Getting Rental Prices Right for Computers[1]

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Abstract

National statistical agencies frequently assume very high geometric depreciation rates in order to capture the fact that computers are usually retired after 3 or 4 years of use. However, typically the service flow that a computer generates over its useful life is roughly constant, which contradicts the geometric model of depreciation where the service flow falls at a constant rate forever. Thus a one hoss shay or light bulb model of depreciation seems to be more appropriate for computers. The paper uses Australian data on computer investment over the past 25 years to construct one hoss shay estimates of computer capital stocks and flows and considers how best to approximate these more realistic models of depreciation with a geometric model. The paper shows that under certain simplifying assumptions, a geometric model of depreciation can provide an exact approximation to an underlying one hoss shay model. This exactness result is extended to a more general model of depreciation, the Constant Efficiency Profile model. Finally, using Australian data, the paper shows how well the geometric approximation fits a one hoss shay model when the simplifying assumptions are not satisfied.

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Key Words

Geometric model of depreciation, one hoss shay model of depreciation, the Constant Efficiency Profile model of depreciation, user cost formulae, capital stocks and service flows.

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1. Introduction

Many statistical agencies use the geometric model of depreciation. But the geometric model of depreciation does not seem to be very realistic for computers: the service flow of a new computer rarely lasts longer than 4 years due to obsolescence. Moreover, since constant quality computer prices decline somewhere around 15% per year, this high negative rate of asset price change becomes a positive addition to a typically high geometric depreciation rate in the geometric user cost formula, leading to the possibility that the resulting user cost is too high which in turn could lead to a value of computer capital services which is also too high.[2]

Basically, the geometric model of depreciation is not plausible for computers. A one hoss shay model of depreciation with a short length of life (equal to 3 or 4 years) seems to be much more plausible.[3] In the present paper, we attempt to determine how well the geometric model of depreciation can approximate one hoss shay models under somewhat idealized conditions (steady growth in asset investments, steady rates of growth in constant quality prices and constant nominal costs of capital). Somewhat surprisingly, we find that under these idealized conditions, the geometric model of depreciation can provide a very good approximation to a one hoss shay model, provided that the "right" geometric depreciation rate is chosen.[4]

The geometric and one hoss shay models of depreciation are described in sections 2 and 3 respectively. In section 4, we assume that a one hoss shay model of depreciation is the "truth" and we show how a geometric depreciation rate can be picked so that either geometric capital services will be exactly equal to one hoss shay capital services or so that geometric capital stocks will be exactly equal to the corresponding one hoss shay capital stocks. However, it turns out that under our simplifying assumptions, these two equalizing depreciation rates are identical. We explain why this puzzling result holds in sections 5 and 6. It turns out that we can show that this result will hold in a much more general class of models. Thus in section 5, we introduce a generalization of the one hoss shay model of depreciation, the *Constant Efficiency Profile* (CEP) model of depreciation. In this model, a new asset delivers services for only a finite lifetime but it is assumed that the service flow of an older asset relative to a new asset remains constant.[5] In section 6, we again show how a geometric model of depreciation can approximate this model exactly under our simplifying assumptions and the puzzle encountered in section 4 is explained in the context of this more general model. Section 7 concludes. An Appendix uses economy wide data on computer investments in Australia for the past 25 years and

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[2] Diewert (2012; 63) suggested this possibility but also suggested that more research was required.

[3] The one hoss shay or light bulb model of depreciation assumes that the service flow of the asset is constant over the lifetime L of the asset and then it is retired at the end of L periods of use.

[4] In the Appendix, we show that under more realistic conditions, our geometric model approximation is adequate to approximate one hoss shay capital services but not so good at approximating one hoss shay capital stocks.

[5] For a one hoss shay asset, an older asset (that is less than L periods old) delivers the same service flow as a new asset. The CEP model allows for an arbitrary pattern of service flows as the asset ages.

shows how good the geometric approximations are to one hoss shay models of depreciation when our simplifying assumptions are not satisfied.

2. The Geometric Model of Depreciation

In this section, we develop the algebra that describes the geometric model of depreciation under some simplifying assumptions, which are as follows:

- The rate of growth of investment g in the asset is constant over time;
- The rate of growth in the price of a constant quality unit of the asset i is constant over time;
- The cost of financial capital for firms or the nominal rate of interest r is constant over time;
- When the geometric model of depreciation is used, it is assumed that the depreciation rate δ is constant over time.

In what follows, we will consider levels of prices, quantities and values for two periods, 0 and 1, and rates of change going from period 0 to 1. We assume that the amount of investment in the asset under consideration in period 0, q_I^0, is 1 and that the corresponding period 0 investment price, p_I^0, is also 1.[6] Thus the corresponding value of investment in period 0, v_I^0, is also equal to 1. Under the above assumptions, the period 1 quantity, price and value of investment in the asset are given by $p_I^1 \equiv (1+i)p_I^0 = 1+i$; $q_I^1 \equiv (1+g)q_I^0 = 1+g$ and $v_I^1 \equiv p_I^1 q_I^1 = (1+i)(1+g)$.[7] The price, quantity and value of investment data for the two periods under consideration are summarized in equations (1) below.

$$
\begin{aligned}
(1)\quad & p_I^0 = 1;\ p_I^1 = (1+i); && p_I^1/p_I^0 = (1+i); \\
& q_I^0 = 1;\ q_I^1 = (1+g); && q_I^1/q_I^0 = (1+g); \\
& v_I^0 = 1;\ v_I^1 = (1+i)(1+g); && v_I^1/v_I^0 = (1+i)(1+g).
\end{aligned}
$$

We assume that the investment in the prior period becomes an addition to the capital stock of the current period. Under our constant rate of growth of investment assumption and using the constant geometric depreciation rate assumption, the capital stock at the beginning of period 0, q_K^0, is given by the following expression:[8]

$$
\begin{aligned}
(2)\quad q_K^0 &\equiv [1/(1+g)] + [(1-\delta)/(1+g)^2] + [(1-\delta)^2/(1+g)^3] + \ldots \\
&= (1+g)^{-1}[1 + \alpha + \alpha^2 + \alpha^3 + \ldots\ ] \\
&= 1/(g+\delta)
\end{aligned}
$$

where we assume that $\alpha \equiv (1-\delta)/(1+g)$ is between 0 and 1. Under our constant rate of growth of investment assumption and using the constant geometric depreciation rate

[6] We can choose units of measurement for the investment good that justify these assumptions.
[7] We assume that $1+i > 0$ and $1+g > 0$ so that prices, quantities and values are positive for the two periods.
[8] This method for obtaining a starting value for the geometric capital stock is due to Griliches (1980; 427) and Kohli (1982); see also Fox and Kohli (1998).

assumption, the capital stock at the beginning of period 1, q_K^1, is given by the following expression:

$$\begin{aligned}(3)\ q_K^1 &\equiv 1 + [(1-\delta)/(1+g)] + [(1-\delta)^2/(1+g)^2] + \ldots \\ &= [1 + \alpha + \alpha^2 + \alpha^3 + \ldots] \\ &= (1+g)/(g+\delta) \\ &= (1+g)q_K^0.\end{aligned}$$

Thus the starting capital stock for period 1 will be equal to q_K^0 times (1+g). The period 0 and 1 prices of the starting capital stocks, p_K^0 and p_K^1, are equal to the corresponding investment prices, p_I^0 and p_I^1. The period t values for geometric capital stocks, v_K^t, are defined as $v_K^t \equiv p_K^t q_K^t$ for t = 0,1. Using these definitions and (1)-(3), the capital stock price, quantity and value information for the geometric depreciation model is summarized in equations (4) below:

$$\begin{aligned}(4)\ &p_K^0 = 1; && p_K^1 = (1+i); && p_K^1/p_K^0 = (1+i); \\ &q_K^0 = 1/(g+\delta); && q_K^1 = (1+g)q_K^0; && q_K^1/q_K^0 = (1+g); \\ &v_K^0 = 1/(g+\delta); && v_K^1 = (1+i)(1+g)v_K^0; && v_K^1/v_K^0 = (1+i)(1+g).\end{aligned}$$

Note that the rates of growth for investment prices, quantities and values for the geometric depreciation model are equal to the corresponding rates of growth for capital stock prices, quantities and values.

We turn now to user costs or rental prices for the geometric depreciation model. The beginning of the period user cost of using the *services* of a unit of capital for period 0, u_S^0, is defined as the cost of purchasing one unit of capital at the beginning of period 0, using the services of the asset during period 0 and then subtracting the discounted market value of the used asset at the end of the period. Thus the period 0 user cost of capital is equal to the following expression:[9]

$$\begin{aligned}(5)\ u_S^0 &\equiv p_K^0 - (1-\delta)p_K^1/(1+r) \\ &= 1 - (1-\delta)(1+i)/(1+r)\end{aligned}$$

where the second equation in (5) follows using equations (4). Rather than discounting end of period prices to the beginning of the period, it is more convenient to revalue beginning of the period prices to their end of period equivalents.[10] Thus the period 0 *end of period user cost*, p_S^0, is defined as $(1+r)u_S^0$. Thus using (5), p_S^0 is equal to the following (familiar) expression for the geometric model of depreciation:[11]

[9] The concept of the user cost of capital dates back to Walras (1954; 267-269) but the modern development of the user cost concept is due to Jorgenson (1963) (1989). This discrete time method for deriving the user cost (5) is due to Diewert (1974; 504) (1980; 471).

[10] See Diewert (2005; 485-486) for a more detailed discussion on the merits of discounting to either the beginning or end of an accounting period. End of period user costs are more consistent with accounting conventions; see Peasnell (1981; 56).

[11] Jorgenson and his coworkers derived this user cost formula in a continuous time framework; see Jorgenson (1963) (1989), Jorgenson and Griliches (1967; 256) and Christensen and Jorgenson (1969).

(6) $p_S^0 \equiv (1+r) - (1-\delta)(1+i) = r - i + (1+i)\delta$.

The corresponding quantity of capital services for period 0, q_S^0, is equal to the period 0 starting capital stock, $q_K^0 = 1/(g+\delta)$ and thus the period 0 value of capital services is $v_S^0 \equiv p_S^0 q_S^0 = 1/(g+\delta)$. The period 1 user cost of capital, u_S^1, is defined as $p_K^1 - (1-\delta)p_K^2/(1+r) = (1+i)p_K^0 - (1-\delta)(1+i)p_K^1/(1+r)$ using our assumption that investment (and asset) prices grow at the constant inflation rate i. Thus $u_S^1 = (1+i)u_S^0$. Since the quantity of capital services for period 1, q_S^1, is equal to the period 1 starting capital stock, q_K^1, using (4), we have $q_S^1 = q_K^1 = (1+g)q_K^0 = (1+g)q_S^0$. The period t value of capital services, v_S^t, is defined as $p_S^t q_S^t$ for t = 0,1. The price, quantity and value of capital services data for the two periods under consideration are summarized in equations (7) below.

(7) $p_S^0 = r - i + (1+i)\delta$; $\quad p_S^1 = (1+i)p_S^0$; $\quad p_S^1/p_S^0 = (1+i)$;
$q_S^0 = 1/(g+\delta)$; $\quad q_S^1 = (1+g)q_S^0$; $\quad q_S^1/q_S^0 = (1+g)$;
$v_S^0 = [r - i + (1+i)\delta]/(g+\delta)$; $v_S^1 = (1+i)(1+g)v_S^0$; $\quad v_S^1/v_S^0 = (1+i)(1+g)$.

Note that the rates of growth for capital services prices, quantities and values for the geometric depreciation model are equal to the corresponding rates of growth for investment and capital stock prices, quantities and values. The geometric model of depreciation is easy to implement and has the advantage that it is not necessary to compute separate prices, quantities and values for each vintage of the assets in use. We turn now to the one hoss shay or light bulb model of depreciation, where it is necessary to keep track of vintages of the asset.

3. The One Hoss Shay Model of Depreciation

We will illustrate the computations for the one hoss shay model of depreciation[12] assuming a length of life for a new asset of 3 or 4 years. This is the likely length of time that a computer lasts before it is retired. We will start off with the 4 year length of life.

We make the same long run basic assumptions about the price and quantity of investments that were made in the previous section. Thus equations (1) in the previous section can still be used in order to describe the price, quantity and value of investments in the asset for periods 0 and 1. However, the algebra that describes the evolution of capital stocks and service flows for the one hoss shay model is different (and more complicated).

The basic idea behind the one hoss shay model of depreciation and capital services is that a new unit of the asset provides a constant flow of services for L periods and then is retired. In the present section, we will assume that the asset class is computers and we assume initially that the length of life is 4 years so that the period length is one year.

[12] The one hoss shay model of depreciation is due to Böhm-Bawerk (1891; 342). For a more detailed analysis of this model, see Hulten (1990) (1996), Diewert and Lawrence (2000) and Diewert (2005).

We will start our analysis by assuming that the length of life of an asset is 4 periods.[13] In general, the value of an asset should equal the discounted flow of the service flows that it yields over its useful life. Denote the expected value of the services provided by a unit of the asset purchased at the beginning of period 0 over periods 0, 1, 2 and 3 by v^t for t = 0,1,2,3. Let the price of the asset purchased at the beginning of period 0 be p_K^0. Then p_K^0 should equal the discounted value of its future service flows so that the following relationship between the v^t and p_K^0 should hold:

$$(8)\ p_K^0 = v^0 + (1+r)^{-1}v^1 + (1+r)^{-2}v^2 + (1+r)^{-3}v^3$$

where r > 0 is the one period nominal discount rate or cost of capital which is assumed to remain constant over time. The above equation assumes that the rental payments v^0, v^1, v^2 and v^3 are received at the beginning of each period. It is more convenient to assume that the rental payments are made at the end of each period. Denote these end of period expected rental prices by u^0, u^{1*}, u^{2*} and u^{3*}. Using these end of period rental prices, we replace equation (8) by equation (9) below:

$$(9)\ p_K^0(1+r) = u^0 + (1+r)^{-1}u^{1*} + (1+r)^{-2}u^{2*} + (1+r)^{-3}u^{3*}.$$

The period 0 rental price for a new unit of this fixed life asset, u^0, will be a counterpart to the end of period 0 geometric model period 0 rental price p_S^0 defined in the previous section by (6).

We assume that the period 0 rental prices for units of the asset that are 1, 2 and 3 years old at the beginning of period 0 are u_1^0, u_2^0 and u_3^0. The *relative efficiency* or utility, e_i, of an older asset of age i relative to a new asset in period 0 is defined as the ratio of the period 0 older asset rental price u_i^0 to the period 0 rental price of a new asset u^0:[14]

$$(10)\ e_i \equiv u_i^0/u^0; \qquad i = 1,2,3.$$

We assume that this pattern of relative efficiencies will persist through all future periods. Using the asset price growth assumptions made in section 2, the price of a new asset at the beginning of period 0 was unity and this price was expected to grow at the inflation rate i so that the price of a new asset over the next 3 periods would be (1+i), $(1+i)^2$ and $(1+i)^3$. With these assumptions, $u^{1*} = u^0(1+i)e_1$, $u^{2*} = u^0(1+i)^2e_2$, and $u^{3*} = u^0(1+i)^3e_3$. Substituting these equations into equation (9) leads to the following equation, which relates the period 0 new asset price p_K^0 to the corresponding period 0 rental price for a unit of the new asset u^0:

$$(11)\ p_K^0(1+r) = u^0[1 + (1+r)^{-1}(1+i)e_1 + (1+r)^{-2}(1+i)^2e_2 + (1+r)^{-3}(1+i)^3e_3].$$

For the *one hoss shay model of depreciation* with length of life L = 4, the relative efficiencies of the assets of age 0, 1, 2 and 3 are all equal to one:

[13] The special assumptions that define the one hoss shay model will be made below in equations (12).
[14] The sequence of (cross sectional) vintage rental prices u^0,u_1^0,u_2^0,u_3^0 is called the *age-efficiency profile* of the asset; see Schreyer (2001) (2009).

(12) $e_1 = e_2 = e_3$.

Substitute equations (12) into (11) and using the fact that $p_K^0 \equiv 1$, we obtain the following expression for the (end of period) one hoss shay user cost or rental price for a new (and old) unit of capital:

(13) $u^0 = (1+r)/(1+\beta+\beta^2+\beta^3)$

where β is defined as follows:

(14) $\beta \equiv (1+i)/(1+r) > 0$.

The price of a new unit of capital at the beginning of period 0, p_K^0, is equal to the investment price for a new unit of the asset, p_I^0, and both of these prices are set equal to 1. The one hoss shay asset prices at the beginning of period 0 for assets that are 1, 2 and 3 years old are defined to be p_{K1}^0, p_{K2}^0 and p_{K3}^0. These vintage asset prices are also set equal to their discounted stream of future expected rentals and so the older is the asset, the fewer terms will be in the stream of discounted rentals. It turns out that the period 0 vintage asset prices can be defined as follows:

$$
\begin{aligned}
(15)\ p_K^0 &\equiv (1+r)^{-1}u^0(1+\beta+\beta^2+\beta^3) = 1; \\
p_{K1}^0 &\equiv (1+r)^{-1}u^0(1+\beta+\beta^2) = (1+\beta+\beta^2)/(1+\beta+\beta^2+\beta^3) \equiv f_1; \\
p_{K2}^0 &\equiv (1+r)^{-1}u^0(1+\beta) = (1+\beta)/(1+\beta+\beta^2+\beta^3) \equiv f_2; \\
p_{K3}^0 &\equiv (1+r)^{-1}u^0 = 1/(1+\beta+\beta^2+\beta^3) \equiv f_3
\end{aligned}
$$

where β is defined by (14). Note that the price for a one period old asset is the fraction f_1 of the new asset price, which is $p_K^0 = 1$, and the asset prices for 2 and 3 year old assets at the beginning of period 0 are the progressively smaller fractions f_2 and f_3.

The beginning of period 0 total value, v_K^0, of the one hoss shay capital stock can now be calculated using the vintage asset prices defined in (15). The quantity of new assets at the start of period 0 is equal to the previous period's quantity of investment, $1/(1+g)$, and the quantity of 1, 2 and 3 period old assets at the start of period 0 is $1/(1+g)^2$, $1/(1+g)^3$ and $1/(1+g)^4$ under our assumptions. Thus v_K^0 is equal to the following expression:

$$
\begin{aligned}
(16)\ v_K^0 &= p_K^0(1+g)^{-1} + p_{K1}^0(1+g)^{-2} + p_{K2}^0(1+g)^{-3} + p_{K3}^0(1+g)^{-4} \\
&= (1+g)^{-1}[1 + f_1(1+g)^{-1} + f_2(1+g)^{-2} + f_3(1+g)^{-3}] \\
&\equiv q_K^0
\end{aligned}
$$

where we have defined the period 0 price of the one hoss shay capital stock to be $p_K^0 = 1$ and hence the value of the period 0 capital stock v_K^0 is equal to the period 0 quantity, q_K^0. Hence we have defined the vintage quantity components of the period 0 capital stock in terms of equivalent units of new capital stock components using the f_i as relative weights.[15]

Repeating the above analysis for period 1 shows that $p_K^1 = (1+i)p_K^0 = 1+i$, $q_K^1 = (1+g)q_K^0$ and $v_K^1 = (1+i)(1+g)v_K^0$. Using the above analysis, the one hoss shay capital stock price, quantity and value data are summarized in equations (17) below.

$$(17)\quad p_K^0 = 1; \qquad p_K^1 = (1+i);$$
$$q_K^0 = (1+g)^{-1}[1+f_1(1+g)^{-1}+f_2(1+g)^{-2}+f_3(1+g)^{-3}]; \qquad q_K^1 = (1+g)q_K^0;$$
$$v_K^0 = q_K^0;\ v_K^1 = (1+i)(1+g)v_K^0; \qquad v_K^1/v_K^0 = (1+i)(1+g)$$

where the f_i were defined in equations (15) and β was defined by (14). Note that the rates of growth for one hoss shay capital stock prices, quantities and values are equal to the corresponding rates of growth for investment prices, quantities and values and these rates of growth are also equal to the corresponding rates of growth for investments and stocks for the geometric model of depreciation. However, *it is not the case that the period 0 capital stock quantities and values necessarily coincide for the geometric and one hoss shay models*. Later, we will look for conditions that make the models consistent with each other.

Finally, we need to compute the prices, quantities and values for the one hoss shay service flows for periods 0 and 1. This is relatively straightforward. The user cost u^0 defined by (13) is the one hoss shay price of capital services, p_S^0, for period 0. The quantity of capital services that corresponds to this user cost is q_S^0 and it is equal to the sum of the lagged investments for 4 periods, $(1+g)^{-1}+(1+g)^{-2}+(1+g)^{-3}+(1+g)^{-4}$. The period 1 one hoss shay price of capital services under our assumptions turns out to be $p_S^1 = u^0(1+i)$ and the corresponding quantity of capital services, q_S^1, is equal to $(1+g)q_S^0$. The one hoss shay capital services price, quantity and value data are summarized in equations (18) below.

$$(18)\quad p_S^0 = (1+r)/(1+\beta+\beta^2+\beta^3); \qquad p_S^1 = (1+i)p_S^0;$$
$$q_S^0 = (1+g)^{-1}+(1+g)^{-2}+(1+g)^{-3}+(1+g)^{-4}; \qquad q_S^1 = (1+g)q_S^0;$$
$$v_S^0 = p_S^0 q_S^0; \qquad v_S^1 = (1+i)(1+g)v_S^0$$

where $\beta \equiv (1+i)/(1+r)$. As usual, the rates of growth for capital services prices, quantities and values for the one hoss shay depreciation model are equal to the corresponding rates of growth for investment and capital stock prices, quantities and values. Comparing equations (18) with equations (7), it can be seen that the rates of growth for capital services prices, quantities and values for the one hoss shay depreciation model are equal

[15] In Diewert and Lawrence (2000) and Diewert (2005), index number methods were used to aggregate the various vintages of the one hoss shay capital stock. This is not necessary in the present situation due to our assumptions about the persistence of growth rates of investment prices; i.e., under our assumptions, the vintage asset prices, the p_{Ki}^t, will all vary proportionally to the variations in the new asset prices. Under these conditions, all standard index number formulae will lead to aggregate prices of capital that move proportionally to the p_K^t. In the Appendix where our simplifying assumptions are not satisfied, we will use index number techniques to aggregate across vintages.

to the corresponding rates of growth for capital services prices, quantities and values for the geometric depreciation model. However, *the period 0 (and period 1) values of capital services for the geometric model are in general not equal to the corresponding period 0 (and period 1) values of capital services for the one hoss shay model*.[16]

We conclude this section by working out the prices, quantities and values for the one hoss shay model when the length of life of the asset is L = 3. Equations (1) still describe prices and quantities for investments in the asset. Equations (15) are replaced by the following equations which define the vintage asset prices for period 0 and the relative asset weights f_1 and f_2:

$$
\begin{aligned}
(19)\quad p_K^0 &\equiv (1+r)^{-1}u^0(1+\beta+\beta^2) = 1;\\
p_{K1}^0 &\equiv (1+r)^{-1}u^0(1+\beta) \quad = (1+\beta)/(1+\beta+\beta^2) \equiv f_1;\\
p_{K2}^0 &\equiv (1+r)^{-1}u^0 \quad = 1/(1+\beta+\beta^2) \quad \equiv f_2.
\end{aligned}
$$

The counterparts to equations (17) are equations (20) which list the one hoss shay capital stock price, quantity and value data for L = 3:

$$
\begin{aligned}
(20)\quad p_K^0 &= 1; & p_K^1 &= (1+i)p_K^0 = 1+i\\
q_K^0 &= (1+g)^{-1}[1+f_1(1+g)^{-1}+f_2(1+g)^{-2}]; & q_K^1 &= (1+g)q_K^0;\\
v_K^0 &= q_K^0; & v_K^1 &= (1+i)(1+g)v_K^0.
\end{aligned}
$$

The L = 3 counterparts to the L = 4 service flow equations (18) are equations (21) which list the one hoss shay capital services price, quantity and value data for the 3 period length of asset life:

$$
\begin{aligned}
(21)\quad p_S^0 &= (1+r)/(1+\beta+\beta^2); & p_S^1 &= (1+i)p_S^0;\\
q_S^0 &= (1+g)^{-1}+(1+g)^{-2}+(1+g)^{-3}; & q_S^1 &= (1+g)q_S^0;\\
v_S^0 &= p_S^0q_S^0; & v_S^1 &= (1+i)(1+g)v_S^0.
\end{aligned}
$$

In the following section, we will look for conditions which will reconcile the geometric depreciation model to a one hoss shay model.

4. Reconciling the Geometric Model of Depreciation to a One Hoss Shay Model.

Suppose that the one hoss shay model of depreciation is the "truth" for L = 4. Then under our stationary growth rate assumptions, the geometric model of depreciation will generate exactly the same capital stocks, provided that the geometric capital stock for period 0, q_K^0, defined in equations (4) is equal to the corresponding period 0 one hoss shay capital stock defined in equations (17). This leads to the following equation:

[16] Thus when forming input aggregates for a sector or the economy, the choice of depreciation model will in general lead to different estimates for aggregate input growth even under our somewhat restrictive assumptions. Although the geometric and one hoss shay depreciation models generate identical rates of growth of prices and quantities for a capital services component under our assumptions on stationary growth rates, the alternative depreciation models will in general generate different *weighting* of these component growth rates which will lead to different overall input growth rates.

(22) $1/(g+\delta) = (1+g)^{-1}[1+f_1(1+g)^{-1}+f_2(1+g)^{-2}+f_3(1+g)^{-3}] \equiv \gamma_4$

where the f_i are defined in equations (15). Equation (22) can be solved for the geometric depreciation rate δ^* that will make the capital stocks in the two models identical:

(23) $\delta^* \equiv \gamma_4^{-1} - g$.

Using Australian data on investment in computers for the past 25 years, we find that the average real growth rate of investment over this period was $g^* \equiv 0.20378$ so that real investment in computers grew at an annual average (geometric) rate of 20.4%. The corresponding (geometric) average rate of change in investment prices was $i^* \equiv -0.14096$. For an approximation to the beginning of the year cost of capital r, we chose the average yield on 5 year Australian government bonds at the beginning of each year. The geometric average of these rates over the past 25 years was $r^* \equiv 0.06627$.[17] With these values for the parameters in our model, we find that the depreciation rate that solves equation (22) for the Australian data is $\delta^* \equiv \gamma_4^{-1} - g^* = 0.32055$. This rate is considerably below the average of the official real depreciation rates over the past 25 years, which was $\delta_{ABS} \equiv 0.39220$.[18]

Instead of choosing a geometric depreciation rate that makes the one hoss shay and geometric capital stocks at the beginning of period 0 equal, we could choose the geometric rate δ_S that makes the period 0 geometric value of capital services equal to the corresponding one hoss shay value of capital services. Using equations (7) and (18), this leads to the following equation:

(24) $[r^* - i^* + (1+i^*)\delta_S]/(g^*+\delta_S)$
$= [(1+g^*)^{-1}+(1+g^*)^{-2}+(1+g^*)^{-3}+(1+g^*)^{-4}](1+r^*)/(1+\beta^*+\beta^{*2}+\beta^{*3}) \equiv \phi_4$

where $\beta^* \equiv (1+i^*)/(1+r^*)$. Equation (24) can be solved for the geometric depreciation rate δ_S^* that will make the value of capital services in the two models identical:

(25) $\delta_S^* \equiv [r^* - i^* - g^*\phi_4]/[\phi_4 - (1+i^*)]$.

Again using the Australian data on investment in computers for the past 25 years, we find that the depreciation rate that solves equation (24) for the Australian data is $\delta_S^* \equiv 0.32055$, which is precisely equal to δ^*, the solution to (22) which equated the quantities (and values) of period 0 geometric and one hoss shay capital stocks.

Now suppose that the one hoss shay model of depreciation is the "truth" for L = 3. Again, we equate the period 0 geometric capital stock to the period 0 one hoss shay capital stock

[17] The calculation of g^*, i^* and r^* is explained in more detail in the Appendix.
[18] The precise method for computing this average ABS depreciation rate is explained in the Appendix. It should be noted that the ABS does not use the geometric model of depreciation.

with length of life equal to three years. This leads to the following counterpart to equation (22):

$$(26)\ 1/(g^*+\delta) = (1+g^*)^{-1}[1+f_1{}^*(1+g^*)^{-1}+f_2{}^*(1+g^*)^{-2}] \equiv \gamma_3$$

where $f_1{}^* \equiv (1+\beta^*)/(1+\beta^*+\beta^{*2})$, $f_2{}^* \equiv 1/(1+\beta^*+\beta^{*2})$ and $\beta^* \equiv (1+i^*)/(1+r^*)$. Equation (26) can be solved for the geometric depreciation rate δ^{**} that will make the capital stocks in the two models identical:

$$(27)\ \delta^{**} \equiv \gamma_3{}^{-1} - g^*.$$

The depreciation rate that solves equation (26) for the long run Australian data is δ^{**} = 0.43240 , which is 10.2% above the official average depreciation rate of 0.39220.

Instead of choosing a geometric depreciation rate that makes the one hoss shay and geometric *capital stocks* at the beginning of period 0 equal, we could choose the geometric rate that makes the period 0 geometric value of *capital services* equal to the corresponding one hoss shay value of capital services. Using equations (7) and (21), this leads to the following equation:

$$(28)\ [r^* - i^* + (1+i^*)\delta_S]/(g+\delta_S)$$
$$= [(1+g^*)^{-1}+(1+g^*)^{-2}+(1+g^*)^{-3}+(1+g^*)^{-4}](1+r^*)/(1+\beta^*+\beta^{*2}+\beta^{*3}) \equiv \phi_4.$$

Equation (28) can be solved for the geometric depreciation rate $\delta_S{}^{**}$ that will make the value of capital services in the two models identical:

$$(29)\ \delta_S{}^{**} \equiv [r^* - i^* - g^*\phi_4]/[\phi_4 - (1+i^*)].$$

The depreciation rate that solves equation (28) for the Australian data is $\delta_S{}^{**} \equiv 0.43240$, which is precisely equal to δ^{**}, the solution to (26) which equated the quantities (and values) of period 0 geometric and one hoss shay capital stocks.

The above results suggest that if the true depreciation model is a one hoss shay model with length of life half way between L = 3 and L = 4 years, then a geometric depreciation model that sets δ equal to the average of $\delta^* \equiv 0.43240$ and $\delta^{**} \equiv 0.32055$ (which is 0.37648 which in turn is reasonably close to the 0.39220 geometric depreciation rate which best approximates the official ABS depreciation rates over the past 25 years) will approximate the Australian computer capital stock data fairly well. This is an encouraging result; it shows that if the growth rate of investment in an asset and the rate of constant quality price change and the nominal discount rate are reasonably constant, then an appropriate geometric model of depreciation can approximate a one hoss shay model of depreciation fairly well.[19] This is an important result because geometric models

---

[19] As will be seen in the Appendix, while the i^t and g^t for Australia do not have definite trends over the past 25 years, the nominal interest rates r^t have a very strong downward trend from about 14.5% in 1989 to

of depreciation are very easy to implement; one does not need to keep track of separate vintages of investment and depreciate each vintage separately and then aggregate the vintage capital stocks and flows.

A remaining puzzle is: why are the δ^* solutions to equations (22) and (24) exactly the same when the equations look very different? And why are the δ^{**} solutions to equations (26) and (28) exactly the same? In the following section, we will consider a general family of depreciation models that contain one hoss shay models as a special case and show that for this class of models, a similar "puzzling" result occurs. We will show why the exact equality holds for this class of models.

5. The CEP Depreciation Model

In this section, we consider a generalized version of the one hoss shay model of depreciation and in the following section, we show that under our constant growth rate assumptions, a geometric model of depreciation can provide an exact approximation to this more general model.

The more general model that we will consider here is the *Constant Efficiency Profile* (CEP) model of depreciation. This model makes two main assumptions:

- The length of life of the asset under consideration is L periods (a finite number greater than 2) and
- The relative efficiency of an asset that is i periods old relative to a new asset remains fixed over time.

Denote the end of period 0 rental price for a new unit of the asset by u^0. We assume that the end of period 0 rental price for units of the asset that are i periods old at the beginning of period 0 is u_i^0 for $i = 1,2,...,L-1$. The *relative efficiency* or utility, e_i, of an older asset of age i relative to a new asset in period 0 is defined as the ratio of the older asset rental price u_i^0 to the period 0 rental price of a new asset u^0:

$$(30)\quad e_i \equiv u_i^0/u^0; \qquad i = 1,2,...,L-1.$$

We assume that this pattern of relative efficiencies will persist through all future periods.[20]

Denote the beginning of period 0 price of a new unit of the asset by p_K^0 and the period 0 price of the same asset that is i periods old by p_{Ki}^0 for $i = 1,2,...,L-1$. As usual, these asset values are set equal to the discounted stream of expected rentals that they are expected to generate. Again assuming a constant nominal cost of capital equal to r and a constant

2.5% in 2013. It will be shown in the Appendix that our geometric approximation method works well for capital services but it does not work so well for the capital stocks.

[20] Of course, this model contains the one hoss shay model as the special case where all of the e_i are equal to one.

expected asset price inflation rate of i, the sequence of period 0 asset prices by age of asset at the beginning of period 0 are defined as follows:

$$
\begin{aligned}
(31)\ p_K^0 &\equiv (1+r)^{-1}[u^0 + \beta u_1^0 + \beta^2 u_2^0 + ... + \beta^{L-1} u_{L-1}^0] ; \\
p_{K1}^0 &\equiv (1+r)^{-1}[u_1^0 + \beta u_2^0 + \beta^2 u_3^0 + ... + \beta^{L-2} u_{L-1}^0] ; \\
&... \\
p_{KL-1}^0 &\equiv (1+r)^{-1} u_{L-1}^0
\end{aligned}
$$

where $\beta \equiv (1+i)/(1+r)$ as usual. Now set $p_K^0 = 1$ and substitute equations (30) into (31) in order to obtain the following system of equations which define the period 0 asset values by age in terms of the CEP user cost for a new asset at the beginning of period 0, u^0, and the relative efficiencies of the assets by their ages, the e_i:

$$
\begin{aligned}
(32)\ p_K^0 &= (1+r)^{-1} u^0[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}] &= 1 ; \\
p_{K1}^0 &= (1+r)^{-1} u^0[e_1 + \beta e_2 + \beta^2 e_3 + ... + \beta^{L-2} e_{L-1}] &= f_1 ; \\
&... \\
p_{KL-1}^0 &= (1+r)^{-1} u^0 e_{L-1} &= f_{L-1}
\end{aligned}
$$

where the fractions f_i are defined as follows:[21]

$$
\begin{aligned}
(33)\ f_1 &\equiv [e_1 + \beta e_2 + \beta^2 e_3 + ... + \beta^{L-2} e_{L-1}]/[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}] ; \\
f_2 &\equiv [e_2 + \beta e_3 + \beta^2 e_4 + ... + \beta^{L-3} e_{L-1}]/[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}] ; \\
&... \\
f_{L-1} &\equiv e_{L-1}/[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}].
\end{aligned}
$$

Under our assumptions on the constancy of r, i and the efficiency profile parameters (the e_i), it can be seen that the f_i are also constant. Under these assumptions, it can also be seen that the sequence of asset prices by age for period 1 are equal to (1+i) times the period 0 counterparts; i.e., the following equations will be satisfied:

$$
\begin{aligned}
(34)\ p_K^1 &= (1+i)p_K^0 ; \\
p_{Ki}^1 &= (1+i)p_{Ki}^0 = f_i p_K^1 = (1+i)f_i ; \qquad i = 1,2,...,L-1.
\end{aligned}
$$

Note that given r, i and the e_i, u^0 and the f_i are determined by equations (32). Note also, that given r, i and the f_i (or equivalently, given the sequence of cross sectional depreciation rates δ_i), then equations (32) can be used to determine the relative efficiency parameters e_i and the CEP user cost u^0 for a new unit of the asset at the beginning of period 0. Thus under our constant rates of growth assumptions, the CEP model of depreciation is consistent with both an arbitrary (but finite) pattern of asset relative efficiencies as well as well as with an arbitrary pattern of cross sectional depreciation rates.

[21] These fractions f_i can be used to define the sequence of one period *cross sectional depreciation rates* for the assets as they age. Define these deprecation rates δ_i using the following equations: $1-\delta_1 \equiv p_{K1}^0/p_K^0$ and $1-\delta_i \equiv p_{Ki}^0/p_{Ki-1}^0$ for $i = 2,3,...,L-1$. Then $1-\delta_1 = f_1$, $(1-\delta_1)(1-\delta_2) = f_2$, ..., $(1-\delta_1)(1-\delta_2)...(1-\delta_{L-1}) = f_{L-1}$.

Using our steady growth of investments at the rate $(1+g)$ both going forward and backward, the sequence of asset quantities that are available at the beginning of period 0 are given by $(1+g)^{-1}$, $(1+g)^{-2}$, ... , $(1+g)^{-L}$. Using these quantities and the asset prices defined in equations (32), we can calculate the beginning of period 0 aggregate asset value for the capital stock, v_K^0, as follows:

(35) $v_K^0 = (1+g)^{-1} + f_1(1+g)^{-2} + ... + f_{L-1}(1+g)^{-L} \equiv \gamma$.

Equation (35) is the CEP counterpart to the corresponding one hoss shay capital stock valuation equation (16). As usual, we will define the period 0 price of the capital stock, p_K^0, to be equal to the corresponding investment price for a new asset which we have normalized to equal 1. Thus as in section 3 above, we can define the vintage quantity components of the period 0 capital stock in terms of equivalent units of new capital stock components using the f_i as relative weights. Thus we define $q_K^0 \equiv v_K^0$ with $p_K^0 \equiv 1$.

Repeating the above analysis for period 1 shows that $p_K^1 = (1+i)p_K^0 = 1+i$, $q_K^1 = (1+g)q_K^0$ and $v_K^1 = (1+i)(1+g)v_K^0$. Using the above analysis, the CEP capital stock price, quantity and value data are summarized in equations (36) below.

(36) $p_K^0 = 1$; $\quad p_K^1 = (1+i)$;
$q_K^0 = (1+g)^{-1} + f_1(1+g)^{-2} + ... + f_{L-1}(1+g)^{-L} \equiv \gamma$; $\quad q_K^1 = (1+g)q_K^0$;
$v_K^0 = q_K^0$; $\quad v_K^1 = (1+i)(1+g)v_K^0$;

where the f_i were defined in equations (33). Note that the rates of growth for the CEP capital stock prices, quantities and values are equal to the corresponding rates of growth for investment prices, quantities and values and these rates of growth are also equal to the corresponding rates of growth for investments and stocks for the geometric model of depreciation. However as was the case for the one hoss shay model, *it is not the case that the period 0 capital stock quantities and values necessarily coincide for the geometric and one CEP models*.

Note that v_K^1 is the value of the capital stock at the beginning of period 1. This value is made up of two components:

- v_K^{1*}, the beginning of period 1 value of the capital stocks that were in place at the beginning of period 0;
- The quantity of investment during period 0 (which is 1) but valued at the beginning of the period price of investment made in period 1, which is $(1+i)$. Thus this value is also equal to $(1+i)$.

Thus v_K^{1*} and v_K^1 are equal to the following expressions:

(37) $v_K^{1*} = (1+i)[f_1(1+g)^{-1} + f_2(1+g)^{-2} + ... + f_{L-1}(1+g)^{-(L-1)}]$;
(38) $v_K^1 = (1+i) + v_K^{1*}$.

We turn now to the determination of the value of capital services for the CEP model for period 0. The sequence of period 0 user costs or rentals by age of asset is u^0, u_1^0, u_2^0, ... , u_{L-1}^0, Under our constant growth rate assumptions, the corresponding quantities are $(1+g)^{-1}$, $(1+g)^{-2}$, ... , $(1+g)^{-L}$. Thus the period 0 value of capital services for the CEP model, v_S^0, is defined as follows:

$$(39)\ v_S^0 \equiv u^0(1+g)^{-1} + u_1^0(1+g)^{-2} + ... + u_{L-1}^0(1+g)^{-L}$$
$$= u^0[(1+g)^{-1} + e_1(1+g)^{-2} + ... + e_{L-1}(1+g)^{-L}] \qquad \text{using (30)}$$

where u^0, the period 0 user cost for a new unit of the asset, can be defined as follows using the first equation in (32):

$$(40)\ u^0 \equiv (1+r)/[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}].$$

We define the aggregate price and quantity of period 0 CEP capital services, p_S^0 and q_S^0, as follows:

$$(41)\ p_S^0 \equiv u^0;\ \ q_S^0 \equiv (1+g)^{-1} + e_1(1+g)^{-2} + ... + e_{L-1}(1+g)^{-L}.$$

To determine u^1, we use the following equations, which are period 1 counterparts to the first equations in (32):

$$(42)\ p_K^1 = (1+r)^{-1} u^1[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}] \quad = 1+i.$$

It can be seen that the u^1 solution to (42) satisfies $u^1 = (1+i)u^0$ where u^0 is defined by (40). It is easy to see that under our steady growth rate assumptions, the aggregate quantity capital services in period 1 is $(1+g)q_S^0$ and the value of CEP capital services in period 1, v_S^1, is equal to $(1+i)(1+g)v_S^0$.

The CEP capital services price, quantity and value data are summarized in equations (43) below.

$$(43)\ p_S^0 = (1+r)/[1 + \beta e_1 + \beta^2 e_2 + ... + \beta^{L-1} e_{L-1}];\ \ p_S^1 = (1+i)p_S^0;$$
$$q_S^0 = (1+g)^{-1} + e_1(1+g)^{-2} + ... + e_{L-1}(1+g)^{-L};\ \ q_S^1 = (1+g)q_S^0;$$
$$v_S^0 = p_S^0 q_S^0 \equiv \phi \qquad v_S^1 = (1+i)(1+g)v_S^0.$$

In the following section, we will approximate the CEP model defined in the present section by a geometric model of depreciation.

6. Approximating a CEP Depreciation Model by a Geometric Depreciation Model

From equations (4) in section 2, we know that the period 0 starting capital stock for a geometric model of depreciation (under our regularity conditions on growth rates) is $1/(g+\delta)$. Using the analysis presented in the previous sections, we know that we can obtain a geometric depreciation model with depreciation rate, δ^*, that will generate exactly the same capital stock prices, quantities and values that the CEP model generates[22] provided that δ^* is the solution to the following equation:

(44) $1/(g+\delta) = (1+g)^{-1} + f_1(1+g)^{-2} + ... + f_{L-1}(1+g)^{-L} \equiv \gamma$

where the f_i are defined by equations (33). Thus the solution to (44) is[23]

(45) $\delta^* \equiv \gamma^{-1} - g$.

Assume that the depreciation rate for the geometric model of depreciation is defined by (45). Denote the value of the geometric capital stock at the beginning of periods 0 and 1 by v_{KG}^0 and v_{KG}^1 respectively and denote the corresponding values of the CEP capital stocks by v_K^0 and v_K^1. Then for the geometric depreciation rate δ^* defined by (45), we have the following equalities:

(46) $v_{KG}^0 = v_K^0$; $v_{KG}^1 = v_K^1$.

Note that v_{KG}^1 is the value of the geometric capital stock at the beginning of period 1. This value is made up of two components:

- v_{KG}^{1*}, the beginning of period 1 value of the geometric capital stock that was in place at the beginning of period 0;
- The quantity of investment during period 0 (which is 1) but valued at the beginning of the period price of investment made in period 1, which is (1+i). Thus this value is also equal to (1+i).

Thus v_{KG}^{1*} and v_{KG}^1 are equal to the following expressions:

(47) $v_{KG}^{1*} = (1+i)(1-\delta)q_{KG}^0 = (1+i)(1-\delta)v_{KG}^0$
(48) $v_{KG}^1 = (1+i) + v_{KG}^{1*}$.

Now compare equations (38) and (48). Since $v_{KG}^1 = v_K^1$, it can be seen that the following equality must also hold:

(49) $v_{KG}^{1*} = v_K^{1*}$.

Thus the end of period 0 value of the depreciated beginning of the period 0 capital stocks coincide for the geometric and CEP models provided that the geometric depreciation rate δ^* is defined by (45).

We now turn our attention to the possible equality of capital services for the geometric and CEP models of depreciation. Using equations (7) and (39), it can be seen that we

[22] See equations (36) above for the CEP capital stock prices, quantities and values for periods 0 and 1.
[23] In order to ensure that δ^* is between 0 and 1, it is necessary that γ satisfy the following inequalities: $1/(1+g) < \gamma < 1/g$.

want the value of geometric capital services for period 0, v_{SG}^0, using the geometric depreciation rate defined by (45), to equal the value of CEP capital services, v_S^0, defined by (39); i.e., we want the following equation to hold:

$$(50)\ v_{SG}^0 = [r - i + (1+i)\delta]/(g+\delta) = u^0(1+g)^{-1} + u_1^0(1+g)^{-2} + ... + u_{L-1}^0(1+g)^{-L} = v_S^0.$$

At this point, it is necessary to develop some alternative expressions for v_{SG}^0 and v_S^0. Recall equations (31) which relate the sequence of period 0 CEP asset prices by age, p_K^0 and the p_{Ki}^0, to the period 0 CEP user costs by age, u^0 and the u_i^0. These equations can be differenced to provide the following expressions for the sequence of CEP user costs in terms of CEP asset prices:[24]

$$\begin{aligned}
(51)\ u^0 &= (1+r)p_K^0 - (1+r)\beta p_{K1}^0 = (1+r)p_K^0 - (1+i)f_1p_K^0 ;\\
u_1^0 &= (1+r)p_{K1}^0 - (1+r)\beta p_{K2}^0 = (1+r)f_1p_K^0 - (1+i)f_2p_K^0 ;\\
u_2^0 &= (1+r)p_{K2}^0 - (1+r)\beta p_{K3}^0 = (1+r)f_2p_K^0 - (1+i)f_3p_K^0 ;\\
&...\\
u_{L-1}^0 &= (1+r)p_{KL-1}^0 = (1+r)f_{L-1}p_K^0
\end{aligned}$$

where we have used equations (32), $p_{Ki}^0 = f_ip_K^0$, to derive the second set of equations in (51). Now set $p_K^0 = 1$ and substitute equations (51) into the first equation in (39) in order to obtain the following expression for the value of CEP capital services in period 0:

$$\begin{aligned}
(52)\ v_S^0 &= u^0(1+g)^{-1} + u_1^0(1+g)^{-2} + ... + u_{L-1}^0(1+g)^{-L}\\
&= (1+r)[(1+g)^{-1}+f_1(1+g)^{-2}+ f_2(1+g)^{-3}+...+ f_{L-1}(1+g)^{-L}]\\
&\quad - (1+i)[f_1(1+g)^{-1}+ f_2(1+g)^{-2}+...+ f_{L-1}(1+g)^{-(L-1)}]\\
&= (1+r)v_K^0 - v_K^{1*}
\end{aligned}$$

where we have used equations (35) and (37) in order to derive the last equation in (52). Equation (52) says that the value of period 0 CEP capital services, v_S^0, is equal to (1+r) times the period 0 CEP value of the beginning of the period capital stock, v_K^0, minus the beginning of period 0 value of the depreciated period 0 starting capital stock, v_K^{1*}.

We need to obtain a counterpart to the CEP equation (52) for the geometric model of depreciation. Using equations (6) and (7), we find that the period 0 value of capital services for the geometric model, v_{SG}^0, is equal to the following expression:

$$\begin{aligned}
(53)\ v_{SG}^0 &= [r - i + (1+i)\delta]/(g+\delta)\\
&= [(1+r) - (1-\delta)(1+i)]/(g+\delta)\\
&= (1+r)v_{KG}^0 - v_{KG}^{1*}
\end{aligned}$$

where the last equation follows using equations (4) and (47). Recall that v_{KG}^{1*} was defined as the beginning of period 1 value of the geometric capital stock that was in place at the beginning of period 0.

---

[24] See Diewert (2005;) for similar expressions.

All the pieces that are necessary to establish the equivalence of the CEP model to the geometric model of depreciation under our constant growth rate assumptions are in place. Choose the geometric depreciation rate δ^* equal to $\gamma^{-1} - g$ where γ is defined in (44). This will ensure that the geometric capital stock prices, quantities and values are equal to their CEP counterparts and it will also ensure that geometric value of the depreciated capital stock at the beginning of period 1, v_{KG}^{1*}, is equal to its CEP counterpart value, v_K^{1*}. Using (52) and (53), it can be seen that (50) is also satisfied; i.e., the value of capital services will be the same in periods 0 and 1 for the two models provided that we choose the geometric depreciation rate defined by (45).

7. Conclusion

What conclusions can we draw from the above computations? For computers, the geometric model of depreciation is a priori implausible. The one hoss shay model of depreciation is much more plausible with an expected length of life of 3 or 4 years.

However, under the assumption that the rate of growth of investments in computers is constant, the rate of decline in constant quality computer prices is constant and the nominal discount rate is constant, then by choosing the "right" geometric depreciation rate, the geometric model of depreciation can closely approximate the price and quantity behavior of the one hoss shay model of depreciation. Somewhat surprisingly, when the "right" geometric depreciation rate is chosen, then the geometric and one hoss shay values of capital services and stocks are exactly matched under our stationarity assumptions. A similar result holds for the CEP model of depreciation.

Unfortunately, the above results do not justify the use of the geometric model of depreciation under all circumstances. The equivalence of the geometric and CEP models will fail if:

- Rates of investment in the asset are far from being constant.
- Rates of change in the price of constant quality investment are far from being constant.
- The (nominal) cost of capital for users of the asset is far from being constant over time.

In the Appendix, using Australian data on computer investment over the past 25 years, we find that while the first two assumptions listed above are approximately satisfied, the third assumption is not justified: Australian interest rates have had a very strong downward trend during the past 25 years. Nevertheless, when we use our "best" geometric approximations to one hoss shay models of depreciation with length of life equal to either 3 or 4 years, we find that the approximating geometric model generates capital services data that are quite close to the corresponding one hoss shay model. However, the approximating geometric capital stocks are not nearly as close to their one hoss shay counterparts. These results suggest that national statistical agencies should consider moving to one hoss shay models of depreciation for computers. These models are not that difficult to implement but they do have the disadvantage that they may be a bit difficult to explain to users.

Finally, it is not only computers where one hoss shay models of depreciation are more plausible than geometric models of depreciation: many long lived infrastructure assets could be better approximated by one hoss shay models. Such assets include pipelines, sewers, electricity and telecommunication networks, railway lines, docking facilities and some commercial structures. Even more assets could be better described by CEP models, which are just as easy to implement as one hoss shay models, provided one has reasonable estimates for the efficiency profiles.

Appendix: Approximating a One Hoss Shay Model for Computers with a Geometric Model: The Case of Australia

Our suggested method for obtaining a geometric model of depreciation that will approximate a one hoss shay model relies on the underlying assumptions that we used to derive our equivalence results; namely: constant nominal discount rates, constant rates of growth for investments and constant rates of price change for the asset. Of course, these assumptions will not be satisfied in practice so it will be useful to test out our results on a real data set and see how well our equivalent geometric model of depreciation approximates a one hoss shay model of depreciation for computers using Australian data on economy wide investments in computers over the past 30 years. We will implement our approximating geometric models assuming that the "truth" is a one hoss shay model with length of life equal to either 3 or 4 years. We will consider the 4 year life model first.

We use Australian Bureau of Statistics (2014) national accounts data on computers and peripherals, which are sourced from the official statistics on information technology (IT). The ABS publishes separate data in time series spreadsheets on IT gross capital formation (investment) (Table 70) , consumption of fixed capital (depreciation) (Table 71) and net capital stock (Table 69), both in current prices and chain volume measures. These data are compiled by individual industries as well as all industries and we will use the all industries data. The published data provide chained dollar estimates (base year is 2009) as well as current dollar estimates for investment, capital stocks and depreciation. The original data cover periods from July 1, 1961 to June 30, 2014. Our calculations are based on data for periods from July 1, 1985 to June 30, 2014.[25] Thus our June years run from 1985 to 2013. We calculated price indexes by dividing the current dollar estimates by the chained dollar estimates. We then renormalized the price series so that all prices were unity for 1985 (and the quantity data were then calculated as the value data divided by the new price indexes). The renormalized ABS price and quantity data are listed in Table 1 below.[26]

[25] The published Tables do not contain enough significant digits for the period 1961-1985 for us to accurately determine prices and volumes for the earlier period.

[26] The capital stock information in Table 1 refers to end of June year estimates. In the subsequent Tables, we will switch to beginning of June year capital stock estimates.

Table 1: ABS Price and Quantity (Volume) of Computer Investment, Capital Stocks and Depreciation, 1985-2013, Millions of 1985 Dollars

Year	P_I	P_K	P_D	Q_I	Q_K	Q_D
1985	1.00000	1.00000	1.00000	3702	8346	2190
1986	0.89421	0.85411	0.90689	4791	10184	2758
1987	0.71591	0.68662	0.71087	6460	12940	3569
1988	0.57849	0.60136	0.58118	8928	16998	4623
1989	0.55463	0.55803	0.55713	9799	20520	6002
1990	0.49638	0.49938	0.49925	9291	22282	7381
1991	0.44296	0.45135	0.44372	10380	23966	8598
1992	0.40818	0.41255	0.40665	12921	26799	9896
1993	0.36897	0.35566	0.36951	15389	30628	11356
1994	0.30176	0.29162	0.30145	18946	36064	13140
1995	0.24875	0.22586	0.24747	25624	45558	15736
1996	0.18035	0.17363	0.17839	35713	60872	19872
1997	0.15296	0.14430	0.14943	54078	87671	26929
1998	0.13029	0.11311	0.12283	64458	117380	36906
1999	0.09601	0.09462	0.09052	89211	159493	49964
2000	0.09429	0.08567	0.08636	94582	195557	65051
2001	0.08363	0.07146	0.07534	107431	232386	80219
2002	0.06630	0.05417	0.05957	146193	292340	99199
2003	0.04772	0.04175	0.04270	207457	390960	127426
2004	0.04026	0.03571	0.03614	259721	507804	165629
2005	0.03652	0.03070	0.03129	314234	652366	213566
2006	0.03082	0.02540	0.02668	381814	812472	271398
2007	0.02471	0.02174	0.02127	528370	1063464	347967
2008	0.02276	0.01939	0.01977	531927	1234519	430943
2009	0.01876	0.01675	0.01683	586658	1377473	502483
2010	0.01652	0.01489	0.01480	596603	1474332	563236
2011	0.01496	0.01358	0.01331	673764	1605495	618553
2012	0.01378	0.01267	0.01233	677684	1689797	667950
2013	0.01296	0.01173	0.01160	921435	1968201	734704

From viewing Table 1, it can be seen that while the implicit prices of investment and depreciation are very close, these price indexes end up about 10.5% higher in 2013 than the corresponding capital stock price. These differences may be caused by aggregation over finer asset classifications than are published.

Note also that we used the information in the above Table to calculate $\delta_{ABS}{}^t \equiv Q_D{}^t/Q_K{}^{t-1}$ for years t = 1989, ... , 2013. Since $Q_K{}^{t-1}$ is the official ABS end of year computer capital stock, it is also equal to the beginning of year t capital stock and so $\delta_{ABS}{}^t$ is an ABS estimated depreciation rate for computers for year t. The average of these rates over the past 25 years is $\delta_{ABS}{}^* \equiv 0.39220$. We made use of this average depreciation rate in section 4 of the main text.

We will choose the price and quantity (volume) series for investment as our key data and we will not use the remaining data on capital stocks and depreciation in the analysis which follows. A key decision has to be made on how to value the beginning of the period capital stock. We will value the beginning of the year t capital stock, $p_K^{t,}$ at the average investment price of the previous year.[27] We also renormalize the price of investment in computers to equal 1 in 1988 (and make the offsetting renormalization of the quantity data). This means that the price of capital at the beginning of the June year 1989 will be equal to unity. Our subsequent Tables will concentrate on the period 1989-2013.[28] Let q_I^t denote the quantity of computer investment in year t. The capital price series p_K^t and the quantity of investment series q_I^t are listed in Table 2 below. The rates of growth in the price of computer capital over year t, the inflation rates i^t, are defined by (A1) below and the growth rate of investment in year t over the previous year, g^t is defined by (A2):

(A1) $i^t \equiv (p_K^{t+1}/p_K^t) - 1$; t = 1989, 1990, ... , 2013.

(A2) $g^t \equiv (q_I^t/q_I^{t-1}) - 1$; t = 1989, 1990, ... , 2013.

All of the variables p_K^t, q_I^t, i^t and g^t can be defined for t = 1989-2013 using the data in Table 1. We require information on one additional variable and that is the financial cost of capital for the purchasers of computers. The cost of capital is likely to vary across users and it is difficult to determine an appropriate economy wide cost of capital. As an approximation to the economy wide "true" cost of capital, we use the July bond yield for 5 year Australian government bonds r^t as compiled by the Reserve Bank of Australia (2014).[29] The interest rate series r^t is also listed in Table 2 below.

Table 2: Real Investment in 1988 Dollars q_I^t, Beginning of the Year Price of Capital p_K^t, Growth Rate of Investment over Previous Year g^t, Annual Inflation Rate for the Price of Capital i^t, Nominal 5 Year Government Bond Yield at the Beginning of the Year r^t, Smoothed Inflation Rate i_S^t and Smoothed Bond Rate r_S^t.

Year	q_I^t	p_K^t	g^t	i^t	r^t	i_S^t	r_S^t
1989	5669	1.00000	0.09756	-0.04126	0.1400	-0.04859	0.14579
1990	5375	0.95874	-0.05185	-0.10502	0.1351	-0.08751	0.12643
1991	6005	0.85805	0.11719	-0.10761	0.1085	-0.09854	0.10524
1992	7475	0.76572	0.24476	-0.07852	0.0705	-0.09187	0.07960
1993	8902	0.70559	0.19101	-0.09606	0.0643	-0.11569	0.07332
1994	10960	0.63781	0.23113	-0.18216	0.0896	-0.15565	0.08087
1995	14823	0.52163	0.35249	-0.17565	0.0844	-0.20595	0.08563
1996	20660	0.43000	0.39377	-0.27498	0.0835	-0.21129	0.07746
1997	31284	0.31176	0.51423	-0.15186	0.0615	-0.18608	0.06551
1998	37289	0.26442	0.19195	-0.14826	0.0535	-0.18217	0.05696
1999	51608	0.22522	0.38401	-0.26309	0.0576	-0.16002	0.05731
2000	54715	0.16596	0.06021	-0.01792	0.0607	-0.11536	0.05915

[27] In theory, balance sheet items at the beginning of a period should be valued at the prices prevailing at the end of the previous period. For simplicity, we will use the prices of the entire previous year instead of the prices of the previous quarter or month. This simplification will not materially affect our estimates.
[28] We need four years of investment data in order to determine the starting stock of one hoss shay capital at the start of the fifth year.
[29] As the July 2013 bond yield was not yet available on the RBA website at the time of writing, we used the May 2013 bond yield as an approximation to the July 2013 yield.

2001	**62148**	**0.16299**	**0.13584**	**-0.11308**	**0.0584**	**-0.11279**	**0.05843**
2002	**84572**	**0.14456**	**0.36081**	**-0.20723**	**0.0562**	**-0.20116**	**0.05474**
2003	**120013**	**0.11460**	**0.41907**	**-0.28019**	**0.0489**	**-0.22385**	**0.05291**
2004	**150247**	**0.08249**	**0.25192**	**-0.15637**	**0.0556**	**-0.17366**	**0.05262**
2005	**181783**	**0.06959**	**0.20989**	**-0.09293**	**0.0519**	**-0.12919**	**0.05491**
2006	**220877**	**0.06312**	**0.21506**	**-0.15613**	**0.0587**	**-0.15006**	**0.05798**
2007	**305659**	**0.05327**	**0.38384**	**-0.19815**	**0.0630**	**-0.15204**	**0.06206**
2008	**307716**	**0.04271**	**0.00673**	**-0.07912**	**0.0640**	**-0.14080**	**0.06031**
2009	**339378**	**0.03933**	**0.10289**	**-0.17554**	**0.0521**	**-0.13192**	**0.05430**
2010	**345131**	**0.03243**	**0.01695**	**-0.11959**	**0.0479**	**-0.12830**	**0.04873**
2011	**389769**	**0.02855**	**0.12933**	**-0.09403**	**0.0466**	**-0.09715**	**0.04070**
2012	**392036**	**0.02587**	**0.00582**	**-0.07935**	**0.0247**	**-0.07780**	**0.03251**
2013	**533045**	**0.02381**	**0.35968**	**-0.05924**	**0.0279**	**-0.05989**	**0.02428**

There is one additional problem that requires discussion. When Jorgenson (1989) and his coworkers apply the geometric model of depreciation, they calculate user costs using ex post asset inflation rates in the user cost formula. This choice to use ex post inflation rates leads to quite volatile user costs.[30] But at the beginning of each period, producers cannot anticipate with certainty the actual end of period price of the asset under consideration. At best, they will only be able to anticipate the general trend of asset prices. A similar problem arises with our choice of discount rate. The central bank can change interest rates quite abruptly and these abrupt changes could induce unwarranted fluctuations in our user costs. Thus if we want to obtain user costs that will approximate market rents for the asset (which are generally very smooth), it is better to use expected capital gains and smoothed interest rates in the user cost formula rather than actual ex post capital gains and actual interest rates Hence at the end of this Appendix, we will repeat our calculations, replacing the i^t and r^t with smoothed approximations i_S^t and r_S^t. Our smoothed inflation and interest rates are listed in Table 2 above.[31] Comparing the i^t with their smoothed counterparts (the i_S^t), it can be seen that the i^t are tremendously volatile while the r^t do not differ all that much from their smoothed counterparts, the r_S^t.[32]

We now carry out the computations that are necessary to calculate the one hoss shay computer capital stocks and service flows for Australia over the June years 1989-2013 when the length of life is equal to 4 years. Define the year t discount factor $\beta^t \equiv (1+i^t)/(1+r^t)$ where the i^t and r^t are listed in Table 2 above. The one hoss shay user cost for year t, u^t, is defined by the year t counterpart to equation (13) in the main text:

(A3) $u^t = p_K^t(1+r^t)/(1+\beta^t+(\beta^t)^2+(\beta^t)^3)$.

The one hoss shay price of a new unit of capital at the beginning of period t is p_K^t (listed in Table 2), which in turn is equal to last year's investment price for a new unit of the asset. The one hoss shay asset prices at the beginning of period t for assets that are 1, 2

[30] The use of ex post inflation rates in the one hoss shay user costs will also lead to volatile user costs.
[31] We used the LOWESS nonparametric smoothing method in Shazam. See White (2004) for a description of the method, which was originally due to Cleveland (1979). We used the cross validation criterion to pick the smoothing parameters.
[32] The variances of the i^t and i_S^t are 0.0049 and 0.0023 respectively, while the variances of the r^t and the r_S^t are 0.00077 and 0.00073 respectively.

and 3 years old are defined to be $p_{K1}{}^t$, $p_{K2}{}^t$ and $p_{K3}{}^t$. These vintage asset prices are defined as follows:

$$\text{(A4)}\quad p_{K1}{}^t \equiv (1+r)^{-1}u^t(1+\beta^t+(\beta^t)^2)$$
$$p_{K2}{}^t \equiv (1+r)^{-1}u^t(1+\beta^t)$$
$$p_{K3}{}^t \equiv (1+r)^{-1}u^t$$

The beginning of year t quantity of new computers is lagged investment $q_I{}^{t-1}$, of one year old computers is $q_I{}^{t-2}$, of two year old computers is $q_I{}^{t-3}$ and of three year old computers is $q_I{}^{t-4}$. The corresponding asset prices are $p_K{}^t$ and the $p_{K1}{}^t$, $p_{K2}{}^t$ and $p_{K3}{}^t$ defined by (A4) above. We form a one hoss shay capital stock of computers at the beginning of year t, $q_{KH}{}^t$ with corresponding asset price $p_{KH}{}^t$ as Fisher (1922) ideal chained price and quantity aggregates. These one hoss shay aggregate asset prices and quantities are listed in Table 3 below along with the corresponding one hoss shay asset values, $v_{KH}{}^t \equiv p_{KH}{}^t q_{KH}{}^t$ for the June years 1989-2013.[33]

The one hoss shay user cost for year t, u^t, has already been defined by (A3) above. In Table 4 below, we relabel u^t as $p_{SH}{}^t$. The corresponding quantity $q_{SH}{}^t$ for year t is simply the sum of lagged investments over the previous 4 years:

$$\text{(A5)}\quad q_{SH}{}^t \equiv q_I{}^{t-1} + q_I{}^{t-2} + q_I{}^{t-3} + q_I{}^{t-4}.$$

The corresponding year t value of one hoss shay capital services is $v_{SH}{}^t \equiv u^t q_{SH}{}^t$. We normalized the one hoss shay user costs $u^t \equiv p_{SH}{}^t$ to equal 1 in 1989 with an offsetting normalization of the $q_{SH}{}^t$ so that the values $v_{SH}{}^t$ are preserved. These one hoss shay user costs and service flows are listed in Table 4 below.

We turn our attention to the details on how to construct the asset value and service flow data for the geometric model of depreciation that will best approximate the above one hoss shay model using the theory that was developed in section 4 of the main text. Our first task is to determine the best geometric depreciation rate δ that can approximate the one hoss shay model of depreciation with length of asset life equal to 4. In order to do this, we need to insert long run average values for g, i and r into equations (22) or (24). We define one plus these long run values g^*, i^* and r^* as geometric means of the $1+g^t$, $1+i^t$ and $1+r^t$:

$$\text{(A6)}\quad 1+g^* \equiv [(1+g^{1989})(1+g^{1990})...(1+g^{2013})]^{1/25} = 1 + 0.20378\ ;$$
$$\text{(A7)}\quad 1+i^* \equiv [(1+i^{1989})(1+i^{1990})...(1+i^{2013})]^{1/25} = 1 - 0.14096$$
$$\text{(A8)}\quad 1+r^* \equiv [(1+r^{1989})(1+r^{1990})...(1+r^{2013})]^{1/25} = 1 + 0.06627.$$

[33] We impose the normalization $p_{KH}{}^{1989} = 1$ on these one hoss shay asset prices and quantities. Note that because the discount factors β^t are no longer constant (as they were in the main text), the vintage asset prices defined by equations (A4) will no longer vary proportionally to the new asset price $p_K{}^t$ and thus it is necessary to use an index number formula in order to aggregate the vintage assets.

Thus g^* is approximately equal to 20.4%, i^* to −14.1% and r^* to 6.6%. Using the values for g^*, i^* and r^* listed in (A6)-(A8) above, we solve the following counterpart to equation (22) for our "best" geometric depreciation rate δ^*:

(A9) $1/(g^*+\delta^*) = (1+g^*)^{-1}[1+f_1(1+g^*)^{-1}+f_2(1+g^*)^{-2}+f_3(1+g^*)^{-3}]$

where $\beta^* \equiv (1+i^*)/(1+r^*)$, $f_1 \equiv (1+\beta^*+\beta^{*2})/(1+\beta^*+\beta^{*2}+\beta^{*3})$, $f_2 \equiv (1+\beta^*)/(1+\beta^*+\beta^{*2}+\beta^{*3})$ and $f_3 \equiv 1/(1+\beta^*+\beta^{*2}+\beta^{*3})$. The solution to (A9) is $\delta^* = 0.32055$. We set the starting geometric capital stock at the beginning of 1989, q_{KG}^{1989}, to be equal to the corresponding starting capital stock for the one hoss shay model, q_{KH}^{1989}, which is listed in Table 3 below. The remaining geometric constant dollar capital stocks are constructed using this starting value and the investment data for computers q_I^t listed in Table 2 above, using the following recursive equations:

(A10) $q_{KG}^t \equiv (1-\delta^*)q_{KG}^{t-1} + q_I^t$; t = 1990, 1991,...,2013.

The beginning of year t price of the geometric capital stock, p_{KG}^t is defined as the lagged ABS investment price which we have listed as p_K^t in Table 2 and the corresponding geometric beginning of year t asset value is defined as $v_{KG}^t \equiv p_{KG}^t q_{KG}^t$. The p_{KG}^t, q_{KG}^t and v_{KG}^t are listed below in Table 3 and can be compared with their one hoss shay (L = 4) capital stock counterparts, p_{KH}^t, q_{KH}^t and v_{KH}^t

Table 3: Comparison of Geometric and One Hoss Shay (L = 4) Capital Stock Prices, Quantities and Values

Year	p_{KG}^t	p_{KH}^t	q_{KG}^t	q_{KH}^t	v_{KG}^t	v_{KH}^t
1989	1.000000	1.000000	10499.7	10499.7	10499.7	10499.7
1990	0.958739	0.979610	12802.9	12939.5	12274.7	12675.7
1991	0.858053	0.870021	14073.9	14241.2	12076.2	12390.2
1992	0.765716	0.754901	15567.3	15389.4	11920.2	11617.5
1993	0.705594	0.699550	18051.8	17375.8	12737.2	12155.3
1994	0.637814	0.661986	21167.6	20259.0	13501.0	13411.2
1995	0.521630	0.539068	25342.2	24533.1	13219.2	13225.0
1996	0.430004	0.462414	32041.9	31306.6	13778.1	14476.6
1997	0.311762	0.317763	42430.9	41822.5	13228.3	13289.7
1998	0.264417	0.268534	60113.6	59987.7	15895.0	16108.7
1999	0.225215	0.238769	78133.0	79385.2	17596.7	18954.8
2000	0.165963	0.160655	104695.4	106653.0	17375.5	17134.4
2001	0.162989	0.163623	125850.7	128023.8	20512.3	20947.7
2002	0.144558	0.151046	147657.2	148385.1	21345.1	22413.0
2003	0.114601	0.123373	184897.3	182644.8	21189.4	22533.5
2004	0.082491	0.084627	245641.3	240206.0	20263.2	20327.9
2005	0.069592	0.069636	317147.9	314008.8	22071.0	21866.2
2006	0.063125	0.064872	397268.9	397307.2	25077.5	25774.0
2007	0.053269	0.055770	490801.6	490640.9	26144.7	27363.0
2008	0.042714	0.042646	639134.0	633644.5	27300.2	27022.2
2009	0.039335	0.040687	741976.0	736939.8	29185.6	29983.9
2010	0.032430	0.032669	843513.9	832378.9	27355.1	27192.6
2011	0.028551	0.028403	918256.7	891264.9	26217.6	25314.4

2012	**0.025867**	**0.025317**	**1013678.0**	**959816.3**	**26220.4**	**24299.7**
2013	**0.023814**	**0.023118**	**1080780.0**	**1010693.0**	**25737.8**	**23365.1**

The geometric model capital stock price, p_{KG}^{2013} ended up about 3.0% higher than its one hoss shay counterpart, p_{KH}^{2013}, q_{KG}^{2013} ended up about 6.9% higher than, q_{KH}^{2013} and v_{KG}^{2013} ended up about 10.2% higher than its one hoss shay counterpart, v_{KH}^{2013}.[34] Thus the geometric model capital stocks can only provide a rough approximation to the one hoss shay capital stocks using our Australian data set.[35]

The Jorgensonian geometric model user cost (which uses ex post asset capital gains in the user cost formula) for year t, p_{SG}^t, is defined as follows:

$$\text{(A11)}\quad p_{SG}^t \equiv p_K^t[(1+r^t) - (1-\delta^*)(1+i^t)]\ ; \qquad t = 1989,...,2013$$

where δ^* is equal to 0.32055 and the p_K^t, r^t and i^t are listed in Table 2 above. The year t quantity of capital services for the geometric model, q_{SG}^t, is defined as the corresponding capital stock q_{KG}^t defined above by (A10). The corresponding year t value of capital services is defined as $v_{SG}^t \equiv p_{SG}^t q_{SG}^t$ for t = 1989,...,2013. Finally, we normalize the price of geometric capital services p_{SG}^t to equal unity for t = 1989 and redefine q_{SG}^t to equal v_{SG}^t/p_{SG}^t. The resulting p_{SG}^t, q_{SG}^t and v_{SG}^t are listed below in Table 4 and can be compared with their one hoss shay (L = 4) capital service counterparts, p_{SH}^t, q_{SH}^t and v_{SH}^t.

Table 4: Comparison of Geometric and One Hoss Shay (L = 4) Capital Service Prices, Quantities and Values

Year	p_{SG}^t	p_{SH}^t	q_{SG}^t	q_{SH}^t	v_{SG}^t	v_{SH}^t
1989	1.000000	1.000000	5130.0	5010.8	5130.0	5010.8
1990	1.034132	1.034539	6255.3	6290.2	6468.8	6507.4
1991	0.881907	0.881526	6876.3	7234.4	6064.2	6377.4
1992	0.696467	0.697618	7606.0	8056.9	5297.3	5620.6
1993	0.650043	0.650410	8819.8	8894.6	5733.3	5785.1
1994	0.696995	0.700110	10342.1	10067.3	7208.4	7048.2
1995	0.559759	0.561529	12381.8	12092.9	6930.8	6790.5
1996	0.520038	0.531574	15655.2	15291.3	8141.3	8128.5
1997	0.309623	0.309561	20731.1	20073.7	6418.8	6214.0
1998	0.256948	0.256813	29370.6	28191.5	7546.7	7239.9
1999	0.256710	0.260543	38174.6	37741.0	9799.8	9833.1
2000	0.133639	0.135692	51152.6	51082.8	6836.0	6931.5
2001	0.152046	0.152022	61488.7	63434.7	9349.1	9643.4
2002	0.153129	0.153814	72143.0	74629.0	11047.2	11479.0

[34] The corresponding ABS net capital stock estimate (all industries, computers and peripherals, current prices) for the June year that starts in July 1, 2013 is $21,414 million which is well below our geometric and one hoss shay estimates of $25,738 and $23,365 respectively. However, the ABS uses a different model of depreciation which follows the methodology used by the US Bureau of Labor Statistics.

[35] The reason why the approximation is not better is due to the fact that our nominal interest rate series had a strong downward trend throughout the sample period. In order for the geometric model to provide a good approximation to the one hoss shay model, we require that g^t, i^t and r^t have no strong trends and this condition is clearly violated for the interest rate series r^t. Nevertheless, our geometric approximation is not terrible.

2003	**0.131311**	**0.133616**	**90338.0**	**91778.5**	**11862.4**	**12263.0**
2004	**0.081446**	**0.081429**	**120016.5**	**116588.8**	**9774.9**	**9493.7**
2005	**0.062044**	**0.062157**	**154953.5**	**151238.0**	**9614.0**	**9400.5**
2006	**0.062704**	**0.062697**	**194099.4**	**194629.4**	**12170.9**	**12202.7**
2007	**0.056496**	**0.056719**	**239798.1**	**244067.3**	**13547.7**	**13843.2**
2008	**0.038319**	**0.038403**	**312271.0**	**311401.0**	**11965.8**	**11958.7**
2009	**0.039604**	**0.039633**	**362518.0**	**368515.0**	**14357.1**	**14605.5**
2010	**0.029849**	**0.029848**	**412127.8**	**425674.8**	**12301.8**	**12705.6**
2011	**0.025189**	**0.025245**	**448646.0**	**470741.6**	**11300.9**	**11883.8**
2012	**0.021133**	**0.021300**	**495267.4**	**501248.1**	**10466.3**	**10676.5**
2013	**0.018946**	**0.019176**	**528052.2**	**531830.7**	**10004.3**	**10198.4**

The one hoss shay model capital services price, p_{SH}^{2013} ended up about 1.1% higher than its geometric counterpart, p_{SG}^{2013}, q_{SH}^{2013} ended up about 0.7% higher than, q_{SG}^{2013} and v_{SH}^{2013} ended up about 1.9% higher than its geometric counterpart, v_{SG}^{2013}. Thus for capital services, the geometric approximation turns out to be quite close to the one hoss shay model.[36]

We now repeat the above comparisons but we use the one hoss shay model of depreciation with length of asset life L equal to 3 years as the "truth". Define the year t discount factor $\beta^t \equiv (1+i^t)/(1+r^t)$ as before where the i^t and r^t are listed in Table 2 above. The one hoss shay (L = 3) user cost for year t, u^t, is defined as follows:

(A12) $u^t = p_K^t(1+r^t)/(1+\beta^t+(\beta^t)^2)$.

The one hoss shay asset prices at the beginning of period t for assets that are 1 and 2 years old are defined to be p_{K1}^t and p_{K2}^t. These vintage asset prices are defined as follows:

(A13) $p_{K1}^t \equiv (1+r)^{-1}u^t(1+\beta^t)$
$p_{K2}^t \equiv (1+r)^{-1}u^t$

The beginning of year t quantity of new computers is lagged investment q_I^{t-1}, of one year old computers is q_I^{t-2} and of two year old computers is q_I^{t-3}. The corresponding asset prices are p_K^t and p_{K1}^t and p_{K2}^t defined by (A13) above. We form a one hoss shay capital stock of computers at the beginning of year t, q_{KH}^t with corresponding asset price p_{KH}^t as Fisher (1922) ideal chained price and quantity aggregates. These one hoss shay aggregate asset prices and quantities are listed in Table 5 below along with the corresponding one hoss shay asset values, $v_{KH}^t \equiv p_{KH}^t q_{KH}^t$ for the June years 1989-2013.[37]

[36] We cannot compare our geometric and one hoss shay estimates of computer capital services with an official ABS estimate because the ABS does not provide economy wide estimates for the value of computer services.
[37] We impose the normalization $p_{KH}^{1989} = 1$ on these one hoss shay asset prices and quantities. Note that because the discount factors β^t are no longer constant (as they were in the main text), the vintage asset prices defined by equations (A4) will no longer vary proportionally to the new asset price p_K^t and thus it is necessary to use an index number formula in order to aggregate the vintage assets.

The one hoss shay (L = 3) user cost for year t, u^t, has already been defined by (A12) above. In Table 6 below, we relabel u^t as p_{SH}^t. The corresponding quantity q_{SH}^t for year t is simply the sum of lagged investments over the previous 4 years:

(A14) $q_{SH}^t \equiv q_I^{t-1} + q_I^{t-2} + q_I^{t-3}$.

The corresponding year t value of one hoss shay capital services is $v_{SH}^t \equiv u^t q_{SH}^t$. We normalized the one hoss shay user costs $u^t \equiv p_{SH}^t$ to equal 1 in 1989 with an offsetting normalization of the q_{SH}^t so that the values v_{SH}^t are preserved. These one hoss shay user costs and service flows are listed in Table 6 below.

We now consider the approximating geometric model of depreciation. We need to determine the best geometric depreciation rate δ that can approximate the one hoss shay model of depreciation with length of asset life equal to 3. Again, we define the long run average values for g, i and r as the g^*, i^* and r^* defined by equations (A6)-(A8). Now solve the following counterpart to equation (26) for our "best" geometric depreciation rate δ^{**}:

(A15) $1/(g^*+\delta^{**}) = (1+g^*)^{-1}[1+f_1(1+g^*)^{-1}+f_2(1+g^*)^{-2}]$

where $\beta^* \equiv (1+i^*)/(1+r^*)$, $f_1 \equiv (1+\beta^*)/(1+\beta^*+\beta^{*2})$ and $f_2 \equiv 1/(1+\beta^*+\beta^{*2})$. The solution to (A15) is $\delta^{**} = 0.43240$. Thus when we switch from the L = 4 to the L = 3 one hoss shay depreciation model, the approximating geometric model depreciation rate jumps from about 32% per year to 43% per year.

We set the starting geometric capital stock at the beginning of 1989, q_{KG}^{1989}, to be equal to the corresponding starting capital stock for the one hoss shay model, q_{KH}^{1989}, which is listed in Table 5 below. The remaining geometric constant dollar capital stocks are constructed using this starting value and the investment data for computers q_I^t listed in Table 2 above, using the following recursive equations:

(A16) $q_{KG}^t \equiv (1-\delta^{**})q_{KG}^{t-1} + q_I^t$;   t = 1990, 1991,...,2013.

The beginning of year t price of the geometric capital stock, p_{KG}^t is defined as the lagged ABS investment price which we have listed as p_K^t in Table 2 and the corresponding geometric beginning of year t asset value is defined as $v_{KG}^t \equiv p_{KG}^t q_{KG}^t$. The p_{KG}^t, q_{KG}^t and v_{KG}^t are listed below in Table 5 and can be compared with their one hoss shay (L = 3) capital stock counterparts, p_{KH}^t, q_{KH}^t and v_{KH}^t.

Table 5: Comparison of Geometric and One Hoss Shay (L = 3) Capital Stock Prices, Quantities and Values

Year	p_{KG}^t	p_{KH}^t	q_{KG}^t	q_{KH}^t	v_{KG}^t	v_{KH}^t
1989	1.000000	1.000000	8952.6	8952.6	8952.6	8952.6
1990	0.958739	0.974179	10750.4	10877.1	10306.8	10596.3
1991	0.858053	0.866811	11476.9	11533.9	9847.8	9997.8

1992	**0.765716**	**0.758010**	**12519.1**	**12150.2**	**9586.1**	**9210.0**
1993	**0.705594**	**0.701176**	**14580.4**	**13972.7**	**10287.8**	**9797.3**
1994	**0.637814**	**0.654443**	**17178.1**	**16716.2**	**10956.4**	**10939.8**
1995	**0.521630**	**0.533535**	**20710.2**	**20393.3**	**10803.0**	**10880.5**
1996	**0.430004**	**0.453134**	**26578.2**	**26257.7**	**11428.7**	**11898.3**
1997	**0.311762**	**0.315719**	**35745.8**	**35644.8**	**11144.2**	**11253.7**
1998	**0.264417**	**0.267067**	**51573.3**	**51958.8**	**13636.8**	**13876.5**
1999	**0.225215**	**0.235083**	**66561.8**	**68096.6**	**14990.7**	**16008.3**
2000	**0.165963**	**0.161748**	**89388.5**	**90961.6**	**14835.1**	**14712.9**
2001	**0.162989**	**0.163095**	**105452.3**	**106667.1**	**17187.5**	**17396.8**
2002	**0.144558**	**0.149024**	**122002.7**	**122073.9**	**17636.5**	**18191.9**
2003	**0.114601**	**0.120797**	**153820.4**	**150385.8**	**17628.0**	**18166.1**
2004	**0.082491**	**0.083987**	**207321.2**	**204216.6**	**17102.2**	**17151.5**
2005	**0.069592**	**0.069585**	**267922.4**	**268496.8**	**18645.3**	**18683.3**
2006	**0.063125**	**0.064382**	**333855.6**	**335445.2**	**21074.6**	**21596.5**
2007	**0.053269**	**0.055090**	**410373.7**	**408834.1**	**21860.4**	**22522.7**
2008	**0.042714**	**0.042685**	**538586.9**	**533147.2**	**23005.4**	**22757.4**
2009	**0.039335**	**0.040333**	**613418.4**	**611696.6**	**24128.8**	**24671.4**
2010	**0.032430**	**0.032606**	**687554.5**	**678149.6**	**22297.3**	**22111.8**
2011	**0.028551**	**0.028452**	**735387.1**	**707407.3**	**20996.4**	**20127.0**
2012	**0.025867**	**0.025489**	**807174.3**	**768578.1**	**20878.9**	**19590.4**
2013	**0.023814**	**0.023335**	**850188.2**	**804768.0**	**20246.5**	**18779.3**

The results are similar to the results for the L = 4 comparison but the differences between the one hoss shay estimates and the corresponding geometric estimates are a bit closer. The geometric capital stock price p_{KG}^{2013} ended up 2.1% higher than p_{KH}^{2013}, the geometric capital stock volume q_{KG}^{2013} ended up 5.6% higher than its one hoss shay counterpart q_{KH}^{2013} and the geometric capital stock value v_{KG}^{2013} ended up 7.8% higher than v_{KH}^{2013}.

The Jorgensonian geometric model user cost for year t, p_{SG}^t, is defined as follows:

(A17) $p_{SG}^t \equiv p_K^t[(1+r^t) - (1-\delta^{**})(1+i^t)]$; t = 1989,...,2013

where δ^{**} is equal to 0.43240 and the p_K^t, r^t and i^t are listed in Table 2 above. The year t quantity of capital services for the geometric model, q_{SG}^t, is defined as the corresponding capital stock q_{KG}^t defined above by (A16). The corresponding year t value of capital services is defined as $v_{SG}^t \equiv p_{SG}^t q_{SG}^t$ for t = 1989,...,2013. Finally, we normalize the price of geometric capital services p_{SG}^t to equal unity for t = 1989 and redefine q_{SG}^t to equal v_{SG}^t/p_{SG}^t. The resulting p_{SG}^t, q_{SG}^t and v_{SG}^t are listed below in Table 6 and can be compared with their one hoss shay (L = 3) capital service counterparts, p_{SH}^t, q_{SH}^t and v_{SH}^t.

Table 6: Comparison of Geometric and One Hoss Shay (L = 3) Capital Service Prices, Quantities and Values

Year	p_{SG}^t	p_{SH}^t	q_{SG}^t	q_{SH}^t	v_{SG}^t	v_{SH}^t
1989	**1.000000**	**1.000000**	**5334.1**	**5222.4**	**5334.1**	**5222.4**

1990	**1.009088**	**1.009338**	**6405.3**	**6518.6**	**6463.5**	**6579.4**
1991	**0.866926**	**0.866705**	**6838.1**	**7251.2**	**5928.2**	**6284.7**
1992	**0.703575**	**0.704252**	**7459.1**	**7626.9**	**5248.1**	**5371.3**
1993	**0.652782**	**0.652999**	**8687.3**	**8434.7**	**5670.9**	**5507.8**
1994	**0.669474**	**0.671346**	**10235.1**	**10012.7**	**6852.1**	**6722.0**
1995	**0.539736**	**0.540800**	**12339.5**	**12229.4**	**6660.1**	**6613.6**
1996	**0.484967**	**0.491973**	**15835.8**	**15516.8**	**7679.9**	**7633.9**
1997	**0.303535**	**0.303500**	**21298.0**	**20776.8**	**6464.7**	**6305.8**
1998	**0.252981**	**0.252902**	**30728.4**	**29869**	**7773.7**	**7553.9**
1999	**0.241662**	**0.243980**	**39658.8**	**39919.2**	**9584.0**	**9739.5**
2000	**0.140184**	**0.141387**	**53259.4**	**53764.1**	**7466.1**	**7601.6**
2001	**0.151818**	**0.151804**	**62830.5**	**64246.4**	**9538.8**	**9752.8**
2002	**0.147083**	**0.147494**	**72691.6**	**75367.5**	**10691.7**	**11116.3**
2003	**0.123163**	**0.124559**	**91649.2**	**90114.1**	**11287.8**	**11224.5**
2004	**0.079852**	**0.079842**	**123526.0**	**119325.6**	**9863.8**	**9527.2**
2005	**0.062728**	**0.062794**	**159633.4**	**158737.6**	**10013.4**	**9967.8**
2006	**0.061419**	**0.061415**	**198917.7**	**202226.0**	**12217.3**	**12419.7**
2007	**0.054347**	**0.054480**	**244508.6**	**247348.8**	**13288.2**	**13475.6**
2008	**0.038806**	**0.038856**	**320900.6**	**316874.0**	**12452.9**	**12312.4**
2009	**0.038564**	**0.038582**	**365486.6**	**373211.7**	**14094.6**	**14399.2**
2010	**0.029837**	**0.029836**	**409658.4**	**426224.4**	**12223.0**	**12716.9**
2011	**0.025511**	**0.025544**	**438158.0**	**443882.8**	**11177.9**	**11338.6**
2012	**0.021800**	**0.021898**	**480930.2**	**480589.7**	**10484.1**	**10523.9**
2013	**0.019742**	**0.019877**	**506558.7**	**504146.7**	**10000.2**	**10020.8**

For capital services, the approximating geometric model is very close to the "true" one hoss shay model with length of life equal to 3 years. The one hoss shay price of capital services p_{SH}^{2013} ended up 0.7% higher than its geometric counterpart p_{SG}^{2013}, the geometric quantity of capital services q_{SG}^{2013} ended up 0.5% higher than its one hoss shay counterpart q_{SH}^{2013} and the value of one hoss shay capital services v_{SH}^{2013} ended up 0.2% higher than its geometric counterpart v_{SG}^{2013}.

Finally, we present counterparts to Tables 3 and 4 where instead of using the actual asset inflation rates i^t and the actual interest rates r^t, we use the smoothed inflation rates and interest rates i_S^t and r_S^t that are listed in Table 2 and we recompute the one hoss shay (L = 4) and best approximating geometric capital stock and service flow prices and quantities. The results are listed on Tables 7 and 8 below.[38]

Table 7: Comparison of Geometric and One Hoss Shay (L = 4) Capital Stock Prices, Quantities and Values using Smoothed Asset Inflation and Interest Rates i_S^t and r_S^t

Year	p_{KG}^t	p_{KH}^t	q_{KG}^t	q_{KH}^t	v_{KG}^t	v_{KH}^t
1989	**1.000000**	**1.000000**	**10543.3**	**10543.3**	**10543.3**	**10543.3**
1990	**0.958739**	**0.966732**	**12832.6**	**12990.7**	**12303.1**	**12558.5**
1991	**0.858053**	**0.862991**	**14094.0**	**14282.1**	**12093.4**	**12325.3**

[38] When calculating the geometric stocks and flows, we continue to use our "best" approximating geometric depreciation rate $\delta^* \equiv 0.32055$.

1992	**0.765716**	**0.760267**	**15581.0**	**15434.8**	**11930.6**	**11734.6**
1993	**0.705594**	**0.706574**	**18061.1**	**17411.4**	**12743.8**	**12302.5**
1994	**0.637814**	**0.651481**	**21173.9**	**20302.6**	**13505.0**	**13226.8**
1995	**0.521630**	**0.544851**	**25346.5**	**24586.0**	**13221.5**	**13395.7**
1996	**0.430004**	**0.449029**	**32044.8**	**31384.6**	**13779.4**	**14092.6**
1997	**0.311762**	**0.321304**	**42432.8**	**41942.4**	**13228.9**	**13476.2**
1998	**0.264417**	**0.271478**	**60114.9**	**60100.9**	**15895.4**	**16316.1**
1999	**0.225215**	**0.229403**	**78133.9**	**79466.1**	**17596.9**	**18229.8**
2000	**0.165963**	**0.166326**	**104696.1**	**106748.5**	**17375.6**	**17755.1**
2001	**0.162989**	**0.163140**	**125851.1**	**128388.5**	**20512.3**	**20945.4**
2002	**0.144558**	**0.150146**	**147657.5**	**148795.3**	**21345.1**	**22341.1**
2003	**0.114601**	**0.120150**	**184897.5**	**183261.4**	**21189.5**	**22018.9**
2004	**0.082491**	**0.084741**	**245641.4**	**241239.9**	**20263.3**	**20442.9**
2005	**0.069592**	**0.070348**	**317148.0**	**315384.1**	**22071.0**	**22186.7**
2006	**0.063125**	**0.064393**	**397269.0**	**399196.3**	**25077.5**	**25705.6**
2007	**0.053269**	**0.054456**	**490801.6**	**492774.8**	**26144.7**	**26834.5**
2008	**0.042714**	**0.043444**	**639134.0**	**636312.3**	**27300.2**	**27643.7**
2009	**0.039335**	**0.039781**	**741976.0**	**740266.8**	**29185.6**	**29448.5**
2010	**0.032430**	**0.032678**	**843513.9**	**835730.8**	**27355.1**	**27310.0**
2011	**0.028551**	**0.028259**	**918256.7**	**894946.2**	**26217.6**	**25290.0**
2012	**0.025867**	**0.025278**	**1013678.0**	**963758.2**	**26220.4**	**24362.2**
2013	**0.023814**	**0.022994**	**1080780.0**	**1014858.0**	**25737.8**	**23335.3**

The capital stock results in the above Table from using the smoothed i_S^t and r_S^t differ little from the unsmoothed results listed in Table 3. This is not a surprise but we expected more differences in the smoothed capital services estimates which are presented in the following Table 8.

Table 8: Comparison of Geometric and One Hoss Shay (L = 4) Capital Service Prices, Quantities and Values using Smoothed Asset Inflation and Interest Rates i_S^t and r_S^t

Year	p_{SG}^t	p_{SH}^t	q_{SG}^t	q_{SH}^t	v_{SG}^t	v_{SH}^t
1989	**1.000000**	**1.000000**	**5264.8**	**5118.9**	**5264.8**	**5118.9**
1990	**0.972338**	**0.972242**	**6408.0**	**6425.8**	**6230.7**	**6247.5**
1991	**0.846689**	**0.846545**	**7037.9**	**7390.5**	**5958.9**	**6256.4**
1992	**0.709322**	**0.709656**	**7780.4**	**8230.7**	**5518.8**	**5840.9**
1993	**0.667614**	**0.66752**	**9018.8**	**9086.4**	**6021.1**	**6065.4**
1994	**0.647801**	**0.648713**	**10573.2**	**10284.4**	**6849.4**	**6671.7**
1995	**0.570481**	**0.575084**	**12656.8**	**12353.8**	**7220.5**	**7104.5**
1996	**0.466359**	**0.470072**	**16001.6**	**15621.1**	**7462.5**	**7343.1**
1997	**0.319964**	**0.320978**	**21188.9**	**20506.6**	**6779.7**	**6582.2**
1998	**0.265442**	**0.265996**	**30018.5**	**28799.5**	**7968.2**	**7660.6**
1999	**0.219458**	**0.219565**	**39016.3**	**38555.0**	**8562.5**	**8465.3**
2000	**0.152247**	**0.152274**	**52280.2**	**52184.6**	**7959.5**	**7946.3**
2001	**0.148713**	**0.148759**	**62844.1**	**64802.9**	**9345.7**	**9640.0**
2002	**0.148211**	**0.148802**	**73733.2**	**76238.7**	**10928.1**	**11344.5**
2003	**0.120614**	**0.121465**	**92329.0**	**93758.0**	**11136.2**	**11388.4**

2004	**0.081138**	**0.081228**	**122661.6**	**119103.5**	**9952.6**	**9674.5**
2005	**0.064558**	**0.064551**	**158368.6**	**154500.0**	**10224.0**	**9973.1**
2006	**0.060741**	**0.060746**	**198377.3**	**198827.3**	**12049.6**	**12078.0**
2007	**0.051835**	**0.051850**	**245083.0**	**249331.5**	**12703.9**	**12927.8**
2008	**0.040762**	**0.040758**	**319153.1**	**318117.5**	**13009.2**	**12965.7**
2009	**0.036588**	**0.036583**	**370507.6**	**376463.4**	**13556.3**	**13772.2**
2010	**0.029644**	**0.029644**	**421210.8**	**434856.0**	**12486.2**	**12891.1**
2011	**0.024429**	**0.024501**	**458533.8**	**480894.8**	**11201.7**	**11782.3**
2012	**0.021027**	**0.021179**	**506182.6**	**512059.3**	**10643.4**	**10844.9**
2013	**0.018386**	**0.018632**	**539690.0**	**543301.6**	**9922.6**	**10122.6**

Comparing the service prices and quantities in Table 8 with the corresponding entries in Table 4 shows that the trends in the series are much the same. However, the year to year volatility in the Table 8 series is far less than the volatility in the corresponding Table 4 series. It is likely that users will be more comfortable using the smoothed series in Table 8 than the unsmoothed series in Table 4.

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