Sunk Costs and the Measurement of Commercial Property Depreciation

Revised May 27, 2015.

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Abstract

Developments in property markets greatly influence economic growth, monetary policy, productivity measurement, inflation measurement and hence welfare payments to the disadvantaged. Property price bubbles often lead to financial crises; those experienced during the 20th century were often triggered by commercial property price movements. Yet property poses significant challenges for national accountants in producing key economic variables used in informing policy assessment and formulation. To address these challenges, this paper formalizes a framework for measuring prices and quantities of capital inputs for a commercial property. In particular, it addresses problems associated with obtaining separate estimates for the land and structure components of a property, a decomposition of property value which is important for the national accounts, productivity measurement and taxation. A key contribution is to address the problem of estimating structure depreciation taking into account the fixity of the structure. We find that structure depreciation is determined primarily by the cash flows that the property generates rather than physical deterioration of the building. Finally, we provide a framework for the determination of the optimal length of life for a structure.

JEL Classification Numbers: C2, C23, C43, D12, E31, R21.

Key Words: Property price indexes, net operating income, discounted cash flow, System of National Accounts, Balance Sheets, land and structure prices, goodwill amortization, intangible assets.

[1] *Acknowledgements*: The authors thank Robert Cairns, David Geltner and Nigel Stapledon for helpful comments and gratefully acknowledge financial support from the Social Sciences and Humanities Research Council of Canada, the Australian Research Council (LP0884095), and the CAER Real Estate Initiative. The paper is forthcoming in the *Canadian Journal of Economics*.

1. Introduction

Property is an important asset class in developed economies, yet as noted by the Deputy Director of Statistics at the OECD Paul Schreyer (2009; 267), "residential and non-residential structures and land pose significant challenges for national accountants and price statisticians." These challenges can distort the accuracy of key information being used in policy assessment and formulation. Besides their direct relevance for informing investment and policy decisions, price and quantity indexes are required for calculating stocks of commercial properties in the Balance Sheets of a country. Related price and quantity indexes are also required for the services of the land and structure components of a commercial property in the production accounts of the country. Hence, with the aim of improving the basic information that is used by policy makers and applied economists in a multitude of fields, the main purpose of the paper is to provide a framework for commercial property capital measurement that would be suitable for national income accounting purposes.

In particular, the System of National Accounts (SNA) requires separate measures for the input contributions of a commercial property structure and the associated land plot.[2] For the most part, this decomposition problem has been neglected in the commercial property academic literature, which has focused on the total investment return of a commercial property project; see for example Gatzlaff and Geltner (1998), Fisher, Geltner and Pollakowski (2007) and Bokhari and Geltner (2012) (2014). The decomposition of property value into land and structure components is also important for tax purposes; i.e., many countries impose differential property taxes on the land and structure values of the property so it is important to provide conceptually sound methods for accomplishing this decomposition. Furthermore, once structure values for a property have been determined, then depreciation of the structure can be defined. The measurement of depreciation is

[2] For countries that provide estimates of Multifactor or Total Factor Productivity (TFP) growth, business land is an important input that is measured poorly for most OECD countries (with the exceptions of Korea and Japan). The Penn World Tables and the World KLEMS data bases do not even have land as an input in their estimates of country and sectoral productivity. See Jorgenson and Griliches (1967) for the concept of TFP.

important for the implementation of a business income tax.[3] A further contribution of the present paper is that it addresses the difficult problems associated with the amortization of property goodwill; i.e., a commercial property may generate profits that more than cover the project's cost of capital. The capitalized excess profits or goodwill need to be amortized over the lifetime of the project. The paper draws on the recent contributions of Cairns (2013) to address this problem of amortizing an intangible asset.

Once a plot of land has been purchased and a building is constructed on it, these initial fixed costs must be distributed across the lifetime of the structure in some way. Diewert (2005; 480) and Cairns (2013; 634) called this distribution or amortization problem *the fundamental problem of accounting*. For assets which actively trade in second hand markets, this intertemporal cost allocation problem was solved in a satisfactory way using the user cost concept that was developed by Jorgenson (1963) (1989) and his coworkers.[4] However, commercial buildings do not trade freely in second hand markets and moreover, taking into account the fact that each building sits on a unique plot of land, commercial properties are far from being homogeneous and so the usual user cost methodology does not work in this context. Thus we will develop an alternative methodology for solving this amortization problem, one that is based on the work of Hicks (1946) and Cairns (2013).

The reader should be warned at the outset that there are some important limitations of our analysis in that we assume that the property developer can form expectations about future cash flows that the property can generate along with expectations about future interest rates that it will face. We also assume that the developer can form expectations about what the undeveloped land price for the property would be in future periods in its best use, excluding the use of the property by the developer.[5] These assumptions are similar to the

[3] The problems of decomposing property value into land and structure components has recently emerged as a problem area for the determination of international accounting standards. Depreciation of sunk cost assets also plays an important role in regulatory economics.
[4] See also the influential paper by Hulten and Wykoff (1981).
[5] This assumption is a crucial one and is not consistent with the residual land approach (which is discussed in section 6) to decomposing property values into land and structure components. However, we note that many local governments around the world are able to form commercial property assessments on a periodic basis that have structure and land components. These assessments are contestable; i.e., the assessment

assumptions made by Hicks (1946) when he developed his intertemporal production theory.

The rest of the paper is structured as follows. Section 2 introduces the notation and assumptions that define our simplified commercial property project. Section 3 studies the problem of determining when the structure should be demolished. We also discuss the problem of defining obsolescence in this section. Section 4 shows how period-by-period asset values, user costs and depreciation schedules can be determined for the project as a whole. Section 5 decomposes these value aggregates into land and non-land components. The case where the project earns pure profits is analyzed in Section 6 where the aggregate values are decomposed into additive land, structure and goodwill components. Section 7 further decomposes the value components derived in section 6 into price and quantity components and section 8 concludes.

2. The Commercial Property Project

We assume that a group of investors has either purchased a commercial property building at the end of period 0 (or the beginning of period 1) or has constructed a new building which is just ready for occupancy at the end of period 0.[6] We assume that the total actual cost of the structure at the beginning of period 1 is known to the investor group and is $C_S^0 > 0$, and the opportunity cost value of the land plot at the beginning of period 1 is $V_L^0 > 0$. The *total initial cost of the commercial property*, C^0, is then defined as

$$(1)\ C^0 \equiv C_S^0 + V_L^0.$$

authority will want to make the assessments as high as possible (to maximize property tax revenue) while the owners of commercial properties will want to make the assessments as small as possible to minimize tax payments. Appeal procedures should ensure that the assessments are at least approximately “fair”. Finally, without this assumption, we cannot determine the optimal length of life for a structure.

[6] The length of each period will typically be a quarter if our analysis is applied empirically because, usually, information on building cash flows will only be available on a quarterly (or annual) basis. Our model is a formalization of a standard commercial investment property appraisal model where the cash flow holding period is assumed to coincide with the redevelopment point.

Time is divided up into discrete periods, t = 0, 1, 2, ... and we assume that the *end of period t value of the land plot* is expected to be $V_L{}^t$ for t = 1, 2, ... This end of period t land price is for the raw land (without the structure). We assume that the *demolition cost* for tearing down the structure at the end of period t is $C_D{}^t$. Thus the investors form definite expectations about demolition costs and the price movements for the land plot that the structure utilizes.[7] It will be convenient (when forming user costs) to relate these expected land values to period-by-period *land price inflation rates* i_t; i.e., we assume that the period t land prices $V_L{}^t$ and land inflation rates i_t satisfy the following equations, with $1+i_t > 0$ for all t:

$$(2)\ V_L{}^t = (1+i_1)(1+i_2)...(1+i_t)V_L{}^0\ ; \qquad t = 1,2, ...$$

We assume that the *beginning of period t cost of capital* (or interest rate) that the investors face is $r_t > 0$ for t = 1, 2, ... Finally, we assume that the building is expected to generate Net Operating Income (or cash flow) equal to $N^t \geq 0$, which following Peasnell (1981) and Diewert (2005; 485) we assume to be realized at the end of each period t = 1, 2, ... Thus the information set that we assume is known to the investors consists of the building cost $C_S{}^0$, the sequence of end of period land values $V_L{}^t$ (or equivalently $V_L{}^0$ and the sequence of land inflation rates i_t), the sequence of end of period t demolition costs $C_D{}^t$, the sequence of one period interest rates r_t and the sequence of cash flows N^t.

Using the above information set, we can define an expected discounted profit maximization problem for each choice of time period t = 1, 2, ... Problem t assumes that the firm demolishes the structure at the end of period t, at which time the structure has no value, but of course the land will have (expected) value $V_L{}^t$. The resulting *expected discounted profit* (Π^t) for the investor group will then be defined as follows:[8]

[7] As mentioned in the previous section, this is a crucial assumption. Without making this assumption, we cannot determine the optimal expected life of the structure. Basically, we assume that the expected price of land at any future date is the expected value for the land in its next best alternative use.

[8] Our analysis utilizes the intertemporal production plan methodology that was pioneered by Hicks (1946). It should be noted that we include planned capital expenditures for period t as a negative contribution to period t cash flow, N^t. In principle, capital expenditures should be capitalized since they affect cash flows in subsequent periods. However, the process of amortizing capital expenditures requires estimates of how much future cash flows are increased due to the capital expenditure. Since capitalizing capital expenditures

$$(3)\ \Pi^t \equiv -C_S{}^0 - V_L{}^0 + \alpha_1 N^1 + \alpha_2 N^2 + ... + \alpha_t N^t + \alpha_t\beta_t V_L{}^0 - \alpha_t C_D{}^t\ ; \qquad t = 1,2, ...$$

where the α_t and β_t are defined recursively as follows:

$$(4)\ \alpha_1 \equiv (1+r_1)^{-1}\ ;\ \alpha_t \equiv (1+r_t)^{-1}\alpha_{t-1} \text{ for } t = 2, 3, ...\ ;$$
$$(5)\ \beta_1 \equiv (1+i_1)\ ; \quad \beta_t \equiv (1+i_t)\beta_{t-1} \quad \text{for } t = 2, 3, ...\ .$$

Thus Π^t is the sum of the discounted cash flows that the property is expected to generate over time periods 1 to t, $\alpha_1 N^1 + \alpha_2 N^2 + ... + \alpha_t N^t$, plus the discounted expected land value of the property at the end of period t, $\alpha_t\beta_t V_L{}^0 = \alpha_t V_L{}^t = (1+r_1)^{-1}(1+r_2)^{-1}... (1+r_t)^{-1}V_L{}^t$, less the initial value of the structure at the beginning of period 1, $C_S{}^0$, less the market value of the land at the beginning of period 1, $V_L{}^0$ and less expected discounted demolition costs, $\alpha_t C_D{}^t$.

3. Choosing the Length of Life of the Structure

We assume that the sequence of Π^t is maximized at t equal to $T \geq 1$.[9] We also assume that Π^T is nonnegative:[10]

$$(6)\ \ \Pi^T \equiv -C_S{}^0 - V_L{}^0 + \alpha_1 N^1 + \alpha_2 N^2 + ... + \alpha_T N^T + \alpha_T(V_L{}^T - C_D{}^T) \geq 0.$$

Thus T is the *endogenously determined expected length of life for the structure*. Note that the determination of the length of life of the structure is not a simple matter of determining when the building will collapse due to the effects of aging and use: it is an

does not affect our analysis for the determination of the date of structure demolition, we will not deal with the added complexity associated with the capitalization process in the present paper. We assume that expected insurance costs and property tax payments are included in the cash flows N^t as negative items.

[9] If there is more than one maximizing t, choose the smallest one. Later we will look at conditions that ensure a finite maximizing T.

[10] Our analysis largely follows that of Cairns (2013; 639) who noted that unless the inequality in (6) is satisfied, investors will not participate in the project: "This participation constraint provides that the cash flows of the project allow investors to recover their sunk investment as a stream of quasi-rents or user costs."

economic decision that depends on all of the variables which were defined in the previous section.

Return to the general expression for Π^t defined by (3) and evaluate $\Pi^{t+1} - \Pi^t$ using definitions (4) and (5):

$$(7)\ \Pi^{t+1} - \Pi^t = \alpha_{t+1} N^{t+1} + \alpha_{t+1} (V_L^{t+1} - C_D^{t+1}) - \alpha_t (V_L^t - C_D^t)$$
$$= \alpha_t(1+r_{t+1})^{-1} [N^{t+1} - (r_{t+1} - i_{t+1})V_L^t + (r_{t+1} - i_{D,t+1})C_D^t].$$

where $i_{D,t+1} \equiv C_D^{t+1}/C_D^t - 1$ is the period t+1 inflation rate for the cost of demolishing the building at the end of period t+1 over the cost of demolishing the building at the end of period t. Thus $\Pi^{t+1} - \Pi^t$ will be negative if the following inequality is satisfied:

$$(8)\ N^{t+1} < (r_{t+1} - i_{t+1})V_L^t - (r_{t+1} - i_{D,t+1})C_D^t .$$

If $r_t > i_t$ and $N^t = 0$ for large t and the r_t and demolition costs C_D^t remain bounded as t tends to infinity while the value of land V_L^t increases indefinitely, then (8) will be satisfied for all large t.[11] The inequalities (8) for large t imply that a finite T will exist where Π^t is maximized and thus the optimal length of life for the building will be well determined.[12] Typically, the cash flows generated by the structure will decline over time due to wear and tear depreciation and increased maintenance and renovation expenses. However, if the building is a retail outlet, changes in the demand for the products that are sold can also affect cash flows. We do not attempt to model the factors that influence (expected) cash flows: we simply work out the implications of these expectations.

The above algebra shows that the eventual decline in the asset value of the property as the life of the building is extended depends entirely on the sequences of cash flows N^t,

[11] This assumption is a reasonable *long run* assumption since if the expected land inflation rate i_t exceeded the period t cost of capital for all large t, it would pay investors to simply purchase land (and not build a structure) and make an infinite stream of period-by-period (imputed) profits. If all investors held expectations such that $i_t > r_t$, the price of land would be bid up to eliminate these effortless profits. In the *short run*, land price bubbles tend to occur from time to time.

[12] If the optimal T is unique, then the inequality (8) evaluated at t =T will hold.

demolition costs C_D^t, one period interest rates r_t and one period expected land inflation rates i_t. However, the initial land value V_L^0 and the initial structure cost C_S^0 do play a role once the optimal length of building life has been determined since we also require that the project be profitable; i.e., that $\Pi^T \geq 0$ for the project to proceed.

The above analysis shows that the decision to retire a commercial property structure is an endogenous one that is not determined exogenously by wear and tear physical deterioration of the building. The retirement decision depends crucially on the intertemporal pattern of cash flows generated by the building and on the movements in the price of the land plot over time. Thus our theory of structure retirement (and depreciation as will be seen below) is somewhat different from existing theories in the real estate literature about the retirement and depreciation of a commercial property structure.[13]

The property development project that we have just analyzed is just one of many projects that the developer may consider. There will be a feasible universe of structures that the developer could place on the land plot. Choose the structure that generates the highest possible discounted profits over the lifetime of each such building. Now apply the algebra that was just developed to this best use structure.

A final problem that we address in this section is: what happens if expectations change? We first look at what happens if expectations are realized but time marches forward. Suppose the optimal project has length T periods and the expected discounted profits generated by this project are Π^T defined by (6). Suppose that expectations about interest

[13] Baum (1991; 59) and Dixon, Crosby and Law (1999; 162) distinguished *physical deterioration* and *obsolescence* of the structure as the primary causes of depreciation (decline in the value of the building over time). In our approach, it is increases in the price of land along with falls in cash flows that drives obsolescence. Dixon, Crosby and Law (1999; 168-170) also noted that *rental decline* (i.e., falls in net operating income as the building ages) contributed to building depreciation and of course, this effect is also part of our approach. Crosby, Devaney and Law (2012) investigate the rental decline phenomenon for UK commercial properties and they also take into account post construction capital expenditures on the properties. When depreciation rates for commercial properties are reported in the real estate literature, they are generally reported as fraction of property value (which includes the value of the land plot). Thus these reported property depreciation rates will understate depreciation rates on the structure by itself. It should be noted that the determination of structure depreciation rates was not the main focus of this research.

rates, inflation rates cash flows for period 1 are realized. Now look at the project from the perspective of time period 2. Let the expected discounted profits of the project from the perspective of the beginning of period 2, lasting to the end of period t when the structure is demolished, Π_2^t, be defined as follows:

$$
\begin{aligned}
(9)\ \Pi_2^t &\equiv -(C_S^0+V_L^0)(1+r_1) + N^1 + (1+r_1)[\alpha_2 N^2 + \ldots + \alpha_t N^t + \alpha_t (V_L^t - C_D^t)]\ ; \quad t = 1,2, \ldots \\
&= -(C_S^0+V_L^0)(1+r_1) + N^1 + (1+r_2)^{-1}N^2 + \ldots + (1+r_2)^{-1}\ldots(1+r_t)^{-1}[N^t + V_L^t - C_D^t] \\
&= (1+r_1)\,\Pi^t
\end{aligned}
$$

where Π^t is expected discounted profits (from the perspective of the beginning of period 1) for the project defined by (3). The interpretation of the first two equations in (9) runs as follows. At the beginning of period 1, the financial capital invested in the project is $C_S^0+V_L^0$. At the end of period 1, this investor liability has grown to the original investment value times one plus the period 1 cost of capital, $(C_S^0+V_L^0)(1+r_1)$.[14] However, this end of period liability is reduced by the amount of period 1 cash flow, N^1, which we assume is paid back to the investors.[15] Thus the net amount of financial capital that investors have committed to the project at the beginning of period 2 is $(C_S^0+V_L^0)(1+r_1) - N^1$. The remaining terms in the definition of Π_2^t are the future discounted cash flows generated by the project, $(1+r_2)^{-1}N^2 + \ldots + (1+r_2)^{-1}\ldots(1+r_t)^{-1}[N^t + (V_L^t - C_D^t)]$, where the cash flows start at period 2 and end at period t and are discounted to the beginning of period 2. The last equation in (9) shows that when the expectations held at the beginning of period 1 are realized for period 1 and the expectations for subsequent periods are unchanged, then $\Pi_2^t = (1+r_1)\Pi^t$ where Π^t equals the expected value of the project discounted to the beginning of period 1 and Π_2^t equals the expected value of the project discounted to the beginning of period 2. Thus it can be seen that if expectations about period 1 variables are realized and expectations about future variables remain unchanged, then looking at the project from the perspective of the beginning of period 2 (instead of

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[14] For simplicity of interpretation, suppose all financial capital is raised by one period debt instruments.

[15] It could be the case that period 1 cash flow, N^1, is negative. In this case, we assume that the project investors cover this cash shortfall at the end of period 1 and so the total financial investment in the project at the beginning of period 2 is $(C_S^0+V_L^0)(1+r_1) - N^1$ where $-N^1 > 0$. This investment must earn the period 2 cost of capital, r_2.

period 1) will not change the optimality of demolishing the structure at the end of period T.[16]

The above result about the invariance of the optimal demolition date can be generalized to situations where expectations have not been realized for past periods but remain the same for future periods. Let the expected discounted profits of the project from the perspective of the beginning of say period 3, lasting to the end of period t when the structure is demolished, $\Pi_3{}^t$, be defined for t > 3 as follows, assuming that expectations about period 1 and 2 variables have been realized:[17]

$$
\begin{aligned}
(10)\ \Pi_3{}^t &\equiv -\,(C_S{}^0+V_L{}^0)(1+r_1)(1+r_2)+ N^1(1+r_2) + N^2 \\
&\qquad + (1+r_3)^{-1}N^3 + \ldots + (1+r_3)^{-1}\ldots(1+r_t)^{-1}(N^t + V_L{}^t - C_D{}^t) \\
&= (1+r_1)(1+r_2)\Pi^t
\end{aligned}
$$

where Π^t is defined by (3). Now suppose that expectations for period 1 and 2 interest rates and cash flows have not necessarily been realized but expectations for these variables remain unchanged for periods greater than 2. Denote the realized interest rates by $r_1{}^*$ and $r_2{}^*$ and the realized period 1 and 2 cash flows by N^{1*} and N^{2*}. Define the expected discounted profits of the project from the perspective of the beginning of period 3, lasting to the end of period t when the structure is demolished, $\Pi_3{}^{t*}$, for t > 3 as follows, but where the beginning of period 3 project liabilities are defined using the realized interest rates and cash flows for periods 1 and 2:

$$
\begin{aligned}
(11)\ \Pi_3{}^{t*} &\equiv -\,(C_S{}^0+V_L{}^0)(1+r_1{}^*)(1+r_2{}^*)+ N^{1*}(1+r_2{}^*) + N^{2*} \\
&\qquad + (1+r_3)^{-1}N^3 + \ldots + (1+r_3)^{-1}\ldots(1+r_t)^{-1}(N^t + V_L{}^t - C_D{}^t).
\end{aligned}
$$

[16] The same conclusion holds for subsequent periods if expectations are realized for subsequent periods; i.e., under these conditions, the optimal date for demolition of the structure remains unchanged.

[17] Suppose T > 3 is the optimal project length, all expectations are realized, the project is financed by debt and $\Pi^T = 0$. Then net debt at the beginning of period 3 is equal to $(C_S{}^0+V_L{}^0)(1+r_1)(1+r_2) - N^1(1+r_2) - N^2$. Then (10) for t = T and $\Pi^T = 0$ implies that this net debt is equal to project future expected cash flows, $(1+r_3)^{-1}N^3 + \ldots + (1+r_3)^{-1}\ldots(1+r_T)^{-1}(N^T + V_L{}^T - C_D{}^T)$. Thus under these conditions, the project's value of liabilities at the beginning of period 3 is equal to the value of project future expected cash flows. This result can be generalized to periods t other than period 3 if t < T.

Using (10) and (11), it can be shown that Π_3^{t*} is equal to a constant plus $(1+r_1)(1+r_2)\Pi^t$ for all t > 3. Thus if expectations for interest rates, cash flows and the price of raw land remain unchanged for these variables for periods t > 2, then it can be shown that the optimal date for tearing down the structure will remain unchanged (and is equal to T).[18]

If expectations about future cash flows and interest rates change, then the optimal time to demolish the structure may change. But even if expectations about the project cash flows are realized, it may become profitable to demolish the structure before the optimal time period T due to the emergence of new technologies or new uses for the land plot that were not anticipated when the property was developed. We will conclude this section by developing a criterion for unanticipated demolition due to the introduction of a new use for the property.

Suppose an (optimal as of time 0) project is implemented at the beginning of period 1 and the expected profit of the project if the structure is demolished at time t > 1 is Π^t defined by (3). Let the optimal life of the structure be defined by choosing the t that maximizes Π^t over all t > 1. Suppose the optimal t is T > 2.[19] Now consider a period τ between 1 and T and define the stream of future expected cash flows that the project will yield, discounted to the beginning of period τ, say Γ_τ^T:

$$(12)\ \Gamma_\tau^T \equiv (1+r_\tau)^{-1}N^\tau + (1+r_\tau)^{-1}(1+r_{\tau+1})^{-1}N^{\tau+1} + ... + (1+r_\tau)^{-1}...(1+r_T)^{-1}[N^T + V_L^T - C_D^T].$$

The investors in the project own the land and structure and so these costs are fixed as time goes forward. Suppose that during period $\tau-1$, a new unanticipated use for the land plot is found and a new structure that embodies this new use or technology could be built

[18] This result is subject to some practical limitations: (i) if period 1 and 2 actual cash flows are significantly below expectations, then the cost of capital for subsequent periods is likely to increase; (ii) if period 1 and 2 cash flows are negative, then investors may not want to cover the deficits and the project may go bankrupt. However, in the second case, if the new owners of the project have the same expectations about future cash flows and interest rates as the initial owners, then the optimal date of demolition does not change.

[19] As indicated above, if expectations about future cash flows, interest rates and the price of land do not change over time, the optimal T will remain unchanged as well.

at a cost of $c_S^{\tau-1}$ and the cost of demolishing the structure at the end of a future period t is c_D^t. Suppose further that the period t expected cash flow that this new structure could generate is n_τ^t for $t \geq \tau$. Now consider the expected net discounted cash flow, γ_τ^t, that the new structure could generate if it were built in period $\tau-1$ and demolished at the end of period t. The existing structure would have to be demolished at a cost of $C_D^{\tau-1}$ and the new structure built at a cost of $c_S^{\tau-1}$. The benefits that the new structure would be able to generate are the discounted cash flows up to and including period t plus the value of the land plot at the end of period t, V_L^t, less the expected demolition costs for the new structure at the end of period t, c_D^t. Thus the net benefits that the new structure built during period $\tau - 1$ could generate if it were demolished at the end of period t are as follows:[20]

$$(13)\ \gamma_\tau^t \equiv - C_D^{\tau-1} - c_S^{\tau-1} + (1+r_\tau)^{-1} n_\tau^\tau + ... + (1+r_\tau)^{-1} ... (1+r_t)^{-1} [n_\tau^t + V_L^t - c_D^t]\ ; \quad t \geq \tau \geq 2.$$

Let $T(\tau)$ be the smallest t which maximizes the γ_τ^t over all $t \geq \tau$. This time period defines the optimal life of the new structure if it is built. Compare the net benefits at time τ of sticking to the existing structure, Γ_τ^T, to the net benefits $\gamma_\tau^{T(\tau)}$ of demolishing the old structure at time $\tau-1$ and building the new structure.[21] It will be optimal to demolish the old structure before its initial optimal time T if there exists a τ such that $T(\tau) < T$ and the following inequality holds:

$$(14)\ \gamma_\tau^{T(\tau)} > \Gamma_\tau^T.$$

If the inequality (14) holds, then the expected cash flow generated by the new structure discounted to the beginning of period τ, $(1+r_\tau)^{-1} n_\tau^\tau + ... + (1+r_\tau)^{-1} ... (1+r_{T(\tau)})^{-1} [n_{T(\tau)}^{T(\tau)} + V_L^{T(\tau)} - c_D^{T(\tau)}]$, exceeds the expected cash flow for the old structure discounted to the

[20] In order to simplify notation, we have assumed that the sequence of expected interest rates (or costs of capital) r_t that the property firm faces is the same for the old and new projects. However, the new project may have different risk characteristics and if so, the discount rates in (13) should be replaced by the sequence of interest rates that are appropriate for the new project.

[21] Note that both of these net benefit measures do not take into account the opportunity cost of land at the beginning of period τ. But this opportunity cost is common to both options and so it does not need to be considered.

beginning of period τ, $(1+r_\tau)^{-1}N^\tau + ... + (1+r_\tau)^{-1}...(1+r_T)^{-1}[N^T + V_L{}^T - C_D{}^T]$, plus the cost of demolishing the old structure during period $\tau-1$, $C_D{}^{\tau-1}$, plus the cost of constructing the new structure during period $\tau-1$, $c_S{}^{\tau-1}$. If the inequality (14) is reversed, then it will be optimal to keep the old structure for at least another period.

Suppose that $\gamma_\tau{}^{T(\tau)} < \Gamma_\tau{}^T$ so that it is not profitable for the developer to tear down the old structure during period $\tau-1$ and replace it with a new structure. To see whether it would be optimal to demolish the old structure one period later during period τ, we need to compare the optimal cash flow generated by the old structure discounted to the beginning of period $\tau+1$, $\Gamma_{\tau+1}{}^T \equiv (1+r_\tau)^{-1}N^\tau + ... + (1+r_\tau)^{-1}...(1+r_T)^{-1}[N^T + V_L{}^T - C_D{}^T]$, to $\gamma_{\tau+1}{}^{T(\tau+1)} \equiv - C_D{}^{\tau-1} - c_S{}^{\tau-1} + (1+r_{\tau+1})^{-1}n_{\tau+1}{}^{\tau+1} + ... + (1+r_{\tau+1})^{-1} ... (1+r_{T(\tau+1)})^{-1}[n_{T(\tau+1)}{}^{T(\tau+1)} + V_L{}^{T(\tau+1)} - c_D{}^{T(\tau+1)}]$, the optimal net benefits that the new structure built during period τ could generate. Comparing $\Gamma_{\tau+1}{}^T$ to $\Gamma_\tau{}^T$, it can be seen that generally, $\Gamma_\tau{}^T$ will be less than $\Gamma_{\tau+1}{}^T$ because the cash flow N^τ that was generated by the old structure during period τ is dropped from the discounted sum of cash flows that define $\Gamma_{\tau+1}{}^T$. Looking at the cash flows that could be generated by the new structure starting at period $\tau+1$, it is likely that the new sequence of cash flows, $n_{\tau+1}{}^{\tau+1}$, $n_{\tau+1}{}^{\tau+2}$, ... , $n_{\tau+1}{}^{t+1}$, is approximately equal to the sequence of cash flows that would have been generated if the new building were built in period $\tau-1$, $n_\tau{}^\tau$, $n_\tau{}^\tau$, ... , $n_\tau{}^t$. Thus if interest rates r_t are approximately constant and demolition and building costs do not change much from one period to the next, it can be seen that the net benefits of building the new structure one period later, $\gamma_{\tau+1}{}^{T(\tau+1)}$, will be approximately equal to the earlier net benefits estimate, $\gamma_\tau{}^{T(\tau)}$. Thus it can be seen that even if it is not optimal to demolish the old structure in the first period τ when the new technology becomes known, over time, it will become more likely that it will be optimal to demolish the old structure before the initial optimal date of demolition due to the fact that the discounted stream of cash flows that the old structure is expected to generate will tend to decrease over time whereas the net benefits generated by the new project will tend to remain roughly constant over time. We regard the premature demolition of a structure due to the emergence of a new technology (or new use for the land plot) as the early retirement of the building due to (unanticipated) *obsolescence*.[22] We note that our theory of obsolescence is rather complex!

In the following sections, we will consider the problems associated with the measurement of depreciation under the assumption that expectations about future variables are realized.

4. Period-by-Period Aggregate Asset Values, User Benefits and Depreciation

We assume that the optimal length of life of the structure T has been determined and that the nonnegative discounted profits constraint (6) holds. Our task in this section is to determine the sequence of project asset values and the changes in asset value over each time period.

The sequence of expected *end-of-period t project asset values* A^t can be defined as follows:

$$
\begin{aligned}
(15)\quad A^0 &\equiv \alpha_1 N^1 + \alpha_2 N^2 + \ldots + \alpha_T N^T + \alpha_T(V_L{}^T - C_D{}^T) ;\\
A^1 &\equiv (1+r_1)[\alpha_2 N^2 + \alpha_3 N^3 + \ldots + \alpha_T N^T + \alpha_T(V_L{}^T - C_D{}^T)] ;\\
A^2 &\equiv (1+r_1)(1+r_2)[\alpha_3 N^3 + \alpha_4 N^4 + \ldots + \alpha_T N^T + \alpha_T(V_L{}^T - C_D{}^T)] ;\\
&\ldots\\
A^{T-1} &\equiv (1+r_1)(1+r_2)\ldots(1+r_{T-1})[\alpha_{T-1} N^{T-1} + \alpha_T N^T + \alpha_T(V_L{}^T - C_D{}^T)]\\
&= N^{T-1} + (1+r_{T-1})^{-1}(N^T + V_L{}^T - C_D{}^T) ;\\
A^T &\equiv N^T + V_L{}^T - C_D{}^T
\end{aligned}
$$

Thus at the end of period t, the expected property asset value A^t is equal to the expected period t cash flow N^t plus the discounted to the end of period t cash flow for period t+1, $(1+r_{t+1})^{-1}N^{t+1}$, plus the discounted to the end of period t cash flow for period t+2,

[22] Following Hayek (1941; 278) in his discussion with Pigou (1941) on the concept of depreciation, *anticipated or foreseen obsolescence* should be treated in the same way as wear and tear depreciation is treated in economic accounting theory. Hayek gave the example of a machine that produces a fashion good where the machine has a useful life of 10 years but after one year, has only a scrap value due to the produced good going out of fashion. Hayek asserted that this form of obsolescence can be foreseen and hence the machine should be fully depreciated in one year instead of over 10 years.

$(1+r_{t+1})^{-1}(1+r_{t+2})^{-1}N^{t+2}$, ... , plus the discounted to the end of period t cash flow for period T, $(1+r_{t+1})^{-1}(1+r_{t+2})^{-1}...(1+r_T)^{-1}N^T$, plus the discounted to the end of period t expected value of the land plot less the cost of demolishing the structure at the end of period T, $(1+r_{t+1})^{-1}(1+r_{t+2})^{-1}...(1+r_T)^{-1}(V_L{}^T - C_D{}^T)$.

Note that the last T equations in (15) can be rearranged to give us the following relationships between the end of period t asset values A^t and the period t cash flows N^t:

$$
\begin{aligned}
(16)\; N^t &= (1+r_t)A^{t-1} - A^t && t = 1,...,T \\
&= r_t A^{t-1} + (A^{t-1} - A^t) \\
&= r_t A^{t-1} + \Delta^t
\end{aligned}
$$

where r_tA^{t-1} reflects the opportunity costs of the capital that is tied up in the project at the beginning of period t, and Δ^t is the period t expected *asset value change* for the project defined by (14):

$$(17)\; \Delta^t \equiv A^{t-1} - A^t\,; \qquad t = 1,...,T.$$

Thus Δ^t is simply the anticipated decline in asset value of the project from the beginning of period t to the end of period t. In the zero discounted profits case where $\Pi^T = 0$, then Δ^t can be interpreted as *time series depreciation* for the project.[23]

It can be seen that the expressions on the right hand side of (16) are analogous to expressions for the traditional user cost of capital; see Jorgenson (1963)(1989).[24] As our later discussion will show, the expression $r_tA^{t-1} + \Delta^t$ is not necessarily equal to a user cost

[23] The term *time series depreciation* is due to Hill (2000) but the concept dates back to Hotelling (1925; 341). We note that Δ^t incorporates both the effects of wear and tear depreciation (or cross sectional depreciation) and anticipated revaluation; see Hill (2000; 6), Hill and Hill (2003; 617), Diewert (2009; 9) and Cairns (2013; 640). Later in section 7, we will see that time series depreciation for land consists of just revaluation while time series depreciation for the structure will be decomposed into cross sectional depreciation and revaluation components.

[24] The expressions on the right hand side of equations (16) are analogous to end of period user costs; see Diewert (2005; 485) (2009; 8). Baumol, Panzar and Willig (1982; 384) identify $r_t A^{t-1} + \Delta^t$ as the period t *payment to capital*; see also Cairns (2013; 640).

if the project makes profits that are above and beyond the cost of capital for the project. In this latter case, $r_tA^{t-1} + \Delta^t$ can be interpreted as a *user benefit* expression rather than a user cost expression.

Diewert (2009; 3) noted that measuring depreciation for a *sunk cost asset* like a commercial structure is difficult since there are no second hand asset markets for a sunk cost asset that can provide period-by-period opportunity costs in order to value the structure asset as it ages. Sales of commercial properties can provide some information but are infrequent and the sale price is for the combined land and structure. It then seems difficult to obtain a sequence of objective measures of period-by-period depreciation or amortization amounts over the life of the building. Let $N^{t*} \geq 0$ be a *period t amortization amount* for the commercial property for t = 1,2,...,T where the N^{t*} satisfy the following equation:

$$(18)\ \alpha_1 N^{1*} + \alpha_2 N^{2*} + ... + \alpha_T N^{T*} = C_S{}^0 + V_L{}^0.$$

N^{t*} can be interpreted as a payment made to the owners of the project at the end of period t for t = 1,2,...,T. Equation (18) says that the initial project cost, $C_S{}^0 + V_L{}^0$, can be distributed across the T time periods before the building is demolished by the series of period-by-period cost allocations N^{t*} where the discounted value of these cost allocations (to the beginning of period 1) is equal to the project cost. Note that the amortization schedules N^{t*} which satisfy (18) are largely arbitrary; the indeterminancy of amortization schedules for sunk cost assets was noticed by e.g. Peasnell (1981; 54), Schmalensee (1989; 295-296) and Diewert (2009; 9).[25]

In the case where $\Pi^T = 0$, it can be shown that the following intertemporal cost allocations satisfy equation (18):

$$(19)\ N^{t*} \equiv N^t \text{ for } t = 1,2,...,T-1 \text{ and } N^{T*} \equiv N^T + V_L{}^T - C_D{}^T.$$

[25] As shown below, if $\Pi^T = 0$ and we value assets at their market values at the beginning of each period, then the indeterminancy disappears .

Thus if the period t cash flow N^t is distributed back to the owners at the end of each period t and the end-of-period T market value of the land plot $V_L{}^T$ less demolition costs $C_D{}^T$ is also distributed to the owners at the end of period T, then the present value of the resulting sequence of distributions will just be equal to the initial project cost. This distribution pattern is consistent with the sequence of end of period asset values A^t defined by (15) and the depreciation amounts Δ^t defined by (17). This intertemporal allocation of project cost is preferred to any other N^{t*} that satisfies (18) as it is useful for the property firm to value its assets at the end of each period at *market values*. As shown by Diewert (2009; 9-10) and Cairns (2013; 640-641), at the end of period t the market value of the firm's assets will be A^t defined by equation t in (15) (if anticipations are realized) and thus the project depreciation schedule defined by (17) will be uniquely determined.[26]

The Δ^t defined by equations (17) can be interpreted as *aggregate period t time series depreciation allocations* for the project as a whole. In the $\Pi^T = 0$ case, the period t cash flow N^t can be interpreted as a period t *aggregate user cost of capital* value for the property (except that for period T, N^T is replaced by $N^T + V_L{}^T - C_D{}^T$).[27] But for national income accounting purposes, it is necessary to decompose the aggregate asset values A^t into land and structure components, and for productivity accounts, it is also necessary to decompose the aggregate user cost values N^t into land and structure components.

5. The Decomposition of Asset Values into Land and Non-Land Components

We make use of the assumption that the firm forms expectations of the market value of the project land plot at the end of each period t, $V_L{}^t$ for t = 0,1,2,... ,T.[28] The availability

[26] Thus our theory of property depreciation is similar to the definition suggested by Blazenko and Pavlov (2004; 57): "Economic depreciation is the reduced ability of an asset to generate future cash flows."
[27] If $\Pi^T > 0$, then the N^t are not equal to user *cost* allocations since they will contain a pure profit component.
[28] If the project is a Real Estate Investment Trust (REIT), then typically assessed values for the property will be available at the end of each reporting period. The assessed value will often have a decomposition into structure and land values. Assessed values for property taxation purposes always have a structure-land

of this information enables us to define the following end-of-period t expected *user cost for the use of the land* during period t, U_L^t, using the approach of Diewert (1974; 504):

$$(20)\ U_L^t \equiv (1+r_t)V_L^{t-1} - V_L^t \qquad t = 1,\ldots,T$$
$$= r_tV_L^{t-1} + \Delta_L^t$$
$$= (1+r_t)V_L^{t-1} - (1+i_t)V_L^{t-1} \qquad \text{using (2)}$$
$$= (r_t - i_t)V_L^{t-1}\ ,$$

where the period t *change in the asset value of land* (*land time series depreciation*) Δ_L^t is defined as:

$$(21)\ \Delta_L^t \equiv V_L^{t-1} - V_L^t\ ; \qquad t = 1,\ldots,T.$$

Definitions (20) and (21) could be used to provide estimates for the user cost and depreciation of land for national accounts purposes or for productivity studies for the commercial property sector. Note that the last equation in (20) shows that the user cost of land in period t will be negative if the anticipated period t land inflation rate i_t is greater than the period t cost of capital r_t, and Δ_L^t will also be negative. In the short run, this situation can occur but over long periods of time, we expect r_t to exceed i_t.

By making repeated use of the first set of equations in (20), it can be shown that the asset values for land, V_L^t, and the land user costs, U_L^t, satisfy the following discounted present value relationships:

$$(22)\ V_L^0 = \alpha_1U_L^1 + \alpha_2U_L^2 + \ldots + \alpha_TU_L^T + \alpha_TV_L^T\ ;$$
$$V_L^1 = (1+r_1)[\alpha_2U_L^2 + \alpha_3U_L^3 + \ldots + \alpha_TU_L^T + \alpha_TV_L^T]\ ;$$
$$V_L^2 = (1+r_1)(1+r_2)[\alpha_3U_L^3 + \alpha_4U_L^4 + \ldots + \alpha_TU_L^T + \alpha_TV_L^T]\ ;$$
$$\ldots$$
$$V_L^{T-1} = (1+r_1)(1+r_2)\ldots(1+r_{T-1})[\alpha_{T-1}U_L^{T-1}+\alpha_TU_L^T+\alpha_TV_L^T]$$

decomposition but these official assessed values may not be based on market values and they may not be up to date.

$$= U_L^{T-1} + (1+r_{T-1})^{-1}(U_L^T+V_L^T) ;$$

$$V_L^T \quad = V_L^T.$$

Thus the period-by-period discounted present values of the user cost charges defined by (20) are consistent with the exogenously given sequence of expected land values.

We use the aggregate period t project asset values A^t of equations (15) along with the land asset values V_L^t in order to define the *end-of-period t non-land or residual asset value* V_R^t as follows:

$$(23)\ V_R^t \equiv A^t - V_L^t ; \qquad t = 0,1,2,...,T.$$

Once the project residual asset values V_R^t have been defined by (23), period t *residual user benefit* U_R^t and *asset value change* Δ_R^t can be defined as follows:

$$(24)\ U_R^t \equiv (1+r_t)V_R^{t-1} - V_R^t ; \qquad t = 1,2,...,T$$

$$= r_tV_R^{t-1} + \Delta_R^t ,$$

where Δ_R^t is defined as

$$(25)\ \Delta_R^t \equiv V_R^{t-1} - V_R^t ; \qquad t = 1,...,T.$$

It is straightforward to show that definitions (23)-(25) and our previous definitions provide us with additive decompositions of aggregate asset values A^t, user benefits N^t and asset value changes Δ^t; i.e., the above definitions imply the following equations:

$$(26)\ A^t = V_L^t + V_R^t ; \qquad t = 0,1,...,T;$$

$$(27)\ N^t = U_L^t + U_R^t ; \qquad t = 1,...,T;$$

$$(28)\ \Delta^t = \Delta_L^t + \Delta_R^t ; \qquad t = 1,...,T.$$

If expected discounted profits Π^T are equal to zero, then the end-of-period t non-land asset value V_R^t can be interpreted as the end-of-*period t structure asset value* $V_S^t \equiv V_R^t$, the period t non-land user benefit term U_R^t can be interpreted as the period t *user cost of the structure* and the period t non-land change in asset value Δ_R^t can be interpreted as period t *time series depreciation for the structure* $\Delta_S^t \equiv \Delta_R^t$. Thus in the case where $\Pi^T = 0$, we have decomposed property values, user costs and time series depreciation amounts into land and structure components.[29] In the following section, we tackle the more difficult case where Π^T is positive.

6. The Decomposition of Asset Values into Land, Structure and Goodwill Components

In this section, we assume that expected discounted project profits are positive; i.e., we assume that:

$$(29)\ \Pi^T = -C_S^0 - V_L^0 + A^0 > 0$$

where C_S^0 is the structure cost, V_L^0 is the end of period 0 land cost and A^0 is end of period 0 expected asset value (defined in equations (15)). Thus the end of period 0 value of liabilities, $C_S^0 + V_L^0$, is less than the end of period 0 value of project asset, A^0. In this situation, it is natural to define an *intangible goodwill asset*, V_G^0, that is equal to the value of assets less tangible liabilities; i.e., define V_G^0 as follows:

$$\begin{aligned}(30)\ V_G^0 &\equiv A^0 - C_S^0 - V_L^0 \\ &= \Pi^T.\end{aligned}$$

[29] A property project has at least two sunk costs: a land component and a structure component. When there are two or more sunk cost assets in a project, Cairns (2013; 644) showed that asset values, user costs and depreciation schedules could not be uniquely determined for the *separate* assets. The reason why we are able to avoid the Cairns impossibility result for the case where $\Pi^T = 0$ is that we assume extra information; that period-by-period exogenous asset values for one of our two assets are available.

Equation (26) for t = 0 implies that $A^0 = V_R^0 + V_L^0$ where V_R^0 is the non-land initial asset value for the project. Equation (30) implies that $A^0 = C_S^0 + V_G^0 + V_L^0$ and thus we deduce that the initial non-land asset value V_R^0 is equal to the sum of the initial goodwill asset V_G^0 and the structure cost C_S^0:

(31) $V_R^0 = V_G^0 + C_S^0$.

Define the *shares of the initial goodwill asset and structure cost* in initial non-land asset value V_R^0, s_G^0 and s_S^0, as follows:

(32) $s_G^0 \equiv V_G^0/V_R^0$; $s_S^0 \equiv C_S^0/V_R^0 = (1-s_G^0)$.

Recall that the non-land user benefits U_R^t were defined in the previous section by equations (24). We will use the shares defined by (32) above to decompose these user benefits into *goodwill and structure user cost components*, U_G^t and U_S^t, as follows:

(33) $U_G^t \equiv s_G^0 U_R^t$; $U_S^t \equiv s_S^0 U_R^t = (1-s_G^0)\, U_R^t$; t = 1,...,T.

The shares defined by (32) can also be used to decompose the non-land asset values V_R^t defined by (23) into end of period t *goodwill and structure asset values*, V_G^t and V_S^t:

(34) $V_G^t \equiv s_G^0 V_R^t$; $V_S^t \equiv s_S^0 V_R^t$; t = 0, 1,...,T.

It can be shown that (30)-(34) and the definitions in the previous section can be used to derive the following relationships between the user costs and the asset values defined by equations (33) and (34):

(35) $U_G^t = (1+r_t)V_G^{t-1} - V_G^t = r_t V_G^{t-1} + \Delta_G^t$; $U_S^t = (1+r_t)V_S^{t-1} - V_S^t = r_t V_S^{t-1} + \Delta_S^t$; t = 1,...,T,

where period t *time series depreciation for goodwill and structures*, Δ_G^t and Δ_S^t are defined as follows:

(36) $\Delta_G{}^t \equiv V_G{}^{t-1} - V_G{}^t$; $\Delta_S{}^t \equiv V_S{}^{t-1} - V_S{}^t$; $\quad t = 1,...,T.$

We have labelled $U_G{}^t$ and $U_S{}^t$ as *user costs* (instead of user benefits) because we can show that these user costs provide for an intertemporal allocation of the initial goodwill asset value $V_G{}^0$ and for the initial structure cost $C_S{}^0$; i.e., the following equations are satisfied by the $U_G{}^t$ and $U_S{}^t$:

(37) $V_G{}^0 = (1+r_1)^{-1}U_G{}^1 + (1+r_1)^{-1}(1+r_2)^{-1}U_G{}^2 + ...+ (1+r_1)^{-1}...(1+r_T)^{-1}U_G{}^T$;

(38) $C_S{}^0 = (1+r_1)^{-1}U_S{}^1 + (1+r_1)^{-1}(1+r_2)^{-1}U_S{}^2 + ...+ (1+r_1)^{-1}...(1+r_T)^{-1}U_S{}^T$.

It can be shown that the following additive decompositions for the period t aggregate commercial property project values hold:

(39) $A^t = V_G{}^t + V_S{}^t + V_L{}^t$; $\quad t = 0,1,...,T;$

(40) $N^t = U_G{}^t + U_S{}^t + U_L{}^t$; $\quad t = 1,...,T;$

(41) $\Delta^t = \Delta_G{}^t + \Delta_S{}^t + \Delta_L{}^t$; $\quad t = 1,...,T.$

We have now succeeded in decomposing commercial property values into land, structure and goodwill components. In the case where the property project makes profits that more than cover the cost of financial capital, we ended up with a depreciable goodwill asset that absorbs up these excess returns.[30]

To further complicate our discussion of the goodwill asset, it should be noted that the Cairns (2013; 644) Impossibility Theorem applies to the residual asset;[31] i.e., only the aggregate value of the joint goodwill and structure asset is uniquely determined under our

[30] There are no definitive guidelines on how this goodwill asset should be treated in the System of National Accounts. It could be treated as a separate intangible asset in the Balance Sheet Accounts or it could be absorbed into the structure component of the accounts or into the land component of the accounts. Any one of these three treatments could be justified.

[31] "If there are two or more types of comprehensive capital (possibly including a form of intangible capital as a source of profit), economic rental schedules and their implied economic depreciation schedules are not unique. Such schedules apply to all forms of comprehensive capital. The sum of the rentals in any period is the cash flow or producer's surplus." R.D. Cairns (2013; 644).

assumptions. Thus instead of using the constant shares s_G^0 and s_S^0 defined by (32) in order to decompose the non-land user benefits U_R^t into goodwill and structure components by equations (33), we could use the following equations for the decomposition:

$$(42)\ U_G^t \equiv s_G^t U_R^t\ ;\ U_S^t \equiv (1-s_G^t)U_R^t\ ; \qquad t = 1,...,T$$

where the period t goodwill shares of U_R^t, the s_G^t, satisfy the following restrictions:

$$(43)\ 0 \leq s_G^t \leq 1\ ; \qquad t = 1,...,T;$$
$$(44)\ V_G^0 = s_G^1(1+r_1)^{-1}U_R^1 + s_G^2(1+r_1)^{-1}(1+r_2)^{-1}U_R^2 + ...+ s_G^T(1+r_1)^{-1}... (1+r_T)^{-1}U_R^T.$$

We know V_G^0 and V_S^0 (= C_S^0). Given V_G^0, V_S^0 and the U_G^t and U_S^t defined by (42), use equations (35) to define the new end of period t asset values V_G^t and V_S^t iteratively for t = 1,...,T. Once the V_G^t and V_S^t have been determined, use equations (36) to define time series depreciation for period t, Δ_G^t and Δ_S^t, for the goodwill and structure assets. These newly defined user costs, asset values and time series depreciation amounts will also satisfy equations (35)-(41) and thus these new values also provide an additive decomposition of asset values, user costs and depreciation for the project. Thus the particular allocation that we initially generated in this section is not unique. However, it does seem to be a sensible one. It simply allocates the period t free cash flow less the market determined user cost of land ($U_R^t = N^t - U_L^t$) to the goodwill and structure assets in a manner that is proportional to the initial asset values for goodwill and the structure (which can be observed).[32] The proportional allocation method defined by (34) is consistent with the *matching principle* that has been suggested in accounting theory,

[32] The proportional allocation of V_R^t to the goodwill and structure asset defined by (34) corresponds to Cairns' (2013; 645) first method for constructing a definite depreciation schedule for two sunk cost assets. He also noted that any standard depreciation model could be applied to the structure and then depreciation for the goodwill asset could be defined residually. Finally, Cairns noted that *a minimum economic payback allocation method* could be used where all available cash flows are allocated to tangible capital until their present value is equal to the initial investment value.

where initial costs are distributed over time in proportion to future revenues that are generated by the initial investment.[33]

There is some controversy in the real estate literature regarding the classification of goodwill (and project rates of return in excess of their cost of capital). Fisher and Lentz (1990) and Fisher and Kinnard (1990) argued that goodwill should be recognized as an asset[34] whereas Miller and Jones (1995) and Geltner, Miller, Clayton and Eichholtz (2014; 61-67 and 94-98), argued that any goodwill component should be absorbed into the price of land. The latter authors justified their position by appealing to the *residual theory of land value* which argues that the price of land at any point in time immediately absorbs any excess profits that the land plot could earn in its best (known) use. If this theory is true, then it can be accommodated in our framework using the analysis in the previous section with $\Pi^T = 0$. The theory is true in many instances (e.g., valuing agricultural land) but we do not think it is generally true. For example, think of a sawmill which locates in a forested area and suppose that the price of lumber relative to the price of logs is very high so that the mill earns profits well in excess of its cost of capital. It does not make sense to impute those profits to the land that the mill sits on; if the original owner of the land plot tried to capture the monopoly profits that the mill is expected to earn, the mill owners would simply purchase a readily available alternative land plot for the mill. Thus we believe that a goodwill asset will exist in many practical applications of our theory of property value. However, in some contexts, the residual theory of land will work in a satisfactory manner.[35]

[33] The matching principle in accounting theory can be traced back to Church (1917; 193); see also Paton and Littleton (1940; 123) and Diewert (2005; 533-540).

[34] Recent papers by Fisher and Kinnard (1990) and by Fisher and Lentz (1990) argue strongly that retail property valuations should include three major components: land, improvements and business value." Miller, Jones and Roulac (1995; 203). We interpret "improvements" as the value of the structure and "business value" as goodwill.

[35] Diewert and Shimizu (2013) applied the residual land approach to a sample of Tokyo commercial properties. They used assessed property values as proxies for market prices for the properties and they were able to derive separate land and structure price and quantity indexes. However, they estimated a common depreciation schedule and they were not able to determine why structure lives vary so much.

Up to this point, our analysis of a commercial property project has focused on values. In the following section, we turn our attention to the problems that are associated with splitting project values into price and quantity components.

7. The Decomposition of Values into Price and Quantity Components

The decomposition of land values into price and quantity components is reasonably straightforward since the quantity of land is constant over the life of the project.[36] Let P_L^t and Q_L^t be the (asset) *price and quantity of project land* at the end of period t. Suitable definitions for these variables are the following ones:

$$(45)\ P_L^t \equiv V_L^t\ ;\ Q_L^t \equiv 1\ ; \qquad t = 0,1,\ldots,T.$$

Let p_L^t and q_L^t denote the period t *user cost price and quantity of project land*. Again, we can set the period t quantity of land equal to 1 and the corresponding price can be set equal to the user cost U_L^t defined by (20). Thus we have the following definitions:

$$(46)\ p_L^t \equiv (1+r_t)V_L^{t-1} - V_L^t\ ;\ q_L^t \equiv 1\ ; \qquad t = 1,\ldots,T.$$

Goodwill does not have to be decomposed into price and quantity components if it is simply regarded as a repository for pure profits.[37] The decomposition of structure values into price and quantity components is more complex. It would seem that we could treat the decomposition of end-of-period t structure asset value V_S^t in a manner that is analogous to our treatment of land value in (45); i.e., simply define the *end-of-period t asset price of structures* P_S^t as the corresponding value V_S^t and define the corresponding

[36] We will not attempt to adjust the price of land for changes in quality that are due to changes in the external environment surrounding the land plot. Thus the price of a plot of land may increase due to increased development in the neighbourhood or it may decrease due to increases in air pollution. From our perspective, what counts is the sequence of expected future selling prices for the land plot (restored to a vacant lot with no structure on it). We do not attempt to model the possible causes for the expected trajectory of land prices. Thus our perspective is a partial equilibrium one which focuses on the theory of the firm.

[37] Other treatments of goodwill are possible but more discussion and research is needed in order to definitively decompose goodwill values into price and quantity components.

asset quantity Q_S^t as 1. However, the resulting prices do not give us the price of a constant quality amount of structure over time: the structure at the end of period t+1 is not the same as a structure at the end of period t, since its useful life has been reduced by one period. This changing quality problem does not apply to land (in our framework) and so the land prices defined by (45) and (46) can be regarded as constant quality prices.

Our suggested solution is to decompose structure asset value V_S^t at the end of period t into the price of a new structure of the same type at the end of period t times a corresponding quantity Q_S^t that is measured in equivalent units of new structure. Let P_S^{t*} be an appropriate (for the type of structure under consideration) *exogenous construction price index* for the end of period t and define the end of period *asset price and quantity* (in constant quality units of measurement) for the project structure as follows:[38]

$$(47)\ P_S^t \equiv P_S^{t*}\ ;\ Q_S^t \equiv V_S^t/P_S^{t*}\ ; \qquad t = 0,1,...,T.$$

We turn now to the problems associated with the decomposition of the *user cost value for structures* for period t, U_S^t, given by equations (32). Define the period t *structure inflation rate* i_S^t for the exogenous end of period t structure price index P_S^{t*} as follows:

$$(48)\ 1+i_S^t \equiv P_S^{t*}/P_S^{t-1*}\ ; \qquad t = 1,...,T.$$

Define the period t (constant quality) *structure depreciation rate* δ_t as follows:

$$(49)\ 1-\delta_t \equiv Q_S^t/Q_S^{t-1}\ ; \qquad t = 1,...,T.$$

Now use (47) to solve for $V_S^t = P_S^{t*}Q_S^t$ for t = 0,1,...,T and substitute these relationships into equations (35). We obtain the following expressions for the U_S^t:

$$(50)\ U_S^t = (1+r_t)V_S^{t-1} - V_S^t \qquad t = 1,...,T$$

[38] This simple method of quality adjustment is essentially the same method that was suggested by Diewert (2009; 16) in a different context.

$$= (1+r_t)\, P_S^{t-1*} Q_S^{t-1} - P_S^{t*} Q_S^{t}$$
$$= (1+r_t)\, P_S^{t-1*} Q_S^{t-1} - (1+i_S^{t})(1-\delta_t) P_S^{t-1*} Q_S^{t-1} \qquad \text{using (48) and (49)}$$
$$= [r_t - i_S^{t} + (1+i_S^{t})\delta_t] P_S^{t-1*} Q_S^{t-1}.$$

Using the last equation in (50), the decomposition of the period t *user cost value for the structure*, U_S^t, into *price and quantity components*, p_S^t and q_S^t, is thus fairly simple:

$$(51)\ p_S^t \equiv [r_t - i_S^t + (1+i_S^t)\delta_t] P_S^{t-1*}\ ;\ q_S^t \equiv Q_S^{t-1}\ ; \qquad t = 1,\ldots,T.$$

Note that p_S^t has the same form as the traditional user cost of capital. Equations (46) and (51) show that traditional capital measurement techniques can be adapted to measure the land and structure input contributions of a commercial property project.[39]

How can our framework for constructing estimates for the price and quantity of commercial property assets (both flow and stock prices with a decomposition into land, structure and goodwill components) and for structure depreciation rates be implemented in practice? We will consider two scenarios about the availability of data below.

For Case 1, we assume that either government agencies or private data providers can obtain information on individual commercial properties over the time period spanning the construction of a new structure (period 0) and its sale to a purchaser at the end of period T. Assume that the seller demolishes the old structure at the end of period T. Thus looking at the data requirements for one of these properties, we require information on the cost of construction of the new building in period 0, C_S^0, as well as the purchase price for the lot, V_L^0. We also require information on the sequence of cash flows that the property generates, the N^t, for the cost of demolition, C_D^T, and on the sale price for the property at the end of period T, which we interpret as the purchase price of the land plot,

[39] See Diewert and Shimizu (2013) for the description of a measurement framework that uses traditional methods. They used a one hoss shay model to describe structure depreciation in their framework. When the fixity of the structure is taken into account, it can be seen that the intertemporal pattern of project cash flows plays a decisive role in determining the time series depreciation of the structure. Thus actual structure depreciation is likely to be much more volatile than one hoss shay depreciation.

V_L^t. All of these costs and benefits are market prices and can potentially be collected by private firms that aggregate commercial property data or by various branches of the government tax and property authorities. What we do not know is the sequence of the relevant costs of capital (the r_t) that the property owners faced during periods 0 to T. It will be necessary to make some guesses in order to approximate the true costs of capital that the commercial property firm faced. After these approximations are formed, it can be verified that we now have enough information to form an ex post version of equation (6), which defined expected discounted profits Π^T for the optimal project. Now use equation (6) to define actual ex post discounted profits for the project. Now check whether ex post discounted profits defined by the ex post version of (6) are nonnegative.[40] If so, we can skip to equations (15) in section 4 and these equations evaluated at the ex post cash flows serve to define the ex post sequence of project asset values which we still denote by A^t. The remainder of the equations in section 4 can now be used using ex post cash flows (and asset values) instead of anticipated cash flows. However, when we turn to section 5 and attempt to decompose period by period ex post asset values for the project into land and non-land components, we find that we cannot accomplish this task without making further assumptions. Under the assumptions made thus far, we know the price of land at the end of period 0, V_L^0, and the price of the same land plot at the end of period T, V_L^T, but we do not know the price of land during the intervening periods. Thus at this point, we need to form approximations to the price of land at the end of each period so that we can decompose overall (ex post) project asset values at the end of each period into land, structure and goodwill components. A simple approximation would be to assume that the land price grew at a constant geometric rate of growth over the T periods but other exogenous information on land prices could be used to improve this approximation. In any case, once end of period land values have been established, the rest of the analysis in sections 5 and 6 goes through. If the ex post Π^T is equal to 0, then there is no goodwill asset. If the ex post Π^T is greater than 0, then a goodwill asset exists, which will absorb up the excess profits that the project made during the life of the building.

[40] If ex post discounted project profits are negative, then we have a project that does not cover its costs of capital and we could either introduce a negative goodwill asset or adjust the r_t sequence downward in some manner so that ex post profits using the adjusted r_t become 0.

For Case 2, we again assume that we can collect information on the cost of construction of the new building in period 0, C_S^0, on the purchase price for the lot, V_L^0 and on the sequence of cash flows $N^1, N^2, ..., N^\tau$ that the property generated up to period $\tau < T$.[41] As in Case 1, we assume that we can form approximations to the sequence of period by period costs of capital for the property firm up to period τ, $r_1, r_2,...,r_\tau$. The new assumption is that periodic appraisals or assessments of the property are made (either by professional appraisers or the property tax authorities) and let us suppose that such an assessment is made at the end of period τ, and the assessed value is A^τ. This is in principle a forward looking assessment of the potential for the property to generate discounted cash flows and thus we take this assessment[42] as an estimate of future cash flows that the property could generate. We can now use this estimate along with our information on realized cash flows to work backwards using equation (16) to solve for $A^{\tau-1}$ in terms of A^τ and then solving for $A^{\tau-2}$ in terms of $A^{\tau-1}$ and so on. This procedure generates the following sequence of end of period t asset values $A^0, A^1,..., A^{T-1}$ that are consistent with the expectations that are held at the end of period τ and the actual cash flows that materialized in periods 1 up to τ:

$$(52)\quad A^0 \equiv (1+r_1)^{-1}N^1 + (1+r_1)^{-1}(1+r_2)^{-1}N^2 +...+ (1+r_1)^{-1}...(1+r_\tau)^{-1}[N^\tau + A^\tau] ;$$
$$A^1 \equiv (1+r_2)^{-1}N^2 + (1+r_2)^{-1}(1+r_3)^{-1}N^3 +...+ (1+r_2)^{-1}...(1+r_\tau)^{-1}[N^\tau + A^\tau] ;$$
$$...$$
$$A^{\tau-1} \equiv (1+r_\tau)^{-1}[N^\tau + A^\tau].$$

The above equations are counterparts to equations (15) in section 4 except that the sequence of aggregate asset values stops at period τ instead of period T. Aggregate time series depreciation for the property for period $t \le \tau$ can be defined by the following counterparts to equations (17) using the A^t defined by (52):

[41] Note that $N^1, N^2,...,N^\tau$ are now interpreted as actual cash flows instead of anticipated cash flows.

[42] We could also interpret A^τ as the sale price of a property if it actually sold as a going concern at the end of period τ. Alternatively, if we have a single property REIT that trades on a stock market, we could interpret A^τ as a stock market based value estimate at time τ for the future cash flows of the property.

$$(53)\ \Delta^t \equiv A^{t-1} - A^t\ ; \qquad t = 1,2,...,T^*.$$

Given the imputed property asset value for the end of period 0, A^0, defined by the first equation in (52), we can form an estimate for the discounted profits generated by the property at the end of period 0 using observed data on cash flows up to period τ and expectations for future period cash flows as of time τ, say $\Pi(\tau)$, as follows:

$$(54)\ \Pi(\tau) \equiv - C_S{}^0 - V_L{}^0 + A^{0*}.$$

It can be seen that $\Pi(\tau)$ is an estimate for the optimal discounted profits of the project, Π^T, defined by (6). If $\Pi(\tau) > 0$, then there will be a goodwill asset while if $\Pi(\tau) = 0$, there is no goodwill asset as of the expectations and project realizations at time t^*.[43]

When we attempt to decompose the asset values A^t defined by (52) into land and non-land components, we find that we need to make further assumptions in order to accomplish this task. Our extra assumption is that the market based assessed or appraised asset value for the property at the end of period τ, A^τ, also has a decomposition into a land component, $V_L{}^\tau$ plus a non-land residual component, $V_R{}^\tau$.[44] As in Case 1, we use the period 0 value of land for the property $V_L{}^0$ along with the period τ estimated land value $V_L{}^\tau$, and an interpolation method in order to estimate end of period land values for the property for all periods t between periods 0 and τ. With these land values in hand, the analysis in sections 5-7 can be adapted to give us a complete sequence of asset values and user costs for the property up to the end of period τ with a decomposition into land, structure and goodwill components. The structure asset values can be used to construct a sequence of structure depreciation rates up to the end of period τ using the algebra in section 7. These depreciation rates are provisional. When a new property appraisal is made in a subsequent period, say period τ^*, then the above Case 2 analysis must be

[43] If $\Pi(\tau) < 0$, then there is a negative goodwill asset.

[44] Note that stock market based valuations for the property at time τ do not provide us with this decomposition.

redone and a new sequence of depreciation rates for the structure up to time τ^* can be computed.[45]

It can be seen that the information burden that is required in order to implement our suggested method for estimating depreciation rates for commercial properties is quite heavy. However, our description of Cases 1 and 2 above shows that our approach can be implemented if we are willing to make a few additional assumptions about the variables in our conceptual framework.[46] The construction of price and quantity indexes for commercial property structures and the land that the structures sit on requires a theoretical framework that can generate idealized target indexes. Then approximations to these target indexes can be constructed using available data along with simplifying assumptions.

8. Conclusion

The main purpose of the present paper has been to provide a framework for measuring capital input for a commercial property. A commercial property has a structure that produces space services and uses intermediate inputs (like electricity), labour and capital. The two major capital inputs are structure services and the services of the land plot that supports the structure. There is a production function (or more generally a production possibilities set) that relates the inputs and outputs for a commercial property. The production accounts in the SNA are based on this production theory. More specifically, the SNA requires a decomposition of the input and output value flows for a commercial property into price and quantity components. It is fairly straightforward to construct these

[45] The resulting structure depreciation rates will tend to be quite variable over time periods for the same property and of course, different commercial properties will generate quite different depreciation sequences. In order to apply these estimated structure depreciation rates in the context of tax policy or for application by statistical agencies to other commercial properties where information on cash flows, construction costs and costs of capital are not available, it will be necessary to smooth and average these estimated depreciation rates.

[46] We note that our approach can generate structure depreciation rates at the level of the individual property. The residual land approach cannot generate structure depreciation rates for an individual property but it can generate depreciation rates (equal to declines in property asset values as the structure ages) for the entire property asset. However, our approach does require an exogenous period by period value of the land plot in its best alternative use. Essentially, at the level of an individual property, the Residual Land approach requires an exogenous estimate of structure value.

prices and quantities using observable data except for the case of structure and land services. Structure and land services for a commercial property are not like other capital services like the services generated by a machine or a vehicle. In the latter case, one can go to markets for used machinery and vehicles and determine how these assets lose value at a point in time due to the effects of ageing. Thus for these tradable assets, depreciation can be determined in an objective manner and with these market based depreciation rates in hand, one can construct Jorgensonian user costs and also value the used assets in the balance sheets of the firm in an objective manner. But the situation is far different for the land and structure assets of a commercial property firm: there are no external markets for the value of a particular structure in a particular location that can be used to provide objective depreciation rates for that structure. Our approach can be viewed as an attempt to provide "objective" depreciation rates for a commercial property structure. Our suggested framework provides a decomposition of aggregate capital input into land, structure and goodwill components. What is new in the present paper is that the fixity of the structure and the endogeneity of the useful life of the structure are taken into account.

Three main practical conclusions emerge from the paper. First, taking the fixity of the structure into account does not lead to a dramatically different measurement framework as compared to more traditional approaches which ignore the fixity problem in the sense that we still obtain user costs for the structure that look familiar; see equations (51).

Second, the pattern of time series depreciation allocations for a commercial property structure is largely (but not exclusively) determined by the cash flows that the property generates over the lifetime of the structure. These cash flow patterns are likely to be very different over different property classes, leading to measurement challenges. In particular, traditional depreciation models for structures (such as the one hoss shay, geometric or straight line models) are unlikely to provide adequate descriptions of economic reality for the commercial property sector.

Finally, our theoretical measurement framework requires a great deal of data for implementation and these data are unlikely to be available. Thus further assumptions will have to be made in order to obtain a practical measurement framework.[47] However, we believe that our framework does capture many realities of the commercial property market, and thereby significantly advances the capacity to understand this important market.

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[47] At the end of section 7, we outline in more detail two measurement scenarios and what extra assumptions are needed in order to implement our measurement framework.

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