

Optimal Debt Maturity Structure, Rollover Risk and Strategic Uncertainty*

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Abstract

This paper analyzes debt maturity structure for a borrower in a setting where creditors are faced with strategic uncertainty. In contrast to the existing literature, I examine the effects of strategic uncertainty on the issuance of debt in an environment where face values of debt are determined endogenously and directly affect investors' rollover decisions. I find that strategic uncertainty has a strong effect on the decisions of both the firm and investors, especially at the rollover stage. As strategic uncertainty increases investors are less willing to roll over short-term debt and the borrower shifts towards long-term debt. Finally, I use the model to study the effects of a sudden deterioration in secondary markets and debt overhang issues on the debt maturity structure, face value of debt and default decisions.

Key words: debt overhang, global games, maturity structure, rollover risk, strategic uncertainty

JEL codes: G32, G33

1 Introduction

The recent crisis of 2007-2009 emphasized the importance of refinancing risk.¹ While the crisis started in the financial sector it quickly spilled over to the real sector severely affecting non-financial firms' access to credit. During that time many firms were forced to default being suddenly unable to repay or roll over their existing debt. Figure 1 shows that years 2008-2009 witnessed an unprecedented number of corporate defaults and defaulted debt

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¹Refinancing risk is the risk that a borrower cannot roll over its existing debt.

volume both among financial and non-financial institutions. Moreover, during that time even healthy corporations faced much higher cost of external finance. Figure 2 shows a dramatic increase of spreads on investment grade and speculative grade bonds during that period.

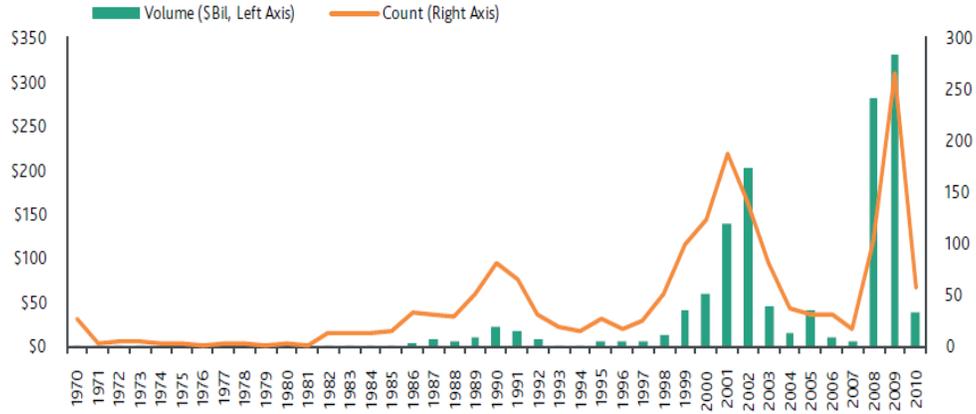


Figure 1: Corporate Default Rates 1970-2010 - source: Moody's Investors Service, 2009, Corporate default and recovery rates, 1920-2008. Moody's Global Credit Policy Special Comment, Annual update

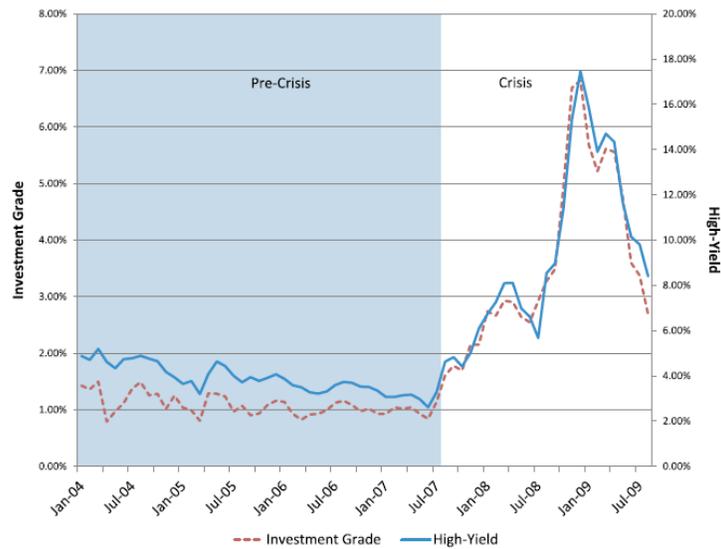


Figure 2: Corporate Bond Spreads During 2004-2009 - source: Almeida et al. (2012)

Following the crisis, a lot of attention was given to the over-reliance of firms and financial institutions on short-term financing as one of reasons for severity of the crisis. This resulted in a fast growing body of literature that provides explanations why the firms and financial institutions exposed themselves to the maturity mismatch (e.g. Brunnermeier and Oehmke [6], Diamond and He [16], Eisenbach [17], or Hubermann and Repullo [24] to name a few). The main goal of this paper is to shed more light on this problem by studying an optimal debt maturity structure in an environment with four important components: (1) firm's default risk depending on the rollover decisions of investors, (2) the short-term investors being subject to strategic uncertainty² when making their rollover decisions, (3) the firm being able to choose face values freely in order to minimize the rollover and default risk, and (4) the face values of debt directly affecting investors decisions. Incorporating all these ingredients into a model leads to a complex but realistic refinancing problem for the borrower that is key for understanding firms' maturity structure choices. To the best of my knowledge this is the first paper that analyzes rollover decisions and debt structure in such environment.

The second goal of this paper is to use a model with the above ingredients to better understand performance of financial and non-financial firms during the most recent recession. More precisely, I use my model to study the effects of a deterioration in secondary market conditions and the role of debt overhang on the firms' investment and financing decisions.³ I find that a shock to secondary market conditions or to profitability of future investment results in a significant increase in both defaults and face values of debt. Therefore, the model provides a potential explanation for the patterns reported in figures 1 and 2 that emphasizes the role of strategic uncertainty. Interestingly, I find that the firm may rely more on short-term finance in the presence of debt overhang compared to the model without debt overhang despite the fact that short-term debt leads to a stronger debt overhang. The reason is that having future investment opportunity decreases the extent of strategic uncertainty among investors.

I consider a problem of a firm that faces a risky but profitable investment opportunity and contemplates financing it by a combination of short-term and long-term debt. The long-term debt is safe (from the firm's perspective) in the sense that it does not require to be rolled over. On the other hand, short-term debt is cheaper since it allows investors to withdraw their loans if they think that the project will pay little, avoiding losses that will be incurred by the long-term creditors. Unfortunately, the short-term debt leads to refinancing risk⁴ and hence to illiquidity risk.

Refinancing risk has its roots in the coordination problem faced by short-term creditors at the time they make their rollover decisions. In particular, the difference between the

²Strategic uncertainty is the uncertainty regarding actions and beliefs of others (see Brandenburger [5] or Morris and Shin [30]).

³Numerous authors have argued that both of these factors are important reasons behind the depth of the 2007-2009 crisis as well as the poor recovery that followed. See Gorton [21] or Gorton and Metrick [22] and its discussion by Shleifer [37] for the role of secondary markets. The issues of debt overhang and its effects on the recovery has been raised in financial press (e.g. "Debt Overhang Slows Down US Economy" Wall Street Journal, July 5th, 2009) as well as in academic literature (Mian, Sufi and Trebbi [28]).

⁴In what follows I use refinancing risk and rollover risk interchangeably.

return from rolling over versus the return from withdrawing funds early depends crucially on the proportion of investors that takes each of these actions. If only a few creditors choose to withdraw, the firm will have to liquidate only a small fraction of its investment and will have enough funds to repay its remaining debt. However, if the fraction of investors that withdraw early is high, the firm will be forced to liquidate a large fraction of its project and will be left with insufficient resources for repaying remaining investors, triggering default. I show that this coordination problem is to large extent dynamic: investors withdraw early because they are afraid of default tomorrow due to large withdrawals today.⁵

The fact that investors face a coordination problem implies that at the heart of their rollover decisions lies strategic uncertainty regarding actions of other investors. In order to model strategic uncertainty we use the insights from the theory of global games. Global games were introduced to the literature by Carlsson and van Damme [8] and Morris and Shin [29] and since then have been applied extensively to model coordination problems among economic agents. In a global game model each agent privately observes an imperfect signal of the future return from firm's investment. The signal plays a dual role. First, it provides information regarding the return from the investment. Second, it provides information regarding signals observed by other agents allowing investors to form beliefs about the actions of others. It is this second feature of the signal that captures the strategic uncertainty.⁶

I find that modeling strategic uncertainty affects significantly the outcome of the model. I show that for any positive amount of short-term debt issued there is a range of returns from investment for which the firm is forced to default due to excessive withdrawals by short-term debt holders, even though the firm is solvent (i.e. if faced with no withdrawals it would be able to repay all of its debt). Moreover, in the presence of strategic uncertainty, the refinancing costs of short-term debt increases making short-term debt more expensive. These two effects discourage the firm from issuing short-term debt. I show that the strength of these effects hinges on the liquidation value of the investment. As liquidation value decreases, strategic uncertainty among investors increases making short-term debt more costly to the firm. As a result the firm increases the proportion of the investment financed with long-term debt compared to the case without strategic uncertainty.

My work is related to the literature on the optimal maturity structure of firm's debt. Early papers related to our work are Bolton and Scharfstein [7], Diamond [13] and [14] or Leland [26] among others. More recent papers include Brunnermeier and Oehmke [6], Diamond and He [16] and Hubermann and Repullo [24]. In contrast to our paper, these models study optimal debt structure in an environment without strategic uncertainty. There are now several papers that study rollover risk in a global game framework. However, none of these papers (with exception of Eisenbach [17]) studies optimal maturity structure, and

⁵For a maturity structure with a large amount of short-term debt there is also a contemporaneous coordination problem where agents withdraw today because they are afraid of default today.

⁶To see this, note that conditional on the observed signal the agent will form beliefs about the distribution of signals received by the others and hence will form a belief about the proportion of agents rolling over. The proportion of agents that decides to roll over becomes an endogenous outcome of the game and will depend now on the face values promised by the firm as well as the return from the investment.

they take the payoffs of the rollover game as exogenous (see Morris and Shin [32] and [33], and Rochet and Vives [36]).

The most related paper to this work is Eisenbach [17] who uses a global game model to study the optimal debt structure choice by banks. However, in his model the rollover decisions are made by fund managers with an exogenously given payoff structure, rather than actual creditors. In contrast, in our model it is the investors who make the rollover decisions and they are directly affected by the face values set by the firm. Therefore, in our framework face values become important choice variables of the firm that affect the rollover outcomes.

The remainder of the paper is structured as follows. In section 2 we present the basic framework. In section 3 we solve the model and characterize the equilibria of the rollover game. In section 4 I investigate the implications of a deterioration in the secondary markets conditions on the optimal debt maturity structure and investors' rollover decisions. Finally, in section 5 I extend the model by adding an extra investment opportunity for the firm that results in a debt overhang problem. Section 6 concludes. All the proofs are relegated to the appendix.

2 The Model

There are three periods $t = 0, 1, 2$ and two types of agents: a single borrower (the firm) and a continuum of lenders (investors) with mass normalized to one. At $t = 0$ the borrower has an investment opportunity that requires one unit of funds. The project is risky and matures at $t = 2$. For simplicity we assume that the borrower has no funds and has to raise everything by issuing short-term and long-term debt. Below I describe in detail the borrower's problem, the nature of the investment opportunity and the investors' problem.

2.1 The Firm

At $t = 0$ the borrower can invest in a project that yields a risky return s , where $s \sim N(\mu, \tau^{-1})$, and requires an investment of one unit of funds. The firm has to raise all the funds by issuing debt. The project can be financed with a combination of short-term and long-term debt. Long-term debt matures at $t = 2$ (with no coupon payment at $t = 1$) and has the same maturity as the project. The short-term debt has to be repaid at $t = 1$. More precisely, when the short-term debt is due, the firm offers short-term creditors a new short term debt contract, who then decide whether to roll over their debt (i.e. purchase newly issued short-term debt) or withdraw their funds (i.e. refuse to purchase new short-term debt). Short-term debt issued at $t = 1$ matures at $t = 2$ at the same time as long-term debt. In what follows I denote by α the fraction of the project financed with short-term debt and $1 - \alpha$ the fraction of the project financed with long-term debt. The face value of the long-term debt is denoted by F_L and of short-term debt is F_S^i , $i = 1, 2$ where the superscript i denotes the period in which short-term debt is to be repaid.

If the firm cannot roll over all of its short-term debt then the early withdrawals are financed by a partial liquidation of the project. The liquidation value of the project is $V < 1$, independent of s . Therefore, if a proportion m of short-term creditors decides to withdraw their funds early and the face value of short-term debt due at $t = 1$ is F_S^1 the firm has to liquidate a fraction x of the initial investment where x satisfies

$$xV = m\alpha F_S^1$$

If $m\alpha F_S^1 > V$, that is if at $t = 1$ the demanded repayments exceed the liquidation value of the entire project the firm defaults. For simplicity, I assume that if the firm goes bankrupt no investors get any compensation.⁷ Therefore, I make the following assumption:

Assumption 1 *If firm defaults all creditors get nothing*

If the firm does not default at $t = 1$ then at the beginning of period $t = 2$ it receives a payoff from the project equal to $(1 - x)s$ - the value of the project scaled down due to the early withdrawals. If the value of the claims exceeds the proceeds from investment the firm defaults and, as explained above, all investors get nothing. Otherwise agents are paid the promised face values, F_S^2 (short-term debt) and F_L (long-term debt). Hence, the profit of the firm is

$$\max \{0, (1 - x)s - \alpha(1 - m)F_S^2 - (1 - \alpha)F_L\}$$

2.2 Investors

Investors are risk-neutral and maximize their total consumption. For simplicity I assume that they do not discount the future.

At $t = 0$ investors choose whether to purchase long-term or short-term debt offered by the firm. Since each investor is infinitesimally small he has no market power and takes the face-value promised by the firm as given. I also assume that the agents have access to a risk-free asset that pays out 1 in period $t = 2$ for each unit of funds invested at $t = 0$. Therefore, investors will purchase debt if and only if, in expectation, the gross return from holding it is at least one.

Investors who have purchased long-term debt issued by the firm are passive observers of the events.⁸ On the other hand, short-term debt holders need to decide at $t = 1$ whether to roll over their loans or not.

⁷This assumption simplifies an otherwise complex payoff structure and has been used recently in Gale and Yorulmazer [19] and Morris and Shin [33], among others. What is important for the results of the model is the assumption that default leads to a large enough deadweight loss of resources.

⁸It is possible to introduce a secondary market for the long-term debt into the model. Such an extension complicates the model, however, does not change the main implications of the model.

2.3 Information structure

The return from the project, s , is random with $s \sim N(\mu, \tau^{-1})$. The ex-ante distribution of returns is a common knowledge among all the agents. The return s is realized at the beginning of $t = 1$ but is unobserved by the firm and creditors. Instead, at $t = 1$, before the face value F_S^2 is chosen, both the firm and investors observe a public signal s_p where

$$s_p = s + \tau_p^{-1/2} \varepsilon_p, \quad \varepsilon_p \sim N(0, 1)$$

Here, τ_p is the precision of the public signal. In addition, before making rollover decisions, but after F_S^2 was chosen,⁹ each short-term creditor receives a private signal x_i

$$x_i = s + \tau_x^{-1/2} \varepsilon_i, \quad \varepsilon_i \sim N(0, 1)$$

I assume that ε_i is *i.i.d.* across individuals and independent of ε_p . Both ε_i and ε_p are assumed to be independent of s .

The presence of a public signal makes the firm's problem dynamic and implies that, as time passes, the firm and the investors learn more about profitability of investment. Not only this assumption seems natural but in its absence the firm would have the same information at $t = 0$ and $t = 1$ and hence it could choose all the face values in the initial period making the firm's problem static.

Private signals received by creditors capture the fact that investors may have better understanding of the aggregate market conditions, overall demand for firm's products, or strategic issues such as the extent of competition in the market. For empirical evidence that outside investors possess information unknown to firms see Chen et al. [9] or Luo [27].

Finally, note that the above information structure abstracts from the case where the firm has its own private information. This case, while interesting and relevant, leads to a very complex signaling game between the firm and the short-term creditors since the firm can use the face value of short-term debt issued at $t = 1$ to signal its private information.¹⁰

2.4 Coordination Problem and Strategic Uncertainty

A crucial feature of the model is the coordination problem faced by the investors at the time they make their rollover decisions. In particular, short-term creditors are aware that for a range of returns from the investment, whether the firm will default or not depends on the proportion of investors that decides to roll over their loans. This in turn makes them roll over their loans if and only if they expect large enough proportion of creditors to roll over. Under complete information, when s is known to everyone at the beginning of period $t = 1$, this leads to multiplicity of equilibria.

⁹Allowing investors to observe the public and private signals simultaneously does not change the results.

¹⁰For more details regarding signaling in global games see Angeletos, Hellwig and Pavan [3] and Angeletos and Pavan [4]. As shown in these papers signaling has a potential to reintroduce multiplicity of equilibria into the model. Corsetti et al. [11] and Zwart [38] are examples of global games models with signaling in which there is a unique equilibrium.

To understand why the coordination motive is present note that high withdrawals can push the firm into default in two ways. First, a high proportion of short-term creditors withdrawing at $t = 1$, i.e. high m , can lead to default today if the total claims of investors that withdraw early are higher than the liquidation value of the project (i.e. if $m\alpha F_S^1 > V$). This is the standard coordination problem that has been first emphasized by Diamond and Dybvig [15] and has been studied extensively in the banking literature. However, the model described above features also a “dynamic” coordination problem. High withdrawals today imply that a large fraction of the project has to be liquidated to pay these claims. Thus, the project has to be rescaled down leading to lower revenues earned by the firm at $t = 2$. If the fraction of the project liquidated is large enough the firm will not have enough funds to pay the remaining creditors the promised amount at $t = 2$ and hence it will default. This happens if the revenue of the firm after rescaling of the project at $t = 1$ (given by $s(1 - m\alpha F_S^1/V)$) is less than the amount of the outstanding debt due at $t = 2$ ($\alpha(1 - m)F_S^2 - (1 - \alpha)F_L$).¹¹

If investors had complete information (i.e. they knew the value of s) the coordination problem among them would lead to multiplicity of equilibria at the rollover stage. Figure 3 shows how the multiplicity region of a complete information game varies with α . The shaded area corresponds to the pairs (α, s) for which a rollover game with complete information regarding the investment’s return has two symmetric pure strategy equilibria, one in which everyone rolls over and the other one in which everyone withdraws. The lines \underline{s} and \bar{s} correspond to the lower bound and upper bound of that region. Above \bar{s} there is a unique equilibrium in which agents always roll over their loans. Below \underline{s} it is always strictly dominant for the investors to withdraw their funds early. Note that as α increases the multiplicity region expands.

At the heart of illiquidity risk lies the strategic uncertainty faced by the short-term debt holders. If the investors were certain that all short-term creditors will always roll over their loans whenever the firm is solvent then no solvent firm would ever become illiquid. However, even a small doubt that an investor has regarding the collective behavior of other creditors can make him withdraw early. Since all creditors think in this way, the uncertainty regarding actions’ of others from the perspective of an individual investor increases. This makes each investor even more likely to choose to “play safe” and to withdraw his loan. When many investors suddenly lose confidence about the ability of the firm to roll over its debt because they are afraid that not enough other investors will provide financing the firm becomes illiquid. This is despite the fact that given enough financing today the firm would be able to meet all of its current and future obligations.

A model with complete information (or with public but imperfect information) cannot capture the strategic uncertainty present among short-term debt holders when deciding whether to roll over and withdraw their loans. This is because, when computing equilibria it is always assumed that agents know with certainty what other agents will do (see Brandenburger [5]). Therefore, to model illiquidity risk and capture strategic uncertainty

¹¹Dynamic coordination motives are absent in other global game models of rollover risk (Eisenbach [17], Morris and Shin [32] or Rochet and Vives [36]). This is because in those papers the coordination game is played (for simplicity) by “fund managers” who care only about default today.

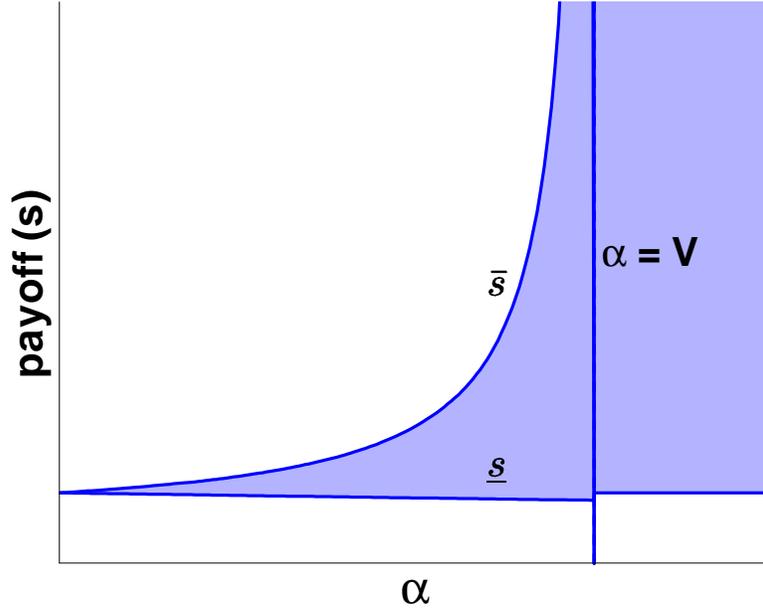


Figure 3: Multiplicity region as α varies

among investors, I follow the global games literature and consider the information structure as described in section 2.3.

2.5 Assumptions

To solve the game given the above information structure I make two additional assumptions.

Assumption 2 *There exists $\bar{s} \in \mathbb{R}$ such that for all $s \geq \bar{s}$ the project matures early and the firm can repay all of its debt.*

Assumption 3 *If indifferent a short-term creditor chooses to rollover his funds.*

To solve the model I use insight from the theory of global games. Applying these techniques requires the model to have regions in which each of the actions strictly dominates the other. For small s withdrawing is always dominant and therefore the model features a “lower” dominance region. Moreover, one can show that as long as $\alpha F_S^1 < V$ (and $F_S^2 > F_S^1$) there exists \bar{s} such that for all $s \geq \bar{s}$ it is dominant to roll over and hence the model also has an “upper” dominance region. However, when $\alpha F_S^1 \geq V$ this is not the case. Without Assumption 2 it can be shown that for each $\alpha \geq \frac{V}{F_S^1}$, in addition to the equilibrium characterized below, there is another equilibrium in which every short-term creditor withdraws his funds early regardless of the signals they observe. Assumption 2 ensures that ignoring

the signals is never optimal and hence eliminates this equilibrium.¹² Assumption 3 is the tie breaking assumption - this assumption allows a simple characterization of the solution to the firm's problem at $t = 1$ when the noise in the public signal becomes vanishingly small (section 3.3)

2.6 Timing

The timing of the model is summarized in Figure 4:

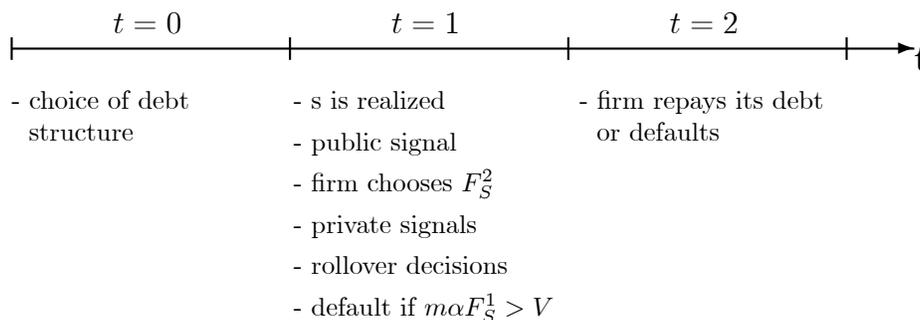


Figure 4: Timeline

At $t = 0$ the firm raises a unit of funds by issuing α of short-term debt with face value F_S^1 and $(1 - \alpha)$ of long-term debt with face value F_L . At the beginning of period $t = 1$ nature draws s from a normal distribution $N(\mu, \tau^{-1})$. The realization of s is unobserved by the firm and by the investors. Instead, all agents observe a public signal about s , s_P . Once the public signal is observed, the firm chooses F_S^2 , the face value of the short-term debt offered to investors at $t = 1$. Additionally, before making their roll over decisions, each short-term creditor observes a private signals of s , x_i . Then given F_S^2 and the information he has, each investor decides whether to purchase newly issued debt (which is equivalent to rolling over his loan) or not (which means early withdrawal of his funds) - in what follows I will refer to this stage of the model as the “rollover game”. The firm satisfies early withdrawals by rescaling the project. If withdrawals are larger than the liquidation value the firm defaults and the game ends. If the firm successfully rolls over enough of the short-term debt I move to $t = 2$. At the beginning of $t = 2$ the project matures and pays out $(1 - x)s$. The firm uses the proceeds from the project to repay its debt. If it does not have enough funds it defaults.

¹²See section A.1 in the appendix for more detailed discussion.

3 Solving the model

3.1 Period $t = 2$

I solve the model using backward induction starting by characterizing the firm's default decision at $t = 2$. Take the maturity structure, $(\alpha, 1 - \alpha)$, the face values, (F_S^1, F_S^2, F_L) and the proportion of creditors that withdrew early, m , as given. Suppose that the firm did not default at $t = 1$. At the beginning of period $t = 2$ the project matures and the firm receives $\left(1 - m \frac{\alpha F_S^1}{V}\right) s$. The firm uses its revenues to repay its outstanding debt which is given by $(1 - m) \alpha F_S^2 + (1 - \alpha) F_L$. The firm defaults at $t = 2$ if and only its revenue is less than its obligations towards investors, i.e. if and only if

$$\left(1 - m \frac{\alpha F_S^1}{V}\right) s - (1 - m) \alpha F_S^2 - (1 - \alpha) F_L < 0$$

Otherwise, the firm repays the debt.

3.2 Rollover Game

Having characterized conditions under which the firm defaults at $t = 2$ I move to analyze investors' rollover decisions. At this stage short-term creditors have to decide whether to roll over their loans or withdraw their funds early. In order to solve for the optimal rollover decisions I need to analyze separately two cases: (1) a case with no rollover risk ($\alpha F_S^1 \leq V$) and (2) a case with rollover risk ($\alpha F_S^1 > V$). This is due to the fact that the investors face different payoffs from withdrawing early depending on the total value of short-term debt, αF_S^1 . In particular, when $\alpha F_S^1 \leq V$ by withdrawing early the short-term debt holder can secure for himself a payoff of F_S^1 . This is not the case when the opposite inequality holds. When $\alpha F_S^1 > V$ withdrawing early yields payoff F_S^1 only if the firm does not default at $t = 1$. However, despite this difference the analysis of both cases is very similar. Therefore, below I consider only the case of no rollover risk and an interested reader is directed to section A.3 of the appendix for the detailed solution to the rollover game when $\alpha F_S^1 > V$.

Take all the face values F_S^1, F_S^2 and, F_L , the maturity structure $(\alpha, 1 - \alpha)$ and the value of the public signal s_p as given. I define a public belief as the posterior belief about s conditional on the public signal. Therefore, the public belief is given by

$$s|s_p \sim N(y, \tau_y^{-1})$$

where $y \equiv \frac{\tau \mu + \tau_p s_p}{\tau + \tau_p}$ and $\tau_y \equiv (\tau + \tau_p)^{-1}$.

I focus on the equilibria in monotone strategies, that is equilibria such that a short-term creditor rolls over his loan if and only if his private signal x_i is greater than a threshold signal x^* and the firm defaults if and only if the return from the project, s , is less than s^* . An equilibrium in monotone strategies is characterized by a payoff indifference and a critical mass conditions. The payoff indifference condition states that at a critical signal x^* a

creditor has to be indifferent between rolling over and not rolling over his loan. The critical mass condition states that at the critical return from investment s^* the withdrawals are such that the firm is left with just enough resources to repay its debt. These two conditions are described below.¹³

Assume that all agents use threshold strategies with cutoff x^* . Then, given x^* , the proportion of investors withdrawing early when the return from investment is s is given by

$$m(x^*, s) = \Pr(x_i < x^* | s) = \Phi\left(\frac{x^* - s}{\tau^{-1.2}}\right)$$

It follows that for a given s the firm repays its debt if and only if

$$\left(1 - m(x^*, s) \frac{\alpha F_S^1}{V}\right) s - (1 - m(x^*, s)) F_S^2 - (1 - \alpha) F_L \geq 0$$

Hence the critical threshold s^* below which the firm defaults at $t = 2$ is the unique solution to:

$$\left(1 - m(x^*, s) \frac{\alpha F_S^1}{V}\right) s - (1 - m(x^*, s)) F_S^2 - (1 - \alpha) F_L = 0 \quad (1)$$

I will refer to the above condition as the critical mass condition.^{14,15}

Consider now an investor who received a signal $x_i = x^*$. Since x^* is the threshold signal it has to be the case that this investor is indifferent between rolling over and withdrawing. Thus, at signal x^* , the expected payoff from rolling over must be equal to the expected payoff from withdrawing early, or:

$$F_S^2 \Pr(s \geq s^* | x^*, s_p) = F_S^1 \quad (2)$$

A pair (x^*, s^*) that solves simultaneously equations (1) and (2) constitutes an equilibrium in monotone strategies. The following proposition characterizes equilibrium when the noise in signals is vanishingly small and private signals become arbitrary more precise compared to the public signal.

Proposition 1 *Let $\tau_x \rightarrow \infty$, $\tau_p \rightarrow \infty$ and $\frac{\tau_x}{\tau_p} \rightarrow \infty$. Then the rollover game has a unique equilibrium in monotone strategies in which all investors use threshold strategies with cutoff x^* and the firm defaults if $s < s^*$ where*

$$x^* = s^* = \frac{(1 - \alpha) F_L + \alpha (F_S^2 - F_S^1)}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}}$$

¹³To keep notation simple I suppress below the dependence of s^* and x^* on $(\alpha, F_L, F_S^1, F_S^2)$.

¹⁴Since I consider here the case when $\alpha F_S^1 \leq V$ the firm will not default at $t = 1$ and, therefore, investors worry only about default at $t = 2$.

¹⁵A careful reader might notice that the equation which determines whether the firm defaults in period $t = 2$ may be non-monotonic in s for high face values F_S^2 . In the appendix I show that the firm will always choose F_S^2 such that the "critical mass" condition is monotonic in s in the relevant range of returns from the project.

The above proposition characterizes the default threshold, s^* (i.e. the threshold return from investment above which the firm is able to repay all of its debt at $t = 2$), as a function of the debt maturity structure $(\alpha, 1 - \alpha)$, the face values (F_S^1, F_S^2, F_L) and the liquidation value V . Note that a larger the liquidation value always lowers the default threshold while higher long-term debt face value, F_L , always makes s^* increase. A change in either of these two values affect the critical thresholds in two ways. First, it directly leads to a change in the default threshold holding investors strategies constant. For example, higher liquidation value implies that given the same withdrawals the firm has to liquidate less of its project resulting in higher revenues at $t = 2$ and hence lower default threshold. Second, a change in either F_L or V has an indirect effect through the change of the investors strategies. In the case of higher liquidation value, investors know that, due to the direct effect of a change in V , the firm is less likely to default and hence they rollover for a wider range of signals. This leads to a further decrease in s^* .

In contrast, the effect of a higher face value, F_S^2 , on the default threshold, s^* , is more complicated. On the one hand, higher F_S^2 makes it more profitable for the investor to roll over the loan, conditional on the firm not defaulting. This increases investors' incentives to roll over and hence tends to decrease s^* . On the other hand, higher F_S^2 increases the amount of the debt to be repaid at $t = 2$ which makes default more likely. This discourages investors from rolling over their loans. It turns out that there exists a threshold value F_S^2 , which I denote \bar{F}_S^2 , such that if $F_S^2 < \bar{F}_S^2$ then an increase in F_S^2 leads to a decrease in the default threshold while if $F_S^2 > \bar{F}_S^2$ then a further increase in F_S^2 results in an increase in s^* . It follows that s^* is minimized at $F_S^2 = \bar{F}_S^2$.

Corollary 1 *The threshold return s^* is minimized at \bar{F}_S^2 where*

$$\bar{F}_S^2 = \frac{\alpha F_S^1}{V} \left(F_S^1 + \sqrt{(F_S^1)^2 + \frac{V}{\alpha^2} [(1 - \alpha) F_L - \alpha F_S^1]} \right)$$

Moreover, s^ decreases with F_S^2 when $F_S^2 < \bar{F}_S^2$ and increases with F_S^2 when $F_S^2 > \bar{F}_S^2$.*

The above corollary implies that the firm will never choose $F_S^2 > \bar{F}_S^2$ because this will result in both higher probability of default (higher s^*) and lower profits conditional on not defaulting.

Above I assumed that $\alpha F_S^1 \leq V$. In section A.3 of the appendix I show that similar results also hold when $\alpha F_S^1 > V$. In particular, I show that for a given maturity structure $(\alpha, 1 - \alpha)$ and given the face values (F_S^1, F_S^2, F_L) there exists a unique threshold s^* such the firm will default if and only if $s < s^*$. Moreover, there also exists a unique value of \bar{F}_S^2 such that s^* is minimized at \bar{F}_S^2 .

3.3 Optimal choice of F_S^2

Having solved the rollover game for any given maturity structure $(\alpha, 1 - \alpha)$ I consider now firm's optimal choice of face value of the short-term debt issued at $t = 1$, F_S^2 . As before, I

take the maturity structure $(\alpha, 1 - \alpha)$ and the associated face values F_S^1 and F_L as given. When choosing the face value of the short-term debt issued in period $t = 1$ the firm faces the following trade-off. On the one hand, higher face value increases incentives for the investors to roll over and hence it decreases the default threshold. On the other hand, a higher F_S^2 decreases profits of the firm conditional on not defaulting. When choosing the optimal face value in period $t = 1$ the firm weighs this positive effect of lower default threshold against the decrease in profits due to a higher promised face value.

The firm's maximization problem at $t = 1$ is given by

$$\max_{F_S^2} \int_{s^*}^{\infty} (s - \alpha F_S^2 - (1 - \alpha) F_L) \tau_y^{-1/2} \phi \left(\frac{s - y}{\tau_y^{-1/2}} \right) ds$$

$$s.t. F_S^2 \geq F_S^1$$

where s^* is a solution to the rollover game

We saw above that the firm can influence the threshold return from the investment above which it survives, s^* , by adjusting the face value of the debt issued at $t = 1$. Recall that \bar{F}_S^2 is the face value that minimizes the default threshold s^* and $s_{\min}^* = \min_{F_S^2} s^*(F_S^2; \alpha)$ (i.e. s_{\min}^* is the default threshold associated with the face value \bar{F}_S^2). It follows that for $s < s_{\min}^*$ the firm defaults regardless of its actions at time $t = 1$.

Define $\bar{s}(\alpha)$ as the investment return above which the firm will not default even if all investors withdraw. It can be shown that

$$\bar{s}(\alpha) = \begin{cases} \min \left\{ \frac{(1-\alpha)F_L}{1-\alpha F_S^1/V}, \bar{\bar{s}} \right\} & \text{if } \alpha F_S^1 < V \\ \bar{\bar{s}} & \text{if } \alpha F_S^1 \geq V \end{cases}$$

where $\bar{\bar{s}}$ is a threshold above which the project matures early and the return is high enough so that the firm can repay all of its debt (see Assumption 2 and its discussion in section 2.5 and section A.1 in the appendix). Then $\bar{s}(\alpha)$ is the threshold above which the firm repays its debt regardless of the actions by the short-term debt holders.

The above discussion implies that if $s < s_{\min}^*$ then the firm will default regardless of the face value F_S^2 . On the other hand, if $s > \bar{s}(\alpha)$ the firm will never default. Thus we see that adjusting the face value F_S^2 allows the firm to avoid default if and only if $s \in [s_{\min}^*, \bar{s}(\alpha)]$. The following proposition characterizes the optimal value of F_S^2 in the limit as the firm's information becomes perfect.¹⁶

¹⁶One should think about this limiting case in the following way. For any given $\tau_p < \infty$ the firm's choice of F_S^2 and the outcome of the rollover game are well defined (as describe in Propositions 1 and 2). Therefore, one can ask a question what happens as $\tau_p \rightarrow \infty$. Proposition 3 simply states what happens to the firm's choice of F_S^2 in the limit as the information it has becomes more and more precise. Conceptually, this is the same as considering the limiting case as the noise is vanishing in a standard global game.

Proposition 2 Suppose that $\tau_x \rightarrow \infty$, $\tau_p \rightarrow \infty$ and that $\frac{\tau_p}{\tau_x} \rightarrow 0$. Then, in the limit, the optimal choice of face value F_S^2 is given by:

- $F_S^2 = F_S^1$ if $s > \bar{s}(\alpha)$
- F_S^2 is the smallest solution to $s^*(F_S^2; \alpha) = s$ if $s \in [s_{\min}^*(\alpha), \bar{s}(\alpha)]$
- if $s < s_{\min}^*(\alpha)$ then the firm defaults regardless of F_S^2

The above result has a simple interpretation. When $s > \bar{s}(\alpha)$ then the firm knows it will survive for sure because all investors will choose to roll over regardless of the face value offered by the firm (as long as $F_S^2 \geq F_S^1$). Therefore, the firm finds it optimal to set the lowest feasible face value, i.e. $F_S^2 = F_S^1$. When $s \in [s_{\min}^*(\alpha), \bar{s}(\alpha)]$ then in order to avoid default the firm has to make use of the face value F_S^2 to lower short-term debt holders' threshold below which they withdraw. Profit maximization implies that the firm will choose the smallest F_S^2 which will make agents roll over their loans. Thus, for realizations of s in this range, the optimal F_S^2 is the smallest solution to $s^*(F_S^2; \alpha) = s$. Corollary 1 implies then that for all $s \in [s_{\min}^*(\alpha), \bar{s}(\alpha)]$ the face value F_S^2 increases as s decreases and at s_{\min}^* the optimal face value is given by \bar{F}_S^2 (the face value that minimizes s^*). Finally, if $s < s_{\min}^*(\alpha)$ then the firm will default regardless of the face value it chooses.

3.4 Optimal Maturity Structure

In this section I study firm's choice of the optimal maturity structure. I impose the requirement that $F_S^1 = 1$.¹⁷ In that case the firm's problem at $t = 0$ is given by

$$\max_{\alpha, F_L} \int_{s_{\min}^*(\alpha)}^{\infty} (s - \alpha F_S^2 - (1 - \alpha) F_L) \tau^{-1/2} \phi\left(\frac{s - \mu}{\tau^{-1/2}}\right) ds$$

s.t. investors' participation constraints

where $s^*(\alpha)$ is implied by the rollover game and F_S^2 is chosen optimally according to proposition 2. The participation constraint for the long-term debt holders is given by

$$F_L \Pr(s \geq s^*) \geq 1$$

for all $\alpha \in [0, 1]$. The participation constraint for the short-term debt holders is

$$\begin{aligned} F_S^1 \Pr(s < s_{\min}^*) + \Pr(s > s_{\min}^*) E[F_S^2(s) | s > s_{\min}^*] &\geq 1 \text{ if } \alpha \in [0, V] \\ E[F_S^2(s) | s > s_{\min}^*] &\geq 1 \text{ if } \alpha \in (V, 1] \end{aligned}$$

¹⁷The same assumption is made by Eisenbach [17]. In the complete information version of the model this restriction would be implied by the rationality of the investors and profit maximization by the firm. In the global game model this is not necessarily true. In period $t = 1$ the firm now has incentives to set $F_S^2 > F_S^1$ in order to decrease the strategic uncertainty faced by the investors and provide them with stronger incentives to roll over their loans. Therefore, it is possible that the firm can credibly commit to setting high F_S^2 in order to compensate the short-term creditors for offering $F_S^1 < 1$ at $t = 0$. For simplicity, however, in what follows I assume that $F_S^1 = 1$.

It is easy to see that the participation constraint of long-term debt holders will hold with equality. This is because increasing the face value of long-term debt leads to lower profits conditional on the firm not defaulting and an increase in $s_{\min}^*(\alpha)$ leading to a higher probability of default (see the discussion of proposition 1). On the other hand, the participation constraint of the short-term debt holders may hold with strict inequality. This is because, when faced with illiquidity risk, the firm may choose to offer short-term debt holders a positive return in order to reduce the default risk due to early excessive withdrawals.

The firm chooses the maturity structure of its debt to maximize its expected profit. The long-term debt has the advantage of having the same maturity as the project. Therefore, by issuing more long-term debt the firm can decrease the risk of illiquidity. On the other hand long-term debt is more expensive than short-term debt. This is because, short-term creditors can act based on their signals. In particular, if they expect default, they can withdraw their funds early securing a positive payoff. This leads to a trade-off between short-term and long-term debt. It is possible to show that regardless of the parameters the firm will always want to finance a positive fraction of the project with short-term debt.

Proposition 3 *For any $\tau < 2\pi$ the optimal proportion of short-term debt is strictly positive, that is $\alpha^* > 0$*

Unfortunately, an analytical solution to the maturity structure choice is unavailable. Therefore, in the next section I turn to a numerical analysis.

3.5 Numerical Analysis

There are three exogenous parameters in the model: liquidation value V and the mean and the standard deviation of the prior belief. Since I consider the case when the signals become arbitrarily precise I do not need to specify the precisions of the signals. In what follows I set the liquidation value $V = 0.75$ and assume that s is distributed according to a normal distribution with mean $\mu = 4$ and standard deviation $\sigma = 2$.

I compare the results from simulating the above model with the solution to the complete information version of the model. I assume that in the complete version of the model both the firm and the investors learn the true return from investment at the beginning of period $t = 1$, before they make any decisions (but after the maturity structure has been determined at $t = 0$). Moreover, in order to deal with the multiplicity of equilibria, I assume that in the rollover game investors always coordinate on the Pareto efficient equilibrium.¹⁸ In what follows I refer to the complete information model as the model without strategic uncertainty and a model with incomplete information structure as the model with strategic uncertainty. A solution to the complete information model is described in Appendix C.

¹⁸This is a standard assumption in the literature. It implies that investors do not face strategic uncertainty when they make their decisions. Note also that in the model with strategic uncertainty we focus on the case when the signals becomes infinitely precise. Therefore, in both models agents face no fundamental uncertainty (see Morris and Shin [32]).

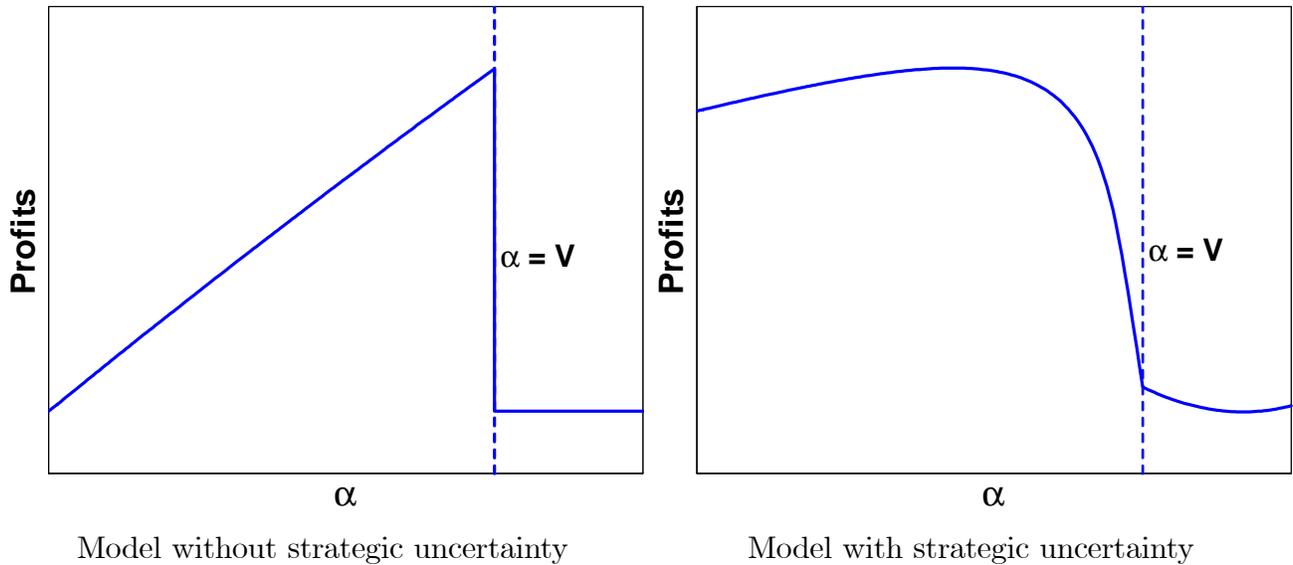


Figure 5: Expected Profits

I start by comparing firm's optimal choice of maturity in each of the models. Figure 5 depicts the expected profit of the firm as a function of α in a model without strategic uncertainty (left panel) and in the model with strategic uncertainty (right panel). We see that in the former model the profit maximizing maturity structure prescribes financing a fraction V of the project with short-term debt and the remaining part, $1 - V$, with the long-term debt. In contrast, in the model with strategic uncertainty, the optimal amount of short-term debt is always strictly less than V . Below I explain why the presence of strategic uncertainty discourages the firm from issuing short-term debt by comparing the outcomes of the rollover game and the expected cost of financing the project in each model.

In the complete information model I assumed that in the rollover game investors always coordinate on the Pareto efficient equilibrium. Therefore, in this model the default threshold coincides with the solvency threshold \underline{s} . In contrast, in the model with strategic uncertainty, the default threshold s^* is in general different than \underline{s} . Figure 6 depicts both the solvency threshold \underline{s} (solid blue line) and the default threshold s^* implied by the model with strategic uncertainty (the red dashed line). We see that in the model with strategic uncertainty the default threshold is an increasing function of short-term debt α and is always above the solvency threshold. This discourages the firm from issuing short-term debt when investors face strategic uncertainty.

Figure 7 shows the expected cost of financing the project as a function of α in the model with strategic uncertainty (left panel) and for a model without strategic uncertainty (right

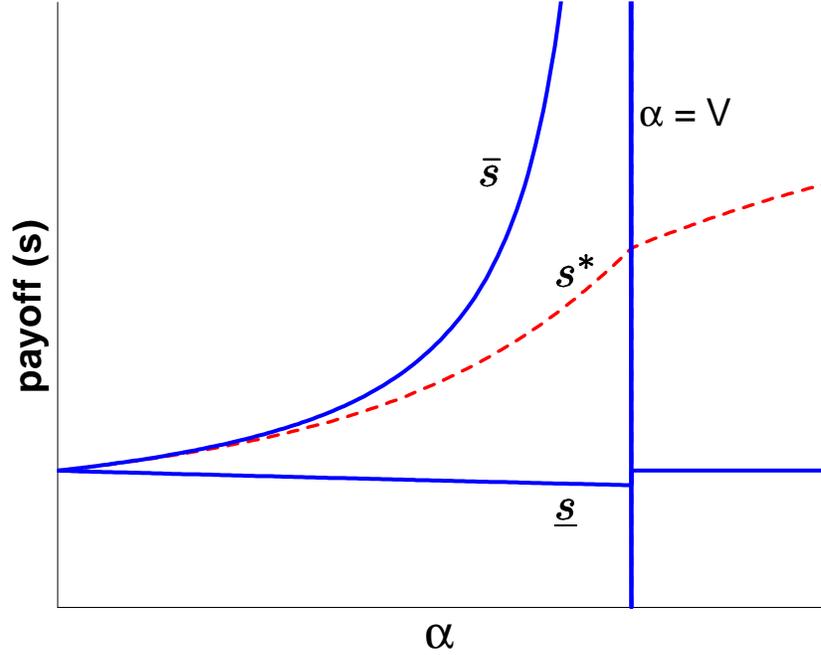


Figure 6: Default Threshold

panel).^{19,20} The vertical dashed line indicates the level of short-term debt that maximizes the expected profits. We see that in the case of the model without strategic uncertainty the optimal amount of short-term debt coincides with the level of short-term debt that minimizes the expected cost of financing the project. On the other hand, in the model with strategic uncertainty, optimal α^* is smaller than the amount that minimizes the expected cost of financing the project. The reason is that, as we have seen above, when α increases the default threshold increases. This in turn tends to decrease firm's expected profits despite the drop in the cost of financing the project. Nevertheless, the firm still finds it optimal to issue a positive amount of short-term debt. The reason is that for small values of α the expected cost of debt decreases rapidly enough to compensate the firm for the increase in the default probability.

4 Secondary Market Conditions

In this section I show how a change in secondary market conditions affects: (1) firm's optimal choice of maturity and (2) the default threshold and the associated face values. Within the model, secondary market conditions are captured by the liquidation value V . A low

¹⁹The expected cost of financing the project for a fixed α is given by $\int_{s^*}^{\infty} [\alpha F_S^2 + (1 - \alpha) F_L] \phi\left(\frac{s - \mu}{\tau^{-1/2}}\right) ds$.

²⁰Both panels use the same scale and hence they are directly comparable.

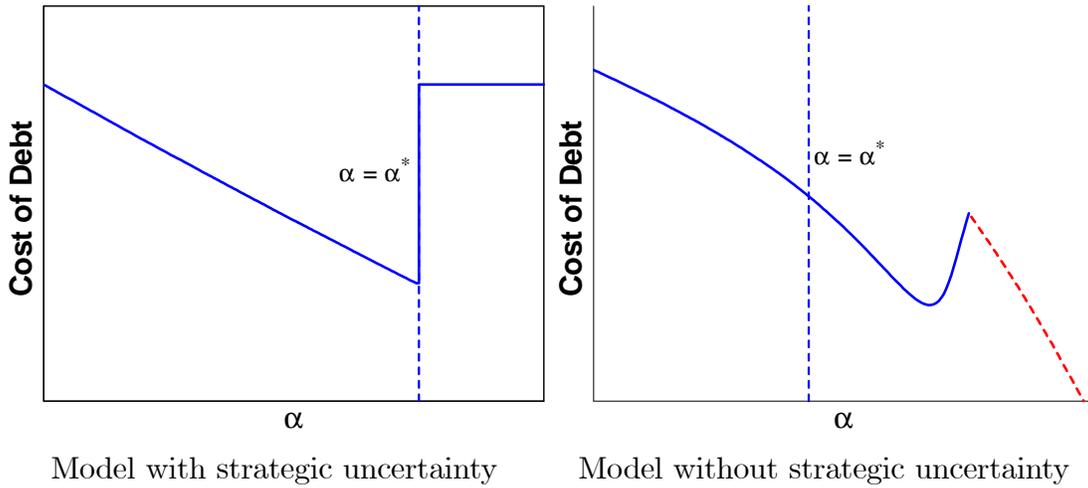


Figure 7: Expected Cost of Servicing Debt

liquidation value means that the firm can sell its capital only at a high discount compared to its fundamental value while a high V implies that the capital can be sold at a low discount. In section 4.1 I consider the effects of different secondary market conditions in period $t = 0$ on firm's choice of maturity structure. In section 4.2 I consider the effect of unexpected deterioration in secondary market condition at the beginning of period $t = 1$ (unexpected drop in V).

4.1 The Role of Liquidation value

Figure 8 shows that optimal amount of short-term debt issued, α^* , increases as the liquidation value V increases. There are two channels through which a higher liquidation value encourages the firm to finance higher proportion of the project with short-term debt. First, higher V decreases the illiquidity region for any $\alpha < V$ by shifting the curve $\bar{s}(\alpha)$ downwards. This implies that ex-ante the firm faces a lower probability of defaulting due to liquidity issues. Second, the decrease in the illiquidity region makes individual investors less concerned about actions of the others. As V increases investors are less worried about withdrawals by other creditors because they know that these withdrawals have a smaller effect on the final return from investment (for given withdrawals the firm has to liquidate less of its project.). Therefore, short-term debt holders are more willing to roll over their loans when s falls in the illiquidity region. This in turn reduces the incidents of defaults due to liquidity problems further decreasing the cost of short-term debt.

As V increases the fraction of the project financed with short-term debt becomes closer and closer to the prediction of the model with complete information (that is the distance

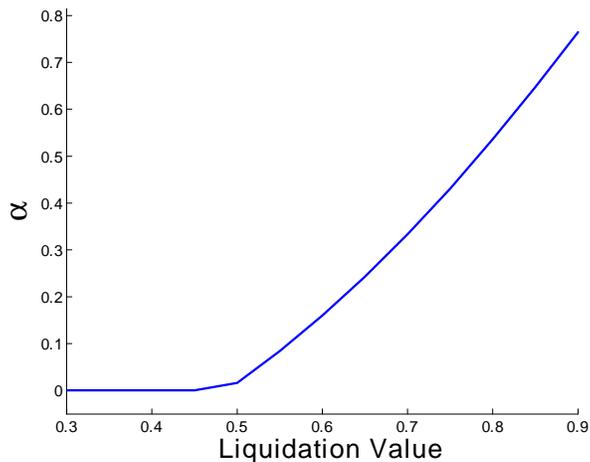


Figure 8: Profits as V varies

between α^* and V decreases). On the other hand, for low enough V the firm will choose to finance its project almost fully with long-term debt. This suggests that a complete information model might be a good approximation of the model with strategic uncertainty when the secondary market is working well and the liquidation value of the project is high. On the other hand, when the secondary market is severely disturbed the presence of strategic uncertainty among investors becomes a dominant force determining the optimal debt maturity structure.

4.2 Unexpected deterioration in market conditions

Consider now an unexpected deterioration in markets conditions at $t = 1$ (after the firm issued short-term and long-term debt). This exercise is motivated by the recent crisis which featured sudden deterioration in secondary markets conditions and tighter access to credit (see for example Ivashina and Scharfstein [25]).

I consider a situation where the liquidation value V falls unexpectedly at $t = 1$ from V^H to V^L , where $V^H > V^L$. Since this fall is unforeseen, the firm issued debt at $t = 0$ expecting that at the time it will have to roll over its short-term debt the liquidation value will be high. I associate V^H with normal time and V^L with a crisis.

As explained in section 4.1 a deterioration in secondary market conditions increases the importance of strategic uncertainty making investors less willing to rollover their debt. When such deterioration happens after the firm issued debt it has two effects on the outcome of the rollover game. First, for low enough returns from investment the firm is forced into default even though it would be able to refinance its debt successfully if the market conditions did not deteriorate. Second, the firms that continue operating are forced to offer higher face values compared to the situation in which liquidation value was high.

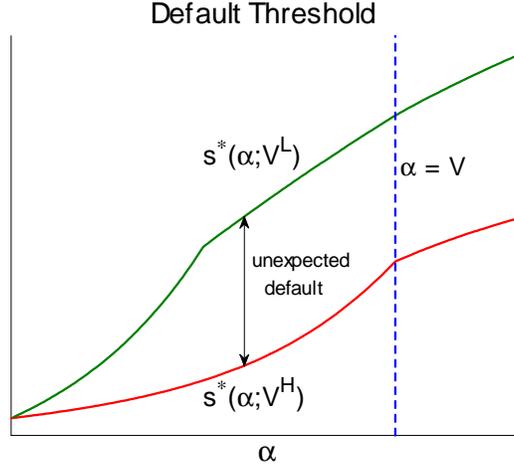


Figure 9: Increase of the default threshold

Figure 9 shows the increase in the default threshold for each α . If liquidation value stayed at the level V^H then the firm would default only if the return from investment was lower than $s^*(\alpha; V^H)$. However, when the secondary market conditions deteriorate, the minimal return that investors require in order to refinance the firm increases significantly. This pushes the default threshold up for each α to $s^*(\alpha; V^L)$ and results in a range of returns for which the firm defaults.

Figure 10 shows that for each α a decrease in liquidation value increases the maximum face value that the firm offers. Recall from section 3.3 that determines the threshold investment return below which the firm will default (since the firm does not default if and only if $s \geq s_{\min}^* = s^*(\bar{F}_S^2; \alpha)$). As we can see, an unexpected deterioration in market conditions leads to a large increase in \bar{F}_S^2 . The reason is that a lower liquidation value makes investors more concerned about other investors' behavior and less willing to roll over their loans. In order to avoid a default the firm offers a higher face value than it would if the market conditions were unchanged. A higher face value implies a higher payoff from rolling (conditional on no default) encouraging them to roll over their loans. Note, however, that despite a large increase in \bar{F}_S^2 the default threshold also increased significantly (10). This is because at one point offering higher face value becomes counterproductive. A higher face value increases debt burden of the firm discouraging the investors from rolling over. For high values of F_S^2 this effect dominates the positive effect of higher payoff conditional on no default.

It is interesting that when there is a sudden deterioration in market conditions, the firms that have projects with payoff $s \in (s^*(\alpha; V^L), s^*(\alpha; V^H))$ are being “trapped” by the pessimistic expectations of investors. When the liquidation value decreases each investors knows that the firm becomes more vulnerable to withdrawals and become more concerned regarding withdrawals by other investors. This in turn makes each investor more likely

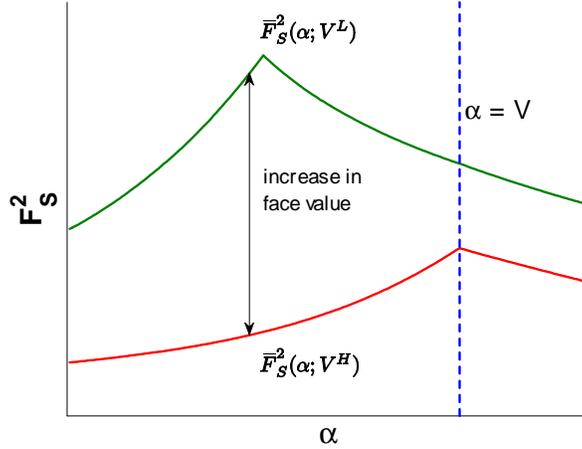


Figure 10: Increase in the face value F_S^2

to withdraw his loan. Since all investors follow the same reasoning they all become more pessimistic about prospect of the firm and even more likely to withdraw, and so on. The firm can try to coordinate investors on rolling over by offering a high face value F_S^2 . However, for $s \in (s^*(\alpha; V^L), s^*(\alpha; V^H))$ the required increase in the face value is so large that it would lead to a huge increase in the debt burden making future default very likely further justifying the initial pessimism of the investors. As a result the firm finds itself being forced to default. It is important to note that while this outcome is due to self-fulfilling beliefs it is the unique outcome of the game since the beliefs of each investor are uniquely determined.

The above predictions of the model are in line with the experience of many firms and financial institutions during the recent subprime crisis. In this period the cost of refinancing the debt went up even for these companies that were considered safe while others, seemingly solvent firms and financial institutions were cut from the credit markets and forced to default (see figures 1 and 2).

5 Debt Overhang and Future Investment Opportunities

In this section I use the model to investigate the effects of debt overhang on firm's maturity choice and investors' rollover decisions. The primary motivation for studying this problem is the amount of attention debt overhang has received as one of the main factors behind slow recovery from the last recession.

The other motivation behind this extension are the recent finding by Diamond and He [16]. They showed that, contrary to the common perception, in the presence of refinancing risk short-term debt can lead to a stronger debt overhang than long-term debt. However,

their model assumes perfect coordination among debt holders and does not take into account strategic uncertainty. Below I investigate if they conclusions change in the setup with strategic uncertainty.

5.1 Debt Overhang Problem in the Model

In order to introduce the debt overhang problem into the model I make the following modification to the setup described in section 2. Namely, I assume that after agents make their roll over decisions the firm can undertake a new project that is financed fully with equity. I assume that the cost of the project is $I < 1$ and it pays a sure net return of $b > 0$. This extra investment opportunity is the only change compared to the benchmark model described in section 2.²¹

Adding a new investment opportunity at $t = 1$ introduces a debt overhang problem into the model. To see this note that if the firm could commit to always invest then its expected profits at $t = 0$ would be higher compared to the case when the investment decision is made at $t = 1$. Committing to always undertake the new investment at $t = 1$ implies higher expected revenues at $t = 2$ regardless of the payoff from the initial investment. This in turn makes investors less worried about default in the future and hence makes them accept lower face values at $t = 0$ and $t = 1$. Thus, not only the firm defaults less often but also it enjoys higher profits than before due to the lower cost of financing its initial project. Unfortunately, the firm cannot commit credibly to always invest. Once the firm learns s it knows that in the case of default the proceeds from the new investment will go to the debt holders. Therefore, the firm will never invest if it expects to default. Anticipating such behavior debt holders request higher face values compared to the case when they are sure that the firm would invest increasing the default threshold and decreasing firm's profits when it does not default.

5.2 Rollover Game in the Presence of Debt Overhang

Consider the firm's decision to undertake the new investment at the end of period $t = 1$. Note that this decision is taken after the firm observes the withdrawals made by investors which allows the firm to infer the true value of s .²² Therefore, the firm will invest if and only if

$$\left(1 - m \frac{\alpha F_S^1}{V}\right) s + b - (1 - m) \alpha F_S^2 - (1 - \alpha) F_L \geq 0 \quad (3)$$

This implies that if the firm suffers high withdrawals it will choose not to undertake the profitable investment because it knows that all the proceeds from the new investment will be used to repayment of the remaining outstanding debt. This constitutes the debt overhang problem in the model.

²¹This specification of debt overhang follows closely Diamond and He [16] and it has advantage of being particularly simple.

²²In the environment considered here observing the mass of short-term debt holders withdrawing is equivalent to observing s .

Equation (3) holding with equality is the critical mass condition for the model with debt overhang and is the only change compared with the rollover game analyzed in section 3.2. Therefore, we know that the equilibrium of the rollover game is determined by the payoff indifference condition for investors (which stays unchanged) and the “new” critical mass condition. The next proposition characterizes the equilibrium of the rollover stage.

Proposition 4 *Let $\tau_x \rightarrow \infty$, $\tau_p \rightarrow \infty$ and $\frac{\tau_p}{\tau_x} \rightarrow 0$. Then the unique equilibrium in monotone strategies is characterized by a pair of thresholds (x^*, s^*) where:*

$$x^*(\alpha) = s^*(\alpha) = \frac{(1 - \alpha) F_L + \alpha (F_S^2 - F_S^1) - b}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}} \quad \text{if } \alpha F_S^1 \leq V$$

$$x^*(\alpha) = s^*(\alpha) = \frac{(1 - \alpha) F_L + \alpha \left(F_S^2 - \frac{V}{\alpha} \right) - b}{1 - \frac{F_S^1}{F_S^2}} \quad \text{if } \alpha F_S^1 > V$$

We see that the presence of the additional investment opportunity decreases the default threshold s^* . To understand how this new investment opportunity changes the outcome of the rollover game I consider its effect on the default threshold when $\alpha F_S^1 \leq V$.²³ This effect can be decomposed into a direct and an indirect effects:

$$\underbrace{\frac{1}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}} b}_{\text{total effect}} = \underbrace{b}_{\text{direct effect}} + \underbrace{\frac{\frac{\alpha F_S^1 F_S^1}{V F_S^2}}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}} b}_{\text{indirect effect}}$$

The direct effect is simply the effect of a higher total return from the firm’s investments. Holding investors’ behavior constant the threshold s^* would simply decrease by b . However, the fact that the firm survives now for lower values of s makes each short-term investor more willing to roll over their loans. This in turn decreases strategic uncertainty among investors since they now expect each other to withdraw less often. Therefore, they roll over their loans for a wider range of signals which further decreases the default threshold, s^* . This is the “indirect effect” implied by the additional investment. Note that the indirect effect becomes stronger as $\alpha F_S^1/V$ increases or the distance between F_S^2 and F_S^1 decreases. Finally, note that the indirect effect can be stronger than direct effect (this can happen when F_S^2 is close to F_S^1 and α close to V).

Above we saw how debt overhang changes the outcome of the rollover game. I focus now on its effect on the optimal maturity structure. As shown in Proposition 4 the extra investment opportunity makes the short-term debt holders more willing to roll over their loans decreasing the probability of the default due to illiquidity. This encourages the firm to issue more short-term debt. On the other hand, the default threshold is increasing in α implying that as the the firm relies more heavily on short-term debt it will forgo the investment opportunity at $t = 1$ for a larger range of payoffs from its initial investment.

²³Similar decomposition can be made for the case with rollover risk.

This effect discourages it from relying on short-term finance. Thus, it is ambiguous whether in the presence of additional investment opportunity the firm relies more or less on short-term debt. While analytical solutions to the optimal maturity structure are unavailable, numerical simulations suggest that the first effect tends to dominate.

Figure 11 depicts the optimal amount of short-term debt issued as a function of the net present value from the additional investment b in the model without strategic uncertainty (left panel) and in the model with strategic uncertainty (right panel). I associate a higher b with a stronger debt overhang problem since for high returns from new investment not investing is particularly harmful for the firm's ex-ante value. We see that in the model with complete information introducing a new investment and changing its return has no effect on the optimal maturity structure. This is in contrast to the model with incomplete information where the optimal amount of short-term debt issued increases with b .

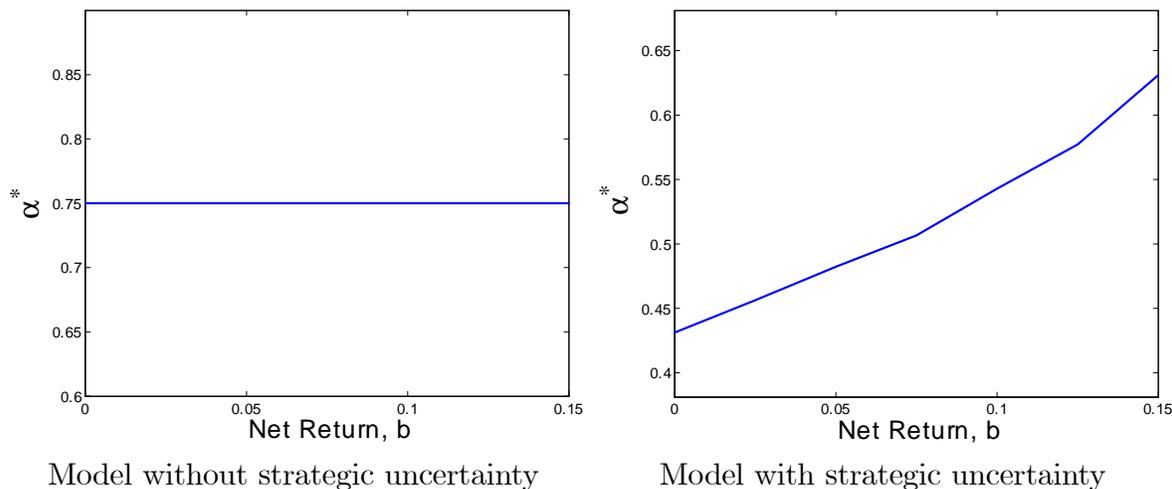


Figure 11: Optimal maturity structure as a function of b

To understand that note that when the firm has an additional investment opportunity, the illiquidity risk associated with short-term debt decreases substantially allowing the firm to offer a lower face value F_S^2 and making short-term debt cheaper. As long as the probability of default is small enough for all α (limiting the debt overhang) the existence of a new investment opportunity will make the firm issue more short-term debt. Therefore, the key parameters determining whether proportion of short-term debt increases or not are the ones controlling the ex-ante belief about investment (i.e. values of μ and τ).

It is interesting to compare the above results to the findings of Diamond and He [16]. They show that the short-term debt imposes a stronger debt overhang than long-term debt in bad states. Thus, they conclude that when the bad states are sufficiently likely in the

future firms with future investment opportunities should rely more on the long-term debt. My model can be thought of as an extreme example of their setup. In particular, in my model neither long-term debt nor short-term debt imposes debt overhang in good states (high s) while both impose an extreme debt overhang in bad states (low s).²⁴ However, short-term debt leads to a higher default threshold and, as we saw above, investment at $t = 1$ is undertaken only in the absence of default. It follows that in my model short-term debt impose severe debt overhang for larger set of states and hence leads to a stronger ex-ante debt overhang.²⁵ Therefore, the main finding of Diamond and He holds also in my setup. However, in contrast to their conclusion, I find that a firm with a future investment opportunity may find it optimal to rely more heavily on short-term debt compared to firm a firm without such an opportunity. The reason behind our different conclusions is the presence of strategic uncertainty in my setup.

5.3 Unexpected lack of new investment opportunity

Note that the model with debt overhang can shed some more light on the default experience of many firms in the crisis. The above model suggests that when the firm and investors are optimistic about the future (high μ and large b) the firm may decide to issue a lot of short-term debt hoping for the extra revenues from the new investment opportunities to cover the shortfall of revenues from the initial investment if the initial project turns out not to be as profitable as expected. However, in the crisis, the new investment opportunity may become less profitable or even disappear. This would cause many firms to default while others would face a higher cost of rolling over its debt. The effects of decrease in profitability of the new investment (or its disappearance) are the same as the effects of deterioration in the secondary markets (see in section 4.2).

6 Conclusion

In this paper I analyzed the choice of debt maturity structure and the refinancing decisions by the firm and investors in an environment where: (1) the firm default risk depends on the rollover decisions of investors (even in the absence of refinancing risk), (2) the short-term investors are subject to strategic uncertainty when making their rollover decisions, (3) the firm can choose freely face values in order to minimize the rollover and default risks and (4) the face values of debt affect directly investors' decisions. These features of the model, in particular strategic uncertainty and endogeneity of the face values, lead to very rich refinancing decisions by the firm and the creditors.

To capture strategic uncertainty I model the rollover game as a global game. I show that in the global game model, even in the absence of rollover risk, the strategic uncertainty

²⁴This zero-one nature of debt overhang follows from the fact that in my model all uncertainty is resolved at $t = 1$ while in Diamond and He [16] the resolution of uncertainty is gradual.

²⁵That is debt overhang measured at $t = 0$ at the time the firm is deciding its maturity structure.

among investors can make them withdraw their funds early, forcing the firm to liquidate a large fraction of its investment and resulting in future default. In order to provide incentives to investors to roll over their loans the firm sets a high face value for the debt issued in the intermediate period. I show that the higher face value has two opposite effects on the rollover decisions of the creditors. On the one hand, a higher face value makes the payoff from rolling over conditional on survival of the firm higher making rolling over more attractive. On the other hand, a higher face value increases the debt burden of the firm making a default more likely. I find that the former effect dominates up to a given threshold level (corollary 1).

The model does not admit a closed form solution and hence, when analyzing the firm's choice of debt maturity, I resort to numerical analysis. The numerical examples presented in the paper suggest that the presence of strategic uncertainty discourages the firm from issuing short-term debt - the optimal amount of short-term debt is lower than in the model without strategic uncertainty. How much lower depends on the parameters of the model. In particular, I explore the effects of different liquidation values of the project on the optimal maturity structure in section 4.1 and find that a lower liquidation value results in less short-term debt being issued up to the point where all of the investment at $t = 0$ is financed with long-term debt.

The model presented in the paper proves to be very flexible and I use it to answer two additional questions. First, I study the effect of a deterioration in secondary market conditions. I show that if the deterioration is unexpected, it can push solvent firms into default and increase the costs of refinancing to others (section 4.2). Since the maturity structure in this situation is ex-post inefficient this may lead an outside observer to conclude that firms do not choose their maturity optimally. This, however, ignores the fact that maturity was chosen when no agent expected a sudden deterioration in market conditions. Second, I use the model to contribute to the literature on debt overhang (section 5.1). I confirm the finding of Diamond and He [16] that short-term debt leads to a stronger debt overhang problem than long-term debt in the presence of default risk. Despite this, I find that the presence of new investment also decreases strategic uncertainty faced by investors making short-term debt effectively cheaper. Which effect dominates depends on the parameters of the model.

The model abstracts from two important issues. First, I do not consider the signaling effect of the firm's choice of face value in the intermediate period. This is an important problem since firms have access to information that is unavailable to outside investors and can use the face values to influence investors' rollover decisions. Secondly, the assumed market structure during the rollover game is very simple. In reality, the firm can issue debt to new investors and long-term investors can sell their debt in the secondary market. Allowing for these options may change the outcome of the rollover game in interesting ways. For example, the price in the secondary market for the long-term debt may contain useful information for the short-term creditors influencing their decisions. Such a model may lead to testable predictions between the prices of the long-term and short-term debt. I leave these issues for further research.

A Benchmark Model: Proofs

In this Appendix I provide the proofs of the results stated in section 3 when analyzing the global game model. In section A.1 I discuss assumption 2 made in the paper and explain why it is necessary. Section A.2 contains the solution to the rollover game when $\alpha F_S^1 \leq V$ while in section A.3 I provide the solution to the rollover game when $\alpha F_S^1 > V$. Finally, section A.4 contains a proof of Proposition 2 while section A.5 contains a proof of Proposition 3.

A.1 The upper dominance region

The usual argument that is used to prove uniqueness of equilibrium (as developed by Carlsson and van Damme [8], and Morris and Shin [29]) requires presence of global strategic complementarities and dominance regions.²⁶ For $\alpha F_S^1 < V$ the model has both lower dominance and upper dominance regions. In particular, when $s < \alpha F_S^2 + (1 - \alpha) F_L$ it is always optimal to withdraw funds early while when $s > \frac{(1-\alpha)F_L}{1-\frac{\alpha F_S^1}{V}}$ (and $F_S^2 > F_S^1$ but F_S^2 small enough) it is strictly dominant to rollover the loan. However, as $\alpha \rightarrow \frac{V}{F_S^1}$ this upper region disappears. The lack of upper dominance region implies that in addition to the equilibrium stated in the section 3 there is an additional equilibrium. In this equilibrium everyone withdraws early regardless of their signal and the firm always defaults. There are two ways to deal with this problem. First, I can impose additional restriction on equilibrium, namely if I require that the creditors use strategies that depend on their signal, this equilibrium disappears (see Goldstein and Pauzner [18]). The other way is to impose upper dominance region exogenously. The latter strategy is followed by Dasgupta [12] and Goldstein and Pauzner [18]. I use their approach in this appendix, that is I assume that there exists an exogenous \bar{s} such that if $s \geq \bar{s}$ then the project matures early, and the firm is able to repay all of its debt. Therefore, for $s \geq \bar{s}$ rolling over is a dominant strategy (and strictly dominant whenever $F_S^2 > F_S^1$). This assumption ensures that rolling over always is optimal for large enough s .

Assumption 4 *There exists \bar{s} such that for all $s \geq \bar{s}$ the project matures at $t = 1$ and the firm can repay its debt.*

It is important to note though, that whether I impose the additional restriction on the equilibrium strategies or I change the model to add the upper dominance region the outcome of the game does not change substantially. This is because the upper dominance region can be made arbitrary far from the mean return - what matters is the fact that it exists rather than its size.

The other technical issue I face is the fact that our game does not feature global strategic complementarities but rather a much weaker single-crossing property. This requires us to confine our attention to equilibria in monotone strategies. While Dasgupta [12] and Goldstein and Pauzner [18] proved uniqueness of equilibria in games with single-crossing property in

²⁶existence of regions in which one of the two actions that can be chosen in the coordination game is dominant.

their particular setups, as pointed out by Morris and Shin [31] single crossing condition is in general not sufficient to ensure uniqueness. Moreover, their result make use the fact that the noise is uniformly distributed and it remains an open question whether similar results can be proven for the case of the other distribution.

A.2 Rollover Game With No Rollover Risk ($\alpha F_S^1 \leq V$)

Take all the maturity structure $(\alpha, 1 - \alpha)$ and the face values (F_S^1, F_S^2, F_L) as given. I show first that there, under some conditions on F_S^2 there exists a region where if everyone rolls over the firm is able to repay its debt but if everyone withdraws then the firm will default.²⁷ I assume first that these conditions are satisfied and prove uniqueness of equilibrium in monotone strategies. Finally, I show that in any equilibrium the conditions under which there exists multiplicity region are necessary satisfied.

We know that if everyone rolls over the firm will not default if and only if

$$s - \alpha F_S^2 - (1 - \alpha) F_L \geq 0$$

On the other hand, if everyone withdraws then the firm is left with $(1 - \alpha F_S^1/V) s$ and thus it will repay its debt if and only if

$$\left(1 - \frac{\alpha F_S^1}{V}\right) s - (1 - \alpha) F_L \geq 0$$

Therefore, the existence of the intermediate region with multiple equilibria requires that

$$F_S^2 < \frac{F_S^1 (1 - \alpha) F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)} \quad (4)$$

If F_S^2 satisfies the above condition then for all $s \geq \alpha F_S^2 + (1 - \alpha) F_L$ the expression

$$\left(1 - m \frac{\alpha F_S^1}{V}\right) s - (1 - m) \alpha F_S^2 - (1 - \alpha) F_L$$

is strictly decreasing in m and crosses zero exactly once.²⁸

Suppose for now that $F_S^2 < \frac{F_S^1 (1 - \alpha) F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)}$. The equilibrium in monotone strategies is characterized by two thresholds, s^* and x^* , such that an agent rolls over his loan if and only if he receives a signal $x_i \geq x^*$ and the firm defaults if and only if $s < s^*$ and repays its debt otherwise.

²⁷In other words, if all investors had perfect information then there will be region of s with two pure strategy equilibria, one in which everyone withdraws and one in which everyone rolls over.

²⁸In the case when $s < \alpha F_S^2 + (1 - \alpha) F_L$ it is possible that $(1 - m \alpha F_S^1/V) s - (1 - m) \alpha F_S^2 - (1 - \alpha) F_L$ is increasing, but it is then negative for all m . Hence the game satisfies the single crossing condition in m .

If all agents follow monotone strategies with a symmetric cutoff x^* then, for a given s , the proportion of short-term debt holders that withdraws early is given by

$$\Phi\left(\frac{x^* - s}{\tau_x^{-1/2}}\right)$$

I saw above that the firm will default regardless of m whenever $s < \alpha F_S^2 + (1 - \alpha) F_L$. Therefore, it has to be the case that $s^* \geq \alpha F_S^2 + (1 - \alpha) F_L$. But then it follows that

$$\left(1 - \Phi\left(\frac{x^* - s}{\tau_x^{-1/2}}\right) \frac{\alpha F_S^1}{V}\right) s - \left(1 - \Phi\left(\frac{x^* - s}{\tau_x^{-1/2}}\right)\right) \alpha F_S^2 - (1 - \alpha) F_L$$

is increasing in s and so for every x^* there exists unique s^* .²⁹

The equilibrium in monotone strategies is then characterized by the following two conditions:

$$\left(1 - \Phi\left(\frac{x^* - s^*}{\tau_x^{-1/2}}\right) \frac{\alpha F_S^1}{V}\right) s^* - \left(1 - \Phi\left(\frac{x^* - s^*}{\tau_x^{-1/2}}\right)\right) \alpha F_S^2 - (1 - \alpha) F_L = 0 \quad (5)$$

$$F_S^2 \int_{s^*}^{\infty} (\tau_x + \tau_y)^{-1/2} \phi\left(\frac{s - \frac{\tau_x x^* + \tau_y y}{\tau_x + \tau_y}}{\tau_x^{-1/2}}\right) ds = F_S^1 \quad (6)$$

Solving (??) for x^* I get

$$x^* = \frac{\tau_x + \tau_y}{\tau_x} s^* - \frac{\tau_x}{\tau_y} y + \frac{(\tau_x + \tau_y)^{1/2}}{\tau_x} \Phi^{-1}\left(\frac{F_S^1}{F_S^2}\right)$$

implying that at $s = s^*$ the proportion of short-term investors that withdraw early is given by

$$\Phi\left(\frac{\tau_y (s^* - y)}{\tau_x^{1/2}} + \left(\frac{\tau_x + \tau_y}{\tau_x}\right)^{1/2} \Phi^{-1}\left(\frac{F_S^1}{F_S^2}\right)\right)$$

Substituting this into the Critical Mass condition and taking the limit as $\frac{\tau_y}{\tau_x} \rightarrow 0$ we get

$$\left(1 - \frac{\alpha F_S^1}{V} \frac{F_S^1}{F_S^2}\right) s^* - \left(1 - \frac{F_S^1}{F_S^2}\right) \alpha F_S^2 - (1 - \alpha) F_L = 0$$

Rearranging the above equation yields:

$$s^* = \frac{(1 - \alpha) F_L + \alpha (F_S^2 - F_S^1)}{\left(1 - \frac{\alpha F_S^1}{V} \frac{F_S^1}{F_S^2}\right)}$$

²⁹Here, we use the fact that the above condition is decreasing in m and, in a monotone equilibrium, m is decreasing in s .

Moreover,

$$\lim_{\substack{\tau_x \rightarrow \infty \\ \tau_y/\tau_x \rightarrow 0}} x^* = s^*$$

This shows that if $F_S^2 < \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)}$ then as $\frac{\tau_y}{\tau_x} \rightarrow 0$ there is a unique equilibrium in monotone strategies such that

$$x^* = s^* = \frac{(1-\alpha)F_L + \alpha(F_S^2 - F_S^1)}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}}$$

It remains to show that $F_S^2 \geq \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)}$ can never be optimal. To see that note that if $F_S^2 \geq \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)}$ then the profits of the firm are maximized if everyone rolls over whenever $s > \frac{V F_S^2}{F_S^1}$ and nobody rolls over if $s < \frac{V F_S^2}{F_S^1}$ so that the firm's profit is bounded above by

$$\int_{\bar{s}}^{\frac{V F_S^2}{F_S^1}} \left[\left(1 - \frac{\alpha F_S^1}{V}\right) s - (1-\alpha)F_L \right] \tau^{\frac{1}{2}} \phi \left(\frac{s - \mu}{\tau^{-1/2}} \right) ds + \int_{\frac{V F_S^2}{F_S^1}}^{\infty} [s - \alpha F_S^2 - (1-\alpha)F_L] \tau^{\frac{1}{2}} \phi \left(\frac{s - \mu}{\tau^{-1/2}} \right) ds$$

where $\bar{s} = \frac{(1-\alpha)F_L}{1 - \frac{\alpha F_S^1}{V}}$. On the other hand, note that for any $1 < \widehat{F}_S^2 < \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)}$ the equilibrium expected profit is given by

$$\int_{s^*}^{\infty} [s - \alpha \widehat{F}_S^2 - (1-\alpha)F_L] \tau^{1/2} \phi \left(\frac{s - \mu}{\tau^{-1/2}} \right) ds$$

where $s^* < \bar{s}$ is a threshold determined by equations (5) and (6). Moreover, $\forall s \geq \bar{s}$ we have

$$\begin{aligned} & \left[\left(1 - \frac{\alpha F_S^1}{V}\right) s - (1-\alpha)F_L \right] - [s - \alpha \widehat{F}_S^2 - (1-\alpha)F_L] \\ &= -\frac{\alpha F_S^1}{V} s + \alpha \widehat{F}_S^2 < -\frac{\alpha F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)} + \alpha \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)} = 0 \end{aligned}$$

implying that the firm earns higher profits by setting $\widehat{F}_S^2 < \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)} \leq F_S^2$ and hence will never find it optimal to set $F_S^2 > \frac{F_S^1 (1-\alpha)F_L}{V \left(1 - \frac{\alpha F_S^1}{V}\right)}$.

Proposition 1 follows immediately from the above discussion.

Proposition 1 *Let $\frac{\tau_y}{\tau_x} \rightarrow 0$ as $\tau_x \rightarrow \infty$ and $\tau_y \rightarrow \infty$. Then the rollover game has a unique equilibrium in monotone strategies in which all investors use threshold strategies with cutoff*

x^* and the firm defaults if and only if $s < s^*$ where

$$x^* = s^* = \frac{(1 - \alpha) F_L + \alpha (F_S^2 - F_S^1)}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}}$$

Corollary 1 follows immediately by minimizing s^* with respect to F_S^2 .

A.3 Rollover Game With Rollover Risk ($\alpha F_S^1 > V$)

I assume now that $\alpha F_S^1 > V$ and therefore if $m > \frac{V}{\alpha F_S^1}$ the firm defaults at $t = 1$. In this case, the firm repays its debt if and only if

$$\left(1 - \min \{m, 1\} \frac{\alpha F_S^1}{V}\right) s - (1 - \min \{m, 1\}) \alpha F_S^2 - (1 - \alpha) F_L \geq 0 \quad (7)$$

Note that all $m \in \left[0, \frac{V}{\alpha F_S^1}\right]$ the derivative of the l.h.s. of the above equation is given by

$$-\frac{\alpha F_S^1}{V} s + \alpha F_S^2$$

and, thus, for a given s , the above equation is either decreasing or increasing in m . Since $\frac{\alpha F_S^1}{V} > 1$ it follows that for all $s \geq \alpha F_S^2 + (1 - \alpha) F_L$ the derivative of equation (7) is strictly negative for all $m \in \left[0, \frac{V}{\alpha F_S^1}\right]$. If $s < \alpha F_S^2 + (1 - \alpha) F_L$ then regardless of the sign of the derivative, I have $\left(1 - \min \{m, 1\} \frac{\alpha F_S^1}{V}\right) s - (1 - \min \{m, 1\}) \alpha F_S^2 - (1 - \alpha) F_L < 0$. Therefore, it follows that the Critical Mass condition satisfied the single crossing property.

It is easy to show that given assumption A.2 all short-term debt holders always withdrawing irrespective of signals is an equilibrium. Thus, I assume that investors follow monotone strategies with threshold $x^* < \infty$. The main difference between the case when $\alpha F_S^1 > V$ and the case of no rollover risk is that now the firm can default at $t = 1$. Let s_D be the return from investment such that if $s < s_D$ then the firm defaults early at $t = 1$. The threshold s_D is defined by

$$\Phi \left(\frac{x^* - s_D}{\tau_x^{-1/2}} \right) = \frac{V}{\alpha F_S^1}$$

that is at $s = s_D$ the fraction of short-term debt holders withdrawing early is such that in order to satisfy early withdrawals the firm has to liquidate the whole project. Rearranging the above equation, we get

$$s_D = x^* - \tau_x^{-1/2} \Phi^{-1} \left(\frac{V}{\alpha F_S^1} \right) \quad (8)$$

In the case of the rollover risk the Critical mass condition is given by

$$\left(1 - \min \left\{ m(s^*, x^*), \frac{V}{\alpha F_S^1} \right\} \frac{\alpha F_S^1}{V} \right) s^* - \left(1 - \min \left\{ m(s^*, x^*), \frac{V}{\alpha F_S^1} \right\} \right) \alpha F_S^2 - (1 - \alpha) F_L = 0$$

and the Payoff indifference condition is³⁰

$$F_S^2 \int_{s^*}^{\infty} (\tau_x + \tau_y)^{-1/2} \phi \left(\frac{s - \mu}{\tau_x^{-1/2}} \right) ds = F_S^1 \int_{s_D}^{\infty} (\tau_x + \tau_y)^{-1/2} \phi \left(\frac{s - \mu}{\tau_x^{-1/2}} \right) ds$$

Consider first the Payoff Indifference condition. Evaluating the integrals and rearranging we get

$$x^* = \frac{\tau_x + \tau_y}{\tau_x} s^* - \frac{\tau_y}{\tau_x} y + (\tau_x + \tau_y)^{-\frac{1}{2}} \Phi^{-1} \left(\frac{F_S^1 \Phi \left(\frac{\frac{\tau_y}{\tau_x + \tau_y} (y - x^*) + \tau_x^{-\frac{1}{2}} \Phi^{-1} \left(\frac{V}{\alpha F_S^1} \right)}{(\tau_x + \tau_y)^{-\frac{1}{2}}} \right)}{F_S^2} \right) \right)$$

Therefore, at s^* , the proportion of agents that withdraw early, $m(s^*, x^*)$, is given by

$$m(s^*, x^*) = \Phi \left(\frac{\frac{\tau_y}{\tau_x^{\frac{1}{2}}} (s^* - y) + \left(\frac{\tau_x}{\tau_x + \tau_y} \right)^{\frac{1}{2}} \Phi^{-1} \left(\frac{F_S^1 \Phi \left(\frac{\frac{\tau_y}{\tau_x + \tau_y} (y - x^*) + \tau_x^{-\frac{1}{2}} \Phi^{-1} \left(\frac{V}{\alpha F_S^1} \right)}{(\tau_x + \tau_y)^{-\frac{1}{2}}} \right)}{F_S^2} \right)}{\tau_x^{\frac{1}{2}}} \right) \right)$$

Taking limit as $\frac{\tau_y}{\tau_x} \rightarrow 0$ we get

$$\lim_{\frac{\tau_y}{\tau_x} \rightarrow 0} m(s^*, x^*) = \frac{V}{\alpha F_S^2}$$

Using this in the Critical mass we get

$$\left(1 - \frac{V}{\alpha F_S^2} \frac{\alpha F_S^1}{V} \right) s^* - \alpha \left(1 - \frac{V}{\alpha F_S^2} \right) F_S^2 - (1 - \alpha) F_L = 0$$

or

$$s^* = \frac{(1 - \alpha) F_L - \alpha \left(F_S^2 - \frac{V}{\alpha} \right)}{1 - \frac{F_S^1}{F_S^2}}$$

It is important to note that the above Critical Mass condition characterizes s^* only if $s^* \leq \bar{s}$ since by assumption the firm cannot default for $s \geq \bar{s}$. If the solution to the critical mass condition is greater than \bar{s} then the monotone equilibrium has to be such that $s^* = \bar{s}$ implying that in the unique monotone equilibrium I have

$$s^* = \min \left\{ \frac{(1 - \alpha) F_L - \alpha \left(F_S^2 - \frac{V}{\alpha} \right)}{1 - \frac{F_S^1}{F_S^2}}, \bar{s} \right\}$$

Finally, from the expression for x^* I have

$$\lim_{\substack{\tau_x \rightarrow \infty \\ \tau_y / \tau_x \rightarrow 0}} x^* = s^*$$

The above discussion is summarized in the following proposition:

³⁰It is easy to see that the Critical mass condition implies that $s^* > s_D$, since at $s^* = s_D$ the left hand side of the "critical mass" condition is less than zero. Moreover, for all $s > \alpha F_S^1 + (1 - \alpha) F_L$ the Critical Mass condition is decreasing in m , for $m \in \left[0, \frac{V}{\alpha F_S^1} \right]$.

Proposition 2 Let $\frac{\tau_x}{\tau_y} \rightarrow 0$, $\tau_x \rightarrow \infty$ and $\tau_y \rightarrow \infty$. Then the rollover game has a unique equilibrium in monotone strategies in which all investors use threshold strategies with cutoff x^* and the firm defaults if $s < s^*$ where

$$x^* = s^* = \min \left\{ \frac{(1 - \alpha) F_L + \alpha (F_S^2 - F_S^1)}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}}, \bar{s} \right\}$$

Again, Corollary 2 follows immediately from the above Proposition.

A.4 Optimal choice of F_S^2

In this section I provide a proof of the following proposition:

Proposition 3 Suppose that $\tau_x \rightarrow \infty$, $\tau_p \rightarrow \infty$ and that $\frac{\tau_p}{\tau_x} \rightarrow 0$. Then the optimal choice of face value F_S^2 converges to

- $F_S^2 = F_S^1$ if $s > \bar{s}(\alpha)$
- F_S^2 that solves $s^*(F_S^2; \alpha) = s$ if $s \in [s_{\min}^*(F_S^2; \alpha), \bar{s}(\alpha)]$

Proof. First note that as $\tau_p \rightarrow \infty$ I have $y \rightarrow s$ and $\tau_y \rightarrow \infty$.

Firm's problem at $t = 1$ is given by

$$\begin{aligned} \max_{F_S^2} \int_{s^*}^{\infty} (s - \alpha F_S^2 - (1 - \alpha) F_L) \tau_y^{1/2} \phi \left(\frac{s - y}{\tau_y^{-1/2}} \right) ds \\ \text{s.t. } F_S^2 \geq F_S^1 \end{aligned}$$

Note that maximizing profits with respect to F_S^2 is equivalent with maximizing profits with respect to s^* . This is because the firm will never find it optimal to set $F_S^2 > \bar{F}_S^2$ where $\bar{F}_S^2 = \arg \min_{F_S^2} s^*(F_S^2)$ and for $F_S^2 \in [F_S^1, \bar{F}_S^2]$ there is one-to-one mapping from a choice of F_S^2 to s^* .

Let $s_{\min}^* = s^*(\bar{F}_S^2)$ and $s_{\max}^* = \bar{s}$ where \bar{s} is the threshold above which agents always roll over (see discussion of the rollover game). Then the firm's problem can be written as

$$\max_{s^* \in [s_{\min}^*, s_{\max}^*]} \int_{s^*}^{\infty} (s - \alpha F_S^2 - (1 - \alpha) F_L) \tau_y^{1/2} \phi \left(\frac{s - y}{\tau_y^{-1/2}} \right) ds$$

Before proceeding, define a profit function $\pi : [s_{\min}^*, s_{\max}^*] \rightarrow \mathbb{R}$ by

$$\begin{aligned} \pi(s^*) &= \int_{s^*}^{\infty} (s - \alpha F_S^2 - (1 - \alpha) F_L) \tau_y^{1/2} \phi \left(\frac{s - y}{\tau_y^{-1/2}} \right) ds \\ &= (y - \alpha F_S^2 - (1 - \alpha) F_L) \left[1 - \Phi \left(\frac{s^* - y}{\tau_y^{-1/2}} \right) \right] + \tau_y \phi \left(\frac{s^* - y}{\tau_y^{-1/2}} \right) \end{aligned}$$

Consider the optimal choice of s^* as $\tau_y \rightarrow \infty$. Denote by s_∞^* the limit of s^* as $\tau_y \rightarrow \infty$. I first argue that if $s \in (s_{\min}^*, s_{\max}^*)$ then $\lim_{\tau_y \rightarrow \infty} s^* = s$.

Consider a sequence of s^* such that $\lim_{\tau_y \rightarrow \infty} s^* = s$ and $\frac{s^* - y}{\tau_y^{-1/2}} \rightarrow 0$. Then, as $\tau_y \rightarrow \infty$ firm's profit converges to

$$\pi(s) = s - \alpha F_S^2(s) - (1 - \alpha) F_L$$

I show now that any sequence with limit other than s generates lower profits for high enough τ_y . To see that take a sequence of s^* that converges to $s_\infty^* < s$. Then along this sequence, as $\tau_y \rightarrow \infty$, firm's profit converges to

$$\begin{aligned} \pi(s_\infty^*) &= s - \alpha F_S^2(s_\infty^*) - (1 - \alpha) F_L \\ &< s - \alpha F_S^2(s) - (1 - \alpha) F_L \\ &= \pi(s) \end{aligned}$$

where the inequality follows from the fact that $s_\infty^* < s$ and therefore $F_S^2(s_\infty^*) > F_S^2(s)$ (a lower threshold is associated with higher face value F_S^2). It follows that this sequence generates lower profit to the firm for large enough τ_y compared to the proposed sequence that converges to s .

Consider now a sequence that converges to $s_\infty^* > s$. Along this sequence the profit to the firm converges to 0 since $y \rightarrow s < s_\infty^*$ implying that $1 - \Phi\left(\frac{s^* - y}{\tau_y^{-1/2}}\right)$ converge to 0. Therefore, any such sequence is dominated for large enough τ_y by a sequence of s^* that has limit $s^* \leq s$.

Therefore, I conclude that for $s \in (s_{\min}^*, s_{\max}^*)$ I have $\lim_{\tau_y \rightarrow \infty} s^* = s$ implying that in the limit F_S^2 solves $s^*(F_S^2; \alpha) = s$.

I now consider a situation where $s \notin (s_{\min}^*, s_{\max}^*)$. Suppose first that $s = s_{\min}^*$. Then the same argument as above establishes that $\lim_{\tau_y \rightarrow \infty} s^* = s$ (with the difference that in this case I only need to show $\lim_{\tau_y \rightarrow \infty} s^* > y$ is not optimal).

Finally, consider $s \geq s_{\max}^*$. Note that in this case, the firm will always survive for any $F_S^2 > F_S^1$. Moreover, note that higher F_S^2 always decreases profits. Therefore, by the same argument as in the case when $s \in (s_{\min}^*, s_{\max}^*)$ I can show that for any sequence s^* that converges to $s_\infty^* < s_{\max}^*$ I can find a sequence of s^* that converges to \hat{s}^* where $s_\infty^* < \hat{s}^* \leq s_{\max}^*$ and for large enough τ_y it generates higher profits than the sequence converging to s_∞^* . Since this holds for any $s_\infty^* < s_{\max}^*$ it follows that for $s \geq s_{\max}^*$ it has to be the case that $\lim_{\tau_y \rightarrow \infty} s^* = s_{\max}^*$. The proposition then follows from the fact that $F_S^2(s_{\max}^*) = F_S^1$. ■

B Debt Overhang: Proofs

The only difference between the rollover game in the model with debt overhang compared to the benchmark global game model is the fact that the firm will now default at $t = 2$ if and only if

$$\left(1 - m \frac{\alpha F_S^1}{V}\right) s + b - \alpha F_S^2 - (1 - \alpha) F_L \geq 0$$

Otherwise, the rollover game is the same as before and therefore following the same steps as in Appendix B I arrive at the following proposition:

Proposition 4 *Let $\tau_x \rightarrow \infty$, $\tau_p \rightarrow \infty$ and $\frac{\tau_s}{\tau_p} \rightarrow 0$. Then the unique equilibrium in monotone strategies is characterized by a pair of thresholds (x^*, s^*) where:*

$$\begin{aligned} x^*(\alpha) = s^*(\alpha) &= \frac{(1-\alpha)F_L + \alpha(F_S^2 - F_S^1) - b}{1 - \frac{\alpha F_S^1 F_S^1}{V F_S^2}} \quad \text{if } \alpha F_S^1 \leq V \\ x^*(\alpha) = s^*(\alpha) &= \frac{(1-\alpha)F_L + \alpha\left(F_S^2 - \frac{V}{\alpha}\right) - b}{1 - \frac{F_S^1}{F_S^2}} \quad \text{if } \alpha F_S^1 > V \end{aligned}$$

C Model without strategic uncertainty

In this appendix I describe briefly the solution to the model without strategic uncertainty (i.e. the model where both the firm and investors learn the true s at $t = 1$).

To solve the model I use backward induction. In section C.1 I characterize firm's optimal default decision at time $t = 2$. In section C.2 I analyze jointly firm's optimal choice of face value for short-term debt issued at $t = 1$ and investors rollover decisions. Finally, in section C.3 I solve for the optimal structure. To save on space the proofs of the results stated below are omitted and are available by request.

C.1 Period $t = 2$

Consider period $t = 2$ and let m be the fraction of short-term debt holders that chose to withdraw early. Then given the maturity structure $(\alpha, 1 - \alpha)$ and the face values F_S^2 and F_L the firm will default at $t = 2$ if and only if

$$\left(1 - \frac{m\alpha F_S^1}{V}\right) s - (1 - m)\alpha F_S^2 - (1 - \alpha)F_L < 0$$

where $\frac{m\alpha F_S^1}{V}$ is the fraction of investment that has been liquidated to cover early withdrawals at $t = 1$.³¹

C.2 Period $t = 1$: Rollover game and firm's choice of F_S^2

At the beginning of period $t = 1$ all agents learn the payoff from the project, s . The firm proposes a payoff F_S^2 to all short-term creditors who roll over their debt. Knowing s , and taking as given the promised face values F_S^2 and F_S^1 as well as the maturity structure, investors choose whether to roll over their loans or whether to withdraw their funds early. As in the case of the model with strategic uncertainty, I need to analyze separately agents choices in the case of no rollover risk ($\alpha F_S^1 \leq V$) and in the case of rollover risk ($\alpha F_S^1 > V$).

³¹Note that as long as the face value of short-term debt issued at $t = 1$ is not very high, i.e. $F_S^2 < \frac{s}{V} F_S^1$, more withdrawals means that the firm defaults at $t = 2$ for higher values of s .

C.2.1 No Rollover Risk ($\alpha F_S^1 \leq V$)

I consider first the case with no rollover risk. In this case there exist two types of pure strategy Nash equilibria. In the first one each agent expects all other agents to withdraw their funds early pushing the firm into default and therefore he finds it optimal to withdraw his funds. Faced with large withdrawals the firm is forced to rescale project so much that it will not be able to repay its long-term debt at $t = 2$ and hence will be forced to default. Therefore, agents expectations are fulfilled and their actions are indeed optimal. In the other equilibrium, each agent expects everyone to roll over and the firm to repay its debt and hence he finds it optimal to rollover his loan. Since no agent withdraws early, firm has enough funds to repay all of its debt justifying agents' decision to rollover. It turns out that for a range of payoffs from the project, s , both equilibria exist. The choice of F_S^2 , depends then the firm's expectation regarding which equilibrium will be played.

The proposition below fully characterizes all possible equilibria at $t = 1$ when there is no rollover risk.

Proposition 5 *Suppose that there is no rollover risk ($\alpha F_S^1 \leq V$). Then:*

1. *If $s < \underline{s}(\alpha, F_S^1, F_L)$ then in any symmetric pure strategy equilibrium, investors choose to withdraw their funds early regardless of F_S^2 , the firm defaults at $t = 2$ and $F_S^2 \in \mathbb{R}_+$;*
2. *If $s \in [\underline{s}(\alpha, F_S^1, F_L), \bar{s}(\alpha, F_S^1, F_L)]$ then there is continuum of equilibria, indexed by F_S^2 , in which agents roll over and an equilibrium with default:*
 - (a) *Firm sets $F_S^2 = \hat{F}_S^2 \in \left[F_S^1, \frac{s - (1 - \alpha)F_L}{\alpha} \right]$, all agents rollover their loans if and only if $F_S^2 \geq \hat{F}_S^2$ and the firm pays back all of its debt at $t = 2$*
 - (b) *$F_S^2 \in \mathbb{R}_+$, regardless of F_S^2 investors choose to withdraw their funds early, and the firm defaults at $t = 2$*
3. *If $s > \bar{s}(\alpha, F_S^1, F_L)$ then there is a unique pure strategy equilibrium in which firm sets $F_S^2 = F_S^1$, all agents rollover and the firm repays all its loans at $t = 2$*

The above proposition shows that I can partition the space of the returns from the project into three regions. First, for low returns ($s < \underline{s}$) irrespective of actions of the firm all investors choose to withdraw early and the firm defaults regardless whether agent choose to withdraw or not. When $s < \underline{s}$ the firm is insolvent. Then, there is an intermediate range ($s \in [\underline{s}, \bar{s}]$) where whether the firm defaults or not depend on the investors. If investors expect the firm to default they will withdraw their loans early and the firm will be forced to default. On the other hand if investor believe that the firm will repay its debt then the will roll over their loans and the firm will repay its debt. In this intermediate range the firm is solvent but it faces illiquidity risk. Finally, if $s > \bar{s}$ the firm will repay its debt regardless of the actions of the short-term investors.

C.2.2 Rollover Risk ($\alpha F_S^1 > V$)

In this case, for any s , the actions of the investors can push the firm into default at $t = 1$. This is because regardless of how high the return is, if everyone decides to withdraw then the firm will not be able to repay all of its short-term debt and hence it will be forced to default. The firm need αF_S^1 to repay its short-term debt while the most it can raise at $t = 1$ is V and by assumption I have $V < \alpha F_S^1$. The proposition below characterizes all possible symmetric pure strategy Nash equilibria in period $t = 1$.³²

Proposition 6 *Suppose that the firm faces rollover risk ($\alpha F_S^1 > V$). Then:*

1. *If $s < \underline{s}(\alpha, F_S^1, F_L)$ then in any symmetric pure strategy equilibrium, all investors choose to withdraw their funds early regardless of F_S^2 , the firm defaults at $t = 1$, and $F_S^2 \in \mathbb{R}_+$*
2. *if $s \geq \underline{s}(\alpha, F_S^1, F_L)$ then there is continuum of equilibria, indexed by F_S^2 , in which agents roll over and an equilibrium with default:*

- (a) *Firm sets $F_S^2 = \widehat{F}_S^2 \in \left[F_S^1, \frac{s-(1-\alpha)F_L}{\alpha} \right]$, all agents rollover their loans if and only if $F_S^2 \geq \widehat{F}_S^2$ and the firm pays back all of its debt at $t = 2$*
- (b) *the firm is indifferent between any $F_S^2 \in \mathbb{R}_+$, regardless of F_S^2 all agents withdraw early and the firm defaults at $t = 1$*

Note that in contrast to the no rollover risk case, here the firm is always vulnerable to early withdrawals and hence regardless of the profitability of the investment if all short-term debt holders coordinate on not rolling over they will push the firm into default.³³

C.3 Optimal Debt Maturity Structure

In this section I study the maturity structure choice of the firm. The firm maximizes its profit taking investors strategies and its own future behavior as given. The multiplicity of equilibria makes this problem ill-posed because without any further assumptions the firm is unable to form expectations regarding future play. To circumvent that problem I assume that whenever possible, investors coordinate on rolling over. This is a standard assumption in the literature, often made implicitly.

³²In contrast to the case when $\alpha F_S^1 \leq V$ there are also additional pure-strategy equilibria in which agents choose different strategy. In particular, for any s proportion $m > \frac{V}{\alpha F_S^1}$ withdrawing and $(1 - m)$ rolling over is equilibrium. This is a consequence of the assumption that when the firm defaults at $t = 1$ no investor gets anything.

³³In the case the firm default there are additional equilibria in which the fraction of investors that rolls over is strictly less than $1 - \frac{V}{\alpha F_S^1}$. This is the result of the assumption that the all the value from investment is lost in the bankruptcy procedure. In the analysis below we ignore these types of equilibria since in terms of the outcomes of the model they are equivalent to the equilibrium in which everyone withdraws early.

Assumption 5 *Whenever possible all agents coordinate on rolling over their loans.*

Under the above assumption the firm's problem at $t = 0$ can be written as³⁴

$$\begin{aligned} \max_{\alpha, F_S^1, F_L} \int_{\underline{s}}^{\infty} (s - \alpha F_S^1 - (1 - \alpha) F_L) \tau^{1/2} \phi \left(\frac{s - \mu}{\tau^{-1/2}} \right) ds \\ \text{s.t. participation constraints and} \\ \underline{s} = \alpha F_S^1 + (1 - \alpha) F_L \end{aligned}$$

that is the firm maximizes its profits subject to the participation constraints and taking into account that it will default if $s < \underline{s}$. The participation constraints are given by

$$\begin{aligned} F_S^1 &\geq 1 \\ F_L \Pr(s \geq \underline{s}) &\geq 1 \end{aligned}$$

in the case of no rollover risk, and

$$\begin{aligned} F_S^1 \Pr(s \geq \underline{s}) &\geq 1 \\ F_L \Pr(s \geq \underline{s}) &\geq 1 \end{aligned}$$

in the case of the rollover risk. Profit maximization implies that all participation constraint hold as equalities - higher promised face values decrease profits without having any positive influence on investors' behavior.

Note that the fact that in the equilibrium all investors break even implies that as long as the fraction of the project financed with short-term debt, α , is smaller than the liquidation value, V , the face value of short-term debt offered by the firm at $t = 0$ is $F_S^1 = 1$. Moreover, when $\alpha > V$ the participation constraints imply that $F_S^1 = F_L > 1$ since it has to be the case that $\underline{s} > 1$. It follows that as α increases above V the cost of short-term debt jumps upwards suggesting that the optimal maturity structure will have $\alpha \leq V$. It turns out that the optimal maturity structure is given by $(V, 1 - V)$.

Proposition 7 *The face values of the debt are:*³⁵

1. *No rollover case ($\alpha \leq V$): $F_S^2 = F_S^1 = 1$ and F_L is the smallest solution to*

$$F_L \left(1 - \Phi \left(\frac{\alpha + (1 - \alpha) F_L - \mu}{\tau^{-1/2}} \right) \right) = 1$$

2. *Rollover case ($\alpha > V$): $F_S^1 = F_S^2 = F_L$ where F_L is the smallest solution to*

$$F_L \left(1 - \Phi \left(\frac{F_L - \mu}{\tau^{-1/2}} \right) \right) = 1$$

³⁴To make notation easier we suppress the dependence of \underline{s} on α , F_S^1 and F_L throughout this section.

³⁵Here, we assumed with out the loss of generality that when $s \leq \underline{s}$ then the firm chooses $F_S^2 = F_S^1$. As explained above, in this case any choice of F_S^2 is consistent with equilibrium. Since the firm gets nothing when it defaults this assumption does not affect firm's behavior at any point in time.

The above face values imply that the optimal maturity structure is given by $(V, 1 - V)$, that is it is optimal to finance the fraction V of the project with short-term debt and the remaining part with the long-term debt.

Finally, Theorem 1 characterizes the unique subgame perfect equilibrium of the model.

Theorem 1 *Under Assumption 1 the unique subgame perfect Nash equilibrium of the game is given by*

1. the maturity structure $(V, 1 - V)$
2. the face values specified in Proposition 3
3. agents rollover decisions $r^*(s)$ such that:

$$r^*(s) = \begin{cases} \text{roll over if } s \geq \underline{s} \\ \text{withdraw if } s < \underline{s} \end{cases}$$

where $\underline{s} = \alpha F_S^1 + (1 - \alpha) F_L$

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