A Note on Reconciling Gross Output TFP Growth with Value Added TFP Growth

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Abstract

The paper obtains relatively simple exact expressions that relate value added Total Factor Productivity growth (TFP growth or Multifactor Productivity Growth) in a value added framework to the corresponding measures of TFP growth in a gross output framework when Laspeyres or Paasche indexes are used to aggregate outputs and inputs. Basically, as the input base becomes smaller, the corresponding estimates of TFP growth become larger. A fairly simple approximate relationship between Fisher indexes of gross output TFP growth and the corresponding Fisher index of value added TFP growth is also derived. The methodology developed in this note can be applied in other situations.

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Key Words

Total Factor Productivity growth, TFP growth, Multifactor Productivity growth, MFP growth, Laspeyres, Paasche and Fisher index number formulae, magnification factors.

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1. Introduction

Schreyer (2001; 26) developed an approximate formula to relate Total Factor Productivity Growth (or Multifactor Productivity Growth) in a gross output model of production to TFP (or MFP) growth in a value added setting. In this note, we take another look at this problem[2] and develop an exact relationship between the two measures if Laspeyres (or Paasche) output and input indexes are used to compute aggregate growth rates of inputs and outputs. We develop rules relating the two productivity concepts that are simpler than the existing rules that have been developed in the literature.

Sections 2 and 3 assume that aggregate outputs and inputs are constructed using the Laspeyres and Paasche formulae respectively while section 4 uses the Fisher (1922) ideal index number formula to aggregate inputs and outputs. Section 5 concludes.

2. The Laspeyres Case

We first consider the situation where Laspeyres indexes are used to aggregate outputs and inputs. For simplicity, consider a situation where we want to compute gross output and value added productivity growth rates for a production unit that produces gross output $q_Y^t > 0$ at prices $p_Y^t > 0$, uses intermediate input $q_M^t > 0$ at the prices $p_M^t > 0$ and uses primary input $q_X^t > 0$ at the prices $p_X^t > 0$ for $t = 0,1$.[3] Define the period t value of gross output, intermediate input and primary input as $v_Y^t \equiv p_Y^t q_Y^t$, $v_M^t \equiv p_M^t q_M^t$ and $v_X^t \equiv p_X^t q_X^t$ respectively for $t = 0,1$. Define (one plus) the growth rates of output, intermediate input and primary inputs as Γ_Y, Γ_M and Γ_X as follows:

$$(1)\ \Gamma_Y \equiv q_Y^1/q_Y^0\ ;\ \ \Gamma_M \equiv q_M^1/q_M^0 \ \text{ and } \Gamma_X \equiv q_X^1/q_X^0\ .$$

We assume that the value of inputs equals the value of outputs in period 0:

$$(2)\ v_Y^0 = v_M^0 + v_X^0.$$

(One plus) *gross output TFP growth* using Laspeyres quantity indexes, Π_G, is defined as follows:[4]

$$(3)\ \Pi_G \equiv \Gamma_Y/[s_M^0\Gamma_M + s_X^0\Gamma_X]$$

where the period 0 input expenditure shares s_M^0 and s_X^0 are defined as follows:

$$(4)\ s_M^0 \equiv v_M^0/[v_M^0 + v_X^0] = v_M^0/v_Y^0 \qquad \text{using (2);}$$

[2] Balk (2009) has a more recent rather comprehensive paper on this topic. We obtain approximately the same result as he obtains but our method of derivation is much simpler.

[3] In the case where there are many inputs and outputs, the output and input quantity ratio aggregates, q_Y^1/q_Y^0, q_M^1/q_M^0 and q_X^1/q_X^0 can be interpreted as Laspeyres indexes of the micro quantities. The corresponding aggregate price ratios are of course Paasche price indexes.

[4] Note that $s_M^0\Gamma_M + s_X^0\Gamma_X$ is the Laspeyres quantity index of all inputs.

$$(5)\ s_X^0 \equiv v_X^0/[v_M^0 + v_X^0] = 1 - s_M^0 \quad \text{using (4).}$$

We now need to define value added TFP growth. We use the Laspeyres index number formula to form an aggregate of gross output less intermediate input. We assume that the quantities of gross output and intermediate input are positive but we change the sign of the price of intermediate inputs from a positive sign to a negative sign in order to form a Laspeyres index of real value added. Thus (one plus) *real value added TFP growth* using a Laspeyres quantity index to construct real value added growth, Π_{VA}, is defined as follows:

$$(6)\ \Pi_{VA} \equiv [S_Y^0\Gamma_Y + S_M^0\Gamma_M]/\Gamma_X$$

where the period 0 value added output expenditure shares[5] S_Y^0 and S_M^0 are defined as follows:[6]

$$(7)\ S_Y^0 \equiv v_Y^0/[v_Y^0 - v_M^0] = 1/[1 - (v_M^0/v_Y^0)] = 1/[1 - s_M^0] \quad \text{using (4);}$$

$$(8)\ S_M^0 \equiv -v_M^0/[v_Y^0 - v_M^0] = 1 - S_Y^0 \quad \text{using (7)} = -s_M^0/[1 - s_M^0] \quad \text{using the last equation in (7).}$$

Now substitute (7) and (8) into (6) and we obtain the following expression for (one plus) value added TFP growth:

$$(9)\ \Pi_{VA} = [\Gamma_Y - s_M^0\Gamma_M]/(1 - s_M^0)\Gamma_X .$$

We can also substitute (5) into (3) and obtain the following expression for (one plus) gross output TFP growth:

$$(10)\ \Pi_G \equiv \Gamma_Y/[s_M^0\Gamma_M + (1- s_M^0)\Gamma_X].$$

Comparing (9) and (10), it can be seen that both value added and gross output TFP (one plus) growth rates can be expressed in terms of (one plus) the growth rates of output, intermediate input and primary input (Γ_Y, Γ_M and Γ_X) and the share of intermediate input in total input, s_M^0.

[5] These shares sum to one but of course S_M^0 is negative and so the "shares" will not be bounded from below by 0 and above by 1.
[6] Note that we did not change the sign of $v_M^0 > 0$ and $s_M^0 > 0$. Thus using (7), $S_M^0 < 0$. We assume that period 0 nominal value added, $v_Y^0 - v_M^0$, is greater than 0.

Now define the *rate of gross output TFP growth* as π_G equal to Π_G less 1 and the *rate of real value added TFP growth* as π_{VA} equal to Π_{VA} less 1. We can obtain the following alternative expressions for π_G and π_{VA}:

$$
\begin{aligned}
(11)\ \pi_G &\equiv \Pi_G - 1 \\
&= (\Gamma_Y/[s_M^0\Gamma_M + (1- s_M^0)\Gamma_X]) - 1 && \text{using (10)} \\
&= [\Gamma_Y - s_M^0\Gamma_M - (1- s_M^0)\Gamma_X]/[s_M^0\Gamma_M + (1- s_M^0)\Gamma_X];
\end{aligned}
$$

$$
\begin{aligned}
(12)\ \pi_{VA} &\equiv \Pi_{VA} - 1 \\
&= ([\Gamma_Y - s_M^0\Gamma_M]/(1 - s_M^0)\Gamma_X) - 1 && \text{using (9)} \\
&= [\Gamma_Y - s_M^0\Gamma_M - (1- s_M^0)\Gamma_X]/(1 - s_M^0)\Gamma_X.
\end{aligned}
$$

Form the ratio of value added TFP growth to gross output TFP growth and we obtain the following *exact formula* relating these two productivity concepts:[7]

$$
\begin{aligned}
(13)\ \pi_{VA}/\pi_G &= (1- s_M^0)^{-1}[s_M^0\Gamma_M + (1- s_M^0)\Gamma_X]/\Gamma_X \\
&= (s_X^0)^{-1}[s_M^0\Gamma_M + (1- s_M^0)\Gamma_X]/\Gamma_X && \text{using (5)} \\
&= [v_Y^0/v_X^0][s_M^0\Gamma_M + (1- s_M^0)\Gamma_X]/\Gamma_X && \text{using (2) and (5).}
\end{aligned}
$$

Thus the bigger is the share of intermediate inputs in total period 0 input, s_M^0, the bigger will be $(1- s_M^0)^{-1}$ and hence the bigger will be value added TFP growth relative to gross output TFP growth.[8] Similarly, the bigger is (one plus) aggregate input growth $s_M^0\Gamma_M + (1- s_M^0)\Gamma_X$ over the two periods being compared relative to (one plus) primary input growth Γ_X, the bigger will be value added TFP growth relative to gross output TFP growth. Typically, (one plus) aggregate input growth will not be all that different from (one plus) primary input growth (both terms will be close to unity) so the term $(1- s_M^0)^{-1} = 1/s_X^0 = v_Y^0/v_X^0$ will explain almost all of the difference in the two TFP growth rates.[9]

Note that the last equation in (13) shows that π_{VA}/π_G is approximately equal to v_Y^0/v_X^0, the value of output revenues v_Y^0 divided by the value of primary input v_X^0, which in turn is equal to value added $v_Y^0 - v_M^0$ using our assumption (2). The ratio of revenues to value added is frequently called the Domar Factor; see Balk (2009; 249).

The above results can readily be generalized to many outputs and inputs due to the consistency in aggregation properties of the Laspeyres formula.

[7] We need to assume that $\pi_G \neq 0$.

[8] Note that $1-s_M^0$ is equal to s_X^0, the share of primary inputs in total inputs used in period 0. The first equation in (12) is roughly equivalent to Domar's (1961; 725) equation (4.6) while the third equation in (12) is roughly equivalent to Balk's (2009; 248) equation (20).

[9] Thus if (Laspeyres) gross output TFP growth π_G is 0.5 % and the period 0 primary input share of total input s_X^0 is ½, then (Laspeyres) value added TFP growth π_{VA} will be approximately $(s_X^0)^{-1}(0.5\%) = 1.0\%$; if $s_X^0 = 1/3$, then $\pi_{VA} \approx (1/3)^{-1}\ (0.5\%) = 1.5\%$.

3. The Paasche Case

We now consider the situation where Paasche indexes are used to aggregate outputs and inputs. Again, define (one plus) the growth rates of output, intermediate input and primary inputs as Γ_Y, Γ_M and Γ_X by equations (1) but if we are in the case of multiple outputs and inputs, these ratios are to be interpreted as being equal to Paasche indexes of gross output, intermediate input and primary input in this section. We assume that the value of inputs equals the value of outputs in period 1:

$$(14)\ v_Y^1 = v_M^1 + v_X^1.$$

(One plus) *gross output TFP growth* using Paasche quantity indexes, Π_G^*, is defined as follows:[10]

$$(15)\ \Pi_G^* \equiv \Gamma_Y/[s_M^1\Gamma_M^{-1} + s_X^1\Gamma_X^{-1}]^{-1}$$

where the period 0 input expenditure shares s_M^1 and s_X^1 are defined as follows:

$$(16)\ s_M^1 \equiv v_M^1/[v_M^1 + v_X^1] = v_M^1/v_Y^1 \quad \text{using (14);}$$

$$(17)\ s_X^1 \equiv v_X^1/[v_M^1 + v_X^1] = 1 - s_M^1 \quad \text{using (16).}$$

We now need to define Paasche value added TFP growth. We use the Paasche index number formula to form an aggregate of gross output less intermediate input. We assume that the quantities of gross output and intermediate input are positive but we change the sign of the price of intermediate inputs from a positive sign to a negative sign in order to form a Paasche index of real value added. Thus (one plus) *real value added TFP growth* using a Paasche quantity index to construct real value added growth, Π_{VA}^*, is defined as follows:[11]

$$(18)\ \Pi_{VA}^* \equiv [S_Y^1\Gamma_Y^{-1} + S_M^1\Gamma_M^{-1}]^{-1}/\Gamma_X$$

where the period 1 value added output expenditure shares[12] S_Y^1 and S_M^1 are defined as follows:[13]

$$(19)\ S_Y^1 \equiv v_Y^1/[v_Y^1 - v_M^1] = 1/[1 - (v_M^1/v_Y^1)] = 1/[1 - s_M^1] \quad \text{using (16);}$$

[10] Note that $[s_M^0\Gamma_M^{-1} + s_X^0\Gamma_X^{-1}]^{-1}$ is the Paasche quantity index of all inputs.
[11] Note that $[S_Y^1\Gamma_Y^{-1} + S_M^1\Gamma_M^{-1}]^{-1}$ is the Paasche real value added index.
[12] These shares sum to one but of course S_M^1 is negative and so the "shares" will not be bounded from below by 0 and above by 1. We assume that $v_Y^1 - v_M^1 > 0$.
[13] Note that we did not change the sign of $v_M^1 > 0$ and $s_M^1 > 0$. Thus using (20), $S_M^1 < 0$.

$$(20)\ S_M^1 \equiv -v_M^1/[v_Y^1 - v_M^1]$$
$$= 1 - S_Y^1 \qquad \text{using (19)}$$
$$= -s_M^1/[1 - s_M^1] \qquad \text{using the last equation in (19).}$$

Now substitute (19) and (20) into (18) and we obtain the following expression for (one plus) value added Paasche TFP growth:

$$(21)\ \Pi_{VA}^* = [S_Y^1\Gamma_Y^{-1} + S_M^1\Gamma_M^{-1}]^{-1}/\Gamma_X$$
$$= \Gamma_X^{-1}/[S_Y^1\Gamma_Y^{-1} + S_M^1\Gamma_M^{-1}]$$
$$= (1-s_M^1)^{-1}\Gamma_X^{-1}/[\Gamma_Y^{-1} - s_M^1\Gamma_M^{-1}].$$

We can also substitute (17) into (15) and obtain the following expression for (one plus) gross output Paasche TFP growth:

$$(22)\ \Pi_G^* \equiv \Gamma_Y/[s_M^1\Gamma_M^{-1} + (1- s_M^1)\Gamma_X^{-1}]^{-1} = [s_M^1\Gamma_M^{-1} + (1- s_M^1)\Gamma_X^{-1}]/\Gamma_Y^{-1}.$$

Comparing (21) and (22), it can be seen that both value added and gross output Paasche TFP (one plus) growth rates can be expressed in terms of (one plus) the growth rates of (Paasche) output, intermediate input and primary input (Γ_Y, Γ_M and Γ_X) and the period 1 share of intermediate input in total input, s_M^1.

Now compute $(\Pi_{VA}^*)^{-1} - 1$ and $(\Pi_G^*)^{-1} - 1$ using (21) and (22):

$$(23)\ (\Pi_{VA}^*)^{-1} - 1 = \{[\Gamma_Y^{-1} - s_M^1\Gamma_M^{-1}]/(1-s_M^1)^{-1}\Gamma_X^{-1}\} - 1$$
$$= [\Gamma_Y^{-1} - s_M^1\Gamma_M^{-1} - (1-s_M^1)\Gamma_X^{-1}]/[(1-s_M^1)\Gamma_X^{-1}];$$

$$(24)\ (\Pi_G^*)^{-1} - 1 = \{\Gamma_Y^{-1}/[s_M^1\Gamma_M^{-1} + (1- s_M^1)\Gamma_X^{-1}]\} - 1$$
$$= [\Gamma_Y^{-1} - s_M^1\Gamma_M^{-1} - (1-s_M^1)\Gamma_X^{-1}]/[s_M^1\Gamma_M^{-1} + (1- s_M^1)\Gamma_X^{-1}].$$

Now take the ratio of (23) to (24) and we obtain the following identity:[14]

$$(25)\ [(\Pi_{VA}^*)^{-1}-1]/[(\Pi_G^*)^{-1}-1] = [s_M^1\Gamma_M^{-1} + (1- s_M^1)\Gamma_X^{-1}]/[(1-s_M^1)\Gamma_X^{-1}].$$

Define the *Paasche rate of gross output TFP growth* as π_G^* equal to Π_G^* less 1 and the *Paasche rate of real value added TFP growth* as π_{VA}^* equal to Π_{VA}^* less 1:

$$(26)\ \pi_G^* \equiv \Pi_G^* - 1;$$
$$(27)\ \pi_{VA}^* \equiv \Pi_{VA}^* - 1.$$

Now multiply both sides of (25) by Π_{VA}^*/Π_G^* and we obtain the following equation:

$$(28)\ \pi_{VA}^*/\pi_G^* = [\Pi_{VA}^*/\Pi_G^*]\{[s_M^1\Gamma_M^{-1} + (1- s_M^1)\Gamma_X^{-1}]/[(1-s_M^1)\Gamma_X^{-1}]\} \qquad \text{using (25)}$$

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[14] We need to assume that $(\Pi_G^*)^{-1}-1 \neq 0$ which will imply $\pi_G^* \equiv \Pi_G^*-1$ is also not equal to 0 since $\Pi_G^* > 0$.

$$= (1-s_M{}^1)^{-1}[S_Y{}^1\Gamma_Y{}^{-1} + S_M{}^1\Gamma_M{}^{-1}]^{-1}/\Gamma_Y \qquad \text{using (21) and (22)}$$
$$= (s_X{}^1)^{-1}[S_Y{}^1\Gamma_Y{}^{-1} + S_M{}^1\Gamma_M{}^{-1}]^{-1}/\Gamma_Y \qquad \text{using (17)}$$

$$= (v_Y{}^1/v_X{}^1)[S_Y{}^1\Gamma_Y{}^{-1} + S_M{}^1\Gamma_M{}^{-1}]^{-1}/\Gamma_Y \qquad \text{using (17) and (14).}$$

Thus the bigger is the share of intermediate inputs in total period 1 input, $s_M{}^1$, the bigger will be $(1- s_M{}^1)^{-1}$ and hence the bigger will be value added Paasche TFP growth relative to Paasche gross output TFP growth. Of course, $(1- s_M{}^1)^{-1}$ is equal to $1/s_X{}^1$ which in turn is equal to the period 1 Domar Augmentation Factor, $v_Y{}^1/v_X{}^1$, which in turn is equal to the period 1 value of gross output divided by the period 1 value of primary input.[15] Similarly, the bigger is (one plus) real value added growth $[S_Y{}^1\Gamma_Y{}^{-1} + S_M{}^1\Gamma_M{}^{-1}]^{-1}$ over the two periods being compared relative to (one plus) gross output growth Γ_Y,[16] the bigger will be Paasche value added TFP growth relative to Paasche gross output TFP growth. Note the similarity of the Paasche formula (28) to our earlier Laspeyres formula (13). Typically, (one plus) real value added growth will not be all that different from (one plus) gross output growth (both terms will be close to unity) so the Paasche Augmentation Factor $(1- s_M{}^1)^{-1} = 1/s_X{}^1 = v_Y{}^1/v_X{}^1$ will explain almost all of the difference in the two Paasche TFP growth rates.

The above results can readily be generalized to many outputs and inputs due to the consistency in aggregation properties of the Paasche formula.

4. The Fisher Case

One plus the Fisher (1922) index of value added productivity growth, $\Pi_{VA}{}^F$, is defined as the geometric mean of the corresponding Laspeyres and Paasche measures of (one plus) value added TFP growth, Π_{VA} and $\Pi_{VA}{}^*$:

$$(29)\ \Pi_{VA}{}^F \equiv [\Pi_{VA}\,\Pi_{VA}{}^*]^{1/2}.$$

Similarly, (one plus) the Fisher (1922) index of gross output productivity growth, $\Pi_G{}^F$, is defined as the geometric mean of the corresponding Laspeyres and Paasche measures of (one plus) gross output TFP growth, Π_G and $\Pi_G{}^*$:

$$(30)\ \Pi_G{}^F \equiv [\Pi_G\,\Pi_G{}^*]^{1/2}.$$

Finally, define the Fisher (1922) indexes of value added and gross output productivity growth, $\pi_{VA}{}^F$ and $\pi_G{}^F$, as follows:

$$(31)\ \pi_{VA}{}^F \equiv \Pi_{VA}{}^F - 1;$$

15 Assumption (14) ensures that $v_Y{}^1/v_X{}^1 > 1$.

16 Note that $[S_Y{}^1\Gamma_Y{}^{-1} + S_M{}^1\Gamma_M{}^{-1}]^{-1}$ is the Paasche index of real value added growth and Γ_Y is to be interpreted as the Paasche index of gross output growth if there are many outputs and we are aggregating outputs in two stages using the Paasche formula.

(32) $\pi_G^F \equiv \Pi_G^F - 1$.

Equations (9) and (10) give exact expressions for the Laspeyres indexes Π_{VA} and Π_G while equations (21) and (22) give exact expressions for the Paasche indexes Π_{VA}^* and Π_G^*. Hence we could use these expressions to calculate the Fisher variables (29)-(32) and we would end up with an exact expression for the ratio of Fisher index value added TFP growth to Fisher gross output growth, π_{VA}^F / π_G^F. However, the resulting expression is difficult to interpret and so we will resort to a different strategy that makes use of equations (13) and (28) but also involves approximating the geometric means that define Π_{VA}^F and Π_G^F by corresponding arithmetic means.[17]

The first step in our strategy is to define the right hand sides of equations (13) and (28) as the constants γ and γ^*:[18]

(33) $\pi_{VA}/\pi_G = (s_X^0)^{-1}[s_M^0\Gamma_M + (1- s_M^0)\Gamma_X]/\Gamma_X \equiv \gamma$;
(34) $\pi_{VA}^*/\pi_G^* = (s_X^1)^{-1}[S_Y^1\Gamma_Y^{-1} + S_M^1\Gamma_M^{-1}]^{-1}/\Gamma_Y \equiv \gamma^*$.

Using definitions (11) and (12) and (26) and (27), equations (33) and (34) imply the following relations:

(35) $(\Pi_{VA} - 1) = \gamma(\Pi_G - 1)$;
(36) $(\Pi_{VA}^* - 1) = \gamma^*(\Pi_G^* - 1)$.

Using definitions (29) and (31), we have the following equations:

(37) $\pi_{VA}^F = [\Pi_{VA}\Pi_{VA}^*]^{1/2} - 1$
$\approx ½\, \Pi_{VA} + ½\, \Pi_{VA}^* - 1$
where we have approximated the geometric mean by an arithmetic mean
$= ½\, [\Pi_{VA} - 1] + ½\, [\Pi_{VA}^* - 1]$
$= ½\, \gamma[\Pi_G - 1] + ½\, \gamma^*[\Pi_G^* - 1]$

where the last equation follows using (35) and (36). Using definitions (30) and (32), we have the following equations:

(38) $\pi_G^F = [\Pi_G\Pi_G^*]^{1/2} - 1$
$\approx ½\, \Pi_G + ½\, \Pi_G^* - 1$
where we have approximated the geometric mean by an arithmetic mean
$= ½\, [\Pi_G - 1] + ½\, [\Pi_G^* - 1]$.

If $½\, [\Pi_G - 1] + ½\, \gamma[\Pi_G^* - 1]$ is not equal to zero, we can take the ratio of π_{VA}^F to π_G^F and using (37) and (38), we obtain the following approximate relationship of Fisher value added TFP growth to Fisher gross output TFP growth:

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[17] These approximations will typically be very close.
[18] These constants can be regarded as *magnification factors*; they magnify the gross output TFP growth rates into the corresponding real value added TFP growth rates.

$$(39)\quad \pi_{VA}{}^F/\pi_G{}^F \approx \{\gamma[\Pi_G - 1] + \gamma^*[\Pi_G{}^* - 1]\}/\{[\Pi_G - 1] + \gamma[\Pi_G{}^* - 1]\}$$
$$= w\gamma + (1-w)\gamma^*$$

where the weight $w \equiv [\Pi_G - 1]/\{[\Pi_G - 1] + [\Pi_G{}^* - 1]\}$. Thus $\pi_{VA}{}^F/\pi_G{}^F$ is approximately equal to a weighted average of the Laspeyres and Paasche magnification factors, γ and γ^*.[19] If we are willing to make a further approximation that the Laspeyres and Paasche indexes of gross output growth are approximately equal so that $\Pi_G \approx \Pi_G{}^*$, then we obtain the following very simple approximate relationship between Fisher value added TFP growth and Fisher gross output TFP growth:

$$(40)\quad \pi_{VA}{}^F/\pi_G{}^F \approx \tfrac{1}{2}\gamma + \tfrac{1}{2}\gamma^*.$$

5. Conclusion

We have obtained relatively simple exact expressions for the relationship between the rate of gross output Multifactor Productivity Growth and the corresponding rate of real value added MFP growth if the Laspeyres or Paasche index number formulae are used to aggregate inputs and outputs. We also obtained a simple approximate expression relating value added TFP growth to gross output TFP growth if the Fisher formula is used to aggregate inputs and outputs. Generally speaking, gross output TFP growth is *magnified* when we move to value added TFP growth and the magnification factor is approximately equal to the reciprocal of the share of primary inputs in total input use.

The same methodology can be used in other situations. For example, we may want to compare value added productivity growth to net value added productivity growth where the latter concept takes depreciation out of capital services and treats it as an intermediate input expense. The resulting net value added TFP growth will be equal to a magnification factor times the corresponding traditional value added TFP growth and the magnification factor will be approximately equal to the reciprocal of the share of labour and waiting services in traditional labour and capital services (which include depreciation in the user costs of capital).[20] In some regulatory contexts, we may want to compute TFP growth with labour added regarded as an intermediate input and only capital services in the primary input base and compare this TFP growth with more traditional measures. Again a magnification factor will emerge.[21] Generally speaking, the smaller we make the input base in the productivity measure, the bigger will be the rate of TFP growth. Other examples of this magnification effect can be found in Schreyer (2001) and in Balk (2009).

References

[19] The weights for γ and γ^* will sum to unity but they need not be nonnegative unless $\Pi_G - 1$ and $\Pi_G{}^* - 1$ have the same sign. This will almost always be the case empirically.

[20] For applications of this net value added approach and estimates of the resulting magnification factors, see Diewert (2014) and Diewert and Yu (2012).

[21] For an application of this approach, see Lawrence, Diewert and Fox (2006).

Balk, B.M. (2009), “On the Relation between Gross Output and Value Added Based Productivity Measures: The Importance of the Domar Factor”, *Macroeconomic Dynamics*, **13** (Supplement 2), 241–267.

Diewert, W.E. (2014), “US TFP Growth and the Contribution of Changes in Export and Import Prices to Real Income Growth”, *Journal of Productivity Analysis* 41, 19-39.

Diewert, W.E. and E. Yu (2012), “New Estimates of Real Income and Multifactor Productivity Growth for the Canadian Business Sector, 1961-2011”, *International Productivity Monitor*, Number 24:Fall, 27-48.

Domar, E.D. (1961), “On the Measurement of Technological Change”, *Economic Journal* 71, 709–729.

Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.

Lawrence, D.A., W.E. Diewert and K.J. Fox (2006), “Who Benefits from Economic Reform: The Contribution of Productivity, Price Changes and Firm Size to Profitability”, *Journal of Productivity Analysis* 26, 1-13.

Schreyer, P. (2001), *OECD Productivity Manual: A Guide to the Measurement of Industry-Level and Aggregate Productivity Growth*, Paris: OECD.