The Measurement of Business Capital, Income and Performance

Tutorial presented at the University Autonoma of Barcelona, Spain, September 21-22, 2005, revised December, 2005.

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Chapter 1

The Measurement of Capital: Traditional User Cost Approaches

1.1 Introduction

"Capital (I am not the first to discover) is a very large subject, with many aspects; wherever one starts, it is hard to bring more than a few of them into view. It is just as if one were making pictures of a building; though it is the same building, it looks quite different from different angles." John Hicks (1973; v)[117].

"Perhaps a more realistic motive for reading earlier writers is not to rediscover forgotten truths,

*[1] The author is indebted to Kevin Fox, Emili Grifell, Peter Hill, Ning Huang, Ulrich Kohli, Knox Lovell, Alice Nakamura, Paul Schreyer and Frank Wykoff for helpful comments in developing the material in this chapter. Much of the material in this chapter is taken from Diewert (2001)[62] (2005a)[67].

> but to gain a perspective of how present day ideas have evolved and, perhaps, by reading the original statements of important ideas, to see them more vividly and understand them more clearly." Geoffrey Whittington (1980; 240)[227].

In this chapter, we discuss some of the problems involved in constructing price and quantity series for both capital stocks and the associated flows of services when there is general (and specific) price change in the economy. This chapter will show how rental prices, stock prices and depreciation rates for capital assets used in production are all related under somewhat strong assumptions. Some of these assumptions will be relaxed in chapter **??** below.[*2] In addition, the assumptions underlying the user cost formulae developed in this chapter will be examined again in chapter **??** when we discuss income concepts.

Our general purpose in this chapter is to show how approximations to market rental prices for various types of capital services can be formed, using data on purchases of new assets of the same type and information on the prices of second hand assets. Thus firms typically purchase capital inputs and use their services over many periods. As outside observers, all we can see is the initial purchase price of the asset. But for many purposes, we require information on how that initial purchase cost is allocated across time so that period by period estimates of the value of capital input can be formed.[*3] Obviously, if firms rented or leased all of their capital input period by period, we would not need to discuss the above problem of intertemporal cost allocation. Thus our primary purpose in this chapter is to suggest methods for forming approximations to period by period market rents for the use of capital assets in production when the corresponding rental markets do not exist.

Before getting into the algebra of capital, we first discuss some of the problems that occur when an economy is experiencing very high inflation. Under these conditions, it will be necessary for the national price statistician to shorten the accounting period (or give up price measurement altogether).

Section 1.3 is a key section; it presents the basic equations relating stocks and flows of capital assuming that data on the prices of vintages of a homogeneous capital good are available. Our goal here is to show how information on used asset prices, combined with information on the relevant opportunity cost of capital and on expected future price movements in asset prices, can enable us to form rental prices for the assets used by a firm or industry, even if these assets are not actually rented. Once these rental prices have been estimated, they can be used to provide period costs for the assets and the contributions of these assets to period by period production can be evaluated. The framework presented in this chapter is not applicable under all circumstances[*4] but it is a framework that will allow us to disentangle the effects of general price change, asset specific price change and depreciation.

Section 1.4 continues the theoretical framework that was introduced in section 1.3. We show how information on vintage asset prices, vintage rental prices and vintage depreciation rates are all

[*2] Basically, in this chapter, we take what might be called the *separable opportunity cost approach* to the measurement of capital services; i.e., we assume that at the beginning of each period of production, the firm (or higher aggregates of firms such as industries or even entire economies) can find relevant market prices for its stocks of used capital equipment or it can find market prices for renting the services of the used stocks of machinery, equipment, structures, land and inventories that it holds for the coming period, so that opportunity costs for the services of its capital stock components exist. We say this is a separable approach because we assume that these market prices for either the stocks or flows of the assets held by the firm at the beginning of each production period exist independently of the actions of the firm on output markets and other input markets. In chapter **??**, we consider *nonseparable approaches* where some of these assumptions are relaxed.

[*3] Some of these purposes are: (i) the estimation of firm or industry one period production functions or their dual representations; (ii) the estimation of output supply elasticities or input demand elasticities; (iii) the estimation of firm or industry cost functions, particularly in a regulatory context and (iv) the estimation of periodic industry capital cost so that industry information on Gross Operating Surplus in the System of National Accounts can be decomposed into depreciation, tax and return to capital components. We will look at some of the issues surrounding this last point in more detail in chapter **??** below.

[*4] Most notably, the framework laid out in this chapter cannot deal with unique or one of a kind assets, which by definition, do not have vintages. In chapter 3, we will attempt to deal with this type of problem.

equivalent under certain assumptions; i.e., knowledge of any one of these three sequences or profiles is sufficient to determine the other two.*5

The previous two sections relied on assumptions about the production unit's nominal interest rate and its expectations about the future course of asset prices. But how are national income accountants and applied economists to determine these variables as outsiders? Section 1.5 discusses the problems involved in making these choices. Section 1.6 discusses the problems associated with the empirical determination of depreciation rates.

Section 1.7 discusses a topic that is of great interest to national income accountants: namely, what is the "correct" concept for depreciation that should be entered into the System of National Accounts (SNA)? In particular, we discuss whether anticipated asset price decline should be an element of depreciation as understood by national income accountants.

Section 1.8 discusses the problems involved in aggregating over vintages of capital, both in forming capital stocks and capital services. Instead of the usual perpetual inventory method for aggregating over vintages, which assumes perfectly substitutable vintages of the same stock, we follow Diewert and Lawrence (2000)[73] and suggest the use of a superlative index number formula to do the aggregation.

Section 1.9 looks at the consistency of our suggested treatment of capital with various production function concepts. This section is quite important because it shows how the general Hicksian (1946)[114] intertemporal production function can be built up using conceptually simpler "Austrian" one period production functions, where goods in process are distinguished both as inputs (at the beginning of the period) and outputs (at the end of the period).

Section 1.10 shows how our basic user cost formulae can be adapted to deal with the business income tax.

Chapter 1 concludes with an Appendix which develops the material presented in the main text in more detail. Thus Appendix A shows how the general algebra presented in sections 1.3 and 1.4 can be adapted to deal with four specific models of depreciation. The four models considered are the one hoss shay model, the straight line depreciation model, the geometric model of depreciation and the linear efficiency decline model. The final section of Appendix A considers a fifth type of depreciation model, one that is based on the assumption that each vintage of the asset has a specific maintenance and operating cost requirement associated with it. We show that this type of model can lead to the linear efficiency decline model studied earlier in Appendix A. However, the main use of the analysis presented this section of Appendix A is to suggest a reason why accelerated depreciation assumptions*6 are quite reasonable and likely to occur empirically.

1.2 Inflation and the Measurement of Economic Activity

Our goal in this chapter is twofold:

- To measure the price and quantity of the *stock* of reproducible capital held by a production unit (an establishment, a firm, an industry or an entire economy) at a *point* in time and
- To measure the price and quantity of the *flow* of reproducible capital services utilized by a production unit over a *period* of time.

In particular, we want to extend the procedures for measuring capital stocks and flows to cover situations where there is general price level change or *inflation*. In this section, we shall review some of the general measurement problems that arise when inflation is high.

*5 This point is due to Jorgenson (1989)[135] and Hulten (1990; 127-129)[127] (1996; 152-160)[128].

*6 An accelerated depreciation schedule for an asset means that depreciation increases as the asset ages.

When capital flows are measured, the normal period of time is either a year or a quarter. Under conditions of high inflation, the aggregation of homogeneous commodity flows within a quarter or a year is complicated by the fact that the within period transactions are valued at very different prices. The recent national income accounting literature explains the problem as follows:

> "Conventional index number theory is mostly concerned with comparisons between *points* of time whereas, in national accounts, price and quantity comparisons have to be made between discrete *periods* of time. Significant changes in price and quantity flows may occur not only between different periods but also within a single accounting period, especially one as long as a year. Indeed, the central problem of accounting under high inflation is that prices are much higher at the end of the accounting period than at the beginning." Peter Hill (1996; 11)[118].

> "The underlying problem is not a traditional index number problem. It stems from the use of current value data as inputs into the calculation of indirect price or quantity measures under high inflation. Current accounts permit identical quantities of the same homogeneous product to be valued at very different prices during the course of the same year. Implicitly, quantities sold at higher prices later in the year are treated as if they were superior qualities when they are not." Peter Hill (1996; 12)[118].

> "Under high inflation, the monetary value of flows of goods and services at different points of time within the same accounting period are not commensurate with each other because the unit of currency used as the *numeraire* is not stable. Adding together different quantities of the same good valued at different prices is equivalent, from a scientific point of view, to using different units of measurement for different sets of observations on the same variable. In the case of physical data, however, it is rather more obvious that adding quantities measured in grams to quantities measured in ounces is a futile procedure." Peter Hill (1996; 32)[118].

> "Before the preparation of the 1993 SNA, issues connected with high or significant inflation had not been dealt with at all in international recommendations concerning national accounts. Uneasiness especially with the recording of nominal interest had been often expressed, for instance in Europe and North America at the time of two digit inflation and above all in countries, like in Latin America, experiencing high or hyper inflation. In relation with the latter situations, uneasiness extended to the whole set of accounts, because, due to the significant rate of inflation within each year, annual accounts in current values could no longer be deemed homogeneous as regards the level of prices in each year. They combine intra-annual flows that are valued at very different prices and are not, strictly speaking, additive. The effect of the intra-annual change in the general price level can be neglected for the sake of simplicity only when the rate of inflation is low. When it is high, the meaning of annual accounts in current values becomes fuzzy." André Vanoli (1998)[218].

> "When inflation is high, the aggregation of flows from different periods becomes very much a case of 'adding apples and bananas'— the flows at the end of the period will carry a much greater weight than the flows at the beginning of the period, so that the change on average will reflect development at the end of the period disproportionately. Annual national accounts at current prices become virtually meaningless and computation of national accounts at constant prices becomes very problematic." Ezra Hadar and Soli Peleg (1998; 2)[104].

Of course, concern over the effects of general price level change has a much longer history in the general cost accounting literature; see Baxter (1984)[9], Tweedie and Whittington (1984)[217] and Whittington (1992)[228] for example.*7

*7 The inflation accounting literature extends back to the accountant Middleditch: "Today's dollar is, then, a totally different unit from the dollar of 1897. As the general price level fluctuates, the dollar is bound to become a unit of different magnitude. To mix these units is like mixing inches and centimeters or measuring a field with a rubber tape-line." Livingston Middleditch (1918; 114-115)[164].

We now discuss in more detail the accounting problems caused by high inflation that are referred to in the above quotations. The basic problem is this: all discrete time economic theories and most of index number theory assumes that all of the transactions of a production unit with respect to a homogeneous commodity within the accounting period can be represented by a *single price* and a *single quantity*. It is natural to let the single quantity be the *sum* of the quantities sold (in the case of an output) or the *sum* of the quantities purchased (in the case of an input). But then, if we want the single price times the single quantity to equal the *value* of transactions for the commodity in the period, the single price *must* equal the value of transactions divided by the sum of quantities purchased or sold; i.e., the single price must equal a *unit value*.[*8] But when there is substantial inflation within the accounting period, unit values give a much higher weight to transactions that occur near the end of the period compared to transactions that occurred near the beginning; it is as if the end of period transactions are being *artificially quality adjusted* to be more valuable than the beginning of the period transactions.

The obvious solution to this artificial implicit weighting problem is to choose the accounting period to be small enough so that the general inflation within the period is small enough to be ignored. This is precisely the solution suggested by the index number theorist Fisher[*9] and the great measurement economist Hicks: the length of the accounting period should be the Hicksian "week":

> "I shall define a week as that period of time during which variations in price can be neglected."
> John R. Hicks (1946; 122)[114].

Thus it seems that there is a simple solution to the problem of constructing meaningful accounting period prices and quantities for homogeneous commodities when there is high inflation: simply shorten the accounting period!

Hill (1996)[118] however notes that there are at least three classes of problems associated with the above solution:

> "In order to keep these issues in perspective, it is useful to summarise the problems created by continually shortening the accounting period.
>
> 1. The compilation of accounts for shorter time periods requires more information about the times at which various transactions take place. Enquiries may have to be conducted more frequently thereby creating additional costs for the data collectors. More burdens are also placed on the respondents supplying the information. In many cases, they may be unable to supply the necessary information because their own internal records and accounts do not permit them to do so, especially when they traditionally report their

[*8] The early index number theorists Walsh (1901; 96)[222] (1921; 88)[223], Fisher (1922; 318)[91] and Davies (1924; 96)[40] all suggested unit values as the prices that should be inserted into a bilateral index number formula. Walsh nicely sums up the case for unit values as follows: "Some nice questions arise as to whether only what is consumed in the country, or only what is produced in it, or both together are to be counted; and also there are difficulties as to the single price quotation that is to be given at each period to each commodity, since this, too, must be an average. Throughout the country during the period a commodity is not sold at one price, nor even at one wholesale price in its principle market. Various quantities of it are sold at different prices, and the full value is obtained by adding all the sums spent (at the same stage in its advance towards the consumer), and the average price is found by dividing the total sum (or the full value) by the total quantities." Correa Moylan Walsh (1921; 88)[223].

[*9] "Essentially the same problem enters, however, whenever, as is usually the case, the data for prices and quantities with which we start are averages instead of being the original market quotations. Throughout this book, 'the price' of any commodity or 'the quantity' of it for any one year was assumed given. But what is such a price or quantity? Sometimes it is a single quotation for January 1 or July 1, but usually it is an average of several quotations scattered through the year. The question arises: On what principle should this average be constructed? The *practical* answer is *any* kind of average since, ordinarily, the variations during a year, so far, at least, as prices are concerned, are too little to make any perceptible difference in the result, whatever kind of average is used. Otherwise, there would be ground for subdividing the year into quarters or months until we reach a small enough period to be considered practically a point." Irving Fisher (1922; 318)[91].

accounts for longer time periods, such as a year.

2. As production is a process which can extend over a considerable period of time, its measurement becomes progressively more difficult the shorter the accounting period. The problem is not confined to agriculture or forestry where many production processes take a year or more. The production of large fixed assets such as large ships, bridges, power stations, dams or the like can extend over several years. The output produced over shorter periods of time then has to be measured on the basis of work in progress completed each period. ...
3. Because many transactions, especially large transactions, are not completed within the day, there are typically many receivables and payables outstanding at any given moment of time. They assume greater importance in relation to the flows as the accounting period is reduced. This makes it more difficult to reconcile the values of different flows in the accounts, especially if the two parties to the transaction perceive it as taking place at different times from each other and do not record it in the same way required by the system. ... Peter Hill (1996; 34-35)[118].

Thus shortening the accounting period leads to increased costs for the statistical agency and the businesses being surveyed. Moreover, firm accounting is geared to years and quarters and it may not be possible for production units to provide complete accounting information for periods shorter than a quarter. As the accounting period becomes shorter, it is less likely that production, shipment, billing and payment for the same commodity will all coincide within the accounting period. Also as the accounting period becomes shorter, work in progress will tend to become ever more important relative to final sales, creating difficult valuation problems.[*10] Put another way, more and more inputs will shift from being intermediate inputs (inputs that are used up within the accounting period) to being durable inputs (inputs whose contribution to production extends over more than one period). In addition to these difficulties, there are others. For example, as the accounting period becomes shorter, transactions tend to become more erratic and sporadic. Many goods will not be sold in a supermarket in a particular day or week. *Normal index number theory breaks down under these conditions*: it is difficult to compare a positive amount of a good sold in one period with a zero amount sold in the next period.[*11] A related difficulty is that many commodities are produced or demanded on a *seasonal basis*. If the accounting period is a year, then there are no seasonal commodity difficulties but as we shorten the period from a year, we will run into the problem of seasonal fluctuations in prices and quantities. In many cases, a seasonal commodity will not be available in all seasons and we again run into the problem of comparing positive values with zero values in the periods when the commodity is out of season. Even if the seasonal commodity does not disappear, the application of standard index number theory is not straightforward.[*12]

Nevertheless, even in the face of the above difficulties, it seems that the only possible solution to the artificial implicit weighting problem that is generated by high inflation is to shorten the accounting period so that normal index number theory can be applied in order to construct meaningful economic

[*10] There are problems in constructing price indexes for work in progress! This is to be expected since there are very few transactions involving partially completed products. National income accountants solve this problem by using an appropriate construction price index or by construction an index of the materials and labour that are used to produce the work in progress. The problem with the latter strategy is that over time, the effects of technical progress in the industry cumulate and price indexes based on inputs used by the industry are unable to capture these effects.

[*11] We will attempt to deal with this problem in chapter 3 below.

[*12] Hadar and Peleg (1998; 5)[104] comment on the importance of seasonal adjustment procedures in the context of high inflation: "As a by-product of the emphasis on quarterly estimates at constant prices the seasonal adjustment got large attention and many resources were spent to improve the adjustment." Diewert (1996)[59] (1998)[60] (1999)[61] reviews possible approaches to the problems involved in treating seasonal commodities (and suggests solutions) when there is high inflation.

aggregates.*13

In addition to the above general problems associated with economic measurement of flow variables under conditions of high inflation, there are some additional problems associated with the measurement of capital. These additional problems are associated with the stock and flow aspects of capital. We will conclude this section by explaining these problems.

Given an accounting period of some predetermined length, we can associate with it at least *three separate points in time*:

- The beginning of the accounting period;
- The middle of the accounting period; and
- The end of the accounting period.

In interpreting the national accounts or the accounts of a business unit, we generally think of all flow variables as being concentrated in the middle of the period. If we follow this convention in the context of high inflation, then we require one (nominal) interest rate to index the value of money or financial capital going from the beginning of the period to the middle of the period and we require another (nominal) interest rate to index the value of money going from the middle of the period to the end of the period. Given these two interest rates, we could construct *centered user costs of capital* for each type of reproducible capital, which would be the appropriate flow variables that would match up with the other flow variables in the production accounts of the production unit. However, in order to reduce the notational complexity of this annex, we do not construct centered user costs in what follows. Instead, for each type of asset, we construct either a *beginning of the period user cost*, which measures the cost of using the asset for the period under consideration but under the assumption that payments and receipts for all flow variables are made at the beginning of the period, or an *end of the period user cost*, which measures the cost of using the asset for the period under consideration but under the assumption that all inputs used and outputs produced are paid at the end of the period.*14

In the following section, we explain the fundamental equations relating stocks and flows of capital.

1.3 The Fundamental Problem of Accounting and the Equations Relating Stocks and Flows of Capital

Before we begin with our algebra, it seems appropriate to explain why accounting for the contribution of capital to production is more difficult than accounting for the contributions of labour or materials. The main problem is that when a reproducible capital input is purchased for use by a production unit at the beginning of an accounting period, we cannot simply charge the entire purchase cost to the

*13 Our discussion in the previous paragraph indicates that this cannot be done if the economy is experiencing a hyperinflation. Thus meaningful economic measurement becomes impossible under very high inflation. This is a hidden cost of inflation that is not discussed very much in the literature on the costs of inflation.

*14 Put another way, what we are doing is somewhat artificially moving the payments and receipts for the flow transactions of the production unit under consideration from the middle of the period (where they would occur on average if the revenue from the outputs sold was received as production was delivered and if the production unit paid inputs as they were used) to the beginning of the period or to the end of the period. Already, it can be seen that as soon as we have to deal with durable inputs, there are complications that do not arise with flow inputs, which are used up during the period of production. Hicks recognized that there were index number complications caused by the durable nature of capital that occurred *within* the basic period: "In all probability these prices will have changed during the year, so that we have a kind of index number problem, parallel to the index number problem of comparing real income in different years. The characteristics of that other problem are generally appreciated; what is not so generally appreciated is the fact that before we can begin to compare real income in different years, we have to solve a similar problem within the single year—we have to reduce the Capital stock at the beginning and end of the year into comparable real terms." J.R. Hicks (1942; 176)[113]. We will discuss this within the period aggregation problem raised by Hicks in more detail in chapter **??**.

period of purchase. Since the benefits of using the capital asset extend over more than one period, the initial purchase cost must be distributed somehow over the useful life of the asset. This is *the fundamental problem of accounting.*[*15] Hulten (1990)[127] explained the consequences for accounting for the durability of capital as follows:

> "Durability means that a capital good is productive for two or more time periods, and this, in turn, implies that a distinction must be made between the value of using or renting capital in any year and the value of owning the capital asset. This distinction would not necessarily lead to a measurement problem if the capital services used in any given year were paid for in that year; that is, if all capital were rented. In this case, transactions in the rental market would fix the price and quantity of capital in each time period, much as data on the price and quantity of labor services are derived from labor market transactions. But, unfortunately, much capital is utilized by its owner and the transfer of capital services between owner and user results in an implicit rent typically not observed by the statistician. Market data are thus inadequate for the task of directly estimating the price and quantity of capital services, and this has led to the development of indirect procedures for inferring the quantity of capital, like the perpetual inventory method, or to the acceptance of flawed measures, like book value." Charles R. Hulten (1990; 120-121)[127].

Hicks made similar observations about the necessity for making imputations when attempting to value the contribution of a capital input to production during an arbitrarily chosen accounting period:

> "Thus it seems true to say that while the valuation of income goods is characteristically a market valuation, the values of the goods which enter into the capital stock are characteristically imputed values. We cannot take over a market valuation for them; we have to set values upon them ourselves." John R. Hicks (1961; 19)[115].

The above quotations should alert us to the fact that the durability of capital inputs is going to lead to many measurement difficulties that are not present for goods and services whose contribution to production takes place within the accounting period.

Most economists agree that the value of an asset at the beginning of an accounting period is equal to the discounted stream of future rental payments that the asset is expected to yield. Thus the *stock value* of the asset is set equal to the discounted future *service flows*[*16] that the asset is expected to

[*15] "A preceding article [Sweeney (1933a)[207]] declared that correct demarcation between capital and income constitutes the fundamental problem of accounting, and then went on to explain the various meanings and aspects of capital. The present article does the same for income." Henry W. Sweeney (1933b; 323)[208].
"The difficulty of imputing expenses to individual sales or even to the gross earnings of the accounting period, the month or year, is an ever present problem for the accountant in the periodic determination of enterprise income. The longer the period for which the income is to be determined, the smaller the relative amount of error. Absolute accuracy can be attained only when the venture is completed and the enterprise terminated." William T Crandell (1935; 388-389)[36].
"Early enterprises and partners working in the main in isolated trading ventures, needed only an irregular determination of profit. But before the business corporation had been very long in operation it was evident that it needed to be treated as a continuing enterprise. For example, calculating dividends by separate voyages was found impractical in the East India Company by 1660. Profit calculation therefore became a matter of periodic estimates in place of the known results of completed ventures." A.C. Littleton (1933; 270)[150].
"The third convention is that of the annual accounting period. It is this convention which is responsible for most of the difficult accounting problems. Without this convention, accounting would be a simple matter of recording completed and fully realized transactions: an act of primitive simplicity." Stephen Gilman (1939; 26)[99].
"*All* the problems of income measurement are the result of our desire to attribute income to arbitrarily determined short periods of time. Everything comes right in the end; but by then it is too late to matter." David Solomons (1961; 378)[196]. Note that these authors do not mention the additional complications that are due to the fact that future revenues and costs must be discounted to yield values that are equivalent to present dollars.

[*16] Walras (1954)[224] (first edition published in 1874) was one of the earliest economists to state that capital stocks are demanded because of the future flow of services that they render. Although he was perhaps the first

yield in future periods. Let the price of a new capital input purchased at the beginning of period t be P_0^t. In a noninflationary environment, it is reasonable to assume that the (potentially observable) sequence of (cross sectional) rental prices classified by the age of the asset that prevails at the beginning of period t will prevail in future periods.*17 Thus there is no need to have a separate notation for future expected rental prices for a new asset as it ages. However, in an inflationary environment, it is necessary to distinguish between the observable rental prices for the asset at different ages at the beginning of period t and future *expected* rental prices for assets of various ages.*18 Thus let f_0^t be the (potentially observable) rental price of a new asset at the beginning of period t, let f_1^t be the (potentially observable) rental price of a one period old asset at the beginning of period t, let f_2^t be the (potentially observable) rental price of a 2 period old asset at the beginning of period t, etc. Then *the fundamental equation* relating the *stock value of a new asset* at the beginning of period t, P_0^t, to the sequence *of cross sectional rental prices by age* prevailing at the beginning of period t, $\{f_n^t : n = 0, 1, 2, ...\}$*19 is:

$$P_0^t = f_0^t + [(1 + i_1^t)/(1 + r_1^t)]f_1^t + [(1 + i_1^t)(1 + i_2^t)/(1 + r_1^t)(1 + r_2^t)]f_2^t + \cdots \tag{1.1}$$

In the above equation,*20 $1 + i_1^t$ is the *rental price escalation factor* that is *expected* to apply to a one period old asset going from the beginning of period t to the end of period t (or equivalently, to the beginning of period $t+1$), $(1 + i_1^t)(1 + i_2^t)$ is *the rental price escalation factor* that is *expected* to apply to a 2 period old asset going from the beginning of period t to the beginning of period $t + 2$, etc. Thus the i_n^t *are expected rental price by age inflation rates* that are formed at the beginning of period t. The term $1 + r_1^t$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period $t + 1$, the term $(1 + r_1^t)(1 + r_2^t)$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period $t+2$, etc. Thus the r_n^t are one period *nominal interest rates* that represent the *term structure of interest rates* at the beginning of period t.*21

We now generalize equation (1.1) to relate the *stock value of an n period old asset* at the beginning of period t, P_n^t, to the sequence *of cross sectional rental prices by age* prevailing at the beginning of

economist to formally derive a user cost formula as we shall see, he did not work out the explicit discounting formula that Böhm-Bawerk (1891; 342)[18] was able to derive.

*17 This is an oversimplification because it neglects the obsolescence problem, which will be addressed in section 1.7 below and in chapters **??** and **??**.

*18 Note that these future expected rental prices are not generally observable due to the lack of futures markets for these future rentals of the asset by the age of the asset.

*19 The sequence of (cross sectional) rental prices at the beginning of period t for an asset that is n periods old, $\{f_n^t; n = 0, 1, 2, ...\}$, is called the period t *age efficiency profile* of the asset.

*20 Note that equation (1.1) assumes that the rentals are paid for at the beginning of each period. It should also be noted that Irving Fisher (1897; 365)[89] seemed to be well aware of the complexities that are imbedded in equation (1.1): "There is not space here to discuss the theory in greater detail, nor to apply it to economic problems. A full treatment would take account of the various standards in which income is or may be expressed, of the case in which the rates of interest at different dates and for different periods does not remain constant, of the fact that the services of capital which are discounted in its value are only *expected* services, not those which actually materialise, and of the consequent discrepancy between income anticipated and income realised, of the propriety or impropriety of including man himself as a species of income-bearing capital, and so on." Hicks (1939; 179)[112] (1942; 177)[113] and Hill and Hill (2003)[122] provide additional discussions on the role of expectations in capital and income theory.

*21 Peter Hill has noted a major problem with the use of equation (1.1) as the starting point of our discussion: namely, *unique assets* will by definition not be reproduced in future periods and so the cross sectional rental prices by age f_n^t will not exist for these assets! In this case, the f_n^t should be interpreted as expected future rentals that the unique asset is expected to generate at today's prices. The $(1 + i_n^t)$ terms then summarize expectations about the amount of asset specific price change that is expected to take place. Thus in this case, $(1 + i_1^t)f_1^t$ is the period $t + 1$ nominal expected rental for a new unit of the durable that is expected to prevail at the beginning of period $t + 1$. This reinterpretation of equation (1.1) is more fundamental but we chose not to make it our starting point because it does not lead to a straightforward method for national statisticians to form reproducible estimates of these future rental payments. Hill (2000)[120] works with this more general model and we will as well in chapter **??** below.

period t, $\{f_n^t\}$. Thus for $n = 0, 1, 2, \ldots$, we assume:

$$P_n^t = f_n^t + [(1 + i_1^t)/(1 + r_1^t)]f_{n+1}^t + [(1 + i_1^t)(1 + i_2^t)/(1 + r_1^t)(1 + r_2^t)]f_{n+2}^t + \cdots \qquad (1.2)$$

Thus older assets discount fewer terms in the above sum; i.e., as n increases by one, we have one less term on the right hand side of (1.2). However, note that we are applying the same price escalation factors $(1 + i_1^t), (1 + i_1^t)(1 + i_2^t), \ldots$, to escalate the cross sectional rental prices prevailing at the beginning of period t, $f_1^t, f_2^t, \ldots$, and to form estimates of future expected rental prices for each unit of the capital stock that is in use at the beginning of period t.

These rental prices by age of asset prevailing at the beginning of period t, $f_0^t, f_1^t, \ldots$ are potentially observable.*22 These *cross sectional rental prices* reflect the relative efficiency of the various ages of the capital good under consideration at the beginning of period t. For now, we assume that these rentals are paid (explicitly or implicitly) by the users at the beginning of period t.*23 Note that the sequence of used asset stock prices at the beginning of period t, $P_0^t, P_1^t, \ldots$ is not affected by general inflation provided that the general inflation affects the expected asset inflation rates i_n^t and the nominal interest rates r_n^t in a proportional manner. We will return to this point later.

The physical productivity characteristics of a unit of capital of each age are determined by the sequence of cross sectional rental prices in our present model. Thus a brand new asset is characterized by the vector of current rental prices by age, $f_0^t, f_1^t, f_2^t, \ldots$, which are interpreted as "physical" contributions to output that the new asset is expected to yield during the current period t (this is f_0^t), the next period (this is f_1^t), and so on. An asset which is one period old at the start of period t is characterized by the vector $f_1^t, f_2^t, \ldots$, etc.*24

We have not explained how the expected rental price inflation rates i_n^t are to be estimated. We shall deal with this problem in section 1.5 below. However, it should be noted that there is no guarantee that our expectations about the future course of rental prices are correct.

At this point, we make some simplifying assumptions about the expected rental inflation rates i_n^t and the interest rates r_n^t. We assume that these anticipated vintage rental inflation factors at the beginning of each period t are all equal; i.e., we assume:

$$i_n^t = i^t; \qquad n = 1, 2, \ldots \qquad (1.3)$$

We also assume that the term structure of interest rates at the beginning of each period t is constant; i.e., we assume:

$$r_n^t = r^t; \qquad n = 1, 2, \ldots \qquad (1.4)$$

However, note that as the period t changes, r^t and i^t can change.

Using assumptions (1.3) and (1.4), we can rewrite the system of equations (1.2), which relate the sequence or profile of *stock prices by age* at the beginning of period t $\{P_n^t\}$ to the sequence or profile of (cross sectional) *rental prices by age* at the beginning of period t $\{f_n^t\}$, as follows:

$$\begin{aligned}
P_0^t &= f_0^t + [(1 + i^t)/(1 + r^t)]f_1^t + [(1 + i^t)/(1 + r^t)]^2 f_2^t + [(1 + i^t)/(1 + r^t)]^3 f_3^t + \cdots \\
P_1^t &= f_1^t + [(1 + i^t)/(1 + r^t)]f_2^t + [(1 + i^t)/(1 + r^t)]^2 f_3^t + [(1 + i^t)/(1 + r^t)]^3 f_4^t + \cdots \\
P_2^t &= f_2^t + [(1 + i^t)/(1 + r^t)]f_3^t + [(1 + i^t)/(1 + r^t)]^2 f_4^t + [(1 + i^t)/(1 + r^t)]^3 f_5^t + \cdots \\
&\cdots \\
P_n^t &= f_n^t + [(1 + i^t)/(1 + r^t)]f_{n+1}^t + [(1 + i^t)/(1 + r^t)]^2 f_{n+2}^t + [(1 + i^t)/(1 + r^t)]^3 f_{n+3}^t + \cdots \qquad (1.5)
\end{aligned}$$

*22 This is the main reason that we use the vintage approach to capital measurement rather than the more fundamental discounted future expected rentals approach advocated by Hill and many other economists.

*23 If they are paid at the end of the period, then we must discount these payments by an appropriate nominal interest rate. We consider this case later.

*24 Triplett (1996; 97)[215] used this characterization for capital assets of various vintages.

On the left hand side of equations (1.5), we have the sequence of asset prices by age at the beginning of period t starting with the price of a new asset, P_0^t, moving to the price of an asset that is one period old at the start of period t, P_1^t, then moving to the price of an asset that is 2 periods old at the start of period t, P_2^t, and so on. On the right hand side of equations (1.5), the first term in each equation is a member of the sequence of rental prices by age that prevails in the market at the beginning of period t. Thus f_0^t is the rent for a new asset, f_1^t is the rent for an asset that is one period old at the beginning of period t, f_2^t is the rent for an asset that is 2 periods old, and so on. This sequence of current market rental prices for the assets of various ages is then extrapolated out into the future using the anticipated price escalation rates $(1+i^t), (1+i^t)^2, (1+i^t)^3$, etc. and then these future expected rentals are discounted back to the beginning of period t using the discount factors $(1+r^t), (1+r^t)^2, (1+r^t)^3$, etc. Note that given the period t expected asset inflation rate i^t and the period t nominal discount rate r^t, we can go from the (cross sectional) sequence of rental prices $\{f_n^t\}$ to the sequence of asset prices $\{P_n^t\}$ using equations (1.5). We shall show below how this procedure can be reversed; i.e., we shall show how given the sequence of asset prices by age at the beginning of period t, we can construct estimates for the sequence of rental prices by age at the beginning of period t.

It seems that Böhm-Bawerk was the first economist to use the above method for relating the future service flows of a durable input to its stock price:

> "If the services of the durable good be exhausted in a short space of time, the individual services, provided that they are of the same quality— which, for simplicity's sake, we assume— are, as a rule, equal in value, and the value of the material good itself is obtained by multiplying the value of one service by the number of services of which the good is capable. But in the case of many durable goods, such as ships, machinery, furniture, land, the services rendered extend over long periods, and the result is that the later services cannot be rendered, or at least cannot be rendered in a normal economic way, before a long time has expired. As a consequence, the value of the more distant material services suffers the same fate as the value of future goods. A material service, which, technically, is exactly the same as a service of this year, but which cannot be rendered before next year, is worth a little less than this year's service; another similar service, but obtainable only after two years, is, again, a little less valuable, and so on; the values of the remote services decreasing with the remoteness of the period at which they can be rendered. Say that this year's service is worth 100, then next year's service— assuming a difference of 5 % per annum— is worth in today's valuation only 95.23; the third year's service is worth only 90.70; the fourth year's service, 86.38; the fifth, sixth and seventh year's services, respectively, worth 82.27, 78.35, 74.62 of present money. The value of the durable good in this case is not found by multiplying the value of the current service by the total number of services, but is represented by a sum of services decreasing in value." Eugen von Böhm-Bawerk (1891; 342)[18].

Thus Böhm-Bawerk considered a special case of (1.5) where all service flows f_n were equal to 100 for $n = 0, 1, \ldots, 6$ and equal to 0 thereafter, where the asset inflation rate was expected to be 0 and where the interest rate r was equal to .05 or 5%.*25 This is a special case of what has come to be known as the *one hoss shay model* and we shall consider it in more detail in Appendix A of this chapter.*26

*25 Böhm-Bawerk (1891; 343)[18] went on and constructed the sequence of vintage asset prices using his special case of equations (1.5).

*26 Solow, Tobin, von Weizsäcker and Yaari (1966; 81)[198] used the term "one hoss shay".

Note that equations (1.5) can be rewritten as follows:*27

$$\begin{aligned} P_0^t &= f_0^t + [(1+i^t)/(1+r^t)]P_1^t; \\ P_1^t &= f_1^t + [(1+i^t)/(1+r^t)]P_2^t; \\ P_2^t &= f_2^t + [(1+i^t)/(1+r^t)]P_3^t; \\ &\cdots \\ P_n^t &= f_n^t + [(1+i^t)/(1+r^t)]P_{n+1}^t; \cdots \end{aligned} \tag{1.6}$$

The first equation in (1.6) says that the value of a new asset at the start of period t, P_0^t, is equal to the rental that the asset can earn in period t, f_0^t,*28 plus the expected asset value of the capital good at the end of period t, $(1+i^t)P_1^t$, but this expected asset value must be divided by the discount factor, $(1+r^t)$, in order to convert this future value into an equivalent beginning of period t value.*29

Now it is straightforward to solve equations (1.6) for the sequence of period t rental prices by age n, $\{f_n^t\}$, in terms of the sequence of asset prices by age n, $\{P_n^t\}$:

$$\begin{aligned} f_0^t &= P_0^t - [(1+i^t)/(1+r^t)]P_1^t = (1+r^t)^{-1}[P_0^t(1+r^t) - (1+i^t)P_1^t] \\ f_1^t &= P_1^t - [(1+i^t)/(1+r^t)]P_2^t = (1+r^t)^{-1}[P_1^t(1+r^t) - (1+i^t)P_2^t] \\ f_2^t &= P_2^t - [(1+i^t)/(1+r^t)]P_3^t = (1+r^t)^{-1}[P_2^t(1+r^t) - (1+i^t)P_3^t] \\ &\cdots \\ f_n^t &= P_n^t - [(1+i^t)/(1+r^t)]P_{n+1}^t = (1+r^t)^{-1}[P_n^t(1+r^t) - (1+i^t)P_{n+1}^t]; \cdots \end{aligned} \tag{1.7}$$

Thus equations (1.5) allow us to go from the sequence of rental prices by age $\{f_n^t\}$ to the sequence of asset prices by age $\{P_n^t\}$ while equations (1.7) allow us to reverse the process.

Equations (1.7) can be derived from elementary economic considerations. Consider the first equation in (1.7). Think of a production unit as purchasing a unit of the new capital asset at the beginning of period t at a cost of P_0^t and then using the asset throughout period t. However, at the end of period t, the producer will have a depreciated asset that is expected to be worth $(1+i^t)P_1^t$. Since this offset to the initial cost of the asset will only be received at the end of period t, it must be divided by $(1+r^t)$ to express the benefit in terms of beginning of period t dollars. Thus the net cost of *using* the new asset for period t*30 is $P_0^t - [(1+i^t)/(1+r^t)]P_1^t$.

The above equations assume that the actual or implicit period t rental payments f_n^t for assets of different ages n are made at the *beginning* of period t. It is sometimes convenient to assume that the

*27 Christensen and Jorgenson (1969; 302)[28] do this for the geometric depreciation model except that they assume that the rental is paid at the end of the period rather than the beginning. Variants of the system of equations (1.6) were derived by Christensen and Jorgenson (1973)[29], Jorgenson (1989; 10)[135], Hulten (1990; 128)[127] and Diewert and Lawrence (2000; 276)[73]. Irving Fisher (1908; 32-33)[90] derived these equations in words as follows: "Putting the principle in its most general form, we may say that for any arbitrary interval of time, the value of the capital at its beginning is the discounted value of two elements: (1) the actual income accruing within that interval, and (2) the value of the capital at the close of the period."

*28 Note that we are implicitly assuming that the rental is paid to the owner at the beginning of period t.

*29 Another way of interpreting say the first equation in (1.6) runs as follows: the purchase cost of a new asset P_0^t less the rental f_0^t (which is paid immediately at the beginning of period t) can be regarded as an investment, which must earn the going rate of return r^t. Thus we must have $[P_0^t - f_0^t](1+r^t) = (1+i^t)P_1^t$ which is the (expected) value of the asset at the end of period t. This line of reasoning can be traced back to Walras (1954; 267)[224]: "A man who buys a house for his own use must be resolved by us into two individuals, one making an investment and the other consuming directly the services of his capital."

*30 This explains why the rental prices f_n^t are sometimes called *user costs*. This derivation of a user cost was used by Diewert (1974; 504)[45], (1980; 472-473)[49], (1992a; 194) and by Hulten (1996; 155)[128]. Diewert based his derivation on the general intertemporal model of production due to Hicks (1939)[112] but specialized to one period.

rental payments are made at the *end* of each accounting period. Thus define u_n^t as the *end of period t rental price or user cost* for an asset that is n periods old at the beginning of period t. In this case, it can be seen that we can rewrite the system of equations (1.5), which relate the sequence of stock prices by age at the beginning of period t $\{P_n^t\}$ to the sequence of end of period rental prices by age at the beginning of period t $\{u_n^t\}$, as follows:

$$
\begin{aligned}
P_0^t &= u_0^t/(1+r^t) + [(1+i^t)/(1+r^t)^2]u_1^t + [(1+i^t)^2/(1+r^t)^3]u_2^t + \cdots \\
P_1^t &= u_1^t/(1+r^t) + [(1+i^t)/(1+r^t)^2]u_2^t + [(1+i^t)^2/(1+r^t)^3]u_3^t + \cdots \\
P_2^t &= u_2^t/(1+r^t) + [(1+i^t)/(1+r^t)^2]u_3^t + [(1+i^t)^2/(1+r^t)^3]u_4^t + \cdots \\
&\cdots \\
P_n^t &= u_n^t/(1+r^t) + [(1+i^t)/(1+r^t)^2]u_{n+1}^t + [(1+i^t)^2/(1+r^t)^3]u_{n+2}^t + \cdots
\end{aligned} \tag{1.8}
$$

where r^t is the relevant period t nominal interest rate or opportunity cost of capital facing the production unit and i^t is the period t anticipated end of period rental price escalation factor.

It can be seen that equations (1.8) can be rewritten in the form (1.5) if we convert the end of period rental prices u_n^t, into corresponding beginning of period t rental prices f_n^t, by defining the f_n^t as follows:

$$f_n^t \equiv u_n^t/(1+r^t); \qquad n = 0, 1, 2, \ldots \tag{1.9}$$

Thus if the rental payment u_n^t is made at the end of the period instead of the beginning, then the corresponding beginning of the period rental f_n^t is equal to u_n^t divided by the discount factor $(1+r^t)$.*31

Inserting equations (1.9) into the second set of equations in (1.7), it can be seen that the sequence of end of period t user costs by age n, $\{u_n^t\}$, can be defined in terms of the period t sequence of asset prices by age, $\{P_n^t\}$, as follows:

$$
\begin{aligned}
u_0^t &= P_0^t(1+r^t) - (1+i^t)P_1^t \\
u_1^t &= P_1^t(1+r^t) - (1+i^t)P_2^t \\
u_2^t &= P_2^t(1+r^t) - (1+i^t)P_3^t \\
&\cdots \\
u_n^t &= P_n^t(1+r^t) - (1+i^t)P_{n+1}^t; \cdots
\end{aligned} \tag{1.10}
$$

Equations (1.10) can also be given a direct economic interpretation. Consider the following explanation for the user cost for a new asset, u_0^t. At the end of period t, the business unit expects to have an asset worth $(1+i^t)P_1^t$. Offsetting this benefit is the beginning of the period asset purchase cost, P_0^t. However, in addition to this cost, the business must charge itself either the explicit interest cost that occurs if money is borrowed to purchase the asset or the implicit opportunity cost of the equity capital that is tied up in the purchase. Thus offsetting the end of the period benefit $(1+i^t)P_1^t$ is the initial purchase cost and opportunity interest cost of the asset purchase, $P_0^t(1+r^t)$, leading to a end of period t net cost of $P_0^t(1+r^t)-(1+i^t)P_1^t$ or u_0^t. In chapter **??**, we will explore these relationships in more depth.*32

*31 It is interesting that Böhm-Bawerk (1891; 343)[18] carefully distinguished between rental payments made at the beginning or end of a period: "These figures are based on the assumption that the whole year's utility is obtained all at once, and, indeed, obtained in anticipation at the beginning of the year; e.g., by hiring the good at a year's interest of 100 payable on each 1st January. If, on the other hand, the year's use can only be had at the end of the year, a valuation undertaken at the beginning of the year will show figures not inconsiderably lower. ... That the figures should alter according as the date of the valuation stands nearer or farther from the date of obtaining the utility, is an entirely natural thing, and one quite familiar in financial life."

*32 For national income accounting purposes, the end of period user costs are more useful as we shall see in chapter **??**.

It is interesting to note that in both the accounting and financial management literature of the past century, there was a reluctance to treat the opportunity cost of *equity capital* tied up in capital inputs as a genuine cost of production.*33 However, more recently, there is an acceptance of an imputed interest charge for equity capital as a genuine cost of production.*34

In the following section, we will relate the asset price profiles $\{P_n^t\}$ and the user cost profiles $\{u_n^t\}$ to *depreciation profiles*. However, before turning to the subject of depreciation, it is important to stress that the analysis presented in this section is based on a number of restrictive assumptions, particularly on future price expectations. Moreover, we have not explained how these asset price expectations are formed and we have not explained how the period t nominal interest rate is to be estimated (we will address these topics in sections 1.5 and 1.6 below). We have not explained what should be done if the sequence of second hand asset prices $\{P_n^t\}$ is not available and the sequences of rental prices or user costs by age, $\{f_n^t\}$ or $\{u_n^t\}$, are also not available (we will address this problem in chapter **??**). *We have also assumed that asset values and user costs are independent of how intensively the assets are used and these asset values and user costs are independent of the firm's decisions about producing outputs and using other inputs.* Finally, we have not modeled uncertainty (about future prices and the useful lives of assets) and attitudes towards risk on the part of producers. Thus the analysis presented in this chapter is only a start on the difficult problems associated with measuring capital input.

1.4 Relationships between Depreciation, Asset Prices and User Costs

Recall that in the previous section, P_n^t was defined to be the price of an asset that was n periods old at the beginning of period t. Generally, the decline in asset value as we go from an asset of a particular age to the next oldest at the same point in time is called *depreciation*. More precisely, we define the *cross sectional depreciation* D_n^t*35 of an asset that is n periods old at the beginning of period t as

$$D_n^t \equiv P_n^t - P_{n+1}^t; \qquad n = 0, 1, 2, \ldots \tag{1.11}$$

Thus D_n^t is the value of an asset that is n periods old at the beginning of period t, P_n^t, minus the value of an asset that is $n+1$ periods old at the beginning of period t, P_{n+1}^t.*36

*33 This literature is reviewed in Diewert and Fox (1999; 271-274)[72] and in chapter 2 below.

*34 Stern Stewart & Co. has popularized this concept of EVA, Economic Value Added. In a newspaper advertisement in the Financial Post in 1999, it described this "new" concept as follows: "EVA measures your company's after tax profits from operations minus the cost of all the capital employed to produce those profits. What makes EVA so revealing is that it takes into account a factor no conventional measures include: the cost of the operation's capital— not just the cost of debt but the cost of equity capital as well."

*35 This terminology is due to Hill (1999)[119] who distinguished the decline in second hand asset values due to aging (cross section depreciation) from the decline in an asset value over a period of time (*time series depreciation*). Triplett (1996; 98-99)[215] uses the cross sectional definition of depreciation and shows that it is equal to the concept of capital consumption in the national accounts but he does this under the assumption of no expected real asset inflation. The early accounting literature also defined depreciation as time series depreciation with the implicit assumption that the general price level was constant. We will examine the relationship of cross sectional to time series depreciation in section 1.7 below as well as in chapter **??**.

*36 Of course, the objections to the use of second hand market data to determine depreciation rates are very old: "We readily agree that where a market is sufficiently large, generally accessible, and continuous over time, it serves to coordinate a large number of subjective estimates and thus may impart a moment of (social) objectivity to value relations based on prices forced on it. But it can hardly be said that the second-hand market for industrial equipment, which would be the proper place for the determination of the value of capital goods which have been in use, satisfies these requirements, and that its valuations are superior to intra-enterprise valuation." L.M. Lachmann (1941; 376-377)[144]. "Criticism has also been voiced about the viability of used asset market price data as an indicator of in use asset values. One argument, drawing on the Ackerlof Lemons Model, is that assets resold in second hand markets are not representative of the underlying population of assets, because only poorer quality units are sold when used. Others express concerns about the thinness of resale markets, believing that it is sporadic in nature and is dominated by dealers who under-bid." Charles R. Hulten and Frank C. Wykoff

Obviously, given the sequence of period t asset prices by age, $\{P_n^t\}$, we can use equations (1.11) to determine the period t sequence of declines in asset values by age, $\{D_n^t\}$. Conversely, given the period t cross sectional depreciation sequence or *profile*, $\{D_n^t\}$, we can determine the period t asset prices by age by adding up amounts of depreciation:

$$
\begin{aligned}
P_0^t &= D_0^t + D_1^t + D_2^t + \cdots \\
P_1^t &= D_1^t + D_2^t + D_3^t + \cdots \\
&\cdots \\
P_n^t &= D_n^t + D_{n+1}^t + D_{n+2}^t + \cdots
\end{aligned} \tag{1.12}
$$

Rather than working with first differences of asset prices by age, it is more convenient to reparameterize the pattern of cross sectional depreciation amounts by defining the sequence of *period t depreciation rates* δ_n^t for an asset that is n periods old at the start of period t as follows:

$$\delta_n^t \equiv 1 - [P_{n+1}^t / P_n^t] = D_n^t / P_n^t; \qquad n = 0, 1, 2, \ldots \tag{1.13}$$

In the above definitions, we require n to be such that P_n^t is positive.*37

Obviously, given the sequence of period t asset prices by age, $\{P_n^t\}$, we can use equations (1.13) to determine the period t sequence of *cross sectional depreciation rates*, $\{\delta_n^t\}$. Conversely, given the sequence of period t depreciation rates, $\{\delta_n^t\}$, as well as the price of a new asset in period t, P_0^t, we can determine the period t asset prices by age as follows:

$$
\begin{aligned}
P_1^t &= (1 - \delta_0^t) P_0^t \\
P_2^t &= (1 - \delta_0^t)(1 - \delta_1^t) P_0^t \\
&\cdots \\
P_n^t &= (1 - \delta_0^t)(1 - \delta_1^t) \cdots (1 - \delta_{n-1}^t) P_0^t; \cdots
\end{aligned} \tag{1.14}
$$

The interpretation of equations (1.14) is straightforward. At the beginning of period t, a new capital good is worth P_0^t. An asset of the same type but which is one period older at the beginning of period t is less valuable by the amount of depreciation $\delta_0^t P_0^t$ and hence is worth $(1 - \delta_0^t) P_0^t$, which is equal to P_1^t. An asset which is two periods old at the beginning of period t is less valuable than a one period old asset by the amount of depreciation $\delta_1^t P_1^t$ and hence is worth $P_2^t = (1 - \delta_1^t) P_1^t$ which is equal to $(1 - \delta_1^t)(1 - \delta_0^t) P_0^t$ using the first equation in (1.14) and so on. Suppose $L - 1$ is the first integer which is such that δ_{L-1}^t is equal to one. Then P_n^t equals zero for all $n \geq L$; i.e., at the end of L periods of use, the asset no longer has a positive rental value. If $L = 1$, then a new asset of this type delivers all of its services in the first period of use and the asset is in fact a nondurable asset.

Now substitute equations (1.14) into equations (1.10) in order to obtain the following formulae for the sequence of the *end of the period user costs by age* $\{u_n^t\}$ in terms of the price of a new asset at

(1996; 17-18)[131].

*37 This definition of depreciation dates back to Hicks (1939)[112] at least and was used extensively by Hulten and Wykoff (1981a)[129] (1981b)[130], Diewert (1974; 504)[45] and Hulten (1990; 128)[127] (1996; 155)[128]: "If there is a perfect second hand market for the goods in question, so that a market value can be assessed for them with precision, corresponding to each particular degree of wear, then the value-loss due to consumption can be exactly measured…" John R. Hicks (1939; 176)[112]. Current cost accountants have also advocated the use of second hand market data (when available) to calculate "objective" depreciation rates: "But as a practical matter the quantification and valuation of asset services used is not a simple matter and we must fall back on estimated patterns as a basis for current cost as well as historic cost depreciation. For those fixed assets which have active second hand markets the problem is not overly difficult. A pattern of service values can be obtained at any time by comparing the market values of different ages or degrees of use. The differences so obtained, when related to the value of a new asset, yield the proportions of asset value which are normally used up or foregone in the various stages of asset life." Edgar O. Edwards and Philip W. Bell (1961; 175)[81].

the beginning of period t, P_0^t, and the sequence of cross section depreciation rates, $\{\delta_n^t\}$:

$$\begin{aligned} u_0^t &= [(1+r^t)-(1+i^t)(1-\delta_0^t)]P_0^t \\ u_1^t &= (1-\delta_0^t)[(1+r^t)-(1+i^t)(1-\delta_1^t)]P_0^t \\ &\cdots \\ u_n^t &= (1-\delta_0^t)\cdots(1-\delta_{n-1}^t)[(1+r^t)-(1+i^t)(1-\delta_n^t)]P_0^t \end{aligned} \tag{1.15}$$

Thus given P_0^t (the beginning of period t price of a new asset), i^t (the new asset inflation rate that is expected at the beginning of period t), r^t (the one period nominal interest rate that the business unit faces at the beginning of period t) and given the sequence of cross section vintage depreciation rates prevailing at the beginning of period t (the δ_n^t), then we can use equations (1.15) to calculate the sequence of end of the period user costs by age of asset for period t, the u_n^t. Of course, given the u_n^t, we can use equations (1.8) to calculate the beginning of the period asset prices P_n^t and finally, given the P_n^t, we can use equations (1.13) in order to calculate the sequence of depreciation rates by age of asset, the δ_n^t. Thus *given any one of these sequences or profiles, all of the other sequences are completely determined*. This means that assumptions about depreciation rates, the pattern of user costs by age or the pattern of asset prices by age *cannot be made independently of each other.*[*38]

It is useful to look more closely at the first equation in (1.15), which expresses the user cost or rental price of a new asset at the beginning of period t (but payment for the asset service is received at the end of period t), u_0^t, in terms of the depreciation rate δ_0^t, the one period nominal interest rate r^t, the new asset inflation rate i^t that is expected to prevail at the beginning of period t and the beginning of period t price for a new asset, P_0^t:

$$u_0^t = [(1+r^t)-(1+i^t)(1-\delta_0^t)]P_0^t = [r^t - i^t + (1+i^t)\delta_0^t]P_0^t. \tag{1.16}$$

Thus the user cost of a new asset u_0^t that is purchased at the beginning of period t (and the actual or imputed rental payment is made at the end of the period) is equal to $r^t - i^t$ (a nominal interest rate minus an asset inflation rate which can be loosely interpreted[*39] as a *real interest rate*) times the initial asset cost P_0^t plus $(1+i^t)\delta_0^t P_0^t$ which is *depreciation* on the asset at beginning of the period prices, $\delta_1^t P_0^t$, times *the inflation escalation factor*, $(1+i^t)$.[*40] If we further assume that the expected asset inflation rate is 0, then (1.16) further simplifies to:

$$u_0^t = [r^t + \delta_0^t]P_0^t. \tag{1.17}$$

Under these assumptions, the user cost of a new asset is equal to the interest rate plus the depreciation rate times the initial purchase price.[*41] This is essentially the user cost formula that was obtained by Walras in 1874:

[*38] This point was first made explicitly by Jorgenson and Griliches (1967; 257)[138]: "An almost universal conceptual error in the measurement of capital input is to confuse the aggregation of capital stock with the aggregation of capital service." See also Jorgenson and Griliches (1972; 81-87)[139]. Much of the above algebra for switching from one method of representing capital inputs by age of asset to another was first developed by Christensen and Jorgenson (1969; 302-305)[28] (1973)[29] for the geometrically declining depreciation model. The general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989)[135] and Hulten (1990; 127-129)[127] (1996; 152-160)[128].

[*39] We will provide a more precise definition of a real interest rate later.

[*40] This formula was obtained by Christensen and Jorgenson (1969; 302)[28] for the geometric model of depreciation but it is valid for any depreciation model. Griliches (1963; 120)[103] also came very close to deriving this formula in words: "In a perfectly competitive world the annual rent of a machine would equal the marginal product of its services. The rent itself would be determined by the interest costs on the investment, the deterioration in the future productivity of the machine due to current use, and the expected change in the price of the machine (obsolescence)."

[*41] Using equations (1.14) and (1.15) and the assumption that the asset inflation rate $i^t = 0$, it can be shown that the user cost of an asset that is n periods old at the start of period t can be written as $u_n^t = (r^t + \delta_n^t)P_n^t$ where P_n^t is the beginning of period t second hand market price for the asset.

"Let P be the price of a capital good. Let p be its gross income, that is, the price of its service inclusive of both the depreciation charge and the insurance premium. Let μP be the portion of this income representing the depreciation charge and νP the portion representing the insurance premium. What remains of the gross income after both charges have been deducted, $\pi = p - (\mu + \nu)P$, is the net income. We are now able to explain the differences in gross incomes derived from various capital goods having the same value, or conversely, the differences in values of various capital goods yielding the same gross incomes. It is, however, readily seen that the values of capital goods are rigorously proportional to their net incomes. At least this would have to be so under certain normal and ideal conditions when the market for capital goods is in equilibrium. Under equilibrium conditions the ratio $[p - (\mu + \nu)P]/P$, or the rate of net income, is the same for all capital goods. Let i be this common ratio. When we determine i, we also determine the prices of all landed capital, personal capital and capital goods proper by virtue of the equation $p - (\mu + \nu)P = iP$ or $P = p/[i + \mu + \nu]$." Léon Walras (1954; 268-269)[224].

However, the basic idea that a durable input should be charged a period price that is equal to a depreciation term plus a term that would cover the cost of financial capital goes back much further*42. For example, consider the following quotation from Babbage:

"Machines are, in some trades, let out to hire, and a certain sum is paid for their use, in the manner of rent. This is the case amongst the frame-work knitters: and Mr. Hensen, in speaking of the rate of payment for the use of their frames, states, that the proprietor receives such a rent that, besides paying the full interest for his capital, he clears the value of his frame in nine years. When the rapidity with which improvements succeed each other is considered, this rent does not appear exorbitant. Some of these frames have been worked for thirteen years with little or no repair." Charles Babbage (1835; 287)[6].

Babbage did not proceed further with the user cost idea. Walras seems to have been the first economist who formalized the idea of a user cost into a mathematical formula. However, the early industrial engineering literature also independently came up with the user cost idea; Church described how the use of a machine should be charged as follows:

"No sophistry is needed to assume that these charges are in the nature of rents, for it might easily happen that in a certain building a number of separate little shops were established, each containing one machine, all making some particular part or working on some particular operation of the same class of goods, but each shop occupied, not by a wage earner, but by an independent mechanic, who rented his space, power and machinery, and sold the finished product to the lessor. Now in such a case, what would be the shop charges of these mechanics? Clearly they would comprise as their chief if not their only item, just the rent paid. And this rent would be made up of: (1) Interest. (2) Depreciation. (3) Insurance. (4) Profit on the capital involved in the building, machine and power-transmitting and generating plant. There would also most probably be a separate charge for power according to the quantity consumed. Exclude the item of profit, which is not included in the case of a shop charge, and we find that we have approached most closely to the new plan of reducing any shop into its constituent production centres. No one would pretend that there was any insuperable difficulty involved in fixing a just rent for little shops let out in this plan." A. Hamilton Church (1901; 907-908)**??**.

"A production centre is, of course, either a mechanic, or a bench at which a hand craftsman works. Each of these is in the position of a little shop carrying on one little special industry, paying rent for the floor space occupied, interest for the capital involved, depreciation for the

*42 Solomons (1968; 9-17)[197] indicates that interest was regarded as a cost for a durable input in much of the nineteenth century accounting literature. The influential book by Garcke and Fells (1893)[98] changed this.

> wear and tear, and so on, *quite independently of what may be paid by other production centres* in the same shop." A. Hamilton Church (1901; 734)**??**.

Church was well aware of the importance of determining the "right" rate to be charged for the use of a machine in a multiproduct enterprise. This information is required not only to price products appropriately but to determine whether an enterprise should make or purchase a particular commodity. Babbage and Canning were also aware of the importance of determining the right machine rate charge:[*43]

> "The great competition introduced by machinery, and the application of the principle of the subdivision of labour, render it necessary for each producer to be continually on the watch, to discover improved methods by which the cost of the article he manufactures may be reduced; and, with this view, it is of great importance to know the precise expense of every process, as well as of the wear and tear of machinery which is due to it." Charles Babbage (1835; 203)[6].

> "The question of 'adequate' rates of depreciation, in the sense that they will ultimately adjust the valuations to the realities, is often discussed as though it had no effect upon ultimate profit at all. Of some modes of valuing, it is said that they tend to overvalue some assets and to undervalue others, but the aggregate of book values found is nearly right. If the management pay no attention at all to the unit costs implied in such valuations, no harm is done. But if the cost accountant gives effect to these individually bad valuations through a machine-rate burden charge, and if the selling policy has regard for apparent unit profits, the valuation may lead to the worst rather than to the best possible policy." John B. Canning (1929; 259-260).

The above equations relating asset prices by age P_n^t, beginning of the period user costs f_n^t, end

[*43] Under moderate inflation, the difficulties with traditional cost accounting based on historical cost and no proper allowance for the opportunity of capital, the proper pricing of products becomes very difficult. Diewert and Fox (1999; 271-274)[72] argued that this factor contributed to the great productivity slowdown that started around 1973 and persisted to the early 1990's. The traditional method of cost accounting can be traced back to a book first published in 1887 by the English accountants, Garcke and Fells, who suggested allocating the "indirect costs" of producing a good proportionally to the amount of labour and materials costs used to make the item: "In some establishments the direct expenditures in wages and materials only is considered to constitute the cost; and no attempt is made to allocate to the various working or stock orders any portion of the indirect expenses. Under this system the difference between the sum of the wages and materials expended on the articles and their selling price constitutes the gross profit, which is carried in the aggregate to the credit of profit and loss, the indirect factory expenses already referred to, together with the establishment expenses and depreciation, being particularised on the debit side of that account. This method has certainly simplicity in its favour, but a more efficient check upon the indirect expenses would be obtained by establishing a relation between them and the direct expenses. This may be done by distributing all the indirect expenses, such as wages of foremen, rent of factory, fuel, lighting, heating, and cleaning, etc. (but not the salaries of clerks, office rent, stationery and other establishment charges to be referred to later), over the various jobs, as a percentage, either upon the wages expended upon the jobs respectively, or upon the cost of both wages and materials." Emile Garcke and John Manger Fells (1893; 70-71)[98]. Compare this rather crude approach to cost accounting to the masterful analysis of Church! Garcke and Fells endorsed the idea that deprecation was an admissible item of cost that should be allocated in proportion to the prime cost (i.e., labour and materials cost) of manufacturing an article but they explicitly ruled out interest as a cost: "The item of Depreciation may, for the purpose of taking out the cost, simply be included in the category of the indirect expenses of the factory, and be distributed over the various enterprises in the same way as those expenses may be allocated; or it may be dealt with separately and more correctly in the manner already alluded to and hereafter to be fully described. The establishment expenses and interest on capital should not, however, in any case form part of the cost of production. There is no advantage in distributing these items over the various transactions or articles produced. They do not vary proportionately with the volume of business. ... The establishment charges are, in the aggregate, more or less constant, while the manufacturing costs fluctuate with the cost of labour and the price of material. To distribute the charges over the articles manufactured would, therefore, have the effect of disproportionately reducing the cost of production with every increase, and the reverse with every diminution, of business. Such a result is greatly to be deprecated, as tending to neither economy of management nor to accuracy in estimating for contracts. The principles of a business can always judge what percentage of gross profit upon cost is necessary to cover fixed establishment charges and interest on capital." Emile Garcke and John Manger Fells (1893; 72-73)[98]. The aversion of accountants to include interest as a cost can be traced back to this quotation.

of the period user costs u_n^t and the (cross sectional) depreciation rates by age of asset δ_n^t are the fundamental ones that we will specialize in Appendix A below in order to measure both capital stocks and capital services under conditions of inflation. In the following section, we shall consider several options that could be used in order to determine empirically the interest rates r^t and the asset inflation rates i^t that appear in these user cost formulae.

Note that all of the algebra developed above can be applied not only to reproducible capital stock components (equipment and machinery and structures) but also to stocks of inventories and land that are used by the production unit. Typically, we set the depreciation rates for inventory and land components equal to zero. This is only approximately correct, since theft and spoilage of some inventory components can give rise to positive depreciation rates and environmental degradation could be regarded as a depreciation component for some land stocks.

1.5 The Empirical Determination of Interest Rates and Asset Inflation Rates

What is the "correct" nominal interest rate r^t that should be used in the various user cost formulae that were developed in the previous section? We consider eight theoretical approaches that might be used to answer this question.

If the production unit raises financial capital by a combination of debt and equity financing, then it would seem to be appropriate to choose the reference nominal interest rate r^t for a particular period t to be a weighted average of its anticipated period cost of debt and equity for that period. Since determining the average interest rate for debt would seem to be a reasonably straightforward exercise,[*44] in the first two approaches, we will focus on various alternative approaches that have been suggested in the literature for the determination of the equity opportunity cost of capital. In the subsequent approaches, we look at methods that have been suggested to determine a relevant opportunity cost of capital for the entire stock of nonfinancial capital held by the firm.

Approach 1: Discounted Cash Flow

Suppose that a company's current period t dividends D^t are expected to grow at the constant real rate g for the indefinite future and that the expected inflation rate for the indefinite future is ρ. The company's current share price S^t should equal the discounted future expected dividends. The discount rate should be the long run cost of equity capital r^t minus the anticipated inflation rate ρ. Under these assumptions, we should have the following relationship between the company's current share price S^t and current dividend rate D^t:

$$S^t = D^t/(r^t - g - \rho). \tag{1.18}$$

Formula (1.18) can be rearranged to give the following formula for the cost of equity capital:

$$r^t = (D^t/S^t) + g + \rho. \tag{1.19}$$

This method for determining the opportunity cost of equity capital is due to Williams (1938)[229] and Gordon and Shapiro (1956)[101]. According to Myers (1992; 489)[168], this method is widely used to determine allowed equity rates of return for regulated utilities in the United States.

There are many problems with this method. The determination of the anticipated future inflation rate ρ will be problematical given that past inflation rates have been very variable during the past

[*44] If all bonds were one period bonds, then there would be no major problems. However, when some debt is floated using multiperiod bonds, there are some difficult problems involved in determining the appropriate period by period interest rate when there are changes in the market price of the bond over time.

three decades. Dividend growth rates are also variable over the business cycle. Finally, dividend price ratios of the form D^t/S^t are also tremendously variable and moreover, this method is not suitable for the determination of an economy wide equity cost of capital since many businesses are not incorporated and many incorporated businesses do not have publicly traded shares.

Approach 2: The Capital Asset Pricing Model

Under certain assumptions, the *capital asset pricing model* of Sharpe (1964)[194], Lintner (1965)[149] and Mossin (1966)[166] yields the following relationship between the expected cost of capital for a company r^t, a safe or riskless interest rate r_s^t and the expected return on a market portfolio of assets r_m^t:

$$r_e^t = r_s^t + \beta(r_m^t - r_s^t) \tag{1.20}$$

where β is the covariance between the company's equity rate of return and the market portfolio rate of return divided by the variance of the market portfolio rate of return. Given a time series of ex post company rates of return r^t, market rates of return r_m^t and the safe rate of return r_s^t, ex post returns can be substituted into (1.20) in place of the anticipated returns r_e^t and β can be estimated in a regression model.*45 Alternatively, an estimate for β can be constructed by taking an average of past covariances $\mathrm{Cov}(r_e^t; r_m^t)$ divided by $\mathrm{Var}(r_m^t)$. Given this estimator for β, an ex ante r_e^t can be calculated as the right hand side of (1.20) where r_m^t is a forecast for the period t market rate of return.

Some of the assumptions that are required to derive (1.20) are: (i) each investor is a von Neumann and Morgenstern (1947)[221] expected utility maximizer with the same preferences over current period consumption and end of the period wealth; (ii) a riskless one period asset actually exists; (iii) all investors have preferences over the same set of risky assets and the common riskless asset; (iv) all investors have the same expectations about the returns, variances and covariances of the risky assets and (v) there are no transactions costs. All of these assumptions are somewhat suspect from the empirical point of view. Machina (1992; 860-862)[154] documents some of the empirical evidence which contradicts the expected utility model. In particular, Mehra and Prescott (1985)[163] show that the equity premium over the safe asset seems to be too large for generally agreed upon values of relative risk aversion. Epstein and Zinn (1990)[85] explain this equity premium by a generalization of the usual expected utility model that allows for first order risk aversion.*46 Assumption (ii), the assumption that a perfectly safe one period asset exists, is also problematical: nominal government bonds are not risk free due to inflation risk. Assumption (iii) is also dubious: what is the relevant set of risky assets facing any investor? Should we include housing or foreign stock markets? Our β estimates will generally change as we change our definition of the "market" for risky assets. Assumption (iv) is also problematical: what will happen to our estimate for β as we include or exclude data for 1987, the year of the great worldwide stock market crash? Finally, assumption (v) is also far from being satisfied. Although the capital asset pricing model could be used to estimate the cost of equity capital for some companies whose shares are traded in a stock market, it cannot be used to estimate the cost of equity capital for many companies and for the economy as a whole since a large proportion of private sector companies are not listed on any stock exchange.

Approach 3: The Ex Post Return Method

To illustrate this approach, we suppose for simplicity that the production unit uses only K_0^t units of a new capital asset at the beginning of period t and u_0^t defined by (1.16) above is the corresponding user cost concept that we wish to use. Suppose further that we have somehow estimated the relevant

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*45 For an example of this approach, see Nagorniak (1972; 351)[170].

*46 For more recent discussions of the equity premium puzzle, see Benninga and Protopapadakis (1992; 770)[14], Burnside and McCurdy (1992)[22] and Diewert (1993a; 427-432)[57].

depreciation rate δ_0^t as well as the expected rate of asset price inflation i^t. Let CF^t be the period t cash flow for the production unit.*47 The period t ex post return to capital for this production unit, r^t, can be obtained by setting cash flow equal to the value of capital services and then solving for the balancing r^t; i.e., we solve the following equation for r^t:

$$CF^t = [r^t - i^t + (1+i^t)\delta_0^t]P_0^t K_0^t. \tag{1.21}$$

The resulting r^t is called the *ex post nominal rate of return to capital employed* and it could be used as the period t reference opportunity cost of capital for this production unit.

The ex post cost of capital method for determining the opportunity cost of capital that is based on solving equation (1.21) (and its generalizations to the case of many assets) for r^t is due to Jorgenson and Griliches (1967)[138] (1972)[139] and Christensen and Jorgenson (1969)**??**.*48 This method has been used frequently in the regulatory context, starting with Christensen, Schoech and Meitzen (1995)[30]. In addition to the simplicity of this method, Christensen, Schoech and Meitzen (1995; 10)[30] note that this method can be applied in a symmetric manner to both a single enterprise as well as to the economy as a whole. National statistical agencies that have programs that measure the productivity of market sector industries generally use this method.*49 From a national income accounting perspective, this method has the great advantage for statistical agencies that it preserves the present structure of the System of National Accounts 1993; i.e., the resulting user costs just sum to the present Gross Operating Surplus that is already in SNA 1993. Thus this method can be viewed as a straightforward elaboration of the present system of accounts which does not change its basic structure; it only provides a decomposition of Gross Operating Surplus or Cash Flow into more basic components; see (1.21) above.*50

The problem with the ex post return method for estimating r^t is that it does not correspond to a true opportunity cost of capital for the business unit; instead, it corresponds to an ex post measure of period t *performance* for the business unit.*51 In the present chapter, what we are attempting to do is to find a way to construct an approximation to a market rental price for an asset (using information on used asset prices) when information on market rental prices is not available. Put another way, what we are doing is looking at the factors that would assist an owner of an asset to decide what price to charge for the services of the asset in the rental market. Hence, our measure of user cost should be *forward looking* or an *ex ante measure* rather than an ex post measure of how things actually turned out during the period. Hicks had some relevant thoughts on the issue of ex ante versus ex post measures:

> "*Ex post* calculations of capital accumulation have their place in economic and statistical *history*; they are a useful measuring rod for economic progress; but they are of no use to theoretical economists, who are trying to find out how the economic system works, because

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*47 Cash flow for period t is defined as the value of goods and services sold during period t less the value of intermediate inputs used in period t less the gross value of labour costs. In national income accounting terminology, cash flow is called gross operating surplus.

*48 These authors set their expected period t asset inflation rates i^t equal to the ex post asset inflation rate that occurred over period t. This use of ex post asset inflation rates will generally lead to very volatile user costs and even negative user costs in many cases which means that these user costs are not suitable for some purposes. Some official productivity programs replace negative user costs by tiny positive ones. We discuss these issues below.

*49 The Bureau of Labor Statistics (1983)[21] in the U.S. was the first country to introduce an official program to measure Multi Factor Productivity or Total Factor Productivity; see Dean and Harper (2001)[41] Other countries with TFP programs now include Canada, Australia, the UK and New Zealand.

*50 However, this advantage of this ex post method for determining an endogenous balancing rate of return is not decisive: if an exogenous r^t were used in the user cost formula, we could simply add another line to the list of primary inputs called "pure profits or losses" and have a reconciling item that would allow gross output less intermediate input to equal the sum of primary input payments.

*51 See chapter **??** below.

> they have no significance for conduct. The income *ex post* of any particular week cannot be calculated until the end of the week, and then it involves a comparison between present values and values which belong wholly to the past. On the general principle of 'bygones are bygones', it can have no relevance to present decisions." J.R. Hicks (1939; 179)[112].

Thus in our present context where we are trying to construct counterparts to market rental rates for durable inputs, ex ante measures are preferred to ex post measures (which could be useful in evaluating ex post economic performance—a different context).

Approach 4: The Average of Past Ex Post Nominal Returns

Instead of using the ex post returns to capital employed method outlined in Approach 3 above, we could switch to a forecasting framework, using some sort of an average of past ex post returns to capital to *forecast* a current period opportunity cost of capital. The problem with methods of this type is their arbitrariness: which ex post approach to the determination of the opportunity cost of capital should be used? Which forecasting method should be used? How far back in time should we go? It may be difficult to reach agreement on what is the most reasonable specific method in this general class of methods. Statistical agencies will be reluctant to use this method because of its lack of reproducibility: different statistical agencies would no doubt use somewhat different forecasting or smoothing methods and obtain different results—perhaps leading to a lack of international comparability in the System of National Accounts. Moreover, as we argued at the end of Approach 3 above, ex post returns incorporate pure profits (or losses) and hence are not true ex ante opportunity costs for equity capital. Nevertheless, over long periods of time, averages of ex post rates of return should approximate an ex ante rate of return that is appropriate to use in a user cost formula, at least when general inflation is not too variable.

Approach 5: The Use of An Exogenous Nominal Market Interest Rate

In this method, a relevant market interest rate is used as a proxy for the equity opportunity cost of capital. This market interest rate could be: (i) the prime lending rate that banks or other financial institutions charge borrowers in "similar" lines of business; or (ii) an index of ex ante interest rates constructed by some reputable private or public agency. As an example of (ii), Christensen, Schoech and Meitzen (1995)[30] used the Moody's public utility bond as a proxy for the cost of capital for a regulated utility. These authors noted that this method has the advantages that the Moody bond yield is publicly available and is updated annually.

This method has some attractions for both regulators and national statistical agencies since it is simple and (somewhat) reproducible.

Approach 6: The Use of An Official Nominal Rate of Return

In this approach, a government or regulatory agency would set an "official" interest rate that could be used to approximate a business unit's cost of equity capital. For example, the official rate might be: (i) the interest rate that is used by the taxation authorities to assess late payment of income taxes; (ii) an equity interest rate that is recommended by the country's accounting standards board or (iii) the midpoint of a regulator's range of acceptable returns to equity capital for a regulated firm. A problem with this method is that there is no guarantee that the official rate set by a taxation authority, accounting standards board or regulator will be "reasonable"; i.e., this method gives no guidance on how the authority will in fact determine the "official" rate. In practice, official rates determined by the tax authorities are probably based on Approach 5(ii) outlined above.

Approach 7: The Use of an Official Real Rate of Return

This approach kills two birds with one stone; i.e., it determines not only the reference nominal interest rate r^t but it also determines the anticipated asset inflation rate i^t that appears in the user cost formulae developed above.

This approach works as follows. Let the consumer price index for the economy at the beginning of period t be c^t say. Then the ex post general consumer inflation rate for period t is ρ^t defined as:

$$1 + \rho^t \equiv c^{t+1}/c^t. \tag{1.22}$$

We assume that the production unit under consideration faces the constant real interest rate r^*. Then by the Fisher (1896)[88] effect*52, the relevant period t nominal interest rate that the producer faces should be approximately equal to r^t defined as follows:

$$r^t \equiv (1 + r^*)(1 + \rho^t) - 1. \tag{1.23}$$

The Australian Bureau of Statistics assumes that producers face an annual real interest rate of 4%, so that $r^* = .04$. This is consistent with long run observed economy wide real rates of return for most OECD countries which fall in the 2 to 5 per cent range.

The final assumption made in order to implement this approach is to assume that the producer anticipates that each asset inflation rate is equal to the general inflation rate ρ^t defined by (1.22); i.e., we assume:

$$i^t = \rho^t. \tag{1.24}$$

With this assumption, the equations relating stock and flow prices simplify dramatically. Thus if we replace $1 + i^t$ by $1 + \rho^t$ and $1 + r^t$ by $(1 + r^*)(1 + \rho^t)$, equations (1.5), which relate the vintage asset prices Pnt to the vintage rental prices f_n^t, become:

$$\begin{aligned}
P_0^t &= f_0^t + [1/(1 + r^*)]f_1^t + [1/(1 + r^*)]^2 f_2^t + [1/(1 + r^*)]^3 f_3^t + \cdots \\
P_1^t &= f_1^t + [1/(1 + r^*)]f_2^t + [1/(1 + r^*)]^2 f_3^t + [1/(1 + r^*)]^3 f_4^t + \cdots \\
&\cdots \\
P_n^t &= f_n^t + [1/(1 + r^*)]f_{n+1}^t + [1/(1 + r^*)]^2 f_{n+2}^t + [1/(1 + r^*)]^3 f_{n+3}^t + \cdots
\end{aligned} \tag{1.25}$$

Note that only the constant real interest rate r^* appears in these equations.

If we substitute equations (1.14) into equations (1.7) and replace $1 + i^t$ by $1 + \rho^t$ and $1 + r^t$ by $(1 + r^*)(1 + \rho^t)$ in the resulting equations, we obtain the following equations, which relate the *beginning of period user costs by age* f_n^t to the price of a new asset P_0^t, the real interest rate r^* and the asset depreciation rates by age δ_n^t:

$$\begin{aligned}
f_0^t &= (1 + r^*)^{-1}[r^* + \delta_0^t]P_0^t \\
f_1^t &= (1 + r^*)^{-1}(1 - \delta_0^t)[r^* + \delta_1^t]P_0^t \\
&\cdots \\
f_n^t &= (1 + r^*)^{-1}(1 - \delta_0^t)\cdots(1 - \delta_{n-1}^t)[r^* + \delta_n^t]P_0^t; \cdots
\end{aligned} \tag{1.26}$$

Note that only the constant real interest rate r^* appears in equations (1.26) and the anticipated asset inflation rates i^t have disappeared.

Now replace $1 + i^t$ by $1 + \rho^t$ and $1 + r^t$ by $(1 + r^*)(1 + \rho^t)$ in equations (1.15) in order to obtain the following formulae for the sequence of the *end of the period user costs by age* $\{u_n^t\}$ in terms of the

*52 The Fisher effect was independently derived by several economists: "Thornton, Marshall, Wicksell, Fisher, Keynes and others have known that the own rate and money rate of interest must diverge by a term equal to the percentage price of the good in terms of which the own rate is measured." Paul A Samuelson (1961; 40)[188].

real interest rate r^*, the price of a new asset at the beginning of period t, P_0^t, the sequence of cross section depreciation rates, $\{\delta_n^t\}$ and the period t general inflation rate ρ^t:

$$\begin{aligned}
u_0^t &= (1+\rho^t)[r^* + \delta_0^t]P_0^t \\
u_1^t &= (1+\rho^t)(1-\delta_0^t)[r^* + \delta_1^t]P_0^t \\
&\cdots \\
u_n^t &= (1+\rho^t)(1-\delta_0^t)\cdots(1-\delta_{n-1}^t)[r^* + \delta_n^t]P_0^t; \cdots
\end{aligned} \tag{1.27}$$

Comparing (1.26) and (1.27), we see that it is slightly easier to work with the f_n^t rather than the u_n^t, since the former user costs do not contain the nuisance variable ρ^t, the general inflation rate.*[53]

Although the assumptions that were made to derive equations (1.26) and (1.27) were quite strong, the end result is very attractive to statistical agencies that may want a simple reproducible approach to the determination of user costs. The user costs defined by (1.26) are particularly attractive because *they will always be positive* and *they will be relatively smooth*; the main source of intertemporal variation is the variation in the beginning of the period stock price of a new asset, P_0^t.

Approach 8: The Use of a Long Run Average Ex Post Real Rate of Return

Approach 7 can be combined with Approach 4 to give us approach 8. Make the same simplifying assumptions that we made in Approach 3 except redefine the period t cash flow CF^t of the business unit to be realized at the beginning of period t. Then determine the period t balancing ex post real rate of return for the business unit, r^{*t}, to be the real interest rate that solves the following counterpart to (1.21):

$$CF^t = f_0^t K_0^t = (1+r^{*t})^{-1}[r^{*t} + \delta_0^t]P_0^t K_0^t \text{ or} \tag{1.28}$$

$$CF^t(1+r^{*t}) = [r^{*t} + \delta_0^t]P_0^t K_0^t. \tag{1.29}$$

Note that equation (1.29) is linear in the balancing real rate of return, r^{*t}, and so it is easy to determine this period t balancing r^*. Now obtain a time series of these balancing ex post real rates of return for the business unit, take the average of these rates (or use some other prediction or smoothing method) and use this average as the reference real rate.

Any of the Approaches 3-8 could be used in empirical applications. The relative simplicity and reproducibility of Approaches 7 and 8 is appealing.

We turn now to a discussion of possible methods for determining the expected asset inflation rates, the i^t rates, which appear in the f_n^t user costs defined by (1.7) and (1.14) and the u_n^t user costs defined by (1.15). There are four possible methods that could be used.

Method 1: Assume All Anticipated Asset Inflation Rates are Zero

This is one very simple reproducible method for determining the i^t that could be used if the general rate of inflation is low. In addition to simplicity and reproducibility, it has the advantage of leading to always positive user costs.*[54] However, if general inflation is somewhat variable and greater than zero, then the resulting user costs will be consistently above market rental prices for the services of the asset and the variability in the general inflation rate will generally lead to variability in the nominal reference interest rate r^t via the Fisher (1896)[88] effect, leading to unduly variable user costs. Thus this method can only be recommended if general inflation is close to zero.

*[53] However, for most practical accounting purposes, the end of period user costs will generally be preferred.

*[54] Recall that we want our user costs to be a close approximation to the one period rental rate for the asset, if such rental markets existed. A negative user cost can usually *not* be a very close approximation to a market rental price because if an asset had a negative rental price, then the seller of the service is *paying* renters of the service to use it! Under these conditions, unless transactions costs were very high, the negative rental price would soon be bid up to zero.

Method 2: Use the Actual Ex Post Asset Inflation Rates

Thus the ex ante anticipated asset inflation rate i^t for a new asset is approximated by the actual ex post asset inflation rate for the new asset that materialized over period t. This method also has the advantages of simplicity and reproducibility but it has two very big disadvantages:

- The resulting user costs can often turn out to be negative and as we pointed out earlier, a negative user cost is not a good approximation to a market rental rate.
- The resulting time series of user costs is invariably very volatile, suggesting again that this type of user cost is not usually going to be a very close approximation to a market rental rate, which will usually be much smoother.

Thus this method cannot be recommended if we want our user costs to be a reasonably close approximation to market rental rates.

Method 3: Use the Ex Post Asset Inflation Rates to Forecast the Ex Ante Rate

This method follows up on Method 2 in that past ex post asset inflation rates are used to predict what the current period asset inflation rate will be. A variant of this method is to simply smooth the time series of ex post asset inflation rates. This method has much to recommend it in that it probably captures the actual dynamics of market rentals (if the "right" forecasting method were used). It will generally lead to considerably smoother user costs than the user costs generated by Method 2.[*55] However, this method suffers from two problems:

- The method will sometimes generate negative user costs, leading to somewhat ad hoc methods for solving this problem.
- The method is not completely reproducible; i.e., different forecasting or smoothing methods will generally lead to different estimates for the anticipated asset inflation rates.

In spite of the above difficulties, this method can be recommended for assets which are "generally" known to be consistently increasing in price (land in some cases although it is easy to run into the negative user cost problem in this case) or consistently decreasing in price over time (any equipment that uses computer chips in a predominant manner).

Method 4: Expected Asset Inflation Rates are Equal to the General Inflation Rate

This is assumption (1.24) above; i.e., the assumption that the expected specific asset inflation rate i^t is equal to the general inflation rate ρ^t. Approaches 7 and 8 explained above showed how this assumption led to fairly sensible, smooth and reproducible user costs so this material does not have to be repeated here.

To sum up: Approaches 7 and 8 and Methods 3 and 4 are our favored methods for determining the r^t and i^t terms that appeared in our user cost formula.[*56]

The present system of national accounts, SNA 1993, does not recognize interest as an explicit cost of production in the production accounts. Also the accounting profession has generally been unwilling

[*55] See Harper, Berndt and Wood (1989)[108] and Diewert (2001)[62] for empirical evidence on this point. Harper, Berndt and Wood (1989; 351)[108] noted that the use of time series techniques to smooth ex post asset inflation rates and the use of such estimates as anticipated price change dates back to Epstein (1977)[83].

[*56] Other methods for determining the appropriate interest rates that should be inserted into user cost formulae are discussed by Harper, Berndt and Wood (1989)[108] and Schreyer (2001)[193]. Harper, Berndt and Wood (1989)[108] evaluated empirically 5 alternative rental price formulae using geometric depreciation but making different assumptions about the interest rate and the treatment of asset price change. They showed that the choice of formula matters. Diewert (2001)[62] (2005a)[67] reached a similar conclusion in his comparison of various user cost formulae.

to have imputed interest on equity capital appear as a cost in historical accounting. Thus although it seems fairly obvious to most economists that interest is a cost of production, there is still some controversy on this point in the accounting profession. We will explore this topic more fully in Chapter 2 below. However, over the years, there have been many accountants who agree with including imputed interest on equity capital as a cost and we conclude this section with some quotes that illustrate this point of view.

> "Once again, the basic reason why interest on the use of total capital should be recorded as a cost is that interest is a cost ... . A company has not performed satisfactorily, either for its shareholders or for society, if it has not generated enough revenue to cover all its costs, including the cost of using capital. The current income statement does not show whether or not the company has met this fundamental test. Its implication is that any earnings above the cost of debt interest are a 'plus'." Robert N. Anthony (1973; 96)[4].

> "The argument in favor of including interest as an element of cost is twofold. From the viewpoint of the business as a whole, it helps to point out an important fact to the managers of any enterprise which persistently fails to return a normal current rate of interest on the investment. From the more detailed cost accounting viewpoint, it is said to make an important cost distinction between those manufacturing departments using costly machinery and those using inexpensive machinery or none at all." Stephen Gilman (1939; 322)[99].

1.6 The Empirical Determination of Depreciation Rates

The empirical determination of asset depreciation rates is not an easy task. The OECD (1993)[171] conducted a survey of average asset lives used by national statisticians for various asset classes in 14 OECD countries. For machinery and equipment (excluding vehicles) used in manufacturing activities, the average life ranged from 11 years for Japan to 26 years for the United Kingdom. For vehicles, the average service lives ranged from 2 years for passenger cars in Sweden to 14 years in Iceland and for road freight vehicles, the average life ranged from 3 years in Sweden to 14 years in Iceland. For buildings, the average service lives ranged from 15 years (for petroleum and gas buildings in the U.S.) to 80 years for railway buildings in Sweden. Faced with this wide range of possible lives, Angus Madison (1993)[156] simply assumed an average service life of 14 years for machinery and equipment and 39 years for nonresidential structures.

In addition to just guessing average asset lives and converting this information into depreciation rates, there are four possible evidence based methods for determining depreciation rates, which we will now outline.

Method 1: Approaches Based on Used Asset Prices

> "If there is a perfect second hand market for the goods in question, so that a market value can be assessed for them with precision, corresponding to each particular degree of wear, then the value-loss due to consumption can be exactly measured ...". John R. Hicks (1946; 176)[114].

> "Some depreciation patterns have very little economic justification (except accounting convenience), but most of them at least purport to approximate the decline in the economic value of the remaining services (i.e., market value). Of the various possible depreciation schemes (net stock measures), two measures seem to be of the most interest: (a) a net stock concept based on a purely physical deterioration depreciation scheme, and (b) the market value of the existing stock of capital. The latter figure can be approximated by the use of depreciation rates derived from studies of used machinery prices." Zvi Griliches (1963; 120)[103].

Economists (like Hicks and Griliches) and accountants (like Bell and Edwards)[*57] have long realized that a possible method for estimating the decline in value of a durable input due to its use over an accounting period is to use information on the market prices of used assets at a point in time and to compare differences in price as a function of the age of the input. This is the method we used in section 1.4 above. Thus given market data on the prices of used assets at any point in time, period by period depreciation rates δ_n^t can be obtained by using equations (1.13).[*58] If we have information on used asset prices for many different time periods t and we are willing to make the assumption that depreciation rates are stable over time, so that δ_n^t equals δ_n, then a stochastic specification of a variant of (1.13) can be made and econometric techniques can be used to estimate the sequence of one period depreciation rates. If the market data on used asset prices is sparse, then instead of estimating a completely general pattern of period to period depreciation rates, various restrictions on these parameters can be imposed.[*59] The simplest such restriction is that δ_n be constant from period to period. Empirical studies of depreciation rates using second hand asset prices have been made by the accountant Beidleman (1973)[11] (1976)[12][*60] and the economists Hall (1971)[106], Hulten and Wykoff (1981a)[129] (1981b)[130] and Oliner (1996)[172]. The literature on this used asset approach is ably reviewed by Hulten and Wykoff (1996)[131][*61] and Jorgenson (1996a)[136].

Many economists and accountants have objected to the use of second hand data to estimate depreciation rates for a variety of reasons:

> "We readily agree that where a market is sufficiently large, generally accessible, and continuous over time, it serves to coordinate a large number of subjective estimates and thus may impart a moment of (social) objectivity to value relations based on prices formed on it. But it can hardly be said that the second-hand market for industrial equipment, which would be the proper place for the determination of the value of capital goods which have been in use, satisfies these requirements, and that its valuations are superior to intra-enterprise valuation." L.M. Lachmann (1941; 376-377)[144].

> "But why, if market values are the key to asset values, does not the accountant find depreciation by direct reference to market quotations for assets of different ages, and abandon his formulae? Various answers suggest themselves ... . Second-hand markets tend to be small and scrubby, so that quotations may be hard to find and harder to trust. Many assets are built specifically

[*57] "But as a practical matter the quantification and valuation of asset services used is not a simple matter and we must fall back on estimated patterns as a basis for current cost as well as historic cost depreciation. For those fixed assets which have active second hand markets the problem is not overly difficult. A pattern of service values can be obtained at any time by comparing the market values of assets of different ages or degrees of use. The differences so obtained, when related to the value of a new asset, yield the proportions of asset value which are normally used up or foregone in the various stages of asset life." Edgar O. Edwards and Philip W. Bell (1961; 175)[81].

[*58] Beidleman (1973)[11] (1976)[12] and Hulten and Wykoff (1981a)[129] (1996; 22)[131] showed that equations (1.13) must be adjusted to correct for early retirement of assets; i.e., equations (1.13) assume that all units of the asset are retired at the same time. Schmalenbach (1959; 91)[190] noted that neglect of the survival problem leads to serious errors in the estimation of depreciation rates.

[*59] See Jorgenson (1996a; 27-28)[136] for a nice summary of the methods that have been used to date.

[*60] "The findings of this chapter provide extensive evidence regarding the predominant role of age in the decline in value of certain fixed assets and the relative unimportance of valuation parameters other than age. The initial rapid decline in second hand value calculated for the regression models supports the use of accelerated depreciation techniques and the approach to finite scrap value favors declining balance methods of depreciation. The range of possible asset lives endorses the need for probability life depreciation." Carl R. Beidleman (1973; 51-52)[11].

[*61] "We have used this approach to study the depreciation patterns of a variety of fixed business assets in the United States ... . The straight-line and concave patterns are strongly rejected; geometric is also rejected, but the estimated patters are extremely close to (though steeper than) the geometric form, even for structures. Although it is rejected statistically, the geometric pattern is far closer to the estimated pattern than either of the other two candidates. This leads us to accept the geometric pattern as a reasonable approximation for broad groups of assets ... ." Charles R. Hulten and Frank C. Wykoff (1996; 16)[131].

for the one firm, and therefore worn replicas do not exist. An owner usually regards his own worn assets as different from, and better than, replicas in the market, because he knows their history, condition and foibles." William T. Baxter (1971; 31)[7].

"One argument, drawing on the Akerlof Lemons Model, is that assets resold in second hand markets are not representative of the underlying population of assets, because only poorer quality units are sold when used. Others express concerns about the thinness of resale markets, believing that it is sporadic in nature and is dominated by dealers who under-bid." Charles R. Hulten and Frank C. Wykoff (1996; 17-18)[131].

In spite of the above objections to the use of the second hand market method for estimating depreciation rates, this method seems more "objective" than simply guessing at the appropriate rates.

A more serious objection to the above model of depreciation rate determination is that the method includes only length of asset service or time in use as an explanatory variable and thus the method neglects variations in the intensity of use of the durable input.*62 There are at least two ways of meeting this criticism:

- We can follow the advice of Edwards and Bell*63 and estimate separate sequences of depreciation rates that pertain to assets that are used with approximately the same intensity and have similar maintenance policies or
- We can incorporate utilization and maintenance variables as explanatory variables in stochastic versions of (1.13).*64

Jorgenson (1996a, 27-29)[136] reviews the literature on these extensions of the basic used asset model of depreciation rate determination.

Method 2: Approaches Based on Rental Prices

Suppose that a rental market for a durable input exists so that we can observe in period t the beginning of the period rental prices for assets of age n, f_n^t, which appear in the user cost formulae (1.7) or with (1.14) substituted into (1.7). Then given information on the firm's cost of capital r^t, on the rental price for a new asset f_0^t, and on the expected rate of price inflation for a new asset i^t, we can use the first equation in (1.7) and the first equation in (1.14) to solve for the period t depreciation rate for a new asset, δ_0^t. Once δ_0^t has been determined, then given information on the rental price for a one period old asset f_1^t, then we can use the second equation in (1.7) and the second equation in (1.14) to solve for the period t depreciation rate for a one period old asset, δ_1^t, etc. Thus given that rental markets exist for durable inputs being used by a business unit, these

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*62 This criticism of depreciation theory dates back to Saliers (1922; 172-174)[187] at least. Many other authors noted this problem: "The question of charging depreciation as a function of output rather than of time has been discussed of late. It is more natural to consider the depreciation of an automobile in terms of miles than of years." Harold Hotelling (1925; 352)[126]. "While there is much to be said in favor of depreciating automobiles and trucks on a mileage basis, rather than by the number of years of use, and while a similar expedient may well be adopted in distributing the depreciation of other machinery and equipment, it must be observed that depreciation is seldom a sole function of use or time. Generally it is a combination of the two, and it is often desirable to check one method by applying the other." Stephen Gilman (1939; 345-346)[99]. "The two main defects of depreciation data are found to be that they ignore variations in the degree of utilisation, and that they are largely based on original rather than reproduction cost." L.M. Lachmann (1941; 375)[144].

*63 "A truck used to haul logs in timber country is not likely to yield the same pattern of services as one used to haul produce over superhighways. Physically identical assets having sharply different uses should be placed in separate categories and treated as different assets, for example, logging trucks and produce trucks. How fine a distinction should be drawn is a matter of practicality." Edgar O. Edwards and Philip W. Bell (1961; 174)[81].

*64 The first approach could be viewed as a special case of the second if we allowed discrete classification variables in place of continuous ones. In his study of used automobile prices, Hall (1971)[106] used the first approach, which Jorgenson (1996a; 27)[136] termed the "analysis of variance approach". Beidleman (1973)[11] (1976)[12] used the second approach, which Jorgenson (1996c; 28) termed the "hedonic approach".

rental prices can be equated to the corresponding user costs and depreciation rates can be derived from the resulting equations.

Of course, if rental markets do not exist (and for the most part they will not), then this method will not work, an obvious point made by Hulten and Wykoff (1996;15)[131]. Jorgenson (1996a; 32)[136] reviewed the few studies that have used this method. Even when rental markets exist, this method may not generate very accurate depreciation rates, because it is very sensitive to the assumptions made about the "correct" nominal opportunity cost of capital r^t and the "correct" expected asset inflation rate i^t.*65 Also the transactions costs in the rental and leasing markets may be very high, creating additional complications and measurement errors. However, in some markets (perhaps the leasing of structures market), the method may work well. Finally, even though the rental price method is unlikely to be a useful method for the empirical determination of depreciation rates, rental prices are useful when they exist since they can be used as period t (opportunity) costs for the use of the corresponding durable inputs during period t.

Method 3: Approaches Based on Production Function Estimation

Suppose a durable input is used by a business unit and it purchases I^t units of it at the beginning of period t in order to produce y^t units of output using the vector $\boldsymbol{x}^t$ of nondurable inputs*66 during period t as well as the services of past purchases of the durable input. Suppose further that the durable input lasts N periods and that after adjusting for physical loss of efficiency, all unretired units of the durable input are perfect substitutes in production. The production function which relates output flow to inputs used during a period is F and if there is no technological progress, we have the following relationship between output produced and inputs used during period t:

$$y^t = F[\boldsymbol{x}^t; I^t + (1-\delta_0)I^{t-1} + (1-\delta_0)(1-\delta_1)I^{t-2} + \cdots + (1-\delta_0)(1-\delta_1)\cdots(1-\delta_{N-1})I^{t-N}] \quad (1.30)$$

where $\delta_0, \delta_1, ..., \delta_{N-1}$ are the one period depreciation rates for a new asset, a one period old asset, etc. Note that we are assuming that these depreciation rates are constant across time periods t.

The *production function method* for determining depreciation rates works as follows:

- Collect data on output produced during period t, y^t, nondurable inputs used, $\boldsymbol{x}^t$, and durable input purchases, I^t, for a number of periods t.
- Assume a functional form for the production function F.
- Add a stochastic specification to equations (1.30) for $t = 0, 1, ..., T$.
- Use econometric techniques to simultaneously estimate the unknown parameters which appear in the production function F as well as the depreciation rates $\delta_0, \delta_1, ..., \delta_{N-1}$.

Variants of this basic method include:

- Restricting the depreciation parameters $\delta_0, \delta_1, ..., \delta_{N-1}$ in some a priori fashion (e.g., make them all equal to a common δ).
- Using the assumption of short run profit maximizing or cost minimizing behavior on the part of the business unit in order to add extra estimating equations involving period t prices to the single estimating equation (1.30).
- Instead of estimating the production function F, estimate the unknown parameters in the dual cost or profit function.*67

*65 Diewert (1983a; 212)[51] (1983b; 1100-1102)[52] discussed these expectational difficulties with ex ante user cost formulae and noted that given the user cost formula for u_0^t and information on r^t, P_0^t and as estimate for the depreciation rate δ_0^t, then the first equation in (1.15) could be used to estimate the producer's anticipated rate of asset price inflation i^t.

*66 If the business unit produces more than one output, the additional outputs can be absorbed into the $\boldsymbol{x}^t$ vector as negative inputs.

*67 For a review of duality theory and the associated functional form problems, see Diewert (1993b)[58].

Empirical studies using this approach to the estimation of depreciation rates include Epstein and Denny (1980)[84], Pakes and Griliches (1984)[174], Nadiri and Prucha (1996)[169] and Doms (1996)[78]. It should be noted that the depreciation rates which are estimated using this production function approach may be different from the estimates that result from the used asset approach studied above. The latter approach incorporates the effects of exhaustion, deterioration and obsolescence (to use Griliches' terminology), while the production function approach typically incorporates only the effects of physical deterioration and exhaustion.*68

There are some problems with the production function approach:

- The approach will only work in a highly aggregated model with a small number of outputs, nondurable inputs and durable inputs due to the difficulties involved in estimating the parameters pertaining to a general production function when there are numerous inputs and outputs.*69
- The assumption that the different vintages of capital can be combined together in the additive capital aggregate that appears as the last term on the right hand side of (1.30) is restrictive; i.e., it is assumed that the depreciation adjusted different vintages of capital are perfect substitutes in production, an assumption which may or may not be true.
- The estimates of the depreciation rates may not be invariant to the degree of disaggregation of the other inputs and outputs used by the business unit.
- Econometric estimates are often sensitive to the stochastic specification of the model and the method of estimation. In other words, econometrically based estimates are often not reproducible: different econometricians using the same data base will often come up with very different answers, particularly in models with many parameters.

Thus as a good general method for the empirical determination of depreciation rates, the production function method is not entirely satisfactory. However, it can provide a check on whether other assumption based estimates for depreciation rates are consistent with the data; i.e., the production function method is at least evidence based.

Method 4: Approaches Based on Insured Values or Other Expert Appraisals

Since most businesses insure their structures against accidental loss, insurance appraisals on the value of structures (and other property) provide an objective source of information.*70 Engineers and other experts may be able to provide reasonably accurate estimates for the value of machinery and equipment components.

The main problem with insurance based value information is that it is not readily made available to the outside observer.

Our overall conclusion in evaluating different methods for the empirical determination of depreciation rates is that the used asset price method seems best when the relevant second hand markets actually exist. However, for firm specific assets that are not traded in second hand markets, it appears

*68 Pigou (1935; 238)[181] clearly distinguished exhaustion (or decline in useful life) from physical deterioration: "A distinction should be drawn between physical changes which, while leaving the element as productive as ever, bring nearer the day of sudden and final breakdown, and physical changes which reduce its current productivity and so rentable value."

*69 If we assume a flexible functional form for the production function F, the number of parameters to be estimated will grow approximately as the square of the number of inputs and outputs that are distinguished in the model. If we do not assume a flexible functional form for F, then our a priori restrictive assumptions on the substitution possibilities for the technology will generally lead to biased estimates for the depreciation rates. These difficulties with the production function approach are discussed in more detail by Diewert (1992a; 177)[54].

*70 Insurance companies have an incentive to avoid appraised values that are too high and the owner of the asset being insured has an incentive to avoid appraised values that are too low. Hence appraised values for insurance purposes are likely to be more reproducible than other expert appraisals.

that depreciation will have to be determined on the basis of engineering estimates or other expert appraisals.

In the next section, we discuss a topic of some current interest: namely the interaction of (foreseen) obsolescence and depreciation. We also discuss cross sectional versus time series depreciation.

1.7 Time Series versus Cross Sectional Depreciation

We begin this section with a definition of the time series depreciation of an asset. Define the *ex post time series depreciation* of an asset that is n periods old at the beginning of period t, E_n^t, to be its second hand market price at the beginning of period t, P_n^t, less the price of an asset that is one period older at the beginning of period $t+1$, P_{n+1}^{t+1}; i.e.,

$$E_n^t \equiv P_n^t - P_{n+1}^{t+1}; \qquad n = 0, 1, 2, \ldots \tag{1.31}$$

Definitions (1.31) should be contrasted with our earlier definitions (1.11), which defined the *cross section amounts of depreciation* for the same assets at the beginning of period t, $D_n^t \equiv P_n^t - P_{n+1}^t$.

We can now explain why we preferred to work with the cross sectional definition of depreciation, (1.11), over the time series definition, (1.31). The problem with (1.31) is that time series depreciation captures the effects of changes in *two* things: changes in *time* (this is the change in t to $t+1$)[*71] and changes in the *age* of the asset (this is the change in n to $n+1$).[*72] Thus time series depreciation aggregates together two effects: the asset specific price change that occurred between time t and time $t+1$ *and* the effects of asset aging (depreciation). Thus the time series definition of depreciation combines together two distinct effects. At first sight, the fact that time series depreciation combines two effects does not seem to be a problem. But there is always a potential problem when we compare values at two different time periods: we must be aware that *general changes in the purchasing power of money can make comparisons across time very misleading*. Thus as a general rule, when comparing prices or values across time, in order to achieve comparability, the two values must either be adjusted for general price level change or the later value should be deflated by 1 plus the relevant nominal interest rate.[*73]

The above definition of ex post time series depreciation is the original definition of depreciation and it extends back to the very early beginnings of accounting theory:

> "[There are] various methods of estimating the Depreciation of a Factory, and of recording alteration in value, but it may be said in regard to any of them that the object in view is, so to treat the nominal capital in the books of account that it shall always represent as nearly as possible the real value. Theoretically, the most effectual method of securing this would be, if it were feasible, to revalue everything at stated intervals, and to write off whatever loss such valuations might reveal without regard to any prescribed rate... The plan of valuing every year instead of adopting a depreciation rate, though it might appear the more perfect, is too tedious and expensive to be adopted ... The next best plan, which is that generally followed ... is to establish average rates which can without much trouble be written off every year, to check the result by complete or partial valuation at longer intervals, and to adjust the depreciation rate if required." Ewing Matheson (1910; 35)[160].

Hotelling, in the first mathematical treatment of depreciation in continuous time, defined time series depreciation in a similar manner:

[*71] This change could be captured by either $P_n^t - P_n^{t+1}$ or $P_{n+1}^t - P_{n+1}^{t+1}$.

[*72] This change could be captured by either $P_n^t - P_{n+1}^t$ or $P_n^{t+1} - P_{n+1}^{t+1}$.

[*73] Of course, a nominal interest rate should contain within it a general price change component; i.e., it contains the Fisher effect.

> "Depreciation is defined simply as rate of decrease of value." Harold Hotelling (1925; 341)[126].

However, what has to be kept in mind that these early authors who used the concept of time series depreciation were implicitly or explicitly assuming that *prices were stable across time*, in which case, time series and cross sectional depreciation will normally coincide.

Hill (2000)[120] recently argued that a form of time series depreciation was to be preferred over cross section depreciation for national accounts purposes:

> "The basic cost of using an asset over a certain period of time consists of depreciation, the decline in the value of that asset, plus the associated financial, or capital cost. An alternative definition of depreciation has been proposed in recent years in what may be described as the vintage accounting approach to depreciation. In the context of vintage accounting, depreciation is defined as the difference between the value of an asset of age k and one of age $k+1$ *at the same point of time*, the two assets *being identical except for their ages*. This concept, although superficially the same as the traditional concept, is in fact radically different because it effectively rules out obsolescence from depreciation *by definition*.
>
> The issue is not a factual one about whether obsolescence does or does not cause the value of assets to decline over time. The question is how should such a decline be interpreted and classified. Whereas the traditional concept of depreciation treats expected obsolescence as an integral part of depreciation, in the vintage approach it is treated as a separate item, a revaluation, which has to be treated as real holding loss in the SNA. Reclassifying part of what has always been treated as depreciation in both business and economic accounting as a holding loss would reduce depreciation and increase every balancing item in the SNA from Net National Product and Income to net saving." Peter Hill (2000; 6)[120].

Thus Hill argued that cross sectional depreciation does not capture the effects of obsolescence and hence it is not the "right" concept for business and national accounts depreciation since obsolescence charges are equivalent to the effects of wear and tear and hence should be regarded as depreciation. Hill's observations on this topic date back to the controversy between Pigou and Hayek in the 1940's. Pigou argued that depreciation should be measured relative to a concept that maintained capital intact from a physical point of view:

> "I accept too the view that, if maintaining capital intact has to be defined in such a way that capital need not be maintained intact even though every item in its physical inventory is unaltered, the concept is worthless. But the inference I draw is, not that we should abandon the concept; rather that we should try to define it in such a way that, when the physical inventory of goods in the capital stock is unaltered, capital *is* maintained intact; more generally, in such a way that, not indeed the quantity of capital—which, with heterogeneous items, can only a conventionalised number—is independent of the equilibrating process, but changes in its quantity are independent of changes in that process." A.C. Pigou (1941; 273)[182].

Pigou (1941; 274)[182] went on to suggest that the Paasche quantity index for capital could be used to determine whether capital was maintained intact between two points in time; i.e., the price weights of the second point in time should be used to value the two capital stocks. Hence if the two capital stocks were unchanged in each component, the Paasche quantity index would be equal to unity, correctly indicating that there was no physical change in the capital stock between the two points in time. However, Hayek responded, correctly, that Pigou's concept of maintaining capital intact would neglect *foreseen obsolescence*:

> "Professor Pigou's answer to the question of what is meant by 'maintaining capital intact' consists in effect of the suggestion that for this purpose we should disregard obsolescence and require merely that such loses of value of the existing stock of capital goods be made good as are due to physical wear and tear. ... If Professor Pigou's criterion is to be of any help, it

would have to mean that we have to disregard *all* obsolescence, whether it is due to foreseen or foreseeable causes, or whether it is brought about by entirely unpredictable causes, such as the 'acts of God or the King's enemy', which alone he wanted to exclude in an earlier discussion of this problem." F.A. v. Hayek (1941; 276)[110].

Hayek went on to give a clear example of where Pigou's point of view would lead to a mismeasurement of depreciation and income:

> "Assume three entrepreneurs, X,Y, and Z, to invest at the same time in equipment of different kinds but of the same cost and the same potential physical duration, say ten years. X expects to be able to use his machine continuously throughout the period of its physical 'life'. Y, who produces some fashion article, knows that at the end of one year his machine will have no more than its scrap value. Z undertakes a very risky venture in which the changes of employing the machine continuously so long as it lasts and having to scrap it almost as soon as it starts to produce are about even. According to Professor Pigou the three entrepreneurs will have to order their investments in such a way that during the first year they can expect to earn the same gross receipts: since the wear and tear of their respective machines during the first years will be the same, the amount they will have to put aside during the first year to 'maintain their capital intact' will also be the same, and this procedure will therefore lead to their earning during that year the same 'net' income from the same amount of capital. Yet it is clear that the foreseen result of such dispositions would be that at the end of the year X would still possess the original capital, Y one tenth of it, while Z would have an even chance of either having lost it all or just having preserved it. ... To treat all receipts except what is required to make good physical wear and tear as net income for income tax purposes would evidently discriminate heavily against industries where the rate of obsolescence is high and reduce investment in these industries below what is desirable." F.A. v. Hayek (1941; 276-277)[110].[*74]

Since the depreciation rates δ_n^t defined by (1.13) are cross sectional depreciation rates (and thus seemingly reflect only wear and tear depreciation) and since they play a key role in the definitions of the beginning of period t user costs f_n^t defined by (1.7) and (1.14) and the end of period user costs u_n^t defined by (1.15), it is necessary to clarify their use in the context of Hayek and Hill's point that simple wear and tear depreciation rates should not be used to measure depreciation in the national accounts (or in business accounts), since they neglect anticipated obsolescence.

Our response to the Hayek and Hill critique is threefold:

- Our user costs f_n^t and u_n^t that are based on cross sectional depreciation rates δ_n^t *are* affected by anticipated obsolescence in principle but
- Hill is correct in arguing that cross section depreciation *will not* generally equal ex post time series depreciation or anticipated time series depreciation and
- Hill is correct in arguing that our cross sectional depreciation based user costs defined by f_n^t and u_n^t *will not* capture obsolescence effects that cause asset lives to shrink[*75] (but f_n^t and u_n^t *will* capture obsolescence effects that work by reducing the real price of the asset in future periods).

[*74] The accounting literature has been wrestling with the appropriate treatment of expected obsolescence for a long time as well: "In a number of industries development has been so rapid and revolutionary over a period of years that the capitalization of losses due to so-called premature retirements would have led to an absurd inflation of asset values. In such situations, the use of higher depreciation rates, rather than capitalization of losses, is indicated." W.A. Paton (1931; 93)[178]. This issue was important in Paton's time with respect to the regulation of public utilities and it is still important today.

[*75] Ahmad, Aspden and Schreyer (2005)[1] make this point as well in a recent paper. We further illustrate this point in chapter **??** below.

Let us consider the first point above. Provisionally, we define *anticipated obsolescence* as a situation where the expected new asset inflation rate (adjusted for quality change) i^t is *negative.*[*76] For example, everyone anticipates that the quality adjusted price for a new computer next quarter will be considerably lower than it is this quarter.[*77] Now turn back to equations (1.5) above, which define the sequence of asset prices by age P_n^t at the start of period t. It is clear that the negative i^t plays a role in defining the sequence of asset prices by age as does the sequence of rental prices by age that is observed at the beginning of period t, the f_n^t. Thus in this sense, cross sectional depreciation rates certainly embody assumptions about anticipated obsolescence.

Zvi Griliches had a nice verbal description of the factors which explain the pattern of used asset prices, which will cast some light on our present subject:

> "The net stock concept is motivated by the observed fact that the value of a capital good declines with age (and/or use). This decline is due to several factors, the main ones being the decline in the life expectancy of the asset (it has fewer work years left), the decline in the physical productivity of the asset (it has poorer work years left) and the decline in the relative market return for the productivity of this asset due to the availability of better machines and other relative price changes (its remaining work years are worth less). One may label there three major forces as exhaustion, deterioration and obsolescence." Zvi Griliches (1963; 119)[103].

Thus for an asset that has a finite life, as we move down the rows of equations (1.5), the number of discounted rental terms decline and hence asset value declines, which is Griliches' concept of *exhaustion*. If the cross sectional rental prices are monotonically declining with age (due to their declining efficiency), then this corresponds Griliches' concept of *deterioration*. Finally, a negative anticipated asset inflation rate will cause all future period rentals to be discounted more heavily, which could be interpreted as Griliches' concept of *obsolescence*.[*78] Thus all of these explanatory factors are imbedded in equations (1.5).[*79]

[*76] Paul Schreyer and Peter Hill noted a problem with this provisional definition of anticipated obsolescence as a negative value of the expected asset inflation rate: it will not work in a high inflation environment. In a high inflation environment, the nominal asset inflation rate i^t will generally be positive but we will require this nominal rate to be less than general inflation in order to have anticipated obsolescence. Thus our final definition of *anticipated obsolescence* is that the real asset inflation rate i^{*t} defined later by (1.37) be negative; see the discussion just above equation (1.39) below.

[*77] Our analysis assumes that the various vintages of capital are adjusted for quality change (if any occurs) as they come on the market. We also need to assume that the form of quality change affects all future efficiency factors (i.e., the f_n^t) in a proportional manner. Thus we are assuming that technical change is of the capital augmenting type. This is obviously only a rough approximation to reality: technical change may increase the durability of a capital input or it may decrease the amount of maintenance or fuel that is required to operate the asset. These changes can lead to nonproportional changes in the f_n^t. We will consider more general models of technical change in chapter **??**.

[*78] However, our present model does not capture obsolescence that may be caused by a future abrupt decline in the rents earned by the asset due to say rising real wage rates or anticipated changes in tastes (recall Hayek's example of the Y entrepreneur); i.e., our present model has the weakness that it projects the present pattern of rents earned by the various vintages of the asset into the future using a single geometric price escalation factor, i^t. Thus our present model cannot deal with cases where at some future date, all of the assets are expected to earn zero rent due to technological progress or shifts in demand. See chapter **??** for examples of this type of phenomenon. Our original model defined by equations (1.2) is flexible enough to deal with Hayek's example of the Y entrepreneur but it is too general to be used in empirical applications without further information on how expectations are formed.

[*79] "Normal wear-and-tear in the course of production is clearly a reason why the value of a capital instrument should be greater at the beginning of a year than at the end, even if the final value was foreseen accurately. Normal wear-and-tear is therefore an element of true depreciation. So is exceptional wear-and-tear, due to exceptionally heavy usage; if the exceptionally heavy usage had been foreseen, the gap between the beginning-value and the end-value would have been larger. On the other hand, any deterioration which the machine undergoes outside its utilisation does *not* give rise to true depreciation; if such deterioration had been foreseen, the initial capital value would have been written down in consequence; the deterioration which it undergoes

Now let us consider the second point: that cross section depreciation is not really adequate to measure time series depreciation in some sense to be determined.

Define the *ex ante time series depreciation* of an asset that is n periods old at the beginning of period t, Δ_n^t, to be its second hand market price at the beginning of period t, P_n^t, less the *anticipated price* of an asset that is one period older at the beginning of period $t+1$, $(1+i^t)P_{n+1}^t$; i.e.,

$$\Delta_n^t \equiv P_n^t - (1+i^t)P_{n+1}^t; \qquad n = 0, 1, 2, \ldots \tag{1.32}$$

Thus ex ante or anticipated time series depreciation for an asset that is t periods old at the start of period t, Δ_n^t, differs from the corresponding cross section depreciation defined by (1.11), $D_n^t \equiv P_n^t - P_{n+1}^t$, in that the anticipated new asset inflation rate, i^t, is missing from D_n^t. However, note that the two forms of depreciation will coincide if the expected asset inflation rate i^t is zero.

We can use equations (1.13) and (1.14) in order to define the ex ante depreciation amounts Δ_n^t in terms of the cross section depreciation rates δ_n^t. Thus using definitions (1.32), we have:

$$\begin{aligned}
\Delta_n^t &\equiv P_n^t - (1+i^t)P_{n+1}^t \qquad n = 0, 1, 2, \ldots \\
&= P_n^t - (1+i^t)(1-\delta_n^t)P_n^t \qquad \text{using (1.13)} \\
&= [1 - (1+i^t)(1-\delta_n^t)]P_n^t \\
&= (1-\delta_1^t)(1-\delta_2^t)\cdots(1-\delta_{n-1}^t)[1-(1+i^t)(1-\delta_n^t)]P_0^t \qquad \text{using (1.14)} \\
&= (1-\delta_1^t)(1-\delta_2^t)\cdots(1-\delta_{n-1}^t)[\delta_n^t - i^t(1-\delta_n^t)]P_0^t.
\end{aligned} \tag{1.33}$$

We can compare the above sequence of ex ante time series depreciation amounts Δ_n^t with the corresponding sequence of cross sectional depreciation amounts:

$$\begin{aligned}
D_n^t &\equiv P_n^t - P_{n+1}^t \qquad n = 0, 1, 2, \ldots \\
&= P_n^t - (1-\delta_n^t)P_n^t \qquad \text{using (1.13)} \\
&= [1-(1-\delta_n^t)]P_n^t \\
&= (1-\delta_1^t)(1-\delta_2^t)\cdots(1-\delta_{n-1}^t)[\delta_n^t]P_0^t \qquad \text{using (1.14).}
\end{aligned} \tag{1.34}$$

Of course, if the anticipated asset inflation rate i^t is zero, then (1.33) and (1.34) coincide and ex ante time series depreciation equals cross sectional depreciation. If we are in the provisional expected obsolescence case with i^t negative, then it can be seen comparing (1.33) and (1.34) that

$$\Delta_n^t > D_n^t \qquad \text{for all } n \text{ such that } D_n^t > 0; \tag{1.35}$$

i.e., if i^t is negative (and $0 < \delta_n^t < 1$), then ex ante time series depreciation exceeds cross section depreciation over all in use vintages of the asset. If i^t is *positive* so that the rental price of each vintage is expected to rise in the future, then ex ante time series depreciation is *less* than the corresponding cross section depreciation for all assets that have a positive price at the end of period t. This corresponds to the usual result in the user cost literature where capital gains or an ex post price increase for a new asset leads to a negative term in the user cost formula (plus a revaluation of the cross section depreciation rate). Here we are restricting ourselves to *anticipated* capital gains rather than the *actual* ex post capital gains and we are focusing on depreciation concepts rather than the full user cost.

This is not quite the end of the story in the high inflation context. National income accountants often readjust asset values at either the beginning or end of the accounting period to take into account

is therefore not depreciation, but a capital loss." John R. Hicks (1942; 178)[113]. In our view, foreseen price declines in future rentals *are* reflected in initial capital values.

general price level change. At the same time, they also want to decompose nominal interest payments into a real interest component and another component that compensates lenders for general price change.

Recall (1.22), which defined the general period t inflation rate ρ^t and (1.23), which related the period t nominal interest rate r^t to a constant real rate r^* and the inflation rate ρ^t. We rewrite now generalize (1.23) by allowing the *period t real interest rate* r^{*t} to vary over time as follows:

$$1 + r^{*t} \equiv (1 + r^t)/(1 + \rho^t). \tag{1.36}$$

In a similar manner, we define the *period t anticipated real asset inflation rate* i^{*t} as follows:

$$1 + i^{*t} \equiv (1 + i^t)/(1 + \rho^t). \tag{1.37}$$

Recall definition (1.32), which defined the *ex ante time series depreciation* of an asset that is n periods old at the beginning of period t, Δ_n^t. The first term in this definition reflects the price level at the beginning of period t while the second term in this definition reflects the price level at the end of period t. We now express the second term in terms of the beginning of period t price level. Thus we define the *ex ante real time series depreciation* of an asset that is n periods old at the beginning of period t, Π_n^t, as follows:

$$\begin{aligned}
\Pi_n^t &\equiv P_n^t - (1 + i^t)P_{n+1}^t/(1 + \rho^t) \qquad n = 0, 1, 2, \ldots \\
&= P_n^t - (1 + i^t)(1 - \delta_n^t)P_n^t/(1 + \rho^t) \qquad \text{using (1.13)} \\
&= [1 - (1 + i^{*t})(1 + \rho^t)(1 - \delta_n^t)]P_n^t/(1 + \rho^t) \qquad \text{using (1.37)} \\
&= (1 - \delta_0^t)(1 - \delta_1^t) \cdots (1 - \delta_{n-1}^t)[1 - (1 + i^{*t})(1 - \delta_n^t)]P_0^t \qquad \text{using (1.14)} \\
&= (1 - \delta_0^t)(1 - \delta_1^t) \cdots (1 - \delta_{n-1}^t)[\delta_n^t - i^{*t}(1 - \delta_n^t)]P_0^t.
\end{aligned} \tag{1.38}$$

The ex ante real time series depreciation amount Π_n^t defined by (1.38) can be compared to its cross section counterpart D_n^t, defined by (1.34) above. Of course, if the real anticipated asset inflation rate i^{*t} is zero, then (1.38) and (1.34) coincide and real ex ante time series depreciation equals cross section depreciation.[*80]

We are now in a position to provide a more satisfactory definition of expected obsolescence, particularly in the context of high inflation. We now define *expected obsolescence* to be the situation where the *real* asset inflation rate i^{*t} is negative. If the real asset inflation rate is negative, then it can be seen comparing (1.38) and (1.34) that

$$\Pi_n^t > D_n^t \qquad \text{for all } n \text{ such that } D_n^t > 0; \tag{1.39}$$

i.e., real anticipated time series depreciation exceeds the corresponding cross sectional depreciation provided that i^{*t} is negative.[*81]

Thus the general user cost formulae that we have developed from the vintage accounts point of view can be reconciled to reflect the point of view of national income accountants. We agree with Hill's point of view that cross sectional depreciation is not really adequate to measure time series depreciation as national income accountants have defined it since Pigou:

[*80] The ex ante real time series depreciation defined by (1.38) is defined in terms of the perspective of discounting from the beginning of the period. In chapter **??**, we will rework our analysis from the more useful perspective of the end of the period.

[*81] What happens if i^{*t} is positive instead of negative? In this case, real time series depreciation will be less than the corresponding cross sectional depreciation. In chapter **??**, we will argue that real time series depreciation is still an appropriate depreciation concept for accounting purposes even if i^{*t} is positive.

> "Allowance must be made for such part of capital depletion as may fairly be called 'normal'; and the practical test of normality is that the depletion is sufficiently regular to be foreseen, if not in detail, at least in the large. This test brings under the head of depreciation all ordinary forms of wear and tear, whether due to the actual working of machines or to mere passage of time— rust, rodents and so on— and all ordinary obsolescence, whether due to technical advance or to changes of taste. It brings in too the consequences of all *ordinary* accidents, such as shipwreck and fire, in short of all accidents against which it is customary to insure. But it leaves out capital depletion that springs from the act of God or the King's enemies, or from such a miracle as a decision tomorrow to forbid the manufacture of whisky or beer. These sorts of capital depletion constitute, not depreciation to be made good before current net income is reckoned, but capital losses that are irrelevant to current net income." A.C. Pigou (1935; 240-241)[181].

Pigou (1924)[180] in an earlier work had a more complete discussion of the obsolescence problem and the problems involved in defining time series depreciation in an inflationary environment. Pigou first pointed out that the national dividend or net annual income (or in modern terms, real net output) should subtract depreciation or capital consumption:

> "For the dividend may be conceived in two sharply contrasted ways: as the flow of goods and services which is produced during the year, or as the flow which is consumed during the year. Dr. Marshall adopts the former of these alternatives .... Naturally, since in every year plant and equipment wear out and decay, what is produced must mean what is produced on the whole when allowance has been made for this process of attrition. ... In concrete terms, his conception of the dividend includes an inventory of all the new things that are made [i.e., gross investment], accompanied as a negative element, by an inventory of all the decay and demolition of old things [i.e., capital consumption]. A.C. Pigou (1924; 34-35)[180].

Pigou then went on to discuss the roles of obsolescence and general price change in measuring depreciation:

> "The concrete content of the dividend is, indeed, unambiguous— the inventory of things made and (double counting being eliminated) and services rendered, *minus*, as a negative element, the inventory of things worn out during the year. But how are we to *value* this negative element? For example, if a machine originally costing £1000 wears out and, owing to a rise in the general price level, can only be replaced at a cost of £1500, is £1000 or £1500 the proper allowance? Nor is this the only, or, indeed, the principle difficulty. For depreciation is measured not merely by the physical process of wearing out, and capital is not therefore maintained intact when provision has been made to replace what is thus worn out. Machinery that has become obsolete because of the development of improved forms is not really left intact, however excellent its physical condition; and the same thing is true of machinery for whose products popular taste has declined. If, however, in deference to these considerations, we decide to make an allowance for obsolescence, this concession implies that the value, and not the physical efficiency, of instrumental goods [i.e., durable capital inputs] is the object to be maintained intact. But, it is then argued, the value of instrumental goods, being the present value of the services which they are expected to render in the future, necessarily varies with variations in the rate of interest. Is it really a rational procedure to evaluate the national dividend by a method which makes its value in relation to that of the aggregated net product of the country's industry depend on an incident of that kind? If that method is adopted, and a great war, by raising the rate of interest, depreciates greatly the value of existing capital, we shall probably be compelled to put, for the value of the national dividend in the first year of that war, a very large *negative* figure. This absurdity must be avoided at all costs, and we are therefore compelled, when we are engaged in evaluating the national dividend, to leave out of account any change in the value of the country's capital equipment that may have been

> brought about by broad general causes. This decision is arbitrary and unsatisfactory, but it is one which it is impossible to avoid. During the period of the war, a similar difficulty was created by the general rise, for many businesses, in the value of the normal and necessary holding of materials and stocks, which was associated with the general rise of prices. On our principles, this increase of value ought not to be reckoned as an addition to the income of the firms affected, or, consequently, to the value of the national dividend." A.C. Pigou (1924; 39-41)[180].

The above quotation indicates that Pigou was responsible for many of the conventions of national income accounting that persist down to the present day.[*82] He essentially argued that (unanticipated) capital gains or losses be excluded from income and that the effects of general price level change be excluded from estimates of depreciation. He also argued for a physical maintenance of capital concept and ignoring obsolescence although he was not happy about neglecting obsolescence in the depreciation accounts. Unfortunately, he did not spell out *exactly* how all of this could be done in the accounts. Our algebra above can be regarded as an attempt to deal with some of the complications that Pigou raised when measuring depreciation.

It should be noted that the early industrial engineering literature also stressed that the possibility of obsolescence meant that depreciation allowances should be larger than those implied by mere wear and tear:

> "Machinery for producing any commodity in great demand, seldom actually wears out; new improvements, by which the same operations can be executed either more quickly or better, generally superceding it long before that period arrives: indeed, to make such an improved machine profitable, it is usually reckoned that in five years it ought to have paid itself, and in ten to be superceded by a better." Charles Babbage (1835; 285)[6].

> "The possibility of New Inventions, processes, or machines coming into use, which may supercede or render an existing plant Obsolete, is a contingency that presses on most manufacturing trades, principally those which have long established, but sometimes also in new concerns where old methods have been adopted or imitated just as they were being superceded elsewhere." Ewing Matheson (1910; 38)[160].

> "A reserve beyond the ordinary depreciation above described may then become necessary, because the original plant, when once superceded by such inventions, may prove unsaleable as second-hand plant, except in so far as it may have a piecemeal or scrap value. ... This risk sometimes arises, not from improvements in the machinery, but from alterations in the kind of product, rendering new machines necessary to suit new patterns or types. Contingencies such as these should encourage an ample reduction of nominal value in the early years of working, so as to bring down the book value of the plant to a point which will allow even of dismantling without serious loss. In such trades, profits should be large enough to allow for a liberal and rapid writing off of capital value, which is in effect the establishment of a reserve-fund as distinct from depreciation for wear and tear." Ewing Matheson (1910; 39-40)[160].

Thus Matheson considered obsolescence that could arise not only from new inventions but also from shifts in demand.

We will end this section by pointing out another important use for the concept of real anticipated time series depreciation. However, before doing this, it is useful to rewrite equations (1.5), which

[*82] Colin Clark (1940; 31)[33] echoed Pigou's recommendations: "The appreciation in value of capital assets and land must not be treated as an element in national income. Depreciation due to physical wear and tear and obsolescence must be treated as a charge against current income, but not the depreciation of the money value of an asset which has remained physically unchanged. Appreciation and depreciation of capital were included in the American statistics of national income prior to 1929, but now virtually the same convention has been adopted in all countries."

define the beginning of period t asset prices by age P_n^t in terms of the beginning of period t rental prices by age of asset f_n^t and equations (1.7), which define the user costs f_n^t in terms of the asset prices P_n^t, using definitions (1.36) and (1.37), which define the period t real interest rate r^{*t} and expected asset inflation rate i^{*t} respectively in terms of the corresponding nominal rates r^t and i^t and the general inflation rate ρ^t. Substituting (1.36) and (1.37) into (1.5) yields the following system of equations:

$$\begin{aligned}
P_0^t &= f_0^t + [(1+i^{*t})/(1+r^{*t})]f_1^t + [(1+i^{*t})/(1+r^{*t})]^2 f_2^t + [(1+i^{*t})/(1+r^{*t})]^3 f_3^t + \cdots \\
P_1^t &= f_1^t + [(1+i^{*t})/(1+r^{*t})]f_2^t + [(1+i^{*t})/(1+r^{*t})]^2 f_3^t + [(1+i^{*t})/(1+r^{*t})]^3 f_4^t + \cdots \\
&\cdots \\
P_n^t &= f_n^t + [(1+i^{*t})/(1+r^{*t})]f_{n+1}^t + [(1+i^{*t})/(1+r^{*t})]^2 f_{n+2}^t + [(1+i^{*t})/(1+r^{*t})]^3 f_{n+3}^t + \cdots
\end{aligned} \tag{1.40}$$

Similarly, substituting (1.36) and (1.37) into (1.7) yields the following system of equations:

$$\begin{aligned}
f_0^t &= P_0^t - [(1+i^{*t})/(1+r^{*t})]P_1^t = (1+r^{*t})^{-1}[P_0^t(1+r^{*t}) - (1+i^{*t})P_1^t] \\
f_1^t &= P_1^t - [(1+i^{*t})/(1+r^{*t})]P_2^t = (1+r^{*t})^{-1}[P_1^t(1+r^{*t}) - (1+i^{*t})P_2^t] \\
&\cdots \\
f_n^t &= P_n^t - [(1+i^{*t})/(1+r^{*t})]P_{n+1}^t = (1+r^{*t})^{-1}[P_n^t(1+r^{*t}) - (1+i^{*t})P_{n+1}^t]; \cdots
\end{aligned} \tag{1.41}$$

Note that the nominal interest and inflation rates have entirely disappeared from the above equations. In particular, the beginning of the period user costs by age f_n^t can be defined in terms of real variables using equations (1.41) if this is desired. On the other hand, entirely equivalent formulae for the user costs can be obtained using the initial set of equations (1.7), which used only nominal variables. Which set of equations is used in practice can be left up to the judgment of the statistical agency or the user.[*83] The point is that the *careful and consistent use of discounting should eliminate the effects of general inflation from our price variables*; discounting makes comparable cash flows received or paid out at different points of time.

Recall definition (1.38), which defined Π_n^t as the *ex ante real time series depreciation* of an asset that is n periods old at the beginning of period t. It is convenient to convert this amount of depreciation into a *percentage* of the initial price of the asset at the beginning of period t, P_n^t. Thus we define the *ex ante time series depreciation rate for an asset that is n periods old at the start of period t*, π_n^t, as follows:[*84]

$$\begin{aligned}
\pi_n^t &\equiv \Pi_n^t/P_n^t \qquad n = 0,1,2,\ldots \\
&= [P_n^t - (1+i^t)P_{n+1}^t/(1+\rho^t)]/P_n^t \qquad \text{using (1.38)} \\
&= [P_n^t - (1+i^t)(1-\delta_n^t)P_n^t/(1+\rho^t)]/P_n^t \qquad \text{using (1.13)} \\
&= [1 - (1+i^{*t})(1-\delta_n^t)] \qquad \text{using (1.37).}
\end{aligned} \tag{1.42}$$

*83 In particular, it is not necessary for the statistical agency to convert all nominal prices into real prices as a preliminary step before "real" user costs are calculated. The above algebra shows that our nominal user costs f_n^t can also be interpreted as "real" user costs that are expressed in terms of the value of money prevailing at the beginning of period t. However, note that typically, real interest and asset specific inflation rates are likely to be more stable than the corresponding nominal rates.

*84 To see that there can be a very large difference between the cross sectional depreciation rate δ_n^t and the corresponding ex ante time series depreciation rate π_n^t, consider the case of an asset whose vintages yield exactly the same service for each period in perpetuity. In this case, all of the asset prices by age P_n^t would be identical and the cross sectional depreciation rates δ_n^t would all be zero. Now suppose a marvelous new invention is scheduled to come on the market next period which would effectively drive the price of this class of assets down to zero. In this case, i^{*t} would be -1 and substituting this expected measure of price change into definitions (1.42) shows that the ex ante time series depreciation rates would all equal one; i.e., under these conditions, we would have $\pi_n^t = 1$ and $\delta_n^t = 0$ for all ages n.

Now substitute definition (1.13) for the cross sectional depreciation rate δ_n^t into the nth equation of (1.41) and we obtain the following expression for the beginning of period t user cost of an asset that is n periods old at the start of period t:

$$\begin{aligned} f_n^t &= (1+r^{*t})^{-1}[P_n^t(1+r^{*t}) - (1+i^{*t})P_{n+1}^t] \qquad n = 0,1,2,\ldots \\ &= (1+r^{*t})^{-1}[P_n^t(1+r^{*t}) - (1+i^{*t})(1-\delta_n^t)P_n^t] \qquad \text{using (1.13)} \\ &= (1+r^{*t})^{-1}[(1+r^{*t}) - (1+i^{*t})(1-\delta_n^t)]P_n^t \\ &= (1+r^{*t})^{-1}[r^{*t} + \pi_n^t]P_n^t \qquad \text{using (1.42).} \end{aligned} \tag{1.43}$$

Thus the period t user cost for an asset that is n periods old at the start of period t, f_n^t, can be decomposed into the sum of two terms.*85 Ignoring the discount factor, $(1+r^{*t})^{-1}$, the first term is $r^{*t}P_n^t$, which represents the (per unit capital) *real interest cost* of the financial capital that is tied up in the asset, and the second term is $\pi_n^t P_n^t = \Pi_n^t$, which represents a concept of *national accounts depreciation.*

The last line of (1.43) is important if at some stage statistical agencies decide to switch from measures of gross domestic product to measures of net domestic product as their featured output measure. If this change were to occur, then the user cost for each age of capital, f_n^t, should be split up into two terms as in (1.43). The first term, $(1+r^{*t})^{-1}r^{*t}P_n^t$ times the number of units of that age of capital in use, (the real opportunity cost of financial capital) could remain as a primary input charge while the second term, $(1+r^{*t})^{-1}\pi_n^t P_n^t$ times the number of units of that age of capital in use, (real national accounts depreciation) could be treated as an intermediate input charge (similar to the present treatment of imports). The second term would be an offset to gross investment.*86

This completes our discussion of the obsolescence problem.*87 In the next section, we turn our attention to the problem of aggregating across vintages of the same capital good.

1.8 Aggregation over Ages of a Capital Good

In previous sections, we have discussed the beginning of period t *stock price* P_n^t of an asset that is n periods old and the corresponding beginning and end of period *user costs*, f_n^t and u_n^t. The stock prices are relevant for the construction of *real wealth measures* of capital and the user costs are relevant for the construction of *capital services measures.* We now address the problems involved in obtaining quantity series that will match up with these prices.

Let the period $t-1$ investment in a homogeneous asset for the sector of the economy under consideration be I^{t-1}. We assume that the starting capital stock for a new unit of capital stock at the beginning of period t is K_0^t and this stock is equal to the new investment in the asset in the previous period; i.e., we assume:

$$K_0^t \equiv I^{t-1}. \tag{1.44}$$

*85 Alternatively, we could decompose $r^{*t} + \pi_n^t$ into the three terms $r^{*t} + \delta_n^t + i^{*t}(1-\delta_n^t)$ which is equal to a real interest rate term plus a cross sectional depreciation term plus a revaluation term.

*86 Using this methodology, we would say that capital is being *maintained intact* for the economy if the value of gross investments made during the period (discounted to the beginning of the period) is equal to or greater than the sum of the real national accounts depreciation terms over all assets. This is *a maintenance of financial capital concept* as opposed to Pigou's *maintenance of physical capital concept*: "Net income consists of the whole of the annual output minus what is needed to maintain the stock of capital intact; and this stock is kept intact provided that its physical state is held constant." A.C. Pigou (1935; 235)[181]. We will discuss these points in more detail in chapter **??**.

*87 It should be noted that our discussion of the obsolescence issue only provides an introduction to the many thorny issues that make this area of inquiry quite controversial. For further discussion, see Oulton (1995)[173], Scott (1995)[189] and Triplett (1996)[215] and the references in these papers. We will take up the discussion of obsolescence again when we discuss the Solow Harper model of technical change in chapter **??**.

Essentially, we are assuming that the length of the period is short enough so that we can neglect any contribution of investment to current production; a new capital good becomes productive only in the period immediately following its construction. In a similar manner, we assume that the capital stock available of an asset that is n periods old at the start of period t is K_n^t and this stock is equal to the gross investment in this asset class during period $t-n-1$; i.e., we assume:

$$K_n^t \equiv I^{t-n-1}; \qquad n = 0, 1, 2, \ldots \tag{1.45}$$

Given these definitions, the value of the capital stock in the given asset class for the sector of the economy under consideration (*the wealth capital stock*) at the start of period t is

$$\begin{aligned} W^t &\equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \cdots \\ &= P_0^t I^{t-1} + P_1^t I^{t-2} + P_2^t I^{t-3} + \cdots \qquad \text{using (1.45).} \end{aligned} \tag{1.46}$$

Turning now to the capital services quantity, we assume that the quantity of services that an asset of a particular vintage at a point in time is proportional (or more precisely, is equal) to the corresponding stock. Thus we assume that the quantity of services provided in period t by a unit of the capital stock that is n periods old at the start of period t is K_n^t defined by (1.45) above. Given these definitions, the value of capital services for all ages of the asset in the given asset class for the sector of the economy under consideration (*the productive services capital stock*) during period t using the end of period user costs u_n^t defined by equations (1.15) above is

$$\begin{aligned} S^t &\equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \cdots \\ &= u_0^t I^{t-1} + u_1^t I^{t-2} + u_2^t I^{t-3} + \cdots \qquad \text{using (1.45).} \end{aligned} \tag{1.47}$$

Now we are faced with the problem of decomposing the value aggregates W^t and S^t defined by (1.46) and (1.47) into separate price and quantity components. If we assume that each new unit of capital lasts only a finite number of periods, L say, then we can solve this value decomposition problem using normal index number theory. Thus define the period t *vintage stock price and quantity vectors*, $\boldsymbol{P}^t$ and $\boldsymbol{K}^t$ respectively, as follows:

$$\boldsymbol{P}^t \equiv [P_0^t, P_1^t, \ldots, P_{L-1}^t];\ \boldsymbol{K}^t \equiv [K_0^t, K_1^t, \ldots, K_{L-1}^t] = [I^{t-1}, I^{t-2}, \ldots, I^{t-L-1}]; \quad t = 0, 1, \ldots, T. \tag{1.48}$$

Fixed base or chain indexes may be used to decompose value ratios into price change and quantity change components. In the empirical work which follows, we have used the chain principle.[*88] Thus the value of the capital stock in period t, W^t, relative to its value in the preceding period, W^{t-1}, has the following index number decomposition:

$$W^t/W^{t-1} = P(\boldsymbol{P}^{t-1}, \boldsymbol{P}^t, \boldsymbol{K}^{t-1}, \boldsymbol{K}^t) Q(\boldsymbol{P}^{t-1}, \boldsymbol{P}^t, \boldsymbol{K}^{t-1}, \boldsymbol{K}^t); \qquad t = 1, 2, \ldots, T \tag{1.49}$$

where P and Q are *bilateral price and quantity indexes* respectively.

In a similar manner, we define the period t *vintage end of the period user cost price and quantity vectors*, $\boldsymbol{u}^t$ and $\boldsymbol{K}^t$ respectively, as follows:

$$\boldsymbol{u}^t \equiv [u_0^t, u_1^t, \ldots, u_{L-1}^t];\ \boldsymbol{K}^t \equiv [K_0^t, K_1^t, \ldots, K_{L-1}^t] = [I^{t-1}, I^{t-2}, \ldots, I^{t-L-1}]; \quad t = 0, 1, \ldots, T. \tag{1.50}$$

We ask that the value of capital services in period t, S^t, relative to its value in the preceding period, S^{t-1}, has the following index number decomposition:

$$S^t/S^{t-1} = P(\boldsymbol{u}^{t-1}, \boldsymbol{u}^t, \boldsymbol{K}^{t-1}, \boldsymbol{K}^t) Q(\boldsymbol{u}^{t-1}, \boldsymbol{u}^t, \boldsymbol{K}^{t-1}, \boldsymbol{K}^t); \qquad t = 1, 2, \ldots, T \tag{1.51}$$

[*88] Given smoothly trending price and quantity data, the use of chain indexes will tend to reduce the differences between Paasche and Laspeyres indexes compared to the corresponding fixed base indexes and so chain indexes are generally preferred; see Diewert (1978; 895)[48] for a discussion.

where again P and Q are *bilateral price and quantity indexes* respectively.

There is now the problem of choosing the functional form for either the price index P or the quantity index Q.[*89] For empirical work, we recommend the *Fisher* (1922)[91] *ideal price and quantity indexes*. These indexes appear to be "best" from the axiomatic viewpoint[*90] and can also be given strong economic justifications.[*91]

It should be noted that our use of an index number formula to aggregate both stocks by age and services by age is more general than the usual aggregation over age procedures, which essentially assume that the different ages of the same capital good are perfectly substitutable so that linear aggregation techniques can be used.[*92] However, as we shall see in the Appendix, the more general method of aggregation suggested here frequently reduces to the traditional linear method of aggregation provided that the vintage asset prices all move in strict proportion over time.

Many researchers and statistical agencies relax the assumption that an asset lasts only a fixed number of periods, L say, and make assumptions about the distribution of retirements around the average service life, L. In Appendix A below that considers different assumptions about the form of cross sectional depreciation, for simplicity, we will stick to the sudden death assumption; i.e., that all assets in the given asset class are retired at age L. However, this simultaneous retirement assumption can readily be relaxed (at the cost of much additional computational complexity) using the following methodology suggested by Hulten:[*93]

> "We have thus far taken the date of retirement T to be the same for all assets in a given cohort (all assets put in place in a given year). However, there is no reason for this to be true, and the theory is readily extended to allow for different retirement dates. A given cohort can be broken into components, or subcohorts, according to date of retirement and a separate T assigned to each. Each subcohort can then be characterized by its own efficiency sequence, which depends among other things on the subcohort's useful life T_i." Charles R. Hulten (1990; 125)[127].

We now have all of the pieces that are required in order to decompose the capital stock of an asset class and the corresponding capital services into price and quantity components. However, in order to construct price and quantity components for capital services, we need information on the relative efficiencies f_n^t of the various vintages of the capital input or equivalently, we need information on cross sectional vintage depreciation rates δ_n^t in order to use (1.49) and (1.51) above. The problem is that we do not have accurate information on either of these series so in Appendix A below, we will assume a standard asset life L and make additional *assumptions* on the either the pattern of vintage efficiencies or depreciation rates. Thus in a sense, we are following the same somewhat mechanical strategy that was used by the early cost accountants:[*94]

> "The function of depreciation is recognized by most accountants as the provision of a means for spreading equitably the cost of comparatively long lived assets. Thus if a building will be of use during twenty years of operations, its cost should be recognized as operating expense, not of the first year, nor the last, but of all twenty years. Various methods may be proper in so allocating cost. The method used, however, is unimportant in this connection. The

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[*89] Obviously, given one of these functional forms, we may use (1.40) to determine the other.

[*90] See Diewert (1992b; 214-223)[55].

[*91] See Diewert (1976; 129-134)[46].

[*92] This more general form of aggregation was first suggested by Diewert and Lawrence (2000)[73]. For descriptions of the more traditional linear method of aggregation, see Jorgenson (1989; 4)[135] or Hulten (1990; 121-127)[127] (1996; 152-165)[128].

[*93] Recall that Edwards and Bell (1961; 174)[81] made a similar suggestion.

[*94] Canning (1929; 204) criticized this strategy as follows: "The interminable argument that has been carried on by the text writers and others about the relative merits of the many formulas for measuring depreciation has failed, not only to produce the real merits of the several methods, but, more significantly, it has failed to produce a rational set of criteria of excellence whereby to test the aptness of any formula for any sub-class of fixed assets."

> important matter is that at the time of abandonment, the cost of the asset shall as nearly as possible have been charged off as expense, under some systematic method." M.B. Daniels (1933; 303)[38].

However, our mechanical strategy is more complex than that used by early accountants in that we translate assumptions about the pattern of cross sectional depreciation rates into implications for the pattern of vintage rental prices and asset prices, taking into account the complications induced by discounting and expected future asset price changes.

In the following section, we discuss the relationship of our suggested procedures with assumptions about the form of the production unit's production function.

1.9 The Production Function Framework

1.9.1 Introduction

> "Thus far, however, we have left out of consideration the fact that commodities are products which result from the combination of productive factors such as land, men and capital goods." Leon Walras (1954; 211)[224].

> "Almost all of our theorizing about investment and the desired stock of capital rests implicitly on some technological considerations and is derived from some kind of general production function. As long as we stick to the production function framework, it is clear that quantity rather than value is the relevant dimension, since the production function is defined as a relationship between the quantity of output and the quantity of various inputs." Zvi Griliches (1963; 118)[103].

In order to measure the contribution of capital to the production of outputs, it is useful to have an idealized model of how capital inputs interact with other flow inputs to produce outputs. The idealized models that economists utilize are based on production functions, or more specifically, on production possibilities sets which are technologically feasible sets of inputs and outputs that can be produced by a specified business unit in a specified time period. There are a number of different production function concepts that can be distinguished. Thus in section 1.9.2, we discuss the short run production function which distinguishes capital as an input at the beginning of the accounting period and (depreciated) capital as an output at the end of an accounting period. In section 1.9.3, we consider an intertemporal production function which relates inputs to outputs over many accounting periods. In this production function concept, the capital stocks the firm has available at the start of the first accounting period are distinguished as inputs and the (depreciated) capital stocks at the end of the last accounting period (when the assets of the firm are sold) are distinguished as outputs, but there is no apparent necessity to keep track of used capital inputs in intermediate accounting periods in this framework (unless they are sold before the final period). Purchases of new capital inputs over intermediate periods are distinguished in this framework. In section 1.9.3, we also attempt to reconcile this intertemporal production function concept with the one period "Austrian" production function concept in section 1.9.2. In section 1.9.4, we indicate how the usual one period production function that treats capital just as an input in each accounting period can be extracted from the Austrian production function framework explained in section 1.9.2.

1.9.2 The Austrian Production Function

> "We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that

> are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23)[115].

> "The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72)[81].

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business or production unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components, which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).*95 The model could be viewed as an Austrian model of production in honour of the Austrian economist Böhm-Bawerk (1891)[18] who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.*96

It should be noted that the neo-Austrian model of Hicks (1973)[117] is different from the model that we are describing in this section. Hicks (1973; 7)[117] interpreted Böhm-Bawerk's production model as follows:

> "Like Böhm-Bawerk (or Hayek) I think of the general productive process as being composed of a number (presumably a large number) of separable elementary processes ... . The elementary process, of the older Austrian theory, was of a simple, too simple, type. There was associated with a unit of output, forthcoming at a particular date, a sequence of units of input at particular previous dates. The sequence of inputs, and the single output, constituted the process."

Hicks (1973; 8)[117] then asserted that Böhm-Bawerk's production model was inconsistent with the existence of fixed capital inputs:

> "For the only kind of capital-using production which will fit into the old Austrian scheme is production without fixed capital, production that uses working capital (or circulating capital) only. Fixed capital (plant and machinery) will not fit in. For fixed capital goods are 'durable-use goods'; their essential characteristic is that they contribute, not just to one unit of output, at one date, but to a sequence of units of output, at a sequence of dates."

*95 For more on this model of production and additional references to the literature, see Malinvaud (1953)[157] and the Appendices in Diewert (1977)[47] (1980)[49]. Diewert derived the Malinvaud (1953)[157] Hicks (1961)[115] and Edwards and Bell (1961)[81] model of production by specializing the general intertemporal production model of Hicks (1939)[112] to the case of only one period.

*96 "They (entrepreneurs) buy goods of remoter rank, such as raw materials, tools, machines, the use of land, and, above all, labour, and, by the various processes of production, transform them into goods of first rank, finished products ready for consumption. ... Goods of remoter rank ... are incapable of satisfying human want; they require first to be changed into consumption goods; and since this process, naturally, takes time, they can only render their services to the wants of a future period—at the earliest, that period distant by the time which the productive process necessarily takes to change them into consumption goods." Eugen von Böhm-Bawerk (1891; 299-300)[18].

However, Böhm-Bawerk (1891; 299-300)[18] certainly mentioned various durable capital inputs such as “tools”, “machines” and “agricultural implements” as inputs in his model of production; what he did not explain explicitly is how time and use (i.e., depreciation) would transform these inputs into less valuable outputs at the end of a production period. What Böhm-Bawerk emphasized was the transformation of partly finished goods into more valuable partly finished goods and final products.[*97]

It will be useful in what follows to develop some notation to describe the one period Austrian model of production of this section. We suppose that there are M durable inputs that the business unit is using at the beginning of period 0. These durable inputs include machines, transportation equipment, other equipment, computers, plant structures, office buildings, tools, office furnishings and furniture, etc. These fixed capital stock components are classified into discrete categories according to their age and other relevant physical characteristics. The list of durable inputs also includes circulating capital stock components: inventories of raw materials, finished goods and partly finished goods (goods in process). Finally, we include in the business unit's list of initial capital stock components any patents or other marketable knowledge products as well as any holdings of land or other natural resources that it might possess. We denote the business unit's beginning of period 0 holdings of durable capital inputs by the nonnegative vector $\boldsymbol{k}^0 \equiv [k_1^0, k_2^0, ..., k_M^0]$ where $k_m^0 \geq 0$ denotes the initial stock of durable input m for $m = 1, ..., M$. We also suppose that $P_m^0 > 0$ is the beginning of period 0 market opportunity cost for a unit of durable input m for $m = 1, ..., M$ and the vector of these initial market values is $\boldsymbol{P}^0 \equiv [P_1^0, P_2^0, ..., P_M^0]$.[*98]

Next, we suppose that there are N outputs or inputs that the business unit can sell or purchase in the marketplace during period 0. The vector of average market prices that the business unit faces for the N commodities in period 0 is $\boldsymbol{p}^0 \equiv [p_1^0, p_2^0, ..., p_N^0]$ where $p_n^0 > 0$ is the average market price for commodity n in period 0 for $n = 1, ..., N$. The vector of *net outputs* that the business unit produces during period 0 is denoted by $\boldsymbol{y}^0 \equiv [y_1^0, y_2^0, ..., y_N^0]$. If $y_n^0 > 0$, then y_n^0 units of commodity n are being produced by the business unit during period 0 while if $y_n^0 < 0$, then $-y_n^0 > 0$ units of commodity n are being used as inputs during period 0. The list of period 0 “flow” inputs includes: various types of labour services, including both establishment employees and contracted professional services, purchases of electricity, heating fuels and telecommunications services. In principle, the entire list of durable capital stock components could be included in the list of flow inputs, since the business unit could purchase additional units of capital during period 0 to add to its initial stocks. The list of “flow” outputs will include the usual outputs that the business unit produces, classified as finely as seems necessary for the purpose at hand.[*99] In principle, all of the initial capital stock components held by the firm at the start of period 0 could appear in the list of flow outputs, since these stocks *could* be sold in the marketplace during period 0.[*100] Note that we are distinguishing as separate flow commodities sales of initial capital stock components from additional purchases of

[*97] Hicks (1973; 5-6)[117] (1965; 238-250)[116] attributed the Austrian model of this section (i.e., the model of Hicks (1961)[115] and Edwards and Bell (1961)[81]) to von Neumann (1937)[220] and Malinvaud (1953)[157], but von Neumann's model is at best only a special case of our Austrian model. Diewert (1977; 108-111)[47] (1980; 472-475)[49] made extensive use of this Austrian model of production but he regarded it as a special case of the intertemporal production model of Hicks (1946; 193-194)[114] to be studied in the following section.

[*98] In chapter 3 below, we will consider various alternative market opportunity costs concepts that might be used. However for now, we think of P_m^0 as being the net realizable value at the beginning of period 0 for a unit of durable input m if no additional units of m are purchased by the business unit during period 0 (this will typically be the case for fixed capital stock components) or P_m^0 is the beginning of period 0 purchase price for a unit of durable input m if additional units of m are purchased during period 0 (this will typically be the case for circulating capital stock components).

[*99] In some cases, it may be necessary to distinguish the same “physical” commodity by its time of production within the accounting period; e.g., electricity produced during the peak time of a day is more valuable than off-peak electricity; strawberries supplied during off seasons are more valuable than strawberries supplied during the local growing season, etc.

[*100] The underlying accounting framework for inventory items is explained in detail in Diewert and Smith (1994)[76].

capital stock components during period 0 for two reasons: (i) the selling price of an asset will usually differ from the purchase price of a similar unit of the asset due to transactions costs and (ii) the technological impact of the sale of a fixed capital stock component is often quite different from the purchase of an additional unit due to internal transactions costs (such as installation and training costs for purchases and dismantling and renovation costs for sales). Thus the dimensionality of the "flow" commodity space N will generally be much greater than the dimensionality of the initial durable input "stock" space M.

Finally, at the end of period 0, the business unit will have at its disposal a vector $\boldsymbol{k}^1$ of durable inputs which can be valued at the end of period 0 (or beginning of period 1) nonnegative market opportunity costs vector $\boldsymbol{P}^1 \equiv [P_1^1, P_2^1, ..., P_M^1]$. The components of $\boldsymbol{k}^1$ consist of depreciated units of the business unit's beginning of period 0 vector of capital stocks $\boldsymbol{k}^0$ that were not sold during period 0 plus any additional units of capital that might have been purchased during period 0.[*101]

We now turn our attention to the definition of the period 0 Austrian production function or more generally, the period 0 Austrian production possibilities set for the production unit under consideration. Several definitions are possible. The broadest definition for the period 0 production possibilities set S^0 is to define S^0 as the set of all technologically feasible vectors of the form $(\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{k}^1)$ where $\boldsymbol{k}^0 \geq \mathbf{0}_M$[*102] is a nonnegative beginning of period 0 vector of capital inputs, $\boldsymbol{y}^0$ is an N dimensional vector of period 0 net outputs that can be produced given $\boldsymbol{k}^0$ and $\boldsymbol{k}^1 \geq \mathbf{0}_M$ is a nonnegative vector of capital stocks that are left over at the end of period 0. In this broadest definition of the period 0 production possibilities set, we allow the production unit to choose its initial vector of capital stocks $\boldsymbol{k}^0$. In our next definition of the period 0 production possibilities set, we restrict the production unit's choices for the initial capital stock vector $\boldsymbol{k}^0$ to be an observed vector of capital stocks $\boldsymbol{k}^0$ for the production unit under consideration. In this case, the period 0 production possibilities set for the business unit can be described as a feasible set of net outputs and end of the period capital stocks $\boldsymbol{y}^0, \boldsymbol{k}^1$ that could be produced using the observed initial vector of capital stocks $\boldsymbol{k}^0$; i.e., the technology set now has the form

$$\{(\boldsymbol{y}^0, \boldsymbol{k}^1) : (\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{k}^1) \in S^0\}. \tag{1.52}$$

The production possibilities set defined by (1.52) is a smaller set than the entire set S^0 where $\boldsymbol{k}^0$ is also freely chosen.

The business unit's competitive profit maximization problem that corresponds to the broadest definition of the period 0 production possibilities set S^0 can be formalized as:

$$\max_{\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{k}^1} \{-\boldsymbol{P}^0 \cdot \boldsymbol{k}^0 + (1 + r^0)^{-1} \boldsymbol{p}^0 \cdot \boldsymbol{y}^0 + (1 + r^0)^{-1} \boldsymbol{P}^1 \cdot \boldsymbol{k}^1 : (\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{k}^1) \in S^0\}. \tag{1.53}$$

where $\boldsymbol{P}^0 \cdot \boldsymbol{k}^0 = \sum_{m=1}^{M} P_m^0 k_m^0$, r^0 is the period 0 nominal interest rate or opportunity cost of capital prevailing at the beginning of the period and S^0 is the period 0 production possibilities set for the business unit. Note that we have divided the net "flow" revenues for period 0, $\boldsymbol{p}^0 \cdot \boldsymbol{y}^0$, and the market value of the business unit's end of the period holdings of capital stocks, $\boldsymbol{P}^1 \cdot \boldsymbol{k}^1$, by one plus the interest rate, $(1 + r^0)$. Thus we are assuming that period 0 "flow" revenues and costs $\boldsymbol{p}^0 \cdot \boldsymbol{y}^0$ are "realized" at the end of period 0 along with the end of period 0 value of the business unit's capital stocks, $\boldsymbol{P}^1 \cdot \boldsymbol{k}^1$. These end of period 0 capital stocks are discounted to make them equivalent to beginning of period 0 values, as is traditional in economics.

However, from the perspective of accounting theory, it is more natural to express all values in terms of end of the period values and thus from this perspective, the business unit's period 0 profit

[*101] These new purchases would show up as (negative) components of the vector $\boldsymbol{y}^0$. Note that we have assumed that the number of components of $\boldsymbol{k}^0$ is equal to the number of components of $\boldsymbol{k}^1$; M. Since we allow components of $\boldsymbol{k}^0$ and $\boldsymbol{k}^1$ to be zero, this restriction involves no real loss of generality.

[*102] Notation: $\boldsymbol{k}^0 \geq \mathbf{0}_M$ means that each component of the M dimensional vector $\boldsymbol{k}^0$ is nonnegative.

maximization problem becomes:

$$\max_{\boldsymbol{k}^0,\boldsymbol{y}^0,\boldsymbol{k}^1}\{-(1+r^0)\boldsymbol{P}^0\cdot\boldsymbol{k}^0+\boldsymbol{p}^0\cdot\boldsymbol{y}^0+\boldsymbol{P}^1\cdot\boldsymbol{k}^1:(\boldsymbol{k}^0,\boldsymbol{y}^0,\boldsymbol{k}^1)\in S^0\}. \tag{1.54}$$

Note that the objective function in (1.54) is $(1+r^0)$ times the objective function in (1.53) so that the maximization problems (1.53) and (1.54) have the same solution sets.

Now let us shift our focus to the business unit's profit maximization problem in period 1. Let $S^1 \equiv \{(\boldsymbol{k}^1,\boldsymbol{y}^1,\boldsymbol{k}^2)\}$ denote the business unit's unrestricted period 1 production possibilities set, which consists of feasible vectors of starting capital stocks $\boldsymbol{k}^1$, period 1 "flow" inputs and outputs $\boldsymbol{y}^1$ and end of period 1 finishing capital stock vectors $\boldsymbol{k}^2$. If there is no technological progress or managerial improvement in the organization of production, S^1 will equal S^0; i.e., the period 1 and period 0 production possibilities sets will be the same (but typically, there is technical progress so that the set S^1 is bigger than the set S^0).

The period 1 counterpart to the period 0 unrestricted profit maximization problem (1.54) is:

$$\max_{\boldsymbol{k}^1,\boldsymbol{y}^1,\boldsymbol{k}^2}\{-(1+r^1)\boldsymbol{P}^1\cdot\boldsymbol{k}^1+\boldsymbol{p}^1\cdot\boldsymbol{y}^1+\boldsymbol{P}^2\cdot\boldsymbol{k}^2:(\boldsymbol{k}^1,\boldsymbol{y}^1,\boldsymbol{k}^2)\in S^1\} \tag{1.55}$$

where r^1 is the beginning of period 1 nominal opportunity cost of capital; $\boldsymbol{P}^1$ is the vector of beginning of period 1 opportunity costs for capital stock components; $\boldsymbol{P}^2$ is the vector of end of period 1 opportunity costs for capital stock components; $\boldsymbol{p}^1$ is the vector of period 1 average prices for units of outputs and inputs and $\boldsymbol{y}^1$ is a period 1 net output vector (positive components of $\boldsymbol{y}^1$ denote outputs, negative components denote inputs).

Obviously, one period profit maximization problems that are analogous to (1.54) and (1.55) can be defined for each accounting period t that the business unit is in operation.

We can also define a one period profit maximization problem that has the same structure as (1.54) except that the restricted production possibilities set defined by (1.52) is used in place of S^0. This restricted period 0 profit maximization problem (with $\boldsymbol{k}^0$ restricted to equal the fixed initial capital stock vector $\boldsymbol{k}^0$) is:

$$\max_{\boldsymbol{y}^0,\boldsymbol{k}^1}\{-(1+r^0)\boldsymbol{P}^0\cdot\boldsymbol{k}^0+\boldsymbol{p}^0\cdot\boldsymbol{y}^0+\boldsymbol{P}^1\cdot\boldsymbol{k}^1:(\boldsymbol{k}^0,\boldsymbol{y}^0,\boldsymbol{k}^1)\in S^0\}. \tag{1.56}$$

Suppose $\boldsymbol{y}^{0*}$ and $\boldsymbol{k}^{1*}$ solves (1.56). Then the end of period 0 capital stock vector $\boldsymbol{k}^{1*}$ can serve as a vector of fixed starting capital stocks for the business unit's period 1 restricted profit maximization problem which is analogous to (1.55) except that $\boldsymbol{k}^1$ is fixed at $\boldsymbol{k}^{1*}$:

$$\max_{\boldsymbol{y}^1,\boldsymbol{k}^2}\{-(1+r^1)\boldsymbol{P}^1\cdot\boldsymbol{k}^{1*}+\boldsymbol{p}^1\cdot\boldsymbol{y}^1+\boldsymbol{P}^2\cdot\boldsymbol{k}^2:(\boldsymbol{k}^{1*},\boldsymbol{y}^1,\boldsymbol{k}^2)\in S^1\}. \tag{1.57}$$

The differences between the two period 0 profit maximization problems (1.54) and (1.56) can be explained as follows: in (1.54), the business unit is allowed to sell its initial holdings of capital (the components of the vector $\boldsymbol{k}^0$) or buy additional units of capital at the beginning of period 0 at the prices $\boldsymbol{P}^0$; in (1.56), the business unit is stuck with its initial holdings of capital $\boldsymbol{k}^0$ at the beginning of period 0 (but still values these initial holdings at the prices $\boldsymbol{P}^0$). Thus the different period 0 profit maximization problems reflect different assumptions about what options are open to the business unit at the beginning of period 0. However, for each of the problems, the Austrian production possibilities set S^0 plays a crucial role.

In the following subsection, we no longer assume that the business unit's decision horizon is limited to a sequence of single periods; we will allow the business unit to make production plans that extend over a number of periods.

1.9.3 The Fisher-Hicks Intertemporal Production Function

"An option is any possible income stream open to an individual by utilizing his resources, capital, labor, land, money, to produce or secure said income stream. An investment opportunity is the opportunity to shift from one such option, or optional income stream, to another ... . Some of the optional income streams, however, would never be chosen, because none of their respective present values could possibly be the maximum." Irving Fisher (1930; 151)[94].

"The problem of the firm, dynamically considered, is to find that stream of outputs, capable of being produced from the initial equipment, which shall have the maximum capital value ... . If we write $x_{r0}, x_{r1}, x_{r2}, ..., x_{rv}$ for the [net] outputs of x_r planned to be sold in successive 'weeks' from the present, then the production function takes the form $f(x_{10}, x_{20}, ..., x_{n0}; x_{11}, x_{21}, ..., x_{n1}; x_{12}, x_{22}, ..., x_{n2}; ...; x_{1v}, x_{2v}, ..., x_{nv}) = 0$ assuming that the plan extends forward for v weeks. The capitalized value of the plan is $C = \sum_{r=1}^{n}\sum_{t=0}^{v}(\beta_t p_{rt} x_{rt})$ where $\beta_t = 1/(1+i_t)$ and i_t is the rate of interest per week for loans of t weeks; p_{r0} is the current price of x_r and p_{rt} is the price the entrepreneur expects to rule in the week beginning t weeks hence." John R. Hicks (1946; 326)[114].

In this section, we will utilize the intertemporal production function concepts developed by Fisher (1930; 151)[94] and Hicks (1946; 136)[114]. As in section 1.9.2, we assume that there are M types of durable capital equipment and that the business unit's initial holdings of capital stock components at the beginning of period 0 is $\boldsymbol{k}^0 \equiv [k_1^0, k_2^0, ..., k_M^0]$ where $k_m^0 \geq 0$ denotes the initial stock of durable input m for $m = 1, ..., M$. We now assume that the business unit's time horizon extends over T periods. Denote a vector of planned net outputs for period t by $\boldsymbol{y}^t \equiv [y_1^t, y_2^t, ..., y_N^t]$ for $t = 0, 1, 2, ..., T-1$ and denote the corresponding vector of anticipated average prices for period t by $\boldsymbol{p}^t \equiv [p_1^t, p_2^t, ..., p_N^t] \geq \mathbf{0}_N$ for $t = 0, 1, 2, ..., T-1$. At the end of period $T-1$ (or equivalently, at the beginning of period T), we assume that the business unit is sold;[*103] i.e., the components of its beginning of period T capital stock vector $\boldsymbol{k}^T \equiv [k_1^T, k_2^T, ..., k_M^T] \geq \mathbf{0}_M$ are sold at the prices $\boldsymbol{P}^T \equiv [P_1^T, P_2^T, ..., P_M^T] \geq \mathbf{0}_M$. The production unit's *intertemporal production possibilities set* $S \equiv \{(\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{y}^1, ..., \boldsymbol{y}^{T-1}, \boldsymbol{k}^T)\}$ is the feasible set of net output vectors $\boldsymbol{y}^0, \boldsymbol{y}^1, ..., \boldsymbol{y}^{T-1}$ for periods $0, 1, ..., T-1$ and beginning of period T (depreciated) capital stock components $\boldsymbol{k}^T$ that can be produced by an initial vector of capital stocks $\boldsymbol{k}^0$ and existing technology (and technology that can be anticipated to exist over the time horizon of the business unit).

Let r^t be the interest rate or opportunity cost of financial capital that is relevant to the business unit at the beginning of period t for $t = 0, 1, 2, ..., T-1$ and let $\boldsymbol{P}^0 \equiv [P_1^0, P_2^0, ..., P_M^0] \geq \mathbf{0}_M$ be the vector of opportunity costs for capital stock components at the start of period 0. Then assuming that period t cash flow or net revenues from variable inputs and outputs, $\boldsymbol{p}^t \cdot \boldsymbol{y}^t = \sum_{n=1}^{N} p_n^t y_n^t$, are "realized" at the end of period t, the business unit's intertemporal planned profit maximization problem can be written as follows:

$$\begin{aligned}&\max_{\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{y}^1, ..., \boldsymbol{y}^{T-1}, \boldsymbol{k}^T}\{-\boldsymbol{P}^0 \cdot \boldsymbol{k}^0 + (1+r^0)^{-1}\boldsymbol{p}^0 \cdot \boldsymbol{y}^0 + (1+r^0)^{-1}(1+r^1)^{-1}\boldsymbol{p}^1 \cdot \boldsymbol{y}^1 \\ &+ (1+r^0)^{-1}(1+r^1)^{-1}(1+r^2)^{-1}\boldsymbol{p}^2 \cdot \boldsymbol{y}^2 + \cdots + (1+r^0)^{-1}(1+r^1)^{-1}\cdots(1+r^{T-1})^{-1}\boldsymbol{p}^{T-1} \cdot \boldsymbol{y}^{T-1} \\ &+ (1+r^0)^{-1}(1+r^1)^{-1}\cdots(1+r^T)^{-1}\boldsymbol{P}^T \cdot \boldsymbol{k}^T : (\boldsymbol{k}^0, \boldsymbol{y}^0, \boldsymbol{y}^1, ..., \boldsymbol{y}^{T-1}, \boldsymbol{k}^T) \in S\}. \qquad (1.58)\end{aligned}$$

Note that all values in the objective function of (1.58) that are realized after the beginning of period 0 are discounted by interest rate terms $(1+r^t)$. Thus all values are expressed in beginning of period 0 equivalent values. Note also that the intertemporal profit maximization problem (1.58) reduces to the single period Austrian profit maximization problem (1.53) if the business unit's time horizon is

[*103] This assumption was used by Hicks (1946; 193-194)[114].

only one period; i.e., if $T = 1$. Note also that in both (1.53) and (1.58), we allowed the initial vector of beginning of period 0 capital stocks $\boldsymbol{k}^0$ to be variable. A counterpart to (1.58) which freezes $\boldsymbol{k}^0$ to equal the business unit's historically determined capital stocks is:[*104]

$$\begin{aligned}
&\max_{\boldsymbol{y}^0,\boldsymbol{y}^1,\ldots,\boldsymbol{y}^{T-1},\boldsymbol{k}^T}\{-\boldsymbol{P}^0\cdot\boldsymbol{k}^0+(1+r^0)^{-1}\boldsymbol{p}^0\cdot\boldsymbol{y}^0+(1+r^0)^{-1}(1+r^1)^{-1}\boldsymbol{p}^1\cdot\boldsymbol{y}^1\\
&+(1+r^0)^{-1}(1+r^1)^{-1}(1+r^2)^{-1}\boldsymbol{p}^2\cdot\boldsymbol{y}^2+\cdots+(1+r^0)^{-1}(1+r^1)^{-1}\cdots(1+r^{T-1})^{-1}\boldsymbol{p}^{T-1}\cdot\boldsymbol{y}^{T-1}\\
&+(1+r^0)^{-1}(1+r^1)^{-1}\cdots(1+r^T)^{-1}\boldsymbol{P}^T\cdot\boldsymbol{k}^T:(\boldsymbol{k}^0,\boldsymbol{y}^0,\boldsymbol{y}^1,\ldots,\boldsymbol{y}^{T-1},\boldsymbol{k}^T)\in S\}. \qquad (1.59)
\end{aligned}$$

If we divide the objective function in (1.56) by $(1+r^0)$, it can be seen that the resulting version of (1.56) is the same problem as (1.59) if $T = 1$; i.e., the intertemporal profit maximization problem (1.59) is equivalent to our restricted one period Austrian profit maximization problem (1.56) when the business unit's time horizon is only a single period.

In the case where the business unit has a multiperiod planning horizon (i.e., the case where $T > 1$), it is possible to relate the one period technology sets of section 1.9.2 to the intertemporal technology set S of the present section. Suppose that the one period Austrian technology sets $S^0, S^1, \ldots, S^{T-1}$ are given for periods $0, 1, \ldots, T-1$. Then these Austrian technology sets can be used to define a Hicksian intertemporal technology set S as follows:

$$S \equiv \{(\boldsymbol{k}^0,\boldsymbol{y}^0,\boldsymbol{y}^1,\ldots,\boldsymbol{y}^{T-1},\boldsymbol{k}^T):(\boldsymbol{k}^0,\boldsymbol{y}^0,\boldsymbol{k}^1)\in S^0,(\boldsymbol{k}^1,\boldsymbol{y}^1,\boldsymbol{k}^2)\in S^1,\ldots,(\boldsymbol{k}^{T-1},\boldsymbol{y}^{T-1},\boldsymbol{k}^T)\in S^{T-1}\}; \qquad (1.60)$$

i.e., in defining (1.60), we simply force the end of period t capital stocks $\boldsymbol{k}^{t+1}$ to be equal to the starting capital stocks for period $t+1$ for $t = 0, 1, 2, \ldots, T-1$. Thus we do not allow the business unit to sell or purchase any units of capital at the very end of each period t in definition (1.60) for each time period.[*105]

We conclude section 1.9.3 by indicating that under certain conditions, solutions to the Hicksian intertemporal profit maximization problem (1.59) are also solutions to a sequence of Austrian single period profit maximization problems, provided that the period by period capital stock valuation vectors $\boldsymbol{P}^1, \boldsymbol{P}^2, \ldots, \boldsymbol{P}^{T-1}$ that appear in the Austrian problems (but do not appear in (1.59)) are chosen appropriately. In order to minimize notational complexity, we will demonstrate the above assertion for the case of a two period intertemporal technology; i.e., we will assume $T = 2$ in (1.59).[*106]

Suppose that $\boldsymbol{y}^{0*}, \boldsymbol{y}^{1*}, \boldsymbol{k}^{2*}$ solves (1.59) when $T = 2$. Under certain conditions, we can define end of period 0 or beginning of period 1 capital stock price and quantity vectors $\boldsymbol{P}^{1*}$ and $\boldsymbol{k}^{1*}$ such that: (i) $\boldsymbol{y}^{0*}$ and $\boldsymbol{k}^{1*}$ solve the period 0 profit maximization problem (1.56) provided that $\boldsymbol{P}^1 = \boldsymbol{P}^{1*}$ and (ii) $(\boldsymbol{k}^{1*}, \boldsymbol{y}^{1*}, \boldsymbol{k}^{2*})$ solves the period 1 profit maximization problem (1.55) provided that the $\boldsymbol{P}^1$ which appears in (1.55) is equal to $\boldsymbol{P}^{1*}$. The translation of the last rather technical sentence is this: period by period "Austrian" profit maximization can be consistent with the intertemporal Hicksian profit maximization model (1.59) provided that the correct "economic" capital stock prices $\boldsymbol{P}^{t*}$ are used in the single period profit maximization problems.

In order to establish the above assertion, it is necessary to introduce *the period t variable profit function* π^t that is dual to the Austrian technology set S^t for $t = 0, 1$:[*107]

$$\pi^t(\boldsymbol{p}^t,\boldsymbol{k}^t,\boldsymbol{k}^{t+1}) \equiv \max_{\boldsymbol{y}}\{\boldsymbol{p}^t\cdot\boldsymbol{y}:(\boldsymbol{k}^t,\boldsymbol{y},\boldsymbol{k}^{t+1})\in S^t\}; \qquad t = 0, 1. \qquad (1.61)$$

[*104] This is the intertemporal profit maximization problem that Hicks (1946; 326)[114] considered.

[*105] Of course units of capital can be bought or sold during period t; these purchases or sales are components of $\boldsymbol{y}^t$.

[*106] The reader who is not interested in the technical details of our demonstration can skip to the end of this section.

[*107] We assume that the technology sets S^t are nonempty closed convex sets subject to free disposal. This will imply that $\pi^t(\boldsymbol{p}^t, \boldsymbol{k}^t, \boldsymbol{k}^{t+1})$ will be jointly concave in the components of $\boldsymbol{k}^t$ and $\boldsymbol{k}^{t+1}$, nondecreasing in the components of $\boldsymbol{k}^t$ and nonincreasing in the components of $\boldsymbol{k}^{t+1}$; see Diewert and Lewis (1982; 303)[75] or Diewert (1985; 226)[53]. For duality theorems between S^t and π^t and references to the literature, see Diewert (1973)[44] (1993b; 165-168)[58].

Using definition (1.61), the single period constrained profit maximization problem (1.56) can be rewritten as the following unconstrained profit maximization problem involving only the components of $\boldsymbol{k}^1$:

$$\max_{\boldsymbol{k}^1}\{-(1+r^0)\boldsymbol{P}^0\cdot\boldsymbol{k}^0+\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^1)+\boldsymbol{P}^1\cdot\boldsymbol{k}^1\}. \tag{1.62}$$

Suppose that $\boldsymbol{k}^{1**}$ is a solution to (1.62) and each component of $\boldsymbol{k}^{1**}$ is positive; i.e., $\boldsymbol{k}^{1**}>>\boldsymbol{0}_M$. If $\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1**})$ is differentiable with respect to the components of $\boldsymbol{k}^1$ at $\boldsymbol{k}^1=\boldsymbol{k}^{1**}$, then the vector of first order partial derivatives of $\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1**})$ with respect to the components of $\boldsymbol{k}^1$, $\nabla_{\boldsymbol{k}^1}\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1**})\equiv[\partial\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1**})/\partial k_1^1,...,\partial\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1**})/\partial k_M^1]$ will satisfy the following first order necessary conditions to solve (1.62):[*108]

$$\nabla_{\boldsymbol{k}^1}\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1**})+\boldsymbol{P}^1=\boldsymbol{0}_M. \tag{1.63}$$

Now use definition (1.61) for $t=1$ and rewrite the period 1 constrained profit maximization problem (1.55) as the following unconstrained profit maximization problem involving the vector of beginning of period 1 capital stocks $\boldsymbol{k}^1$ and the vector of end of period 1 capital stocks $\boldsymbol{k}^2$:

$$\max_{\boldsymbol{k}^1,\boldsymbol{k}^2}\{-(1+r^1)\boldsymbol{P}^1\cdot\boldsymbol{k}^1+\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^1,\boldsymbol{k}^2)+\boldsymbol{P}^2\cdot\boldsymbol{k}^2\}. \tag{1.64}$$

Suppose that $\boldsymbol{k}^{1***}>>\boldsymbol{0}_M$ and $\boldsymbol{k}^{2***}>>\boldsymbol{0}_M$ are solution vectors for (1.64) and that $\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^1,\boldsymbol{k}^2)$ is differentiable with respect to the components of $\boldsymbol{k}^1$ and $\boldsymbol{k}^2$ at $(\boldsymbol{k}^1,\boldsymbol{k}^2)=(\boldsymbol{k}^{1***},\boldsymbol{k}^{2***})$. Then the vector of first order partial derivatives of π^1 with respect to the components of $\boldsymbol{k}^1$, $\nabla_{\boldsymbol{k}^1}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^{1***},\boldsymbol{k}^{2***})$ and the vector of first order partial derivatives of π^1 with respect to the components of $\boldsymbol{k}^2$, $\nabla_{\boldsymbol{k}^2}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^{1***},\boldsymbol{k}^{2***})$ will satisfy the following first order necessary conditions to solve (1.64):[*109]

$$\nabla_{\boldsymbol{k}^2}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^{1***},\boldsymbol{k}^{2***})+\boldsymbol{P}^2=\boldsymbol{0}_M; \tag{1.65}$$

$$\nabla_{\boldsymbol{k}^1}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^{1***},\boldsymbol{k}^{2***})-(1+r^1)\boldsymbol{P}^1=\boldsymbol{0}_M. \tag{1.66}$$

Now assume that the intertemporal production possibilities set S is constructed using the one period technology sets S^0 and S^1 and definition (1.60) when $T=2$. Using definitions (1.61), we can rewrite the constrained intertemporal profit maximization problem (1.59) (when $T=2$) as the following unconstrained profit maximization problem involving the beginning and end of period 1 capital stock vectors $\boldsymbol{k}^1$ and $\boldsymbol{k}^2$ as decision variables:[*110]

$$\max_{\boldsymbol{k}^1,\boldsymbol{k}^2}\{-(1+r^0)\boldsymbol{P}^0\cdot\boldsymbol{k}^0+\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^1)+(1+r^1)^{-1}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^1,\boldsymbol{k}^2)+(1+r^1)^{-1}\boldsymbol{P}^2\cdot\boldsymbol{k}^2\}. \tag{1.67}$$

Assume that $\boldsymbol{k}^{1*}>>\boldsymbol{0}_M$ and $\boldsymbol{k}^{2*}>>\boldsymbol{0}_M$ solves (1.67) and that π^0 and π^1 are differentiable with respect to the components of $\boldsymbol{k}^1$ and $\boldsymbol{k}^2$ when $(\boldsymbol{k}^1,\boldsymbol{k}^2)=(\boldsymbol{k}^{1*},\boldsymbol{k}^{2*})$. Then $\boldsymbol{k}^{1*}$ and $\boldsymbol{k}^{2*}$ will satisfy the following first order necessary conditions for solving (1.67):

$$\nabla_{\boldsymbol{k}^1}\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1*})+(1+r^1)^{-1}\nabla_{\boldsymbol{k}^1}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^{1*},\boldsymbol{k}^{2*})=\boldsymbol{0}_M; \tag{1.68}$$

$$(1+r^1)^{-1}\nabla_{\boldsymbol{k}^2}\pi^1(\boldsymbol{p}^1,\boldsymbol{k}^{1*},\boldsymbol{k}^{2*})+(1+r^1)^{-1}\boldsymbol{P}^2=\boldsymbol{0}_M. \tag{1.69}$$

[*108] The assumption that S^0 is a convex set will imply that $\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^1)$ is a concave function in the components of $\boldsymbol{k}^1$; see Diewert (1973)[44] (1985; 226)[53]. Thus conditions (1.63) are also sufficient to imply that $\boldsymbol{k}^{1*}$ solves (1.62).

[*109] The convexity of S^1 implies that conditions (1.65) and (1.66) are sufficient for $\boldsymbol{k}^{1***}>>\boldsymbol{0}_M$ and $\boldsymbol{k}^{2***}>>\boldsymbol{0}_M$ to solve (1.64).

[*110] Intertemporal profit maximization problems of this type are studied in much greater detail in Diewert and Lewis (1982)[75] and Diewert (1985; 225-228)[53].

We use the vector of partial derivatives $\nabla_{\boldsymbol{k}^1}\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1*})$ in order to define a vector of end of period 0 “economic prices” or shadow prices of capital $\boldsymbol{P}^{1*}$:[*111]

$$\boldsymbol{P}^{1*} \equiv -\nabla_{\boldsymbol{k}^1}\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^{1*}). \tag{1.70}$$

Problem 1 Let $\boldsymbol{k}^{1*} >> \mathbf{0}_M$ and $\boldsymbol{k}^{2*} >> \mathbf{0}_M$ solve (1.67) in the differentiable case. Replace the $\boldsymbol{P}^1$ which occurs in the Austrian maximization problems (1.62) and (1.64) by the $\boldsymbol{P}^{1*}$ defined by (1.70).
(a) Show that $\boldsymbol{k}^{1*}$ also solves (1.62).
(b) Show that $\boldsymbol{k}^{1*}$ and $\boldsymbol{k}^{2*}$ also solve (1.64).

Problem 2 Derive counterparts to the results in problem 1 when T is increased from 2 to 3.

The thrust of the above algebra is this: under some regularity conditions,[*112] single period Austrian profit maximization is perfectly consistent with the long run intertemporal maximization of profits, provided that the business unit uses “economic” prices to value its end of period capital stock components. The problem with this result is that it is usually difficult to determine these economic prices as outside observers of the business unit (or even as insiders); i.e., at the end of period 0, how can we determine $\boldsymbol{P}^{1*}$ defined by (1.70)? In the earlier part of this chapter, we implicitly assumed that these shadow prices could be adequately be approximated by user costs based on information about resale prices for used assets but this approximation may not be adequate. The difficulties involved in the practical determination of “economic prices” explain why most accountants dismiss the use of economic prices as a practical alternative for the valuation of a business unit's end of the period capital stocks. However, as we shall see in the next two chapters, accountants' methods for valuing the assets of the firm are often even more arbitrary than those used by economists.

Traditional production function models do not distinguish capital as an input at the beginning of a period and capital as an output at the end of the same period as was done in the Austrian production function. In the following section, we indicate how a traditional production function can be derived from an Austrian production function.

1.9.4 The Traditional Production Function

> “I belong to the party which is still looking to find, at the end of its journey, a rehabilitation of the so-called ‘Production Function’ $P = f(L, C)$ [where P is product, L is labour input and C is capital input] in some form or other; what I am looking for is a concept of capital which will ultimately allow us to think, more or less, in those terms.” John R. Hicks (1961; 18)[115].

> “In the context of the Hicksian model, it is clear that we can construct several capital aggregates that must be carefully distinguished: (a) a current period capital stock aggregate (an input from the viewpoint of the current period) using current period capital stock prices as weights in the aggregation procedure; (b) a (depreciated) following period capital stock aggregate (an output from the viewpoint of the current period) using discounted expected following period capital stock prices as weights; (c) a current period investment aggregate (an output) using current period investment goods prices as weights in the aggregation procedure; and (d) a capital aggregate that is an aggregate of (a) and (b) where capital as an input and capital as an output are oppositely signed in the index number formula that is used.” W. Erwin Diewert (1980; 474-475)[49].

[*111] Since $\pi^0(\boldsymbol{p}^0,\boldsymbol{k}^0,\boldsymbol{k}^1)$ is nonincreasing in the components of $\boldsymbol{k}^1$, $\boldsymbol{P}^{1*} \geq \mathbf{0}_M$.

[*112] These regularity conditions are not insignificant. In particular, the following assumptions may not be satisfied: (i) convexity of the one period technology sets S^t; (ii) differentiability of the variable profit functions π^t and (iii) the assumption of interior solutions to (1.67); i.e., that $\boldsymbol{k}^{t*} >> \mathbf{0}_M$ for each t.

We return to the Austrian model of section 1.9.2 and note that there is an easy way of simplifying the model so that we do not have to distinguish each durable commodity as both an input and an output: we need only use Leontief's (1936; 54-57)[146] Aggregation Theorem. This result says that if two commodities are always used or produced in fixed proportions by a production unit in each period t, then the two commodities can be aggregated into a single composite commodity. More specifically, let x_1^t and x_2^t denote the quantities of say two inputs used during period t and let $p_1^t > 0$ and $p_2^t > 0$ denote the period t average price for each commodity. If $x_1^t = \alpha x_2^t$ for all periods t under consideration, then the two commodities can be aggregated into a composite commodity with period t aggregate input X^t equal to the quantity of input 1 during period t; x_1^t and with period t composite price P^t equal to the period t value of the two commodities divided by x_1^t; i.e.,

$$X^t \equiv x_1^t; P^t \equiv [p_1^t x_1^t + p_2^t x_2^t]/x_1^t = p_1^t + \alpha p_2^t. \tag{1.71}$$

Definitions (1.71) can still be used to aggregate the two commodities even if say commodity 1 is an input and commodity 2 is an output; in this case, x_1^t and x_2^t have opposite signs and α, the factor of proportionality, is negative.

Now consider the case of a single durable input that lasts 2 or more periods and whose productivity declines only with age (and not use). Suppose that $k^0 > 0$ units of the (new) durable input were purchased at the start of period 0 at price $P_0^0 > 0$, and suppose that the end of period 0 price for depreciated units is $P_1^1 > 0$. Then from the perspective of the end of period 0, the net cost of using k^0 units of the durable input during period 0 is

$$(1 + r^0)P_0^0 k^0 - P_1^1 k^0 = u_0^0 k^0 \tag{1.72}$$

where u_0^0 is an end of period 0 user cost similar to those defined by (1.10) above. Now define $x_1^0 = k^0, x_2^0 \equiv -k^0; p_1^0 \equiv (1 + r^0)P_0^0$ and $p_2^0 \equiv P_1^1$ and apply Leontief's Aggregation Theorem. It can be seen that $X^0 = k^0$ and $P^0 = u_0^0$; i.e., the ex post (ex ante if P_1^1 is an expected price) end of period 0 user cost u_0^0 can be viewed as the period 0 price for the use of one unit of an aggregate of capital where the two capitals are capital input at the beginning of period 0 and capital output at the end of period 0. The resulting aggregate capital can be viewed as the capital input which appears in a "traditional" production function and a user cost is the price which is associated with the capital aggregate.

Obviously, the above aggregation technique can be applied to all vintages of a capital input provided that declines in value over the period are independent of use. If declines in value are not independent of use, then we need to distinguish different end of period prices that depend on the intensity of use of the durable input over the accounting period. This disaggregation of each type of beginning of the period capital input into separate categories depending on period 0 use can be carried out as finely as seems empirically necessary.*113 Thus Leontief's Aggregation Theorem can be applied to aggregate capital inputs in an Austrian production function even if the value of the assets declines with use as well as with age.

The above aggregation technique will not work for assets that lose their identity during the period 0 production process; e.g., a computer chip on hand at the beginning of the period emerges as part of a computer at the end of the period or a concrete foundation at the beginning of the period becomes part of a building at the end of the period, etc. We will look at the treatment of goods in process and other inventory items in chapter 5.

The production theory framework explained in this section will be helpful in addressing some specific measurement issues in subsequent chapters (such as the measurement of income and inventory change).

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*113 Again, this observation is due to Edwards and Bell (1961; 174)[81].

1.10 The Treatment of Business Income Taxes

There are a number of possible methods for incorporating business income taxes into a user cost formula. We consider three approaches in this section.

The first approach is the simplest. Assume that the firm sets up a leasing unit that purchases a capital input at price P^0 at the beginning of the period, rents out the services of the capital to the firm during the period and then "sells" the depreciated asset at the end of the period[*114] at the price $P^1(1-\delta^0)$. The problem for the leasing unit is to determine an appropriate rental price p_K^0 for the use of the durable input during the period.[*115] We need a few more assumptions in order to accomplish this task. Thus we assume that the leasing firm is financed by equity capital and the appropriate (nominal) beginning of the period opportunity cost of capital is r^0. We assume that the rental is paid at the end of the period as are (possible) property taxes (or other specific taxes on the asset) and the business income tax. We assume that the per unit capital property tax is $\tau^0 P^0$ (where τ^0 is the appropriate specific tax rate) and the business income tax rate is t^0. Taxable income per unit of capital employed is defined as rental income, p_K^0, less allowable depreciation expense, $\alpha^0\delta^0 P^0$, where economic depreciation is $\delta^0 P^0$ and α^0 is the proportion of economic depreciation that the tax authorities allow the firm to deduct from taxable income in period 0, less property taxes, $\tau^0 P^0$; i.e., we have:

$$\text{Period 0 taxable income per unit capital employed } \equiv p_K^0 - \alpha^0\delta^0 P^0 - \tau^0 P^0. \tag{1.73}$$

Now the leasing unit can determine the appropriate rental price for a unit of capital, p_K^0, by solving the following equation for p_K^0: the initial beginning period purchase price P^0 times one plus the opportunity cost of capital should equal the depreciated end of period value $P^1(1-\delta^0)$ plus the rental income p_K^0 less the specific taxes payable $\tau^0 P^0$ less the business income tax rate t^0 times taxable income; i.e., solve the following equation for p_K^0:

$$P^0(1+r^0) = P^1(1-\delta^0) + p_K^0 - \tau^0 P^0 - t^0[p_K^0 - \alpha^0\delta^0 P^0 - \tau^0 P^0]. \tag{1.74}$$

The solution to (1.74) is:

$$p_K^0 = (1-t^0)^{-1}\{P^0(1+r^0) - P^1(1-\delta^0) - t^0\alpha^0\delta^0 P^0\} + \tau^0 P^0. \tag{1.75}$$

If there is no asset price inflation during the course of period 0 so that $P^0 = P^1$ and if the tax authorities set the allowed depreciation equal to economic depreciation so that $\alpha^0 = 1$, then (1.75) simplifies dramatically to the following formula for the rental price:

$$p_K^0 = [\delta^0 + (1-t^0)^{-1}r^0 + \tau^0]P^0. \tag{1.76}$$

Thus in this case, the rental price is equal to the asset purchase price P^0 times the economic depreciation rate δ^0 plus the tax augmented opportunity cost of capital $(1-t^0)^{-1}r^0$ plus the asset specific tax rate τ^0. In this simple case, the intent of the business income tax is made clear: the intent is to tax the real return to capital. Unfortunately, in the general case when there is asset price inflation during the period or when the tax authorities do not know what the economic depreciation rate is (or set tax depreciation rates artificially low or high for other purposes), the original purpose

[*114] The "sale" can be back to the leasing unit.

[*115] This approach to determining the user cost of capital in the context of a business income tax was explained in Diewert (1980; 470-471)[49]. The original approaches to incorporating the business income tax into a user cost formula (using a continuous time formulation) are due to Jorgenson (1963)[134] and Hall and Jorgenson (1967)[107].

of the business income tax is lost and a considerable amount of deadweight loss to the economy can result.*116

We turn now to our second approach for deriving a user cost formula when there is a business income tax. In this second approach, we introduce the production function, $Y = F(L, K)$ into the model, where F is the production function, Y is the output that can be produced by L units of variable nondurable inputs (labour say) and K units of durable capital. We make the same assumptions as above with respect to capital and the tax regime and in addition assume that the end of period price for output and labour is p_Y^1 and w^1 respectively. Taxable income for the producer is now defined (as functions of Y, L and K) as follows:

$$\text{Taxable income } \equiv p_Y^1 Y^1 - w^1 L^1 - [\alpha^0 \delta^0 P^0 + \tau^0 P^0]K. \tag{1.77}$$

The net cost of buying one unit of capital at the beginning of the period, using it during the period and selling it (possibly to itself) at the end of the period, ignoring income tax, is

$$\text{End of period user cost ignoring income tax } \equiv P^0(1 + r^0) - P^1(1 - \delta^0) + \tau^0 P^0 \tag{1.78}$$

Using (1.77) and (1.78), the firm's period 0 profit maximization problem can be written as follows:

$$\begin{aligned} &\max_{Y,L,K}\{p_Y^1 Y^1 - w^1 L^1 - [P^0(1 + r^0) - P^1(1 - \delta^0) + \tau^0 P^0]K \\ &\qquad - t^0\{p_Y^1 Y^1 - w^1 L^1 - [\alpha^0 \delta^0 P^0 + \tau^0 P^0]K\} : Y = F(L, K)\} &(1.79)\\ &= (1 - t^0)\max_{Y,L,K}\{p_Y^1 Y^1 - w^1 L^1 - p_K^0 K : Y = F(L, K)\} &(1.80)\end{aligned}$$

where (1.80) follows from (1.78) by substituting the tax adjusted rental price of capital p_K^0 defined by (1.75) into (1.78). Thus our second approach is equivalent to our first approach.*117

Problem 3 Modify the above two approaches by assuming that the firm is financed entirely by debt and interest is tax deductible.

Problem 4 Modify the above two approaches by assuming that capital purchases are financed partly by debt and partly by equity.

Obviously the above two approaches to dealing with capital taxation in a user cost context become much more complicated as we model in more detail the intricacies of the business income tax in most countries. Hence a third approach is sometimes the only feasible one and that is to assume that business income taxes fall on the rate of return to capital and to simply treat them as specific taxes on each capital stock component; i.e., we treat business income taxes in much the same manner as we treated specific property taxes in the above two approaches. This approach is not as theoretically sound as the first two approaches but sometimes data limitations will force us to adopt it.

1.11 Appendix A: Alternative Models of Depreciation

A1. The One Hoss Shay Model of Efficiency and Depreciation

In section 1.3 above, we noted that Böhm-Bawerk (1891; 342)[18] postulated that an asset would yield a constant level of services throughout its useful life of L years and then collapse in a heap to yield no services thereafter. This has come to be known as the one hoss shay or light bulb model of depreciation. Hulten noted that this pattern of relative efficiencies has the most intuitive appeal:

*116 This is another large topic which we will not examine here.
*117 This equivalence result may be found in Diewert (1980; 471)[49].

"Of these patterns, the one hoss shay pattern commands the most intuitive appeal. Casual experience with commonly used assets suggests that most assets have pretty much the same level of efficiency regardless of their age— a one year old chair does the same job as a 20 year old chair, and so on." Charles R. Hulten (1990; 124)[127].

Thus the basic assumptions of this model are that the period t efficiencies and hence cross sectional rental prices f_n^t are all equal to say f^t for vintages n that are less than L periods old and for older vintages, the efficiencies fall to zero. Thus we have:

$$f_n^t = \begin{cases} f^t & \text{for } n = 0, 1, 2, \ldots, L-1; \\ 0 & \text{for } n = L, L+1, L+2, \ldots. \end{cases} \tag{A1}$$

Now substitute (A1) into the first equation in (1.5) and get the following formula*118 for the rental price f^t in terms of the price of a new asset at the beginning of year t, P_0^t:

$$f^t = P_0^t/[1 + (\gamma^t) + (\gamma^t)^2 + \cdots + (\gamma^t)^{L-1}] \tag{A2}$$

where the period t discount factor γ^t is defined in terms of the period t nominal interest rate r^t and the period t expected asset inflation rate i^t as follows:

$$\gamma^t \equiv (1 + i^t)/(1 + r^t). \tag{A3}$$

Now that the period t rental price f^t for an unretired asset has been determined, substitute equations (A1) into equations (1.5) and determine the sequence of period t vintage asset prices, P_n^t:*119

$$P_n^t = \begin{cases} f^t[1 + (\gamma^t) + (\gamma^t)^2 + \cdots + (\gamma^t)^{L-1-n}] & \text{for } n = 0, 1, 2, \ldots, L-1 \\ 0 & \text{for } n = L, L+1, L+2, \ldots. \end{cases} \tag{A4}$$

Finally, use equations (1.9) to determine the end of period t rental prices, u_n^t, in terms of the corresponding beginning of period t rental prices, f_n^t:

$$u_n^t = (1 + r^t) f_n^t; \qquad n = 0, 1, 2, ... \tag{A5}$$

Given the vintage asset prices defined by (A4), we could use equations (1.13) above to determine the corresponding vintage cross section depreciation rates δ_n^t.

A2. The Straight Line Depreciation Model

The straight line method of depreciation is very simple in a world without price change: one simply makes an estimate of the most probable length of life for a new asset, L periods say, and then the original purchase price P_0^t is divided by L to yield as estimate of period by period depreciation for the next L periods. In a way, this is the simplest possible model of depreciation, just as the one hoss shay model was the simplest possible model of efficiency decline.*120 The accountant Canning summarizes the straight line depreciation model as follows:

*118 This formula simplifies to $P_0^t[1 - (\gamma^t)^L]/[1 - \gamma^t]$ provided that γ^t is less than 1 in magnitude. However, this restriction on the magnitude of γ^t does not always hold empirically. However, (A2) is still valid under these conditions.

*119 Note that all of the asset prices P_n^t will vary in strict proportion over time provided that the discount factor γ^t is constant over time, which will be the case if we can assume that the real interest rate r^{t*} is constant over time. In this constant real interest rate case, we can apply Hicks' (1946; 312-313)[114] Aggregation Theorem in order to form a capital stock aggregate over vintages. The Theorem says that if all prices in the aggregate move in strict proportion over time, then any one of these prices can be taken as the price of the aggregate. The corresponding quantity aggregate is equal to the value aggregate divided by the chosen price.

*120 In fact, it can be verified that if the nominal interest rate r^t and the nominal asset inflation rate i^t are both zero, then the one hoss shay efficiency model will be entirely equivalent to the straight line depreciation model.

> "*Straight Line Formula* ... In general, only two primary estimates are required to be made, viz., scrap value at the end of n periods and the numerical value of n. ... Obviously the number of periods of contemplated use of an asset can seldom be intelligently estimated without reference to the anticipated conditions of use. I the formula is to be respectable at all, the value of n must be the most probable number of periods that will yield the most economical use." John B. Canning (1929; 265-266).

The following quotations indicate that the use of straight line depreciation dates back to the 1800's at least:

> "Sometimes an equal installment is written off every year from the original value of the plant; sometimes each machine or item of plant is considered separately; but it is more usual to write off a percentage, not of the original value, but from the balance of the plant account of the preceding year." Ewing Matheson (1910; 55)[160].

> "In some instances the amount charged to revenue account for depreciation is a fixed sum, or an arbitrary percentage on the book value." Emile Garcke and John Manger Fells (1893; 98)[98].

The last two quotations indicate that the declining balance or geometric depreciation model (to be considered in the next section) also dates back to the 1800's as a popular method for calculating depreciation.

We now set out the equations which describe the straight line model of depreciation in the general case when the anticipated asset inflation rate i^t is nonzero. Assuming that the asset has a life of L periods and that the cross sectional amounts of depreciation $D_n^t \equiv P_n^t - P_{n+1}^t$ defined by (1.11) above are all equal for the assets in use, then it can be seen that the beginning of period t vintage asset prices P_n^t will decline linearly for L periods and then remain at zero; i.e., the P_n^t will satisfy the following restrictions:

$$P_n^t = \begin{cases} P_0^t[L-n]/L & n = 0, 1, 2, \ldots, L \\ 0 & n = L+1, L+2, \ldots \end{cases} \tag{A6}$$

Recall definition (1.13) above, which defined the cross sectional depreciation rate for an asset that is n periods old at the beginning of period t, δ_n^t. Using (A6) and the nth equation in (1.13), we have:

$$(1-\delta_0^t)(1-\delta_1^t)\cdots(1-\delta_{n-1}^t) = P_n^t/P_0^t = 1-(n/L) \qquad \text{for } n = 1, 2, \ldots, L. \tag{A7}$$

Using (A7) for n and $n+1$, it can be shown that

$$(1-\delta_n^t) = [L-(n+1)]/[L-n] \qquad n = 0, 1, 2, \ldots, L-1. \tag{A8}$$

Now substitute (A7) and (A8) into the general user cost formula (1.15) in order to obtain the *period t end of the period straight line user costs*, u_n^t:*121

$$\begin{aligned} u_n^t &= (1-\delta_0^t)\cdots(1-\delta_{n-1}^t)[(1+r^t)-(1+i^t)(1-\delta_n^t)]P_0^t \qquad n = 0, 1, 2, \ldots, L-1 \\ &= [1-(n/L)][(1+r^t)-(1+i^t)([L-(n+1)]/[L-n])]P_0^t. \end{aligned} \tag{A9}$$

Equations (A6) give us the sequence of asset prices by age that are required to calculate the wealth capital stock while equations (A9) give us the user costs by age that are required to calculate capital services for the asset. It should be noted that if the anticipated asset inflation rate i^t is large enough compared to the nominal interest rate r^t, then the user cost u_n^t can be negative. This means that the corresponding asset becomes an output rather than an input for period t.*122

*121 The user costs for $n = L, L+1, L+2, \ldots$ are all zero.

*122 However, one is led to wonder if the model is reasonable if some vintages of the asset have negative user costs while other vintages have positive one. As we noted before, it is not really reasonable to have ex ante negative

A3. The Declining Balance or Geometric Depreciation Model

The declining balance method of depreciation dates back to Matheson (1910; 55)[160].[*123] In terms of the algebra presented in section 1.4 above, the method is very simple: all of the cross sectional vintage depreciation rates δ_n^t defined by (1.13) are assumed to be equal to the same rate δ, where δ a positive number less than one; i.e., we have for all time periods t:

$$\delta_n^t = \delta; \qquad n = 0, 1, 2, \ldots . \tag{A10}$$

Substitution of (A10) into (1.15) leads to the following formula for the sequence of period t user costs by age:

$$\begin{aligned} u_n^t &= (1-\delta)^{n-1}[(1+r^t) - (1+i^t)(1-\delta)]P_0^t; \qquad n = 0, 1, 2, \ldots \\ &= (1-\delta)^{n-1} u_0^t; \qquad n = 1, 2, \ldots . \end{aligned} \tag{A11}$$

The second set of equations in (A11) says *that all of the vintage user costs are proportional to the user cost for a new asset.* This proportionality means that we do not have to use an index number formula to aggregate over assets by age to form a capital services aggregate. To see this, using (A11), the period t services aggregate S^t defined earlier by (1.47) can be rewritten as follows:

$$\begin{aligned} S^t &\equiv u_0^t K_0^t + u_1^t K_1^t + u_2^t K_2^t + \cdots \\ &= u_0^t [K_0^t + (1-\delta) K_1^t + (1-\delta)^2 K_2^t + \cdots] \\ &= u_0^t K_A^t \end{aligned} \tag{A12}$$

where the *period t capital aggregate* K_A^t is defined as

$$K_A^t \equiv K_0^t + (1-\delta)K_1^t + (1-\delta)^2 K_2^t + \cdots \tag{A13}$$

If the depreciation rate δ and the vintage capital stocks are known, then K_A^t can readily be calculated using (A13). Then using the last line of (A12), we see that the value of capital services (over all vintages), S^t, decomposes into the price term u_0^t times the quantity term K_A^t. Hence, it is not necessary to use an index number formula to aggregate over vintages using this depreciation model.

A similar simplification occurs when calculating the wealth stock using this depreciation model. Substitution of (A10) into (1.14) leads to the following formula for the sequence of period t vintage asset prices:

$$P_n^t = (1-\delta)^{n-1} P_0^t; \qquad n = 1, 2, \ldots . \tag{A14}$$

Equations (A14) say *that all of the vintage asset prices are proportional to the price of a new asset.* This proportionality means that again, we do not have to use an index number formula to aggregate over vintages to form a capital stock aggregate. To see this, using (A14), the period t wealth aggregate W^t defined earlier by (1.46) can be rewritten as follows:

$$\begin{aligned} W^t &\equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \cdots \\ &= P_0^t [K_0^t + (1-\delta) K_1^t + (1-\delta)^2 K_2^t + \cdots] \\ &= P_0^t K_A^t \end{aligned} \tag{A15}$$

user costs (but it is quite reasonable to have negative ex post user costs).

[*123] Matheson (1910; 91)[160] used the term “diminishing value” to describe the method. Hotelling (1925; 350)[126] used the term “the reducing balance method” while Canning (1929; 276) used the term the “declining balance formula”.

where K_A^t was defined by (A13). Thus K_A^t can serve as both a capital stock aggregate or a flow of services aggregate, which is a major advantage of this model.[*124]

There is a further simplification of the model which is useful in applications. If we compare equation (1.55) for period $t+1$ and period t, we see that the following formula results using equations (1.39):

$$K_A^{t+1} \equiv K_0^{t+1} + (1-\delta)K_A^t. \tag{A16}$$

Thus the period $t+1$ aggregate capital stock, K_A^{t+1}, is equal to the investment in new assets that took place in period t, which is K_0^{t+1}, plus $1-\delta$ times the period t aggregate capital stock, K_A^t. This means that given a starting value for the capital stock, we can readily update it just using the depreciation rate δ and the new investment in the asset during the prior period.

We now look at the ancient accounting and engineering literature for some hints on how to determine the geometric depreciation rate δ for a particular asset class. Matheson was perhaps the first engineer to address this problem. On the basis of his experience, he simply postulated some approximate rates that could be applied:

> "In most [brick or stone] factories an average of 3 per cent for buildings will generally be found appropriate, if due attention is paid to repairs. Such a rate will bring down a value of £ 1000 to £ 400 in thirty years." Ewing Matheson (1910; 69)[160].
>
> "Buildings of wood or iron would require a higher rate, ranging from 5 to 10 per cent, according to the design and solidity of the buildings, the climate, the care and the regularity of the painting, and according also, to the usage they are subjected to." Ewing Matheson (1910; 69)[160].
>
> "Contractors' locomotives working on imperfect railroads soon wear out, and a rate of 20 per cent is generally required, bringing down the value of an engine costing £ 1000 to £ 328 in five years." Ewing Matheson (1910; 86)[160].
>
> "In engineering factories, where the work is of a moderate kind which does not strain the machines severely, and where the hours of working do not average more than fifty per week, 5 per cent written off each year from the diminishing value will generally suffice for the wear-and-tear of machinery, cranes and fixed plant of all kinds, if steam engines and boilers be excluded." Ewing Matheson (1910; 82)[160].
>
> "The high speed of the new turbo generators introduced since 1900, and their very exact fitting, render them liable to certain risks from variations in temperature and other causes. Several changes in regard to speed and methods of blading have occurred since their first introduction and if these generators are taken separately, only after some longer experience has been acquired can it be said that a depreciation rate of 10 per cent on the diminishing value will be too much for maintaining a book-figure appropriate to their condition. Such a rate will reduce £ 1000 to £ 349 in ten years." Ewing Matheson (1910; 91)[160].

How did Matheson arrive at his estimated depreciation rates? He gave some general guidance as follows:

> "The main factors in arriving at a fair rate of depreciation are:
>
> 1. The Original value.
> 2. The probable working Life.
> 3. The Ultimate value when worn out or superceded.

[*124] This advantage of the model has been pointed out by Jorgenson (1989)[135] (1996b)[137] and his coworkers. Its early application dates back to Jorgenson and Griliches (1967)[138] and Christensen and Jorgenson (1969)[28] (1973)[29].

> Therefore, in deciding upon an appropriate rate of depreciation which will in a term of years provide for the estimated loss, it is not the original value or cost which has to be so provided for, but that cost less the ultimate or scrap value." Ewing Matheson (1910; 76)[160].

The algebra corresponding to Matheson's method for determining δ was explicitly described by the accountant Canning (1929; 276). Let the initial value of the asset be V_0 and let its scrap value n years later be V_n. Then V_0, V_n and the depreciation rate δ are related by the following equation:

$$V_n = (1 - \delta)^n V_0. \tag{A17}$$

Canning goes on to explain that $1 - \delta$ may be determined by solving the following equation:

$$\log(1 - \delta) = [\log V_n - \log V_0]/n. \tag{A18}$$

It is clear that Matheson used this framework to determine depreciation rates even though he did not lay out formally the above straightforward algebra.

However, Canning had a very valid criticism of the above method:

> "This method can be summarily rejected for a reason quite independent or the deficiencies of formulas 1 and 2 above [(A17) and (A18) above]. Overwhelming weight is given to V_n in determining book values. ... Thus the least important constant in reality is given the greatest effect in the formula." John B. Canning (1929; 276).

Thus Canning pointed out that the scrap value, V_n, which is not determined very accurately from an a priori point of view, is the tail that is wagging the dog; i.e., this poorly determined value plays a crucial role in the determination of the depreciation rate.*125

An effective response to Canning's criticism of the declining balance method of depreciation did not emerge until relatively recently when Hall (1971)[106], Beidelman (1973)[11] (1976)[12] and Hulten and Wykoff (1981a)[129] (1981b)[130] used an entire array of used asset prices at point in time in order to determine the geometric depreciation rate which best matched up with the data.*126 As we noted in section 1.6 above, another theoretical possibility would be to use information on vintage rental prices in order to deduce the depreciation rate.*127 Hulten and Wykoff summarize their experience in estimating depreciation rates from used asset prices by concluding that the assumption of geometric or declining balance depreciation described their data relatively well:

> "We have used the approach to study the depreciation patterns of a variety of fixed business assets in the United States (e.g., machine tools, construction equipment, autos and trucks, office equipment, office buildings, factories, warehouses, and other buildings). The straight line and concave patterns [i.e., one hoss shay patterns] are strongly rejected ; geometric is also rejected, but the estimated patterns are extremely lose to (though steeper than) the geometric

*125 "There may be cases in which the formula fits the facts, but ... the chance of its being a formula of close fit is remote indeed. Its chief usefulness seems to be to furnish drill in the use of logarithms for students in accounting." John B. Canning (1929; 277).

*126 Jorgenson (1996a)[136] has a nice review of most of the empirical studies of depreciation. It should be noted that Beidelman (1973)[11] (1976)[12] and Hulten and Wykoff (1981a)[129] (1996; 22)[131] showed that equation (A17) must be adjusted to correct for the early retirement of assets. The accountant Schmalenbach (1959; 91)[190] (the first German edition was published in 1919) also noticed this problem: "The mistake should not be made, however, of drawing conclusions about useful life from those veteran machines which are to be seen in most businesses. Those which one sees are but the rare survivors; the many dead have long lain buried. This can be the source of serious errors."

*127 This possibility is mentioned by Hulten and Wykoff (1996; 15)[131]: "In other words, if there were active rental markets for capital services as there are for labor services, the observed prices could be used to estimate the marginal products. And the rest of the framework would follow from these estimates. But, again, there is bad news: most capital is owner utilized, like much of the stock of single family houses. This means that owners of capital , in effect, rent it to themselves, leaving no data track for the analyst to observe."

form, even for structures. Although it is rejected statistically, the geometric pattern is far closer than either of the other two candidates. This leads us to accept the geometric pattern as a reasonable approximation for broad groups of assets, and to extend our results to assets for which no resale markets exist by imputing depreciation rates based on an assumption relating the rate of geometric decline to the useful lives of assets." Charles C. Hulten and Frank C. Wykoff (1996; 16)[131].

This brings us to our next problem: how can we convert our asset lives expressed in years until retirement into geometric rates?

One possible method for converting an average asset life, L periods say, into a comparable geometric depreciation rate is to argue as follows. Suppose that we believe that the straight line model of depreciation is the correct one and the asset under consideration has a useful life of L periods. Suppose further that investment in this type of asset is constant over time at one unit per period and asset prices are constant over time. Under these conditions, the long run equilibrium capital stock for this asset would be[*128]:

$$1 + [(L-1)/L] + [(L-2)/L] + \cdots + [2/L] + [1/L] = L(L+1)/2L = (L+1)/2. \quad \text{(A19)}$$

Under the same conditions, the long run equilibrium geometric depreciation capital stock would be equal to the following sum:

$$1 + (1-\delta) + (1-\delta)^2 + \cdots = 1/[1-(1-\delta)] = 1/\delta. \quad \text{(A20)}$$

Now find the depreciation rate δ which will make the two capital stocks equal; i.e., equate (61) to (62) and solve for δ. The resulting δ is:

$$\delta = 2/(L+1). \quad \text{(A21)}$$

Obviously, there are a number of problematical assumptions that were made in order to derive the depreciation rate δ that corresponds to the length of life L[*129] but (A21) gives us at least a definite method of conversion from one model to the other.

As an example of how the conversion formula (A21) might work, consider the case of nonresidential construction. Maddison (1993)[156] assumed that the average length of life for nonresidential construction L was equal to 39 years. Applying the conversion formula (A21) in this case implies that the corresponding geometric depreciation rate equals .05. Similarly, Maddison's assumed life of 14 years for machinery and equipment translates into a geometric depreciation rate δ equal to a 13 1/3% for this asset class.

There is one remaining problem to deal with in the context of the geometric depreciation model that is not present in the other models which assume finite lives for the assets. The problem is this: since the geometric model implies that the effects of past investments linger on forever, no finite

[*128] Recall equations (A6), which imply that the vintage asset prices are proportional. Hence Hicks' Aggregation Theorem will imply that the capital aggregate will be the simple sum on the left hand side of (A19).

[*129] The two assumptions that are the least justified are: (1) the assumption that the straight line depreciation model is the correct model to do the conversion and (2) the assumption that investment has been constant back to minus infinity. Hulten and Wykoff (1996; 16)[131] made the following suggestions for converting an L into a δ: "Information is available on the average service life, L, from several sources. The rate of depreciation for non-marketed assets can be estimated using a two step procedure based on the 'declining balance' formula $\delta = X/L$. Under the 'double declining balance' formula, $X = 2$. The value of X can be estimated using the formula $X = \delta L$ for those assets for which these estimates are available. In the Hulten-Wykoff studies, the average value for of X for producer's durable equipment was found to be 1.65 (later revised to 1.86). For nonresidential structures, X was found to be 0.91. Once X is fixed, δ follows for other assets whose average service life is available."

set of investment data can give us a completely accurate starting value for geometric capital stock components. To solve this problem, we outline an approximate method used by Kohli (1982)[140].[*130]

Suppose that our investment data on an asset class start at year 0 and end at year $T+1$. Calculate the average geometric rate of growth in (real) investment for this asset class over the sample time period, g as:

$$1 + g \equiv (I^T/I^0)^{(1/T)}. \tag{A22}$$

Now assume that investments in this asset class prior to time period 0 grew at the same rate as the geometric rate of growth within the sample period. Thus investment in period -1 is assumed to be $I^0/(1+g)$, in year -2 is assumed to be $I^0/(1+g)^2$ and so on. Assuming that the geometric depreciation rate for the asset class under consideration is δ, we see that the capital stock K^0 at the start of period 0 should be equal to the following sum under our assumptions:

$$\begin{aligned} K^0 &= I^{-1} + (1-\delta)I^{-2} + (1-\delta)^2 I^{-3} + \cdots \\ &= (1+g)^{-1}I^0 + (1-\delta)(1+g)^{-2}I^0 + (1-\delta)^2(1+g)^{-3}I^0 + \cdots \\ &= (1+g)^{-1}I^0[1 + (1-\delta)(1+g)^{-1} + (1-\delta)^2(1+g)^{-2} + \cdots] \\ &= (1+g)^{-1}I^0[1 - \{(1-\delta)(1+g)^{-1}\}]^{-1} \qquad \text{assuming } (1-\delta)(1+g)^{-1} \text{ is less than } 1 \\ &= (1+g)^{-1}I^0(1+g)/(\delta+g) \\ &= I^0/(\delta+g). \end{aligned} \tag{A23}$$

A4. The Linear Efficiency Decline Model

Recall that our first class of models (the one hoss shay models) assumed that the efficiency (or cross section user cost) of the asset remained constant over the useful life of the asset. In our second class of models (the straight line depreciation models), we assumed that the cross sectional depreciation of the asset declined at a linear rate. In our third class of models (the geometric depreciation models), we assumed that cross section depreciation declined at a geometric rate. Comparing the third class with the second class of models, it can be seen that geometric depreciation is more *accelerated* than straight line depreciation; i.e., depreciation is relatively large for new vintages compared to older ones. In this section, we will consider another class of models that gives rise to an accelerated pattern of depreciation: the class of models that exhibit *a linear decline in efficiency.*

It is relatively easy to develop the mathematics of this model. Let f_0^t be the period t rental price for an asset that is new at the beginning of period t. If the useful life of the asset is L years and the efficiency decline is linear, then the sequence of period t cross sectional user costs f_n^t is defined as follows:

$$f_n^t \equiv \begin{cases} f_0^t[L-n]/L; & n = 0, 1, 2, \ldots, L-1; \\ 0; & n = L, L+1, L+2, \ldots. \end{cases} \tag{A24}$$

Now substitute (A24) into the first equation in (1.5) and get the following formula for the rental price f_0^t in terms of the price of a new asset at the beginning of year t, P_0^t:

$$f_0^t = LP_0^t/[L + (L-1)(\gamma^t) + (L-2)(\gamma^t)^2 + \cdots + 1(\gamma^t)^{L-1}] \tag{A25}$$

where the period t discount factor γ^t is defined in terms of the period t nominal interest rate r^t and the period t expected asset inflation rate i^t in the usual way:

$$\gamma^t \equiv (1+i^t)/(1+r^t). \tag{A26}$$

[*130] See also Fox and Kohli (1998)[96].

Now that f_0^t has been determined, substitute (A25) into (A24) and substitute the resulting equations into equations (1.5) and determine the sequence of period t vintage asset prices, P_n^t:

$$P_n^t = \begin{cases} P_0^t[(L-n)+(L-n-1)(\gamma^t)+\cdots+1(\gamma^t)^{L-1-n}]/[L+(L-1)(\gamma^t)+\cdots+1(\gamma^t)^{L-1}] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } n=0,1,2,\ldots,L-1 \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } n=L,L+1,L+2,\ldots. \end{cases} \tag{A27}$$

Finally, use equations (1.9) to determine the end of period t rental prices, u_n^t, in terms of the corresponding beginning of period t rental prices, f_n^t:

$$u_n^t = (1+r^t)f_n^t; \qquad n = 0,1,2,\ldots \tag{A28}$$

Given the vintage asset prices defined by (A27), we could use equations (1.13) above to determine the corresponding vintage cross section depreciation rates δ_n^t. We will not table these depreciation rates since our focus is on constructing measures of the capital stock and of the flow of services that the stocks yield.

There is a relationship between the linear efficiency decline model and another model of depreciation that appears in the tax literature. Assume that the nominal interest rate r^t and the nominal asset inflation rate i^t are both zero, then using (A27), it can be shown that

$$D_n^t \equiv P_n^t - P_{n+1}^t = P_0^t[L-n]/[L(L+1)/2] \qquad \text{for } n=0,1,2,\ldots,L; \tag{A29}$$

i.e., when $r^t = i^t = 0$, *depreciation declines at a linear rate* for the linear efficiency decline model. When depreciation declines at a linear rate, the resulting formula for depreciation is called *the sum of the year digits formula.*[*131] Thus just as the one hoss shay and straight line depreciation models coincide when $r^t = i^t = 0$, so too do the linear efficiency decline and sum of the digits depreciation models coincide.

In the final section of this Appendix, we indicate why it is reasonable to expect depreciation to be accelerated even when, at first glance, it appears that the asset is of the one hoss shay type.

A5. The Linearly Increasing Maintenance Expenditures Model

Many years ago, the accountant Canning raised the following interesting problem that bears on our topic:[*132]

> "By spending enough for parts replacements (repairs), it is possible to keep any machine running for an indefinitely great length of time, but it does not pay to do so. Query: How does one know just when a machine is worn out?" John B. Canning (1929; 251).

In other words, Canning notes that the choice of when to retire an asset is really an *endogenous* decision[*133] rather than an *exogenous* one as we have assumed up to now. In this section, we attempt to model the retirement decision in a preliminary way using the concept of a *maintenance profile.*

[*131] Canning (1929; 277) describes the method in some detail so it was already in common use by that time.

[*132] Matheson (1910; 76-77)[160] raises the same sort of issues in a less focused manner: "But this principle has to be applied with considerable qualification where repairs really renew the life of a machine and prolong greatly its period of useful work. For instance, a locomotive during its life may have its wheel tires renewed four times, its boiler three times, and be painted seven times, so that before the framework, the wheels and other more durable parts fail, and the engine is broken up, much more than its original cost will have been expended on it. The value of any such serious renewals of this kind should be duly credited in a proper system of depreciation. Another course, followed more often in the United States than in Great Britain, is to prefer the substitution of new machines with all modern improvements rather than to renew or repair old plant, which even rendered serviceable may not be so economical in working."

[*133] We consider some additional models of endogenous retirement later in this course. In this section, we are relaxing the strong separability assumptions that we have made up to now; i.e., up to now, we have assumed that used asset prices are independent of the actions of the firm and other prices in the economy.

For most new machines and new structures, engineers are able to devise a *maintenance schedule* that will ensure that the asset delivers its services during the period under consideration. Thus in the Queensland Competition Authority (2000; Chapter 13)[186],*[134] a schedule of costs per kilometer of rail track that is required to keep the rails in working order as a function of the age of the track is laid out. These maintenance expenditures will enable the track to deliver transportation services over its lifetime. This schedule has a fixed cost aspect to it and then as the track ages, the maintenance expenditures increase linearly up to a certain point and then flatten out. Similarly, a new truck will have a schedule of recommended maintenance operations that the owner is urged to follow. In addition to these maintenance expenditures, we could also include operating costs like fuel and driver inputs since these inputs are necessary to deliver ton miles of output. Finally, an office building will also have maintenance expenditures associated with it and some operating expenditures such as heat since the renters of offices typically want square meters of space maintained at a comfortable temperature. In any case, we assume that at the beginning of period t, we know the period t maintenance and operating expenditures necessary to operate an asset that is n periods old at the beginning of period t, m_n^t, $n = 0, 1, 2, \ldots$ .We say that $\{m_n^t\}$ is the *period t (cross section) maintenance profile.*

We now have to distinguish between the *gross and net rental prices* of an asset that is n periods old at the beginning of period t, g_n^t and f_n^t, respectively. An office or an apartment is typically rented on a gross basis; i.e., a tenant rents an office that is t periods old at the beginning of period t and pays the gross rent g_n^t at the beginning of the period and the landlord is responsible for the period t maintenance costs m_n^t.*[135] On the other hand, a truck (on a long term lease) is usually rented on a net basis; i.e., the user of the truck is responsible for operating costs and maintenance. In any case, the relationship between the gross and net rental prices is:

$$g_n^t = f_n^t + m_n^t; \qquad n = 0, 1, 2, \ldots. \tag{A30}$$

Thus our present notation is consistent with our previous notation where we valued an asset by the discounted stream of its net rentals; i.e., previously, we used the cross sectional profile of net rentals f_n^t (extrapolated to future periods) in order to value assets by age.

Our new asset valuation by age equation is the following equation, which gives the value of a new asset at the beginning of period t, assuming that the asset will be retired after L periods of use:

$$\begin{aligned} P_0^t(L) \equiv\ & g_0^t + (\gamma^t) g_1^t + (\gamma^t)^2 g_2^t + \cdots + (\gamma^t)^{L-1} g_{L-1}^t \\ & - [m_0^t + (\beta^t) m_1^t + (\beta^t)^2 m_2^t + \cdots + (\beta^t)^{L-1} m_{L-1}^t] \end{aligned} \tag{A31}$$

where the discount factors γ^t and β^t are defined as follows:*[136]

$$\gamma^t \equiv (1 + i^t)/(1 + r^t);\ \beta^t \equiv (1 + \alpha^t)/(1 + r^t). \tag{A32}$$

As in the previous sections, i^t is *the one period anticipated inflation rate for the services of the asset* at the beginning of period t, r^t is *the period t nominal interest rate* and hence γ^t is the same discount factor that has appeared in previous sections. The new parameter is α^t, which is *the one period anticipated inflation rate for maintenance (and operating cost) services* and so β^t is the counterpart to γ^t except that β^t is the discount factor that applies to future anticipated

*[134] Euan Morton of the Queensland Competition Authority brought this work to the author's attention.

*[135] We assume that these costs are converted to beginning of period t costs using present values if necessary.

*[136] We could have presented a more general equation than (A31); i.e., we could have presented a counterpart to the very general equation (1.2) in section 1.3 above, involving the anticipated asset inflation rates i_n^t, the one period nominal interest rates r_n^t and another set of anticipated maintenance cost inflation rates α_n^t. However, we would soon be forced to impose the simplifying assumptions (1.3) and (1.4) along with a new set of simplifying assumptions, $\alpha_n^t = \alpha^t$.

operating and maintenance *costs* while γ^t is the discount factor that applies to future anticipated *gross revenues*.

The interpretation of equation (A31) is straightforward: a new asset that is to be used for L periods is equal to the discounted stream of the gross rentals that it is expected to yield minus the discounted stream of expected maintenance and operating costs.

We now evaluate equation (A31) for $L = 1, 2, \ldots$, and pick the L which gives the highest value of $P_0^t(L)$. We call this optimal value L^*. Once the optimal L^* has been determined, then if used asset markets are in equilibrium, the sequence of period t asset prices by age n, P_n^t, the sequence of period t gross rental prices by age n, g_n^t, and the sequence of period t cross sectional maintenance costs by age of asset n, m_n^t, should satisfy the following system of equations:

$$\begin{aligned}
P_0^t &\equiv g_0^t + (\gamma^t)g_1^t + (\gamma^t)^2 g_2^t + \cdots + (\gamma^t)^{L-1} g_{L^*-1}^t \\
&\quad - [m_0^t + (\beta^t)m_1^t + (\beta^t)^2 m_2^t + \cdots + (\beta^t)^{L-1} m_{L^*-1}^t] \\
P_1^t &\equiv g_1^t + (\gamma^t)g_2^t + (\gamma^t)^2 g_3^t + \cdots + (\gamma^t)^{L-1} g_{L^*-2}^t \\
&\quad - [m_1^t + (\beta^t)m_2^t + (\beta^t)^2 m_3^t + \cdots + (\beta^t)^{L-1} m_{L^*-2}^t] \\
&\cdots \\
P_{L^*-2}^t &= g_{L^*-2}^t + (\gamma^t)g_{L^*-1}^t - [m_{L^*-2}^t + (\beta^t)m_{L^*-1}^t] \\
P_{L^*-1}^t &= g_{L^*-1}^t - m_{L^*-1}^t.
\end{aligned} \tag{A33}$$

Equations (A33) are the counterparts to our earlier system of equations (1.5). Given the cross sectional gross rental prices g_n^t and the cross sectional maintenance costs m_n^t (and the discount factors γ^t and β^t), we can determine the period t asset prices by age n, P_n^t, using equations (A33). Given the asset prices P_n^t and the cross sectional maintenance costs m_n^t, we can also use equations (A33) to determine the period t gross rental prices by age n, g_n^t: start with the last equation in (A33) and determine $g_{L^*-1}^t$; then move up to the second last equation and determine $g_{L^*-2}^t$; etc. Of course, once the g_n^t have been determined, then we may use equations (A30) to determine the net rental prices (or user costs) f_n^t and then we can use these f_n^t as weights for the period t capital stocks by age n and construct a measure of capital services as in the previous sections. Thus equations (A33) are indeed the key equations in this section.

Unfortunately, in general, we cannot derive counterparts to equations (1.6)[*137] using equations (A33). To see why this is so, look at the first equation in (A33) and try to convert it into a counterpart to the first equation in (1.6):

$$\begin{aligned}
P_0^t &= g_0^t - m_0^t + \gamma^t[g_1^t + \gamma^t g_2^t + \cdots + (\gamma^t)^{L-2} g_{L^*-1}^t] - \beta^t[m_1^t + \beta^t m_2^t + \cdots + (\beta^t)^{L-2} m_{L^*-1}^t] \\
&\neq g_0^t - m_0^t + \gamma^t P_1^t.
\end{aligned} \tag{A34}$$

It is also the case that we no longer have the simple formula for anticipated time series depreciation that we derived in equations (1.38) above. Put another way, suppose all expectations held at the beginning of period t turned out to be true. Under this assumption, we can derive the following relationship between the price of a one period old asset at the beginning of period t, P_1^t, and the price of a one period old asset at the beginning of period $t+1$, P_1^{t+1}, as follows:

$$\begin{aligned}
P_1^{t+1} &= (1+i^t)[g_1^t + \gamma^t g_2^t + \cdots + (\gamma^t)^{L-2} g_{L^*-1}^t] - (1+\alpha^t)[m_1^t + \beta^t m_2^t + \cdots + (\beta^t)^{L-2} m_{L^*-1}^t] \\
&\neq (1+i^t)P_1^t.
\end{aligned} \tag{A35}$$

[*137] It was equations (1.6) that allowed us to deduce the pattern of period t user costs from the pattern of period t used asset prices.

However, if we assume that the period t anticipated gross rental price escalation factor $1+i^t$ is equal to the period t anticipated operating and maintenance cost escalation factor $1+\alpha^t$ so that γ^t is equal to β^t, then the two inequalities (A34) and (A35) become equalities. Hence, to make further progress[*138], we make the following simplifying assumption:

$$\alpha^t = i^t \text{ or } \gamma^t = \beta^t. \tag{A36}$$

Using assumption (A36), we can rewrite equations (A33) as follows:

$$\begin{aligned}
&P_0^t \equiv g_0^t - m_0^t + \gamma^t P_1^t = f_0^t + [(1+i^t)/(1+r^t)]P_1^t \\
&P_1^t \equiv g_1^t - m_1^t + \gamma^t P_2^t = f_1^t + [(1+i^t)/(1+r^t)]P_2^t \\
&\ldots \\
&P_{L^*-2}^t = g_{L^*-2}^t - m_{L^*-2}^t + \gamma^t P_{L^*-1}^t = f_{L^*-2}^t + [(1+i^t)/(1+r^t)]P_{L^*-1}^t \\
&P_{L^*-1}^t = g_{L^*-2}^t - m_{L^*-2}^t + 0 = f_{L^*-1}^t + 0
\end{aligned} \tag{A37}$$

where the second set of equations follows using equations (A30). Note that equations (A37) are exact counterparts to equations (1.6) in section 1.3 above.

Obviously, equations (A37) can be rewritten to give us explicit formulae for the gross rental prices g_n^t in terms of the period t asset prices P_n^t and the period t maintenance costs by age n, m_n^t:

$$g_n^t = m_n^t + P_n^t - [(1+i^t)/(1+r^t)]P_{n+1}^t; \qquad n = 0, 1, 2, ..., L^* - 1. \tag{A38}$$

Equations (A38) are exact counterparts to our earlier system of equations (1.7) for the period t user costs, f_n^t.

In order to get a useful, explicit depreciation model, we make some further assumptions:

$$\gamma^t \equiv (1+i^t)/(1+r^t) = \gamma \qquad \text{for all periods } t; \tag{A39}$$

$$g_n^t = \lambda^t g \qquad \text{for all periods } t \text{ and } n = 0, 1, 2, \ldots; \tag{A40}$$

$$m_n^t = \lambda^t [b + nc] \qquad \text{for all periods } t \text{ and } n = 0, 1, 2, \ldots \tag{A41}$$

where g, b and c are positive parameters with $g > b$; i.e., the gross rental must be greater than the fixed maintenance cost. We now explain the meaning of assumptions (A39)-(A41). Assumption (A39) means *that the real interest rate is constant* over all periods. Assumption (A40) is *a one hoss shay type assumption* except that it is applied to the gross output of the asset; i.e., (A40) means that the gross services yielded by a properly maintained asset of any age in period t is the same across all ages. Assumption (A41) says that the period t maintenance costs by age have a *fixed cost component* that is the same across all ages of the asset, $\lambda^t b$, plus another component that *increases linearly* in the age of the asset, $\lambda^t nc$ for an asset that is n periods old at the start of period t. The presence of the scalar factor λ^t in both (A40) and (A41) means that we are assuming that period t rental prices g_n^t and maintenance costs m_n^t are essentially constant except for a common period t *inflation factor* λ^t.

Now substitute assumptions (A36) and (A39)-(A41) into (A31) and obtain the following expression for the function $P_0^t(L)$, which gives the anticipated asset value of a new asset as a function of the

[*138] We do this in order to obtain a simpler set of relations between g_n^t, m_n^t and P_n^t than the rather complex system of relations defined by equations (A33). However, for most industries, assumption (A36) will not be warranted; i.e., operating and maintenance costs in the industry will generally increase at rates that differ from the rate of increase in gross leasing prices (or gross output prices) for that industry. In this case, we are stuck with the general setup represented by equations (A30)-(A33).

number of periods L that it is used:[*139]

$$\begin{aligned} P_0^t(L) &\equiv g_0^t + (\gamma^t)g_1^t + (\gamma^t)^2 g_2^t + \cdots + (\gamma^t)^{L-1} g_{L-1}^t \\ &\quad - [m_0^t + (\beta^t)m_1^t + (\beta^t)^2 m_2^t + \cdots + (\beta^t)^{L-1} m_{L-1}^t] \\ &= \lambda^t[g-b][1+\gamma+\gamma^2+\cdots+\gamma^{L-1}] - \lambda^t c[1+2\gamma+3\gamma^2+\cdots+(L-1)\gamma^{L-2}]. \end{aligned} \tag{A42}$$

Now reparameterize the positive parameter c as follows:

$$c \equiv [g-b]d \tag{A43}$$

where d is another positive parameter. Substitute (A43) into (A42) and the resulting equation can be rewritten as follows:

$$P_0^t(L) = \lambda^t[g-b]h(L) \tag{A44}$$

where the function $h(L)$ is defined as

$$h(L) \equiv [1+\gamma+\gamma^2+\cdots+\gamma^{L-1}] - d[1+2\gamma+3\gamma^2+\cdots+(L-1)\gamma^{L-2}]. \tag{A45}$$

Provided that d is small enough and the real interest rate escalation factor γ is close to one, a positive integer L^* that maximizes $h(L)$ will exist. This exercise determines the optimal age of retirement of a new asset.

However, we now reverse the argument: given an L^*, we look for a positive parameter d such that $h(L)$ will be at a maximum when $L = L^*$. This can be done numerically. For $\gamma = 1/(1.04)$ (this corresponds to a real interest rate of 4%), it can be verified that the d that corresponds to Maddison's assumed life for nonresidential structures, namely $L^* = 39$, is approximately equal to 0.002. This in turn corresponds to the assumption that maintenance costs for nonresidential structures are rising at the rate of 0.2 percentage points per year. The d that corresponds to Maddison's (1993)[156] assumed life for machinery and equipment, $L^* = 14$, is approximately equal to 0.012. This in turn corresponds to the assumption that maintenance costs for machinery and equipment are rising at the rate of 1.2 percentage points per year.

Once the d^* that corresponds to the desired asset life L^* has been found, then the function $h(L)$ defined by (A45) is known. Now set the right hand side of equation (A44) (evaluated at $L = L^*$) equal to the price of a new asset at the beginning of period t, P_0^t, and solve the resulting equation for $\lambda^t[g-b]$. The solution is:

$$\lambda^t[g-b] = P_0^t/h(L^*). \tag{A46}$$

We now have enough information to evaluate the sequence of period t net rental prices, f_n^t, as follows:[*140]

$$\begin{aligned} f_n^t &= g_n^t - m_n^t && n = 0,1,2,\ldots,L-1 \text{ using (A30)} \\ &= \lambda^t g - \lambda^t[b+nc] && \text{using (A40) and (A41)} \\ &= \lambda^t[g-b] - \lambda^t n[g-b]d && \text{using (A43)} \\ &= \lambda^t[g-b][1-nd] && \text{rearranging terms} \\ &= P_0^t[1-nd]/h(L^*) && \text{using (A46).} \end{aligned} \tag{A47}$$

[*139] By examining (A42), it can be seen that as b and c increase (so that either the fixed cost component or the rate of increase in maintenance costs increases), then the optimal age of retirement L^* will decrease. Conversely, as g increases (so that the gross revenue yielded by the asset exogenously increases), then the optimal age of retirement will increase.

[*140] We do not have enough information to obtain the sequence of gross rental prices, g_n^t, but, fortunately, it is the net rental prices that we need in order to calculate a capital services aggregate.

If the price P_0^t of a new asset is known, then everything on the right hand side of (A47) is known and the sequence of net rental prices f_n^t can be calculated. Once the f_n^t are known, then the second set of equations in (A37) can be used in order to obtain the sequence of vintage asset prices, P_n^t: given P_0^t and f_0^t, use the first equation in (A37) to determine P_1^t; then use f_1^t and the second equation to determine P_2^t and so on.

It turns out that this special case of the linearly increasing maintenance expenditures model is virtually equivalent to the linear efficiency decline model explained in section A4 above. The explanation for this result is contained in equations (A47): these equations show that *the (net) user costs decline linearly as older assets are used*. This is precisely the assumption made in the model presented in the previous section, when we assumed constant real interest rates.

The importance of the model presented in this section is that it casts some light on the conditions under which we might expect net rental prices to decline in a linear fashion even though we know the asset is of the *gross* one hoss shay type; i.e., an older truck can deliver the same ton miles in a period as a younger one provided that we spend enough on maintenance. Thus the simplified model presented at the end of this section provides a justification for assuming a quite accelerated form of depreciation, even though the asset essentially delivers one hoss shay type services. Put another way, in the context of assets that are capable of delivering the same services as they age, then if maintenance costs rise as the asset ages, accelerated depreciation is inevitable.[*141] The more general model presented at the beginning of this session could also be used in regulatory contexts where maintenance schedules often exist and the determination of "economic" depreciation is a matter of some importance.

The simple model presented at the end of this section may also help to explain why there is tremendous diversity in the ages at which identical assets are retired in different countries. For example, if maintenance costs are higher or are expected to rise more quickly in a particular country, then the model implies that identical assets in that country will be retired at an earlier age. This observation can help to explain why well maintained assets in developing countries are used much longer than in developed countries. Conversely, assets employed in a country enjoying a boom so that gross rental prices are relatively high will be retired at a later age than assets employed in a country experiencing relatively low rental rates, other things being equal. If future asset rental rates are expected to decline or increase less rapidly than future maintenance costs (i.e., i^t increases less than α^t), then the expected future gross revenues will decline or grow less rapidly than expected future operating costs and the asset will be retired earlier. Thus the models presented in this section can cast some light on why the same asset is retired at different ages across countries and uses.

1.12 References

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Chapter 2

Capital and Accounting Theory: The Early History

2.1 Introduction

In this chapter, we will review what accounting theory has to say on some controversial topics associated with the measurement of capital. To an outsider with an economics background, the accounting treatment of capital, interest and depreciation seems rather strange. Hopefully, this chapter will help to explain why the accounting profession ended up with the positions that it has taken with respect to these measurement issues.

Sections 2.2, 2.3 and 2.5 look at the accounting treatment of interest on equity capital, on depreciation and on the treatment of capital gains respectively. Section 2.4 provides an introduction to section 2.5 and it tries to explain why accountants do not regard capital gains as being "productive", whether these gains were anticipated or not.

2.2 Accounting Theory and Interest as a Cost of Production

> "The stock which is lent at interest is always considered as a capital by the lender. He expects that in due time it is to be restored to him, and that in the mean time the borrower is to pay him a certain annual rent for the use of it." Adam Smith (1776; reprinted 1963; 271)[195].

> "And human nature being what it is, we are justified in speaking of the interest on capital as the reward of the sacrifice involved in the waiting for the enjoyment of material resources, because few people would save much without reward; just as we speak of wages as the reward of labour, because few people would work hard without reward." Alfred Marshall (1920; 232).

*[1] The author thanks Peter Hill for valuable comments on an earlier draft of this chapter.

It is clear that economists from Adam Smith through Alfred Marshall regarded interest as a reward (to the lender of financial capital for abstaining from or deferring consumption and as a cost of production for producers who are recipients of the financial capital.[*2]

However, the problem of explaining the factors that determined the interest rate was a much more difficult task. Böhm-Bawerk (1891; 24-72)[18] summarized the literature on this topic up to his time and provided a verbal description of a modern theory of interest[*3] while Fisher (1930)[94] presented a very convincing algebraic and geometric description of the same theory.[*4] Böhm-Bawerk (1891; 285-6)[18], Fisher (1897; 522)[89] and Hicks (1946; 141-142)[114] explained how the present price of a good purchased now for delivery next period is equal to the (spot) price of the good next period divided by one plus the current period interest rate. Hicks (1946; 136)[114] generalized the simple one (spot) commodity and multiple time period models of Fisher (1930)[94] into a general model with many commodities and many time periods (his "Futures Economy" where all commodities can be bought and sold on forward markets) and Debreu (1959)[42] provided a rigorous proof of the existence of equilibrium in such a model. But Hicks (1946; 119-127)[114] also developed another model of intertemporal equilibrium that had a theory of the interest rate built into it: the temporary equilibrium model.[*5] This second Hicksian model used the same building blocks as the futures economy model, except that instead of assuming the existence of futures markets, Hicks assumed the existence of current period (spot) markets for commodities and financial capital and the existence of definite expectations about future period spot prices (which could depend on current period prices) for all consumers and producers in the economy. In this model, these expected future period spot prices were used by producers and consumers in their intertemporal profit maximization and utility maximization plans.

The above sketch of the role of interest in modern economic theory would seem to indicate that economists generally accept interest as a valid cost of production, and indeed, interest plays a vital role in the intertemporal allocation of resources. However, many accountants and some economists objected to interest as a cost of production; in particular, they objected to interest that is imputed to the equity capital employed by a business unit. A few accountants objected to associating an interest cost with the use of a durable input over an accounting period (in addition to a depreciation

[*2] Böhm-Bawerk (1891; 301-303 and 365-371)[18] and Fisher (1930; 48-52)[94] patiently refuted the Marxian exploitation theory which denied that interest was a valid cost of production.

[*3] "In our science there are three views in circulation as to the formation of capital. One finds its origin in Saving, a second in Production, and a third in both together. Of these, the third enjoys the widest acceptance, and it is also the correct one." Eugene von Böhm-Bawerk (1891; 100)[18].

[*4] For the case of one (spot) commodity and two periods, his geometric analysis was particularly compelling. Fisher (1930; 246)[94] introduced a family of "willingness lines" (i.e., indifference curves over combinations of consumption today and consumption tomorrow) and in Fisher (1930; 265)[94], he introduced the "opportunity line" (i.e., the combinations of consumption today and consumption tomorrow that the economy could produce{the intertemporal production possibilities set). Unfortunately, Fisher (1930; 57)[94] viewed interest as a capital gain and hence interest payments were not negative "income" for a producer and hence could not be a "cost" in his view. Fisher's "income" was what everybody else called "consumption" and so his terminology caused great confusion in the economics literature of his time.

[*5] The Hicksian model of temporary equilibrium can be viewed as a generalization of a model of capital formation due to Walras (1874; translated 1954; 267-306)[225]. In order to justify a demand to save or defer present consumption, Walras (1954; 274)[225] assumed the existence of a single future period commodity that would be consumed in fixed proportions indefinitely and hence had a price which was proportional to the reciprocal of the one period interest rate, which was assumed to be constant over time. In deriving the price of this future composite commodity, Walras assumed that the spot price of the composite commodity would remain constant over future periods; i.e., he made a static expectations assumption. Hicks (1946; 227-232)[114] was able to dispense with these restrictive assumptions by assuming general intertemporal preferences for consumers and definite expectations about future prices. Walras' (1954; 239-240)[225] theory of production was also somewhat primitive. He essentially assumed: no joint production, constant returns to scale and fixed coefficient or no substitution production functions (which have come to be known as Leontief (1941)[147] production functions). On the other hand, Hicks (1946; 325-326)[114] had a perfectly general intertemporal production function. For a modern interpretation of Walras' theory of capital formation and interest rate determination, see Diewert (1977)[47].

cost) on the grounds that such interest rate adjustments are likely to be minor in view of the errors involved in estimating depreciation.*6 However, if interest rates are high and the durable input is long lived, other accountants have pointed out that the neglect of interest can lead to substantial underestimation of costs.*7 On the other hand, the main objection of accountants*8 and some economists*9 to the inclusion of interest on equity capital as a cost is that it is an imputation or estimated value and accountants should stick to recording values rather than creating them. These objectors have a valid point: it is not a trivial matter to determine precisely what are the relevant debt and equity interest rates.*10 However, in our view, imputations associated with capital are inevitable and are due to the durable nature of capital inputs and the fact that not all capital inputs are rented or leased. It is better to be approximately correct than precisely wrong!*11

*6 "In the muddy pool of controversy over the question of 'interest on the investment', one finds all kinds of slippery arguments about what rate is to be employed. Most who discuss the question seem to think that, given a correct rate, there is little to object to except the arbitrariness of the rate chosen and of the valuations that result, and the clerical labor involved in the calculating and recording ... . But the present writer wishes to urge a somewhat different objection. If interest is to be charged at some agreed rate, into what quantity shall that rate be multiplied? Into book value of the assets? But look at these book values. Consider how they are themselves determined and consider how far they are likely, under any valuation rule yet discovered, to miss the ideally useful and convenient figure. To go through the book valuations found by the simple [accounting depreciation] formulas in Appendix A, where exact prevision of events is supposed, and begin charging interest upon them, is as absurd as trying to correct for the earth's rotation in a snowball fight. To attempt a 5% alteration of an amount which may be 50% in error in a direction unknown to us, is what the kindergarten teachers call 'busy work"'. John B. Canning (1929; 297)[23].

*7 "Cost-of-capital is seldom recognized when practical men calculate depreciation. This is not unreasonable if the rate is low or the life is short. But cost-of-capital alters the figures materially if the rate is high or the life is long (as they often are): its recognition then gives useful extra information about asset values, income and costs." William T. Baxter (1975; 159)[8].

*8 "The whole question of whether actual historical costs or imputed costs (this includes all costs that are not actual, from the point of view of explicit transactions entered into between the particular enterprise and other parties) should constitute accounting data is that of whether the accountant can or should attempt to account for enterprise income on the basis of the same set of concepts that the economist uses. The inclusion of imputed costs of any variety, in the accounts, invariably results in a confusion of expenses with aspects of business income or its distribution." William T. Crandell (1935; 387)[36]. "Although few would assert that equity interest is not a cost, many persons argue that it is an imputed cost and that accounting does not record imputed costs." Robert N. Anthony (1973; 90)[4].

*9 Economists who are national income accountants have in the past resisted imputing an interest cost for the use of equity capital as the following quotations from SNA 1993 indicate: "The amounts of rents and interest actually payable on rented land and borrowed funds are recorded in the allocation of primary income account, and the entrepreneurial income account, but the implicit rents on land owned by the enterprise and the implicit interest chargeable on the use of the enterprise's own funds are not recorded in the accounts of the System." Eurostat (1993; 175)[87]. "The volumes of intermediate consumption, consumption of fixed capital and any taxes on production measured at the prices or rates of the previous year or the fixed base year should be added to obtain a comprehensive volume measure covering all inputs." Eurostat (1993; 402-3)[87]. The last quotation indicates that the current System of National Accounts 1993 does not recognize interest, either imputed or paid, as a period cost that should be associated with the use of capital inputs: only depreciation and tax costs are recognized by the System. However, the next international version of the SNA will probably recognize interest as a cost of production in principle, thanks to the efforts of the Canberra Group II on the Measurement of Non-Financial Assets.

*10 Wright noted that the "correct" opportunity cost of equity funds for the firm might depend on whether it was necessary for the firm to raise funds or whether it had excess funds that need to be invested: "Since this article is concerned with the theory rather than the practice of depreciation, however, a few remarks about the determination of the interest rate may not be out of place. The appropriate rate for any period will be the opportunity cost of funds during that period. In the case of a firm whose growth is not restricted by the amount of funds it can raise, the opportunity cost of funds will usually be the average cost of raising additional funds, though in some cases it might be the rate at which surplus funds can be placed in the market." F.K. Wright (1964; 86)[231].

*11 This phrase is attributed to J.M. Keynes. Of course, not all accountants objected to the economists treatment of interest: "On the other hand the capitalization of interest during construction can be supported by reference to the economist's conceptions of effective cost. Our old friend, 'interest on investment as an operating cost', bobs up in this connection. Suffice it to say here on this point that the case against the inclusion of hypothetical interest charges in operating expense must be based on other grounds than the economist's theories of price

In section 2.4 below, we attempt to cast a bit more light on the question about the productiveness of interest by looking at the literature on exactly which types of economic activity are "productive". However, before we do this, we consider what the accounting literature has said about the nature of depreciation in the following section.

2.3 Accounting Theory and Depreciation

2.3.1 Introduction

"Depreciation is defined simply as rate of decrease of value." Harold Hotelling (1925; 341)[126].

"The net stock concept is motivated by the observed fact that the value of a capital good declines with age (and/or use). This decline is due to several factors, the main ones being the decline in the life expectancy of the asset (it has fewer work years left), the declines in the physical productivity of the asset (it has poorer work years left), and the decline in the relative market return for the productivity of this asset due to the availability of better machines and other relative price changes (its remaining work years are worth less). One may label these three major forces as exhaustion, deterioration, and obsolescence." Zvi Griliches (1963; 119)[103].

Chapter 1 above defined the (cross sectional) depreciation rate of a durable input in terms of the decline in value of a "newer" machine or other durable input compared to a "used" machine that had been used for one additional accounting period.*12 The two values that were compared were market values that pertained to the end (or the beginning) of the accounting period. If the price of "new" machines were the same at the beginning and the end of the accounting period (so that $P_0^t = P_0^{t+1}$), then the decline in the value at the end of the period, $P_1^{t+1} - P_0^{t+1}$, would be the same as the decline in value over period t, $P_1^{t+1} - P_0^t$. Many of the early treatments of depreciation implicitly assumed price stability (i.e., $P_0^t = P_0^{t+1}$), and hence depreciation was identified with the decline in value of the durable input over the accounting period.*13

As the above quotation by Griliches (1963; 119)[103] above indicates, economists tried to analyze the factors that determine depreciation rates.*14 Accountants, engineers, statisticians and economists have all made contributions to the literature on depreciation. We shall review some of their approaches below; see Chapter 1 for other approaches.

determination. Virtually all economists view interest on capital, proprietary as well as borrowed, as an effective cost of production." W.W. Paton (1931; 93-94)[178].

*12 However, Chapter 1 indicated that *real time series depreciation* was also a useful depreciation concept. It is this latter concept that fits in nicely into a user cost formula, which will form a hopefully good approximation to a market rental rate for the asset in question for the period under consideration. Thus the purpose to which we want to use the depreciation estimate must be kept in mind.

*13 Recall the above quotation by Hotelling. It should be noted that the economic statistician Hotelling (1925; 345)[126] deduced a continuous time counterpart to the user cost formula (1.7) in chapter 1.

*14 It is interesting to note that Pigou identified more or less the same three factors affecting depreciation as Griliches: "Allowance must be made for such part of capital depletion as may fairly be called 'normal'; and the practical test of normality is that the depletion is sufficiently regular to be foreseen, if not in detail, at least in the large. This test brings under the head of depreciation all ordinary forms of wear and tear, whether due to the actual working of machines or to mere passage of time—rust, rodents and so on—and all ordinary obsolescence, whether due to technical advance or to changes of taste. It brings in too the consequences of all ordinary accidents, such as shipwreck and fire, in short of all accidents against which it is customary to insure. But it leaves out capital depletion that springs from the act of God or the King's enemies, or from such a miracle as a decision tomorrow on the part of this country to forbid the manufacture of whisky or beer. These sorts of capital depletion constitute, not depreciation to be made good before current net income is reckoned, but capital losses that are irrelevant to current net income." A.C. Pigou (1935; 240-241)[181].

2.3.2 The Appraisal Approach

The earliest approaches to depreciation were based on appraisals:

> "[There are] various methods of estimating the Depreciation of a Factory, and of recording alteration in value, but it may be said in regard to any of them that the object in view is, so to treat the nominal capital in the books of account that it shall always represent as nearly as possible the real value. Theoretically, the most effectual method of securing this would be, if it were feasible, to Revalue everything at stated intervals, and to write off whatever loss such valuations might reveal without regard to any prescribed rate ... . The plan of valuing every year instead of adopting a depreciation rate, though it might appear the more perfect, is too tedious and expensive to be adopted ... the next best plan, which is that generally followed ... is to establish average rates which can without much trouble be written off every year, to check the result by complete or partial valuation at longer intervals, and to adjust the depreciation rate if required." Ewing Matheson (1884; 35)[160].

> "One of the first clear references to depreciation accounting was in the annual report of the Baltimore and Ohio Railroad for the year ended September 30, 1835. That report explained that income for the year was determined 'after carrying $75,000 to the debit of profit and loss to make good deterioration of the railway and machinery ... .' During the years following 1835, there was no consistent policy followed by any group of companies or even by any one company. Apparently, some companies made a separate provision for depreciation as did the Baltimore and Ohio Railroad, while other companies charged replacement costs to expense in lieu of depreciation." P.D. Woodward (1956; 71)[230].

Thus the very earliest treatments of the depreciation problem seem to have been on the basis of periodic appraisals of the value of fixed assets. Thus the first early accounting approach to the determination of depreciation is (i) the appraisal approach: changes in appraised values, if negative, were regarded as costs to be charged to the accounting period between appraisals.[*15] However, as the quotation by Woodward above indicates, there were two additional early treatments of depreciation: (ii) engineers made estimates of the value of the physical deterioration and loss of productive life that equipment and machinery might have experienced during an accounting period[*16] and (iii) new purchases of durable inputs were simply expensed in the period of purchase.

Obviously, the third approach (which is consistent with cash flow accounting) is not helpful in the determination of periodic income (period by period income), which is the task at hand.[*17] The other two approaches are reasonable but not helpful in the context of the explicit determination of depreciation rates: approach (i) mixes up (unanticipated) capital gains with the determination of depreciation rates while approach (ii) gives no indication as to how depreciation rates would be determined. However, in general, appraisal approaches are useful, particularly if the purpose of the

[*15] "Bookkeeping Modernized by John Mair (2nd edition, 1768). The same general procedure is described as in Bookkeeping Methodiz'd and there are no illustrations of depreciation but the inference might be made that if the 'value' of the property were less than cost, this decrease in value would be included in the change of Profit and Loss." Perry Mason (1933; 210-211)[159]. "There is little reason to doubt that depreciation was originally calculated on the basis of appraisals. The appraisal, it may be conjectured, was originally on a market price basis in order to obtain a figure roughly equivalent to what would have been realized at the date of the appraisal had the asset actually been sold ... . After general adoption of the accounting period convention, such appraisals were probably made at the end of each accounting period. It must, however, soon have been obvious that such periodic appraisals gave erratic results depending, of course, upon who made them, how they were made and the general state of business at the time they were made." Stephen Gilman (1939; 488)[99].

[*16] This is really a variant of method (i).

[*17] However, if period by period estimates of "income" are not required, the third approach is perfectly consistent with the economist's usual approach, which consists in maximizing the discounted value of future expected profits; e.g., see Hicks (1946)[114].

accounting exercise is to measure ex post income.

2.3.3 Sinking Fund Approaches

Another approach to depreciation accounting used by early accountants was the *sinking fund approach*:

> "The depreciation problem, in general terms, is the problem of writing off from fiscal period to fiscal period sums sufficient to return the capital invested in a property when that property has outlived its usefulness." J.S. Taylor (1923; 1010)[214].

> "After the straight line formula the one perhaps most widely used is the sinking fund formula and the modifications of this method." John B. Canning (1929; 273)[23].

As the above quotations indicate, some early statisticians and accountants viewed the depreciation problem as a method for funding the future replacement of a durable input. In this sinking fund method, the focus shifted from changes in asset values to the problem of setting aside period by period accounting charges into a fund which will cumulate over the useful life of a fixed asset into an amount which will be sufficient to replace the asset on its retirement date. This treatment of depreciation has some elements in common with the maintenance of physical capital approach to income determination, but the two methods are distinct.

In the 1930's, the sinking fund approach to depreciation was successfully attacked by a number of accountants on the grounds that depreciation accounting should be viewed as a method of spreading the initial cost of a durable input over its useful life rather than as a solution to the logically distinct problem of deciding whether the asset should be replaced at the end of its life:

> "There is a much larger number who in some way try to identify the deposits to a sinking fund or the deposits and earnings thereon with 'depreciation expense'. The only source of an 'expense' of depreciation is the outlay or outlays made or agreed to be made for the asset in order to have the enjoyment of the service." John B. Canning (1929; 274)[23].

> "Depreciation exists whether the property being used is to be replaced or not. In no sense does the depreciation allowance ('reserve') account represent the accumulation for the purpose of acquiring future assets." M.B. Daniels (1933; 306)[38].

> "Generally it is conceded that it is the purpose of recording depreciation to recover the original expenditure, the purchase of a new truck being a separate and distinct transaction having no possible connection with, or relation to, the recovery of the original investment. It is simple to point out that the truck at the end of six years may be replaced at a higher price or a lower price, or at the same price, or by teams and wagons, or that no replacement will be made if the need for a truck has disappeared. The above arguments seem sufficiently convincing to discredit this replacement theory in so far as it is related to fixed assets ." Stephen Gilman (1939; 349)[99].

> "The availability of money for replacement may offer serious financial problems. The problem of financing replacements may be sufficiently difficult to tax the resourcefulness and foresight of business men but it is in no sense whatever an accounting problem. The originally acquired asset was a deferred charge and its cost is recovered by the depreciation program. The replacement whether it be an identical item or not, is a fresh transaction resulting in the creation of a new deferred charge the cost of which in turn must, from the accounting viewpoint, be recovered over the years which follow its acquisition." Stephen Gilman (1939; 494)[99].

We turn now to accounting treatments of depreciation that take the *intertemporal cost allocation viewpoint.* These are basically the approaches that were reviewed in the Appendix to chapter 1, except that this early accounting literature implicitly assumed a stable general price level and made no adjustments for changes in the general price level and it avoided the complications introduced by discounting.

2.3.4 Intertemporal Cost Allocation Approaches

> "A plough, for instance, which lasts twenty years, will contribute a twentieth part of its life-work and use to the ingathering of twenty different harvests." Eugen von Böhm-Bawerk (1891; 305)[18].

> "Straight Line Formula ... . In general, only two primary estimates require to be made, viz., scrap value at the end of n periods and the numerical value of n ... . Obviously the number of periods of contemplated use of an asset can seldom be intelligently estimated without reference to the anticipated conditions of use. If the formula is to be respectable at all, the value of n must be the most probable number of periods that will yield the most economical use." John B. Canning (1929; 265-266)[23].

The first method that comes to mind in attempting to determine a sequence of depreciation rates for a durable capital input as it ages is the one suggested by Böhm-Bawerk above: estimate the expected number of accounting periods n that the input is likely to be used in production and assume that the single period depreciation rate is $\delta = 1/n$. This straight line method of depreciation can be used to allocate the initial purchase cost of the asset, say P^0, across the n periods of its life; these historical cost allocations under *straight line depreciation* would be $(1/n)P^0, (1/n)P^0, ..., (1/n)P^0$, a sequence of n equal allocations. This would correspond to a *historical cost method of depreciation*. The straight line depreciation method can also be used in conjunction with current values of new units of the asset, yielding the following sequence of *current value depreciation charges*: $(1/n)P^0, (1/n)P^1, ..., (1/n)P^{n-1}$, where P^t is the price of a new unit of the asset at the beginning of period t, for $t = 0, 1, ..., n-1$.*18

Another commonly used method for the determination of depreciation rates rests on the assumption that depreciation occurs on the undepreciated value of the asset at a *constant geometric rate* δ where $0 < \delta < 1$. The sequence of historical cost allocations of original cost P^0 that this method generates is $\delta P^0, \delta(1-\delta)P^0, \delta(1-\delta)^2P^0, ..., \delta(1-\delta)^nP^0, ...$ while the corresponding stream of periodic current cost accounting charges is $\delta P^0, \delta(1-\delta)P^1, \delta(1-\delta)^2P^2, ..., \delta(1-\delta)^nP^n, ...$. This method of depreciation is sometimes called the *reducing balance method**19 or the *declining balance method*.*20 As we saw in chapter 1, this method of accounting for depreciation (applied to current values) is very convenient when it is necessary to construct capital aggregates for productivity measurement purposes. Empirical estimates for the declining balance depreciation parameter δ generally come from: (i) "official" estimates by broad asset class made by the national tax or regulatory authorities;*21

*18 These depreciation methods neglect the complications that occur when we take into account anticipated asset price changes that are expected to take place over the accounting period.

*19 See Hotelling (1925; 350)[126].

*20 See Canning (1929; 276-277)[23].

*21 "It is undoubtedly true that the methods now in use for the systematic accounting for depreciation and many of the now generally accepted concepts of depreciation have a comparatively recent origin, and that much of the development of the subject has taken place since and as a result of the establishment of governmental regulation of public utilities and the enactment of income tax legislation." Perry Mason (1933; 209)[159]. "Under the impetus of increased income tax rates due to World War I, the interest in depreciation calculation increased. The first edition of Bulletin F issued by the Bureau of Internal Revenue in August 31, 1920, advocated the straight-line age-life method." P.D. Woodward (1956; 74)[230]. "Rates of depreciation must of necessity conform to Internal Revenue edicts ... on documents submitted to the Internal Revenue Service. Unfortunately, the same rates are often used in accounts which do not have to be submitted to the IRS, with attendant undesirable influences on business decisions ... . The writer advises engineering students as follows: 1. Disregard official rates of depreciation. 2. Estimate economical length of life for the type of service originally intended for the asset ... ." William F. Berkow (1964; 380)[15].

(ii) estimates made by the engineers or managers of the business unit,*22 or (iii) statistical studies such as those to be discussed in chapter 1.

Saliers (1922) and Canning (1929; 260-309)[23] list many other rather arbitrary methods that accountants have used to estimate depreciation rates. The arbitrariness of these accounting depreciation methods and the fact that the estimates are generally based on a priori reasoning rather than on empirically observable declines in value*23 has of course attracted comment from many accountants and economists over the years:

> "Accountants immediately discard their own figures and demand an appraisal of the plant and other fixed assets, whenever they are called upon to compute capital value for the purpose of sale, reorganization etc. Apart from such occasions they adhere to their depreciation methods with the proviso that the method itself matters less than consistent adherence to it, once it has been adopted. These methods generally limit guessing to a minimum considered unavoidable in the circumstances." Gabriel A.D. Preinreich (1938; 240)[185].

> "For the past hundred years accountants have been searching for the 'true' depreciation method which would allocate the cost of the machine over its lifetime in accordance with the rate at which it is actually being 'used' up. They have reluctantly concluded that there is no 'true' depreciation method, and that all the methods used or proposed are mere conventions, the choice between which is a matter of convenience." F. Lutz and V. Lutz (1951; 7)[153].

However, historical cost accountants such as Daniels and Ijiri have defended the arbitrariness of accounting cost allocations as follows:

> "The function of depreciation is recognized by most accountants as the provision of a means for spreading equitably the cost of comparatively long lived assets. Thus, if a building will be of use during twenty years of operations, its cost should be recognized as operating expense, not of the first year, nor the last, but of all twenty years. Various methods may be proper in so allocating cost. The method used, however, is unimportant in this connection. The important matter is that at the time of abandonment the cost of the asset shall as nearly as possible have been charged off as expense, under some systematic method." M.B. Daniels (1933; 303)[38].

> "However, there is a diametrically opposite problem in historical cost accounting ... the problem is one of disaggregation or allocation. Suppose that resources A and B are purchased together for $20, but at the end of the year the firm had only Resource A. How much of the $20 should be assigned to Resource A? Depreciation is a typical problem of this kind. However, accountants have devised many methods, however arbitrary they may be, by which such allocations are carried out objectively." Yuji Ijiri (1979; 67)[132].

Both of the above authors recognize the arbitrariness of historical cost accounting allocations of asset cost; the best that can be said of these methods is that they are "systematic". If the tax authorities

*22 "Finally, a word should be said about the professional responsibility for valuation. Many accountants assert that valuation of fixed tangible assets is a job for appraisal engineers. Others say that it is the job of the management themselves and that accountants have discharged their whole duty when they avoid certifying statements in which the assets have been negligently or fraudulently valued. The engineers are not too happy with the burden thrust upon them. They say, at least many of them do, that it is impossible to make valuations unless the operating policy, particularly that of maintenance, upkeep and repairs, is foreknown." John B. Canning (1929; 307)[23].

*23 Wright contrasted the accountant's allocation approach with the economist's change in value approach as follows: "There have been two distinct approaches to the solution of the depreciation problem which might be designated the "accounting approach' and the 'economic approach', respectively. The accounting approach requires the cost of an asset less salvage, if any, to be distributed over the life of the unit 'in a systematic and rational manner'. The economic approach, on the other hand, ignores cost as an irrelevant datum: the value of an asset at any point of time is simply the sum of its discounted future services (including salvage if any). It seems clear that the accounting approach does not really represent an attempt at valuation: indeed, it has been officially described as 'a process of allocation, not of valuation'." F.K. Wright (1964; 81)[231]. Wright was right!

specify that one or more depreciation formulae must be used for tax purposes, then the use of the resulting historical cost allocations might also be characterized as "objective".

Since historical cost accountants regularly criticize current value accountants for their use of imputed or estimated values, it is important to recognize that historical cost accounting is subject to precisely the same criticism: historical cost accounting, by accepting an arbitrary a priori pattern of depreciation rates *imputes* period by period depreciation costs. If we attempt to estimate the period by period durable input costs accruing to a business unit, *then any method of accounting will have to resort to imputed or estimated values.*[*24]

We conclude this section with three additional criticisms of historical cost depreciation allocations.

The first criticism is due to Canning[*25] who asked that criteria be developed to choose among the many depreciation methods that were used by historical cost accountants. This request for a rational criterion for choosing a depreciation formula has not been answered because the answer cannot be given on the basis of a priori reasoning: period by period empirical evaluation of the physical condition and market value of the assets to be depreciated is required.

The second criticism is due to Edwards and Bell[*26] who noted that the historical cost accountant would need to be clairvoyant in order to determine the useful life of an asset; i.e., identical new assets are not all retired at the same time.[*27] On the other hand, current value accounting techniques, by estimating period by period used asset values avoid in principle the difficult problems that arise when durable inputs are used at different intensities.[*28] We can paraphrase this second criticism of historical cost accounting techniques as follows: different historical cost accountants will estimate different lengths of life (and scrap values) for the same asset, leading to variable or "nonobjective" period by period depreciation estimates.

The third criticism of the historical cost accountant's allocation method for determining period by period depreciation is the most important one: historical cost accounting will not preserve the real capital of the firm during periods of inflation; i.e., *historical cost accounting income will be vastly overstated during periods of high inflation* and will lead to unsustainable levels of taxation for firms who face an income tax that is based on income measures generated by historical cost accounting conventions. Thus the unpleasant consequences of the neglect of general or specific price change in the treatment of depreciation include the following items:

- The possibility of crippling income taxes;

[*24] "For a specific asset, objective verifiable values based upon external transactions are available at only two points of time: at the moment of acquisition, and at the moment of disposal. If these two events occur within the same accounting period, no depreciation problem arises. But when the events are widely separated in time (as is usually the case with fixed assets), determination of periodic income is impossible without establishing a value for the asset at the end of each intervening period. The problem of depreciation accounting is the problem of establishing these needed values without the objective verifiable basis which only external transactions can provide." F.K. Wright (1964; 81)[231].

[*25] "The interminable argument that has been carried on by the text writers and others about the relative merits of the many formulas for measuring depreciation has failed, not only to produce the real merits of the several methods, but, more significantly, it has failed to produce a rational set of criteria of excellence whereby to test the aptness of any formula for any sub-class of fixed assets." John B. Canning (1929; 204)[23].

[*26] "If historic cost is to be allocated among the asset's services as time passes, it is necessary to know in advance the total stock of these services. Otherwise there can be no basis for apportionment. Current cost depreciation, on the other hand, requires in theory no such clairvoyance. We need only know the services used or foregone this period and the price this period of those services." Edgar O. Edwards and Philip W. Bell (1961; 175)[81]. The position of Edwards and Bell is precisely the position that we took in chapter 1: we are interested in measuring depreciation in order to form an approximation to the rental value of a capital input for the period under consideration and the opportunity cost approach advocated by Edwards and Bell is one of the best approaches to this measurement problem.

[*27] "A thousand new Ford cars, regardless of prior statistics, may last five years, eight years, or ten years, a fact which no one can determine from examining statistics of old Ford cars." Stephen Gilman (1939; 513)[99].

[*28] However, in practice, it is difficult to determine these opportunity cost values accurately. But the principle is a good one.

- The possibility of unsustainable dividend payments being made; and
- The possibility of products being priced incorrectly (i.e., being priced below cost).

Sweeney noted that the above problems with historical cost accounting methods for determining depreciation became very apparent during the German hyperinflation*29 of 1923:

> "Regarded from the more technical viewpoint of accounting, the problems faced and solved by German accountants during the period of absurd price fluctuations are quite worthy of study by accountants of all other countries. For the problems associated with keeping and interpreting financial records that must be expressed in a monetary unit oscillating even hourly are problems that the rest of the world must face to a less extreme degree." Henry W. Sweeney (1927; 180-181)[204].

Sweeney went on to point out that if traditional historical cost depreciation is used under these conditions, firms will record apparent profits that are really losses and business income taxation will be too high:

> "This kind of profit is an 'apparent profit' (Scheingewinn). But the true result was a loss because the goods were sold at less than actual cost in terms of the current price level." Henry W Sweeney (1927; 184)[204].

> "Finally, such inflation tends to cause profits and income taxes to be paid on merely apparent profits, and, therefore, from capital. What Mahlberg considered very unjust was not merely the fact that apparent profits, which were actually capital losses, were taxed; but especially the fact that many large genuine profits could not be taxed according to law. Such genuine profits were made by concerns with large capital asset holdings and small depreciation thereon on the one side, and with large long term liabilities on the other. The injustice of a taxation system that measures income in a depreciating monetary unit and then taxes it is well stated by F.W. Thornton with regard tot he United States." Henry W. Sweeney (1927; 190-191)[204].

How do accountants (and government tax authorities) cope with the defects of historical cost accounting when inflation becomes so high that it cannot be ignored? Sweeney pointed out what happened in Germany during its hyperinflation:

> "At first various supplementary measures were adopted, such as charging all new fixed asset costs to expense and creating a special reserve to provide for maintenance of plant value and business efficiency ... Later, computation of depreciation on the basis of reproductive cost grew in popularity, which, indeed, is still evident form a survey of cotemporary German depreciation theory." Henry W. Sweeney (1931; 166)[206].

Thus when there is very high inflation, historical cost accounting has to be abandoned with either all asset purchases being immediately expensed (so there is no need for depreciation allowances) or with nominal dollar depreciation values being indexed for general inflation (this is consistent with maintaining real financial capital intact) or for the specific asset inflation rate (this is consistent with maintaining real physical capital intact).*30 Sweeney described the differences between historical cost accounting and the two methods for indexing depreciation (using a general price index—his stabilized

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*29 Sweeney (1927; 181-182)[204] indicated that the number of paper marks need to buy one gold mark went from 1 in January 1914 to the following values:1.353 in January 1916, 1.389 in January 1918, 14.776 in January 1920, 47.911 in January 1922, 11,672 in January 1923 and to 1,000,494,971,000 on December 27, 1923.

*30 Recall the controversy between Pigou (1941)[182] and Hayek (1941)[110] on maintaining capital intact.

accounting*31 —versus a specific price index—reproductive cost depreciation*32) as follows:

> "The fundamental difference between ordinary and stabilized depreciation methods is that, whereas the former is based upon original cost per books, the latter rests upon original cost adjusted for the change in average price levels. And the underlying difference between stabilized depreciation and reproductive cost depreciation is that, whereas the former is based upon the current *general* price level equivalent of original cost, the latter depends upon the current *special* price level equivalent of original cost (viz.., cost of reproduction). Or, otherwise expressed, the ordinary depreciation method is concerned with maintenance of nominal, monetary capital; the stabilized type with preservation of real, economic capital; and the reproductive cost type with keeping physical, material capital intact." Henry W. Sweeney (1931; 174)[206].

It should be mentioned that the type of accounting that adjusts asset values and depreciation amounts from historical cost into current values (either of the general or specific type) is known as *current value accounting* in order to distinguish it from traditional historical cost accounting.*33 Sweeney (1927)[204] (1931)[206] favored general price level adjustments and the related maintenance of real financial capital concept as did Hayek (1941)[110] some years later. Sweeney explained his preference for general price level adjustments as follows:

> "Maintenance of *nominal* capital, which may be kept intact by maintenance of the same mere money amount, is short sighted when money is depreciating in value. Maintenance of *material* capital, which may be maintained by constant ownership of the same amount of material quantities, may be poor business policy if the same goods in the same quantities are not required for business needs, and it may not be capital maintenance according to sensible interpretation, because the original goods being maintained in kind and quantity may be decreasing in general economic desirability (i.e., value) over a period of time. hence, maintenance of value, not of mere physical equivalence, insures preservation of the same economic power over goods and services, and such preservation, which is maintenance of *real* capital, is much more worthwhile." Henry W. Sweeney (1927; 185)[204].

Sweeney's preference for the maintenance of real financial capital is very similar to Hayek's (1941)[110] preference for maintaining real financial capital intact over Pigou's (1941)[182] preference for maintaining physical capital (or material capital to use Sweeney's phrase) intact. However, it is a somewhat tricky business to implement (correctly) the Sweeney and Hayek view; we will address this issue in chapter **??** below.*34

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*31 Middleditch appears to be the first accountant to advocate this general price level adjustment method. He referred to the problem of incomparable units of purchasing power as follows: "Today's dollar is, then, a totally different unit from the dollar of 1897. As the general price level fluctuates, the dollar is bound to become a unit of different magnitude. To mix these units is like mixing inches and centimeters or measuring a field with a rubber-tape." Livingston Middleditch (1918; 114-115)[164]. Sweeney (1927; 183)[204] later made a similar observation: "Such comparison violates the mathematical principle that dissimilar items can not properly be compared with one another."

*32 Zeff (1982; 546)[234] attributed the replacement (or reproductive) cost or specific price level adjustment solution to Paton (1918)[176]. We will deal with replacement cost in more detail in the following chapter.

*33 For additional material on current value accounting, see Baxter (1975)[8], Whittington (1980)[227] and Zeff (1982)[234].

*34 As we shall see in chapter **??**, both types of adjustment are necessary in order to define income under inflationary conditions, which follows the strategy suggested by Sterling: "It follows that the appropriate procedure is to (1) adjust the present statement to current values and (2) adjust the previous statement by a price index. It is important to recognize that *both* adjustments are necessary and that neither is a substitute for the other. Confusion on this point is widespread." Robert R. Sterling (1975; 51)[203].

Given that Sweeney's attacks on the inadequacies of historical cost accounting[*35] were quite convincing in periods of high inflation and hence show that there are problems even when inflation is low, it is somewhat disappointing that historical cost accounting has not been fundamentally reformed in virtually all countries. Fitzgerald, Stickler and Watts made the following observations on the durability of historical cost accounting even when most OECD countries were experiencing substantial inflation:

> "Presumably in response to the high rates of inflation experience, legal requirements have been introduced in Brazil and Chile for the adjustment of historical cost statements to reflect general price level changes. In Argentina, historical cost statements are accompanied by statements similarly adjusted. However, the replacement of historical cost statements with statements adjusted for the effects of *general price-level* changes is prohibited by law or by the accountancy profession in forty-three countries. The replacement of historical cost statements with statements adjusted for the effect of *specific price* changes is similarly prohibited in forty-six countries. It is required in none and minority practice in only one country—the Netherlands." R.D. Fitzgerald, A.D. Stickler and T.R. Watts (1979; 12)[95].

In section 2.5 below, we will review accounting treatments of capital gains. However, before we do this, it is first necessary to review what economists and accountants have classified as "productive activity".

2.4 The Basic Forms of Productive Activity

> "In economics, it is difficult to prove originality; for the germ of every new idea will surely be found over and over again in earlier writers." Irving Fisher (1930; ix)[94].

Over the years, several general forms of productive activity have been identified:

- *Farming* or *harvesting* nature's bounty;
- *Manufacturing* or *physically transforming* less valuable commodities into more valuable commodities;
- The *transportation* of commodities from one location to a more valuable location;
- The *selling* and *marketing* of commodities to final demanders; i.e., the *retailing* of commodities;
- The provision of *personal services*;
- The *storage* or *holding* of goods from one time period to a future time period when they will be more valuable and
- The redistribution of *risk*.

The first four types of productive activity are not controversial and were identified long ago by Smith[*36] in 1776 and Marshall[*37] in 1898. Services are also regarded as being productive today, but

[*35] MacNeal (1939)[155] also provided some convincing attacks on the conventions of historical cost accounting, which were also ignored by the accounting profession; see Zeff (1982)[234].

[*36] "A capital may be employed in four different ways: either, first, in procuring the rude produce annually required for the use and consumption of the society; or, secondly, in manufacturing and preparing that rude produce for immediate use and consumption; or, thirdly, in transporting either the rude or manufactured produce from the places where they abound to those where they are wanted; or, lastly, in dividing particular portions of either into such small parcels as suit the occasional demands of those who want them. In the first way are employed the capitals of all those who undertake the improvement or cultivation of lands, mines, or fisheries; in the second, those of all master manufacturers; in the third, those of all wholesale merchants; and in the fourth, those of all retailers. It is difficult to conceive that a capital should be employed in any way which may not be classed under some or other of those four." Adam Smith (1963; 278)[195].

[*37] "Production, in the narrow sense, changes the form and nature of products. Trade and transport change their external relations." Alfred Marshall (1898; 183)[158]. Marshall elaborates as follows: "Man cannot create

the last two types of productive activity are regarded as being more controversial. Our focus in this section is on the question whether the sixth type of activity is productive, or put another way, we ask whether *holding gains* can be regarded as being productive.

Galliani (1751)[97][*38] noticed the similarity between currency trading and the intertemporal trading of money while Böhm-Bawerk[*39] noticed the analogy between transportation and storage activities. Over the years, many accountants (such as Edwards and Bell[*40] and Chambers[*41]) and economists (such as Lerner[*42] and Debreu[*43]) have argued that the sixth type of productive activity is just as valuable as the other two types. However, when the sixth type of activity is labeled as "speculative activity" instead of "storage activity", many economists and accountants have objected to treating the sixth type of productive activity in exactly the same manner as the first five types. Specifically, these economists and accountants argue against the inclusion of *holding gains* or *capital gains* on assets held by a business unit over an accounting period. In the following section, we look at their arguments in some detail.

2.5 Accounting Theory and the Treatment of Capital Gains

In this section, we shall present some of the arguments that have been advanced by economists and accountants to deny treating speculative gains or capital gains on assets held through an accounting period as net revenues or components of business income in a manner that is symmetric to the treatment of transformation and transportation activities. We shall consider six types of objection.

material things. ... All that he can do in the physical world is either to readjust matter so as to make it more useful, as when he makes a log of wood into a table; or to put it in the way of being made more useful by nature, as when he puts seed where the forces of nature will make it burst out into life. It is sometimes said that traders do not produce: that while the cabinet-maker produces furniture, the furniture-dealer merely sells what is already produced. But there is no scientific foundation for this distinction. They both produce utilities, and neither of then can do more: the furniture-dealer moves and rearranges matter so as to make it more serviceable that it was before, and the carpenter does nothing more. The sailor or the railway-man who carries coal above ground produces it, just as much as the miner who carries it underground; the dealer in fish helps to move on fish where it is of comparatively little use to where it is of greater use, and the fisherman does no more." Alfred Marshall (1898; 132-133)[158].

[*38] "Hence arose exchange and interest, which are brothers. One is the equalizing of present money and money distant in space, made by an apparent premium, which is sometimes added to the present money, and sometimes to the distant money, to make the intrinsic value of both equal, diminished by the less convenience or the greater risk. Interest is the same thing done between present money and money that is distant in time, time having the same effect as space; and the basis of the one contract, as of the other, is the equality of the true intrinsic value." Ferdinando Galliani (1751), reprinted in Monroe (1930; 302)[97].

[*39] "If the difference of the place at which goods are available is a sound economic reason for exchanging fungible goods that are in other respects entirely similar, and if the advantage and convenience of the present place may justify the claim and allowance of a premium, just as much may the difference of the time at which similar goods are available be a sound reason for their exchange, and a guarantee that there will be premium on the – more valuable – present goods. This premium, and nothing else, is Interest." Eugen von Böhm-Bawerk (1891; 295)[18].

[*40] "A business firm can strive to earn profit by combining factors of production having one value into a product which has a greater value, and it can attempt to make gains by holding assets while their prices rise." Edgar O. Edwards and Philip W. Bell (1961; 272)[81].

[*41] "It therefore does not seem to be realistic to suppose that holding gains or cost savings are a class apart from sellers' margins. They all, if positive, make the firm better off, that is, able to command more goods and services than before." R.J. Chambers (1965; 740)[27].

[*42] "There is another kind of speculation, which we may call simple or productive speculation. A man who does not consider himself to have any influence on the market price but who believes that the price is going to rise or is going to fall quite independently of his own actions, and who buys or sells in an attempt to make a profit, is a simple or productive speculator. If he guesses right he makes a profit, if wrong he makes a loss . . . The same thing applies to the man who transports a good from one place to another . . . these are perfectly legitimate production activities." Abba P. Lerner (1946; 69-70)[148].

[*43] "The idea that a good or a service available at a certain date (and a certain location) is a different commodity from the same good or service available at a different date (or a different location) is old." Gerard Debreu (1959; 35)[42].

The *first objection* states that capital gains on assets held through the accounting period are capital losses to someone else and hence there is no net gain to the community. Consider the following quotations by the accountants Schmidt and Crandell:

> "Only in one case can appreciation be real profit to the business man, viz., when he uses money credit to buy goods for speculation outside of his regular business needs. If his selling prices thereafter are higher than the money lent plus interest and costs after selling the goods, the difference will be his realized speculative gain. This kind of profit is especially high in times of rising general price levels. But this kind of private profit is no profit to the community, because the lender of money loses the same buying power on his money that the borrower gains." Fritz Schmidt (1931; 291)[192].

> "What treatment should be accorded the speculative gains and losses realized from trading among individuals in securities? It is obvious that these sorts of transactions cannot increase the national wealth, hence the national income cannot be affected thereby. Whatever one gains the other loses." William T. Crandell (1935; 399)[36].

The argument that the capital gains made by one business unit must be offset by capital losses made by some other consumer or business unit does not seem to be correct. Consider the case of a one person economy that controls a single business unit. Any capital gains made by the business unit that result from an optimal intertemporal allocation of resources are not offset by capital losses.*44

The *second objection* to the inclusion of capital gains in income is more subtle: speculative holding activities do not enhance the productive powers of the economy and hence any increase in revenues resulting from these activities should not be recognized as a benefit to the economy. In fact, focusing on speculative gains may be bad for the economy because it will cause managers to not focus on the other types of productive activities*45 (transformation and transportation). Consider the following quotations which are representative of this point of view:

> "Some theoretical explanation of the reasons why appreciation cannot be profit is needed at this point. For this purpose we must consider the enterprise as a part of the national production machine. It will then be clear that a maintenance of total productivity as of a certain moment will only be possible, if the productive instrumentalities of all individual enterprises concerned are preserved intact. The maintenance of productive power as a whole is not possible if accounting is based on an original value basis. The reason is that pure appreciation would then appear as profit whenever a change of value has taken place between the purchase and selling dates for the materials and wages that compose a product." Fritz Schmidt (1931; 289)[192].

> "The appreciation in value of capital assets and land must not be treated as an element in national income. Depreciation due to physical wear and tear and obsolescence must be treated as a charge against current income, but not the depreciation of the money value of an asset which has remained physically unchanged. Appreciation and depreciation of capital were included in the American statistics of national income prior to 1929, but now virtually the same convention has been adopted in all countries." Colin Clark (1940; 31)[33].

> "Enhancement of asset values as a result of increased market prices does not, without real-

*44 More generally, the reader should refer to Lerner's (1946; chapter 8)[148] analysis of the benefits to society from competitive speculation.

*45 "Net realizable value is considered significant because it purports to reveal an alternative available to a firm in which the firm, as a periodic (year end) opportunistic calculator, ever seeks to exploit by selling the assets. However valid this might be in the minds of many profit seekers, it tends to misdirect the energies of management from production and the creation of values to the trading function alone. Such misdirection could well result in limited production and thus a lower standard of living." Norton M. Bedford and James C. McKeown (1972; 336)[10].

ization of such appreciation through sale, constitute a basis for recognition of revenue to the business enterprise. However, the realization of gain on the sale of a capital asset does not necessarily imply any contribution by the seller to the social product during the period of realization. Because such gains are irrelevant to production of the period, capital gains (and losses) are excluded from calculations of national income and product. It is seen then that, whereas standards of accounting for revenue provide for recognition of capital gains once they have been realized, such gains find no place at all in the accounting for the economy." Gilbert P. Maynard (1952; 190)[162].

"The essence of the difference between financial capital maintenance and all concepts of physical capital maintenance is in the treatment of the effects of price changes while assets are held. Under financial capital maintenance, all such effects are included in income .... Under physical capital maintenance, the effects of price changes are excluded from income on the grounds that, if positive, they do not enable an enterprise to increase its operating capability or, if negative, they do not force a reduction." Bryan Carsberg (1982; 62)[24].

"12.67. Holding gains are sometimes described as 'capital gains'. The term 'holding gain' is widely used in business accounting and is preferred here because it emphasizes the fact that holding gains accrue purely as a result of holding assets over time without transforming them in any way." *System of National Accounts 1993*, Eurostat (1993; 273)[87].

The above authors have implicitly ruled out storage and holding activities as being productive like transformation and transportation activities.*46 However, we can follow Galliani and Böhm-Bawerk and argue that holding activities are completely analogous to transportation activities. Since transportation activities are regarded as being productive, so should holding activities.

The *third objection* to the inclusion of capital gains in the period by period income statements of a business unit has been made by accountants and it is an objection only to the inclusion of unrealized capital gains (i.e., no sale of the asset which has experienced a capital gain over the accounting period has been made) in income, not to the inclusion of realized capital gains (i.e., the appreciating asset has been sold during the accounting period). The objection is that unrealized capital gains should not be included in the period's income due to their hypothetical and unverifiable nature. Consider the following quotations:

"Appreciation, Capital Gains and Losses. A part of the ultimate net income of an enterprise can be assigned in some cases to natural growth and other increases in value. In the case of timber tracts, orchards and similar properties, natural increase, commonly called accretion, is an important factor in financial history. In other cases enhancement of property values due to changing business and general economic conditions, a general rise in the price level, or other factors which result in an increase in effective value over actual cost, usually referred to as appreciation, are of marked significance. At what point, in the succession of events that lead to final fruition of these gains in cash, should the accountant recognize the change? Eventually, if no cognizance is taken of it before, the gain will be realized in cash when the property itself, or the product resulting from its use, is sold. Until such time as the gain is validated by sale, the increased value is commonly characterized as unrealized and the gain as 'unearned' or 'unrealized income'." William T. Crandell (1935; 389)[36].

"The various codifications of accounting doctrine during the past two decades have been in

*46 However, other parts of the *System of National Accounts 1993* explicitly recognize storage and holding gains on storage activities as being productive: "Many goods have to be stored in a properly controlled environment and the activity of storage can become an important process of production in its own right whereby the goods are "transported" from one point of time to another. In economics, it is generally recognized that the same goods available at different times , or locations, may be qualitatively different from each other and command different prices for this reason." Eurostat (1993; 130-131)[87].

general agreement that revenue should be recognized in the accounts only when certain tests of realization have been met. In the vast majority of cases, revenue realization is marked by a discrete event, that of sale and delivery of goods or services. Thus accountants draw a distinction between the earning or accrual of revenue throughout the productive processes and the realization of revenue, giving recognition in the accounts only to the latter ... . The national income accountant is concerned with the creation of product, not alone with its subsequent sale." Gilbert P. Maynard (1952;189)[162].*47

"There is another important respect in which business and social accounting differ which is worthy of comment here. Although business accountants are fully aware of the tentative nature of their measurements of income, they place great emphasis upon the objectivity and verifiability of the business data to which they grant recognition in the accounts." Gilbert P. Maynard (1952; 193)[162].

The accountant's objection to the inclusion of unrealized capital gains as a contribution to the income of an accounting period due to their hypothetical nature is a valid one. However, the traditional accounting solution to the unrealized capital gains problem is to assume that no capital gains occur in any accounting period unless a realization occurs in some period (i.e., the asset is sold) in which case, all of the capital gains that accrued over the many accounting periods that the asset was held are imputed to the period of sale. This historical cost treatment of capital gains can create tremendous distortions (particularly in inflationary environments) to both the periodic income statements and balance sheets of the business unit.*48 Thus the accountant's treatment of unrealized capital gains (i.e., to exclude them from the income statement) is just as hypothetical (and more misleading in an inflationary environment) as including them in periodic income.*49 However, the historical cost accountant's objection to the hypothetical nature of period by period valuations of the capital stock components held by the business unit could be used to justify a separate treatment of unrealized capital gains on income statements rather than simply lumping them in with the more objective

*47 This quotation illustrates one of the three main differences between business accounting and social (or national income) accounting: the former emphasizes the *realization* of revenues or the sale of products while the latter emphasizes the *creation* of production of products. The other two main differences are: (i) social accounting usually does not recognizes capital gains (realized or unrealized) in income statements whereas business accounting recognizes realized capital gains and (ii) business accounting uses a historical cost accounting treatment of depreciation whereas social accounting uses a current price approach to depreciation.

*48 This point is quite old as the following quotations indicate. "Then why not recognize appreciation also, as it accrues, instead of waiting until a sale is made ... . If this appreciation were not recognized in 1914 the item of appreciation would become revenue in 1915; and net revenue would not be correctly stated in either period." William A. Paton (1918; 43)[176]. "The insistence of accountants upon the importance of differentiating between realized and unrealized income has probably proved a wise one. But the artificial showing that it causes should be clearly understood. For, as a consequence, a period may be credited with income that it did not earn, and be charged with a loss that it did not suffer." Henry W. Sweeney (1933; 334)[209]. "Some limitations of accounting profit as a managerial tool can now be briefly indicated ... . Capital gains are counted only when realized. This means that some of the events of past periods, notably price changes and the gains and losses associated with them, are treated as though they were events of the current period. If an asset has been held for five years and then sold, all of the gains and losses arising over the five year period are credited to the year of sale." Edgar O. Edwards and Philip W. Bell (1961; 116)[81]. "In effect, present accounting data are predicated on the assumption that holding activities do not represent a purposeful means by which management can enhance the market position of the firm. To the extent that the firm attempts to make gains in this fashion, traditional accounting data fail to inform management, owners, and outsiders as to the progress the firm has made during the current period. A second consequence of not counting gains when they arise is that when such gains are in fact realized, the gains earned over the entire time span during which the assets where held by the firm are attributed entirely to the period in which the gains are realized." Edgar O. Edwards and Philip W. Bell(1961; 222)[81]. "The third consequence of the failure to report capital gains and losses as they occur is the badly distorted balance sheet values which result." Edgar O. Edwards and Philip W. Bell (1961; 223)[81].

*49 MacNeal (1939)[155] also makes this point very forcefully. However, historical cost accountants can claim that their procedure of excluding unrealized capital gains from periodic income is more "objective" and "verifiable" than any procedure that includes them in periodic income.

(transformation and transportation) sources of income.*50

A *fourth argument* against the inclusion of capital gains in income statements runs as follows: for most businesses, capital gains or losses are an unintended consequence of their normal productive activities and moreover, in the long run, these gains and losses will tend to cancel. Hence it is not worth the bother of including these gains and losses as income, particularly when income may be taxed and hence a large unrealized capital gain may lead to a large tax bill which in turn may lead to a curtailment of the firm's normal productive activities. However, this line of thought led to a difficulty: what if the normal activity of a business unit was speculative (e.g., a commodities trader or a land speculator)? These business units would seem to be excluded from paying any income taxes on their earnings from speculative activities. To get around this difficulty, Plehn and other economists introduced the concept of *recurrence of income*:

> "Income is essentially wealth available for recurrent consumption recurrently (or periodically) received. Its three essential characteristics are: receipt, recurrence, and expendability." Carl C. Plehn (1924; 5)[184].
>
> It will, I think be readily admitted that of those particular gains and profits which are recurrent, expendable receipts are the ones about whose income character there is seldom any doubt. Thus the gains and profits of a merchant are his income. The possible or even probable irregularity or uncertainty which distinguishes them from some other incomes does not seem to militate against their inclusion in income, provided they are expected to be recurrent. The same is true of the gains and profits of dealers in capital assets, for the lands, stocks and bonds, houses and the like are their stock in trade . . . . But it is when gains and profits lack one or two of the three characteristics of income, or have them in less than complete form, that a question arises. The one that is most often lacking is recurrence. Thus gains and profits from transactions outside of one's regular vocation or line of business, like the profit from the sale of a home, are of doubtful income character." Carl C. Plehn (1924; 10)[184].
>
> "The British income tax places very heavy stress upon the annual character of income. For an explanation of this conception, which results in the exclusion from taxable income of gains of an irregular nature, one must go back as far as the fifteenth century, when, with an agricultural society where few fortuitous gains developed, the idea of receipts as being annual in character became deeply impressed upon the minds of the people. It became the habit to think of one's regular receipts as his income, and to consider irregular receipts as additions to capital." Robert Murray Haig (1921; reprinted 1959; 69)[105].

Thus if a business unit *regularly* makes profits on its speculative activities, the resulting profits are regarded as income but any capital gains on *occasional* speculative activities are not regarded as income according to the recurrence criterion for income. Of course, the problem with this concept is that it is difficult to draw the boundaries of recurrence:

> "When is income recurrent? Professor Plehn expressly says it need not be perfectly regular. But how irregular can it be and still be 'recurrent'? The big profit on the sale of an old homestead may well occur twice in a lifetime. Does it not then 'recur'? If we extend the picture through two or more lifetimes 'recurrence' becomes altogether likely. In the case of corporations whose life goes on indefinitely every windfall, or extraordinary profit, may some day be duplicated. Evidently the 'recurrency' concept turns out to be too elusive to pass muster as a basis for analysis." Irving Fisher (1924; 666)[93].

More fundamentally, if holding gains are regarded as being valid additions to income in some contexts, then why should they be excluded in other contexts? When we view speculative activities as being the intertemporal counterpart to transportation activities, it is obvious that a case can be made for

*50 The same logic would justify a separate listing for historical cost depreciation expense since it too is hypothetical.

including them as a valid form of productive activity: goods are transported across time to periods where they will be more highly valued instead of being transported across space to locations where they will be more highly valued.

The *fifth objection* to the inclusion of capital gains in income is related to the last objection: (ex post) holding gains are so variable and transitory, that their inclusion in measures of national output and hence income just leads to a lot of meaningless noise in the accounts. This is probably the most persuasive objection. The obvious solution to this objection is to *smooth* volatile asset prices in order to eliminate the transitory fluctuations. However, this solution leads to a potential *lack of objectivity and reproducibility* of the smoothed estimates of asset prices. Different smoothing methods will generate different trended asset prices.

The final *sixth objection* to the inclusion of capital gains in income is well explained by Hicks, who objected to the inclusion of unanticipated capital gains on the grounds that they are not relevant to economic choices made by producers and consumers with respect to the purchase or holding of durable goods. Thus Hicks distinguished between *ex ante* and *ex post* income concepts:

> "All the definitions of income we have hitherto discussed are *ex ante* definitions—they are concerned with what a person can consume during a week and still *expect* to be as well off as he was. Nothing is said about the realization of this expectation. If it is not realized exactly, the value of his prospect at the end of the week will be greater or less than it was expected to be, so that he makes a 'windfall' profit or loss. If we add this windfall gain to any of our preceding definitions of income (or subtract the loss), we get a new set of definitions, definitions of 'income including windfalls' or 'income *ex post*'. There is a definition of income ex post corresponding to each of our previous definitions of income *ex ante*; but for most purposes it is that corresponding to Income No. 1 which is the most important. Income No. 1 *ex post* equals the value of the individual's consumption *plus* the increment in the money value of his prospect which has accrued during the week; it equals Consumption *plus* Capital accumulation." J.R. Hicks (1946; 178)[114].

Hicks then went on to criticize his definition of ex post Income No. 1 as follows:

> "This is a very convenient property, but unfortunately it does not justify an extensive use of the concept in economic theory. *Ex post* calculations of capital accumulation have their place in economic and statistical *history*; they are a useful measuring rod for economic progress; but they are of no use to theoretical economists, who are trying to find out how the economic system works, because they have no significance for conduct. The income *ex post* of any particular week cannot be calculated until the end of the week, and then it involves a comparison between present values and values which belong wholly to the past. On the general principle of 'bygones are bygones', it can have no relevance to present conditions. The income which is relevant to conduct must always exclude windfall gains; if the occur, they have to be thought of as raising income for future weeks (by the interest on them) rather than as entering into any effective sort of income for the current week. Theoretical confusion between income *ex post* and *ex ante* corresponds to practical confusion between income and capital." J.R. Hicks (1946; 179)[114].

Thus if our purpose is to construct user costs of capital that correspond to market rental prices for units of durable goods (whether used in consumption or production), then it seems that Hicks' is correct: it is the ex ante (or beginning of the period) point of view that is most relevant to the construction of user costs and hence unanticipated capital gains should not be included in the user cost formula.*51 Of course, this leaves open the difficult problems involved in empirically estimating

*51 For additional material on ex ante versus ex post estimates of depreciation and income, see Hicks (1942)[113] and Hill and Hill (2003)[122].

ex ante capital gains.

Our own opinion on this controversial literature is as follows:

- If our goal is to form period by period approximate rental prices for assets that are held by production units for several periods, then only *anticipated holding gains* should be included in the user cost formula.*52
- If our goal is to measure the period by period *ex post performance* of a business unit, then one can make a case for either treatment of holding gains; i.e., exclude unanticipated holding gains from the income measure because the production unit did nothing to earn these gains: they fell from heaven. On the other hand, if we want a measure of the financial position of the production unit at the end of the period, then unanticipated holding gains and losses certainly impact this financial position and hence they could be included.

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*52 This is consistent with the position of Hill (1999)[119] (2000)[120] (2005)[121] and Hill and Hill (2003)[122] on this subject.

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Chapter 3

Accounting Theory and Alternative Methods for Asset Valuation

3.1 Introduction

In this Chapter, we study the following problem: *the determination of period by period values for durable assets that are held by the business unit for multiple accounting periods.* There are many possible methods for asset valuation that could be used. We shall consider seven methods:[*1]

(1) historical cost valuations;
(2) general purchasing power adjusted historical costs;
(3) net realizable values or appraisal or market values;
(4) replacement (or reproduction) costs;
(5) future discounted cash flows;
(6) asset specific index number adjusted historical cost; and
(7) valuations based on intertemporal cost allocation methods.

The main method of valuation that is in general use today by financial accountants is the first method: historical cost accounting.[*2] However, this method assumes that there is no inflation in

[*1] Daines considered 4 of these 7 methods: "There are four possible bases which might be adopted: (1) liquidation value, or that value which is likely to be realized if the assets were thrown onto the market in the process of an orderly or forced liquidation; (2) original cost with proper allowance for decline in value of current assets and allowances for depreciation and depletion of fixed assets; (3) capitalized income producing value; (4) present market price of replacing or reproducing a similar asset in its present state of condition." H.C. Daines (1929; 98)[37].

[*2] National income accountants use essentially method 6, which is thought to approximate methods 3 and 5: "10.13. To ensure consistency between the accumulation accounts and the balance sheets, assets recorded in balance sheets should be valued as if they were being acquired on the date to which the balance sheet relates. For example, if fixed assets were to be acquired on the balance sheet date they would be recorded at their current

the economy (or alternatively, inflation is ignored). The next five methods attempt to deal with the valuation problem when there is general or specific price change over time. The last method deals with the intertemporal allocation of fixed costs and will be explained in section 3.8.

3.2 Historical Cost Valuation

> "Today's dollar is, then, a totally different unit from the dollar of 1897. As the general price level fluctuates, the dollar is bound to become a unit of different magnitude. To mix these units is like mixing inches and centimeters or measuring a field with a rubber tape-line." Livingston Middleditch (1918; 114-115)[164].

Historical cost depreciation (i.e., decline in asset value over an accounting period) is determined as follows: once a useful life for an asset has been estimated and a corresponding depreciation schedule has been determined, the initial purchase cost of the asset is allocated across accounting periods as a sum of periodic depreciation allowances. The corresponding historical cost value of the asset at the end of an intermediate accounting period is simply the initial purchase cost less the accumulated depreciation allowances over prior periods.

As we have seen in chapter 2, the main problem with historical cost valuation of assets shows up if there is a large change in the price of the asset (due to general inflation for example) from the time of its purchase to the end of the current accounting period: the historical cost valuation may bear no resemblance at all to a current market valuation for the asset. Thus in an inflationary situation, historical cost depreciation allowances will be understated, income will be overstated and income taxes may become capital taxes.*3 The problem is that historical cost accounting implicitly assumes that monetary values at the end of an accounting period are comparable to monetary values at the beginning of the accounting period; i.e., there is an implicit assumption of price level stability. The accountant Middleditch (1918)[164] challenged this implicit assumption, having observed the tremendous inflation that occurred during World War I.*4

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purchasers' prices, including any costs of ownership transfer, or at their current basic prices if produced on own account. The valuation of fixed assets that were acquired some time before the balance sheet date is more problematic. In general, they are valued by writing down the current purchasers' or basic prices of new assets by the accumulated consumption of fixed capital on the assets. With good information and efficient markets, the written down values of the assets should equal, or at least approximate, both the present, or discounted, values of the remaining future benefits to be derived from them and their market values when active second hand markets exist. In practice, these values may differ from each other because of lack of information or other imperfections. As already stated, the written down value of the asset is generally the most practical and also the preferred method of valuing an existing fixed asset, bearing in mind that the calculation of consumption of fixed capital should take into account the observed values of secondhand assets when they are actively traded." *System of National Accounts 1993*, page 219[87].

*3 On the other hand, if the business unit actually sold the asset at the end of an intermediate accounting period in inflationary conditions, income would suddenly be much larger for that period under the realization conventions of historical cost accounting. This discrepancy in historical cost incomes, depending on whether an asset is held or sold, should alert us to the possibility that something is seriously wrong with historical cost accounting; see MacNeal (1939)[155] for further criticisms of historical cost accounting.

*4 The accountant William A. Paton (1920; 2-3)[177] was not far behind in making a similar observation: "The significance of the dollar – the accountant's yardstick – is constantly changing . . . . One of the fundamental limitations of accounting arises here. The units of physical science are always the same; and hence direct comparisons of situations and phenomena arising at different times can be made in this field. Accountants deal with an unstable, untrustworthy index; and, accordingly, comparisons of unadjusted accounting statements prepared at different periods are always more or less unsatisfactory and are often positively misleading."

There are two main virtues that are claimed for historical cost accounting: (i) it is objective[*5] and reproducible and (ii) it is conservative. Both of these virtues are subject to criticism. Historical cost asset valuations are not reproducible or objective since different accountants will not necessarily make the same assumptions about the appropriate amounts of historical cost depreciation. But the important problem is that historical cost end of period values will be completely meaningless in a high inflation environment; i.e., they will not reflect current opportunity costs or market values. Thus historical cost accounting values might be objective but at the same time, they are irrelevant.[*6] Conservatism, on the other hand, conflicts with accuracy; i.e., if we wanted to be super conservative, why not assume all intermediate asset values are zero? The absurdity of this statement should make us realize that accuracy is a much more important virtue than conservatism.[*7]

It is perhaps useful to elaborate a bit more on the meaning of "accuracy" in the context of determining period by period values for the assets of a business unit. It seems clear that there cannot be an answer to the problem of constructing period by period values of assets that are in use that are as unambiguous as the actual selling price of an asset; i.e., we can only make estimates of these intermediate values. Thus it might be reasonable to follow the example of Morgenstern (1963; 77)[165] and regard these estimated intermediate values as probability distributions. "Accuracy" in this context could be defined as providing a suitable measure of central tendency (e.g., a mean valuation) along with a measure of dispersion (e.g., a variance). Unfortunately, accounting theory (and practice) has not proceeded along these lines,[*8] although occasionally, accountants recognize that introducing statistical concepts into accounting would be useful.[*9]

[*5] "Its greatest advantage is the fact that an original cost method is most easily subject to objective verification; it is the easiest to use in practice." H.C. Daines (1929; 98)[37]. Littleton comments at greater length on the virtues of *objectivity* and *verifiability* in business financial accounts: "Professional accountants have long struggled with the problems of providing data needed by managements and investors. As a result of an extensive accumulation of experience, accountants have come to several relevant conclusions: that men often want, and need, data which are not within the function of accounting to supply or practitioners to certify; that it should lie within the province of people specially trained and experienced in accounting to set the limits of their technology in the matter of supplying all information a client might find useful; that the professional accountant should confine his professional work of dealing with objectively derived and convincingly verifiable data which have been collected and marshalled by well known procedures from evidence of actual business transactions; that there can be no objection to any desired amount of collateral, interpretive use of properly derived account data, including such devices as averages, ratios, trends and projections." A.C. Littleton (1956; 365)[151]. Thus Littleton reflects the opinion of most financial accountants that historical cost accounting is "best" for financial reporting because of its reproducibility properties and hence adjusting the historical cost valuations of assets is best left to management accounting. Of course, if inflation is high or moderate, this will leave investors stuck with an inadequate historical cost income reported by the firm. On the other hand, Chambers, commenting on Littleton's defence of historical cost accounting, is willing to work with a weakened version of objectivity and verifiability in order to obtain more accurate financial accounts: "Objectivity is, without doubt, a useful notion; one to be used whenever possible. But its use and importance should not be overworked. It seems to be sufficient to stipulate that *initial* entries in accounting records are to be based on documentary (and therefore verifiable) evidence; no such stipulation can be made about subsequent adjustments such as valuation provisions." R.J. Chambers (1956; 588)[25].

[*6] "Insofar as objectivity is regarded as an indispensable quality of an income concept which is to have any claim to being practical, accounting income is practical enough. But this is of little moment if it does not measure what we want it to measure. Objectivity without relevance is not much of a virtue." David Solomons (1961; 378)[196]. The economist Morgenstern (1963; 66)[165] uses the term "meaningless statistics" to describe historical cost incomes during periods of rapid inflation.

[*7] "'Conservatism', especially when it merely means 'highly probable understatement', is not meritorious'." John B. Canning (1929; 105)[23]. "Conservation infers understatement and understatement infers falsity. Falsity cannot be characterized as fundamental truth." Stephen Gilman (1939; 204)[99].

[*8] "It is, of course, unlikely that balance sheets will be drawn up in the indicated manner; this is a matter for the future. But it is clear that present balance sheets already contain an element of expectation and speculation." Oskar Morgenstern (1963; 78)[165].

[*9] "The accountant of the future will be a distinctly different type . . . . Accounting and statistics will be his tools; the entire scope of internal and external business problems that are reducible to mathematical measurement will be his field." H.C. Daines (1929; 109)[37]. "It is necessary for the accountant to realize that his measures of income or financial position are actually probability distributions." Harold Bierman (1963; 504)[16].

We turn now to a discussion of other methods for valuing assets on a periodic basis, methods that will more closely approximate current market values or opportunity costs.

3.3 Purchasing Power Adjusted Historical Cost

"It is obvious, therefore, that if quantities, whether measured in pounds or bushels or dollars, are to be correctly combined or compared, the unit of measurement must be homogeneous . . . . Yet many men who are not measuring their heights with fluctuating rulers, and who would throw verbal stones at such a silly doing, are complacently living in a similar kind of glass house, a business structure where in the substance of value continues to be measured by a dollar of seriously fluctuating size." Henry W. Sweeney (1936; reissued 1964; 11)[213].

"Professor Baxter (1976) has characterized the development of Latin American inflation accounting systems as having two stages: firstly, fixed assets and depreciation are adjusted by reference to a general index, and, secondly, at a later stage, the 'time-log' error on stocks [inventories] and monetary working capital is corrected by the application of an index." David Tweedie and Geoffrey Whittington (1984; 243)[217].

This method of constructing a current value at the end of an accounting period originates with Middleditch (1918)[164]*10 and works as follows. Suppose an asset was purchased at the beginning of accounting period 0 at the price P^0, the period 0 depreciation rate is δ^0 and a general rate of price inflation over period 0 is ρ^0; i.e., the general price level at the end of the period divided by the general price level at the beginning of the period is $1+\rho^0$. Then the *historical cost accounting value of the asset* at the end of the period is $(1-\delta^0)P^0$ but the *General Price Level Adjusted (GPLA) value* is:

$$V_{GPLA} \equiv (1-\delta^0)(1+\rho^0)P^0. \tag{3.1}$$

The advantage of this method for constructing current asset values on a period by period basis is its relative *simplicity* (adjusted historical cost values at the beginning of the period need only be inflated by the common indexation factor $1+\rho^0$) and its *objectivity* (once the appropriate indexation factor $1+\rho^0$ has been chosen).*11

In response to rapid inflation or a hyperinflation, GPLA accounting is the main form of current value accounting that has been used historically.

Note the difference between ρ^0, an ex post general inflation rate, and the asset specific anticipated inflation rate i^0 defined as $1+i^0 \equiv P^1/P^0$ where P^0 and P^1 are the price of the same asset at the beginning and end of the accounting period. In general, ρ^0 will not equal i^0 and hence the GPLA value for the asset will not equal its end of period market value (unless the general inflation rate

*10 Sweeney (1964; 8-11)[213] reviews the early history of this method. He was also an early contributor to the method: "First, the stabilizing procedure is based upon the index of the general price level, or 'general index'. The reason is that 'measurement based on the general price index enables all values to be expressed in accordance with the customary main object of economic activity, namely, increased command over economic commodities and services in general'. Second, stabilized accounting, by its use of price index numbers, estimates the reproductive or replacement costs of merchandise and fixed assets as at any dates for which reliable indexes are available." Henry W. Sweeney (1935; 185)[211].

*11 "Current value accounting is easy to explain and meaningful, but hard to audit. It requires estimates of the current values of all assets and liabilities. More often than not, prices for 'used' assets are hard to get. Auditors would be required to make substantial judgemental decisions in implementing current value accounting. But we live in a litigious age, and auditors are reluctant to exercise judgement in such situations because, occasionally, subsequent events might not bear out these judgements, and costly and embarrassing lawsuits may result . . . . GPLA financial statements are easy to audit and are objective. Two auditors given the same historical records and the same data for the GNP Deflator are likely to derive the same general price level adjusted statements." Sidney Davidson, Clyde P. Stickney and Roman L. Weil (1976; 225)[39].

ρ^0 is equal to the asset specific inflation rate i^0). This is the main weakness of General Price Level Adjusted accounting. However, its strength is that it will adjust for the effects of general inflation.

The remaining topic to be discussed is how to choose the general inflation rate ρ^0.[*12]

One of the simplest choices is to use the inflation rate for a widely traded commodity (such as gold[*13]) as the index of general inflation. Another alternative is to use the rate of increase in the exchange rate of the country against a stable currency.[*14] Instead of using the price of gold or any single commodity as the indicator of inflation, the general inflation between the beginning and the end of the accounting period might be better captured by looking at the price change of a "representative" basket of goods. As a further refinement, we could replace a fixed basket price index by a more general price index such as the Fisher (1922) ideal price index, which allows for substitution in response to price changes.[*15]

Accountants and economists have struggled with the problem of choosing an appropriate price index to represent inflation for approximately a century. Many of the problems have still not been resolved: (i) Which commodities should be included in the index?[*16] (ii) How should the individual price ratios be weighted[*17]; i.e., what is the theoretically correct functional form for the price index? (iii) A related problem is whose weights should be used in the index?[*18] (iv) If the accounting period is shorter than a year, how can we deal with seasonal commodities that might be present in the index?[*19]

Even though the above questions are difficult to answer, we agree with Staubus that adjusting historical costs for general inflation by an imperfect index will generally be an improvement over historical cost accounting:

> "The argument that the corporate accountant cannot use the different purchasing power in-

[*12] "The only problem left is the selection of the index. In view of the motivation of the enterprise, it should be obvious that we think the Consumer Price Index is the most appropriate. It is the closest substitute for a utility measurement that is currently available . . . . The other indices which are often described as general, e.g., the implicit GNP deflator, include intermediate goods. Intermediate goods should be excluded from the purchasing power concept, because they are only indirectly productive of utility." Robert R. Sterling (1970; 340-341)[202]. Diewert (2002; 556)[63] also argued against the use of the GDP deflator as a general measure of price change due to the fact that imports enter the index with negative weights. This negative weight for imports can lead to the perverse result that an *increase* in import prices leads to an immediate *fall* in the index.

[*13] This choice was used by German accountants during the German hyperinflation of 1923; see Sweeney (1927)[204] (1928)[205].

[*14] This alternative has also been used; see Wasserman (1931; 10)[226].

[*15] See Diewert (1976; 117)[46]. Diewert (1992; 214-222)[56] also shows that the Fisher ideal index has very good properties from the viewpoint of the test or axiomatic approach to index number theory.

[*16] See Hill (1996; 97)[118] and Diewert (2002; 555-560)[63] for recent discussions on this issue.

[*17] "The simplest way to convert a money measure into a real measure is through an accepted index of the general price level. No perfectly satisfactory index of the general price level exists, nor can one be conceived. It is not only that price indexes are imperfect because of poor price reporting and inadequate coverage, but even in theory it is impossible to construct a perfect price index no matter how much 555-information one has. Since all prices do not move together, it is necessary to use an average of different price movements. The average must be weighted, and the appropriate weights change as between the beginning and end of the period over which price change is being measured . . . . But for practical purposes, the theoretical imperfection of index numbers need not worry us too much." Sidney S. Alexander (1962; 188)[2].

[*18] "Another practical problem, in the use of purchasing power as a common denominator, is the selection of the index to use . . . . If the individual were interested in purchasing everything in general, as assumption which is highly problematical, a general price index could be used in converting financial statements into equivalent purchasing power and would be adequate for his needs. But no one is interested in purchasing everything in general; most individuals have a more or less limited and fixed class of goods in which their purchases are made." Donald K. Griffith (1937; 128-129)[102]. "Furthermore, the use of a general price index for the purpose of modifying dollar values and dollar results assumes that all investors are alike, having the same purchasing habits." Stephen Gilman (1939; 6)[99].

[*19] In many cases, seasonal commodities are not available in all seasons and thus there will be no prices for these out of season commodities. "Seasonal characteristics rule out any formal accounting period shorter than a year." Stephen Gilman (1939; 77)[99].

dexes of each individual shareholder must be read as either a weak excuse for inaction or an insistence on a degree of perfection that accountants have not reached in the past and are not likely to reach in the future. Surely a broadly based price index provides a better measure of the change in the measuring unit than the assumption that there is no change at all, as the millions of people who base contracts on such indexes recognize." George J. Staubus (1975; 44-45)[201].

It is sometimes asserted that General Price Level Accounting adds no additional information over that which is available from reading historical cost accounting balance sheets[*20]; i.e., if investors know historical cost values and they can look up the relevant general inflation index, then they can readily calculate the adjusted asset values defined by (A1). This would be true if the business unit made the following information available to investors in each accounting period: (i) the value of new investments made in each period and (ii) the historical cost residual value of all assets that are sold or retired during the accounting period. In general, this information is not provided in balance sheets; hence providing investors with an aggregate GPLA asset value will provide new information that could not be calculated by individual investors.

3.4 Net Realizable Values or Exit Values

"Some economists, notably Professor Jacob Viner of the University of Chicago, hold the belief that the value which the assets would bring in the market is the only proper basis of value for use in accounting." H.C. Daines (1929; 98)[37].

"These markets [for assets] can be divided into two kinds, the markets in which the firm could buy the asset in its specified form and at the specified time and the markets in which the firm could sell the asset in its specified form and at the specified time. The prices obtained in markets of the first group we shall call entry prices; the prices obtained in markets in the second group we shall call exit prices." Edgar O. Edwards and Philip W. Bell (1961; 75)[81].

A century ago, it was not unusual for accountants to value the fixed assets of a business unit at the end of an accounting period by appraised values; i.e., estimates of the net realizable values that the assets would bring in the market at the moment in time:

"[There are] various methods of estimating the Depreciation of a Factory, and of recording alteration in value, but it may be said in regard to any of them that the object in view is, so to treat the nominal capital in the books of account that it shall always represent as nearly as possible the real value. Theoretically, the most effectual method of securing this would be, if it were feasible, to Revalue everything at stated intervals, and to write off whatever loss such valuations might reveal without regard to any prescribed rate . . . . The plan of valuing every year instead of adopting a depreciation rate, though it might appear the more perfect, is too tedious and expensive to be adopted . . . the next best plan, which is that generally followed . . . is to establish average rates which can without much trouble be written off every year, to check the result by complete or partial valuation at longer intervals, and to adjust the depreciation rate if required." Ewing Matheson (1884; 35)[160].

"One of the first clear references to depreciation accounting was in the annual report of the Baltimore and Ohio Railroad for the year ended September 30, 1835. That report explained that income for the year was determined 'after carrying $75,000 to the debit of profit and loss to make good deterioration of the railway and machinery . . . '. During the years following

[*20] "When accounts expressed in 'diverse amounts of general purchasing power', as in historical dollar financial statements, are restated in terms of the dollar of a single point of time, nothing new is being said. No 'change' has occurred, except in the size of the units of measurement employed." Maurice Moonitz (1970; 466).

> 1835, there was no consistent policy followed by any group of companies or even by any one company. Apparently, some companies made a separate provision for depreciation as did the Baltimore and Ohio Railroad, while other companies charged replacement costs to expense in lieu of depreciation." P.D. Woodward (1956; 71).

> "There is little reason to doubt that depreciation was originally calculated on the basis of appraisals. The appraisal, it may be conjectured, was originally on a market price basis in order to obtain a figure roughly equivalent to what would have been realized at the date of the appraisal had the asset actually been sold . . . . After general adoption of the accounting period convention, such appraisals were probably made at the end of each accounting period. It must, however, soon have been obvious that such periodic appraisals gave erratic results depending, of course, upon who made them, how they were made and the general state of business at the time they were made." Stephen Gilman (1939; 488)[99].

However, during the first 35 years of the twentieth century, many business firms arbitrarily revalued their fixed assets to suit their immediate purposes.*21 By the 1930's, the accounting profession reacted against these abuses by adopting the historical cost accounting methodology for valuing assets, and the accounting profession as a whole has stuck to this position since that time (except when an economy experienced very rapid inflation in which case General Price Level Adjusted accounting has been temporarily adopted). However, most economists and some accountants, such as Sweeney (1936; 44-53)[213], Staubus*22, Edwards and Bell (1961)[81], Chambers*23 and Sterling*24, have advocated the use of current values to value assets at the end of each accounting period.

The basic problem with the use of current values is that it is difficult to determine exactly what is the "correct" concept for a current value. Edwards and Bell (1961; 75)[81] distinguish between an *entry value* (the minimum cost of purchasing a replacement for a currently held asset) and an *exit value* (the maximum price a currently held asset could be sold for in the market less the transactions costs of the sale; i.e., the net realizable value for the asset).*25 In this section, we will focus on the problems associated with the use of exit values and we will deal with entry values in the next section.

Historical cost accountants have two principle objections to the use of (imputed) net realizable values to value assets held by a business unit at the end of an accounting period:

- they are not objective and
- they are not additive.

On the lack of objectivity of net realizable values, consider the following quotations:

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*21 For a discussion, see Sweeney (1964; 45-47)[213].

*22 "A difference between net realizable value and replacement cost, other than that related to direct costs of buying and selling such as commissions, transportation and taxes, indicates that the firm buys in a different market from that in which it sells . . . . Net realizable value of an asset is the preferable basis for measurement in this type of situation because it takes into consideration the destination of the asset rather than its source." George J. Staubus (1961; 36-37)[200].

*23 We reach the conclusion that opportunity cost, and not the authors' current cost, is the appropriate asset measurement basis. Opportunity costs (market resale prices) are relevant to the firm always." R.J. Chambers (1965; 736)[27].

*24 "Edwards and Bell also build a case for exit prices, but then reject them in favor of entry prices. We were not convinced by their reasons for rejecting exit values, and we particularly disagree with the idea that exit values would be less useful to external users of the data." Robert R. Sterling (1970; 328)[202].

*25 The distinction between entry and exit values was recognized by the Prussian legal system in the 1880's according to Schmalenbach: "There is no basis whatsoever for the opinion held by the old school of tax jurists that the user-value allegedly meant is the value in the open market, i.e., the value on a sale. In Prussian land law the user-value was something quite different; it was the value of the property to the average person for use in its present state and therefore approximated in general to the price at which an equivalent property could be acquired." Eugen Schmalenbach (1959; 20)[191]. Economists have also long made the distinction between entry and exit prices: "There are three entirely separate concepts of the basis on which capital can be measured, namely market value, replacement value and cost price." Colin Clark (1940; 375)[33]. Clark's market, replacement and cost values are the exit, entry and historical cost values of Edwards and Bell respectively.

> "Which alternative should be used as a basis? The highest, or the lowest, or an average? How should the search area, to get offers or find prices, be determined?" Yuji Ijiri (1979; 66)[132].

> "'Forced liquidation value' is also ill defined, but it sometimes seems to mean the price that could be obtained by selling to the first man on the street that one happened to meet. If this is the meaning, then we agree that it would be absurd to report such values. A less radical notion of immediate exit price is obviously called for." Robert R. Sterling (1970; 328)[202].

Thus to find an estimated net realizable value for an asset, it is necessary to determine what is the appropriate set of potential buyers and how their price bids could be elicited. If instead of seeking prices from potential buyers of the asset, we resort to appraisal values for the asset, we again encounter a certain lack of determinancy: how many appraisals should be made; what are the credentials of the appraisers; what criteria do the appraisers use[*26]; etc.

Rather than saying that hypothetical net realizable values or appraised values are not *objective*[*27], it might be more accurate to say that they do not pass the *reproducibility* test; i.e., two accountants attempting to construct net realizable values for a firm's assets would not generally come up with the same values. This is the major advantage of historical cost accounting and general price level adjusted accounting; aside from the major problems involved in defining asset lives and depreciation rates[*28], these two methods of accounting can claim that they pass the reproducibility test.

Turning now to the lack of additivity of net realizable values, consider the following quotation:

> "The second factor which makes current cost income more disputable than historic cost income is the non-additivity of current costs. The historical cost of Resource A and Resource B is by definition the sum of the historical cost of Resource A and Resource B . . . . This additivity does not exist in current cost valuation, insofar as the price of a resource is not necessarily equal to the sum of the prices of its components. If the current cost of Resource A is $20 and Resource B is $30 but that of A and B together is $60, should we use $50 or $60 as the current cost of Resource A and Resource B?" Yuji Ijiri (1979; 67)[132].

Thus if we have two assets that can be combined together to produce an extraordinary revenue stream (e.g., a machine and a building to house the machine that together produce a new product with a high profit margin), then the joint asset may have a net realizable value that is much greater than the sum of the separated net realizable values; i.e., net realizable values for assets are not necessarily additive.[*29]

[*26] Essentially appraisers encounter the same sort of difficulties that were mentioned in the previous sentence.

[*27] "Market values when obtainable are also objective in character." H.C. Daines (1929; 99)[37].

[*28] There are some additional more minor reproducibility problems with historical cost accounting: (i) if certain asset values are "known" to fall below historical cost, then the offending assets are to be valued at "market" value; (ii) there can be some ambiguity as to when exactly a sale is realized; i.e., it is sometimes difficult to allocate revenues to specific accounting periods and (iii) there can be uncertainty about what proportion of overdue payments will eventually become bad debts. Gilman (1939; 541)[99] noted the inconsistency of historical cost accounting practices with respect to point (ii) above: "It would appear that those who condemn revaluations upward should, in all consistency, condemn downward revaluations. With some exceptions, such consistency is not observed."

[*29] This point did not originate with Ijiri as the following quotations indicate, but Ijiri phrased the point in the most elegant fashion: "By and large, the reason why these writers [on asset valuation principles] could not arrive at a satisfactory theory was their premise, that the object of the balance sheet was the ascertainment of the status of capital. They did not realize that it is not possible to arrive at a value for a capital composed of a number of parts, merely by adding together the values of the individual parts." Eugen Schmalenbach (1959; 20-21)[191]. "Capital instruments used jointly with others in turning out goods for sale do not, properly speaking, have separate capital values at all." John B. Canning (1929; 233)[23]. "Although the correspondence between this definition of current cost and the data produced under the above rules of measurement is far from perfect, use of its alternative - market value – would raise far more formidable problems. First, an objective set of rules for measuring the market value of plant assets could not easily be established. Next, although the plant account could be assumed to be at market value, there would still remain the problem that with market value the sum of its parts is not equal to the whole." Myron J. Gordon (1953; 376)[100].

In order to overcome the lack of additivity of net realizable values, it will be necessary to make some rather arbitrary judgments. For example, current values could be obtained for each asset that was purchased separately (or for each group of assets that was purchased jointly) on a stand alone basis; e.g., if a tractor were purchased with several supplementary attachments, then we could attempt to find a net realizable value for the entire asset package. Thus the additivity problem is "solved" by restricting the collection of net realizable values to the asset combinations that were actually purchased by the business unit.[*30]

To overcome the lack of reproducibility objection to the use of net realizable values is a bigger task and might involve considerable costs.[*31] Accounting standards organizations or the government (in its role as a collector of business income taxes) would have to specify acceptable methods for constructing net realizable values. One possible (partial) solution might be to utilize appraised values for property insurance purposes. Insurance companies have an incentive to insure property up to its maximum value to the business unit (if premium revenue is proportional to insured value) but they also have an interest in not allowing overinsurance (in order to minimize carelessness and fraud on the part of the insured business unit). Another possible solution to the lack of reproducibility problem would be for a national Accounting Standards Board or the Government to develop appraisal criteria and to train and license appraisers.

We leave the final words on possible methods for the objective or reproducible determination of net realizable values to Chambers:

> "We will take a more or less common sense view: namely that a statement of financial position as at a date will include singular statements, in respect of plant assets, which are indicative of one or more of the following: the cost at that date of acquiring plant in the condition in which it then stands, the valuation which a lender might place on it as a security for a loan, the valuation which the owner might place on it for insurance purposes, or the price which might be obtained for it if it were decided to change the character of the company's investments. Anyone is at liberty to contend that these would all be different; but they have one thing in common, they are all estimates made in the context of conditions operating about the time at which the financial statements are prepared. They are approximations to contemporary value in the market." Raymond J. Chambers (1964; 270)[26].

We turn now to a discussion of entry values.

3.5 Replacement Costs or Entry Values

> "The replacement cost is the sum of money which would have to be expended at the present time to reproduce a physical property identical with that in existence at the present time and used for the benefit of the public." Hammond V. Hayes (1913; 618)[111].

> "The values which the accountant uses in closing the books and preparing statements ideally should be based upon economic conditions at the moment of closing. If plant and equipment assets were valued at the close of each period on the basis of costs of replacement – effective current costs – depreciation changes would be increased in a period of rising prices and the

[*30] More elaborate solutions to the additivity problem could be obtained by adapting the techniques used in the axiomatic cost allocation literature to this revenue allocation context. For references to the cost allocation literature, see Young (1985)[232] (1994)[233] and Moulin (1995)[167]. Hedonic regression techniques could also be used to solve the additivity problem; see Triplett (2004)[216].

[*31] "I do not object to current cost accounting if one can show that its benefit to society is greater than its cost of implementation. Remember, however, the bill to society for establishing and running such a system can be enormous, considering the cost of assessment, calculation, and auditing (all of which must be done every year) as well as the cost of solving disputes if the firm or the accountants are challenged on the reliability of data or are accused of intending to mislead the public." Yuji Ijiri (1979; 71)[132].

> other concomitant effects would be registered in the accounts in a rational manner." William A. Paton (1920; 6-7)[177].

The description of an *entry price* or *replacement value* of an asset has already been provided while discussing the previous method: it is the current market cost of purchasing a physically identical replacement for an asset currently being held by a business unit. As can be seen from the above quotations, the concept of a replacement value dates back at least 80 years.

Replacement cost as a basis for asset valuation grew in popularity during the 1920,s due to the inflationary upheavals that took place at that time and in the prior decade:

> "In Germany, during the severe inflation period, the orthodox practice of calculating depreciation on the basis of original book costs was eventually swept aside because accountants and business men came to perceive that, in maintaining the substance of capital, it was no longer useful. At first various supplementary measures were adopted, such as charging all new fixed asset costs to expense and creating a special reserve to provide for maintenance of plant value and business efficiency (e.g., the prevalent Werkerhaltungskonto). Later, computation of depreciation on the basis of reproductive cost grew in popularity, which, indeed, is still evident from a survey of contemporary German depreciation theory." Henry W. Sweeney (1931; 166)[206].

> "Prices go up and prices go down, and with each change in the price level the discussion of replacement cost usage recurs. It appears that businessmen and accountants were willing to experiment with the use of replacement cost in the 1920's and early 1930's. But this receptivity to its use has declined steadily since then: in the 1940's practicing accountants were opposed to its use; .. . Thus if past experience holds true for the future, replacement cost will still receive its share of attention from theoreticians while practicing accountants largely ignore it." Germain Boer (1966; 97)[17].

Even though replacement cost accounting is no longer used by business accountants in most low inflation countries, it should be noted that it is still used today by some national income accountants as the basis for computing depreciation on a current cost basis.[*32]

The net realizable value and replacement cost of an asset can be regarded as the selling and buying prices for the asset in the relevant second hand market. Replacement cost will generally exceed the corresponding net realizable value due to the existence of transactions costs.

There is a variant of replacement cost accounting that at first sight seems to eliminate the need to consider second hand markets: find a current purchase price for a new asset that corresponds to the used asset on hand, apply the same method of depreciation to this new asset price (instead of the original historical cost price for the asset) and the resulting depreciated current price is an estimate for replacement value. However, this method of constructing replacement values implicitly assumes that the correct depreciation rates are known.

Replacement cost can exceed the corresponding net realizable value for reasons other than transactions costs. Consider the following example due to Paton:

> "One example will be sufficient to show the ruinous error which may flow from a slavish adherence to the cost-of-replacement theory in appraisals. In 1924, a valuation was made of the properties of the Kansas City Railways by two independent engineers. One of the items to be appraised was three old engines in the power house. These were of the massive type, with enormous flywheels, and were standard equipment twenty or twenty-five years ago, or more. This equipment was in excellent physical shape, but was utterly obsolete, and a couple of the engines were no longer even connected. The company's power at the time of appraisal was

[*32] Usually, the specific index number adjusted method for approximating a replacement value is used by national income accountants; see section 3.7 below.

entirely supplied by other and more modern equipment, although the old units were capable of giving service if required. One of the engineers went to the Westinghouse Company, with complete specifications, and secured an estimate of what it would actually cost, as of the date of the appraisal, to construct these engines, on special order. He then made an estimate of the cost of shipment, installation, etc. The result was a cost of replacement figure considerably over a million dollars. The other engineer treated the units as scrap and gave them a net value of $20,000." William A. Paton (1931; 95).

What happened in the above example is that technical progress occurred which caused the net realizable value for the used asset to plummet, but the replacement value for the asset was high, since the old asset was no longer being produced. Thus, there is a logical difficulty associated with the use of replacement cost values for unique assets such as a specially constructed machine or an engineering structure that is specific to the business unit: no replacement cost values are readily available in the marketplace for unique assets.[*33] A solution to this difficulty is provided by the approach of the first engineer in the above example: simply calculate the estimated cost of building the specific asset using the technology and input prices that pertain to the end of the accounting period.[*34]

Replacement cost values are subject to the same two difficulties that were associated with the use of net realizable values: replacement costs are not generally reproducible (different accountants will generally obtain different estimates of replacement cost) and replacement costs are not generally additive (if a group of assets is replaced, the aggregate replacement cost may be less than the sum of the individual replacement costs). The lack of additivity is not a serious problem: we can again impose additivity by seeking replacement costs for assets according to how they were originally purchased; i.e., if a group of assets were jointly purchased, then we attempt to find a joint replacement cost for the same group of assets. However, the lack of reproducibility is a serious limitation on the use of replacement values.

In this section, we considered the use of replacement costs and in previous section, we considered the use of net realizable values as a basis for valuing the assets held by a firm at the end of an accounting period. Is there a rational basis for choosing between these alternative valuation methods? One way of answering this question is to consider whether the business unit is likely to buy additional units of the asset in the near future (in which case an appropriate opportunity cost would appear to be replacement cost) or whether the business unit is likely to sell the asset in question (in which case the relevant opportunity cost would appear to be net realizable value). Thus several accountants[*35] have argued for the use of replacement values for raw material inventories and for net realizable values for inventories of finished products. Following this same logic, an expanding firm might value its fixed capital stock components at replacement values while a contracting firm might use net realizable values. While this line of reasoning does not provide a complete answer to the question of which valuation base to use, it does seem helpful.

[*33] "In some cases, such as permanent investments, plant sites, construction jobs, etc., almost no reliable data may be obtained for use in market valuation." H.C. Daines (1929; 101)[37]. There is no active trading market for large aggregates of fixed assets which have been put together into a specialized production design for specialized use. Any attempt to assign a market value to the aggregate of land, buildings, machinery, equipment and motive power constituting the average industrial plant is obviously impossible." Stephen Gilman (1939; 80)[99].

[*34] Statistics Canada has used this methodology for years to estimate a construction price index; i.e., engineering and construction firms are asked to provide estimates for the cost of building a specific asset in the current survey period.

[*35] Robert R. Sterling (1970; ix)[202] seems to have been the first accountant to argue along these lines: "It seems clear, for example, that one can postulate a continuing firm which is operating in two different markets (say, a retailer) and make a good case for valuing inventory at replacement cost. Under those circumstances the 'opportunity cost' of a unit is the cost of replacing it, since the firm must restock." Edwards (1975; 240-241)[80] argued for the use of entry values for those markets where a firm is usually a buyer and exit values for those markets where the firm is usually a seller and Davidson, Stickney and Weil (1976; 211)[39] endorsed this argument.

We turn now to a brief discussion of yet another basis for interim asset valuations.

3.6 Future Discounted Cash Flows

> "The flow of services issuing from an article of capital may have any duration and any distribution of rate. In every case the capital value of the article is the discounted value of its anticipated services." Irving Fisher (1897; 527)[89].

> "If one could approximate the whole future series of money outgoes and of money receipts of an enterprise, one could find, given a rate of discount, a direct capital value of that enterprise." John B. Canning (1929; 207)[23].

The view that the appropriate value for an asset is the *discounted stream of the future net revenues* that can be attributed to it was actively advocated by Irving Fisher (1897)[89] (1930)[94].*36 In the accounting literature, estimating a current asset value as the discounted stream of its future expected returns is known as the *economic approach* to asset valuation.

Of course, a current purchase price for an asset can be thought of as representing a lower bound to the asset's economic value to the purchaser, but in this section, we will define an asset's economic value as an estimated discounted stream of net returns that can be attributed to the asset.

Accountants pointed out that this "economic" approach to asset valuation suffers from two flaws:

- future discounted net returns are generally not known with any degree of certainty and hence the resulting estimates will not be reliable*37 and
- even if we did know future revenue flows with certainty, revenue flows are produced by the joint efforts of all assets and it is generally impossible to allocate the resulting joint net revenue flows to individual assets.*38

Another way of phrasing the first objection is to say that economic values will not generally pass the *reproducibility test*; i.e., different accountants will generally obtain different estimates for economic values. In principle, the second objection to the economic approach can be overcome; an econometric model could tease out shadow prices as derivatives of an intertemporal profit function with respect to the components of the fixed capital stock. Needless to say, there would be reproducibility problems with the resulting estimates: the resulting shadow prices would depend on somewhat arbitrary assumptions about future technical progress and about future expected input and output prices that the firm is expected to face plus assumptions about functional forms and stochastic specification that are sure to vary from econometrician to econometrician.

In spite of the above rather negative evaluation of the economic approach to asset valuation, accountants*39 have recognized that for certain unique assets held by a business unit, the economic approach may be the only relevant approach for obtaining current asset values. For example, a reasonable estimate for the value of a unique oil field held by an exploration company might be the estimated discounted net revenues generated by the crude oil pumped out of the field over the life of

*36 In chapter 2, we noted that Böhm-Bawerk (1891; 342)[18] seems to have been the first to notice this principle.

*37 "The non-availability of the future series of data, except for certain fragmentary items attaching to the near future, not only prevents the systematic development of realized income statistics to the point of large usefulness but prevents also a full development of capital valuation. For without reliable estimates of all future series to be discounted, reliable present valuations are impossible." John B. Canning (1929; 321)[23].

*38 "It [the capitalization of the income producing value of the net assets] is impractical of application, since from the very nature of the case, the earnings of a business are the joint product of all the assets, conditions and services which the business possesses and uses. It is impossible, therefore, to impute on the basis of total earnings any particular value to any given asset." H.C. Daines (1929; 98)[37].

*39 "The familiar accounting dilemma of relevance versus reliability emerges for the question of how do you produce another unique asset, such as Snow White or a particular oil field." John Leslie Livingstone and Roman L. Weil (1982; 253)[152].

the field. In order to obtain these estimates, it will be necessary to: (i) estimate how much crude will be extracted in each future period; (ii) estimate future spot prices per barrel of crude (less applicable taxes); (iii) estimate future extraction costs and (iv) provide an appropriate discount rate. In fact, there are engineering firms that will provide such estimates and accountants accept their valuations in order to put an estimated value on oil reserves. As another example, suppose a business unit holds the rights to a movie or a patent; (both are unique assets). Then a reasonable current asset value for the movie might be the discounted value of future expected rental income and for the patent might be the discounted value of future anticipated royalty payments.

In order for economic valuations to pass the objectivity or reproducibility test, it seems necessary that these valuations be done by specialized valuation firms, which could be accredited by the relevant accounting standards board or by the relevant governmental authority.

We turn now to another promising class of methods for valuing assets.

3.7 Specific Price Level Adjusted Historical Cost

> "Actual cost, for example, because of its stability and its consequent effectiveness in attracting capital, might be chosen as the basis on which to compute the return; while reproduction cost, or possibly some index number designed to rise and fall with the general level of commodity prices, might conceivably be chosen as the proper basis by which to regulate charges." James C. Bonbright (1926; 305)[19].

> "On account of the expense involved, to argue for yearly appraisals of fixed assets, would sound impractical. When price levels remain fairly constant they would prove to be unnecessary. During periods of price fluctuation an adjustment could be made in previous appraisals to reveal this condition or an entirely new appraisal resorted to. In this connection, price indexes may prove very helpful in the future to both the accountant and the appraisal engineer." H.C. Daines (1929; 101)[37].

> "Knowing the exact composition of the client's property as at the date for which the new appraisal is to be made, the appraisal company then values such property at the prices prevailing on that date ... . A method that may very conveniently and profitably be used as a quick and cheap substitute under certain conditions is the index-number method. This method is a phase of 'stabilized accounting', which is concerned with the use of index numbers to restate accounting figures in a uniform price level before combining or comparing them." Henry W. Sweeney (1934; 110)[210].

The specific price level method for constructing current values for an asset held by a business unit through successive accounting periods was suggested by Daines (1929; 101)[37], Sweeney (1934; 110)[210] and many other accountants.[*40] The method works as follows. First, assets held by the business unit at the beginning of period 0 are classified into a finite number of distinct asset classes. Secondly, it is supposed that index numbers that pertain to each asset class are available at the beginning and end of each accounting period. Finally, suppose that an asset was purchased at the beginning of accounting period 0 at the price P^0,[*41] the period 0 depreciation rate is δ^0 and the asset inflation rate for the relevant asset class over period 0 is i^0 (i.e., the specific asset index number at the end of the period divided by the specific asset index number at the beginning of the period is

[*40] "Inasmuch as the price level is not stable for any great length of time, and since this calculation is contemplated for each fiscal period, the only feasible procedure for a company with thousands of assets is the use of price index numbers." Albert L. Bell (1953; 49)[13]. "Where no market exists for new fixed assets of the type used by the firm, two means of measuring current costs are available: (1) appraisal, and (2) the use of price index numbers for like fixed assets to adjust the original cost base to the level which would now have to be paid to purchase the asset in question." Edgar O. Edwards and Philip W. Bell (1961; 186)[81].

[*41] More generally, P^0 can be the estimated beginning of period 0 current value for the asset.

$1 + i^0$). Then the *Specific Price Level Adjusted (SPLA) value of the asset* at the end of period 0 is defined as

$$V_{SPLA} \equiv (1 - \delta^0)(1 + i^0)P^0. \tag{3.2}$$

Comparing (3.2) with (3.1), we see that the present specific price index number method for constructing an end of period estimated asset value is very similar to the General Price Level Adjusted asset value defined earlier by (3.1); the only difference is that now a presumably more relevant specific price index is used for revaluation purposes rather than an index of general inflation.

If the same set of asset specific price indexes is given to all accountants, then Specific Price Level Adjusted values will satisfy the reproducibility test. The SPLA asset value should also be closer to its end of period market value (i.e., an end of period purchase cost or net realizable value) since presumably, the index numbers reflect a sample of market transaction prices for new units of the asset (or similar assets) during a time period that includes the end of period 0. Thus SPLA values will tend to be reproducible and relevant.

We also note that Specific Price Level Adjusted accounting is not completely impractical since it has occasionally been used historically in business financial accounting.*42 It is also essentially equivalent to the Perpetual Inventory Method for constructing capital stocks in the National Accounts.*43

There are some problems associated with the use of Specific Price Level Adjusted values:

(i) None of the available specific price indexes may be relevant for the particular asset on hand.*44 A related problem is that different accountants may classify the same asset into different asset classes thus destroying the reproducibility property for the method.*45

(ii) The asset specific index numbers will generally pertain to a discrete interval of time instead of the precise date at which the accounting period ends. Under these conditions, the exact adjustments (if any) that the accountant should make to the specific indexes is ambiguous.*46

*42 “Fixed assets, as distinguished from current assets, had to be revalued by means of index corrections. The indices . . . , which had to be used for any given fiscal year, were published in an official government publication. These index coefficients were computed on the basis of the wholesale price indices for construction materials, lumber and steel products.” H. Peter Holzer and Hans-Martin Schönfeld (1963; 383)[125].

*43 “6.189. The perpetual inventory method requires an estimate to be made of the stock of fixed assets in existence and in the hands of producers. This is done by estimating how many of the fixed assets installed as a result of gross fixed capital formation undertaken in previous years have survived to the current period. Average service lives, or survival functions, based on observations or technical studies may be applied to past investments for this purpose. Fixed assets purchased at different prices in the past have then to be revalued at the prices of the current period. This may be done by utilizing appropriate price indices for fixed assets. The construction of suitable price indices covering long periods of time raises difficult conceptual and practical problems, but these technical problems of price measurement are not peculiar to the PIM method and will not be pursued further in the present context.” *System of National Accounts 1993*, pages 148-149[87].

*44 “The fact that the purchasing power shown will be in terms of the index used, and not in terms of the actual purchasing power available to a given enterprise for making its purchases, is a decided limitation to the use of the index numbers in accounting.” Donald K. Griffith (1937; 126)[102]. “Not many years ago standard telephone cables consisted of numerous wires encased in a lead sheeting. In the present microwave era it would be just as wrong to apply replacement-cost index numbers to the cost of the old cable and call the resulting value for the purpose of arriving at depreciation expense as it would be to apply price-index numbers to the cost of the famous twenty mule team and call the result the cost of automotive transportation.” Charles W. Smith in G.O. May and others (1952; 126).

*45 Note that GPLA accounting is not subject to this problem since there is only one asset class. Of course, the countervailing problem associated with GPLA accounting is that it is less relevant or accurate as an approximation to actual current values: “A simple general purchasing power index is proposed, but that has no real relevance to the value of capital goods.” Solomon Barkin in G.O. May and others (1952; 115).

*46 Suitable rules of thumb would have to be developed. Gilman raises similar timing and domain of definition issues in the context of finding suitable estimates for end of period values for the inventory components of a business unit’s capital stock: “Another cause of profit distortion is to be found in the methods used for determining selling prices as the preliminary basis for the proportional cost calculation. Should market quotations on the last day of each month be used? Should the daily quotations for the entire month be averaged? Should the averages for the past three months be used? Under mercurial market conditions these questions become important. The purpose

(iii) The related issue of the timeliness of the specific indexes should also be raised: annual specific price indexes for capital stock components that appear with a half year time lag will be useless in the context of quarterly accounting.*47

(iv) The construction of SPLA values is mainly suitable for the valuation of fixed capital stock components and not circulating capital stock components.*48 How then should end of period prices for inventory stocks be constructed? The problems involved in constructing current prices for inventory items are generally not as severe because relevant market prices for inventory components held by the business unit are often available in the records of the business unit: market prices for used fixed assets are more difficult to obtain.*49

(v) The SPLA values for assets at the end of the accounting period are still dependent on the rather arbitrary depreciation rates (recall the depreciation rate δ^0 in (3.2)) that are associated with historical cost accounting.*50 To cure this lack of reproducibility in the method, the Agency that provides the asset specific index numbers should also provide "standard" depreciation rates for assets in each class (or alternatively, provide index numbers for not only new assets but also used assets). The adoption of this last suggestion will not only lead to reproducible SPLA values, but it will also lead to reproducible estimates of depreciation.

Which Agency should provide the relevant index numbers and depreciation rates? Three possible choices are: (i) the relevant National Statistical Agency; (ii) the relevant Accounting Standards Board or (iii) an Agency or Department of the relevant National Government (e.g., the income taxation authority).

We note that historical cost valuations for fixed assets have proved to be very resilient from a historical perspective, being temporarily abandoned only in the face of dramatic inflationary shocks when the method clearly became absurd.*51 It seems likely that the alternative asset valuation methods

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of the popular three months' average plan is, according to McKee, 'to eliminate temporary market fluctuations, and reflect costs by market trends instead'." Stephen Gilman (1939; 333)[99].

*47 "Accountants are fully aware of the difference between 'dollar accounting' and a conceivable 'purchasing power accounting', and would prefer just as the economists do, a purchasing power accounting. But the adjustment data can never become available at the time records are originally made, nor do they become available in time for report making. Whether or not it would pay to make such an accounting currently, is doubtful; but the cumulative effect of a depreciating currency upon valuations of long-lived assets and debts may be such as to require partial readjustments at relatively long intervals." John B. Canning (1929; 196-197)[23].

*48 "In order to make the accounts reasonably reflect current conditions and to avoid abrupt value changes, numbers of accountants have recommended that fixed asset accounts be regularly adjusted by means of an index number. Gradual changes thus computed would be better than the irregular revaluations which have occurred in the past, but the recording of index number adjustments on the books conceals historical costs and at best constitutes only a partial solution to the general problem of valuation. Even though fixed asset values were satisfactorily determined by index numbers, the more important problem of inventory valuation would still remain." Ralph C. Jones (1935; 172)[133].

*49 For a worked example of how to deal with inventory index numbers in the context of a user cost approach to the measurement of inventory services, see Diewert and Smith (1994)[76]. We will deal with the treatment of inventories in more detail in chapter 5.

*50 "For each account requiring adjustment the price index is of a homogeneous class of assets which includes those in the account. The use of a specific index for each account rather than a general index for all accounts follows from the use of current cost rather than purchasing power historical cost as the basis of valuation. The appropriateness of the index used for each account is, of course, limited by the knowledge of the assets included in the account, the index numbers available, and by the criterion of objectivity ... . This [specific index number adjusted] quantity differs from market value in that (1) historical deferred cost is arrived at by means of arbitrary, generally straight-line, depreciation charges; (2) an index of the cost of new assets is used to adjust used assets; and (3) the impact of technological change on a firm's assets may differ radically from the recognition of technological change in an index number designed to cover a broader group of assets." Myron J. Gordon (1953; 375)[100].

*51 "These factors may account for the present status of the index number accounting practice in Europe. It had its start in seemingly fertile soil, because the monetary system in Europe at that time was completely broken down, but the index number methodology has failed to develop and bear fruit. It seems reasonable to conclude that, since the index methodology has become dated, it failed to meet the fundamental and lasting needs of based on current values described above as Methods 2-6 have failed to be adopted permanently in business financial accounting for a number of reasons:

(i) the alternative method was thought to be too inaccurate (General Price Level Adjusted valuations);
(ii) the alternative method was thought to be too nonobjective or not reproducible (all other methods) or
(iii) the alternative method was thought to be too expensive or too complex.

However, it seems possible that all of these objections could now be overcome with the use of Specific Price Level Adjusted values, provided that a National Authority could provide the accounting profession with the relevant asset specific index numbers and standard depreciation rates.

Our final method of asset valuation is rather different in nature from the previous methods.

3.8 Prepaid Expense "Assets" and their Allocation

The nature of a capital asset used in production is that a production unit makes an expenditure in the current period but the benefits of this asset expenditure are not confined to the current period. Up to now, the types of asset expenditures that we have been considering were of the *tangible* type; i.e., investments in reproducible capital equipment like structures and machinery and equipment along with investments in land and inventory. However, many investments are in *intangible assets* such as *advertising and marketing expenses, research and development expenditures* and *firm investment in training.* All of these categories of expenditures have the character that the present period outlays will create incremental revenues in the future for the firm that undertakes them. These current period expenditures on intangible assets have a different character than expenditures on tangible durable inputs, which can be used for a number of periods and then sold to other users.*52 The problem with intangible asset expenditures is that they usually have the nature of a fixed cost. Thus these fixed costs, once incurred, are usually of no consequence for a firm's future strategic behavior; i.e., fixed costs are irrelevant to the firm's intertemporal profit maximization problem, provided that the firm is not driven to bankruptcy by these fixed costs. However, again the problem of trying to determine the period by period income of the firm emerges in this context: it is not "fair" to charge all of these intangible asset expenditures to the period when they were incurred: it would be "fairer" to distribute these expenditures over future time periods when the benefits of the investment materialize. Thus the problem emerges of how to *allocate* the cost outlays on intangible investments over future periods. Thus the accounting problems in the present section have a different character than in the previous sections, where a straightforward opportunity cost approach was used. In the present section, the approach taken is one of *matching* current costs with future expected revenues.*53

The problem of intertemporally allocating intangible investment expenditures to future periods when the benefits might be realized is similar to other intertemporal cost allocation problems that are associated with *prepaid expenses* and *transactions costs.*

Prepaid expenses as an accounting asset class occurred quite early in the history of accounting. Thus Hatfield (1927; 16)[109] gave several examples of this type of "asset", including insurance payments which apply to multiple accounting periods, the stripping away of surface rock for a strip mine

business. If it had met a fundamental need it would surely not have disappeared from business usage." Donald K. Griffith (1937; 131)[102].

*52 In some cases, the stream of future revenues created by an intangible investment can be sold on the marketplace (e.g., patents, trademarks and franchises), but this still does not solve the problem of how to distribute the intangible investment costs over future periods if the asset is not sold.

*53 Paton and Littleton (1940; 123)[179] argued that the primary purpose of accounting is to match costs and revenues. For an excellent early discussion on the importance of matching costs to future revenues, see Church (1917; 193)[32].

and prepaid expenses in general. Hatfield (1927; 18)[109] correctly noted that this type of asset is different from the usual sort of tangible asset since this type of asset cannot readily be converted into cash; i.e., it may have no opportunity cost value.

Transactions costs as an asset class are recognized by some national income accountants. Thus in Australia, the transactions costs associated with the purchase of a residential structure are capitalized and written off over the expected length of time that the average resident is held by the same owner.

We will not give a detailed treatment of possible methods for accomplishing this intertemporal cost allocation problem in this chapter. The following chapter will do this for an R&D asset but the same principles can readily be adapted to other types of prepaid asset.*54

We leave our last words on the subject of asset valuation to one of the pioneers of current value accounting:

> "Even crude attempts should result in an improvement over present depreciation practices. During periods of rapidly changing prices crude measurements of a relevant item are likely to be much more meaningful than accurate measurements of an irrelevant one (in this case, historic cost)." Edgar O. Edwards (1954; 268)[79].

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*54 The basic framework is outlined in section 11 of Diewert (2005)[68].

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Chapter 4

Constructing a Capital Stock for R&D Investments

4.1 Introduction

It is likely that the next international version of the System of National Accounts (SNA) will contain recommendations that national expenditures on Research and Development (R&D) be capitalized and then depreciated over subsequent periods. This is quite appropriate from the viewpoint of economic theory, since the R&D expenditures in the present period usually give rise to benefits that are realized in the future periods and hence, these R&D expenditures have the nature of a capital input to production.

R&D expenditures are an example of an *intangible* expenditure. Other examples of expenditures on intangible assets are *advertising and marketing expenses* and *expenditures on training.* All of these categories of expenditures have the character that the present period outlays will create incremental revenues in the future for the firm that undertakes them. These current period expenditures on intangible assets have a different character than expenditures on tangible durable inputs, which can be used for a number of periods and then sold to other users.[*1] The problem is how to *allocate* the cost outlays on intangible investments over future periods. Thus these accounting problems in have a different character than in the treatment of reproducible capital, where a straightforward opportunity cost approach can be taken. In the present chapter, the approach taken is one of *matching* current costs with future expected revenues.[*2]

The problem of intertemporally allocating intangible investment expenditures to future periods when the benefits might be realized is similar to other intertemporal cost allocation problems that are

[*1] In some cases, the stream of future revenues created by an intangible investment can be sold on the marketplace (e.g., patents, trademarks and franchises), but if it is not sold, we still have the problem of how to distribute the intangible investment costs over future periods. If it is sold, then the purchaser has the problem of allocating the purchase cost to future periods.

[*2] Paton and Littleton (1940; 123)[179] argued that the primary purpose of accounting is to match costs and revenues. For an excellent early discussion on the importance of matching costs to future revenues, see Church (1917; 193)[32].

associated with *prepaid expenses* and *transactions costs.* It is often the case that when a business unit purchases a nonresidential structure, there are substantial transactions costs associated with this purchase. From one viewpoint, these costs are sunk and should be written off immediately; i.e., these expenditures should be treated as a business intermediate or primary input expense. However, from the point of view of constructing period by period estimates of the income of the business unit, it is more appropriate to take these transactions costs, capitalize them and then distribute them over the future periods that the associated asset is expected to be held, ensuring that the discounted value of these future cost distributions is equal to the current period transactions costs incurred.[*3] Prepaid expenses[*4] can also be given a similar treatment. In both of these cases, the transactions costs and the prepaid expenses can be given an accounting treatment that is similar to our suggested treatment of R&D expenses.

In section 4.2, we present a reasonably general cost allocation model that assumes a constant structure of real and nominal interest rates. We relate this model to the usual models of depreciation for reproducible capital. In sections 4.3 and 4.4, we relax this simplifying assumption.

In section 4.3, we present the details involved in making various cost imputations for a simple 4 period model, while in section 4.4, we present the algebra of the general model, along with a discussion of the data requirements for implementation of the model.

Section 4.5 concludes with a discussion of some of the difficult conceptual and practical issues that are involved in capitalizing R&D expenditures.

4.2 The Basic Cost Matching Methodology

To fix ideas, suppose that as of at the beginning of period t, a firm has made expenditures on creating an intangible asset, which are equal to C^t:[*5]

$$C^t \equiv \sum\nolimits_{m=1}^{M} P_m^t Q_m^t \tag{4.1}$$

where P_m^t is the period t price for the mth type of input that is used to create the intangible asset and Q_m^t is the corresponding quantity purchased. These expenditures in period t are expected to generate a future stream of incremental revenues for the firm. Let R_0^t denote the immediate period t incremental revenues (which could be zero) and let R_n^t denote the incremental revenues that the period t expenditures C^t are *expected* to generate n periods from the present period t, for $n = 1, 2, \ldots$ Let r^t be the (nominal) period t opportunity cost of financial capital. Then the discounted value of these expected incremental revenues is:[*6]

$$R^t \equiv R_0^t/(1+r^t) + R_1^t/(1+r^t)^2 + R_2^t/(1+r^t)^3 + R_3^t/(1+r^t)^4 + \cdots \tag{4.2}$$

[*3] In the Australian national accounts, household transactions expenses on the purchase of residential housing units are capitalized and distributed over the average length of life that a dwelling unit is held by a single owner in Australia.

[*4] Prepaid expenses as an accounting asset class occurred quite early in the history of accounting. Thus Hatfield (1927; 16)[109] gave several examples of this type of "asset", including insurance payments which apply to multiple accounting periods, the stripping away of surface rock for a strip mine and prepaid expenses in general. Hatfield (1927; 18)[109] correctly noted that this type of asset is different from the usual sort of tangible asset since this type of asset cannot readily be converted into cash; i.e., it may have no opportunity cost value.

[*5] Much of the material in this section is taken from section 11 in Diewert (2005a)[67]. C^t could represent the cumulated costs over a number of periods that it has taken to develop a useful product or process.

[*6] Thus R_0^t is the revenue that the R&D project is expected to generate over the course of period t (and this revenue is not expected to materialize until the end of period 0), R_1^t is the revenue that the R&D project is expected to generate over the course of period $t+1$, and so on. At the beginning of period t, the present value of these expected revenue contributions is R^t. At the beginning of period $t+1$, the expected asset value will decline (if expectations do not change) to $R_2^t/(1+r^t) + R_3^t/(1+r^t)^2 + \cdots$ and so on. Thus as the revenues are realized, the asset value will decline.

The problem is to allocate the current period cost C^t over future periods. Thus let C^t_n be the allocation of C^t to the accounting period that is n periods after period t for $n = 0, 1, 2, \ldots$ At first sight, it seems reasonable that these future cost allocations C^t_n should sum to C^t. However, this turns out *not* to be so reasonable: costs that are postponed to future periods must be escalated by the (nominal) interest rate r^t, so that the present value of discounted future costs is equal to the actual period t costs C^t. Thus the intertemporal cost allocations C^t_n should satisfy the following equation:[*7]

$$C^t = C^t_0/(1+r^t) + C^t_1/(1+r^t)^2 + C^t_2/(1+r^t)^3 + C^t_3/(1+r^t)^4 + \cdots \tag{4.3}$$

To see why discounting is necessary, consider the following simple example where we have a cumulated investment equal to C^t at the beginning of period t and we anticipate the revenue R^t_1 at the end of period 1, which is two periods from the beginning of period 0. The expected discounted profits that this investment will generate are:

$$\Pi \equiv -C^t + R^t_1/(1+r^t)^2. \tag{4.4}$$

The period by period cash flows for this project are $-C^t, 0, R^t_1$. In general, we want to *match* the beginning of period t cost C^t with the end of period $t+1$ revenue flows. Thus we want to convert the cash flow stream $-C^t, 0, R^t_1$ into an equivalent cash flow stream $0, 0, -C^t_1 + R^t_1$. If we choose

$$C^t_1 \equiv C^t(1+r^t)^2, \tag{4.5}$$

then it can be seen that these two cash flow streams have the same present value and C^t_1 is the "right" period $t+1$ cost allocation. Put another way, if we simply carried forward the period t costs C^t and set C^t_1 equal to C^t, we would be neglecting the fact that the costs were in place at the beginning of period t while the return on the investment was deferred until the end of period $t+1$ and hence, we need to charge the opportunity cost of financial capital for two periods on the initial investment (for two periods) until it is expensed in period $t+1$.

How should the intertemporal cost allocations C^t_n be chosen? It is natural to make these cost allocations proportional to the corresponding period anticipated revenues. Thus choose the number α so that the following equation is satisfied:

$$C^t = \alpha R^t. \tag{4.6}$$

Thus we set the observed period t cost associated with the intangible investment C^t equal to the constant α times the discounted value of the anticipated incremental revenue stream R^t that the investment is expected to yield.[*8]

Typically, α will be equal to or less than one, since otherwise, the period t intangible investment expenditures C^t should not be undertaken. If α is less than one, then there will be an expected profit above the opportunity cost of capital, which could be some form of monopoly profit or a reward for risk taking.

Once α has been determined by solving (4.6), then the intertemporal cost allocations C^t_n can be defined to be proportional to the corresponding anticipated incremental revenues R^t_n for future periods:

$$C^t_n \equiv \alpha R^t_n; \qquad n = 0, 1, 2, \ldots \tag{4.7}$$

[*7] We are assuming that the period $t+n$ cost allocation, C^t_n, is written off at the end of period $t+n$.

[*8] Of course, the practical problem that the national income accountant will face is: how can the future stream of incremental revenues be estimated?

We can convert the nominal cost allocation factors C_n^t into constant (period t) dollar cost allocations q_n^t as follows:

$$\begin{aligned} q_n^t &\equiv C_n^t/(1+\rho^t)^n; \qquad n = 0,1,2,\ldots \\ &= \alpha R_n^t/(1+\rho^t)^n \end{aligned} \tag{4.8}$$

where ρ^t is the period t consumer price inflation rate, which is expected to persist into the future.[*9] The q_n^t defined by (4.8) are the constant (beginning of period t) dollar counterparts to the period t nominal cost allocation factors C_n^t that occurred in (4.3). The corresponding future expected spot prices P^{t+n} of the q_n^t are defined to be the expected cumulated consumer price inflation rates; i.e, define the P^{t+n} as follows:

$$P^{t+n} \equiv (1+\rho^t)^n; \qquad n = 0,1,2,\ldots. \tag{4.9}$$

Thus the nominal cost allocation factors C_n^t and the q_n^t and the P^{t+n} satisfy the following equations:

$$C_n^t = P^{t+n} q_n^t; \qquad n = 0,1,2,\ldots. \tag{4.10}$$

We can define the period t real interest rate r^{t*} in terms of the period t nominal rate r^t and the period t general inflation rate ρ^t as follows:

$$1 + r^{t*} \equiv (1+r^t)/(1+\rho^t). \tag{4.11}$$

Substituting (4.8)-(4.11) into (4.3) shows that the constant dollar cost allocations q_n^t satisfy the following equation:

$$C^t = q_0^t/(1+r^{t*}) + q_1^t/(1+r^{t*})^2 + q_2^t/(1+r^{t*})^3 + q_3^t/(1+r^{t*})^4 + \cdots \tag{4.12}$$

We can define a *real capital stock* that corresponds to the R&D asset at the beginning of period t by K^t by setting K^t equal to C^t defined above by (4.12). If expectations about future prices and revenues are realized, then we can define the corresponding real R&D capital stock at the start of period $t+1$ by K^{t+1}, at the start of period $t+2$ by K^{t+2}, and so on, where these stocks are defined to be the expected discounted future real cost allocations; i.e., we have:

$$\begin{aligned} K^t &\equiv q_0^t/(1+r^{t*}) + q_1^t/(1+r^{t*})^2 + q_2^t/(1+r^{t*})^3 + q_3^t/(1+r^{t*})^4 + \cdots \\ K^{t+1} &\equiv q_1^t/(1+r^{t*}) + q_2^t/(1+r^{t*})^2 + q_3^t/(1+r^{t*})^3 + q_4^t/(1+r^{t*})^4 + \cdots \\ K^{t+2} &\equiv q_2^t/(1+r^{t*}) + q_3^t/(1+r^{t*})^2 + q_4^t/(1+r^{t*})^3 + q_5^t/(1+r^{t*})^4 + \cdots \\ &\cdots \end{aligned} \tag{4.13}$$

If expectations about future prices and revenues are realized, then K^t defined by the first line in (4.13) represents the beginning of period t constant dollar R&D capital stock, K^{t+1} defined by the second line in (4.13) represents the beginning of period $t+1$ constant dollar R&D capital stock and so on. The corresponding prices for the K^{t+n} are taken to be the CPI prices P^{t+n} defined by (4.9) above.

Thus accounting for R&D stocks at first glance seems to be relatively straightforward. Given our expectations about the future revenue flows R_n^t that the R&D asset is expected to generate, we use equations (4.7) to calculate our nominal sequence of future cost allocations, the C_n^t. Given the C_n^t and expectations about the future course of the CPI, we form the constant dollar cost allocations q_n^t

[*9] This expectational assumption could be relaxed at the cost of more notational complexity; see section 4.4 below. It should be noted that the use of the CPI as a price for converting a financial cost into a real cost is not without controversy as we have seen in the previous chapter.

using equations (4.8). Then q_n^t gives us the expected constant dollar charge that we should make at the end of period $t+n$ as an R&D real cost for period t. This can be converted into the nominal period $t+n$ cost, $P^{t+n}q_n^t$, where P^{t+n} is defined by (4.9). The corresponding beginning of period $t+n$ real R&D capital stock K^{t+n} is defined by the appropriate equation in (4.13) and this can be converted into the corresponding nominal beginning of period $t+n$ value of the R&D capital stock, $P^{t+n}K^{t+n}$.

There is one more useful relationship that we can develop in this accounting method for allocating a fixed cost over future time periods when the cost produces a benefit. Look at the first two equations in (4.13). Substituting the second equation into the first, we obtain the following equation:

$$K^t = q_0^t/(1+r^{t*}) + K^{t+1}/(1+r^{t*}). \tag{4.14}$$

This equation can be solved for the period t constant dollar cost allocation flow variable, q_0^t, in terms of constant dollar capital stocks as follows:

$$q_0^t = r^{t*}K^t + K^t - K^{t+1} = r^{t*}K^t + D_0^t. \tag{4.15}$$

The remaining equations in (4.13) lead to similar formulae for the remaining constant dollar allocations q_n^t in terms of R&D capital stocks, K^{t+n}, and the real interest rate r^{t*}:

$$\begin{aligned} q_1^t &= r^{t*}K^{t+1} + K^{t+1} - K^{t+2} = r^{t*}K^{t+1} + D_1^t; \\ q_2^t &= r^{t*}K^{t+2} + K^{t+2} - K^{t+3} = r^{t*}K^{t+2} + D_2^t; \\ &\cdots \end{aligned} \tag{4.16}$$

where the anticipated *real time series depreciation amounts* D_n^t for period $t+n$ are defined in terms of the constant dollar capital stocks as follows:[*10]

$$D_n^t \equiv K^{t+n} - K^{t+n+1}; \qquad n = 0, 1, 2, \ldots. \tag{4.17}$$

Equations (4.15) and (4.16) are counterparts to equations (1.7) in chapter 1, except in chapter 1, we held the quantity fixed and allowed the user costs by age to change, whereas in the present chapter, we are fixing the price of the R&D asset (if there is no general inflation) and allowing the quantity flow R&D charges q_n^t to vary with n.

Equations (4.15) and (4.16) allow us to decompose the period t real R&D flow charge q_0^t into a constant dollar depreciation component D_n^t and a real interest rate charge, $r^{t*}K^t$.[*11]

It should be noted that the cost allocation model outlined above can be applied to other forms of "assets"; namely, deferred charges, prepaid expenses and transfer fees when a reproducible asset is acquired. The one hoss shay form of revenue matching is probably the preferred method for dealing with this type of transfer fee "asset".

Of course, the practical problem with all of the above algebra is that it is entirely driven by the pattern and magnitudes of expected future incremental revenues but the statistician will not have very good information on these expected revenues. Moreover, it is not clear what would be a "good" rough approximation to these expected revenues.[*12] This is an important area for future research.

[*10] These definitions are the counterparts to definitions (1.11) in chapter 1, which defined cross sectional depreciation. Obviously the depreciation amounts D_n^t can be transformed into depreciation rates as follows: $\delta_n^t \equiv 1 - [K^{t+n+1}/K^{t+n}] = D_n^t/K^{t+n}$; $n = 0, 1, 2, \ldots$.

[*11] If it is necessary to keep track of nominal interest flows (including imputed interest), then the algebra does not work out so nicely; see the example in the following section.

[*12] Thus the practical measurement problems are much harder in the intangible context compared to the case of reproducible capital where a straightforward capital expenditures survey that includes information on the age of retired capital assets (or their age and sale price if the reproducible assets are sold before they are worthless) will enable the statistician to construct somewhat accurate depreciation rates and user costs.

Finally, if we want to construct measures of *real R&D (flow) input*, we need *additional* information on the prices and quantities of the inputs that are used to create the R&D asset. Thus in addition to information on R&D costs, we need to be able to collect information on the price and quantity components that went into this R&D cost, in order to construct measures of real R&D input.*13 Hence in addition to information on the projected future period revenue flows that will materialize as a result of the R&D investment, we need some additional information in order to construct measures of real R&D input:

- Information on the prices and quantities of the flow inputs that comprise the period 0 cost, C^0; i.e., these are the P_m^t and Q_m^t that appeared in equation (4.1) above.

We agree with Pitzer (2004)[183] that R&D investments are *fundamentally different* from investments in reproducible capital. Pitzer (2004; 2)[183] regards R&D investments as producing "recipes" but a recipe is not the usual type of "productive" input. Pitzer defines "productive" inputs as follows:

> "Fundamental to production is the notion that inputs are proportional in some sense to outputs. Outputs are created by combining a particular collection of inputs in a particular manner. ... If more outputs are desired, then more inputs are necessary, and usually, more of all inputs. It may not be necessary to double the inputs to double the outputs, but more inputs are necessary to produce more outputs." John Pitzer (2004; 2)[183].

We also agree with Pitzer that the asset value of a marketable R&D investment is a discounted monetary flow of payments that will usually have some form of monopoly element to it:

> "A patented entity is an asset that permits the owner to levy an assessment, much like a tax, on other units for the use of a recipe. In some cases, the assessment is paid by another producing unit to acquire the right to produce outputs based on the recipe. In other cases, the owner will produce the outputs and levy the assessments on its customers by charging a monopoly price. The market value of the asset is the present value of the future assessments that are expected to be collected by the asset's owner." John Pitzer (2004; 5)[183].

Thus the treatment of R&D assets will necessarily be quite different in some respects from the treatment of reproducible capital assets. In particular, the treatment of R&D assets involves *two* separate deflation problems:

- The deflation of expenditures on R&D inputs at a point in time into price and quantity components. This requires information on the price and quantity components of the inputs into the creation of the R&D asset.
- The deflation of the nominal intertemporal cost allocation of the current R&D flow expenditures into constant dollar cost allocations. The deflator to be used here is a general purchasing power deflator.

There are some additional complications in dealing with intangible assets in a national income accounting framework that can be best illustrated by a "concrete" example. Thus we consider a simple example in the following section and look at the various accounting transactions that will be necessary to implement the cost matching model outlined in this section.

4.3 A Simple Example

We consider a special case of the model outlined in the previous section where the current period t is set equal to 0, R&D costs equal to C^0 are incurred at the beginning of period 0 and this project

*13 On a national basis, it is important to be able to construct measures of real R&D input so that the resulting real measure can be used as an explanatory variable in regressions that regress measures of economy wide performance (like Total Factor Productivity Growth) on various explanatory variables.

has no further nonfinancial costs in future periods but it is expected to yield nominal revenues of R_1^0 and R_2^0 in periods 1 and 2. Thus there are no revenues generated by the project during period 0. All costs and revenues are transacted at the end of each period. The one period nominal (bond) expected interest rate (or cost of capital) at the beginning of period 0 is r_0, at the beginning of period 1 is r_1 and at the beginning of period 2 is r_2, with all expectations being formed at the beginning of period 0.[*14] We assume that the project is funded by one period bonds and no dividends are paid out so that as the firm gathers revenues, bond debt is retired at the end of periods 1 and 2. From the perspective of the beginning of period 0, the firm's *expected discounted profits* are:

$$\Pi^0 \equiv -C^0 + R_1^0/(1+r_0)(1+r_1) + R_2^0/(1+r_0)(1+r_1)(1+r_2) \geq 0. \tag{4.18}$$

From the perspective of the end of period 2, the firm's expected profits are:

$$\Pi^0(1+r_0)(1+r_1)(1+r_2) = -C^0(1+r_0)(1+r_1)(1+r_2) + R_1^0(1+r_2) + R_2^0 \equiv A^3 \tag{4.19}$$

where A^3 is the firm's net asset value at the end of period 3.

The counterpart to equation (4.3) in the previous section is the following equation:

$$C^0 = C_0^0/(1+r_0) + C_1^0/(1+r_0)(1+r_1) + C_2^0/(1+r_0)(1+r_1)(1+r_2) \tag{4.20}$$

where the C_n^0 are the cost allocations of the actual beginning of period 0 cost, C^0, to periods $n = 0, 1, 2$.[*15] Since there are no expected revenues in period 0, it is natural to set the period 0 cost allocations, C_0^0, equal to 0. We shall also impose inequalities on the period 1 and 2 cost allocations so that the allocated costs do not exceed the corresponding revenues for those periods. Thus we assume that the 3 cost allocations C_n^0 satisfy (4.20) and the following equations and inequalities:

$$C_0^0 = 0;\ 0 < C_1^0 \leq R_1^0;\ 0 < C_2^0 \leq R_2^0. \tag{4.21}$$

This is all of the information that we need to set up a set of (expected) accounts for the firm for the 4 periods under consideration. Table 4.1 below does this. Note that $(-)$ means that the corresponding item is a current period cost to the firm. D^t denotes the net debt of the firm at the end of period t. D^0 is equal to C^0 and so the beginning of period 0 net asset value, counting just debt (negatively) and realized cash flow (positively) is $A^0 \equiv -D^0 = -C^0$.

The beginning of period 0 debt level, D^0, is equal to the end of period 0 expenditures on R&D, C^0. In this example, there are no further real expenditures on R&D in periods $0, 1$ and 2. If there were, we would have to set up tables similar to Table 4.1 and combine the resulting tables by summing up expenditures in the various categories.[*16]

The beginning of period 1 and 2 debt levels are given by:

$$\begin{aligned} D^1 &\equiv (1+r_0)D^0 = (1+r_0)C^0; \\ D^2 &\equiv (1+r_1)D^1 - R_1^0 = (1+r_0)(1+r_1)C^0 - R_1^0. \end{aligned} \tag{4.22}$$

[*14] Thus our model is slightly more general than in the previous section in that we are no longer assuming that the term structure of interest rates is constant.

[*15] Think of C^0 as being the market component of Gross Expenditures on Research and Development (GERD). For now, we exclude the non market component of GERD from our simple model because there are no identifiable revenue streams that are associated with non market R&D, which is given away freely to all who want to use it. Thus there are no future revenues that current costs can be matched to.

[*16] Thus Table 4.1 sets out the basic accounting framework for capitalizing the costs pertaining to a single period, period 0. A similar Table can be constructed if there are additional R&D expenditures in period 1 that generate identifiable incremental future revenues.

Line 0:	(Beginning of) Period t; $t =$	0	1	2	3
Line 1:	Revenues at t, R_t^0	0	0	R_1^0	R_2^0
Line 2:	Nonfinancial costs C^t $(-)$	$-C^0$	0	0	0
Line 3:	Interest paid $(-)$	0	$-r_0D^0$	$-r_1D^1$	$-r_2D^2$
Line 4:	Deferred nonfinancial costs	C^0	0	0	0
Line 5:	Deferred interest	0	r_0D^0	r_1D^1	r_2D^2
Line 6:	Allocated costs C_n^0 $(-)$	0	0	$-C_1^0$	$-C_2^0$
Line 7:	Period t income π^t	0	0	$R_1^0 - C_1^0$	$R_2^0 - C_2^0$
Line 8:	Period t debt D^t	$D^0 = C^0$	D^1	D^2	0
Line 9:	Period t net asset value A^t	$A^0 = -D^0$	A^1	A^2	A^3

Table. 4.1 Abbreviated Income and Balance Sheet Accounts for the R&D Firm

The *beginning of period t net realizable asset values* A^t, counting only beginning of the period debt (negatively) and cash flows (positive for revenues, negative for costs), turn out to equal the negative of the end of period debt levels:

$$\begin{aligned}
A^1 &\equiv -(1+r_0)D^0 = -(1+r_0)C^0;\\
A^2 &\equiv R_1^0 - (1+r_1)D^1 = R_1^0 - (1+r_0)(1+r_1)C^0;\\
A^3 &\equiv R_2^0 - (1+r_2)D^2 = R_2^0 + (1+r_2)R_1^0 - (1+r_0)(1+r_1)(1+r_2)C^0. \qquad (4.23)
\end{aligned}$$

Lines 1-3 and 8-9 in Table 4.1 correspond to real transactions whereas lines 4-6 represent the imputations that are necessary to get the sum of the first 6 lines to equal an appropriate *matched income* π^t for each period t, line 7, which matches costs with revenues in each period. Note that since there are no project revenues within periods 0 and 1, the corresponding matched incomes, π^0 and π^1, are set equal to zero by our imputation scheme.*17

Note that the *net present value of the matched incomes*, π^2 and π^3, *equals the discounted present value of the revenues generated by the R&D investment in period 0*, Π^0, defined by (4.18); i.e., we have:*18

$$\Pi^0 = \pi^2/(1+r_0)(1+r_1) + \pi^3/(1+r_0)(1+r_1)(1+r_2). \qquad (4.24)$$

We now work through the lines in Table 4.1 for period 0. The firm undertakes R&D expenditures prior to the beginning of period 0 that sum to the value C^0. These expenditures show up as a negative entry in line 2.*19 Since there are no period 0 revenues associated with these R&D expenditures, the expenditures C^0 are capitalized at the beginning of period 0 and these capitalized expenditures show up in line 4. These capitalized expenditures are formally identical to an investment, and thus are an imputed output of the firm for period 0. At the beginning of period 0, the firm borrows financial capital of the amount D^0 equal to C^0 (in order to finance the payments associated with the R&D expenditures) and this shows up in the firm's balance sheet at the beginning of period 0 as debt;*20 see line 8. There is no deferred or capitalized interest at the beginning of period 0 because the borrowing took place at the beginning of the period. Also, there are no allocated costs to set

*17 Thus π^t is the income at the beginning of period t or at the end of period $t-1$. There is no income during period 0 and none until the very end of period 1 so we only recognize positive income at the beginning of period 2 or at the end of period 1.

*18 We need to use the matched income entries in Table 4.1 and (4.18) and (4.20) to establish this result.

*19 This input value aggregate can be decomposed into a price and quantity component if prices for R&D employees, R&D intermediate input purchases and R&D capital service inputs are available; see de Haan and van Rooijen-Horsten (2004; 19)[43] for an outline of the methodology for setting up an R&D input price index.

*20 For the sake of simplicity, we have only a single interest rate to measure the opportunity cost of capital in each period; i.e., we have not dealt with the complications due to a mixture of debt and equity financing.

against revenue in period 0, because the R&D project has yet to generate any revenues, and so line 6 has a 0 entry for the beginning of period 0. The sum of the first 6 lines gives us the firm's income for period 0, π^0, and it turns out to be 0 (as desired using our matching principle).

Turning now to the beginning of period 1 and the entries in the beginning of period 1 column, there are still no project revenues at the beginning of period 1, so the entry in line 1 is 0. We assume that there are no additional nonfinancial costs that were incurred in period 0,*[21] so that the entry in line 2 is also 0. However, the firm has period 0 interest expenses equal to $r_0 D^0$, and these interest expenses appear as a negative entry in line 3. These interest costs are also associated with the project and since we still have no project revenues to offset these costs, our matching principle forces us to capitalize these interest expenses as well; see the offsetting entry in line 5 of the period 1 column. Since there were no project revenues in period 0, we still do not allocate any costs to this period, so that line 6 has a 0 entry for the beginning of period 1. The sum of the first 6 lines for the period 1 entries gives us the firm's income for period 0, π^0, and it also sums to 0. Line 8 gives us the total (net) debt of the firm at the beginning of each period, assuming that any revenues received during the previous period are used to reduce debt at the end of that period. The sequence of net beginning of period t debts, D^t, are defined by (4.22). The net debt at the end of period 2 or the beginning of period 3, D^3 is 0 because we assumed that the R&D project was intertemporally profitable. For periods $t = 0, 1, 2$, the negative of D^t is equal to A^t, which is the realizable net asset value of the firm; i.e., it is an asset value that recognizes all costs (with a negative sign) but it recognizes revenues only when they occur; i.e., when they are realized.

Turning now to the entries in the beginning of period 2 column, there are project revenues in period 1, so the entry in line 1 is R_1^0. Again, we assume that there are no additional nonfinancial costs in period 1, so that the entry in line 2 is 0. However, the firm has period 1 interest expenses equal to $r_1 D^1$, and these interest expenses appear as a negative entry in line 3. It turns out that our matching principle forces us to capitalize these interest expenses as well;*[22] see the offsetting entry in line 5 of the period 2 column. Since there are project revenues in period 2, we allocate the cost C_1^0 to this period, so that line 6 has the entry $-C_1^0$ for period 2. The sum of the first 6 lines for the period 2 entries gives us the firm's income for period 1, π^1, and it sums to end of period 1 revenues, R_1^0, less the allocated cost C_1^0. The entries in the beginning of period 3 column are analogous to the period 2 columns.

4.4 A Summary of the Information Needed to Implement the Capitalization Procedure

Examining the example in the previous section, it can be seen that R&D cost capitalization procedure (in nominal terms) has the following informational requirements:

- Information on current period 0 R&D costs in nominal terms, C^0 say.
- Information on the expected future stream of incremental revenues generated by the R&D investment in period 0 in nominal terms, say $R_0^0, R_1^0, R_2^0, \ldots$
- Information on the one period nominal interest rates r_1^0, r_2^0, r_3^0, that are expected to prevail in future periods $1, 2, 3, \ldots$

Given the above information, first find the parameter α by solving the following equation:

$$C^0 = \alpha[R_0^0/(1 + r_0^0) + R_1^0/(1 + r_0^0)(1 + r_1^0) + R_2^0/(1 + r_0^0)(1 + r_1^0)(1 + r_2^0) + \cdots]. \tag{4.25}$$

*[21] If there were additional R&D research expenditures, we would set up another table similar to Table 4.1, which would distribute these costs over future periods.

*[22] This capitalization of interest is required in order to ensure that the present value equation (4.24) will hold.

The *matched end of period n cost* C_n^0 that should be allocated to the end of period n for the R&D project that was finished at the beginning of period 0 can now be defined as follows, once we have determined α:

$$C_n^0 \equiv \alpha R_n^0; \qquad n = 0, 1, 2, \ldots . \tag{4.26}$$

This gives us enough information to fill out all of the real and imputed transactions that correspond to the transactions in the Table 4.1 example. Thus the entire cost matching procedure is driven by our assumptions about the future nominal anticipated incremental revenues, the R_n^0, and our assumptions about future nominal one period interest rates (or opportunity costs of capital), the r_n^0. In order to get constant dollar cost allocations q_n^0 as in section 4.2 above, we need some additional information on expected future rates of general inflation. Thus to the above information set, add:

- Information on expected future rates of CPI inflation for periods $1, 2, 3, \ldots$, say, $\rho_1^0, \rho_2^0, \rho_3^0, \ldots$.

Finally, if we want to construct measures of real R&D (flow) input, we need information on the prices and quantities of the inputs that are used to create the R&D asset. Thus, add to the above information set:

- Information on the prices and quantities of the flow inputs that comprise the period 0 cost, C^0; i.e., these are the P_m^t and Q_m^t that appeared in equation (4.1) above.

Thus there are 5 separate informational components that are required in order to do a complete accounting for R&D investments.

4.5 Discussion of Some Difficult Issues

Pitzer (2004)[183] and de Haan and van Rooijen-Horsten (2004)[43] raise a number of difficult issues that arise if we attempt to capitalize R&D expenditures in the National Accounts or in the accounts of any production unit. In this section, we look at some of these difficulties in the light of the algebra presented in the previous sections.

- How exactly do the various imputations outlined in section 4.2 fit into standard national accounting categories?

At least some new lines to the system of input output framework will have to be created to accommodate some of the imputations. The details need to be worked out.

- What is the "best" set of "standard" assumptions that we can make about the pattern of future expected revenues for market R&D?

There is some review of the empirical literature on R&D depreciation rates*23 in De Haan and van Rooijen-Horsten (2004; 20-24)[43] but it would seem that a more extensive discussion of these issues is required before the Canberra Group can make concrete recommendations to the National Accounts community. Although life is simplest if we assume geometric rates of revenue decay, there may well be more realistic "standard" models for the pattern of future incremental revenues that are quite different from our usual set of assumptions about depreciation for reproducible capital.

- What is the "best" deflator for converting current dollar values into constant dollar values?

De Haan and van Rooijen-Horsten (2004; 20)[43] mention that the Frascati Manual recommends the use of the GDP deflator for constant price comparisons, but Kohli (1982; 211)[141] (1983; 142)[142] and Diewert (2002; 556)[64] argue against this choice, since the GDP deflator has negative weights

*23 A rather critical review of this literature is contained in Diewert (2005b)[71].

for imports and this can cause the deflator to *decrease* if the price of imports *increases* enough.*24 Hill (1996; 94-97)[118] and Diewert (2002; 557)[64] discuss some alternative choices to the GDP deflator.

- What is the "best" set of assumptions to make about interest rates and future inflation rates?

Should we work with the assumption of a constant real interest rate as is convenient in studies of depreciation for reproducible capital? If so, how should we choose this real rate?

- Should Non Market R&D be capitalized?

Aspden (2003)[5] argues that all research potentially provides benefits to society that can accrue over long periods of time. Hence, he advocates capitalizing both private and public R&D. There is no doubt that publicly funded research that is made freely available provides benefits to society. However, there are no "straightforward" market *transactions* that can provide us with guidance on the future distribution of these benefits. The problem is that the freely given benefits may show up in the form of lower output prices, higher input prices or higher profits. To work out the exact nature of the improvements due to the freely available R&D would require some complicated general equilibrium modeling along with many assumptions. Moreover, the cost matching methodology explained above will not work in this context because there will be no easily identifiable revenues that the deferred costs can be matched to. These considerations suggest that it would be simpler to *not* capitalize publicly available R&D expenditures. This is the position taken by de Haan and van Rooijen-Horsten (2004; 18)[43] and provisionally by Pitzer (2004; 9)[183]. However, current National Accounts conventions simply put the non market R&D expenditures into government consumption (i.e., an artificial output is created out of these input expenditures and added to GDP). This is essentially the same treatment that is done for other difficult to measure general government "outputs" but I am not sure that it is completely satisfactory. However, given that these non market R&D expenditures have been shunted over to the government sector, there is nothing to prevent us from inventing a method that would essentially spread this government "output" over future periods.*25 The exact details of how this should be done need to be worked out.

- Should unsuccessful R&D ventures be capitalized?

De Haan and van Rooijen-Horsten (2004; 24)[43] discuss this issue. They note that some experts argue that unsuccessful ventures should be immediately expensed or "written off", while other experts argue that all R&D activities, whether successful or not, contribute to acquiring a commercially valuable knowledge stock.*26 Both points of view are justifiable but it seems to me that the first point of view is more in line with market realities. This issue requires further discussion.

- Is the proposed method of R&D capitalization consistent with the national accounts treatment of other intangible assets, such as mineral exploration, advertising and franchising?

*24 Diewert (2002; 556)[64] gave a recent US example of perverse behavior of the GDP deflator, where the chain type price indexes for C, I, X and M for the third quarter of 2001 decreased over the previous quarter (at annual rates) by 0.4%, 0.2%, 1.4% and 17.4% respectively, but yet the overall US GDP deflator increased by 2.1%.

*25 This possibility was noted by Pitzer (2004; 9)[183]: "If unpatented entities were to be treated as assets, then an accounting treatment needs to be created for them. At first glance, it would appear that unpatented entities should be given the same accounting treatment as patented entities because both affect future income in the same way. The lack of assessments for unpatented entities prevents use of any of the treatments discussed in the previous section, which means that either different treatments should be accorded patented and unpatented entities or a new methodology applicable to both should be developed."

*26 De Haan and van Rooijen-Horsten(2004; 24)[43] also note the analogy of unsuccessful R&D ventures to unsuccessful oil wells: "For mineral exploration, the SNA 1993 recommends that all mineral exploration should be treated as gross fixed capital formation (#166) since both successful and unsuccessful exploration efforts are needed to acquire new reserves. In a similar way, one may conclude that the value of the knowledge capital stock should include both the costs of successful and unsuccessful R&D."

This is an issue for national accounting experts to discuss. However, in my opinion, all (market sector) current period expenditures on any of the above intangible assets have the same character as (market sector) current expenditures on R&D: expenditures are made now in the hope of "creating" future period incremental revenues. Hence, essentially the same matching of costs to future revenues methodology used above could be applied to these activities and overall consistency could be achieved.[*27]

- How should taxes and subsidies and subsidies be treated?

Again, de Haan and van Rooijen-Horsten (2004; 12)[43] discuss this issue. They point out that in the Netherlands, subsidies for R&D are quite substantial. In Canada, there is a favorable business income tax treatment for R&D investments of an approved type. It is not completely clear how to deal with these tax and subsidy complications.

4.6 References

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[*27] The same matching methodology will work for transfer costs (i.e., transactions costs of whatever form) as well.

Chapter 5

Constructing a Capital Stock for Inventories and the Measurement of Inventory Change

5.1 Introduction

The current System of National Accounts (SNA) treatment of inventory change in real terms is very confusing to users. The problem is that it can happen that the value of inventory change has a sign that is *opposite* to the sign of the corresponding constant dollar inventory change. This means that the corresponding implicit price deflator is meaningless. In this paper, the nature of the problem is explained and a solution to the problem is suggested. In the Appendix, a theoretical framework that provides a unified treatment for measuring inventory change and the user cost of inventories is explained.[*1] Appendix 2 gives some background information on the origins of the theoretical framework used in Appendix 1.

In section 5.2, a simple 2 good, 4 period numerical example is introduced and it is explained how a "typical" SNA treatment of inventory change works in the context of this example. The example illustrates the problem described in the previous paragraph: the current dollar aggregate inventory change has a sign opposite to the corresponding constant dollar change.

In section 5.3, the same example is reworked using the methodological approach suggested in Appendix 1. The suggested solution involves treating inventory change in a manner that is symmetric to the current SNA treatment of exports and imports.

Section 5.4 concludes.

[*1] This methodology is based on Diewert and Smith (1994)[77] and Diewert (2004; 36)[66]. The initial accounting methodology can also be found in Diewert (2005; 21-23)[70]. Diewert and Lawrence (2005)[74] used this framework as well. The underlying model of production that is used in this chapter is the Hicks (1961)[115] and Edwards and Bell (1961)[81] model explained in section 1.9.2 of chapter 1.

5.2 The SNA Treatment of Inventory Change

Consider the following data on the end of period stocks of two inventory items for three periods, where p_n^t and q_n^t denote the price and quantity of stock n at the end of period t:

Table. 5.1 Price and Quantity Data for Two Inventory Stocks

	p_1^t	p_2^t	q_1^t	q_2^t
Period 0	1.0	1.0	200	200
Period 1	.9	2.0	260	150
Period 2	.8	3.0	310	110
Period 3	.7	4.0	330	100

Thus the price of the first stock p_1^t is slowly declining while the corresponding end of period stock q_1^t grows from 200 to 330 over the three periods. On the other hand, the price of the second stock p_2^t quadruples over the three periods while the corresponding end of period stock q_2^t steadily falls from 200 to 100 over the three periods. These price changes are more violent than what is usually observed over the course of a year but they would not necessarily be unusual if we think of the first good as computer chip and the second good as crude oil.

The *end of period t SNA constant dollar stock of inventories*, K_{SNA}^t, using the end of period 0 as the base period, can be defined as the following Laspeyres type quantity aggregate:

$$K_{SNA}^t \equiv p_1^0 q_1^t + p_2^0 q_2^t; \qquad t = 0, 1, 2, 3. \tag{5.1}$$

Note that we use the inventory stocks q_n^t at the end of period t along with the prices of the stocks at the end of period 0, p_n^0, in the above definition of the period t constant dollar stock of inventories. Thus for period 0 (the beginning of period 1), the constant dollar stock coincides with the current dollar stock. The *value of the current dollar stock of inventories at the end of period t*, VK^t, is defined in the usual fashion as follows:

$$VK^t \equiv p_1^t q_1^t + p_2^t q_2^t; \qquad t = 0, 1, 2, 3. \tag{5.2}$$

If we divide VK^t by K_{SNA}^t, we obtain P_{SNA}^t, the *end of period t SNA implicit price index for the constant dollar stock of inventories*:

$$P_{SNA}^t \equiv VK^t / K_{SNA}^t = [p_1^t q_1^t + p_2^t q_2^t]/[p_1^0 q_1^t + p_2^0 q_2^t]; \qquad t = 0, 1, 2, 3. \tag{5.3}$$

Note that the SNA implicit price index for the inventory stock is a Paasche price index between period t and 0.

The *SNA constant dollar value of inventory change for period t*, ΔK_{SNA}^t, can be defined in a straightforward manner as the difference between the end of period t and beginning of period t constant dollar stocks defined above by (5.1):

$$\begin{aligned} \Delta K_{SNA}^t &\equiv K_{SNA}^t - K_{SNA}^{t-1} \qquad t = 1, 2, 3 \\ &= p_1^0 q_1^t + p_2^0 q_2^t - [p_1^0 q_1^{t-1} + p_2^0 q_2^{t-1}] \qquad \text{using (5.1)} \\ &= p_1^0 [q_1^t - q_1^{t-1}] + p_2^0 [q_2^t - q_2^{t-1}] \\ &= p_1^0 \Delta q_1^t + p_2^0 \Delta q_2^t \end{aligned} \tag{5.4}$$

where $\Delta q_n^t \equiv q_n^t - q_n^{t-1}$ is the difference in the closing and opening stock of inventory item n over period t. Note that the last equation in (5.4) shows that the aggregate change in the constant dollar change in inventories is equal to the sum of the individual item changes, using the end of period 0 prices as weights.

The (approximate) *SNA current dollar value of inventory change for period t*, ΔVK_{SNA}^t, can be defined as the sum of the individual item changes, Δq_n^t, weighted by the average of the beginning and end of period prices, $(1/2)p_n^{t-1} + (1/2)p_n^t$:[*2]

$$\Delta VK_{SNA}^t \equiv [(1/2)p_1^{t-1} + (1/2)p_1^t]\Delta q_1^t + [(1/2)p_2^{t-1} + (1/2)p_2^t]\Delta q_2^t; \qquad t = 1, 2, 3. \tag{5.5}$$

The corresponding *implicit price index for the SNA inventory change*, ΔP_{SNA}^t, is obtained by dividing the value series ΔVK_{SNA}^t defined by (5.5) by the constant dollar series ΔK_{SNA}^t defined by (5.4):

$$\begin{aligned} \Delta P_{SNA}^t &\equiv \Delta VK_{SNA}^t / \Delta K_{SNA}^t \qquad t = 1, 2, 3 \\ &= (1/2)\{[p_1^{t-1} + p_1^t]\Delta q_1^t + [p_2^{t-1} + p_2^t]\Delta q_2^t\} / \{p_1^0 \Delta q_1^t + p_2^0 \Delta q_2^t\}. \end{aligned} \tag{5.6}$$

The above definition for the change in stocks price index, ΔP_{SNA}^t, looks a bit strange at first sight but *if* the weights Δq_1^t and Δq_2^t are positive, it can be seen that it is a perfectly reasonable price index that compares an average of the beginning and end of period t prices with the base prices (which are the end of period 0 prices for the inventory components).[*3]

The above definitions are used to construct the value, price and quantity of end of period inventory stocks (VK^t, P_{SNA}^t and K_{SNA}^t respectively) for periods $0, 1, 2$ and 3 and the value, price and quantity of the change in inventory stocks ($\Delta VK_{SNA}^t, \Delta P_{SNA}^t$ and ΔK_{SNA}^t respectively) for periods 1-3 using the data in Table 5.1. The results are listed in Table 5.2 below.

Table. 5.2 Values, Prices and Quantities for Aggregate Inventories at Period 0 Prices

	VK^t	P_{SNA}^t	K_{SNA}^t	ΔVK_{SNA}^t	ΔP_{SNA}^t	ΔK_{SNA}^t
Period 0	400	1.000	400	–	–	–
Period 1	534	1.302	410	−18.0	−1.800	10
Period 2	578	1.376	420	−57.5	−5.750	10
Period 3	631	1.467	430	−20.0	−2.000	10

At first glance, the values, prices and quantities for the aggregate inventory stock look reasonable, with the values growing fairly quickly due to rapid increases in the price of the second inventory good but the real stocks growing at the much slower rate of 10 units per year. Turning to the values, prices and quantities for the changes in the aggregate inventory stock, we see that the quantity growth, ΔK_{SNA}^t, is equal to 10 in periods $1, 2$ and 3, which is the difference in the corresponding beginning and end of period stocks, K_{SNA}^t. This is very satisfactory. However, when we calculate the change in the value of inventories at current prices, ΔVK_{SNA}^t, a different picture emerges. Because the price of inventory item 2 increases much more rapidly than the price of item 1 declines, the steady

[*2] This is not quite the theoretically correct measure of inventory change that is suggested in the *System of National Accounts 1993* on pages 130-131 but is regarded as an approximation that is frequently used as the following quotation indicates: "This suggests that even when prices are changing a good approximation to the PIM may be obtained by taking the difference between the quantity of goods held in inventory at the beginning and at the end of the accounting period and valuing the difference at the average prices prevailing within the period. This measure, which may be described as the "quantity" measure, is widely used in practice and is sometimes mistakenly considered to be the theoretically appropriate measure under all circumstances." *SNA 1993*, page 131. For a more complete discussion of the SNA theoretically correct measure of inventory change, see Hill (2005)[124]. However, Hill (2005)[124] notes that the theoretically correct method suffers from the same problems that arise using the approximate method.

[*3] However, note that when we set t equal to zero, in general, ΔP_{SNA}^0 will not equal unity; i.e., the index does not satisfy the identity test.

decline in the quantity of item 2 when valued at current prices outweighs the steady increase in the quantity of item 1 at current prices so that overall, the change in the value of inventories at current prices turns out to be strongly negative (-18 over the course of period 1, -57.5 over the course of period 2 and -20 over the course of period 3). Thus the corresponding implicit price index for inventory change, ΔP^t_{SNA}, turns out to be *negative* in all three periods (since the value change is negative and the corresponding constant dollar quantity change is positive). Looking at definition (5.6) above, it can be seen that the root of the problem is that the quantity weights in the price index number formula, Δq^t_1 and Δq^t_2, are of opposite signs and the value aggregates in the numerator and denominator of (5.6) are of opposite signs and fairly small. Index number theory breaks down under these circumstances and can frequently give rise to meaningless numbers as is the case in the present situation.[*4]

The existence of negative implicit prices for an output component of the national accounts may not create any great conceptual problems for the compilers of the accounts (since the negative implicit prices are just a consequence of definitions that seem reasonable to accountants) but they do create problems for many macroeconomic modelers who base their models on microeconomic theory: negative prices create great difficulties for this class of user. Hence, in the following section, a different theoretical framework (based on microeconomic theory) is suggested that will avoid the negative implicit price problem.[*5]

In addition to the negative implicit price problem, there is another problem with the above approximate SNA methodology: namely, it relies on a *fixed base Laspeyres type methodology.* Note that the inventory stock aggregate is a fixed base Laspeyres quantity index which uses the prices of period 0 as the weights for the individual stock components. Definitions (5.7)-(5.10) below redo definitions (5.1)-(5.6) above but instead of using the prices at the end of period 0 as the base prices, the prices at the end of period 3 are used as the base prices. Definitions (5.2) and (5.5) remain unchanged since they are values but the counterparts to (5.1), (5.3), (5.4) and (5.6) are listed below (the new stock and flow aggregates are denoted by $K^t_{SNA}(3), P^t_{SNA}(3), \Delta K^t_{SNA}(3)$ and $\Delta P^t_{SNA}(3)$):

$$K^t_{SNA}(3) \equiv p^3_1 q^t_1 + p^3_2 q^t_2; \qquad t = 0,1,2,3; \tag{5.7}$$

$$P^t_{SNA}(3) \equiv VK^t/K^t_{SNA}(3) = [p^t_1 q^t_1 + p^t_2 q^t_2]/[p^3_1 q^t_1 + p^3_2 q^t_2]; \qquad t = 0,1,2,3. \tag{5.8}$$

$$\begin{aligned} \Delta K^t_{SNA}(3) &\equiv K^t_{SNA}(3) - K^{t-1}_{SNA}(3) \qquad t = 1,2,3 \\ &= p^3_1 q^t_1 + p^3_2 q^t_2 - [p^3_1 q^{t-1}_1 + p^3_2 q^{t-1}_2] \qquad \text{using (5.7)} \\ &= p^3_1[q^t_1 - q^{t-1}_1] + p^3_2[q^t_2 - q^{t-1}_2] \\ &= p^3_1 \Delta q^t_1 + p^3_2 \Delta q^t_2; \end{aligned} \tag{5.9}$$

$$\begin{aligned} \Delta P^t_{SNA}(3) &\equiv \Delta VK^t_{SNA}/\Delta K^t_{SNA}(3) \qquad t = 1,2,3 \\ &= (1/2)\{[p^{t-1}_1 + p^t_1]\Delta q^t_1 + [p^{t-1}_2 + p^t_2]\Delta q^t_2\}/\{p^3_1 \Delta q^t_1 + p^3_2 \Delta q^t_2\}. \end{aligned} \tag{5.10}$$

The above definitions are used to construct the price and quantity of end of period inventory stocks ($P^t_{SNA}(3)$ and $K^t_{SNA}(3)$ respectively) for periods $0,1,2$ and 3 and the price and quantity of the change in inventory stocks ($\Delta P^t_{SNA}(3)$ and $\Delta K^t_{SNA}(3)$ respectively) for periods $1,2$ and 3 using the data in Table 5.1. The results are listed in Table 5.3 below.

[*4] Hill (1971)[123] noted this problem with traditional index number theory many years ago.

[*5] However, there is a cost to the suggested solution: the single SNA output category, "change in inventories", is replaced by the difference between two output categories: the "end of period stock of inventories" less the "beginning of the period stock of inventories".

Table. 5.3 Values, Prices and Quantities for Aggregate Inventories at Period 3 Prices

	VK^t	$P^t_{SNA}(3)$	$K^t_{SNA}(3)$	ΔVK^t_{SNA}	$\Delta P^t_{SNA}(3)$	$\Delta K^t_{SNA}(3)$
Period 0	400	0.4255	940	–	–	–
Period 1	534	0.6829	782	−18.0	.1139	−158
Period 2	578	0.8798	657	−57.5	.4600	−125
Period 3	631	1.0000	631	−20.0	.7692	−26

From Table 5.2, it was seen that the constant dollar stock of inventories (using the end of period 0 prices as the weights) *grew* from 400 to 430 from the end of period 0 to the end of period 3 whereas from Table 5.3, it appears that the constant dollar stock of inventories *fell* from 940 to 631 over the three periods. Turning to the changes in the constant dollar stocks, Table 5.2 told us that the change in stocks at constant prices was *positive* (equal to 10 in each period) while Table 5.3 tells us that the change in stocks was *strongly negative* in each period (−158, −125 and −26). The corresponding implicit prices are all *negative* in Table 5.2 while they are all *positive* in Table 5.3. The lack of harmony in the two sets of results is due to the large change in relative prices (and the smaller but still significant change in relative quantities) over the three periods and the fact that a quantity index is being used that uses the price weights of only *one* of the two periods being compared. Chapter 16 in the *SNA 1993* recommends the use of symmetrically weighted index number formulae rather than the asymmetrically weighted Laspeyres formula. Thus in the next section, the Fisher (1922)[92] price and quantity index (which is a symmetrically weighted formula) will be used in order to construct inventory stock aggregates. Since the price and quantity data move relatively smoothly over time, chained Fisher indexes will be used rather than fixed base Fisher indexes.[*6]

5.3 A Suggested Alternative Treatment of Inventory Change

Following the advice given in Chapter 16 of *SNA93*, the Fisher price and quantity indexes, P^t_F and K^t_F, are adopted as the measure of the aggregate price and quantity (or volume) for the end of period stocks of inventories. Since the price and quantity data have relatively smooth trends over time, chained Fisher indexes were used.[*7] The value of the end of period aggregate inventory stock, VK^t, is listed in Table 5.4 below along with P^t_F and K^t_F.[*8]

Table. 5.4 Values, Prices and Quantities for Aggregate Inventories using Chained Fisher Indexes

	VK^t	P^t_F	K^t_F	V^t_A	P^t_A	Q^t_A	V^t_I	Error^t
Period 0	400	1.000	400.0	–	1.000	–	–	–
Period 1	534	1.374	388.6	−15.7	1.374	−11.4	−46.0	−30.3
Period 2	578	1.642	352.1	−60.0	1.642	−36.5	−80.0	−20.0
Period 3	631	1.851	340.8	−20.8	1.851	−11.2	−26.0	−5.2

Comparing P^t_F and K^t_F in Table 5.4 with P^t_{SNA} and K^t_{SNA} in Table 5.2, it can be seen that the Fisher price index grows more rapidly (from 1 to 1.851) than the SNA price index (from 1 to 1.467) and the corresponding Fisher volume index for the inventory stock grows more slowly. The SNA volume index uses the prices of period 0 as weights and the decreases in q_2 are just outweighed by

[*6] This is consistent with the advice given in Chapter 16 of the *SNA 1993*, pages 388-389, where fixed base symmetrically weighted indexes are recommended if the data fluctuate or bounce and chained indexes are recommended if the data have trends.

[*7] The Bureau of Economic Analysis uses chained Fisher indexes to calculate inventory stocks; see Parker and Seskin (1996)[175] and Ehemann (2005)[82].

[*8] The entries in the final 4 columns of Table 5.4 will be explained later.

the increases in q_1. However, when the Fisher chained volume index is used, the decreases in q_2 get a higher weight (due to the rapidly increasing price of q_2) than the weight accorded to the increases in q_1, leading to a decrease in the Fisher volume index compared to the increase in the fixed base SNA type index. Note that the differences are not insignificant.

The problem of determining the price and quantity of the change in the aggregate inventory stocks is now addressed. The methodology described in Appendix 1 below is used, which provides a consistent theoretical framework based on economic theory for not only the price and quantity of the change in inventories but also for the user cost of aggregate inventory stocks held at the beginning of each period. According to the model developed in Appendix 1, the *theoretically correct period t value aggregate for the value of inventory change*,*9 V_I^t, is given by (A10), which is rewritten using the notation in Table 5.1 as follows:

$$\begin{aligned} V_I^t &\equiv \sum\nolimits_{j=1}^{2} P_{Kj}^t \Delta K_j^t \\ &= \sum\nolimits_{j=1}^{2} P_{Kj}^t [K_j^t - K_j^{t-1}] \qquad t = 1, 2, 3 \\ &= \sum\nolimits_{j=1}^{2} p_j^t q_j^t - \sum\nolimits_{j=1}^{2} p_j^t q_j^{t-1} \\ &= V_E^t - V_B^t \end{aligned} \tag{5.11}$$

where

$$V_E^t \equiv \sum\nolimits_{j=1}^{2} p_j^t q_j^t \tag{5.12}$$

is the *end of period t inventory stock value aggregate* and

$$V_B^t \equiv \sum\nolimits_{j=1}^{2} p_j^t q_j^{t-1} \tag{5.13}$$

is a (hypothetical) *beginning of period t inventory stock value aggregate* where the beginning of the period stocks are valued at the end of period prices. From the second line in (5.11) above, it would appear that the theoretical inventory change value aggregate for period t, V_I^t, has a straightforward decomposition into prices (the end of period t prices for the stocks p_j^t) times the quantity changes over period t, $q_j^t - q_j^{t-1}$. However, because the quantities in this value aggregate are really quantity differences and hence can be of either sign, index number theory may fail if the prices in the index number formula are taken to be the p_j^t and the quantities are taken to be the $q_j^t - q_j^{t-1}$. The suggested solution to this problem is to regard the inventory change value aggregate as the difference between the end of period value aggregate V_E^t and the hypothetical beginning of period value aggregate V_B^t and then use normal index number theory to decompose V_E^t into the product of the price and quantity components P_E^t and Q_E^t respectively and to decompose V_B^t into the product of the price and quantity components P_B^t and Q_B^t respectively. *In other words, it is suggested that the change in inventories value aggregate be treated in a manner that is symmetric to the treatment of the current trade balance as the difference between the value of exports less the value of imports.**10 This trade balance aggregate has exactly the same type of problem as the inventory change aggregate: it could be positive in one period and negative in the following period. Index number theory cannot decompose this type of difference value aggregate into meaningful price and volume components, unless it is guaranteed that the value differences will remain well away from zero.

*9 This is the theoretically correct value aggregate if it is desired to have a formula for the user cost for inventories that is completely symmetric to the user cost for reproducible capital. If this symmetry property is not regarded as important, then we need only use the first 3 columns in Table 5.4 in order to decompose the beginning and end of period values for the stock of inventories into Fisher ideal price and quantity components. If this second approach is taken, then it is not necessary to perform the computations in the remainder of this section.

*10 This methodological approach was suggested in Diewert (2004; 36)[66].

Before we illustrate our suggested treatment of inventory change using the data in Table 5.1, we will first use these data to illustrate a simple approach that is problematic. *An approximate approach to the treatment of inventory change* can be implemented as follows. First construct the chained Fisher price and quantity indexes for the end of period t inventory stocks, P_F^t and K_F^t respectively. Now define the *period t approximate price for inventory change*, P_A^t, to be the end of period t Fisher stock price for inventory components P_F^t and define the period t *approximate quantity or volume of inventory change* Q_A^t to be the difference between the beginning and end of period t Fisher quantity indexes for the inventory stocks; i.e., we have the following definitions:*11

$$P_A^t \equiv P_F^t; \qquad t = 1, 2, 3; \tag{5.14}$$

$$Q_A^t \equiv [K_F^t - K_F^{t-1}]; \qquad t = 1, 2, 3. \tag{5.15}$$

The corresponding *approximate value of inventory change in period t* is V_A^t defined as the product of the approximate price and quantity defined above:

$$V_A^t \equiv P_A^t Q_A^t; \qquad t = 1, 2, 3. \tag{5.16}$$

We note that definitions (5.14)-(5.16) collapse down to the theoretical model presented in Appendix 1, *provided that there is only one inventory item in the aggregate.* Moreover, the use of these definitions makes the aggregate inventory stocks perfectly consistent with the aggregate value of inventory change; i.e., the stock and flow aggregates are perfectly consistent. V_A^t, P_A^t and Q_A^t are listed in Table 5.4 above for periods $1, 2$ and 3.

However, even though definitions (5.14)-(5.16) are perfectly consistent with the theoretical approach explained in Appendix 1 when there is only one inventory item in the aggregate, this correspondence does not hold in general when there are two or more inventory items in the aggregate.*12 When there are two or more inventory items in the aggregate, it is not necessarily the case that the value of the approximate change in the value of inventories V_A^t is equal to the theoretically correct value of inventory change, V_I^t, and so there will generally be an *aggregation error* between these two value aggregates defined for period t as follows:

$$\text{Error}^t \equiv V_I^t - V_A^t; \qquad t = 1, 2, 3. \tag{5.17}$$

The "true" values of the inventory change aggregate, V_I^t, and the aggregation error between this value and the approximate value V_A^t are listed in the last two columns of Table 5.4. It can be seen that the aggregation errors are too large to be ignored in this case. Hence, we conclude that while under some circumstances, the approximate method for calculating the price and quantity for inventory change *can* be satisfactory, in many cases it will *not* be satisfactory.

*11 This approximate method for the treatment of inventory change is very close to the method presently in use by the BEA to calculate real estimates of inventory change. The BEA method uses the average of the beginning and end of period Fisher stock prices, $(1/2)P_F^{t-1} + (1/2)P_F^t$, in place of P_F^t on the right hand sides of (5.14) and (5.15); see Parker and Seskin (1996)[175] and Ehemann (2005)[82]. However, if there is only one inventory item, then the use of our (5.14) and (5.15) will give the "right" answer if we use the user cost framework developed by Diewert and Smith (1994)[77] whereas the BEA procedure will not. Ehemann (2005)[82] developed a variant of the BEA procedure by constructing Fisher indexes of acquisitions and disposals and taking their difference, say $B_F^t - S_F^t$ using the notation in the Appendix, in place of the difference in Fisher stocks, $K_F^t - K_F^{t-1}$. In the case of only one inventory item, the Ehemann method will coincide with the BEA method, provided that U^t and G^t in the Appendix equations (A3) and (A4) are zero in the two periods being compared. In the many inventory item case, even if U^t and G^t are zero, the BEA and Ehemann methods will differ due to the different weights in the Fisher indexes K_F^t, B_F^t and S_F^t.

*12 If either end of period *prices* of inventory items vary in strict proportion over time or the *quantities* in the end of period inventory stocks vary in strict proportion over time, then the approximate approach will be perfectly consistent with the theoretical approach explained in Appendix 1. This is because the Fisher formula is consistent with both Hicks' (1946; 312-313)[114] and Leontief's (1936; 54-57)[146] Aggregation Theorems; see Allen and Diewert (1981)[3].

Recall the end of period value of inventories aggregate, V_E^t defined by (5.12) and the hypothetical beginning of period value of inventories aggregate V_B^t defined by (5.13). Using chained Fisher indexes for periods 1 to 3, the resulting price and quantity decompositions using the data in Table 5.1 are listed in Table 5.5.

Table. 5.5 Values, Prices and Quantities for Beginning and End of Period Inventory Aggregates using Chained Fisher Indexes

	V_E^t	P_E^t	Q_E^t	V_B^t	P_B^t	Q_B^t
Period 1	534	1.0000	534.0	580	1.0000	580.0
Period 2	578	1.1947	483.8	658	1.2707	517.8
Period 3	631	1.3473	468.4	657	1.4769	444.9

Looking at Table 5.5, it can be seen that the volume of end of period inventories is decreasing more slowly (from 534.0 to 468.4) than the volume of beginning of period inventories (from 580.0 to 444.9). Hence the difference between the two volume aggregates is increasing. It can also be seen that the price of end of period inventories increases more slowly (from 1 to 1.3473) than the corresponding price of beginning of period inventories (from 1 to 1.4769). The relatively large discrepancy in these two rates of price increase explains why the approximate method for dealing with inventory change does not work well for this example.*13 Since the beginning of period inventory stock gets a negative weight when an inventory change aggregate is formed and it has a higher inflation rate than the end of period stock, it can be expected that the price of this net output aggregate will decrease.*14 Although this result is counterintuitive from the perspective of measuring general inflation, it is sensible from the perspective of production theory: the increase in the beginning of period price of inventories acts like an increase in the price of an intermediate input and so the net return to the producer of producing a unit of gross output less a unit of the intermediate has decreased; i.e., the price of net output has decreased.

To indicate how further stages of aggregation might proceed, an *investment aggregate* is introduced, which has price p_5^t and quantity q_5^t in period t. It is assumed that the price and quantity of this investment aggregate is constant during periods 1 to 3 and in particular, it is assumed that:

$$p_5^t = 1;\ q_5^t = 1000; \qquad t = 1, 2, 3. \tag{5.18}$$

The task now is to construct chained Fisher aggregate prices and quantities for each year, P^t and Q^t (with corresponding value $V^t \equiv P^tQ^t$), that aggregate over end of period stocks, q_1^t and q_2^t (with corresponding prices p_1^t and p_2^t), beginning of year hypothetical stocks indexed with negative signs, $q_3^t \equiv -q_1^{t-1}$ and $q_4^t \equiv -q_2^{t-1}$ (with corresponding prices $p_3^t \equiv p_1^t$ and $p_4^t \equiv p_2^t$) and other investment flows, q_5^t (with corresponding price p_5^t). Thus there are 5 commodities in all that are

*13 If the two rates of price increase were equal, then the aggregation errors associated with the approximate method would be zero.

*14 The problem is similar to an analogous problem that occurs when the price of imports increases faster than the price of exports and other output components of GDP. In this case, the *increase* in the price of imports will *reduce* the GDP deflator. Kohli (1982; 211)[141] (1983)[142] (2004; 91)[143] noticed this problem with the GDP deflator many years ago: "Actually, it can easily be seen that any terms of trade change away from the base period price ratio results in a fall in real national product. This clearly reveals the weakness of this measure of real value added, the drawbacks of direct index numbers, and the dangers of aggregating positive with negative quantities." Ulrich Kohli (1983; 142)[142]. An example of this anomalous behavior of the GDP deflator just occurred in the advance release of gross domestic product for the third quarter of 2001 for the US national income and product accounts: the chain type price indexes for C, I, X and M *decreased* (at annual rates) over the previous quarter by 0.4%, 0.2%, 1.4% and 17.4% respectively but yet the overall GDP deflator *increased* by 2.1%. Thus there was general deflation in all sectors of the economy but yet the overall GDP deflator increased. See Table 4 in the Bureau of Economic Analysis (2001)[20].

being aggregated. The results for this *investment plus change in inventories Fisher aggregate*, V^t, P^t and Q^t, are listed in the first three columns of Table 5.6.

Table. 5.6 Values, Prices and Quantities for Aggregate Investment plus Inventory Change using Chained Fisher Indexes

	V^t	P^t	Q^t	P^t_{2S}	Q^t_{2S}	P^t_{AA}	Q^t_{AA}	V^t_{AA}
Period 1	954	1.0000	954.0	1.0000	954.0	1.0000	984.3	984.3
Period 2	920	0.9473	971.2	0.9484	970.1	0.9933	946.4	940.0
Period 3	974	0.9182	1060.8	0.9217	1056.7	0.9881	991.0	979.2

As expected, the price of the investment plus inventory change aggregate, P^t, decreases over time (in a sensible manner) and the corresponding quantity or volume, Q^t, steadily increases. The columns in Table 5.5 that decompose the two inventory aggregates plus the first 3 columns in Table 5.6 are the core of the new suggested presentation of aggregate inventory change. *The key is to decompose the inventory change into two aggregates, show the price and quantity detail for those two aggregates and then move to the next stage of aggregation where the two inventory aggregates are aggregated with other flow aggregates.* All of the columns in Table 5.5 and the first 3 columns in Table 5.6 show sensible prices, quantities and values, which is not the case with the existing SNA method for dealing with inventory change.

In the first 3 columns of Table 5.6, we constructed the aggregate P^t and Q^t by using the Fisher chained formula over the 5 most finely disaggregated prices and quantities in the model.*15 It is also possible to construct this aggregate price and quantity in two stages. In the first stage, the end of period Fisher chained inventory aggregate price and quantities, P^t_E and Q^t_E, and the beginning of period hypothetical inventory aggregate price and quantities, P^t_B and Q^t_B are constructed: see the entries in Table 5.5. In the second stage of aggregation, chained Fisher indexes are calculated using P^t_E, P^t_B and p^t_5 as the period t prices and $Q^t_E, -Q^t_B$ and q^t_5 as the corresponding period t quantities. The results of this two stage aggregation procedure are listed in Table 5.6 under the columns with the headings P^t_{2S} and Q^t_{2S} (the corresponding two stage value aggregate equals V^t and so it is not listed). It can be seen that these two stage estimates are reasonably close to their one stage counterparts, P^t and Q^t.*16

Finally, the approximate price and quantity for inventory change listed in Table 5.4, P^t_A and Q^t_A, can be used, along with p^t_5 and q^t_5, in order to construct approximate investment and inventory change aggregate price, quantity and value for period t, P^t_{AA}, Q^t_{AA} and V^t_{AA} respectively, using the Fisher chain formula. The results are listed in the last 3 columns of Table 5.6. It can be seen that for this particular numerical example, the approximate method is not acceptable.*17 The errors in values, prices and quantities are large compared to the theoretically preferred measures, V^t, P^t and Q^t.

5.4 Conclusion

The SNA method for treating changes in inventories suffers from two major problems:

*15 Diewert and Lawrence (2005)[74] used this strategy to construct investment plus inventory change aggregates in their empirical work for Australia.

*16 This is in accordance with the experience of the U.S. Bureau of Economic Analysis in constructing two stage chained Fisher aggregates. Diewert (1978; 888)[48] derived a theoretical result that showed that normally, the single stage and two stage estimates should approximate each other fairly closely.

*17 Lasky (1998; 106)[145] and Ehemann (2005)[82] essentially used this methodology to evaluate the adequacy of the BEA method for estimating inventory change, except that they used all components of GDP ($C+G+I+X-M$) in place of our use of just I as the outside commodity in the next stage of aggregation. Both Lasky and Ehemann found relatively large aggregation errors in using the BEA approximate method for estimating inventory change. Thus the problem that we are describing is not just a hypothetical one.

- Aggregate real inventory stocks and changes in stocks are evaluated at constant base period prices which leads to difficulties if the relative prices of inventory components are changing over time (and this method is not consistent with the use of symmetrically weighted or superlative indexes which is recommended in *SNA93*);
- The SNA implicit prices for inventory change can be negative and are extremely difficult for users to interpret.

Since the Canberra II Group has recommended that user costs for reproducible capital stocks be added to the SNA production accounts as a recommended decomposition of gross operating surplus and since the Group also recommended that inventory stocks be included as assets that should have user costs in these optional accounts, it is necessary to carefully specify the links between the user cost of inventories and the treatment of the change in inventories in the production accounts. The Appendix to this paper presents a coherent theoretical framework for the treatment of inventory change and for the construction of user costs for inventory items.

In addition to suggesting a consistent accounting framework for the user cost of inventories and the treatment of inventory change, the other main methodological suggestion in this chapter is to treat inventory in a manner that is symmetric to the treatment of the current trade balance as the difference between the value of exports less the value of imports. Although this suggested treatment of inventory leads to sensible price and volume estimates, it has the downside of being somewhat different than the current SNA treatment of inventory change, which is well established. Hence users may find our suggested solution to the problems associated with the current SNA treatment of inventory change to be a bit strange at first.*18 However, if it is explained to users that the suggested treatment of inventory change is entirely analogous to the current SNA treatment of international trade, the suggested new treatment will eventually be regarded as being quite acceptable.

5.5 Appendix: A Theoretical Treatment of Inventory Change

A theoretical framework is needed to measure the contribution of the change inventory stock over a period to production. It is also necessary to work out the user cost of the beginning of the period stock of inventories. A framework to answer these questions is outlined, taken from Diewert and Smith (1994)[77].*19

First consider the theory for a single inventory stock item. Consider a firm that perhaps produces a noninventory output during period t, Y^t, uses a noninventory input X^t, sells the amount S^t of an inventory item during period t and makes purchases of the inventory item during period t in the amount B^t. Suppose that the average prices during period t of Y^t, X^t, S^t and B^t are P_Y^t, P_X^t, P_S^t and P_B^t respectively. Then neglecting balance sheet items, the firm's period t *cash flow* is:*20

$$CF^t \equiv P_Y^t Y^t - P_X^t X^t + P_S^t S^t - P_B^t B^t. \tag{A1}$$

Let the firm's beginning of period t stock of inventory be K^{t-1} and let its end of period stock of inventory be K^t. These inventory stocks are valued at the balance sheet prices prevailing at the

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*18 The problem is that most users are not aware that normal index number theory fails spectacularly as a value aggregate approaches zero.

*19 Their analysis is in turn based on the Austrian model of production explained by Böhm-Bawerk (1891)[18], Hicks (1961)[115] and Edwards and Bell (1961)[81]. For additional material on this model, see the Appendices in Diewert (1977)[47] (1980)[50] and section 1.9.2 of chapter 1.

*20 Note that this framework is flexible enough to allow the firm to either purchase or produce internally inventory items. Note also that firm purchases of inventory items from other domestic firms would appear in the national accounts as intermediate input purchases and purchases from foreign suppliers would appear as imports. On the other hand, sales of inventory items by the firm to domestic producers, households or foreigners would appear in the national accounts as gross outputs, final household consumption or exports respectively.

beginning and end of period t, P_K^{t-1} and P_K^t respectively. Note that all 4 prices involving inventory items, P_S^t, P_B^t, P_K^{t-1} and P_K^t can be different.

The firm's end of period t *economic income* or *net profit* is defined as its cash flow plus the value of its end of period t stock of inventory items less $(1+r^t)$ times the value of its beginning of period t stock of inventory items:

$$EI^t \equiv CF^t + P_K^t K^t - (1+r^t)P_K^{t-1}K^{t-1} \tag{A2}$$

where r^t is the nominal cost of capital that the firm faces at the beginning of period t. Thus in definition (A2), it is assumed that the firm has to borrow financial capital or raise equity capital at the cost r^t in order to finance its initial holdings of inventory items. This cost could be real (in the case of a firm whose initial capital is funded by bonds) or it could be an opportunity cost (in the case of a firm entirely funded by equity capital).

The end of period stock of inventory is related to the beginning of the period stock by the following equation:

$$K^t = K^{t-1} + B^t - S^t - U^t \tag{A3}$$

where U^t denotes inventory items that are lost, spoiled, damaged or are used internally by the firm. In the case of livestock inventories, there is a natural growth rate of inventories over the period so equation (A3) is replaced by:

$$K^t = K^{t-1} + B^t - S^t + G^t \tag{A4}$$

where G^t denotes the natural growth of the stock over period t.*21

Define the *change in inventory stocks* over period t as:

$$\Delta K^t \equiv K^t - K^{t-1}. \tag{A5}$$

Using (A5), both (A3) and (A4) can be written as:

$$K^t = K^{t-1} + \Delta K^t. \tag{A6}$$

Now substitute (A6) into the definition of economic income (A2) and the following expression is obtained:

$$\begin{aligned} EI^t &\equiv CF^t + P_K^t[K^{t-1} + \Delta K^t] - (1+r^t)P_K^{t-1}K^{t-1} \\ &= CF^t + P_K^t \Delta K^t - [r^t P_K^{t-1} - (P_K^t - P_K^{t-1})]K^{t-1}. \end{aligned} \tag{A7}$$

Thus *economic income is equal to cash flow plus the value of the change in inventory (valued at end of period balance sheet prices) minus the user cost of inventories times the starting stocks of inventories* where this *period t user cost* is defined as

$$P_U^t \equiv r^t P_K^{t-1} - (P_K^t - P_K^{t-1}). \tag{A8}$$

Note that the above algebra works for both livestock and ordinary inventory items.

Of course, there can be two versions of the user cost:

- An ex post version where the *actual* end of period balance sheet price of inventories is used or
- An ex ante version where at the beginning of period t, we estimate a *predicted value* for the end of period balance sheet price.

*21 If the firm is constructing inventory items either for direct sale or as an intermediate step in its production processes, then these produced additions to the stock would be included in the term G^t.

For the production accounts in the SNA, the ex ante version is the appropriate version, which means the national income accountant has some leeway in forming estimates of the end of period balance sheet price for the inventory item. Looking at (A7), it is important to note that the change in inventories that occurred over period t, ΔK^t, should be valued at the end of period t price for the inventory item, P_K^t.*22

If the firm is using or selling many inventory items, say J items, then equation (A7) becomes:

$$EI^t \equiv CF^t + \sum\nolimits_{j=1}^{J} P_{Kj}^t \Delta K_j^t - \sum\nolimits_{j=1}^{J} [r^t P_{Kj}^{t-1} - (P_{Kj}^t - P_{Kj}^{t-1})] K_j^{t-1} \tag{A9}$$

where the notation is obvious. The terms involving the value of the change in inventories over the period are the following ones:

$$\sum\nolimits_{j=1}^{J} P_{Kj}^t \Delta K_j^t = \sum\nolimits_{j=1}^{J} P_{Kj}^t [K_j^t - K_j^{t-1}] \tag{A10}$$

$$= \sum\nolimits_{j=1}^{J} P_{Kj}^t K_j^t - \sum\nolimits_{j=1}^{J} P_{Kj}^t K_j^{t-1}. \tag{A11}$$

Looking at (A10), it would appear that normal index number theory could be applied to the sum of terms in the value aggregate on the right hand side, with prices defined as the end of period t balance sheet prices P_{Kj}^t and corresponding quantities defined as the inventory changes $K_j^t - K_j^{t-1}$ over period t. However, this value aggregate is not necessarily of one sign over time: it could be positive, negative or zero. *Normal index number theory breaks down for value aggregates that can be either positive or negative over time.**23 Thus index number theory should not be applied to the value aggregate on the right hand side of (A10). Instead, it is recommended that index number theory be applied *separately* to the two value aggregates on the right hand side of (A11).*24 Thus $\sum_{j=1}^{J} P_{Kj}^t K_j^t$ should be decomposed (using normal index number theory) into $P_{KE}^t K_E^t$ where P_{KE}^t is the scalar end of period t aggregate price of inventories and K_E^t is the corresponding end of period t aggregate stock and $\sum_{j=1}^{J} P_{Kj}^t K_j^{t-1}$ should be decomposed into $P_{KB}^t K_B^t$ where P_{KB}^t is the scalar beginning of period t aggregate price of inventories and K_B^t is the corresponding beginning of period t aggregate stock. Then in place of the current single aggregate for inventory change that is reported in the current System of National Accounts, it is recommended that *inventory change be treated in a manner that is symmetric to the treatment of aggregate exports and imports in the accounts*; i.e., the end of period aggregates P_{KE}^t and K_E^t (the counterparts to the aggregate price of exports and the aggregate quantity of exports) and the beginning of period aggregates P_{KB}^t and K_B^t would be reported separately just as exports and imports are reported separately in the current SNA.

There is another treatment of inventory change that could be used by statistical agencies that is much more straightforward. The definition of economic income, (A2) above, can be rewritten as follows:

$$EI^t \equiv CF^t + P_K^t K^t - P_K^{t-1} K^{t-1} - r^t P_K^{t-1} K^{t-1}. \tag{A12}$$

*22 However, the current SNA methodology requires that inventory change over the production period be evaluated at the average prices of the period. This requirement could be accommodated in our framework by replacing the end of period price of the inventory item, P_K^t, by an appropriate average inventory price for period t. If this is done, and if the actual end of period price of the inventory item is used for balance sheet purposes, then a reconciliation entry will be required in the Revaluation Accounts.

*23 To see why this breakdown occurs, consider a situation where the value aggregate just happens to be zero in the base period. Laspeyres price and quantity indexes will be undefined under these circumstances and nonsensical numbers will be obtained if the value aggregate is very close to zero in the base period. However, if the Laspeyres, Paasche or Fisher formula is used in forming a larger aggregate that is bounded well away from zero, then the right hand side of (A10) can be used when forming this larger aggregate and the same results will be obtained as using the right hand side of (A11) in forming the larger aggregate.

*24 This solution to the aggregation problem was suggested by Diewert (2004; 36)[66].

Using (A12), *the value of inventory change for period t* is simply defined as the end of period t value of the stock, VK^t,*25 less the beginning of period t value of the stock, VK^{t-1}:

$$VK^t - VK^{t-1} = P_K^t K^t - P_K^{t-1} K^{t-1}. \tag{A13}$$

Using this decomposition of economic income, the user cost value aggregate is defined as the last term on the right hand side of (A12) and so *the new user cost of inventories* is:

$$P_U^{t*} \equiv r^t P_K^{t-1}. \tag{A14}$$

The new user cost of inventories, P_U^{t*} defined by (A14), can be compared to the initial user cost of inventories, P_U^t defined by (A8), and the new value of inventory change defined by (A13) can be compared to the earlier expression for the value of inventory change defined by (A11). Both the old and the new decomposition of economic income are theoretically valid. However, note that a nominal interest rate r^t appears in (A14) whereas a type of real interest rate appeared in (A8). Hence for a country experiencing high inflation, the new user cost of inventories will be higher than the old user cost and similarly, the new value of inventory change defined by (A13) will be higher than the old value of inventory change defined by (A11).*26 Thus nominal GDP will tend to be higher using the new decomposition compared to the initial one and it will be substantially higher under conditions of high inflation.

There are advantages and disadvantages of using the second decomposition of economic income compared to the first:

- The *main advantage* of the second decomposition is that it is much more straightforward and will be easier to explain to users. Also, it is much easier to reconcile quarterly changes in inventories to annual changes using the second decomposition.
- The *main disadvantage* of the second decomposition is that the resulting user cost of inventories is *different* from the user cost formula for reproducible capital and so an awkward asymmetry would be introduced into the SNA if a user cost approach to reproducible capital were introduced.*27

Both decompositions of economic income involve a difference in two value aggregates where the sign of the difference cannot be bounded away from zero. Hence for both decompositions, it is recommended that the beginning and end of period values be separately deflated and shown as two items in the real accounts in a manner that is analogous to the present treatment of exports less imports.

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*25 See the first 3 columns of Table 5.4 for the Fisher chain decomposition of the end of period value of the stocks VK^t into price and quantity components for the numerical example.

*26 If the initial decomposition of economic income is used, then the beginning of the period inventory stocks are valued at the higher end of period prices but since this value aggregate is given a minus sign, this will reduce nominal GDP.

*27 The ex ante user cost for a reproducible capital asset contains an anticipated asset inflation rate in it similar to (A8), which offsets the nominal interest rate term. The ex ante user cost concept should be close to an actual rental or leasing price for the asset since it based on the same considerations that an owner would consider in setting a rental price. Hence, it seems desirable to have the user cost of inventories aligned with the user cost of reproducible capital. For additional discussions on ex ante and ex post measures, see Hicks (1946; 178-179)[114] and Hill and Hill (2003)[122].

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