all multiplied by population size or, for alternative \( x \),
\[
n(x) \left[ \sum_{i \in N(x)} g(U_i(x)) - g(\alpha) \right].
\]
(6)
Consequently, these principles may be thought of as trading off average transformed utility against population size. This is illustrated in Figure 4.

![Critical-Level Generalized Utilitarianism](image)

**Figure 4: Critical-Level Generalized Utilitarianism**

Although the label on the vertical axis in Figure 4 is different from its label in Figure 3, the two drawings are otherwise identical.

A version of the social-aggregation theorem exists for generalized-utilitarian principles. Harsanyi assumed that the expected-utility hypothesis requires the Bernoulli hypothesis, with individual ex-ante goodness associated with the expected value of actual utilities. This assumption is not required by the expected-utility hypothesis.\(^5\) All that is required is that the

\(^5\) See Weymark [1991] and Blackorby, Bossert and Donaldson [2005].
von-Neumann-Morgenstern utility functions be an individual-independent increasing transformation of individual utility. The assumption that individual ex-ante utilities satisfy the expected-utility hypothesis is consistent with critical-level generalized utilitarianism as follows.

For same-person comparisons, if there are \( m \) possible outcomes \( m \geq 2 \), Individuals rank a prospect with probabilities \((p_1, \ldots, p_m)\) and outcomes \((x_1, \ldots, x_m)\) is ranked as at least as good as a prospect with probabilities \((q_1, \ldots, q_m)\) and outcomes \((y_1, \ldots, y_m)\) if and only if the expected transformed utility in the first is at least as great as the expected transformed utility in the second. That is, ex-ante utility is given by

\[
\sum_{j=1}^{m} p_j g(U^i(x^i_j)) \geq \sum_{j=1}^{m} q_j g(U^i(y^i_j)).
\]  
(7)

The function \( g(U^i(\cdot)) \) is person \( i \)'s von-Neumann-Morgenstern utility function.\(^6\) The social goodness ordering makes the first prospect as at least as good as the second if and only if

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} p_j \left[ g(U^i(x^i_j)) - g(\alpha) \right] \geq \sum_{i=1}^{n} \sum_{j=1}^{m} q_j \left[ g(U^i(y^i_j)) - g(\alpha) \right]
\]  
(8)

Although this formulation is for a fixed population, it is easily extended to the variable-population environment. In that case, the CLGU value can be calculated for each state: overall value is the expected value of the state-specific values. In mathematical notation, (8) becomes

\[
\sum_{j=1}^{m} p_j \left( \sum_{i \in N(x^i_j)} \left[ g(U^i(x^i_j)) - g(\alpha) \right] \right) \geq \sum_{j=1}^{m} q_j \left( \sum_{i \in N(y^i_j)} \left[ g(U^i(y^i_j)) - g(\alpha) \right] \right).
\]  
(9)

A complication that is present in the uncertainty case is that some individuals may not exist in all states. For a discussion, see Blackorby, Bossert and Donaldson [2005].

### 3. Which member of the CLGU family is appropriate?

The CLGU principles, which include the CU and CLU principles, do not require complete knowledge of all possible individual utility levels. People whose lives are over in any pair of alternatives can be ignored and future people whose existence and well-being are unaffected can be ignored as well. Thus, all the CLGU principles satisfy independence of the existence of the dead (existence independence). In addition, the principles can be expanded to take account of the interests of non-human animals (Blackorby and Donaldson [1992], Blackorby, Bossert and Donaldson [2005, Chapter 11]). Because members of the CLGU family of principles are used in the examples discussed above, the information presented is sufficient information for ranking alternatives.

\(^6\) Although a measure of ex-ante utility is not needed, a reasonable representation is \( g^{-1} \left( \sum_{j=1}^{m} p_j g(U^i(x^i_j)) \right) \).
Suppose that, in the near future, a small group of humans leaves Earth on a spaceship and, after travelling through space for several generations, establishes a colony on a planet that belongs to a distant star. The colonists lose contact with Earth and, in all possible alternatives, the two groups have nothing to do with each other from then on. No decision made by the members of either group affects the other in any way.

Now suppose that the colonists are considering an important social decision and want to know which of the associated alternatives is best. If the population principle belongs to the CLU or CLGU families, the other group can be disregarded: the ranking of the feasible alternatives is independent of its existence and, therefore, of both the number and utility levels of its members. In this case, the population principle can be applied to the colonists alone or, for that matter, to the affected colonists alone.

Existence independence and a few other properties characterize critical-level generalized utilitarianism (CLGU). Among principles that satisfy a few basic axioms—continuity, strong Pareto, and a weak condition on the value of adding people to utility-unaffected populations—existence independence is satisfied by the CLGU principles and by them alone (Blackorby, Bossert and Donaldson [2005; Theorem 6.10]).

The estimation of utilities or transformed utilities is only one step towards application of population principles. A principle is also needed and, if the CLU family is employed, a critical level (α) must be chosen. It should be positive to avoid the trade-off made by classical utilitarianism, which leads to the repugnant conclusion. A positive value for α moves the trade-off upward, so that numbers can substitute for well-being as long as average utility is above α. But how high should α be? It should, in our view, correspond to a decent but basic life. Having said that, there may be some uncertainty about the effect of the critical level on social evaluations.

One possibility is to choose a range for α and allow some pairs of alternatives to be unranked. That approach is followed in Blackorby, Bossert and Donaldson [1996, 2005 ch. 7]. Instead, a sensitivity analysis might be considered more appropriate. A range for the critical level is chosen and policies evaluated for each value of α. In some cases, the ranking will be the same for all α in the range but, in others, disagreement is possible.

A similar procedure can be used in the generalized-utilitarian case. But other principles may be more difficult to apply. Suppose, for example, that critical levels are number-dependent. Such a principle ranks x as at least as good as y if and only if

$$\sum_{i \in N(x)} \left[ U^i(x) - c_{i-1} \right] \geq \sum_{i \in N(y)} \left[ U^i(y) - c_{i-1} \right],$$  \hspace{1cm} (10)

where $c_{i-1}$ is the $i$th critical level and $c_0$ is arbitrary. This formulation increases the information needed for social rankings substantially. If the critical levels are all different, knowledge of the number of people alive in the distant past and far future is needed. Average utilitarianism has similar properties. In the CLU and CLGU cases, however, unaffected people can be disregarded. This may suggest that the CLU and CLGU families are better than other contenders.
Arrhenius [2000] has criticized the CLGU and CLU principles with positive values of $\alpha$ on the grounds that, for any alternative in which all utility levels are negative, there is another in which all utility levels are positive but less than $\alpha$ that is ranked as worse. Principles that have this property are said to lead to the sadistic conclusion. There are principles which do not exhibit this property but they do not satisfy independence of the existence of the dead. As noted above, existence independence and a handful of basic axioms characterize CLGU. It follows that given satisfaction those basic axioms, no principle can avoid the repugnant conclusion, avoid the sadistic conclusion, and satisfy existence independence. A simple modification of CLU is a principle that avoids both the repugnant and sadistic conclusions and, necessarily fails to satisfy existence independence. Its value function coincides with the CLU value function for average utilities above $\alpha$, is equal to average utility minus $\alpha$ when average utility is positive and less than or equal to $\alpha$, and equal to the sum of utilities minus $\alpha$ when average utility is non-positive. This principle is illustrated in Figure 5.

![Figure 5: A Principle that avoids the Repugnant and Sadistic Conclusions](image.png)
The choice of the transformation $g$ for the CLGU principles requires a concave function defined for all utility levels including negative ones. One possibility is given by

$$g(u) = \frac{1 - e^{-\gamma u}}{\gamma}$$

(11)

where $\gamma$ is a positive constant. This has a limit as $\gamma \to 0$, namely $g(u) = u$. If $\gamma$ is positive, $g$ strictly concave.

The properties of this function for inequality aversion are not obvious. What happens, for example if the measurement units for utilities change size? We are sure the transformation can be adjusted, but the change is not obvious.

Toby Ord has suggested another functional form for $g$. It is

$$g(u) = \int \frac{1}{1 + e^u} \, du + c,$$

(12)

where $c$ is a constant. It’s concave, but we haven’t been able to find a closed form for the integral. Again, the function’s properties are not clear.

A very simple concave function is one that gives extra weight to negative utilities. In that case, $g(u) = u$ when $u$ is non-negative, and $g(u) = cu$, $c > 1$, when $u$ is negative.

If a lower bound is placed on utilities, more possibilities immediately appear. With a lower bound of $\bar{u}$ a possible functions include

$$g(u) = \frac{[u + \bar{u}]^r}{r}$$

(13)

where $0 < r < 1$ or $r < 0$, or any cardinally equivalent function.

Our inclination is to accept avoidance of the repugnant conclusion and satisfaction of existence independence as the most important properties and reject avoidance of the sadistic conclusion. In addition, we find the choice of transformation $g$ to be a difficult one. This leads us us to advocate the CLU principles with positive critical levels.

4. Interpersonal comparisons of utility in economic environments

Utilities are not easy to estimate. Not available in social statistics, they are replete with difficulties concerning interpersonal differences, uncertainty about the future, and differences in household composition and preferences. In addition, there are special difficulties with non-human animals.

One way of making interpersonal comparisons is the use of equivalence scales and equivalent expenditures. In the models normally considered, public good provision is ignored or assumed to be constant. The set of households is divided into types along with a reference household type, normally a single individual. If the expenditure of a household of type $z$ is $m$, the equivalent expenditure for household type $z$, $m^e$, is defined by

$$V(p, m, z) = V^r(p, m^e)$$

(14)
where $V(\cdot, \cdot, z)$ is the indirect utility function of household type $z$, $V^r$ is the indirect utility function of the reference household, and $p$ is the vector of prices the household faces. If equivalent expenditure for a household of one adult and two children is $30,000, the the household is equivalent, for utility purposes, to three single reference adults with expenditures of $30,000 each. The underlying assumption is that all members have the same level of well-being. There is at least one sophisticated way to deal with this problem, but data limitations make it impractical.

Equivalent expenditures are closely related to equivalence scales. The relative equivalence scale $E^r$ is just $m/m^e$. If the above household spends $60,000, then its equivalence scale has a value of 2.

Equivalent expenditure $m^e$ in equation (14) cannot be identified from behaviour without restrictions on either the reference indirect utility function $V^r$ or the functional form of $m^e$.

Blackorby and Donaldson [1991, 1993] investigated the case in which equivalence scales are independent of expenditure, a condition they call equivalence-scale exactness. They showed that, if the reference utility function is not linear in the logarithm of expenditure, the equivalent expenditures can be uniquely estimated from behaviour. In this case, $m^e = \nu(p, z)m$.

This result was generalized by Donaldson and Pendakur [2004, 2006]. They considered cases in which equivalence scales depend on expenditure in two ways. Generalized equivalence-scale exactness requires the logarithm of equivalent expenditure to be a price-dependent linear function of the logarithm of expenditure, satisfying

$$\ln m^e = \kappa(p, z)\ln m + \ln \nu(p, z). \tag{15}$$

Generalized absolute equivalence-scale exactness requires equivalent expenditure $m^e$ to be a price-dependent linear function of expenditure, with

$$m^e = \nu(p, z)m + \delta(p, z). \tag{16}$$

In both, given similar conditions on reference preferences, the equivalent expenditures can be uniquely estimated. This means that an economy of heterogeneous households with different budgets can be reduced to an economy of homogeneous people (single reference individuals) with budgets equal to equivalent expenditures.

Identification is of the following sort. Suppose one of the restrictions above holds. Then, behaviour is consistent with an infinite number of equivalent-expenditure functions. However, only one of these equivalent expenditure functions satisfies the restriction. Pendakur [1999] and Donaldson and Pendakur [2004, 2006] further show how to estimate equivalent expenditure functions from consumer demand data. Such data are typically household-level observations: quantities $q$ purchased by households with demographic characteristics $z$ facing budget constraints defined by prices $p$ and budgets $m$.

Given $m^e$ defined by the functional forms in (15) or (16) above, the vector of quantity demand equations $q$ for households with characteristics $z$ are given by (via Roy’s Identity)

$$q(p, m, z) = \left( q^r(p, m^e) + \frac{\partial m^e(p, m, z)}{\partial p} \right) \left( \frac{\partial m^e(p, m, z)}{\partial m} \right)^{-1}, \tag{17}$$
where \( q'(p,m^e) \) is the vector of quantity demand equations for the reference household evaluated at market prices and equivalent expenditure. The other two terms are the price and expenditure derivatives of the equivalent expenditure function, and these terms translate and scale, respectively, the reference quantity demand equations. The equivalent expenditure function may thus be estimated by comparing the demands of reference households to the demand equations of other household types (with varying \( z \)).

Suppose, for example, that equation (16) holds. If we add an error term \( \varepsilon \), to the equation above and substitute in for \( m^e \), we get for households \( i = 1, \ldots, N \):

\[
q_i(p,m_i) = \left( q_i^r(p,m_i^e) + \frac{\partial \nu(p,z_i)}{\partial p} m_i + \frac{\partial \delta(p,z_i)}{\partial p} \right) + \varepsilon_i. \tag{18}
\]

Estimation may proceed given observations \( \langle q_i, m_i, z_i \rangle \) for any price vector \( p \) via non-linear least squares (or any other optimization that suits). In the case where \( \delta(p,z) = 0 \) for all \( (p,z) \), we have equivalence-scale exactness. Donaldson and Pendakur [2006] show that this restriction is rejected against a GAES alternative, implying that equivalence scales are not independent of expenditure \( m \), and find that equivalence scales for large households decline with expenditure. Kouvo, Schröder and Schmidt [2005] corroborate this finding via income-evaluation surveys. However, both Donaldson and Pendakur and Kouvo et al. find that the departures from equivalence-scale exactness are not very large.

The focus of those papers is on short term expenditure and well-being. Pendakur [2005] has, however, been able to extend the analysis to lifetimes in the case of children, permitting the estimation of lifetime equivalence scales and lifetime well-being. As a result, interpersonal comparisons of lifetime utility can be estimated.

Although estimated equivalence scales provide interpersonal comparisons, the single reference utility function is needed to calculate utilities. A functional form for the reference utility function is estimated when estimating the scales but it is only ordinal measurable. To be used for welfare analysis, it must be cardinally measurable.

We suggest that utilities are approximately logarithmic in expenditures. Thus, a 10\% increase in expenditures increases utility by approximately the same amount, no matter what the starting expenditure is. If this property holds for all prices, then

\[
V_r(p,m) = \ln \left[ \frac{m - a(p)}{b(p)} \right] + k(p), \tag{19}
\]

where \( a \) and \( b \) are homogeneous of degree one, \( k \) is homogeneous of degree zero, and \( m > a(p) \).\(^7\) The function is not defined for \( m \leq a(p) \).

If utility is given by equation (19), then utility is approximately linear in the log of money for rich households. For these households, a proportionate increase in expenditure yields the same increase in utility no matter how rich they are. However, for poor households, whose money income is closer to \( a(p) \), a proportionate increase in money income generates

\(^7\) Equation (19) is a special case of equation 5 in Lewbel [2003].
more utility. The presence of the function $a(p)$ also ensures that utility is not affine in $\ln m$, so that equivalence scales may be identified from behaviour.

The functions $a$, $b$ and $k$ can be estimated, but not uniquely. We show, in the appendix that a function that satisfies (19) can be chosen such that a utility level of zero represents neutrality and a level of 100 represents a benchmark high standard of living.\(^8\)

We are willing to assume the utility function of (19) because of the attractiveness of the near-constant marginal utility of the logarithm of expenditures. We do this for three reasons. First, constant marginal utility of the logarithm of expenditures is a very simple formulation that satisfies the requirement that marginal utility is decreasing in money. Second, Amiel and Cowell [1999], among others, establish that scaling-independent inequality and related welfare measures are more intuitively appealing to students in labs and survey respondents than translation-independent measures. If utility is linear in the logarithm of expenditure, scalings of the expenditure distribution lead to translations of the utility distribution which would not affect welfarist social evaluation. This cannot be said for translations of the expenditure distribution (adding a constant to each expenditure). Third, many social indicators are based on scaling. For example, the change in the cost of living (inflation) is always expressed in proportionate terms (percentage increases) rather than absolute terms (dollar increases). Also, wage contracts in collective bargaining are often written in terms of proportionate increases for all members of a bargaining unit. So, for all these reasons, we think equation (19) is an acceptable representation of reference utility.

In addition, estimated reference preferences are likely to be good fits only in the range of prices used for estimation. This second difficulty means that the scales may be of little use if relative prices change a lot in an environmental crisis. Thus the choice of a reference utility function is, at the moment, an open question.

Other possibilities for the reference household that have a similar functional form are

$$V^r(p, m) = \frac{1}{r} \left[ \frac{m - a(p)}{b(p)} \right]^r + k(p)$$  \hspace{1cm} (20)

$r < 1$, $r \neq 0$, $a$ and $b$ are homogeneous of degree one and $k$ is homogeneous of degree zero. If $r$ is near 1, the marginal utility of expenditures is nearly constant. Curvature of the function increases with smaller values of $r$.

There are other ways to make interpersonal comparisons, of course. Budget studies supplemented with researchers’ judgements can provide them. And there are methods that use expenditure data in a crude way, looking at the percentage of expenditure on necessities.

The assumption implicit in the equivalence-scale literature is that all members of a household are equally well off. There is reason to believe the claim to be false, and it has been investigated by Browning, Chiappori and Lewbel [2013] and by Dunbar, Lewbel and Pendakur [2013]. The latter paper finds that, in Malawi, the first child in a household commands nearly 20% of household expenditure, rising by 5–10 percentage per additional child. Fathers command a larger share of resources than mothers, and mothers seem to sacrifice more resources than fathers to their children.

\(^8\) Utility levels below zero or above 100 are possible.
Equivalent expenditures combined with a reference utility function provide a method for computing the sum of utilities, counting each person’s interest (including children). If CLGU is used, a transformed utility function $g(V^r(p, x))$ is needed. Mathematically, this is the same problem as choosing a utility function, but the significance is different. Transformed reference utility measures its social significance.

5. Theoretical considerations and policy

Theory sometimes helps with policy considerations. Blackorby and Donaldson [1992] considered non-human animals used for research and food, and employ a simple model which has several lessons for policy. If, for example, researchers are faced with taxes on animal use, the need to minimize costs will tend to drive animal well-being to the lowest level compatible with satisfactory experimental results, no matter what the tax rate is. In general, that level of animal well-being may be lower than that suggested by CLU with a positive critical level. The exercise suggests, therefore, that animal care committees that enforce standards that set a minimum for animal well-being are needed.

A similar model with farm animals reaches similar conclusions. Regulation is needed to ensure a minimum level of well-being for them.

Blackorby, Bossert and Donaldson [2005] also investigate a very simple model in which a rich country gives foreign aid with population consequences to a poor one. Although the model is too simple for robust policy recommendations, it suggests a direction for further investigation.

At two different times, Blackorby and Donaldson served on the Animal Care Committee at The University of British Columbia. Their attention was drawn to the use of mice to make monoclonal antibodies. The mice are given a kind of cancer in their stomachs, and they produce the antibodies while they suffer. A single mouse can withstand two or three rounds of the procedure, with antibodies withdrawn each time. The question they were asked is whether each mouse should suffer through two rounds or three. If three rounds are chosen, two thirds as many mice are needed.

Assuming the suffering of the mice is the same each time, classical utilitarianism finds the two arrangements equally good because total utility is the same in each. But CLU with a positive critical level prefers the three-round solution. The two-round case subtracts a multiple of $3\alpha$ from total utility, and the three-round case subtracts $2\alpha$. Because total utility (disutility) is the same in both cases, the three-round solution is ranked as better.

6. Are there too many people alive now?

The world now faces a significant warming and climate change, and it is tempting to ask whether there are too many people alive now and whether measures to limit population growth are needed. If we take action by placing a world cap on fossil fuel production, \[^{10}\] we

\[^{9}\] See also Blackorby, Bossert and Donaldson [2005].

\[^{10}\] Berners-Lee and Clark [2014].
can expect humanity to have a higher standard of living along with freedom from dislocations and wars that result from climate change (Dyer [2009]). In addition, the probability that humanity will survive in significant numbers for a couple of centuries will be higher.

That suggests that principles that recommend large populations, such as CU, may well also recommend effective environmental action. And the extreme pro-natalist views of the pope and many others, if applied consistently will also recommend action in order to maintain the number of future people. If the resulting policies produce large-scale misery, such an outcome would not be welcomed by any CLU or CLGU principle with a critical level above misery.

CLU and CLGU with priority recommend action if the critical level is below the average utility of future generations. If, however the critical level is very high, action will not be recommended: levels of utility below $\alpha$ for future people will count against action. Average utilitarianism is different, we think. It asks for high levels of well-being and population limitation (depending on economies of scale).

Monbiot [2008] argues that, although population growth matters, it’s not the most important environmental issue. The article relies mainly on the observation that population growth now occurs mostly in very poor countries. People in those countries have a smaller environmental impact than people in rich countries. We guess that means that environmentalists should welcome misery because it has a small environmental impact. When those miserable people join humanity, however, they become part of us.

Once people are born, whether their well-being is above or below the critical level, they are alive in all feasible histories of the planet, and their interests count. We should be concerned about all human misery and avoid recommending a world filled up with people whose lives are barely worth living. CLU with a positive and reasonable critical level makes a judgement like this.

Monbiot [2006] argues that permissions to burn fossil fuels should be allocated so that every person has the same right to pollute the atmosphere with carbon dioxide. This means that adding poor people diminishes the carbon rights of everyone.

Current projections for peak world population expect 9-13 billion people by 2070. But people born in the next fifty-five years can be expected to have children of their own. This is above most estimates of the carrying capacity of the planet and suggests that we should attempt to reduce our population gradually.

7. Concluding remarks

Our suggestion in this paper is that the estimation of utilities using equivalent expenditures, although incomplete, is a promising direction. In a period of climate change, we must be able to assess the well-being consequences of event such as famines and other natural disasters. There has been some work by economists on the topic and more is needed.

Of course, the most important policy we can follow is an effective way to reduce world greenhouse-gas emissions. That means a cap on production, resulting in changes in present and future population sizes and well-being. Other schemes, such a carbon taxes will not
work, unless they are applied in all countries. When we follow such a drastic plan, we need to know we’re doing the right thing, and population principles, properly applied, can help.

Appendix

If the reference indirect utility function satisfies (19), the utility difference from a percentage change in expenditure is approximately independent of the expenditure level for large values of \( m \). Thus,

\[
V^r(p, m(1 + \rho)) - V^r(p, m) = \ln \left[ \frac{m(1 + \rho) - a(p)}{m - a(p)} \right]
\]

for all \( p, m \), and \( \rho, \rho > 0 \). (\( \rho \) is a percentage expressed as a decimal so, for an increase of 15\%, \( \rho = .15 \).) As \( m \to \infty \), the right side of (21) approaches \( \ln(1 + \rho) \), which is independent of \( m \).

Suppose that the function in \( V^r \) is estimated. All other indirect utility functions that are cardinally equivalent and, therefore, represent the same preferences is \( V^r \) are given by

\[
\hat{V}^r(p, m) = cV^r(p, m) + d
\]

where \( c > 0 \).

Suppose \( \hat{V}^r(p, m) \) is zero (representing neutrality) at \( (\bar{p}, \bar{m}) \) and equal to 100 (representing the benchmark good life) at \( (\bar{p}, \hat{m}) \). If

\[
c = \frac{100}{V^r(\bar{p}, \hat{m}) - V^r(\bar{p}, \bar{m})}
\]

and

\[
d = -\frac{100V^r(\bar{p}, \hat{m})}{V^r(\bar{p}, \hat{m}) - V^r(\bar{p}, \bar{m})},
\]

the function \( \hat{V}^r \) is given by

\[
\hat{V}^r(p, m) = 100 \frac{V^r(p, m) - V^r(\bar{p}, \bar{m})}{V^r(\bar{p}, \hat{m}) - V^r(\bar{p}, \bar{m})}.
\]

This function is cardinally equivalent to \( V^r \), \( \hat{V}^r(\bar{p}, \bar{m}) = 0 \) and \( \hat{V}^r(\bar{p}, \hat{m}) = 100 \).
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