Decompositions of Productivity Growth into Sectoral Effects: Some Puzzles Explained

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Discussion Paper 14-10,
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Abstract

An earlier paper by Diewert (2013) provided some new decompositions of economy wide labour productivity growth and Total Factor Productivity (TFP) growth into sectoral effects. The economy wide labour productivity growth rate turned out to depend on the sectoral labour productivity growth rates, real output price changes and changes in sectoral labour input shares. A puzzle is that empirically, the real output price change effects, when aggregated across industries, did not matter much. The economy wide TFP growth decomposition into sectoral explanatory factors turned out to depend on the sectoral TFP productivity growth rates, real output and input price changes and changes in sectoral aggregate input shares. The puzzle with this decomposition is that empirically all of these price change effects and input share effects did not matter much when they were aggregated over sectors; only the sectoral TFP growth rates contributed significantly to overall TFP growth. The present paper explains these puzzles.

Journal of Economic Literature Classification Numbers

C43, C82, D24.

Key Words

Total Factor Productivity, labour productivity, index numbers, sectoral contributions to growth.

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1. Introduction

Many analysts want to decompose aggregate labour productivity growth (or aggregate Multifactor growth) into sectoral effects so that the industry sources of productivity growth can be determined. However, it turns out that measures of economy wide productivity change cannot be obtained as a simple weighted sum of the corresponding industry measures; Denison (1962) showed that changes in the allocation of resources across the industries also played an important role in contributing to aggregate labour productivity change. However, the Denison decomposition of aggregate labour productivity change into explanatory effects ignored the role of changes in industry output prices. In an important paper, Tang and Wang (2004; 426) extended the Denison decomposition to take into account changes in real output prices in their decomposition of economy wide labour productivity into explanatory contribution effects. However, Tang and Wang combined the effects of changes in real output prices with the effects of changes in input shares and so Diewert (2013) extended their to provide a decomposition of aggregate labour productivity growth into separate contribution terms due to sectoral productivity growth, changes in input shares and changes in real output prices. In section 2 below, we present Diewert's decomposition of aggregate labour productivity growth into explanatory sectoral factors. Diewert (2013) further extended the Tang and Wang methodology in order to provide a decomposition of economy wide Total Factor Productivity (or Multifactor Productivity) growth into industry explanatory factors.[2] Section 3 presents this generalization.

However, in Diewert's (2013) empirical examples using Australian data which illustrated his new decompositions, he found that many of the industry explanatory factors, when summed over industries, were very small. In sections 4 and 5 below, we will explain why these somewhat puzzling results hold. Section 4 explains the labour productivity puzzles while section 5 explains the Total Factor Productivity growth decomposition puzzles.

2. Diewert's Aggregate Labour Productivity Growth Decomposition

Let there be N sectors or industries in the economy.[3] Suppose that for period t = 0,1, the *output* (or real value added or volume) of sector n is Y_n^t with corresponding period t *price* P_n^t [4] and *labour input* L_n^t for n = 1,...,N. We assume that these labour inputs can be added across sectors and that the *economy wide labour input* in period t is L^t defined as follows:

$$(1)\quad L^t \equiv \sum_{n=1}^{N} L_n^t \,; \qquad t = 0,1.$$

[2] Other recent papers dealing with decompositions of labour productivity include Diewert (2010), de Avillez (2012), Dumagan (2013), Balk (2014a) (2014b), Dumagan and Balk (2014) and Reinsdorf (2014).

[3] The material in this section and the following one is taken from Diewert (2013).

[4] These industry real output aggregates Y_n^t and the corresponding prices P_n^t are *indexes* of the underlying micro net outputs produced by industry n. The exact functional form for these indexes does not matter for our analysis (with some mild restrictions to be noted later) but we assume the indexes satisfy the property that for each t and n, $P_n^tY_n^t$ equals the industry n nominal value added for period t.

Industry n labour productivity in period t, X_n^t, is defined as industry n real output divided by industry n labour input:

$$(2)\ X_n^t \equiv Y_n^t/L_n^t\ ; \qquad t = 0,1\ ;\ n = 1,...,N.$$

It is not entirely clear how aggregate labour productivity should be defined since the outputs produced by the various industries are measured in heterogeneous units, which are in general, not comparable. Thus it is necessary to weight these heterogeneous outputs by their prices, sum the resulting period t values and then divide by an appropriate *output price index*, say P^t for period t, in order to make the economy wide nominal value of aggregate output comparable in real terms across periods.[5] Thus with an appropriate choice for the aggregate output price index P^t, the period t *economy wide labour productivity*, X^t, is defined as follows:[6]

$$(3)\ X^t \equiv \textstyle\sum_{n=1}^N P_n^tY_n^t/P^tL^t = \sum_{n=1}^N (P_n^t/P^t)Y_n^t/L^t = \sum_{n=1}^N p_n^tY_n^t/L^t\ ; \qquad t = 0,1$$

where the *period t industry n real output price*, p_n^t, is defined as the industry t output price P_n^t, divided by the aggregate output price index for period t, P^t; i.e., we have the following definitions:[7]

$$(4)\ p_n^t \equiv P_n^t/P^t\ ; \qquad n = 1,...,N\ ;\ t = 0,1.$$

Using definitions (2) and (3), it is possible to relate the period t aggregate productivity level X^t to the industry productivity levels X_n^t as follows:[8]

$$\begin{aligned}(5)\ X^t &\equiv \textstyle\sum_{n=1}^N P_n^tY_n^t/P^tL^t \\ &= \textstyle\sum_{n=1}^N p_n^t[Y_n^t/L_n^t][L_n^t/L^t] \\ &= \textstyle\sum_{n=1}^N p_n^t s_{Ln}^t X_n^t && \text{using definitions (2)}\end{aligned}$$

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[5] An economy wide sequence of real value added output levels Y^t (and the corresponding value added output price levels P^t) is generally defined by aggregating individual industry outputs (and intermediate inputs entered with negative signs) into economy wide output levels, using bilateral Laspeyres, Paasche, Fisher (1922) or Törnqvist indexes. These indexes use the industry P_n^t and Y_n^t for the two periods being compared and either fixed base or chained aggregate price and quantities (the P^t and Y^t) are generated for each period in the sample. If Fisher indexes are used as the basic formula, then for each year t, the product of the Fisher aggregate price and quantity levels, P^tY^t, will equal the sum of industry price times volumes for year t, $\sum_{n=1}^N P_n^tY_n^t$, which in turn is equal to market sector nominal value added for year t. Thus the year t aggregate quantity levels Y^t are equal to $\sum_{n=1}^N P_n^tY_n^t/P^t$ and so equation (3) can be rewritten as $X^t = Y^t/L^t$.

[6] This follows the methodological approach taken by Tang and Wang (2004; 425). As noted in the previous footnote, the aggregate output price index P^t can be formed by applying an index number formula to the industry output prices (or value added deflators) for period t, $(P_1^t,...,P_N^t)$, and the corresponding real output quantities (or industry real value added estimates) for period t, $(Y_1^t,...,Y_N^t)$. The application of chained superlative indexes would be appropriate in this context but again, the exact form of index does not matter for our analysis as long as P^tY^t equals period t aggregate nominal value added.

[7] These definitions follow those of Tang and Wang (2004; 425).

[8] Equation (5) corresponds to equation (2) in Tang and Wang (2004; 426).

where the *share of labour used by industry n in period t*, $s_{Ln}{}^t$, is defined in the obvious way as follows:

$$(6)\ s_{Ln}{}^t \equiv L_n{}^t/L^t\ ; \qquad n = 1,...,N\ ;\ t = 0,1.$$

Thus aggregate labour productivity for the economy in period t is a weighted sum of the sectoral labour productivities where the weight for industry n is $p_n{}^t$, the real output price for industry n in period t, times $s_{Ln}{}^t$, the share of labour used by industry n in period t.

Up to this point, the above analysis follows that of Tang and Wang (2004; 425-426) but now we follow Diewert's (2104) approach.[9]

First, Diewert defined the *value added or output share of industry n in total value added for period t*, $s_{Yn}{}^t$, as follows:

$$\begin{aligned}(7)\ s_{Yn}{}^t &\equiv P_n{}^tY_n{}^t/\textstyle\sum_{i=1}^N P_i{}^tY_i{}^t && t = 0,1\ ;\ n = 1,...,N \\ &= p_n{}^tY_n{}^t/\textstyle\sum_{i=1}^N p_i{}^tY_i{}^t && \text{using definitions (4).}\end{aligned}$$

Diewert noted that the product of the sector n real output price times its labour share in period t, $p_n{}^ts_{Ln}{}^t$, with the sector n labour productivity in period t, $X_n{}^t$, equals the following expression:

$$\begin{aligned}(8)\ p_n{}^ts_{Ln}{}^tX_n{}^t &= p_n{}^t[L_n{}^t/L^t][Y_n{}^t/L_n{}^t]\ ; && t = 0,1\ ;\ n = 1,...,N \\ &= p_n{}^tY_n{}^t/L^t\ .\end{aligned}$$

Using definition (3) and equation (5), *aggregate labour productivity growth* (plus 1) going from period 0 to 1, X^1/X^0, is equal to:

$$\begin{aligned}(9)\ X^1/X^0 &= \textstyle\sum_{n=1}^N p_n{}^1\, s_{Ln}{}^1\, X_n{}^1/\textstyle\sum_{n=1}^N p_n{}^0s_{Ln}{}^0X_n{}^0 \\ &= \textstyle\sum_{n=1}^N [p_n{}^1/p_n{}^0][s_{Ln}{}^1/s_{Ln}{}^0][X_n{}^1/X_n{}^0][p_n{}^0Y_n{}^0/L^0]/\textstyle\sum_{i=1}^N [p_i{}^0Y_i{}^0/L^0] && \text{using (8)} \\ &= \textstyle\sum_{n=1}^N [p_n{}^1/p_n{}^0][s_{Ln}{}^1/s_{Ln}{}^0][X_n{}^1/X_n{}^0]\ s_{Yn}{}^0 && \text{using definitions (7).}\end{aligned}$$

Thus overall economy wide labour productivity growth, X^1/X^0, is an output (or value added) share weighted average of three *growth factors* associated with industry n. The three growth factors are:

- $X_n{}^1/X_n{}^0$, (one plus) the rate of growth in the labour productivity of industry n;
- $s_{Ln}{}^1/s_{Ln}{}^0$, (one plus) the rate of growth in the share of labour being utilized by industry n and

[9] Tang and Wang (2004; 425-426) *combined* the effects of the real price for industry n for period t, $p_n{}^t$, with the industry n labour share $s_{Ln}{}^t$ for period t by defining the relative size of industry n in period t, $s_n{}^t$, as the product of $p_n{}^t$ and $s_{Ln}{}^t$; i.e., they defined the industry n *weight* in period t as $s_n{}^t \equiv p_n{}^ts_{Ln}{}^t$. They then rewrote equation (5), $X^t = \sum_{n=1}^N p_n{}^ts_{Ln}{}^tX_n{}^t$, as $X^t = \sum_{n=1}^N s_n{}^tX_n{}^t$. Thus their analysis of the effects of the changes in the weights $s_n{}^t$ did not isolate the separate effects of changes in industry real output prices and industry labour input shares.

- $p_n^1/p_n^0 = [P_n^1/P_n^0]/[P^1/P^0]$ which is (one plus) the rate of growth in the real output price of industry n.

Thus in looking at the contribution of industry n to overall (one plus) labour productivity growth, start out with a straightforward share weighted contribution factor, $s_{Yn}^0[X_n^1/X_n^0]$, which is the period 0 output or value added share of industry n in period 0, s_{Yn}^0, times the industry n rate of labour productivity growth (plus one), X_n^1/X_n^0. This straightforward contribution factor for industry n will be augmented if real output price growth for industry n is positive (if p_n^1/p_n^0 is greater than one) and if the share of labour used by industry n is growing (if s_{Ln}^1/s_{Ln}^0 is greater than one). The decomposition of overall labour productivity growth given by the last line of (9) seems to be intuitively reasonable and fairly simple as opposed to the decomposition obtained by Tang and Wang (2004; 426) which does not separately distinguish the effects of real output price change from changes in the industry's labour share.

The puzzle with the above decomposition (9) is that empirically, it appears that the effects of real output price change, when aggregated over industries, are insignificant; i.e., Diewert (2013) found that for the case of Australia, the following decomposition of aggregate labour productivity growth provided a close approximation to the exact decomposition (9):

$$(10)\ X^1/X^0 \approx \sum_{n=1}^N [s_{Ln}^1/s_{Ln}^0][X_n^1/X_n^0]\, s_{Yn}^0.$$

Note that the real output price change augmentation factors (the p_n^1/p_n^0) are not present on the right hand side of (10). Thus for the case of Australia, only the labour share augmentation factors s_{Ln}^1/s_{Ln}^0 and the industry labour productivity growth rates X_n^1/X_n^0 proved to be significant determinants of overall labour productivity growth.[10] In section 4 below, we will show why this result holds in general.

3. Diewert's Aggregate Multifactor Productivity Growth Decomposition

As in the previous section, let there be N sectors or industries in the economy and again suppose that for period t = 0,1, the *output* (or real value added or volume) of sector n is Y_n^t with corresponding period t *price* P_n^t. However, it is now assumed that each sector uses many inputs and index number techniques are used to form industry input aggregates Z_n^t with corresponding aggregate industry input prices W_n^t for n = 1,...,N and t = 0,1.[11]

[10] This does not mean that the industry real output price augmentation factors $[p_n^1/p_n^0]$ were all close to one (they were not for Diewert's Australian data); it just means that when aggregating over industries, the factors which were greater than one are balanced by factors less than one so that the effects of real output price change cancel out when aggregating over industries.

[11] These industry input aggregates Z_n^t and the corresponding price indexes W_n^t are *indexes* of the underlying micro inputs utilized by industry n. The exact functional form for these indexes does not matter for our analysis but it is assumed that the indexes satisfy the property that for each t and n, $W_n^tZ_n^t$ equals the industry n input cost for period t.

Industry n Total Factor Productivity (*TFP*) *in period t*, X_n^t, is defined as industry n real output Y_n^t divided by industry n real input Z_n^t:

$$(11)\ X_n^t \equiv Y_n^t/Z_n^t\ ; \qquad t = 0,1\ ;\ n = 1,...,N.$$

As in section 2, *economy wide real output in period t*, Y^t, is defined as total value added divided by the economy wide output price index P^t. Thus for each period t, the following identity holds:[12]

$$(12)\ Y^t = \sum_{n=1}^N P_n^tY_n^t/P^t = \sum_{n=1}^N p_n^tY_n^t\ ; \qquad t = 0,1$$

where the *period t industry n real output price* is defined as $p_n^t \equiv P_n^t/P^t$ for n = 1,...,N and t = 0,1.

Economy wide real input in an analogous manner. Thus an *economy wide input price index* for period t, W^t, is formed in one of two ways:[13]

- By aggregating over all industry micro economic input prices using microeconomic input quantities as weights to form W^t (single stage aggregation of inputs) or
- By aggregating the industry aggregate input prices W_n^t (with corresponding input quantities or volumes Z_n^t) into the aggregate period t input price index W^t using an appropriate index number formula (two stage aggregation of inputs).

Economy wide real input in period t, Z^t, is defined as economy wide input cost divided by the economy wide input price index W^t:[14]

$$(13)\ Z^t = \sum_{n=1}^N W_n^tZ_n^t/W^t = \sum_{n=1}^N w_n^tZ_n^t\ ; \qquad t = 0,1$$

where the *period t industry n real input price* is defined as:

$$(14)\ w_n^t \equiv W_n^t/W^t; \qquad n = 1,...,N \text{ and } t = 0,1.$$

The *economy wide level of TFP* (or *MFP*) *in period t*, X^t, is defined as aggregate real output divided by aggregate real input:

$$(15)\ X^t \equiv Y^t/Z^t\ ; \qquad t = 0,1.$$

[12] As in sections 2, $P_n^tY_n^t$ is nominal industry n value added in period t. Industry n period t real value Y_n^t added is defined as period t nominal industry n value added deflated by the industry n value added price index P_n^t. It need not be the case that $P_n^tY_n^t$ is equal to $W_n^tZ_n^t$; i.e., it is not necessary that value added equal input cost for each industry for each time period.

[13] In either case, it is assumed that the product of the total economy input price index for period t, W^t, times the corresponding aggregate input quantity or volume index, Z^t, is equal to total economy nominal input cost. If Laspeyres or Paasche price indexes are used throughout, then the two stage and single stage input aggregates will coincide. If superlative indexes are used throughout, then the two stage and single stage input aggregates will approximate each other closely using annual data; see Diewert (1978).

[14] Note that W^tZ^t equals period t total economy input cost for each t.

Denote the output share of industry n in period t, $s_{Yn}{}^t$, by (7) again and define the *input share of industry n in economy wide cost in period t*, $s_{Zn}{}^t$, as follows:

$$(16)\ s_{Zn}{}^t \equiv W_n{}^tZ_n{}^t/\textstyle\sum_{i=1}^N W_i{}^tZ_i{}^t \qquad t = 0,1\ ;\ n = 1,\ldots,N$$
$$= w_n{}^tZ_n{}^t/\textstyle\sum_{i=1}^N w_i{}^tZ_i{}^t$$

where the second equation in (16) follows from the definitions $w_n{}^t \equiv W_n{}^t/W^t$.

Substitute (12) and (13) into definition (15) and the following expression for the economy wide level of TFP in period t is obtained:

$$(17)\ X^t = \textstyle\sum_{n=1}^N p_n{}^tY_n{}^t/\sum_{i=1}^N w_i{}^tZ_i{}^t \qquad t = 0,1$$
$$= \textstyle\sum_{n=1}^N p_n{}^t(Y_n{}^t/Z_n{}^t)Z_n{}^t/\sum_{i=1}^N w_i{}^tZ_i{}^t$$
$$= \textstyle\sum_{n=1}^N (p_n{}^t/w_n{}^t)X_n{}^tw_n{}^tZ_n{}^t/\sum_{i=1}^N w_i{}^tZ_i{}^t \qquad \text{using (11)}$$
$$= \textstyle\sum_{n=1}^N (p_n{}^t/w_n{}^t)X_n{}^ts_{Zn}{}^t \qquad \text{using (16).}$$

Using (17), *aggregate TFP growth* (plus 1) going from period 0 to 1, X^1/X^0, is equal to:

$$(18)\ X^1/X^0 = \textstyle\sum_{n=1}^N (p_n{}^1/w_n{}^1)X_n{}^1s_{Zn}{}^1/\sum_{i=1}^N (p_i{}^0/w_i{}^0)X_i{}^0s_{Zi}{}^0$$
$$= \textstyle\sum_{n=1}^N (p_n{}^1/p_n{}^0)(w_n{}^0/w_n{}^1)(X_n{}^1/X_n{}^0)(s_{Zn}{}^1/s_{Zn}{}^0)(p_n{}^0/w_n{}^0)X_n{}^0s_{Zn}{}^0/\sum_{i=1}^N (p_i{}^0/w_i{}^0)X_i{}^0s_{Zi}{}^0$$
$$= \textstyle\sum_{n=1}^N s_{Yn}{}^0(p_n{}^1/p_n{}^0)(w_n{}^0/w_n{}^1)(s_{Zn}{}^1/s_{Zn}{}^0)(X_n{}^1/X_n{}^0).$$

The last equation in (18) follows from the following equations for n = 1,...,N:

$$(19)\ (p_n{}^0/w_n{}^0)X_n{}^0s_{Zn}{}^0 = (p_n{}^0/w_n{}^0)(Y_n{}^0/Z_n{}^0)(w_n{}^0Z_n{}^0/\textstyle\sum_{i=1}^N w_i{}^0Z_i{}^0) \qquad \text{using (11) and (16)}$$
$$= p_n{}^0Y_n{}^0/\textstyle\sum_{i=1}^N w_i{}^0Z_i{}^0.$$

Thus one plus economy wide TFP growth, X^1/X^0, is equal to an output share weighted average (with the base period weights $s_{Yn}{}^0$) of one plus the industry TFP growth rates, times an augmentation factor, which is the product $(p_n{}^1/p_n{}^0)(w_n{}^0/w_n{}^1)(s_{Zn}{}^1/s_{Zn}{}^0)$. Thus formula (18) is very similar to the previous labour productivity growth formula (9) except that now there is an additional multiplicative augmentation factor, which is $w_n{}^0/w_n{}^1$, the *reciprocal* of one plus the rate of growth of real input prices for sector n.

Diewert's empirical results for Australia indicated that while individual industry terms for any of the above contribution terms on the right hand side of (18) can be quite significant, the aggregate effects of the real output and input price augmentation factors as well as the labour input share factors were close to zero; i.e., for Diewert's Australian data, it appeared that the following decomposition of aggregate Multifactor productivity growth provided a close approximation to the exact decomposition given by (18):

$$(20)\ X^1/X^0 \approx \textstyle\sum_{n=1}^N s_{Yn}{}^0\,[X_n{}^1/X_n{}^0].$$

Note that the real output price change augmentation factors (the $p_n{}^1/p_n{}^0$), the reciprocal real input price change augmentation factors (the $w_n{}^0/w_n{}^1$) and the input share augmentation factors (the s_{Zn}^1/s_{Zn}^0) are not present on the right hand side of (20). Thus for the case of Australia, only the industry multifactor productivity growth rates X_n^1/X_n^0 proved to be significant determinants of overall Multifactor productivity growth.[15] In section 5 below, we will show why this somewhat puzzling result holds in general.[16]

4. The Labour Productivity Growth Decomposition Puzzle Explained

We want to show that the exact identity (9) can be approximated by the right hand side of (10) in section 2 above.

The starting point in the derivation of the approximate identity (10) is to note that if the bilateral index number formula that is used to form the period t economy wide output aggregates and the corresponding aggregate output price levels is the direct or implicit Laspeyres, Paasche, Fisher, Törnqvist or any other known superlative index number formula, then it can be shown that the output price levels generated by Cobb-Douglas index number formula will approximate the price levels corresponding to any of the above formulae to the first order around an equal price and quantity point.[17] Recall that the aggregate output price index levels for periods 0 and 1 were defined as P^0 and P^1. Thus using this above approximation result, it can be seen that to the accuracy of a first order Taylor series approximation, the following approximate equality will hold:

(21) $\ln[P^1/P^0] \approx \Sigma_{n=1}^N s_{Yn}^0 \ln[P_n^1/P_n^0]$.

Subtracting $\ln[P^1/P^0]$ from both sides of (21) and making use of definitions (4) for the industry real output prices p_n^t, it can be seen that (21) becomes the following approximate equality:

(22) $0 \approx \Sigma_{n=1}^N s_{Yn}^0 \ln[p_n^1/p_n^0]$.

From (9), we have X^1/X^0 equal to the weighted arithmetic mean of the N numbers, $[p_n^1/p_n^0][s_{Ln}^1/s_{Ln}^0][X_n^1/X_n^0]$ with weights s_{Yn}^0. Approximate this weighted arithmetic mean by the corresponding weighted geometric mean[18] and take logarithms of the resulting approximate equality. We obtain the following approximate equality:

(23) $\ln[X^1/X^0] \approx \sum_{n=1}^N s_{Yn}^0 \ln[p_n^1/p_n^0] + \sum_{n=1}^N s_{Yn}^0 \ln[s_{Ln}^1/s_{Ln}^0] + \sum_{n=1}^N s_{Yn}^0 \ln[X_n^1/X_n^0]$

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[15] This does not mean that the industry augmentation factors p_n^1/p_n^0, w_n^0/w_n^1 and s_{Zn}^1/s_{Zn}^0 were all close to one (they were not for Diewert's Australian data); it just means that when aggregating over industries, the factors which were greater than one are balanced by factors less than one so that the effects of real output and input price changes and of industry cost share changes cancel out when aggregating over industries.

[16] In order to derive the approximate MFP growth decomposition (18), we will require one additional assumption; namely that the value of inputs equals the value of outputs less intermediate inputs for each industry in period 0.

[17] See Diewert (1978) for a proof of this result. Note that this approximation result also holds for indexes built up in two stages.

[18] It is easy to show that a weighted geometric mean of N positive numbers will approximate the corresponding weighted arithmetic mean of the same numbers to the first order around a point where the numbers are all equal.

$$\approx \sum_{n=1}^{N} s_{Yn}^{0} \ln[s_{Ln}^{1}/s_{Ln}^{0}] + \sum_{n=1}^{N} s_{Yn}^{0} \ln[X_n^{1}/X_n^{0}]$$

where we have used (22) to establish the second approximate equality. Now exponentiate both sides of (23) and approximate the geometric mean on the right hand side by the corresponding arithmetic mean and we obtain the following approximate equality:

$$(24)\ X^1/X^0 \approx \sum_{n=1}^{N} s_{Yn}^{0} [s_{Ln}^{1}/s_{Ln}^{0}][X_n^{1}/X_n^{0}]$$

which is the approximate equality (10). Thus to the accuracy of a first order Taylor series approximation, the industry real output augmentation factors p_n^1/p_n^0, in aggregate, do not contribute to overall labour productivity growth.

5. The Multifactor Productivity Growth Decomposition Puzzle Explained

In order to derive the approximate aggregate MFP growth decomposition defined by (20) in section 3, it is necessary to make another assumption. The extra assumption is that for each industry, the value of primary inputs is exactly equal to the value of outputs less intermediate input costs for the base period. This extra assumption implies that the industry output shares will equal the industry primary input shares so that the following equalities will be satisfied:[19]

$$(25)\ s_{Yn}^{0} = s_{Zn}^{0}; \qquad n = 1,\ldots,N.$$

We also assume that the bilateral index number formula that is used to form the period t economy wide input aggregates is the Laspeyres, Paasche, Fisher, Törnqvist or any other known superlative index number formula. Again, it can be shown that the corresponding Cobb-Douglas index number formula for the price index will approximate any of the above formulae to the first order around an equal price and quantity point. Recall that the aggregate input price index levels for periods 0 and 1 were defined as W^0 and W^1. Thus using this above approximation result, it can be seen that to the accuracy of a first order Taylor series approximation, the following approximate equality will hold:

$$(26)\ \ln[W^1/W^0] \approx \Sigma_{n=1}^{N} s_{Zn}^{0} \ln[W_n^{1}/W_n^{0}].$$

Subtracting $\ln[W^1/W^0]$ from both sides of (26) and making use of definitions (14) for the industry real input prices w_n^t, it can be seen that (26) becomes the following approximate equality:

$$\begin{aligned}(27)\ 0 &\approx \Sigma_{n=1}^{N} s_{Zn}^{0} \ln[w_n^{1}/w_n^{0}] \\ &= \Sigma_{n=1}^{N} s_{Yn}^{0} \ln[w_n^{1}/w_n^{0}] && \text{using assumptions (25)} \\ &= -\Sigma_{n=1}^{N} s_{Yn}^{0} \ln[w_n^{0}/w_n^{1}] \\ &= \Sigma_{n=1}^{N} s_{Yn}^{0} \ln[w_n^{0}/w_n^{1}]\end{aligned}$$

[19] Most national statistical agencies that compute Multifactor Productivity use balancing rates of return in their user costs in order to make the value of inputs equal to the value of outputs; i.e., for most statistical agencies that compute TFP growth, this assumption will be satisfied.

where the last equality follows from the fact that −0 = 0.

From (18), we have X^1/X^0 equal to the weighted arithmetic mean of the N numbers, $(p_n^1/p_n^0)(w_n^0/w_n^1)(s_{Zn}^1/s_{Zn}^0)(X_n^1/X_n^0)$ with weights s_{Yn}^0. Approximate this weighted arithmetic mean by the corresponding weighted geometric mean and take logarithms of the resulting approximate equality. We obtain the following approximate equality:

$$\begin{aligned}(28)\ \ln[X^1/X^0] &\approx \textstyle\sum_{n=1}^N s_{Yn}^0 \ln[p_n^1/p_n^0] + \sum_{n=1}^N s_{Yn}^0 \ln[w_n^0/w_n^1] \\ &\quad + \textstyle\sum_{n=1}^N s_{Yn}^0 \ln[s_{Zn}^1/s_{Zn}^0] + \sum_{n=1}^N s_{Yn}^0 \ln[X_n^1/X_n^0] \\ &\approx \textstyle\sum_{n=1}^N s_{Yn}^0 \ln[s_{Zn}^1/s_{Zn}^0] + \sum_{n=1}^N s_{Yn}^0 \ln[X_n^1/X_n^0] \qquad \text{using (22) and (27).}\end{aligned}$$

Exponentiate both sides of (28) and on the right hand side of the resulting approximate identity, we obtain the product of the weighted geometric means of the s_{Zn}^1/s_{Zn}^0 and of the X_n^1/X_n^0 using the output share weights s_{Yn}^0. Approximate both weighted geometric means by their corresponding weighted arithmetic means and we obtain the following approximate equality:

$$\begin{aligned}(29)\ X^1/X^0 &\approx \{\textstyle\sum_{n=1}^N s_{Yn}^0 [s_{Zn}^1/s_{Zn}^0]\}\{\sum_{n=1}^N s_{Yn}^0 [X_n^1/X_n^0]\} \\ &= \{\textstyle\sum_{n=1}^N s_{Yn}^0 [s_{Yn}^1/s_{Yn}^0]\}\{\sum_{n=1}^N s_{Yn}^0 [X_n^1/X_n^0]\} \qquad \text{using (25)} \\ &= \{\textstyle\sum_{n=1}^N s_{Yn}^1\}\{\sum_{n=1}^N s_{Yn}^0 [X_n^1/X_n^0]\} \\ &= \{\textstyle\sum_{n=1}^N s_{Yn}^0[X_n^1/X_n^0]\}\end{aligned}$$

where the last equation follows since $\sum_{n=1}^N s_{Yn}^1 = 1$; i.e., the period 1 industry shares of total economy value added sum up to unity in period 1.[20] Thus we have derived the very simple approximate TFP growth decomposition into industry explanatory factors defined by (20) in section 3.

6. Conclusion

We have explained the rather puzzling empirical results obtained by Diewert (2013) in his analysis of sectoral factors that contributed to economy wide labour productivity growth and to economy wide TFP or MFP growth. The simplification of the general decomposition formulae given by (9) for labour productivity and by (18) for TFP into (10) and (20) respectively are due to the definitions of the productivity concepts as an index number of aggregate output growth divided by either aggregate labour growth or an index number of aggregate input growth. The use of index numbers to define aggregate output and input leads to a cancellation of output and input price effects in the aggregate decompositions. In the case of the TFP decomposition, the assumption that value added equals input cost for each industry in each period leads to a cancellation of input

[20] Note the role of assumptions (25) in the derivation of (29). The assumption that industry primary input costs equal the corresponding industry net revenues is what enables us to deduce the unimportance of industry cost share reallocations in the aggregate when decomposing aggregate TFP growth into industry explanatory factors. Note that in the case of Labour Productivity growth, we did *not* assume that industry labour input shares were equal to industry net revenue shares and so changes in labour input shares did play an important role in the approximate decomposition of aggregate Labour Productivity growth given by (24) in the previous section.

allocation effects in the aggregate TFP decomposition. Thus if analysts want to focus on industry explanatory factors that do not cancel out in the aggregate, the simplified approximate formulae defined by (10) and (20) are recommended. On the other hand, if analysts want to focus on the individual contributions of each industry to productivity growth (whether the effects cancel out in the aggregate or not), then the more complex exact formulae defined by (9) and (18) are recommended.

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