The Operating System Network Effect and Carriers’ Dynamic Pricing of Smartphones

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Abstract

The utility a consumer realizes from owning a smartphone increases with its operating system (OS) network size. Due to this OS network effect, multi-network telecom carriers have a different pricing strategy for smartphones than the single-network manufacturers in a dynamic environment. While manufacturers choose higher prices for larger networks, carriers, who can internalize competition across OSs, have incentives to choose lower prices for larger networks. The carriers’ pricing strategy contributes to the increasing smartphone users and OS concentration. In this paper, I first analyze a theoretical model to compare the pricing strategies of the carriers and manufacturers. Then I design a structural model of consumers’ demand and the carriers’ dynamic pricing game for smartphones, and empirically study the impact of the OS network effect and carriers’ two-year contract policy on the smartphone market penetration and OS concentration. I estimate the model using product level data from August 2011 to July 2013 in the US. I deal with the empirical challenges of dynamic prices for multi-product carriers, high dimension continuous state variables, and asymmetric oligopolistic firms in the estimation. The results show that the OS network size has a positive and significant impact on consumer utility. I then study two counterfactual cases in which I eliminate the OS network effect and the carriers’ pricing strategy, respectively. I find that, without the OS network effect, the smartphone penetration rate would decrease by 54.7% and the largest OS share difference decrease by 31.7% by May 2013. Without the carriers’ pricing strategy, the penetration rate would decrease by 29.1% and the OS market share difference decrease by 11.2%.

Keywords: OS Network Effect, Carrier Dynamic Pricing Game, Two-Year Contract, Asymmetric Multi-Network Sellers, Value Function Approximation, MPEC.

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1 Introduction

In markets with network effects, such as computers and smartphones, consumers value the size of the installed consumer base because it can lead to higher utility. Due to the network effect, the current price of a product affects its network size and thus future demand. This leads the suppliers of the product to make dynamic pricing decisions, no matter whether the products are sold by single-network manufacturers or by multi-network retailers. However, in this paper, I show that the multi-network retailers have a quite different pricing strategy than the manufacturers. The retailers choose lower prices for products with initially larger networks while the manufacturers do the opposite.

In this paper, I study the pricing patterns and incentives of multi-network retailers selling smartphones. These products are subject to a network effect that arises through the operating systems for two reasons. First, application stores themselves generate an indirect network effect. Application developers choose to develop more apps for large OSs. In return, more consumers adopt large OSs because of the greater number and better quality of applications. Second, there is a direct OS network effect. Friends and family members prefer adopting the same OS. The benefits of doing so include convenient communication (e.g., FaceTime, iMessage), as well as ease of sharing files and purchases, and lower learning costs.

Smartphones are sold by telecom carriers who act like multi-network retailers. During the 2011-2013 period, each carrier sold products of all OSs. For example, Verizon sold smartphones with iOS, Android, Blackberry, and Windows Phone. The carriers give discounts on smartphone purchases when sold with a two-year wireless contract. Consumers pay the discounted prices instead of the manufacturer retail prices for smartphones. As will be shown in Section 6, the carrier discounts rise in line with past OS shares.

\footnote{The only exception is that T-Mobile only started to sell iPhones in April 2013.}
The goal of this paper is to study the impact of the OS network effect on the multi-network retailers’ dynamic prices. To build intuition, I solve a two-period, two-OS theoretical model to compare the price for the large OS with that for the small OS in two different settings. In the first setting, a multi-network carrier sells products with both a large and small OS. In the second setting, single-network manufacturers sell their own products. I find that the carrier chooses a lower price for the large OS than the small OS in the first period, while the manufacturers do the opposite.

The intuition of the pricing strategies of the single-network manufacturers and the multi-network carrier is as follows. In a dynamic environment with network effects, both the manufacturers and the carrier would choose low prices to grow the network to attract future customers. The difference between them lies in their competitive environments. A manufacturer faces competition from other manufacturers and its initial network size affects its market power. In equilibrium, manufacturers with larger initial networks choose higher prices. In contrast, the multi-network carrier sells products of all networks. This implies that the carrier is able to internalize the competition across networks. It prefers a very large network to several small networks, so that future consumers will be more likely to buy the product as opposed to the outside option. Therefore, the carrier chooses relatively lower prices for initially larger networks, which will grow fast and gain customers.

I then develop a structural model of consumers’ demand and carriers’ dynamic pricing game for smartphones to empirically quantify the OS network effect and measure its impact on the carriers’ two-year contract discounts. I estimate the model using product level data from the period August 2011-July 2013. In the estimation, I deal with several empirical challenges including dynamic prices of multiple products for each carrier, high dimension continuous state variables, and asymmetric oligopolistic firms.

The estimation results show that the OS network effect is positive and significant. This
implies that consumers’ utility from a smartphone is affected by the number of its OS users, and the carriers’ prices are affected by the OS network sizes. I study two counterfactual scenarios to measure the impact of the OS network effect and the carriers’ discounts on the growth of smartphone users and OS concentration. In the first counterfactual case, I eliminate the OS network effect. I find that, without the OS network effect, the smartphone penetration rate would decrease by 54.69% and the largest OS share difference would decrease by 31.66% when the carriers make static pricing decisions. In the second counterfactual case, I eliminate the carrier discounts on two-year contracts. Without the carriers discounts, the smartphone penetration rate would decrease by 29.06% and the largest OS share difference would decrease by 11.18%. The results show that both the OS network effect and carrier discounts are very important to smartphone penetration and OS concentration, but the OS network effect is more important than the carrier discounts.

The paper proceeds as follows. In Section 2, I discuss the related literature and contributions of this paper. Section 3 provides the background to the US smartphone industry. In Section 4, I study a simple two-period, two-OS model to compare the large OS price with that of the small OS in two different settings. Section 5 sets up consumer demand and the carriers’ supply model for smartphones with two-year contracts. The data used in this paper is described in Section 6. Section 7 discusses identification and estimation details. The estimated results are presented in Section 8. Section 9 studies the two counterfactual scenarios. Section 10 concludes the paper.

2 Literature Review

The main contribution of this paper is to empirically study the impacts of a network effect on the prices charged by multi-network retailers. Neither the theoretical literature nor the empirical literature has studied this topic. Theoretical research on the network effect
has focused on the competition between single-network manufacturers, but not the prices of multi-network retailers. Katz and Shapiro (1985), Farrell and Saloner (1986), Katz and Shapiro (1992), Katz and Shapiro (1994), Shapiro and Varian (1999), Rochet and Tirole (2003), Armstrong (2006), Zhu and Iansiti (2007), Rysman (2009), and Weyl (2010) all study the impact of the network effect on the prices of either monopoly or oligopoly manufacturers. In this paper, I focus on the network effect’s impact on multi-network sellers’ prices, which differs from the impact on single-network sellers’ prices.

The empirical literature also focuses on markets in which manufacturers choose prices. This includes papers that study the network effect in the video game industry (Zhu and Iansiti 2007, Lee 2013, Dubé, Hitsch, and Chintagunta 2010), the DVD player industry (Gowrisankaran, Park, and Rysman 2010), the VCR industry (Park 2004), the yellow page industry (Rysman 2004), the ACH banking industry (Ackerberg and Gowrisankaran 2006), videocalling technology (Ryan and Tucker 2012), and the PDA industry (Nair, Chintagunta, and Dubé 2004). In all these markets, manufacturers sell products directly to consumers. In this paper, I focus on the impact of the network effect upon carrier prices across different OSs.

This paper makes a further contribution by estimating a structural model of dynamic pricing with asymmetric multi-network carriers. I deal with several empirical challenges including dynamic prices of multiple products, high-dimension continuous state variables, and asymmetric oligopolistic firms.

There are literatures on estimating discrete choice dynamic games and continuous choice games with single-product firms. However, none of those methods is ideal to be applied to this paper, where the dynamic game features both continuous choices and multi-product firms.

Aguirregabiria and Mira (2007) and Pakes, Ostrovsky, and Berry (2007) have proposed
estimation methods for dynamic discrete choice games. Bajari, Benkard, and Levin (2007) propose a two-step method that can estimate dynamic games with continuous choices. Their first step estimates the policy functions and the second step estimates the model’s parameters using simulated minimum distance estimator. Their two-step method requires monotonicity of the policy functions on shocks and linearity of the value functions in the model’s parameters. These two requirements make the method not applicable to this paper, because it is hard to prove monotonicity of policy functions and the value function is nonlinear in this paper. In addition, first step noise could arise without a sufficiently large number of observations and a flexible parametric assumptions on policy functions. The noise could bias the second step estimates.


With a small number of single-product firms, the dynamic problem could be solved with value function iteration or policy function iteration. However, when there are many firms and each firm has many products, previous methods cannot be used due to the high dimension choice space (prices of many products).

Instead, I solve the multi-network carriers’ dynamic pricing game within the estimation. In particular, I approximate the carriers’ value functions with basis functions and develop an efficient iterative method to solve the dynamic pricing game. I estimate the model using
Generalized Methods of Moments (GMM) with MPEC, which was introduced by Su and Judd (2012). The moment conditions are based on the orthogonality between unobserved shocks and instrumental variables. The carriers’ Bellman equations are used as constraints. The value function approximation method is motivated by the sieve estimation literature. Ai and Chen (2003) propose a minimum distance estimator with sieve approximation and show its efficiency. Barwick and Pathak (2011) also use sieve approximations to solve a dynamic maximization problem.


This paper focuses on the consumer demand for smartphones with contracts and the impact of the OS network effect and the carrier discounts on those contracts. The carriers’ two-year contract policy is an inseparable part of the carriers’ pricing. Consumers receive discounts on the manufacturers’ smartphone prices if they sign two-year wireless service contracts. Only with a two-year contract to guarantee a revenue flow from wireless service consumption can the carriers offer discounts on smartphone purchases. The impact of the OS network effect upon carrier prices explained above would no longer exist if the carriers don’t make pricing decisions.\(^2\)

\(^2\)The two-year contract policy existed long before the smartphone industry developed. The question of why contracts are two-years long is also interesting, but it is not the focus of this paper. This paper takes the two-year length as given.
3 Background of the U.S. Smartphone Industry

The major manufacturers in the U.S. smartphone market are Apple Inc., Research in Motion Limited (produces Blackberry models), HTC Corporation, Motorola, Inc., Samsung Electronics Co. Ltd., LG Corp., and Nokia Corporation. The top four operating systems in the U.S. smartphone industry are Android, iOS, Blackberry, and Windows Phone. The combined market share of the four increased from 94% to 99% during 2011 to 2014.[3]

Every smartphone operating system has an online store where applications can be purchased and downloaded to extend the functionality of the smartphone. The proprietary OSs have application stores that are exclusive to the operating system. Licensable or open source OSs have application stores that work with any device that runs the OS, no matter the manufacturer. The application stores for the four OSs are: Google Play(for Android), App Store (for Apple iOS), BlackBerry App World (for BlackBerry), and Windows Phone Marketplace (for Windows Phone).

There are indirect and direct network effects at the operating system level[4] The indirect network effect arises through app development. Both the OS developers and third-party firms develop apps for the operating systems. The apps on different operating systems are exclusive. Consumers find value in apps because of the additional functionality of their smartphones. The firms maximize profits by developing apps for the large operating systems due to the large number of users. In this way, the network effect exists at the OS level. There is also a direct OS network effect. Family members and friends prefer adopting the same OS because they can enjoy convenient communication methods (FaceTime, iMessage) and easy ways to share files; they can share their apps purchases; and there is a

[4] I don’t separately model the carriers’ wireless network effect in this paper, because the carrier-OS fixed effects I add in consumers’ utility function can incorporate the carrier specific effects, including the wireless network network.
lower learning cost to use an OS.

A smartphone device is useful when connected to a wireless service provider’s network that allows the consumer to make calls and access data such as email and the internet. Thus, a consumer has to choose both the smartphone model and the service provider when buying a smartphone. The top four service providers (carriers) in the U.S. are Verizon Wireless, AT&T Mobility, Sprint Corporation, and T-Mobile US. Together, they account for 90% of the total market for smartphone sales. They have varying degrees of network coverage and different pricing plans for wireless service. According to [Kantar World Panel](http://www.statista.com/statistics/) data, the average combined share of smartphone sales for the big four carriers (service providers) during Oct. 2011 to Nov. 2013 is 88.72%. The average sales share of Verizon is 33.26%, 28.68% for AT&T, 15.58% for Sprint, and 11.10% for T-Mobile.

The wireless carriers offer discounts on smartphones if consumers purchase with long-term contracts, usually two years. For example, Apple’s retail price of the iPhone 5 was $649 in Oct 2012. Consumers could pay $199 for an iPhone 5 if they sign a two-year wireless service contract with the carriers. If consumers need to terminate the contract within two-years, they have to pay early termination fees. Depending on how many months left in an unfinished contract, the termination fee is between $150 to $350. According to the Statista.com, the average monthly churn rate for the four wireless carriers is 2%[5].

The US Wireless Industry Overview 2011 reported that more than 78% of users are on two-year contracts. This number includes both the feature phones and smartphone subscribers. And the percentage is expected to be even higher for the smartphone market because smartphones are much more expensive if bought without a contract. The data used in this paper is from Aug. 2011 to Oct 2013, during which most smartphone users were on two-year contracts. Because of this, I focus on the consumers’ demand for smartphones

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[5] Churn rate is defined as the proportion of contractual customers or subscribers who leave a supplier during a given time period. Data source is from this link: [http://www.statista.com/statistics/](http://www.statista.com/statistics/)
with two-year contracts.

4 A Two-Period, Two-OS Model of Pricing

In this section, I study two two-period, two-OS theoretical models to show that multi-network sellers (carriers) choose lower smartphone prices for large OSs than for small OSs, while the single-network sellers (manufacturers) do the opposite. In the first model, a monopoly seller sells two smartphone models with different operating systems in the two periods. In the second model, two single-network firms choose prices and play a dynamic pricing game. The key difference between the two models is that the multi-network seller can internalize the competition, while single-network sellers can not.

4.1 Demand Model Setup

There are two smartphone models, A and B, in a two-period economy, period 1 and 2. The two models have different operating systems, which are also denoted by \( \{ A, B \} \). Let \( \delta_j, j \in \{ A, B \} \), be the consumer utility from the characteristics of model \( j \). Let the initial market shares of the two OSs be \((n_{A1}, n_{B1})\) in period \( t \).

**Assumption 1.** Without loss of generality, assume that network A has a higher market share than B initially: \( n_{A1} > n_{B1} \) and that the OS network effect exists: \( \gamma > 0 \).

**Assumption 2.** The two models provide the same characteristics: \( \delta_A = \delta_B \). The two models have the same unit cost, \( c = 0 \).\(^6\)

\(^6\)There are several reasons that one OS has a higher share than the other initially. Different OSs may enter the market at different years. Their companies may have different shocks. Different operating systems may have different openness towards smartphone manufacturers.

\(^7\)By normalizing costs to be zero, the prices in this section can be interpreted as markups that the carrier earns. When the costs are not zero, choosing prices is equivalent to choosing markups.
Assumption 1 and 2 will be assumed for the rest of this section. Assumption 2 implies that the only difference between A and B is their network size difference, so that this model focuses on the network size effect. Let the total mass of consumers be 1. At the beginning of period \( t \), only consumers who do not own smartphones enter the market, so the market size in period \( t \) is

\[
M_t = 1 - n_{At} - n_{Bt}.
\]

Consumer \( i \)'s utility of buying smartphone \( j \in \{A, B\} \) in period \( t \) is:

\[
u_{ijt} = \delta_j + \gamma n_{jt} - \alpha p_{jt} + \epsilon_{ijt},
\]

in which \( n_{jt} \) measures the network size of OS \( j \) at the beginning of period \( t \), \( p_{jt} \) is the carrier’s smartphone price in period \( t \), \( \epsilon_{ijt} \) is the idiosyncratic utility shock of model \( j \) in period \( t \), and \( \gamma \) and \( \alpha \) are marginal utility of OS network size and price, respectively.

An outside option exists in each period and it means not to buy a smartphone. Let the mean utility of the outside option be zero. Consumer \( i \)'s utility of the outside option is \( u_{i0t} = \epsilon_{i0t} \). Assume that the utility shock \( \epsilon_{ijt} \) follows a Type-I extreme value distribution and is i.i.d. across consumers, models, and periods. Then the sales market share of model \( j \) in period \( t \) is

\[
s_{jt} = \frac{e^{(\delta_j + \gamma n_{jt} - \alpha p_{jt})}}{1 + \sum_{k=A,B} e^{(\delta_k + \gamma n_{kt} - \alpha p_{kt})}}.
\]

(1)

An OS network grows due to new sales of smartphones. At the beginning of the second period, the market share of OS \( j \in \{A, B\} \) is

\[
n_{j2} = n_{j1} + M_1 s_{j1}.
\]

(2)
Therefore, the market size in the second period is:

\[ M_2 = 1 - n_{A2} - n_{B2} \]
\[ = 1 - n_{A1} - n_{B1} - M_1(s_{A1} + s_{B1}) \]
\[ = M_1(1 - s_{A1} - s_{B1}) \]
\[ = M_1s_{01}, \]

where \( s_{01} \) is the market share of the outside option in the first period. The market size in the second period is the measure of consumers who do not buy smartphones in the first period.

4.2 Model One: A Multi-Network Seller

Suppose that a seller sells both models and chooses prices to maximize the sum of discounted profits. Next, I solve the problem using backwards induction to analyze the multi-network seller’s pricing decisions.

4.2.1 Optimal Prices in the Second Period

At the beginning of the second period, the multi-network seller observes the OS network sizes \((n_{A2}, n_{B2})\) and chooses the prices \((p_{A2}, p_{B2})\) for A and B. The seller faces a static problem after entering the second period, because it is the last period. For second period prices \((p_{A2}, p_{B2})\), the seller’s profit in the second period is:

\[
\pi_2(p_{A2}, p_{B2}) = M_2(s_{A2}p_{A2} + s_{B2}p_{B2})
= M_2 \left[ \frac{e^{(\delta_A + \gamma n_{A2} - \alpha p_{A2})}p_{A2} + e^{(\delta_B + \gamma n_{B2} - \alpha p_{B2})}p_{B2}}{1 + \sum_{k=A,B} e^{(\delta_k + \gamma n_{k2} - \alpha p_{k2})}} \right].
\]
where $M_2s_{j2}$ is the market demand for phone $j$ in the second period. The first order conditions with respect (FOC) to price $p_{A2}$ is:

$$
M_2 \left[ \underbrace{s_{A2} - \alpha p_{A2}^* s_{A2}(1 - s_{A2})}_{\text{marginal profit from A}} + \underbrace{\alpha p_{B2}^* s_{A2} s_{B2}}_{\text{marginal profit from B}} \right] = 0. \tag{4}
$$

The price of A affects the seller’s profit not only though consumers’ demand for phone A but also though that of phone B, because it sells both A and B. Raising the price of A increases the seller’s profit from phone B, and vice versa. This implies that the multi-network seller can internalize the competition between A and B. Dividing the FOC by $M_2s_{A2}$, we have:

$$
1 - \alpha(p_{A2}^*)(1 - s_{A2}) + \alpha(p_{B2}^*)s_{B2} = 0. \tag{5}
$$

Rewrite equation (5):

$$
1 - \alpha p_{A2}^* + \alpha p_{A2}^* s_{A2} + \alpha p_{B2}^* s_{B2} = 0. \tag{6}
$$

The FOC for $p_{B2}$ can be derived in a similar way:

$$
1 - \alpha p_{B2}^* + \alpha p_{B2}^* s_{B2} + \alpha p_{A2}^* s_{A2} = 0. \tag{7}
$$

By comparing the FOCs for A and B, (6) and (7), one immediately gets: $p_{A2}^* = p_{B2}^*$. That is, in order to have the marginal profits from A and B both be zero, the carrier chooses the same price for A and B.

Denote the second period price for A and B by $p_{2}^*$. According to the sales share equation (1), $s_{A2}$ and $s_{B2}$ are functions of $(p_{2}^*, n_{A2}, n_{B2})$. The FOC in the second period becomes:

$$
0 = 1 - \alpha p_{2}^* + \alpha p_{2}^* \left[ s_{A2}(p_{2}^*, n_{A2}, n_{B2}) + s_{B2}(p_{2}^*, n_{A2}, n_{B2}) \right] \\
= 1 - \alpha p_{2}^* \left[ 1 - s_{A2}(p_{2}^*, n_{A2}, n_{B2}) - s_{B2}(p_{2}^*, n_{A2}, n_{B2}) \right]. \tag{8}
$$
which implies that $p^*_2$ is a function of the OS network sizes $(n_A, n_B)$: $p^*_2(n_A, n_B)$. The FOC also implies that $s_A + s_B = 1 - \frac{1}{\alpha p^*_2}$. The second period profit is:

$$
\pi_2 = p^*_2(s_A + s_B)M_2
= p^*_2(1 - \frac{1}{\alpha p^*_2})M_2
= (p^*_2(n_A, n_B) - \frac{1}{\alpha})M_1s_{01}.
$$

(9)

where $M_2 = M_1s_{01}$ as in equation (3). This implies that the profit in the second period is strictly increasing with price $p^*_2$ and market size $M_2$.

Before moving to the first period, let me first analyze how the OS network sizes $(n_A, n_B)$ affect the optimal price $p^*_2$, since the first period prices will make an impact on $\pi_2$ through $(n_A, n_B)$ and $p^*_2$. Using the FOC (8) and Implicit Function Theorem, the first order derivative of price $p^*_2$ with respect to $n_j$, $j \in \{A, B\}$ can be derived:

$$
\frac{\partial p^*_2(n_A, n_B)}{\partial n_j} = \frac{\gamma}{\alpha} s_j > 0.
$$

(10)

There are two implications of equation (10). First, the price $p^*_2$ increases with the OS network size $n_j$. Second, the larger OS network A has a stronger positive impact on $p^*_2$ than OS B, since the sales share of A is higher than B, $s_A > s_B$. This is because A and B have the same price and A has a larger network than B $n_A > n_B$ (See Appendix A.1 for the proof for $n_A > n_B$), so demand for A is higher than B: $s_A > s_B$. Therefore, $n_A$ has a stronger impact on $p^*_2$ than $n_B$:

$$
\frac{\partial p^*_2(n_A, n_B)}{\partial n_A} > \frac{\partial p^*_2(n_A, n_B)}{\partial n_B} > 0.
$$

(11)

From equation (10), the second order derivatives can also be derived to check the
convexity of price $p^*_2$ as a function of $n_j^2$, $j \in \{A, B\}$:

$$\frac{\partial^2 p^*_2(n_{A2}, n_{B2})}{\partial n_{j2}^2} = \frac{\gamma}{\alpha} s_{j2}(1 - s_{j2}) > 0. \quad (12)$$

$$\frac{\partial^2 p^*_2(n_{A2}, n_{B2})}{\partial n_{A2}\partial n_{B2}} = -\frac{\gamma}{\alpha} s_{A2}s_{B2} < 0. \quad (13)$$

Equation (12) and (13) imply that the price $p^*_2$ is a convex function in the OS network sizes $(n_{A2}, n_{B2})$. Due to the convexity, $p^*_2$ increases as more consumers use the same OS, either OS A or B. For example, consider two possible states in the second period, $(n_{gA}^2, n_{gB}^2) = (0.4, 0.1)$ and $(n_{hA}^2, n_{hB}^2) = (0.25, 0.25)$. These two states have the same total measure of existing users, 0.5. However, the price $p_2$ is higher in state $g$ than $h$ because of the convexity. Notice that, a third state $(n_{g'}_{A2}, n_{g'}_{B2}) = (0.1, 0.4)$ will lead to the same $p^*_2$ as state $g$, because A and B have the same characteristics. Therefore, $p^*_2$ increases with $|n_{A2} - n_{B2}|$. More importantly, since $\pi_2$ increases in $p^*_2$, the convexity implies that the seller maximizes $\pi_2$ when all consumers use the same OS at the beginning of the second period, given the market size $M_2$.

Whether the seller chooses OS A or B to be the larger network in the second period will depend on their initial OS network sizes. Intuitively, the seller would choose the initially larger network A, which is less costly for the seller than to reach the same level of OS concentration by choosing the smaller network B. In Appendix A.1, I prove that, if $n_{A1} > n_{B1}$, then the profit maximizing prices must lead to $n_{A2} > n_{B2}$.

The second period state $(n_{A2}, n_{B2})$ has two more effects on $\pi_2$, in addition to that $\pi_2$ increases with their difference $|n_{A2} - n_{B2}|$. One effect is that, as $n_{A2}$ and $n_{B2}$ increase, consumers’ demand in the second period increases because larger OS network sizes raise consumer utility. Another effect is that, as $n_{A2}$ and $n_{B2}$ increase, the market size $M_2$ decreases, so the demand decreases. Depending on which of the two effects dominates the
other, the seller may increase or decrease the prices for both A and B simultaneously in
the first period. However, the two effects do not change the seller’s preference in the OS
concentration in the second period to increase price.

Next, I study the multi-network seller’s problem in the first period. Equation (11) will
be the key to compare the prices of A and B in the first period.

4.2.2 Optimal Prices in the First Period

In the first period, the seller observes the initial OS market sizes \((n_{A1}, n_{B1})\) and chooses
prices \((p_{A1}, p_{B1})\) to maximize total profit from both periods. The seller’s profit maximiza-
tion problem in the first period is:

\[
\max_{p_{A1}, p_{B1}} \{ \pi_1(p_{A1}, p_{B1}) + \beta \pi_2(n_{A2}, n_{B2}|p_{A1}, p_{B1}) \}
\]

\[
= \max_{p_{A1}, p_{B1}} \{ M_1(s_{A1}p_{A1} + s_{B1}p_{B1}) + \beta \left[ p^*_2(n_{A2}, n_{B2}|p_{A1}, p_{B1}) - \frac{1}{\alpha} \right] M_1 s_{01} \} \tag{14}
\]

where \(\beta\) is the discount factor across periods. The equality in (14) is from the profit
function (9). The OS network sizes \((n_{A2}, n_{B2})\) depend on first period prices \((p_{A1}, p_{B1})\), as
in equations (1) and (2).

The first order condition for \(p_{A1}\) is:

\[
M_1 \left\{ s_{A1} - \alpha p^*_1 s_{A1} (1 - s_{A1}) + \frac{\alpha p^*_2 s_{A1} s_{B1}}{\alpha} + \beta \left[ p^*_2(n_{A2}, n_{B2}|p_{A1}, p_{B1}) - \frac{1}{\alpha} \right] M_1 s_{01} \right\} = 0.
\]

\[
= \frac{\beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{A1} s_{01}}{\alpha} + \beta M_1 s_{01} \left[ -\alpha \frac{\partial p^*_2}{\partial n_{A2}} s_{A1} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{A1} s_{B1} \right] = 0.
\]

The FOC implies that, the price \(p_{A1}\) affects first period profits from A and B, the market
size in the second period, and the price in the second period. Divide the FOC by \(M_1 s_{A1}\),
I get:

\[ 1 - \alpha p_{A1}^* + \alpha p_{A1}^* s_{A1} + \alpha p_{B1}^* s_{B1} + \beta (p_2^* - \frac{1}{\alpha}) \alpha s_{B1} + \beta M s_{B1} [-\alpha \frac{\partial p_2^*}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p_2^*}{\partial n_{B2}} s_{B1}] = 0. \]  

(16)

The first order condition for \( p_{B1} \) can be derived similarly.

\[ 1 - \alpha p_{B1}^* + \alpha p_{B1}^* s_{B1} + \alpha p_{A1}^* s_{A1} + \beta (p_2^* - \frac{1}{\alpha}) \alpha s_{B1} + \beta M s_{B1} [-\alpha \frac{\partial p_2^*}{\partial n_{A2}} (1 - s_{B1}) + \alpha \frac{\partial p_2^*}{\partial n_{B2}} s_{A1}] = 0. \]  

(17)

Comparing the FOCs in (16) and (17), I get:

\[ \alpha p_{A1}^* + \beta M s_{B1} \alpha \frac{\partial p_2^*}{\partial n_{A2}} = \alpha p_{B1}^* + \beta M s_{B1} \alpha \frac{\partial p_2^*}{\partial n_{B2}}. \]  

(18)

According to equation (11), \( \frac{\partial p_2^*}{\partial n_{A2}} > \frac{\partial p_2^*}{\partial n_{B2}} > 0 \), which means that the large OS A has a stronger positive impact on \( p_2 \) than B. Hence, the price of A in the first period should be lower than B, \( p_{A1} < p_{B1} \), so that their marginal profits will both be equal to zero.

To be more specific, plug equation (10) in equation (18), I get:

\[ \alpha p_{A1}^* + \beta M s_{B1} \gamma s_{A2} = \alpha p_{B1}^* + \beta M s_{B1} \gamma s_{B2}, \]

Divide by \( \alpha \), the relationship between the prices is:

\[ p_{A1}^* - p_{B1}^* = \frac{\beta \gamma}{\alpha} M s_{B1} (s_{B2} - s_{A2}). \]  

(19)

Since phone A has a higher sales share than B in the second period, \( s_{A2} > s_{B2} \), the price of A is lower than B in the first period, \( p_{A1}^* < p_{B1}^* \).

The first period prices \( (p_{A1}, p_{B1}) \) have two effects on the discounted total profit. First, as explained in previous subsection, the seller may increase or decrease the two prices to
balance the tradeoff between market sizes across periods, depending on the discount factor \( \beta \), price elasticity \( \alpha \), and OS network effect \( \gamma \). Second and more importantly, given the market size tradeoff, the seller chooses a lower price for the initially larger network A than B in the first period, so that A has as many users as possible and the second period profit is maximized.

4.2.3 The Dynamic Effect and the OS Network Effect

The dynamic model makes an impact on the first period prices. Without the dynamic effect, \( \beta = 0 \), equation (19) implies that the two products have the same price in the first period as well, \( p_{A1} = p_{B1} \), no matter which product has a larger network. However, when the model is dynamic (\( \beta > 0 \)), then the large network has a lower price than the small network in the first period, \( p_{A1} < p_{B1} \). The prices of A and B in the second period are the same, \( p_{A2} = p_{B2} \), no matter whether the problem is dynamic or static.

The OS network effect makes a similar effect on \((p_{A1}, p_{B1})\) as the dynamic effect. Equation (19) implies that, when there is no network effect, \( \gamma = 0 \), A and B will have the same price in the first period. In this case, the seller’s problem is still dynamic, but the OS sizes do not affect pricing. However, when the network effect exists, \( \gamma > 0 \), the larger network A has lower price than B in the first period. Because the seller’s second period profit increases with the OS concentration.

4.2.4 Model One Summary

The following proposition summarizes the properties of the multi-network seller’s optimal prices in both periods \((p_{A1}^{sc}, p_{B1}^{sc}, p_{A2}^{sc}, p_{B2}^{sc})\).

**Proposition 1.** The following statements hold for the carrier’s optimal prices.

1. The price of A is lower than that of B in the first period: \( p_{A1}^{sc} < p_{B1}^{sc} \).
(2) The price difference between the two models $|p_{A1}^{c*} - p_{B1}^{c*}|$ increases as the OS network effect $\gamma$ increases.

(3) The OS market share difference at the beginning of the second period $(n_{A2} - n_{B2})$ increases in the OS network effect $\gamma$.

Proof. See Appendix A.1.

The first statement of Proposition 1 says that, when the OS network effect exists, the multi-network seller chooses a lower price for the initially larger network $A$ than for network $B$ in the first period. As explained above, the intuition is that, to increase consumer demand, the seller prefers for most consumers to use the initially larger network $A$ in the first period, so that the seller can charge higher prices in the second period to increase profit.\footnote{The reason why the carriers sell products with different OSs is that consumers are heterogeneous. Some consumers might prefer $B$ no matter how large network $A$ is. So the carrier still sells $B$, which can increase its profits.}

The second statement implies that price difference between $A$ and $B$ increases with the OS network effect parameter $\gamma$. This is because that the seller’s incentive to choose lower price for $A$ increases with the OS network effect.

The third statement says that the market becomes more concentrated in the large operating system as the network effect increases.\footnote{I do not model the vertical relationship between the carrier and the manufacturers. It is probably not optimal for a carrier to sell only one manufacturer’s model, because in this case, the manufacture has the monopoly power and can charge carriers a high wholesale price. Accordingly, the carrier’s profit could decrease. The question of why the carrier is selling the both operating systems is also interesting, but is beyond the focus of this paper.} This result says that the multi-network seller’s prices contribute to the OS concentration of smartphones.

The results in Proposition 1 are derived using the logit demand model. The logit model is an ideal choice to compare carrier prices in a static and dynamic model. That is because the static logit model provides a clear benchmark to study the carrier’s incentive in a
dynamic model. The static logit demand model predicts that the carrier will choose the same price for both OSs, no matter which one has the initial network advantage.\footnote{This reflects the fact that the cross-derivatives of prices (\(\alpha_{A_t} \beta_{B_t}\)) are symmetric for both OSs, so the carrier’s two first-order conditions are symmetric, as shown in Appendix A.} When moving from a static model to a dynamic one, the changes in the seller’s pricing strategy reflect the impact of the dynamic factors, OS network sizes and market size.

More importantly, the multi-network seller’s incentive to have the large OS dominate the small OS exists no matter which demand model is used in the two-period model. With other demand functions, the seller would also choose a lower price for the large OS than for the small OS, as long as the network effect is strong and the discount factor is sufficiently large.\footnote{It is difficult to theoretically prove this result. However, by solving numerical examples, I find that for demand functions which also reflect the increasing share in model’s mean utility, there exist critical values of (\(\gamma, \beta, \alpha\)) such that at these values, the multi-network seller chooses same price for both models in the first period. And if either \(\gamma\) or \(\beta\) increases, carriers choose a lower price for the large OS than for the small OS in the first period, and vice versa.}

Though I consider a monopoly carrier case, the carrier’s pricing strategy in a dynamic environment carries over to oligopoly carriers, who carry the same set of OSs.\footnote{I show this in Appendix A.3. In addition, I used numerical examples with oligopolistic carriers to check the equilibrium prices. The results find that the carriers still choose lower prices for the large network than the small network in the first period.} The competition among the carriers doesn’t change their incentives to take advantage of the initially larger network.

### 4.3 Model Two: Two Single-Network Sellers

To compare the multi-network seller’s pricing strategy with single-network sellers’ prices, I now consider two single-network sellers, each sell one of the two smartphones and play a dynamic pricing game in the two periods. The demand model setup is the same as before. Denote the two sellers as \(\{A, B\}\). Seller \(j \in \{A, B\}\) produces and sells smartphone \(j\) that uses OS \(j\) in the two periods. In this section, I use superscript \(m\) to denote the sellers. For
seller $j$, the profit in each period $t$ is

$$\pi_{jt}^m(p_{At}^m, p_{Bt}^m) = p_{jt}^m s_{jt}(p_{At}^m, p_{Bt}^m) M_t.$$  

The dynamic problem of seller $j$ is

$$\max_{p_{jt}^m} \left\{ \pi_{jt}^m(p_{A1}^m, p_{B1}^m) + \beta \max_{p_{jt}^m} \left\{ \pi_{jt}^m(p_{A2}^m, p_{B2}^m | p_{A1}^m, p_{B1}^m) \right\} \right\},$$

in which $\beta$ is the discount factor across periods. The Subgame Perfect Nash Equilibrium of the finite period dynamic game can be solved backwards. The following proposition summarizes the properties of the sellers’ equilibrium prices $(p_{A1}^{*m}, p_{B1}^{*m}, p_{A2}^{*m}, p_{B2}^{*m})$.

**Proposition 2.** The following statements hold for the single-network sellers’ equilibrium prices.

1. The optimal price of A is higher than that of B in both periods: $p_{At}^{*m} > p_{Bt}^{*m}$, for $t = 1, 2$.

2. The larger OS A keeps its network advantage in the second period, though model A is more expensive in the first period.

**Proof.** See Appendix A.2.

The first statement of Proposition 2 says that when the single-network sellers choose prices, seller A chooses higher prices than B in both periods, because the initial OS network advantage gives seller A market power. Suppose both models have the same price in the first period in equilibrium, then B’s marginal profit is zero at the price. However, since A’s initial network size is larger, A’s marginal profit at the price is positive. Hence, seller A could increase profit by raising price. As a result, A’s price is higher than B’s price in the first period. The second statement says that A keeps its network advantage over B in the second period, though A is more expensive in the first period.
4.4 Comparing the Two Models

By comparing Proposition 1 and Proposition 2, it is clear that in the first period, the multi-network seller chooses a lower price for A than B, while single-network sellers do the opposite. These price differences in the two models are caused by the fact that the multi-network seller can internalize the competition effect of the two OSs. In the single-network seller model, the initially larger OS maintains the network advantage and has higher prices than the small OS in each period. In the multi-network model, the initially larger OS has a lower price than the small OS in the first period, thus it grows faster than the small OS. Therefore, the multi-network seller’s pricing strategy accelerates the concentration of the OS market. Next, I set up an empirical model of the consumers’ demand and the carriers’ pricing of smartphones, which is more general than the two-period, two-OS setting.

5 An Empirical Model of Demand and Supply of Smartphones

In this section, I design a structural model of consumers’ demand and carriers’ dynamic pricing of smartphones with two-year wireless service contracts. There are four leading wireless carriers, four operating systems, and hundreds of smartphone models. I assume consumers make static purchase decisions of smartphones with a random coefficient model. The carriers sell different sets of smartphone models and they play an infinite horizon dynamic pricing game.13

13In this paper, I focus on the price decisions of the carriers. I do not model the manufacturers’ prices in a Stackelberg leader-and-follower framework. In other words, I analyze the “follower” part of the full game given the manufacturers’ wholesale prices fixed. This simplification is not problematic in terms of model specification and estimation. In a Stackelberg leader-and-follower model, the manufacturers choose prices first and then the carriers choose prices accordingly. So the carriers are only affected by the manufacturers’ prices. Since I do model the carriers’ wholesale costs paid to the manufacturers, the simplification of focusing on the “follower” part is not problematic.
One challenge in modeling the consumers’ demand for smartphones is that there is no available data on smartphone model level market shares. In the data, characteristics and prices are at the model level, while the market shares are observed at carrier-OS level. To deal with this challenge and use data at both levels, I introduce a carrier-OS specific unobserved quality shock, which will be explained in detail in this section.

5.1 Consumer Demand

Each period, consumers who don’t own any smartphone or have ended previous two-year contracts enter the market. Each consumer chooses one option from the available choice set to maximize utility. The choice set in period \( t \) is \( \Omega_t = \{ (j, s, c, t) \}_{jsc} \cup \{ (0, t) \} \), where \( j \) is a smartphone model, \( s \) is an operating system, and \( c \) is a carrier. \( (0, t) \) is the outside option, which means not buying any smartphone. If a consumer purchases model \( (j, s, c, t) \), s/he signs a two-year wireless service contract with carrier \( c \). Let \( J_t \) be the total number of models in \( \Omega_t \).

Assume consumer \( i \)’s utility of buying model \( (j, s, c) \) in period \( t \) to be:

\[
u_{ijsc} = x'_{jsc} \beta_i - \alpha_i (p_{jsc}^c + f_{ct}) + \gamma N_{st} + \psi_{sc} + \xi_{sc} + \epsilon_{ijsc}.
\] (20)

\( x_{jsc} \) is a \( K \) by 1 vector of observed smartphone characteristics. \( p_{jsc}^c \) is the price of the smartphone if purchased with a two-year contract. \( f_{ct} \) is the carrier \( c \)'s plan price for the two years in the contract. \( N_{st} \) is the number of users of OS \( s \) at the beginning of period \( t \). \( \psi_{sc} \) is the carrier-OS \( (s, c) \) dummy, which captures the fixed effect of the carrier and

14I assume that the consumer pays the wireless plan price for two years when signing the contract. This assumption is not really restrictive since the monthly plan price is the same across the different models sold by the same carrier. So this assumption shouldn’t affect consumers’ decisions within each carrier’s models.

15Consumers may also care about future OS network, but they do not observe the future OS network sizes. In my paper, “Consumers’ Dynamic Demand for Smartphones with Two-Year Contracts,” 2014, I find that static model tends to underestimate the OS network effect.
operating system. It captures the carrier-OS quality that is constant across periods. $\xi_{sct}$ is the carrier-OS level unobserved quality shock in period $t$. It represents the shocks both in carrier service quality and the operating system quality. $\epsilon_{ijset}$ is the consumer idiosyncratic utility shock.

The unobserved quality shocks $\xi_{sct}$ is assumed to be carrier-OS specific because the market shares are observed at the carrier-OS level. The implication of the assumption is that the variation in market shares across different models in the same carrier-OS group is determined by their observed characteristics and prices, not by their individual unobserved quality shocks. The normal distribution assumption will used in the carriers’ pricing model in the next section.

The parameters $\theta_i = (\beta_i, \alpha_i)$ are consumer specific, as the random coefficient demand model in Berry, Levinsohn, and Pakes (1995). Let $\theta = (\beta, \alpha)$, where $\beta$ is the mean of $\beta_i$ over all consumers and $\alpha$ is the constant part of $\alpha_i$ that is the same for all consumers. Consumers are heterogeneous in income levels and tastes for smartphone characteristics. Consumer $i$ is described by his/her characteristics $v_i = (y_i, v_{i1}, ..., v_{iK})$, in which $y_i$ is income and $K$ is the dimension of the smartphone characteristics. The elements in $v_i$ are assumed to follow independent standard normal distributions $N(0, I_K)$, where $I_K$ is identity matrix with dimension $K$. Consumer characteristics $v_i$ is assumed to be independent of the unobserved quality shock $\xi_{sct}$. Consumer $i$’s coefficient vector $\theta_i$ is assumed to be:

$$\theta_i = \theta + \Phi v_i,$$

in which $\Phi$ is a diagonal matrix that determines the impact of consumer characteristics on the parameters. Rewrite the utility function (20) as:

$$u_{ijset} = \delta_{jset} + \mu_{ijset} + \epsilon_{ijset},$$
in which
\[ \delta_{jsct} = x'_{jsc} \beta - \alpha (p'_{jsc} + f_{ct}) + \gamma N_{st} + \psi_{sc} + \xi_{sct}, \]
\[ \mu_{ij} = \left[ x'_{jsc}; (p'_{jsc} + f_{ct}) \right]^{'} \Phi v_i. \]

The mean utility of the outside option is normalized to zero. Consumer \( i \)'s utility of the outside option is:
\[ x_{i0t} = \epsilon_{i0t}. \]

Assume that the \( \epsilon_{ij} \) follows a Type-I extreme value distribution and is i.i.d. across \((i, j, s, c, t)\). Then consumer \( i \)'s probability of choosing product \( j \) in period \( t \) is:
\[ s_{ij} = \frac{e^{(\delta_{ij} + \mu_{ij})}}{1 + \sum_{j' \in \Omega_t} e^{(\delta_{j'c} + \mu_{ij'})}}. \]

Let \( A_{j} \) be the set of consumer characteristics such that \( j \) has the highest utility for consumers whose characteristics belong to this set. That is, \( A_{j} = \{ v_i | u_{ij} \geq u_{ij'} \}, \) for all \((j', c') \in \Omega_t\). Then the market share of product \( j \) in period \( t \) is:
\[ s_{j} = \int_{A_j} s_{ij} dF(v_i). \quad (21) \]

I aggregate the smartphone model shares to carrier-OS levels, observe smartphone model level shares in the data, but the carrier-OS group level shares. Let \( \Omega_{st} \) be the set of all models with OS \( s \) by carrier \( c \) in period \( t \). The market share of the carrier-OS group \((s, c)\) in period \( t \) is the sum of shares over models in \( \Omega_{st} \):
\[ s_{st} = \sum_{j \in \Omega_{st}} s_{j}. \quad (22) \]
5.2 The Carriers’ Dynamic Pricing Model

The carriers play a dynamic pricing game because of the evolving OS network sizes and market size. In this section, I model carriers’ costs of a smartphone, the aggregate market size, and the carriers’ dynamic game for choosing smartphone prices. Each period, the carriers observe the market shares of all OSs and choose the prices of smartphones when sold with two-year wireless service contracts. For example, the carriers’ two-year contract price for an iPhone is $199, while its manufacturer retail price is $649.

5.2.1 The Carriers’ Unit Costs

A carrier pays a wholesale cost, a service cost, and an unobserved cost shock on each smartphone model. Carrier $c$’s wholesale cost of model $(j, s, c, t)$ is the product of a manufacturer wholesale price rate $\omega_j$ and the manufacturer retail price $p_{jst}^m$. $\omega_j$ is assumed to be manufacturer specific and the same across carriers. It captures a manufacturer’s bargaining power against the carriers.

In addition to the wholesale cost, carrier $c$ also pays a monthly service cost $\kappa_{sc}$, which is carrier-OS specific. The service cost includes the costs of selling a phone, maintaining wireless coverage, and providing customer services. Different OSs may have different service costs. For example, users of different OSs consume different amounts of data and request different types of customer service. There is also an unobserved cost shock, $\lambda_{jst}$. Hence,

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16 The manufacturers may also affect the two-year contract prices of smartphones by signing price contracts with carriers. However, a carrier will only sign a price contract with a manufacturer if the two-year contract prices can maximize its long run profit, together with other manufacturers’ products. Therefore, the carriers are indirectly choosing the two-year contract prices even when they sign price contracts with manufacturers.

17 Another option is to assume the wholesale discounts be a function of operating system shares. Intuitively, the manufacturers’ wholesale prices could be affected by the OS network sizes. For example, the wholesale costs could be higher for smartphone models with larger OS network sizes. However, this effect could already be reflected in the manufacturer retail prices. In this case, modeling discount rates as depending on OS shares would double count the OS share’s effect. Instead, I use manufacturer specific wholesale discounts to measure the manufacturers’ bargaining power against the carriers.
carrier $c$’s unit cost of selling model $(j, s, c)$ with a two-year contract in period $t$ is:

$$c_{jstc} = \omega_j p_{m jsct} + 24\kappa_{sc} + \lambda_{jstc}, \quad (23)$$

where $p_{m jsct}^m$ is the manufacturer’s retailer price.

### 5.2.2 The Market Size

At the beginning of period $t$, the state variables for the carriers are the cumulative market shares of different OSs: $n_t = (n_{1t}, ..., n_{St})$, where $S$ is the number of OSs. The OS shares are defined relative to the number of potential smartphone users $M$: $n_{st} = N_{st}/M \quad (25)$ The sum of OS shares is less than 1 because some potential consumers do not own smartphones.

Two types of consumers enter the market each period, those who do not own any smartphone yet and those who have finished their previous contracts. The proportion of consumers without any smartphone in period $t$ is $1 - \sum_{s=1}^{S} n_{st}$.

I assume that each existing user has the same probability to end his/her current contract each period. This probability is fixed to be $1/8$ because a contract is two years long and one period is three months in this paper. Let $M_t$ be the market size at the beginning of period $t$. Then given state variable $n_t$, the market size is:

$$M_t = \left\{1 - \sum_s n_{st}\right\} M + \frac{1}{8} \sum_s n_{st} M = \left\{1 - \frac{7}{8} \sum_s n_{st}\right\} M. \quad (24)$$

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18I include the population between age 12-70 as potential smartphone users. It is 75% of the total US population according to 2010 population age distribution.

19In reality, the share of population that have ended their two-year contract in period $t$ should be endogenous. But tracking the distribution of the smartphone owners’ contract status would require this distribution being taken as a state variable for the carriers. This complicates the state space of the dynamic problem a lot.

20Since one period is three months and the contract is 24 months long. The number of periods left in a contract at the end of each period is in $\{0,1,2,...,7\}$. The assumption that a user has $1/8$ probability to end his/her contract is equivalent to that $1/8$ of the users will have 0 period left at the end of the period.
The market size is decreasing in the sum of OS shares. By assumption, the market size will not become zero, because each period one eighth of previous users end their contracts and re-enter the market again.

5.2.3 Timing of the Pricing Game and the Carriers’ Bellman Equations

The timing of the dynamic pricing game is as following. At the beginning of period $t$, all carriers observe the state variables $n_t$. Then the demand and cost shocks ($\xi_t, \lambda_t$) are realized. Each carrier observes all carriers’ shocks and costs. The carriers choose prices simultaneously and then the consumers make choices accordingly. At the end of period $t$, the state variables update to $n_{t+1}$ and the market enters the next period.

At the end of period $t$, each operating system loses $1/8$ of its existing users, who end their current contracts and enter the market at period $t+1$. This implies that, $7/8$ of the existing users of an OS keep using the OS. Meanwhile, the OS also gets new users from the sales in period $t$. Let $\Omega_{st}$ be the set of smartphones with OS $s$ in period $t$. The transition of the cumulative market share of operating system $s$ is:

$$n_{st+1}(n_t, p_t^c) = \frac{7}{8}n_{st} + (1 - \frac{7}{8}\sum_{s'=1}^{S}n_{s't}) \sum_{(j,c)\in\Omega_{st}} s_{jst}(p_t^c(\xi_t, \lambda_t), \xi_t).$$

(25)

Denote the set of carrier $c$’s smartphones in period $t$ by $\Omega_{ct}$. Let $\lambda_t$ be the vector of all cost shocks in period $t$. Then the profit of carrier $c$ in period $t$, given price $p_t^c$, is:

$$\pi_{ct}(p_t^c, \xi_t, \lambda_t) = \sum_{(j,s)\in\Omega_{ct}} (p_{jst}^c + f_{ct} - c_{jst})s_{jst}(p_t^c, \xi_t)M_t,$$

(26)

where $c_{jst}$ is the unit cost as in equation (23) and $f_{ct}$ is carrier $c$’s price of two years’ wireless service.
Carrier c’s Bellman equation is:

\[ V_c(n_t) = E_{\xi,\lambda} \left[ \max_{p_{jcst}(\xi_t, \lambda_t), (j, s) \in \Omega_{ct}} \left\{ \pi_{ct}(p_{jcst}^c(\xi_t, \lambda_t)) + \beta^d V_c(n_{t+1}(n_t, p_{jcst}^c(\xi_t, \lambda_t))) \right\} \right], \] (27)

subject to equation (25). \( \beta^d \) is a discount factor across periods. The expectation is over the unobserved quality shock vector \( \xi \) and the cost shock vector \( \lambda \). To avoid including the shocks as state variables, I define the value functions before the carriers observe the shocks. In addition to the i.i.d. assumption of \( \lambda_{jst} \), I make the following assumption to calculate the carriers’ value functions.

**Assumption 3.** (1) The random cost shock \( \lambda_{jst} \) follows normal distribution \( N(0, \sigma_{\lambda}^2) \).

(2) The unobserved quality shock \( \xi_{st} \) follows the normal distribution \( N(0, \sigma_{\xi}^2) \) and is i.i.d. across carrier-OS groups and periods.

(3) Cost shock \( \lambda_{jst} \) and quality shock \( \xi_{st} \) are independent with each other.

The value functions are carrier specific because the carriers are different in their smartphone sets, wireless service costs, and wireless plan prices. In an ideal model, the evolving sets of smartphone models and their characteristics should also be state variables. However, the number of smartphone models by each carrier is relatively stable and the characteristics of all operating systems are improving in the data, as will be shown in Section 6. To keep the problem tractable, I don’t add them as state variables.

For any \( (\xi_t, \lambda_t) \), carrier c’s first-order conditions with respect to price can be derived. Given the prices of other models in period \( t \), the equilibrium prices \( p_{jcst}^c(\xi_t, \lambda_t) \) must satisfy the following first-order condition:

\[ M_t s_{jst}(p_{jcst}^c(\xi_t)) + M_t \sum_{(j', s') \in \Omega_{ct}} m_{j's't} \frac{\partial s_{j's't}}{\partial p_{jcst}^c} + \beta^d \frac{\partial V_c(n_{t+1}(n_t, p_{jcst}^c(\xi_t)))}{\partial p_{jcst}^c} = 0, \] (28)
where $m_{jst}$ denotes the carrier markup:

$$m_{jst} = p_{jst}^c + f_{ct} - c_{jst}.$$  (29)

The price $p_{jst}^c$ affects both the current sales of all models of carrier $c$ and future state variables. The FOC implies that the marginal long run profit should be zero at the equilibrium prices.

5.3 Equilibrium

The equilibrium concept used in the carriers’ dynamic pricing game is Markov perfect Nash equilibrium (MPNE). In this paper, a MPNE is a subgame perfect equilibrium where the strategies depend only on the past through the OS market shares updated from last period. For any shock vector $(\xi, \lambda)$ in a period, one equilibrium is a price vector $p^c(n, \xi, \lambda)$ and value functions $\{V_c(n)\}$ such that (1) given $p^c(n, \xi, \lambda)$, $\{V_c(n)\}$’s are the expected discounted long run profits and (2) given $\{V_c(n)\}$, the price vector $p^c(n, \xi, \lambda)$ maximizes the long run profit for each carrier, given the rivals’ strategy following the price.

Theoretically, the dynamic game may have multiple equilibria for a set of model parameters. However, as often argued in the empirical literature, the multiple equilibria possibility is not an issue for the identification, as long as all the observed actions are outcomes from the same equilibrium. The assumption implies that the observed prices are the carriers’ best responses to their opponents’ strategies under the true parameters.

5.4 Discussion of the Demand and Supply Model

Ideally, the consumers’ demand for smartphones with contracts should be modeled as dynamic decisions. The main concern with using a static demand model is that it ignores the consumers’ expectation of future prices, new launched models, and OS network effect.
However, the static demand model still gives the carriers the incentive to choose prices dynamically. As long as consumers value the OS network sizes, no matter in a static or dynamic way, the carriers will always endogenously make an impact on the future OS network sizes through current prices. The static model might underestimate the importance of the OS network effect, because forward-looking consumers may postpone their adoption of a large OS which can be even larger in the future. A static model would treat this low current demand for a large OS as if that consumers do not valuing the OS network effect enough. Therefore, the results with the static demand model could be interpreted as lower bounds of the impact of OS network effect on the industry.

Besides, it is quite challenging to nest a dynamic demand model into the dynamic pricing problem of asymmetric players with multiple products. There are two reasons. First, it’s very difficult to have explicit forms of the derivatives of the market shares in a dynamic demand model because consumers’ value functions are unknown. These derivatives will be required to analyze the first-order conditions of the dynamic pricing problem of the carriers.

Second and more importantly, the carriers’ state variables have to include the distribution of smartphone ownership, whose dimension is very high (more than a thousand) in this paper. The consumers’ state variables are their current smartphone ownership status (current model and number of periods left in the current contract) in a dynamic demand model. It is extremely hard to solve a dynamic problem with such high dimension state variables.

In this paper, I don’t endogenously model the carriers’ service prices $f_{ct}$. There are several reasons for this. First, each carrier’s plan price rarely changed during the sample periods. Second, the problem of choosing the optimal service price is not closely related to the OS network effect because the service price is carrier specific, not OS or model
specific. Third, the plan prices are not adjusted periodically in the data and the very few adjustments of the carriers are not made simultaneously. This makes it hard to clearly define a period. Therefore, I take the carriers’ service prices as given in this paper.

6 Data

The data used in this paper comes from several internet sources. The data sample period is from Aug 2011 to Oct 2013. The comScore.com reports the U.S. cumulative smartphone subscriber market shares every month. Sales market shares for the past three months are published every month by the Kantar World Panel. Carrier prices and manufacturer retail prices are collected via the web archive website. The smartphone characteristics data are collected from phonearena.com.

This paper focuses on the leading four carriers and four operating systems. The four carriers are Verizon, AT&T, Sprint, and T-Mobile. Their combined sales market shares during the sample period is around 90%. The four operating systems are iOS, Android, Blackberry, and Windows Phone. Their combined sales market share is more than 99%. During the data period, Verizon carries around 25 models of smartphones a month on average. AT&T, Sprint, and T-Mobile have 28, 19, and 17 models, respectively.

I exclude the population younger than twelve years old and older than 70 years old as smartphone consumers. This assumption makes the potential market size of smartphone consumption to be 75% of the population, according to the 2010 US population distribution by age. The website comScore.com reports the total number of smartphone subscribers and the cumulative market share of each OS every month. I calculate the market size $M_t$ and state variables $n_t$ using these data.

The Kantar World Panel publishes the sales market share of each carrier for the previous three months ending. For example, in Feb 2012, it published the sales market shares for
the three months ending in Jan 2012. In addition, the website also reports the OS sales shares conditional on the sales shares of AT&T and Verizon. Combining the sales market shares by carrier and the OS sales within each carrier, I get the sales market share of each carrier-OS group. One missing piece of the sales data is the conditional OS sales market shares for Sprint or T-Mobile. Since only the combined OS sales market share for the two are observed, I construct the OS sales market shares for them as proportional to the number of models they have on different OSs. In the end, there are sales market shares for 16 carrier-OS groups for 26 months.

The web archive website has been archiving the carriers’ webpages several times every month from 2008. The carriers’ two-year contract price and the listed manufacturer retail price of each model can be collected by month. In the sample period, the data has 2283 model-month observations. The highest two-year contract price is $399 for the 64 GB iPhones from multiple carriers.

I also collect the monthly wireless plan prices from the web archive website. Each carrier offers multiple plans each month. I use the single line price for medium amount of data and minutes. Verizon’s wireless plan price was the highest at $70 and T-Mobile’s was the lowest at $50. The average across carriers is $60 per month during 2011 to 2013. This matches the $61 average reported by New Street Research company for 2013.

A period is three months in the structural model to match the sales shares data. Smartphone models and their prices are observed every month. Thus, to use as much information as possible, I construct consumers’ choice set every period in the following way. If a smartphone model is observed in multiple months in a period, I treat them as different choice options in that period.

21 The web archive website link is: http://archive.org/web/
22 I use the prices for the following minutes and data bundles for the 4 carriers: Verizon (unlimited minutes, 2GB), AT&T (450 minutes, 300MB), Sprint (unlimited minutes, 1GB), T-Mobile (unlimited minutes and data).
The smartphone characteristics include camera pixels, built-in storage, 4G dummy, weight, screen size, resolution, processor speed, system memory, and battery capacity. All the characteristics are scaled so that their values are in similar range to compare their coefficients in the utility function.

Table 1 shows the summary statistics of the number of models, characteristics, average smartphone price, and manufacturer retail price by carrier-OS groups by month. All carriers have more than 10 Android models each month on average. Windows Phone has the fewest number of models, with monthly average lower than 2. iOS models have the highest carrier contract prices and manufacturer retail prices on average. Windows Phone models have the lowest carrier prices and manufacturer prices.

The pattern of hardware characteristics across OSs is mixed. The iOS models outperform other models in camera pixels and screen pixels per square inch. Android models have the best battery capacities, screen sizes, and processor speeds, and Android models dominate the sales. Most new iPhone users signed contracts with Verizon and AT&T. T-Mobile only started to sell iPhone’s in April 2013.

Figure 1: Increase of Smartphone Users by Month
Table 1: Descriptive Statistics by Carrier-OS group (Average over Months)

<table>
<thead>
<tr>
<th>Carrier-OS Group</th>
<th>No. of Models</th>
<th>Carrier Price 100$</th>
<th>Manuf. Price 100$</th>
<th>Battery 1000mAh</th>
<th>Camera megapixels</th>
<th>Screen inches</th>
<th>Pixel 100/inch$^2$</th>
<th>Processor Ghz</th>
<th>Sales Share%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verizon-iOS</td>
<td>4.69</td>
<td>2.26</td>
<td>6.74</td>
<td>1.43</td>
<td>7.14</td>
<td>3.65</td>
<td>3.26</td>
<td>1.72</td>
<td>17.80</td>
</tr>
<tr>
<td>Verizon-And</td>
<td>16.15</td>
<td>1.22</td>
<td>5.04</td>
<td>1.92</td>
<td>6.86</td>
<td>4.24</td>
<td>2.68</td>
<td>2.60</td>
<td>18.94</td>
</tr>
<tr>
<td>Verizon-Bla</td>
<td>3.00</td>
<td>1.45</td>
<td>4.66</td>
<td>1.31</td>
<td>5.03</td>
<td>3.01</td>
<td>2.60</td>
<td>1.34</td>
<td>0.47</td>
</tr>
<tr>
<td>Verizon-Win</td>
<td>1.81</td>
<td>1.03</td>
<td>4.32</td>
<td>1.49</td>
<td>5.68</td>
<td>3.94</td>
<td>2.45</td>
<td>1.61</td>
<td>0.08</td>
</tr>
<tr>
<td>AT&amp;T-iOS</td>
<td>5.07</td>
<td>1.98</td>
<td>6.47</td>
<td>1.41</td>
<td>6.75</td>
<td>3.66</td>
<td>3.11</td>
<td>1.66</td>
<td>20.75</td>
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<tr>
<td>AT&amp;T-And</td>
<td>13.88</td>
<td>0.98</td>
<td>4.69</td>
<td>1.90</td>
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<td>2.66</td>
<td>2.98</td>
<td>9.58</td>
</tr>
<tr>
<td>AT&amp;T-Bla</td>
<td>3.81</td>
<td>0.86</td>
<td>4.48</td>
<td>1.25</td>
<td>5.30</td>
<td>3.07</td>
<td>2.59</td>
<td>1.25</td>
<td>1.4</td>
</tr>
<tr>
<td>AT&amp;T-Win</td>
<td>4.81</td>
<td>0.75</td>
<td>4.36</td>
<td>1.67</td>
<td>7.82</td>
<td>4.26</td>
<td>2.47</td>
<td>1.81</td>
<td>1.36</td>
</tr>
<tr>
<td>Sprint-iOS</td>
<td>4.91</td>
<td>2.04</td>
<td>6.71</td>
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<td>7.36</td>
<td>3.66</td>
<td>3.28</td>
<td>1.79</td>
<td>3.07</td>
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<tr>
<td>Sprint-And</td>
<td>11.38</td>
<td>0.90</td>
<td>4.41</td>
<td>1.80</td>
<td>5.77</td>
<td>3.97</td>
<td>2.45</td>
<td>2.03</td>
<td>11.98</td>
</tr>
<tr>
<td>Sprint-Bla</td>
<td>1.65</td>
<td>1.30</td>
<td>4.53</td>
<td>1.18</td>
<td>5.06</td>
<td>2.68</td>
<td>2.76</td>
<td>1.11</td>
<td>0.38</td>
</tr>
<tr>
<td>Sprint-Win</td>
<td>1.13</td>
<td>0.79</td>
<td>4.42</td>
<td>1.57</td>
<td>5.60</td>
<td>3.77</td>
<td>2.57</td>
<td>1.36</td>
<td>0.45</td>
</tr>
<tr>
<td>T-Mobile-iOS</td>
<td>4.00</td>
<td>—</td>
<td>6.96</td>
<td>1.44</td>
<td>7.94</td>
<td>3.98</td>
<td>3.26</td>
<td>2.55</td>
<td>0.36</td>
</tr>
<tr>
<td>T-Mobile-And</td>
<td>10.62</td>
<td>1.58</td>
<td>4.05</td>
<td>1.77</td>
<td>6.10</td>
<td>5.19</td>
<td>2.43</td>
<td>2.62</td>
<td>11.44</td>
</tr>
<tr>
<td>T-Mobile-Bla</td>
<td>3.69</td>
<td>2.01</td>
<td>4.46</td>
<td>1.39</td>
<td>5.19</td>
<td>2.94</td>
<td>2.61</td>
<td>1.19</td>
<td>0.83</td>
</tr>
<tr>
<td>T-Mobile-Win</td>
<td>1.31</td>
<td>1.15</td>
<td>3.49</td>
<td>1.46</td>
<td>5.80</td>
<td>3.91</td>
<td>2.48</td>
<td>1.89</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Notes: The first three letters are used to denote the operating system. For example, “And” is for Android. T-Mobile eliminated the 2-contract policy since it started to sell iOS models. The sales shares are conditional shares among the listed groups and are recorded for the sales in the past three months every month.
Figure 1 shows the monthly increases in the number of smartphone subscribers. There are spikes in the new smartphone users during holiday seasons (December and January). To address this issue, the data are seasonally adjusted when estimating consumer demand model. The green curve shows the adjusted monthly increases of smartphone subscribers. The monthly increases are adjusted so that (1) the geometric mean of adjusted sales is the same across months of the year and (2) the total increase of smartphone users from Sep 2009 to Jan 2014 is equal to the original data.\footnote{Following Gowrisankaran and Rysman (2009), I first regress the monthly log increases on the month dummies. Then divide each monthly increase by the exponentiated dummy for its month of the year. The adjusted increase is constructed by multiplying the divided increases by a constant such that the total increase during Sep 2009 to Jan 2014 is the same with that in the original data.}

Table 2 shows the reduced form estimation results from regressing carrier two-year contract discounts and manufacturer prices on OS shares in the end of the past month, characteristics, and OS dummies. The contract discount of a smartphone model is defined as the difference between its manufacturer retail price and two-year contract price. Column 2 shows that the carriers give higher discounts to models with larger OS shares in the last month. Column 3 shows that the manufacturers charge higher prices for phones with larger OS shares. These results are consistent with the theoretical results in the two-period-two-OS example.

Figure 2 shows the cumulative market shares of the four operating systems during the August 2011-October 2013 period. The market shares of iOS and Android have been increasing, while Blackberry have been decreasing. The market shares of iOS and Android both increased from below 6% to above 25%. Blackberry’s market share decreased from 10% to less than 5%. The Windows Phone market share is stable and small, at around 3%.\footnote{Following Gowrisankaran and Rysman (2009), I first regress the monthly log increases on the month dummies. Then divide each monthly increase by the exponentiated dummy for its month of the year. The adjusted increase is constructed by multiplying the divided increases by a constant such that the total increase during Sep 2009 to Jan 2014 is the same with that in the original data.}
Table 2: OLS Regressions of Carrier Discounts and Manufacturer Prices ($100)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Carrier Discounts</th>
<th>Manufacturer Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS Share (Lag 1)</td>
<td>1.0331***</td>
<td>0.3651*</td>
</tr>
<tr>
<td></td>
<td>(0.3090)</td>
<td>(0.2113)</td>
</tr>
<tr>
<td>Screen Size</td>
<td>0.1863***</td>
<td>0.2412***</td>
</tr>
<tr>
<td></td>
<td>(0.0360)</td>
<td>(0.0450)</td>
</tr>
<tr>
<td>Dummy 4G</td>
<td>0.2055***</td>
<td>0.5735***</td>
</tr>
<tr>
<td></td>
<td>(0.0328)</td>
<td>(0.0360)</td>
</tr>
<tr>
<td>Screen Pixels</td>
<td>0.1784***</td>
<td>0.4567***</td>
</tr>
<tr>
<td></td>
<td>(0.0269)</td>
<td>(0.0405)</td>
</tr>
<tr>
<td>Camera</td>
<td>0.0149***</td>
<td>0.0769</td>
</tr>
<tr>
<td></td>
<td>(0.0041)</td>
<td>(0.0070)</td>
</tr>
<tr>
<td>CPU (GHz)</td>
<td>-0.0145</td>
<td>0.0335*</td>
</tr>
<tr>
<td></td>
<td>(0.0123)</td>
<td>(0.0189)</td>
</tr>
<tr>
<td>OS Dummies</td>
<td>yes</td>
<td>—</td>
</tr>
<tr>
<td>Manufacturer Dummies</td>
<td>—</td>
<td>yes</td>
</tr>
<tr>
<td>Month Dummies</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Figure 2: The Cumulative OS Market Shares (2009.09-2014.03)
7 Identification and Estimation

The structural model parameters are the demand parameters \( \theta_d = (\alpha; \beta; \gamma; \psi; \Phi; \sigma_\xi) \) in the carrier-OS share equation (22) and the supply parameters \( \theta_s = (\omega; \kappa; \sigma_\lambda) \) in the carrier first-order condition equation (28). I use GMM with MPEC to estimate the parameters. The moment conditions are based on the carrier-OS level unobserved quality shock \( \xi_{sct} \) in equation (22) and the model level cost shock \( \lambda_{jsct} \) in equation (28).

To fit the data into the structural model, I first calculate model level shares in equation (21) using the smartphone model level data on characteristics and prices and data on the number of OS users. The model level shares are then used to calculate \( \xi \) and \( \lambda \). To calculate \( \xi_{sct} \), I aggregate the model level shares to carrier-OS level shares, so that they match the observed carrier-OS shares in the data. To calculate \( \lambda_{jsct} \), I use the model level shares and their derivatives w.r.t. prices to solve the carriers’ first-order conditions, so that the observed prices are the equilibrium prices in the carriers’ dynamic pricing game model.

Next, I discuss how the data provides identification for the structural model parameters and explain how I calculate \( \xi_{sct} \) and \( \lambda_{jsct} \) in details.

7.1 Identification

The identification of the demand side parameters comes from the variation in sales market shares across carrier-OS groups and across periods. The smartphone prices and the plan prices identify the parameters in the supply model.

The identification of the price coefficient \( \alpha \) requires instruments, since the carriers’ price \( p_{jsct} \) is correlated with the unobserved carrier-OS group quality \( \xi_{sct} \). The candidate instruments for model price \( p_{jsct} \) include the average characteristics of models in the carrier-OS group \((s, c, t)\), that of other OSs by the same carrier \((-s, c, t)\), that of other carriers \((-c, t)\), and that of all models in period \( t \). The characteristics are correlated with prices via the
wholesale costs and the competition among the carriers. I assume the characteristics are not correlated with the unobserved quality shock $\xi_{sct}$. This implies that the average characteristics are valid instruments. Individual model characteristics could also be instruments, but I don’t use them because there are many models in one group. Instead, I use the averages by carrier-OS group. Once the endogeneity issue is dealt with, $\alpha$ is identified by the variation in market shares of carrier-OS groups that differ in prices of their models.

The OS network effect coefficient $\gamma$ is identified by the sales market shares and the OS network sizes at the beginning of each period. For example, when everything else is controlled, if a group with a larger OS network size has a higher market share, then the OS network effect is positive and the magnitude can be identified by the market share difference across OSs. By assumption, there is no endogeneity issue in the identification of $\gamma$ because the OS network size at the beginning of period $t$, $N_{st}$, is not correlated with current shock $\xi_{sct}$.

Controlling for prices, the OS network effect, and the carrier-OS dummies, the characteristics parameter $\beta$ is identified by carrier-OS groups that differ in the characteristics of their models and their sales market shares.

The diagonal matrix $\Phi$ is the coefficients on the interaction between consumer characteristics and the prices and model characteristics. It is identified by the market shares in periods with varying distribution of the consumer characteristics. I use the CPS (Current Population Survey) household income distribution each year during 2011 to 2013. The average household income increased from $69,677 to $72,641 and the standard deviation increased from $368 to $499.\[^{24}\] To see why the income distribution variation identifies $\Phi$, suppose that the average consumer income increases from one period to another, then the difference between the two periods’ product shares identifies the income’s impact on the

\[^{24}\text{See Appendix D.1 for the details of simulation and normalization of individual income levels.}\]
price coefficient, everything else equal. If the coefficient on the interaction of income and price is zero, then the two periods will have the same market shares. But if the coefficient is positive (less price elastic), then the period with higher level of income will have higher market shares of more expensive smartphones.

The carrier service cost $\kappa_{sc}$ is identified by the average prices in the carrier-OS groups. The structural model predicts the markup and thus the unit cost $c_{jst}$ for each smartphone model. The unit cost equation (23) then provides the identification of $\kappa_{sc}$ and the wholesale price parameters in $\omega$. With the control of the wholesale costs, the cost $\kappa_{sc}$ is identified by the average difference between the unit costs and the wholesale costs over all models in the same carrier-OS group. Similarly, by controlling the service costs, the wholesale price parameters in $\omega$ are identified by the average difference between unit costs and the service costs over models by the same manufacturer.

I do not directly estimate the standard variances of the unobserved demand and cost shocks, $\sigma_\xi$ and $\sigma_\lambda$. Instead, I calculate them using the demand and cost shocks, which can be calculated given a set of the rest parameters. Following Goettler and Gordon (2011), I use 0.975 as the discount rate $\beta^d$ for a three-month period model.

7.2 Estimation

7.2.1 Moment Conditions and Objective Functions

I construct moment conditions based on the orthogonality between instrumental variables, the unobserved carrier-OS demand shock $\xi_{sct}$, and the unobserved cost shock $\lambda_{jst}$. The details of calculating $\xi(\theta_d)$ and $\lambda(\theta_d, \theta_s)$ are described in the next subsection.

An endogeneity issue exists in the demand model between the carrier-OS-period specific demand shock $\xi_{sct}$ and the carrier prices. So proper instrument variables are required to get consistent estimates. By assumption, there is no endogeneity issue in the supply
model because the cost shocks \( \lambda_{jsc} \)s are not correlated with the carriers’ service costs and wholesale costs.

The moment conditions for the unobserved shocks are:

\[
E[\xi_{sct}(\theta_d)|Z_{1sct}] = 0, \quad (30)
\]

\[
E[\lambda_{jsc}(\theta_s, \theta_d)|Z_{2jsc}] = 0, \quad (31)
\]

where \( Z_{1sct} \) is a vector of variables that are correlated with prices \( p_{jsc} \) but not correlated with the shock \( \xi_{sct} \). It includes the average smartphone characteristics as listed in the identification part. There are 44 moment conditions in (30). \( Z_{2jsc} \) is a vector of variables that are orthogonal to the cost shocks \( \lambda_{jsc} \). It includes the variables in the unit cost (manufacturer dummies, carrier-OS dummies), the characteristics of \((j, s, c, t)\), the average characteristics over models by carrier \( c \) in period \( t \), the average over models with the same operating system \( s \) in period \( t \), and the average over models in the same period \( t \). There are 67 moment conditions in (31). The number of structural parameters is 58 in the demand and supply model together.

In the estimation, I apply the moment conditions on \( \xi_{sct} \) to each model in the carrier-OS group \( \Omega_{sct} \), which implies that there is a vector of moment conditions \([Z_{1sct}\xi_{sct}; Z_{2jsc}\lambda_{jsc}]\) for shocks of model \((j, s, c, t)\). To get efficient estimates, I use two-stage GMM as often used in the literature.

In the first stage, the weight matrix for the demand side moment conditions is \( W_{d1} = (Z_1'Z_1)^{-1} \) and that for the supply side moment conditions is \( W_{s1} = (Z_2'Z_2)^{-1} \). Let the first stage estimates be \((\hat{\theta}_{d1}, \hat{\theta}_{s1})\). The second stage uses the optimal weight matrix estimate.
\[ \hat{W}_2, \text{ estimated using the first stage results. The second stage objective function is:} \]

\[ Q_2(\theta_d, \theta_s) = \left( \frac{1}{N_s} \sum_{j \in \Omega_{sct}} [Z_{1sct} \xi_{sct}(\theta_d); Z_{2jsct} \lambda_{jsct}(\theta_s; \theta_d)] \right)' \]

\[ \hat{W}_2(\theta) = \frac{1}{N_s} \sum_{j \in \Omega_{sct}} [Z_{1sct} \xi_{sct}(\theta_d); Z_{2jsct} \lambda_{jsct}(\theta_s; \theta_d)]. \] (32)

### 7.2.2 Calculating the Unobserved Shocks \( \xi \) and \( \lambda \)

In this subsection, I describe the algorithm for calculating the unobserved carrier-OS specific quality shock \( \xi_{sct} \) and \( \lambda_{jsct} \). I calculate \( \xi_{sct} \) by matching the model predicted carrier-OS market shares to the observed shares. The cost shock \( \lambda_{jsct} \) can be backed out from the carriers’ FOCs.

The model market share in equation (21) has the integration of individual probabilities over the distribution of consumer characteristics \( v_i \). Since \( v_i \) is a high dimension vector, I use numerical approximation to calculate the integration. I simulate \( N_s = 300 \) consumers with different characteristics each period and use the averages of their individual choice probabilities to approximate the sales market share of each model.

\[ \tilde{s}_{jsct}(\theta_d) = \frac{1}{N_s} \sum_{i=1}^{N_s} s_{ijsc}(\theta_d). \] (33)

Denote the set of models with operating system \( s \) and carrier \( c \) in period \( t \) by \( \Omega_{sct} \). The predicted carrier-OS sales share is the sum over all models in the group.

\[ \tilde{s}_{sct}(\theta_d) = \sum_{j \in \Omega_{sct}} \tilde{s}_{jsct}(\theta_d). \]

Following Berry (1994), it can be shown that the quality shocks \( \xi \) are uniquely determined by the observed carrier-OS shares for a given \( \theta_d \). See Appendix C for the proof of
the inversion from the carrier-OS sales shares to $\xi_{sct}(\theta_d)$.

As often used in the random coefficient demand model literature, an iterative procedure is applied to solve for the unobserved shocks $\xi_{sct}(\theta_d)$. Let the observed carrier-OS shares be $s_{sct}^0$. Given an initial guess of the unobserved demand shocks, $\xi^0_{sct} = \{s_{sct}^0\}$, calculate the predicted market shares $\tilde{s}_{sct}(\theta_d, \xi^0_{sct})$. Then compare the predicted shares with the observed shares. The updating rule is to increase $\xi_{sct}$ if the predicted share is less than the observed market share for the group $(s,c,t)$ and decrease it otherwise. Repeat this updating process until the vector $\xi^k$ converges. The updating process is summarized in the following equation:

$$\xi^k_{sct}(\theta_d) = \xi^k_{sct}(\theta_d) + \chi (s_{sct}^0 - \tilde{s}_{sct}(\theta_d, \xi^k_{sct})),$$

where $\chi$ is a constant, set to be 0.9. The proof of unique fixed point in Appendix C guarantees that the iteration converges to the solution of $\xi_{sct}(\theta_d)$.

Next, I describe the algorithm for calculating the cost shock $\lambda_{j_{sct}}$ for a given value of $(\theta_d, \theta_s)$. I first solve for the smartphone model level markup $m_{j_{sct}}$ using data and FOCs. Then the calculation of $\lambda_{j_{sct}}$ is straightforward, using the definition of markup. As in equation (28), the FOC w.r.t. price $p_{j_{sct}}^c$ is:

$$M_t s_{j_{sct}}(p_t^c, \xi_t; \theta_d) + M_t \sum_{(j',s') \in \Omega_{ct}} m_{j's't} \frac{\partial s_{j's't}}{\partial p_{j_{sct}}^c} + \beta_d \frac{\partial V_c(n_{t+1}(n_t, p_{j_{sct}}^c(\xi_t)))}{\partial p_{j_{sct}}^c} = 0. \quad (34)$$

An important feature of the FOC’s is that the left hand sides are linear in the model markups $m_{j_{sct}}$’s, given prices. The first two terms in equation (34) can be calculated using the demand side market share functions. To calculate the last term in equation (34), I approximate each carrier’s value function with a linear combination of basis functions, so that there is explicit functional form of the derivative $\frac{\partial V_c(n_{t+1})}{\partial p_{j_{sct}}^c}$.

Each carrier’s value function is a multivariate function of the four operating systems’
market shares $n_t$. I use the second-order complete polynomials as basis functions to approximate each carrier’s value function\footnote{The Multivariate Adaptive Regression Spline (MARS) is another method used in empirical literature to approximate multivariate functions. Friedman (1991) proposed this algorithm. There is a Matlab package, ARESLab by Jekabsons (2011), to implement the MARS method to find basis functions. Similar method has been used by Jia and Pathak (2010). The problem of using this method is that the approximated functions are not differentiable in a small number of points where the splines connect. But in my paper, the first-order conditions require the value functions to be differentiable on its domain. So the MARS method is not used in this paper.}. Hence, there are 15 basis functions for the four state variables. Denote the basis functions by $B_f(n) = (1, bf_1(n), ..., bf_{14}(n))$. Let $\theta^v_c$ be the basis function coefficient vector for carrier $c$. The coefficient vector $\theta^v_c$ is carrier specific.

$$V_c(n_t) = B_f(n_t) \ast \theta^v_c.$$  

Let $\theta^v$ be the vector containing all carriers’ coefficients for value function approximation. With the approximated value functions, model level markups can be solved using the FOCs. Appendix B derives the function form for markups with the approximated value functions. Let $m_{jset}(\theta_d, \theta_s)$ be the solution for the given $(\theta_d, \theta_s)$. Then the unobserved cost shock is:

$$\lambda_{jset}(\theta_d, \theta_s) = p^e_{jset} + f_{ct} - m_{jset}(\theta_d, \theta_s) - \omega_j p^m_{jset} - 24\kappa_{sc}. \quad (35)$$

Therefore, given an alternative of the parameters $(\theta_d, \theta_s)$, the shocks $\xi_{set}(\theta_d)$ and $\lambda_{jset}(\theta_d, \theta_s)$ can be solved using observed data and approximation of the value functions\footnote{When calculating the GMM objective functions, I exclude observations with carrier prices at $0$ and T-Mobile models in May 2013-July 2013. For the $0$ carrier prices, since $0$ is the lower bound of the carrier prices, so they are corner solutions in the optimal equilibrium prices. The corresponding cost shocks, calculated as if $0$’s are interior solutions, could be far from the actual shocks. For the T-Mobile models after May 2013, they were sold at manufacturer prices without any contract. There are no carrier discounts for those models. So the cost shocks calculated in equation(35) for these observations have a different meaning. Therefore, I exclude the cost shocks of these observations in the GMM objective function.}. Next subsection describes the restrictions on the approximation of value functions, which corresponds to the MPEC constraints in the estimation.
7.2.3 Equilibrium Constraints

The carriers’ Bellman equations are imposed as constraints on the approximation parameters $\theta^v$. The right hand side of the Bellman equations are expectations of the carriers’ discounted profits over $(\xi, \lambda)$, which requires me to solve for the optimal prices for any possible $(\xi, \lambda)$. Instead of integrating over the distributions of $(\xi, \lambda)$, I simulate $R$ vectors of the demand and costs shocks. For each simulated $(\xi^r, \lambda^r)$, I solve for the equilibrium prices $p_{j|sct}^c(\xi^r, \lambda^r)$ using a Newton-Raphson iteration method and take the advantage of linearity of markups in the price FOCs. Then I use the average long run profit across simulations to approximate the right hand side of the Bellman equations:

$$R^\sim H S_{ct}(\theta_d, \theta_s, \theta^v) = \frac{1}{R} \sum_{r=1}^{R} \left( \sum_{(j,s)} (p_{j|sct}^c - \omega_j p_{j|sct}^m - 24 \kappa s c - \lambda_{j|sct}^r) M_t s j|sct(p_t^c, \xi_t^r; \theta_d) + \beta^d V_c(n_{t+1}(n_t, p_t^c), \theta^v) \right)$$

In the estimation, I use inequality constraints which allow some error between the two sides of the Bellman equations. Barwick and Pathak (2011) also imposed Bellman equations as equilibrium constraints when using approximations of the value functions. Specifically, for each carrier $c$ in each period $t$, the following inequality is imposed on the approximated value functions.

$$|V_c(n_t; \theta_d, \theta_s, \theta^v) - R^\sim H S_{ct}(\theta_d, \theta_s, \theta^v)| / |V_c(n_t; \theta^v)| < \tau. \quad (36)$$

27 Due to computation time issue, I use $R = 50$ in the estimation. After I get the parameter estimates, I check the impact of the number of Monte Carlo simulation on the value functions. See Appendix D.2 for details.

28 See Appendix D.3 for the algorithm for solving the equilibrium prices in period $t$ for a simulated shock pair $(\xi^r, \lambda^r)$.

29 If the value function approximation is ideal, for instance, with infinite basis functions, then the Bellman equations should hold exactly. However, when a finite sequence of basis functions are used, there is inevitably some error between the two sides of each Bellman equation.
The constraint (36) implies that the relative difference between the two sides of each carrier’s Bellman equation at any state $n_t$ should be smaller than the tolerance $\tau$. The smaller is $\tau$, the better the value functions are approximated. In the estimation, the value of $\tau$ is set to be 5% and the constraint is satisfied for all carriers in all periods.

I conclude this section by summarizing the GMM estimation objective with MPEC.

$$\min_{\theta_d, \theta_s, \theta^v} \{Q_2(\theta_d, \theta_s, \theta^v)\},$$
subject to (36).

where $Q_2$ is the GMM objective function as in equation (32).

8 Estimation Results

Table 3 shows the estimation results for the demand model parameters. The OS network effect coefficient estimate $\hat{\gamma}$ is positive and significant, which means that a consumer’s utility of buying a smartphone increases with the number of existing OS users. The price coefficient $\hat{\alpha}$ is also significant, meaning that consumer utility decreases with the carrier price of smartphones. The interaction between price and income has positive coefficient $\phi_1$, which means that consumers’ disutility of price decreases with income levels.

The coefficients for characteristics imply that consumers’ utility increases with storage, camera pixels, and 4G feature. The coefficients for battery, screen size, and pixels are negative. There are several reasons for the coefficients being negative. First, the best selling models do not have the most advanced characteristics. For example, iPhone 5’s battery is 1540mAh, while most other top models have capacities more than 2000mAh. Similarly, the average screen size for carrier-iOS groups is 3.66 inches, while that for carrier-Android
groups is 4.24 inches and that for carrier-Windows Phone groups is 3.94 inches.

Table 3: Demand Model Parameter Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Fixed Effects</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS Subscribers (Million), $\hat{\gamma}$</td>
<td>0.0418***</td>
<td>Verizon-iOS, $\psi_{iv}$</td>
<td>4.1023***</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.8155)</td>
<td></td>
</tr>
<tr>
<td>Carrier Price ($100), $-\hat{\alpha}$</td>
<td>-0.2731***</td>
<td>Verizon-Android, $\psi_{av}$</td>
<td>2.5591***</td>
</tr>
<tr>
<td></td>
<td>(0.0120)</td>
<td>(0.7282)</td>
<td></td>
</tr>
<tr>
<td>Price*Income, $-\hat{\phi_1}$</td>
<td>0.01420***</td>
<td>Verizon-Blackberry, $\psi_{bv}$</td>
<td>1.1071*</td>
</tr>
<tr>
<td></td>
<td>(0.0086)</td>
<td>(0.6047)</td>
<td></td>
</tr>
<tr>
<td>Storage (GB), $\hat{\beta_1}$</td>
<td>0.0310***</td>
<td>Verizon-Windows, $\psi_{bw}$</td>
<td>3.6198***</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.7527)</td>
<td></td>
</tr>
<tr>
<td>Battery (1000mAh), $\hat{\beta_2}$</td>
<td>-0.3379***</td>
<td>AT&amp;T-iOS, $\psi_{ia}$</td>
<td>4.6184***</td>
</tr>
<tr>
<td></td>
<td>(0.0704)</td>
<td>(0.8458)</td>
<td></td>
</tr>
<tr>
<td>Camera(100MP), $\hat{\beta_3}$</td>
<td>0.1248***</td>
<td>AT&amp;T-Android, $\psi_{aa}$</td>
<td>0.9682</td>
</tr>
<tr>
<td></td>
<td>(0.0410)</td>
<td>(0.6446)</td>
<td></td>
</tr>
<tr>
<td>Screen Size (inch), $\hat{\beta_4}$</td>
<td>-1.0008***</td>
<td>AT&amp;T-Blackberry, $\psi_{ba}$</td>
<td>1.5645***</td>
</tr>
<tr>
<td></td>
<td>(0.3951)</td>
<td>(0.5366)</td>
<td></td>
</tr>
<tr>
<td>Dummy 4G, $\hat{\beta_5}$</td>
<td>0.3522***</td>
<td>AT&amp;T-Windows, $\psi_{wa}$</td>
<td>2.5305***</td>
</tr>
<tr>
<td></td>
<td>(0.1210)</td>
<td>(0.6644)</td>
<td></td>
</tr>
<tr>
<td>Pixels (100/inch²), $\hat{\beta_6}$</td>
<td>-0.7273***</td>
<td>Sprint-iOS, $\psi_{is}$</td>
<td>1.0753</td>
</tr>
<tr>
<td></td>
<td>(0.0856)</td>
<td>(0.8169)</td>
<td></td>
</tr>
<tr>
<td>RAM (GB), $\hat{\beta_7}$</td>
<td>0.1821</td>
<td>Sprint-Android, $\psi_{as}$</td>
<td>2.0045***</td>
</tr>
<tr>
<td></td>
<td>(0.1515)</td>
<td>(0.7506)</td>
<td></td>
</tr>
<tr>
<td>CPU (Ghz), $\hat{\beta_8}$</td>
<td>0.1908</td>
<td>Sprint-Blackberry, $\psi_{bs}$</td>
<td>0.9243</td>
</tr>
<tr>
<td></td>
<td>(0.3116)</td>
<td>(0.6315)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sprint-Windows, $\psi_{ws}$</td>
<td>3.5882***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.7360)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T-Mobile-iOS, $\psi_{it}$</td>
<td>2.0908***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T-Mobile-Android, $\psi_{at}$</td>
<td>0.6807</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.6694)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T-Mobile-Blackberry, $\psi_{bt}$</td>
<td>0.1648</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.5915)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>T-Mobile-Windows, $\psi_{wt}$</td>
<td>2.5361***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.5969)</td>
<td></td>
</tr>
</tbody>
</table>

Second, there may be model level unobserved quality across smartphone models, which is not included in the demand model due to data limitation. For example, consider two smartphone models which differ in their pixels, but the model with a low pixel density has a
high unobserved quality. Then this model has higher sales market share due to unobserved quality, though consumers do like high pixel density. In this case, the demand model would interpret the data as if consumers do not like high pixel density.\(^{31}\)

The carrier-OS fixed effects, \(\psi_s\), measure the unobserved characteristics of these groups that are constant over time. The results show that iOS has the highest fixed effect among all OSs. The Windows Phone’s fixed effect is higher than Android. This implies that the high sales market shares of Android compared with Windows Phone is not because Android has high fixed effects. Android has high sales shares due to its number of models and the frontier characteristics of its models.

Given the approximated demand market share equation \(^{(33)}\), the own and cross product demand elasticities are:

\[
\frac{\partial s_{j's'c't}}{\partial \bar{p}_{j's'c't}} = \begin{cases} 
-\bar{p}_{j's'c't} s_{j's'c't} s_{ij' s'c't}, & \text{if } (j', s', c') = (j, s, c), \\
\bar{p}_{j's'c't} s_{j's'c't} \sum_{i=1}^{N_t} \alpha_i s_{ij sct} (1 - s_{ij sct}), & \text{if } (j', s', c') \neq (j, s, c).
\end{cases}
\]

Table 4 shows the demand elasticities across carrier-OS groups in May 2013. The \((g, g')\)th element in the table is the sum of model elasticities in carrier-OS group \(g\) when prices of all products in group \(g'\) increase by 1%. The own elasticities are stronger than cross elasticities. The own elasticities of Android are the highest, meaning that consumers’ demand of Android models is the most elastic. The own elasticities of iOS are relatively small. The cross elasticities show that Android and iOS are better substitutes than other OSs.

\(^{31}\)However, ignoring the model level unobserved quality does not affect the identification of the model characteristics. Because the moment conditions on the carrier-OS unobserved quality are valid as long as the characteristics are exogenous. The carrier-OS unobserved quality includes the model level unobserved quality. Although there may be correlation among the carrier-OS unobserved qualities because that the different carriers may sell the same model, the moment conditions are based on the orthogonality between the carrier-OS unobserved qualities and the observed characteristics.
Table 4: Demand Elasticities w.r.t. Prices by Group

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V-iOS</td>
<td>-27.68</td>
<td>2.16</td>
<td>0.004</td>
<td>0.096</td>
<td>2.287</td>
<td>0.968</td>
<td>0.035</td>
<td>0.148</td>
</tr>
<tr>
<td>V-Android</td>
<td>6.805</td>
<td>-75.24</td>
<td>0.012</td>
<td>0.275</td>
<td>6.167</td>
<td>2.655</td>
<td>0.103</td>
<td>0.425</td>
</tr>
<tr>
<td>V-Blackberry</td>
<td>1.195</td>
<td>1.103</td>
<td>-15.45</td>
<td>0.050</td>
<td>1.083</td>
<td>0.467</td>
<td>0.019</td>
<td>0.077</td>
</tr>
<tr>
<td>V-Windows</td>
<td>1.709</td>
<td>1.543</td>
<td>-0.03</td>
<td>-20.33</td>
<td>1.549</td>
<td>0.666</td>
<td>0.026</td>
<td>0.108</td>
</tr>
<tr>
<td>A-iOS</td>
<td>2.522</td>
<td>2.16</td>
<td>0.004</td>
<td>0.096</td>
<td>-27.91</td>
<td>0.968</td>
<td>0.035</td>
<td>0.148</td>
</tr>
<tr>
<td>A-Android</td>
<td>7.804</td>
<td>6.81</td>
<td>0.013</td>
<td>0.304</td>
<td>7.075</td>
<td>-87.35</td>
<td>0.112</td>
<td>0.469</td>
</tr>
<tr>
<td>A-Blackberry</td>
<td>2.026</td>
<td>1.898</td>
<td>0.003</td>
<td>0.086</td>
<td>1.837</td>
<td>0.794</td>
<td>-26.22</td>
<td>0.134</td>
</tr>
<tr>
<td>A-Windows</td>
<td>1.563</td>
<td>1.413</td>
<td>0.002</td>
<td>0.063</td>
<td>1.417</td>
<td>0.610</td>
<td>0.024</td>
<td>-18.02</td>
</tr>
<tr>
<td>Outside</td>
<td>0.121</td>
<td>0.116</td>
<td>0.000</td>
<td>0.005</td>
<td>0.110</td>
<td>0.047</td>
<td>0.002</td>
<td>0.008</td>
</tr>
</tbody>
</table>


The last row in table 4 are elasticities of the outside option. The elasticity of the outside option is relatively small compared with other groups. This implies that when prices of models in a carrier-OS group increase, most consumers switch to other groups but not to the outside option.

Table 5 shows the estimates of the supply side parameters. The wholesale cost coefficients \( \omega \) are significant, which are defined as the carrier’s wholesale cost divided by the manufacturer retail price. The average wholesale cost ratio is 82.83%. Apple’s wholesale price ratio is 85.01%. It implies that the carriers pay $552 to Apple for a $649 iPhone. Blackberry’s wholesale price ratio is the second highest among all manufacturers at 84.21%. Samsung’s wholesale price ratio is the lowest, at 80.84%. It implies that the carriers pay $484 for a Samsung S3 with manufacturer retail price $599.
Table 5: Supply Model Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price ratio ( b ):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apple, ( b_a )</td>
<td>0.8501*** (0.0074)</td>
<td>AT&amp;T-iOS, ( \kappa_a )</td>
<td>0.2410*** (0.0375)</td>
</tr>
<tr>
<td>Samsung, ( b_s )</td>
<td>0.8084*** (0.0027)</td>
<td>AT&amp;T-Android, ( \kappa_a )</td>
<td>0.2566*** (0.0389)</td>
</tr>
<tr>
<td>Motorola, ( b_m )</td>
<td>0.8151*** (0.0028)</td>
<td>AT&amp;T-Blackberry, ( \kappa_a )</td>
<td>0.2418 (0.3795)</td>
</tr>
<tr>
<td>LG, ( b_l )</td>
<td>0.8305*** (0.0029)</td>
<td>AT&amp;T-Windows, ( \kappa_a )</td>
<td>0.2407 (0.3811)</td>
</tr>
<tr>
<td>HTC, ( b_h )</td>
<td>0.8164*** (0.0026)</td>
<td>Sprint-iOS, ( \kappa_s )</td>
<td>0.2185*** (0.0436)</td>
</tr>
<tr>
<td>Blackberry, ( b_b )</td>
<td>0.8421*** (0.0043)</td>
<td>Sprint-Android, ( \kappa_s )</td>
<td>0.2268*** (0.0372)</td>
</tr>
<tr>
<td>Nokia, ( b_n )</td>
<td>0.8354*** (0.035)</td>
<td>Sprint-Blackberry, ( \kappa_s )</td>
<td>0.2305 (0.5933)</td>
</tr>
<tr>
<td>Monthly service cost (100$) ( \kappa ):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verizon-iOS, ( \kappa_{vi} )</td>
<td>0.2462** (0.0207)</td>
<td>Sprint-Windows, ( \kappa_{sw} )</td>
<td>0.2217*** (0.2257)</td>
</tr>
<tr>
<td>Verizon-Android, ( \kappa_{vi} )</td>
<td>0.2360*** (0.0538)</td>
<td>T-Mobile-iOS, ( \kappa_{ti} )</td>
<td>0.2121 (0.0100)</td>
</tr>
<tr>
<td>Verizon-Blackberry, ( \kappa_{vb} )</td>
<td>0.2411 (0.2898)</td>
<td>T-Mobile-Android, ( \kappa_{ta} )</td>
<td>0.2281*** (0.0679)</td>
</tr>
<tr>
<td>Verizon-Windows, ( \kappa_{vw} )</td>
<td>0.2381** (0.1253)</td>
<td>T-Mobiel-Blackberry, ( \kappa_{tb} )</td>
<td>0.2251 (0.6919)</td>
</tr>
<tr>
<td>T-Mobiel-Windows, ( \kappa_{tw} )</td>
<td>0.2262 (0.2060)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The monthly service cost estimates \( \kappa \) are at the carrier-OS levels. The average monthly service cost for all carriers is $23.32. Overall, Verizon and AT&T have higher service costs with an average of $24.27. Their monthly markups on wireless services are $39.47 and $35.50, respectively. The average monthly service cost for Sprint and T-Mobile is $22.36. Their monthly markups on wireless services are $37.56 and $27.71 respectively. Sprint has higher markups than AT&T because it has lower service costs.

Given the estimates above, the carriers’ profits from each two-year contract customer can be calculated. Take AT&T and iPhone 5 in December 2012 for example. On one hand,
AT&T sells the iPhone 5 at $199 while paying $552 to the manufacturers, which implies a $353 net cost on the phone. On the other hand, AT&T earns a net margin of $35.50 each month from the customer’s wireless service, which makes $852’s total net margin in the two years’ contract. Therefore, AT&T earns a net profit of $499 from a two-year contract from an iPhone 5 customer over the two years before counting the shocks. Similarly, I can calculate the carriers’ two-year contract markups on all models. Table 6 shows the carriers’ markups on two-year contract smartphones by carrier-OS.

Table 6: Carriers’ Markups on Two-Year Contract Smartphones by Carrier-OS ($100)

<table>
<thead>
<tr>
<th>Operating System</th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>iOS</td>
<td>5.89</td>
<td>5.13</td>
<td>5.59</td>
<td>—</td>
</tr>
<tr>
<td>Android</td>
<td>6.72</td>
<td>5.50</td>
<td>6.26</td>
<td>4.01</td>
</tr>
<tr>
<td>Blackberry</td>
<td>7.39</td>
<td>5.65</td>
<td>6.52</td>
<td>4.19</td>
</tr>
<tr>
<td>Windows</td>
<td>6.81</td>
<td>5.95</td>
<td>5.98</td>
<td>4.26</td>
</tr>
</tbody>
</table>

Across operating systems, the carriers’ two-year contract markups on Blackberry and Windows Phone models are higher than iOS and Android models in general. This is because that 1) the carriers give lower discounts on smaller operating systems and 2) the carriers pay lower wholesale prices to manufacturers. Among the carriers, Verizon has the highest markups among all carriers. The reason is that Verizon has the highest margins on wireless service. Sprint’s average markups on contracts are higher than AT&T because Sprint has lower service cost estimates. T-Mobile’s markups are the lowest among all carriers due to their low margins on services.

The carriers’ value functions at any state can be calculated with the polynomial approximation. The value functions are plotted in Appendix E. Verizon’s value function has the highest value among all carriers. AT&T has the second highest value function. T-Mobile’s value function has the lowest value.
9 Counterfactuals

In this section, I study two counterfactual scenarios to measure the impact of the OS network effect on consumers’ demand for smartphones and the OS market concentration. I eliminate the OS network effect and the carrier discounts on two-year contracts in the two scenarios, respectively.

9.1 The Impact of the OS Network Effect

In this counterfactual case, the OS network effect is eliminated. The purpose is to find out the impact of the OS network effect on the carriers’ prices and the market concentration of operating systems. The carriers’ prices are affected by two effects after eliminating the OS network effect. First, without the OS network effect, the carriers do not have the incentive to give higher discounts to larger OSs. Second, consumers’ utility of a smartphone decreases when there is no OS network effect.

By fixing the OS network effect to be zero and compensating the consumers, the consumer utility is:

\[ u_{ijset} = x'_{jsc} \beta_i - \alpha_i(p^c_{jt} + f_{ct}) + \psi_{sc} + \xi_{sct} + \epsilon_{ijset} \]

in which \( \xi_{sct} \) is the unobserved carrier-OS specific utility calculated using the estimates. For any given price vector \( p^c_t \), the market share of each model \( s_{jset}(p^c_t) \) can be derived similarly as in the demand model.

The carriers play a static pricing game without the OS network effect, because they do not need to choose prices dynamically such that the future OS network sizes would make an impact on demand. Carrier \( c \)'s profit in period \( t \) is:

\[ \pi_{ct}(p^c_t; \xi_t, \lambda_t) = \sum_{(j,s) \in \Omega_{ct}} (p^c_{jset} + f_{ct} - \omega_j p^m_{jset} - 24 \kappa_{sc} - \lambda_{jset}) s_{jset}(p^c_t; \xi_t) M_t, \]
where \( \bar{M}_t \) is the new market size at the beginning of period \( t \): \( \bar{M}_t = (1 - \frac{7}{8} n_t)M \).

I solve for the new equilibrium prices using the carriers’ first-order conditions. Table 7 shows the average price decreases by carrier-OS group. The carriers increase prices in the static pricing game. The prices of iOS and Android models increase more than Blackberry and Windows prices. Without the OS network effect, the carriers would not give high discounts for large OSs because OS network sizes do not affect consumer demand.

<table>
<thead>
<tr>
<th></th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>iOS</td>
<td>213.38</td>
<td>297.75</td>
<td>243.58</td>
<td>—</td>
</tr>
<tr>
<td>Android</td>
<td>133.63</td>
<td>262.38</td>
<td>175.80</td>
<td>405.99</td>
</tr>
<tr>
<td>Blackberry</td>
<td>78.62</td>
<td>247.73</td>
<td>159.72</td>
<td>389.11</td>
</tr>
<tr>
<td>Windows</td>
<td>129.85</td>
<td>218.95</td>
<td>210.13</td>
<td>377.18</td>
</tr>
</tbody>
</table>

Figure 3 compares the cumulative OS market shares. The bold curves show the OS shares without OS network effect and the dashed curves represent the OS shares in the data. Without the OS network effect, the smartphone penetration rate decreases for two reasons. First, consumers’ utility of smartphone decrease in general because the OS network effect no long exists. Second, the carriers increase their prices in static models. The two effects both reduce consumers’ demand for smartphones. The aggregate smartphone penetration in the counterfactual case decreases from 45.06% in Aug 2011 to 23.67% Aug 2013. While in the data, the smartphone penetration rate increased from 45.06% in Aug 2011 to 78.36% in Aug 2013.
The OS market is less concentrated without the OS network effect. The market shares of iOS and Android decrease from more than 30% to below 10%. The market shares of Blackberry RIM and the Windows Phone do not as much. Their joint market share is 6.75%, compared with 6.39% in the data. The OS market becomes less concentrated for two reasons. First, the large OSs do not have the “initial installed base advantage” anymore. This implies consumers are equally likely to buy products with large or small OS networks. Second, the carriers do not have the incentive to give more discounts to the large networks anymore. Thus the OS concentration slows down.

Table 8 shows the profit comparison by carrier during the sample period. The carriers profit decreases by $53.84 billion (79.13%) combined. This huge profit loss is caused by the
decrease in consumer demand. The profit losses for Verizon, AT&T, Sprint, and T-Mobile are 76.11%, 78.28%, 84.04%, and 85.97% respectively.

Table 8: Profit Comparison by Carrier ($ billion)

<table>
<thead>
<tr>
<th></th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated</td>
<td>28.61</td>
<td>20.95</td>
<td>11.72</td>
<td>6.77</td>
</tr>
<tr>
<td>No OS network effect</td>
<td>6.83</td>
<td>4.55</td>
<td>1.87</td>
<td>0.95</td>
</tr>
</tbody>
</table>

9.2 The Impact of the Carrier Discounts

This counterfactual case studies the importance of the carriers’ discounts of smartphones sold with two-year contracts. I consider a scenario in which the carriers sell smartphones at the manufacturer retail prices and decrease their service prices by $15 each month. To make this counterfactual results comparable with the full model, consumers are assumed to use smartphones for two years. This implies that a consumer can buy the iPhone 5 at the price of $649 and pay $360 less on wireless service for the two years.

Suppose that the new plan price is $p_{ct}$ for carrier $c$. By assumption, $p_{ct} = f_{ct} - 3.6$ (in $100 units). Since the consumers are paying the manufacturer retail price $p_{j}^{m}$, the utility function is:

$$ u_{ij}^{mct} = x_{j}^{c} \beta - \alpha (p_{j}^{m} + p_{ct}^{p}) + \psi_{jc} + \tilde{\xi}_{ct} + \epsilon_{ij}^{mct}, $$

where $\tilde{\xi}_{ct}$ is the unobserved carrier-OS specific utility calculated using estimation results. The market shares can be derived from this utility function. Let the corresponding sales market shares be $\tilde{s}_{j}^{mct}$.

Though the service prices decrease, consumers actually pay more on most smartphones in the counterfactual case. Take the AT&T 16GB iPhone5 in December 2012 for example.

\[32\] In fact, carriers started to sell smartphones at the manufacturer prices and decrease monthly service prices by at least $15 from late 2013.
A consumer gets a $450 discount if they sign the two-year contract, while they only save $360 on the service price. In this case, the consumer pays $90 more without the discount on contract.

Figure 4 compares the OS growth paths in this counterfactual case with the data. The overall smartphone penetration rate without the two-year contract discount is 49.30% (86.52 million), compared with 78.36% (137.51 million) in the data by the end of May 2013. The reason is that consumers now pay higher total prices on the most popular smartphone models. Without the two-year contract, the decrease in service price is only $360, which is less than the two-year contract discount on those models. Therefore, the carriers’ discounts on two-year contracts contributed to the smartphone penetration.

Figure 4: OS Growth without two-year Contract
iOS and Android still dominate the market for several reasons. First, they have initial OS network advantages. Second, iOS has high fixed effects across carriers, so consumers’ demand for iPhones is relatively high compared with other OSs. But the Blackberry RIM and the Windows Phone together share 9.17% of the market, instead of 6.39% in the data. The gap between Android and Windows Phone market shares decreases from 28.57% in the data to 11.18%. It implies that the carrier discount also contributed to the concentration of the OS market shares.

The profit for carrier $c$ in period $t$ is:

$$
\pi_{ct}(p^*_t, \hat{\xi}_t, \hat{\lambda}_t) = \sum_{(j,s) \in \Omega_{ct}} (p^m_{jact} - \omega_j p^m_{jact} + f_{ct} - 24 \hat{\kappa}_{sc} - \hat{\lambda}_{jact}) s_{jact} M_t,
$$

where $(\hat{\xi}_{sc}, \hat{\lambda}_t)$ are the shocks in period $t$.

Table 9 compares the profits in the counterfactual scenario with the profits estimated from the original model. The overall profit decreases by $23.52$ billion (34.91%) for the 4 carriers combined from Aug 2011 to Aug 2013.

<table>
<thead>
<tr>
<th></th>
<th>Verizon</th>
<th>AT&amp;T</th>
<th>Sprint</th>
<th>T-Mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated</td>
<td>28.61</td>
<td>20.95</td>
<td>11.72</td>
<td>6.77</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>18.43</td>
<td>13.26</td>
<td>8.14</td>
<td>4.46</td>
</tr>
</tbody>
</table>

The two counterfactual cases imply that both the OS network effect and the carriers’ discounts on the two-year contracts have contributed significantly to the growth of the smartphone industry in the US and the concentration of the operating system networks. The OS network effect plays a relatively more important role than the two-year contract discounts.
10 Conclusion

The literature on the network effect has focused on prices of single-network manufacturers, but not the prices of multi-network retailers. In this paper, I analyze the impact of a network effect on multi-network retailers’ dynamic prices both theoretically and empirically. I first analyze a two-period, two-OS theoretical model to compare the price for the large OS with that for the small OS. I show that, when network effects exist, manufacturers and multi-network retailers have quite different dynamic pricing strategies. While manufacturers of the products with larger networks choose higher prices, the retailers choose lower prices for the products with larger networks. The difference in their pricing strategies is a result of the difference in their competition environments.

The above pricing strategy of retailers is present in the smartphone industry, where the smartphone operating system network effect exists and telecom carriers act like multi-network retailers. I estimate a structural model of consumers’ demand and carriers’ dynamic pricing of smartphones, to quantify the OS network effect and measure its impact on carrier prices. I use smartphone model level data during 2011 to 2013 to estimate the structural model. The results show that there is positive and significant OS network effect. This implies that consumers’ utility and thus carriers’ prices are affected by the OS network sizes. With the estimates, I study two counterfactual cases in which I eliminate the OS network effect and the carriers’ two-year contract policy, respectively. The results show that both the OS network effect and carriers’ two-year contract discounts significantly accelerated the smartphone industry’s growth and the concentration of operating systems. Furthermore, the OS network effect is relatively more important than the carrier discounts to both smartphone growth and OS concentration.

This paper also makes a contribution by estimating a structural model of dynamic pricing of multiple products with high-dimension continuous state variables and asymmetric
oligopolistic firms. The existing estimation algorithms for discrete choice dynamic games or continuous choice games with single-product firms can not be used in this paper. I solve the carriers’ dynamic pricing game by approximating the carriers’ value functions with basis functions. I develop an iterative procedure to efficiently solve for the equilibrium prices. As a result, the oligopolistic multiple-product carriers’ asymmetric pricing game can be solved efficiently within the estimation algorithm.

References


Appendix

A Proofs for the Two-OS, Two-Period Model

Lemma 1. Given the costs of the two models are the same (not necessarily 0), the multi-network seller chooses the same price for A and B in the second period: $p^*_A = p^*_B$.

Proof. The seller’s pricing problem can be solved backwards. Let the unit cost of the two models be $c$. The seller’s profit in the 2nd period is:

$$\pi_2(p_{A1}, p_{B1}, p_{A2}, p_{B2}) = \sum_{j=A,B} (p_{j2} - c)M_2s_{j2}$$

$$= \sum_{j=A,B} (p_{j2} - c)M_2 \frac{e^{(\delta_j + \gamma n_{j2} - \alpha p_{j2})}}{1 + \sum_{k=A,B} e^{(\delta_k + \gamma n_{k2} - \alpha p_{k2})}}.$$ 

The FOC with respect to $p_{A2}$ is:

$$s_{A2} - \alpha(p^*_A - c)s_{A2}(1 - s_{A2}) + \alpha(p^*_B - c)s_{A2}s_{B2} = 0,$$ (A.1)

which is equivalent to:

$$1 - \alpha(p^*_A - c) + \alpha(p^*_A - c)s_{A2} + \alpha(p^*_B - c)s_{B2} = 0.$$ (A.2)

Similarly for B, we have:

$$1 - \alpha(p^*_B - c) + \alpha(p^*_A - c)s_{A2} + \alpha(p^*_B - c)s_{B2} = 0.$$ (A.3)

Compare equations (A.2) and (A.3), we immediately get $p^*_A = p^*_B$. $\square$

A.1 Proof of Proposition 1

Proof. Let the unit cost of the two models be 0. Let the price in the 2nd period of the two models be $p^*_2$. Then plug $p^*_2$ into equation (A.1). We get:

$$1 - \alpha p^*_2(1 - s_{A2} - s_{B2}) = 0.$$
Plug in the sales market share equations and rearrange the terms. We get:

\[ \alpha p_2^* - 1 = e^{(\delta_A + \gamma n_{A2} - \alpha p_2^*)} + e^{(\delta_B + \gamma n_{B2} - \alpha p_2^*)}. \]  \hspace{1cm} (A.4)

Using total differentiation on (A.4), we get:

\[ \alpha \frac{\partial p_2^*}{\partial n_{A2}} = e^{(\delta_A + \gamma n_{A2} - \alpha p_2^*)}(\gamma - \alpha \frac{\partial p_2^*}{\partial n_{A2}}) - \alpha e^{(\delta_B + \gamma n_{B2} - \alpha p_2^*)} \frac{\partial p_2^*}{\partial n_{A2}}. \]

Then we can solve \( \frac{\partial p_2^*}{\partial n_{A2}} \):

\[ \frac{\partial p_2^*}{\partial n_{A2}} = \frac{\gamma}{\alpha} s_{A2}. \]  \hspace{1cm} (A.5)

Similarly for \( \frac{\partial p_2^*}{\partial n_{B2}} \), we have:

\[ \frac{\partial p_2^*}{\partial n_{B2}} = \frac{\gamma}{\alpha} s_{B2}. \]  \hspace{1cm} (A.6)

Then the profit in the 2nd period is:

\[ \pi_2 = \sum_{j=A,B} p_{j2} M_2 s_{j2} = p_2^* M_2 (s_{A2} + s_{B2}) = p_2^* M_2 (1 - \frac{1}{1 + e^{(\delta_A + \gamma n_{A2} - \alpha p_2^*)} + e^{(\delta_B + \gamma n_{B2} - \alpha p_2^*)}}) \]

\[ = p_2^* M_2 (1 - \frac{1}{\alpha p_2^*}) \]

\[ = \frac{M_2}{\alpha} (\alpha p_2^* - 1). \]  \hspace{1cm} (A.7)

Then the maximization problem in the first period is:

\[ \max_{p_{A1}, p_{B1}} \pi_1(p_{A1}, p_{B1}) + \beta \pi_2(p_{A1}, p_{B1}, p_2^*(p_{A1}, p_{B1})) \]

\[ = \max_{p_{A1}, p_{B1}} \sum_{j=A,B} p_{j1} M_1 s_{j1} + \beta \left( \alpha p_2^*(p_{A1}^*, p_{B1}^*) - 1 \right) M_2 \]

\[ = \max_{p_{A1}, p_{B1}} \sum_{j=A,B} p_{j1} M_1 s_{j1} + \beta \left( p_2^*(p_{A1}, p_{B1}) - \frac{1}{\alpha} \right) M_1 s_{01}. \]  \hspace{1cm} (A.8)

in which \( s_{01} = 1 - s_{A1} - s_{B1} \) (0 means the outside option) and the market size in the second period is the proportion of the first period market size who didn’t buy any smartphone,
\[ M_2 = M_1 s_{01} \]. Then the FOC with respect to \( p_{A1} \) is:

\[
0 = s_{A1} - \alpha p^*_{A1} s_{A1}(1 - s_{A1}) + \alpha p^*_{B1} s_{A1} s_{B1} + \alpha \beta (p^*_2(p_{A1}, p_{B1}) - \frac{1}{\alpha}) s_{A1} s_{01} + \beta \frac{\partial p^*_2}{\partial p_{A1}} s_{01},
\]

(A.9)

in which the partial derivative of price \( p^*_2 \) with respect to 1st period price \( p_{A1} \) is:

\[
\frac{\partial p^*_2}{\partial p_{A1}} = \frac{\partial p^*_2}{\partial n_{A2}} \frac{\partial n_{A2}}{\partial p_{A1}} + \frac{\partial p^*_2}{\partial n_{B2}} \frac{\partial n_{B2}}{\partial p_{A1}}
\]

\[
= \frac{\partial p^*_2}{\partial n_{A2}} M_1 (-\alpha) s_{A1}(1 - s_{A1}) + \frac{\partial p^*_2}{\partial n_{B2}} M_1 \alpha s_{A1} s_{B1}.
\]

Plug \( \frac{\partial p^*_2}{\partial p_{A1}} \) into (A.9), we get:

\[
1 - \alpha p^*_{A1} + \alpha p^*_{A1} s_{A1} + \alpha p^*_{B1} s_{B1} + \beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{01} + \beta M_1 s_{01} [-\alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{B1}] = 0.
\]

(A.10)

Similarly for OS B, the first-order condition of profit with respect to price \( p_{B1} \) gives the following equation:

\[
1 - \alpha p^*_{B1} + \alpha p^*_{B1} s_{B1} + \alpha p^*_{A1} s_{A1} + \beta (p^*_2 - \frac{1}{\alpha}) \alpha s_{01} + \beta M_1 s_{01} [-\alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{A1}] = 0.
\]

(A.11)

Then taking the difference of equations (A.10) and (A.11), we get:

\[
- \alpha p^*_{A1} + \beta M_1 s_{01} [-\alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{A1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{B1}]
\]

\[
= - \alpha p^*_{B1} + \beta M_1 s_{01} [-\alpha \frac{\partial p^*_2}{\partial n_{A2}} (1 - s_{B1}) + \alpha \frac{\partial p^*_2}{\partial n_{B2}} s_{A1}].
\]

Plug in the partial derivatives in (A.5) and (A.6), we get:

\[
p^*_{A1} - p^*_{B1} = \frac{\beta \gamma}{\alpha} M_1 s_{01} (s_{B2} - s_{A2}).
\]

(A.12)

Next we need to prove \( p^*_{A1} < p^*_{B1} \) under the condition that \( n_{A1} > n_{B1} \). That is the 1st period price for the large OS is lower than that for the small OS. Notice that \( s_{B2} < s_{A2} \) \( \Leftrightarrow \) \( n_{B2} < n_{A2} \), given \( p^*_{B2} = p^*_{A2} \) and \( \delta_A = \delta_B = \delta \). Next I discuss three cases of relationships between \( p^*_{A1} \) and \( p^*_{B1} \) to prove that the profit maximization solution satisfies
\( p^*_A < p^*_B \).

First, suppose \( p^*_A = p^*_B \). Then \( n_{B2} < n_{A2} \). Because \( n_{j2} = n_{j1} + M_1 s_{j1} \), \( n_{A1} > n_{B1} \), and \( s_{A1} = s_{B1} \). But this means (A.12) is violated.

Second, suppose \( p^*_A > p^*_B \). If this is the case, (A.12) implies that \( s_{B2} > s_{A2} \), which means that \( n_{B2} > n_{A2} \). That is, the initial OS network advantage of A is reversed in the 2nd period due to high price of A. Since \( n_{j2} = n_{j1} + M_1 s_{j1} \), \( n_{B2} > n_{A2} \) implies that \( s_{B1} > s_{A1} \). That is, the sales share of B is higher than that of A in the first period. Use the equation of sales market shares, we get:

\[
e(\delta + \gamma n_{B1} - \alpha p^*_B) > e(\delta + \gamma n_{A1} - \alpha p^*_A),
\]

(A.13)

Let \((p^*_A, p^*_B, p^*_2)\) bet the profit maximization prices in the two periods. Then the seller’s total profit is:

\[
\Pi^* = M_1 p^*_A e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + p^*_B e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} + \beta / \alpha (\alpha p^*_2 - 1)
\]

Now consider another price plan for the two periods \((p'_A, p'_B, p^*_2)\), in which:

\[
\gamma n_{A1} - \alpha p'_A = \gamma n_{B1} - \alpha p^*_B,
\]

\[
\gamma n_{B1} - \alpha p'_B = \gamma n_{A1} - \alpha p^*_A.
\]

Then \( p'_A = \frac{\gamma n_{A1} - \gamma n_{B1} + \alpha p^*_B}{\alpha} \) and \( p'_B = \frac{\gamma n_{B1} - \gamma n_{A1} + \alpha p^*_A}{\alpha} \). The seller’s total profit with this price plan is:

\[
\Pi' = M_1 p'_A e^{(\delta + \gamma n_{A1} - \alpha p'_A)} + p'_B e^{(\delta + \gamma n_{B1} - \alpha p'_B)} + \beta / \alpha (\alpha p^*_2 - 1)
\]

\[
= M_1 p'_B e^{(\delta + \gamma n_{A1} - \alpha p'_A)} + p'_A e^{(\delta + \gamma n_{B1} - \alpha p'_B)} + \beta / \alpha (\alpha p^*_2 - 1)
\]

Take the difference of the two profits with the two price plans, we have:

\[
\Pi' - \Pi^* = M_1 \frac{\gamma / \alpha (n_{A1} - n_{B1}) (e^{(\delta + \gamma n_{B1} - \alpha p^*_B)} - e^{(\delta + \gamma n_{A1} - \alpha p^*_A)})}{1 + e^{(\delta + \gamma n_{A1} - \alpha p^*_A)} + e^{(\delta + \gamma n_{B1} - \alpha p^*_B)}}.
\]

The according to (A.13), we know that \( \Pi' > \Pi^* \). Hence, there exists another price plan that leads to higher profit than \((p^*_A, p^*_B, p^*_2)\), when \( p^*_A > p^*_B \). Therefore \( p'_A > p'_B \) can not be the profit maximization solution.

Therefore, the seller’s profit maximization prices in the first period must satisfy \( p^*_A <
\( p_{B1}^* \). And this implies that \( n_{A2} > n_{B2} \) and so \( s_{A2} > s_{B2} \). And we have:

\[
p_{A1}^* - p_{B1}^* = \frac{\beta \gamma}{\alpha} M_1 s_0 (s_{B2}^{os} - s_{A2}^{os}),
\]

\[
n_{A2} - n_{B2} = n_{A1} - n_{B1} + M_1 (s_{A1} - s_{B1}).
\]

So we have the following conclusions:

1. The optimal price of A is lower than that of B in the first period: \( p_{A1}^* < p_{B1}^* \).
2. The price gap \( |p_{A1}^* - p_{B1}^*| \) between the two models increase as the OS network effect becomes stronger (\( \gamma \) increases).
3. The OS market share difference \( (n_{A2} - n_{B2}) \) increases in the OS network effect \( \gamma \).

\[\Box\]

### A.2 Proof of Proposition 2

**Proof.** This proof has three steps. The first step shows that the price of the larger OS network is higher in the second period. The second step shows that the price of the larger OS network is higher in the first period. The third step shows that the operating system with initial advantage in the first period keeps its advantage in the second period. Combine the above three steps, the proposition statements are proved.

- **Step 1.** This steps shows that, if \( n_{A2} > n_{B2} \) at the beginning of the second period, then \( p_{A2}^m > p_{B2}^m \). Manufacturer \( j \)'s problem in the second period is:

\[
\max_{p_{j2}^m} \{ \pi_{j2}^m(p_{j2}^m, p_{-j2}^m) \} = p_{j2}^m s_{j2} M_2 e^{(\delta_j + \gamma n_{j2} - \alpha p_{j2}^m)} M_2.
\]

Then the FOC of the problem is:

\[
s_{j2} + p_{j2}^m (-\alpha s_{j2} + \alpha s_{j2}^2) = 0,
\]

which is equivalent to the following equation since \( s_{j2} > 0 \):

\[
p_{j2}^m = \frac{1}{\alpha (1 - s_{j2})}.
\]

By comparing the equations (A.14) for model A and B, we have the following equation:

\[
\frac{p_{A2}^m}{p_{B2}^m} = \frac{1 - s_{B2}}{1 - s_{A2}}.
\]
From equation (A.15) and the assumption that \( n_{A2} > n_{B2} \), the result \( p_{A2}^m > p_{B2}^m \) holds. The proof is as following. Suppose that \( p_{A2}^m \leq p_{B2}^m \). This implies that model A not only has larger OS network size \( (n_{A2} > n_{B2}) \), but also lower prices in the second period. Then \( s_{A2} > s_{B2} \). So the LHS (left hand side) of equation (A.15) is less than 1 but the RHS (right hand side) is greater than 1. This contradiction shows that \( p_{A2}^m > p_{B2}^m \) when \( n_{A2} > n_{B2} \). Therefore, the model with larger OS network size at the beginning of the second period has higher price in the second period.

• Step 2. This steps shows that if \( n_{A1} > n_{B1} \) and \( n_{A2} > n_{B2} \), then \( p_{A1}^m > p_{B1}^m \). That is, the optimal price of the larger OS model is higher in the first period.

From the Step 1, the maximized profit in the second period for \( j \) is:

\[
\pi_{j2}^m = p_{j2}^m s_{j2} M_2
\]

\[
= \frac{s_{j2}}{\alpha(1-s_{j2})} (1-n_{A2}-n_{B2})
\]

\[
= \frac{s_{j2}}{\alpha(1-s_{j2})} M_1 s_{01},
\]

in which \( s_{01} \) is the market share of the outside option in the first period. Then manufacturer \( j \)'s profit maximization problem in the first period is:

\[
\max_{p_{j1}} \{ \pi_{j1}(p_{j1}, p_{-j1}) + \beta \pi_{j2}^m \}
\]

\[
= p_{j1}^m s_{j1} M_1 + \frac{\beta}{\alpha} \frac{s_{j2}}{1-s_{j2}} s_{01} M_1
\]

\[
= p_{j1}^m \frac{s_{j1}}{1} + \sum_{k=A,B} e^{(\delta_j+\gamma n_{j1}-\alpha p_{jk}^m)} e^{(\delta_k+\gamma n_{k1}-\alpha p_{k1}^m)} M_1 + \frac{\beta}{\alpha} \frac{s_{j2}}{1-s_{j2}} s_{01} M_1.
\]

Then the FOC w.r.t. \( p_{j1}^m \) is:

\[
s_{j1} - \alpha s_{j1}(1-s_{j1}) + \beta s_{01} s_{j1} \frac{s_{j2}}{1-s_{j2}} + \frac{\beta}{\alpha} s_{01} \frac{1}{(1-s_{j2})^2} \frac{\partial s_{j2}}{\partial s_{j1}} \frac{\partial s_{j1}}{\partial p_{j1}^m} = 0 \tag{A.16}
\]

Using the definition of \( s_{j2} \) and \( s_{j1} \), we get the following partial derivatives:

\[
\frac{\partial s_{j2}}{\partial s_{j1}} = \gamma s_{j2}(1-s_{j2}) \tag{A.17}
\]

\[
\frac{\partial s_{j1}}{\partial p_{j1}^m} = -M_1 \alpha s_{j1}(1-s_{j1}).
\]
Plug equations in (A.17) into equation (A.16) and rearrange the terms, we get the following equation:

$$1 + \beta s_{01} \frac{s_{j2}}{1 - s_{j2}} = (1 - s_{j1})(\alpha p_{j1}^m - \beta M_1 s_{01} \frac{\gamma s_{j2}}{1 - s_{j2}}).$$

(A.18)

Equation (A.18) can be applied to both model A and model B, then by comparing the two sides for the two models, we get:

$$1 + \beta s_{01} \frac{s_{A2}}{1 - s_{A2}} = 1 - s_{A1} \left[\alpha p_{A1}^m - \beta M_1 s_{01} \frac{\gamma s_{A2}}{1 - s_{A2}}\right],$$

(A.19)

Given the assumption that \(n_{A2} > n_{B2}\), it is shown in Step 1 that \(s_{A2} > s_{B2}\). So for the equation (A.19), \(LHS > 1\). Next I show that \(p_{A1}^m > p_{B1}^m\).

Suppose \(p_{A1}^m \leq p_{B1}^m\), then \(s_{A1} > s_{B1}\) because model A not only has the OS network advantage but also lower or equal price than model B. So on the RHS of equation (A.19), we have \(R1 < 1\). In addition, since \(s_{A2} > s_{B2}\), then \(R2 < 1\) in equation (A.19). Thus, the \(RHS < 1\) for equation (A.19), which contradicts the result that \(LHS > 1\).

Therefore, in the first period, the price of model A is higher than model B, \(p_{A1}^m > p_{B1}^m\), under the condition that \(n_{A2} > n_{B2}\) and the \(n_{A1} > n_{B1}\). In the next step, I show that \(n_{A2} > n_{B2}\) holds if \(n_{A1} > n_{B1}\).

- Step 3. This step shows that the manufacturer A keeps its OS network advantage to the second period: \(n_{A2} > n_{B2}\) given \(n_{A1} > n_{B1}\). Suppose on the contradictory that \(n_{A2} \leq n_{B2}\), then according to Step 1 result, \(s_{B2} > s_{A2}\). So \(LHS < 1\) for equation (A.19). Also \(n_{A2} \leq n_{B2}\) implies that \(s_{A1} < s_{B1}\), so \(R1 > 1\) for equation (A.19). In addition \(p_{A1}^m > p_{B1}^m\) and that \(s_{B2} > s_{A2}\) imply that \(R2 > 1\) for equation (A.19). Hence, the \(RHS > 1\), which contradicts that \(LHS < 1\). Thus, \(n_{A2} \leq n_{B2}\) can’t hold, which means that \(n_{A2} > n_{B2}\) holds.

The three steps above shows that when the two manufacturers choose prices, the one with initial OS network advantage choose higher prices in both periods and keeps its advantage in the second period. So I have proved that: (1), \(p_{A1}^m > p_{B1}^m\), for \(t = 1, 2\), and (2), \(n_{A2} > n_{B2}\).

A.3 Competition among Multi-Network Sellers

In this subsection, I use two steps to argue that even with multiple multi-network sellers in the two-period model, the equilibrium prices would still be that the product with the
large network has a lower price than the small network in the first period. Without loss of
generality, suppose there are two symmetric sellers. Notice that, in the static game in the
second period, the two sellers choose same prices for both A and B and have same profit.
The following arguments is for the prices and profits in the first period.

First of all, there exists a symmetric equilibrium in which the two sellers choose the
same prices for the two models. In such a symmetric equilibrium, the large platform will
have a lower price than the small platform in the first period. Suppose instead, that both
sellers choose a higher price for the large platform in the first period. Then given the rival’s
prices, each seller has the incentive to deviate and chooses a lower price for the product
with the large network. Because the deviating seller gain consumers from both the other
seller and the outside option. This makes the deviation profitable. Hence, in a symmetric
equilibrium, the price of the product with the large network is lower than that of the small
network in the first period.

Second, there doesn’t exist an asymmetric equilibrium. Suppose instead, that seller 1’s
price of the large network is lower than the small network, and seller 2’s price of the large
platform is higher than the small network. Then seller 2 could be better off by choosing
the same price as seller 1. Because to get the same profit as seller 1 in the first period,
seller 2 has to choose a very low price for the small platform to compete for customers,
when seller 1 can easily get consumers with a low price on the initially large platform.

Therefore, when there are multiple sellers, they would coordinate on choosing the same
low price for the large network than the small network. Intuitively, the network effect is
not seller specific, so the sellers can’t exclude others from benefiting from the growing OS
networks. As a result, no seller would like to grow the small platform by sacrificing the
first period profit.

B Solve for the Model Markups

The first-order conditions with respect to carrier prices are:

\[ M_t s_{j \text{set}} + M_t \sum_{(k,s')} (p_{j sct}^e - \omega_j p_{j \text{set}}^M + p_{ct}^f - 24 \kappa_{sc} - \lambda_{jsct}) \frac{\partial s_{k s'ct}}{\partial p_{j sct}^c} + \beta \frac{\partial V_c(n_t+1)}{\partial p_{j sct}^c} = 0. \]  

(B.1)

Next the partial derivatives in equation (B.1) will be explicitly derived. First, given the
model market share in equation (21), the partial derivatives of shares with respect to prices
can be derived. For \( \frac{\partial s_{k s'ct}}{\partial p_{j sct}^c} \), the derivatives depend on whether the products are in the same
group or not.

With the assumption that the coefficient matrix determining how consumer character-
istics affect utility is diagonal, consumer i’s price coefficient can be written as:

\[ \alpha_i = \alpha + \phi_i y_i. \]
If \((j', s', c') = (j, s, c)\), then
\[
\frac{\partial s_{j's'c't}}{\partial p_{jst}} = -\frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} (1 - s_{ijst}). \tag{B.2}
\]

If \((j', s', c') \neq (j, s, c)\), then the partial derivative is:
\[
\frac{\partial s_{j's'c't}}{\partial p_{jst}} = \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ij's'c't} s_{ijst}, \tag{B.3}
\]

where \(N_s\) is the number of simulated consumers.

Second, for \(\frac{\partial V_c(n_{t+1})}{\partial p_{jst}}\), we have:
\[
\frac{\partial V_c(n_{t+1})}{\partial p_{jst}} = \sum_{l=1}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{lt+1}} \frac{\partial n_{lt+1}}{\partial p_{jst}}. \tag{B.4}
\]

Given an approximate of the value function form \(\hat{V}_c(n_t)\), then \(\frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}}\) can be derived. And \(\frac{\partial n_{s't+1}}{\partial p_{jst}}\) can be derived from the network size transition rule. It also depends on whether \(s = s'\) or not. The network size transition rule is:
\[
n_{s't+1} = \frac{7}{8} n_{s't} + \frac{M_t}{\text{pop}} \sum_{(j', c') \in \Omega_{s't}} s_{j's'c't}(p_{jt}).
\]

If \(s = s'\), then
\[
\frac{\partial n_{s't+1}}{\partial p_{jst}} = \frac{M_t}{\text{pop}} \sum_{(j', c') \in \Omega_{s't}} \frac{\partial s_{j's'c't}}{\partial p_{jst}}
= \frac{M_t}{\text{pop}} \sum_{(j', c') \in \Omega_{s't}} \left[ -\frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} + \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} s_{ij's'c't} \right].
\]

If \(s \neq s'\), then the partial derivative of OS shares with respect to carrier price is:
\[
\frac{\partial n_{s't+1}}{\partial p_{jst}} = \frac{M_t}{\text{pop}} \frac{1}{N_s} \sum_{(j', c') \in \Omega_{s't}} \alpha_i s_{ijst} s_{ij's'c't}.
\]
Then we have:

\[
\frac{\partial V_c(n_{t+1})}{\partial p_{jstc}} = \sum_{s' = 1}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \frac{\partial n_{s't+1}}{\partial p_{jstc}} = \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \frac{M_t}{\text{pop}} \sum_{(j', c') \in \Omega_{s't}} \left[-\frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} + \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} s_{ij's'c't} \right] + \sum_{l \neq s} \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \frac{M_t}{\text{pop} N_s} \sum_{(j', c') \in \Omega_{s't}} \alpha_i s_{ijst} s_{ij's'c't} \]

\[
= \frac{M_t}{\text{pop}} S \sum_{s' = 1}^{S} \frac{\partial V_c(n_{t+1})}{\partial n_{s't+1}} \left[ \sum_{(j', c') \in \Omega_{s't}} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} s_{ij's'c't} \right] - \frac{\partial V_c(n_{t+1})}{\partial n_{st+1}} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} \]

(B.5)

Then the equations (B.2)-(B.5) can be plugged back into equation (B.1). Define the markup as \( m_{jstc} = p_{jstc} - \omega_j p_{jstc} + p_{ct} - \kappa_{sc} - \lambda_{jstc} \). Then plug the derivatives into the FOC, we have:

\[
s_{jstc} + \sum_{(j', c') \in \Omega_{ct}} m_{j's'c't} \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} d_{ij's'c't} - \frac{1}{N_s} \sum_{i=1}^{N_s} \alpha_i s_{ijst} m_{jstc} - \frac{\beta^d}{M_t} \frac{\partial V_c(n_{t+1})}{\partial p_{jstc}} = 0.
\]

(B.6)

Since the individual market shares can be calculated from the demand model with estimated parameters \( \hat{\theta}_d \) and the carriers’ value functions are parametrically approximated, the only unknowns are the markups \( m_{jstc} \)'s in equation (B.6), given the parameters. There are \( J_t \) equations and \( J_t \) unknowns. The equations are linear in the unknowns. So the markups \( (m_{jstc})'s \) can be solved using matrix inversion. Then we can calculate the unobserved cost shock:

\[
\lambda_{jstc} = p_{jstc} - \omega_j p_{jstc} + f_{ct} - \kappa_{sc} - m_{jstc}.
\]

(B.7)

C The Inversion of Carrier-OS Shares to Carrier-OS Unobserved Quality

The goal is to prove that the observed vector carrier-OS sales market shares \( s_{stc} \)'s uniquely determine a vector of the carrier-OS unobserved quality \( \xi_{stc} \). This proof is similar to the proof in [Berry 1994]. For notation simplicity, I use a logit model version of the demand model with carrier-OS unobserved quality. This proof goes through for the random coefficient demand model as well.

Consider the consumer utility function:

\[
u_{ijstc} = \delta_{ijstc} + \xi_{stc} + \epsilon_{ijstc},
\]

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where \( \bar{\delta}_{j\text{sc}} \) is the mean observed utility and \( \xi_{\text{sc}} \) is carrier-OS specific unobserved quality. The utility of the outside option is assumed to be \( u_{ij0t} = \epsilon_{0i0t} \). Then given the assumption that \( \epsilon_{ij0t} \)'s follow Type-I extreme value distribution and are i.i.d. across consumers, models, and periods, the market share of carrier-OS group \((c, s)\) in period \(t\) is:

\[
s_{\text{sc}} = \mathcal{J}(\xi_t) = \sum_{j \in \Omega_{\text{sc}}} \frac{e^{(\delta_{j\text{sc}} + \xi_{\text{sc}})}}{1 + \sum_{(j',s',c') \in \Omega_t} e^{(\delta_{j's'c't} + \xi_{j's'c't})}}.
\]

(C.1)

I shall prove that there is a unique \( \xi_t = \mathcal{J}^{-1}(s_t^{\text{data}}) \in \mathbb{R}^K \) for any fixed finite \( \bar{\delta} \) vector, where \( s_t^{\text{data}} \) is the observed vector of carrier-OS market shares in period \(t\). The equation (C.1) has the following properties. (1), \( \partial \mathcal{J}_{\text{sc}} / \partial \xi_{\text{sc}} > 0 \); (2) \( \partial \mathcal{J}_{\text{sc}} / \partial \xi_{j's'c't} < 0 \), if \((s, c) \neq (s', c')\); and (3) \( \mathcal{J}_{\text{sc}} \) approaches to zero as \( \xi_{\text{sc}} \) goes to \(-\infty\) and it approaches to 1 as \( \xi_{\text{sc}} \) goes to \(\infty\).

Define the element-by-element inverse, \( r_{\text{sc}}(\xi_t, s_{\text{sc}}^{\text{data}}) \). This function is defined as the unobserved quality value for carrier-OS \((s, c)\) such that the model predicted carrier-OS share \( \mathcal{J}_{\text{sc}} \) equals the observed share \( s_{\text{sc}}^{\text{data}} \). That is:

\[
s_{\text{sc}}^{\text{data}} = \mathcal{J}(r_{\text{sc}}((\xi_t, s_{\text{sc}}^{\text{data}})), \xi_{\text{sc}}; \bar{\delta}_t).
\]

(C.2)

Since the market share function \( \mathcal{J} \) is continuously differentiable and satisfy the three properties above, the function \( r_{\text{sc}}(\xi_t, s_{\text{sc}}^{\text{data}}) \) is well defined and differentiable. In particular, \( r_{\text{sc}} \) is strictly increasing in \( \xi_{j's'c't} \) for any \((s', c') \neq (s, c)\) and doesn’t depend on \( \xi_{\text{sc}} \). So a vector \( \xi_t \) solves equations in (C.1) if and only if it is a fixed point of the element-by-element inverse: \( \xi_t = r(\xi_t, d^{\text{data}}) \). Next, I first show existence of fixed point of \( r(\xi_t, d^{\text{data}}) \), then show the uniqueness of the fixed point.

First, \( r(\xi_t, d^{\text{data}}) \) has a lower bound \( \xi \). The lower bound for the \((s, c)\)th element is the value of \( r_{\text{sc}}(\xi'_t, d^{\text{data}}) \), with \( \xi'_t = -\infty \), for all \((s', c') \neq (s, c)\). Define \( \xi \) as the smallest value across the products of these lower bounds. Note that there is no upper bound for \( r_{\text{sc}} \), but a slight variant of the element-by-element inverse has.

**Lemma 2.** There is a value \( \bar{\xi} \), with the property that if one element of \( \xi_t \), say \( \xi_{\text{sc}} \) is greater than \( \bar{\xi} \), then there is another carrier-OS pair \((s', c')\) such that \( r_{s'c't}(\xi_t, s_t^{\text{data}}) < \xi_{s'c't} \).

**Proof.** To construct \( \bar{\xi}_t \), set \( \xi_{s'c't} = -\infty \), for all \((s', c') \neq (s, c)\). Then define \( \bar{\xi}_{\text{sc}} \) as the value of \( \xi_{\text{sc}} \) that set the outside option market share \( \mathcal{J}_{0t}(\xi_{\text{sc}}, \xi_{\text{sc}}) = s_{01}^{\text{data}} \). Define \( \bar{\xi} \) as any value greater than the maximum of the \( \xi_{\text{sc}} \). Then, if for the vector \( \xi_t \), there is an element \((s, c)\) such that \( \xi_{\text{sc}} > \bar{\xi}_{\text{sc}} \), then \( \mathcal{J}_{0t}(\xi_{\text{sc}}, \xi_{\text{sc}}) < s_{01}^{\text{data}} \), which implies that \( \sum_{s'c'} \mathcal{J}_{s'c't}(\xi_t; \bar{\delta}_t) > \sum_{s'c'} s_{01}^{\text{data}} \), so there is at least one carrier-OS pair \((s', c')\) such that \( \mathcal{J}_{s'c't}(\xi_t; \bar{\delta}_t) > s_{01}^{\text{data}} \). Then for this pair \((s', c')\), \( r_{s'c't}(\xi_t, s_{\text{sc}}^{\text{data}}) < \xi_{s'c't} \).

Now define a new function which is a truncated version of \( r_{\text{sc}} \): \( \tilde{r}_{\text{sc}}(\xi_t, s_{\text{sc}}^{\text{data}}) = \min \{ r_{\text{sc}}(\xi_t, s_{\text{sc}}^{\text{data}}), \bar{\xi} \} \). Then \( \tilde{r} \) is a continuous function which maps \( [\bar{\xi}, \bar{\xi}]^K \) into itself. Then by Brouwer’s fixed-point theorem, \( \tilde{r} \) is has a fixed point \( \xi^* \). By the definition of \( \bar{\xi} \) and \( \xi^* \) can’t have a value
at the upper bound, so $\xi^*$ is in the interior of $[\xi, \bar{\xi}]^K$. This implies that $\xi^*$ is also a fixed point of the function $r(\xi_t, s_t^{data})$. So there exists a fixed point for the element-by-element inverse function.

Next I show the uniqueness of the fixed point. One sufficient condition for uniqueness is the diagonal dominance of the Jacobian matrix of the inverse functions. That is: $\sum_{(s', c') \neq (s, c)} |\partial r_{sc}/\partial \xi_{s'c't} | < |\partial r_{sc}/\partial \xi_{set} |$. By the implicit function theorem on equation (C.2), we have:

$$\partial r_{sc}/\partial \xi_{s'c't} = -[\partial \mathcal{J}_{set}/\partial \xi_{s'c't}]/[\partial \mathcal{J}_{set}/\partial \xi_{set}],$$

which implies that $\partial r_{sc}/\partial \xi_{set} = 1$. Then the sum is:

$$\sum_{(s', c') \neq (s, c)} |\partial r_{sc}/\partial \xi_{s'c't} | = \frac{1}{|\partial \mathcal{J}_{set}/\partial \xi_{set}|} \sum_{(s', c') \neq (s, c)} |\partial \mathcal{J}_{set}/\partial \xi_{s'c't} |. \tag{C.3}$$

Note that increasing all the unobserved quality levels (including the outside option $\xi_{0t}$) by the same amount wouldn’t change any market share. That is:

$$\sum_{s'c'=0}^{K} \partial \mathcal{J}_{set}/\partial \xi_{s'c't} = 0$$

Then it implies that:

$$\partial \mathcal{J}_{set}/\partial \xi_{set} = -[\partial \mathcal{J}_{set}/\partial \xi_{0t} + \sum_{s'c' \neq (sc)} \partial \mathcal{J}_{set}/\partial \xi_{s'c't}].$$

Since all terms in the right hand side are strictly negative, so

$$|\partial \mathcal{J}_{set}/\partial \xi_{set}| > \sum_{s'c' \neq (sc)} |\partial \mathcal{J}_{set}/\partial \xi_{s'c't} |.$$

Then the sum in equation (C.3) is:

$$\sum_{(s', c') \neq (s, c)} |\partial r_{sc}/\partial \xi_{s'c't} | < 1 = |\partial r_{sc}/\partial \xi_{set}|.$$

Hence the sufficient condition for uniqueness is satisfied. Therefore, the element-by-element inverse function has unique fixed point $\xi^*$, which is the solution of the market share inversion function.
D Computation Appendix

D.1 Simulating the Income Levels of Consumers

In the estimation, I simulate the income levels $Y_{it}$ of 300 individuals for each period $t$ based on household income distributions using yearly Current Population Survey (CPS) data. Each individual’s income level is assumed to independently follow a lognormal distribution, whose mean and standard deviation are from the CPS data.

I normalize the simulated individuals’ income levels by the log mean income in 2011, such that the mean of normalized log income levels is zero in 2011. That is, in the estimation, the log income level of consumer $i$ in year is $y_{it} = \log(Y_{it}) - \mu_{2011}$, where $\mu_{2011}$ is the log of mean income in 2011 and $Y_{it}$ is $i$’s income level in dollars. The income levels in 2012 and 2013 are also normalized by $\mu_{2011}$. In this way, the normalized log-income data keep the pattern of growing household income over time.

D.2 Number of Monte Carlo Draws to Approximate Integration over Shocks

In the estimation, the MPEC constraints are the carriers’ Bellman equations. These constraints are used so that the value functions are well approximated.

$$V_c(n_t) = E_{\xi, \lambda}[\max_{p_{jsct}(\xi_t, \lambda_t), (j,s) \in \Omega_{ct}} \left\{ \pi_{ct}(p^c_t, \xi_t, \lambda_t) + \beta d V_c(n_{t+1}(n_t, p^c_t(\xi_t, \lambda_t))) \right\}]$$

To calculate the expectation over $(\xi, \lambda)$ on the right hand side, I simulate $R$ vectors of quality shocks and cost shocks $(\xi^r, \lambda^r)$, $r = 1, ..., R$ to approximate the integration of discounted profits over $(\xi, \lambda)$.

$$\hat{V}_c^R(n_t) \approx \frac{1}{R} \sum_{r=1}^{R} \left[ \max_{p_{jsct}(\xi^r, \lambda^r), (j,s) \in \Omega_{ct}} \left\{ \pi_{ct}(p^c_t, \xi^r, \lambda^r) + \beta d V_c(n_{t+1}(n_t, p^c_t(\xi^r, \lambda_t))) \right\} \right]$$

Given a set of parameter values, the algorithm solves the equilibrium prices of the carriers’ dynamic pricing game for each $(\xi^r, \lambda^r)$ and use the average over the draws to approximate the value functions.

In the estimation, I use 50 draws ($R=50$) due to computation burden. But since the dimension of $(\xi, \lambda)$ is more than 200, I need to check whether $R = 50$ is generating large error in the value function. To check this, I simulate more draws ($R = 100, 300, 1000$) and use the estimation results to re-calculate the $\hat{V}_c^R(n_t)$’s for all carriers in all periods. I find that when $R$ increases from 50 to 1000, $\hat{V}_c^R$ changes by 1.35% on average with a standard deviation of 1.61%.
D.3 Solving the Carriers’ FOC’s

For each simulated shock \((\xi^r, \lambda^r)\), the algorithm for solving the equilibrium prices in period \(t\) follows four steps.

1. Guess an initial price vector for all models, \(p^{0r}\). With the price, I calculate the consumers’ choice probabilities \(s_{ijset}^{0r}\) for all models and the next period state variable \(n_{t+1}(n_t, p^{0r})\) using \(p^{0r}\) and \(n_t\).

2. Calculate the markups \(m_{jset}^{0r}\) using first-order conditions (B.6) for all models, which are linear in the markup \(m_{jset}\)’s.

3. Update the price for all models simultaneously. The new price for model \((j, s, c, t)\) is:

\[
p_{jset}^{1r} = m_{jset}^{0r} + c_{jset}(\theta_s, \lambda^r).
\]

4. Compare \(p_{jset}^{1r}\) with \(p_{jset}^{0r}\). If the distance between the two prices, in \(L^1\) norm, are larger than \(10^{-3}\), then repeat step 2-4. If the distance is smaller than \(a0^{-3}\), then the prices are solved.

E Plots of the Carriers’ Value Functions

Each carrier’s value function is a function of the 4 state variables, the 4 OS market shares. Figure 5 and 6 are the plots of the value functions. The market shares of iOS and Android are varying in Figure 5. The market shares of Blackberry and Windows Phone are fixed at the July 2013 shares. In Figure 6, the market shares of Blackberry and Windows Phone are varying. The market shares of iOS and Android are fixed at the July 2013 shares.
Figure 5: Value Functions (iOS and Android Market Shares)
Figure 6: Value Functions (Blackberry and Windows Phone Market Shares)