

Chapter 8

**THE TREATMENT OF SEASONALITY IN THE
COST-OF-LIVING INDEX:
AN INTRODUCTION**

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1. Introduction

This paper addresses the problem of constructing a monthly (or annual) consumer price or cost of living index given that there are seasonal fluctuations for important product groups. Strongly seasonal products are available only at certain times of year. Weakly seasonal products are available all year, but have substantial fluctuations in prices or quantities synchronized with the time of year. Strongly seasonal products are usually excluded from official price indexes. The included seasonal products are usually just the weakly seasonal ones. In conventional practice, component indexes are compiled using observed data and then are seasonally adjusted.²

The approach suggested here is a radical departure from conventional practice. Basically, each seasonal commodity is treated as a separate commodity in each of the designated within-year time periods (e.g., months) that the commodity is available, and then normal index number techniques are applied. This removes the need to seasonally adjust the indexes compiled using the observed data; these series are already seasonally adjusted by virtue of their construction. The proposed approach is easier to understand than most seasonal adjustment methods. The results are also replicable in the sense that different individuals who independently and correctly apply the proposed approach to the same data will produce similar results.

The methods are illustrated using the Turvey artificial seasonal dataset. Turvey (1979) invented four years of monthly price and quantity data for five seasonal products: apples, peaches, grapes, strawberries and oranges. His data are shown in table 1. Turvey sent his artificial data to statistical agencies around the world. He asked them to apply their usual techniques to construct monthly, and annual average, price indexes. About 20 countries did as asked, and Turvey tabled their estimated indexes. Turvey (1979, p. 13) finds that: “[T]he monthly indices display very large differences, e.g., a range of 129.12-169.5 in June....”

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² See Dagum (1983) regarding the seasonal adjustment procedures of Statistics Canada using X-11-ARIMA. While the CPI component (and variant) indexes are often seasonally adjusted, in official practice the CPI itself is not.

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W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura,
PRICE AND PRODUCTIVITY MEASUREMENT: Volume 2 -- Seasonality. Trafford Press.

Table 1. The Turvey Artificial Data Set: Prices (p) and Quantities (x) of Fruits Sold

		Months (m)											
		1	2	3	4	5	6	7	8	9	10	11	12
		1970 (t=0)											
Apples	p	1.14	1.17	1.17	1.40	1.64	1.75	1.83	1.92	1.38	1.10	1.09	1.10
	x	3,086	3,765	4,363	4,842	4,439	5,323	4,165	3,224	4,025	5,784	6,949	3,924
Peaches	p	--	--	--	--	--	3.15	2.53	1.76	1.73	1.94	--	--
	x						91	498	6,504	4,923	865		
Grapes	p	2.48	2.75	5.07	5.00	4.98	4.78	3.48	2.01	1.42	1.39	1.75	2.02
	x	82	35	98	26	75	82	1,490	2,937	2,826	1,290	338	
Strawberries	p	--	--	--	--	5.13	3.48	3.27	--	--	--	--	--
	x					700	2,709	1,970					
Oranges	p	1.3	1.25	1.21	1.22	1.28	1.33	1.45	1.54	1.57	1.61	1.59	1.31
	x	10,266	9,656	7,940	5,110	4,089	3,362	3,396	2,406	2,486	3,222	6,958	9,762
		1971 (t=1)											
Apples	p	1.25	1.36	1.38	1.57	1.77	1.86	1.94	2.02	1.55	1.34	1.33	1.30
	x	3,415	4,127	4,771	5,290	4,986	5,869	4,671	3,534	4,509	6,299	7,753	4,285
Peaches	p	--	--	--	--	--	3.77	2.85	1.98	1.80	1.95	--	--
	x						98	548	6,964	5,370	932		
Grapes	p	2.80	3.32	5.48	5.67	5.44	5.30	3.93	2.33	1.66	1.64	2.10	2.35
	x	85	32	10	8	53	80	94	1,583	3,021	2,984	1,308	354
Strawberries	p	--	--	--	--	5.68	3.72	3.78	--	--	--	--	--
	x					806	3,166	2,153					
Oranges	p	1.35	1.36	1.37	1.44	1.51	1.56	1.66	1.74	1.76	1.77	1.76	1.50
	x	10,888	10,314	8,797	5,590	4,377	3,681	3,748	2,649	2,726	3,477	3,548	10,727
		1972 (t=2)											
Apples	p	1.43	1.53	1.59	1.73	1.89	1.98	2.07	2.12	1.73	1.56	1.56	1.49
	x	3,742	4,518	5,134	5,738	5,498	6,420	5,157	3,881	4,917	6,872	8,490	5,211
Peaches	p	--	--	--	--	--	4.69	3.32	2.29	1.90	1.97	--	--
	x						1.04	604	7,378	5,839	1,006		
Grapes	p	3.20	4.03	6.06	6.59	6.01	5.94	4.61	2.79	1.94	1.95	2.46	2.92
	x	88	34	11	8	70	87	103	1,668	3,118	3,043	1,441	373
Strawberries	p	--	--	--	--	6.21	3.98	4.30	--	--	--	--	--
	x					931	3,642	2,533					
Oranges	p	1.56	1.53	1.55	1.62	1.70	1.78	1.89	1.91	1.92	1.95	1.94	1.64
	x	11,569	10,993	9,621	6,063	4,625	3,970	4,078	2,883	2,957	3,759	8,238	11,827
		1973 (t=3)											
Apples	p	1.67	1.79	1.85	1.94	2.06	2.13	2.22	2.25	1.95	1.87	1.88	1.73
	x	4,051	4,909	5,567	6,253	6,101	7,023	5,671	4,187	5,446	7,377	9,283	4,955
Peaches	p	--	--	--	--	--	6.10	4.08	2.80	2.06	2.01	--	--
	x						111	653	7,856	6,291	1,073		
Grapes	p	3.52	4.67	6.48	7.34	6.51	6.43	5.00	3.07	2.20	2.19	2.74	3.13
	x	91	37	11	9	80	92	97	1,754	3,208	3,199	1,646	391
Strawberries	p	--	--	--	--	6.89	4.32	4.91	--	--	--	--	--
	x					1,033	4,085	2,877					
Oranges	p	1.68	1.66	1.70	1.85	1.95	2.03	2.10	2.12	2.07	2.13	2.14	1.79
	x	12,206	11,698	10,438	6,593	4,926	4,307	4,418	3,165	3,211	4,007	8,833	12,558

Source: From Turvey (1979). See text for further details.

2. Background and a Proposed Solution to the Problem of Seasonality

Several approaches to seasonal adjustment have been suggested in the literature. One is to drop seasonal items altogether. A second, and widely used, approach is to use a statistical time series method to smooth the series compiled from observed data.³ A third approach is to estimate shadow prices for seasonal commodities for the periods when they are out of season.⁴ A fourth approach is to directly compare each of the seasons this year with the corresponding seasons the year before (or in a base year),⁵ but this approach does not provide price trend estimates.⁶

For the seasonal fruits covered by the Turvey data set, the apples, grapes and oranges are seasonal in a weak way; that is, these products are available in all months. In contrast, peaches are only sold in June through October, and strawberries are only sold in May, June and July. Hence, for this example, peaches and strawberries are strongly seasonal products. We propose to treat each of the five fruits listed in table 1 as a separate commodity in each month it is sold.⁷ Hence, viewing our data month by month, we have 3 fruit products in the months of January through April, and in November and December; 4 fruit products in May when strawberries become available, and in August through October when peaches are still available; and 5 fruit products in June and July when both strawberries and peaches are in season.

For the period of a year, there are 12 apple, 12 grape, 12 orange, 5 peach, and 3 strawberry products. Thus, January through April each have 12 products, May has 13, June through July have 14, and the months of August through December each have 13. In contrast, viewed year by year, we have the same 44 products (=12+12+12+5+3) each year.

Let p_i^t and x_i^t be the price and quantity of product i ($i = 1, \dots, n_t$) in year t , with the number of products each year denoted by n_t . The annual price and quantity vectors for the Turvey data are represented here as $p^t \equiv (p_1^t, \dots, p_{44}^t)$ and $x^t \equiv (x_1^t, \dots, x_{44}^t)$ for $t = 1, \dots, 4$.⁸ The Laspeyres price index $P_L(p^b, p^t, x^b, x^t) \equiv p^t \cdot x^b / p^b \cdot x^b = P_L^{b,t}$, the Paasche price index, $P_P(p^b, p^t, x^b, x^t) \equiv p^t \cdot x^t / p^b \cdot x^t = P_P^{b,t}$, and the Fisher price index,⁹ $P_F(p^b, p^t, x^b, x^t) \equiv (P_L^{b,t} P_P^{b,t})^{1/2} = P_F^{b,t}$ are the fixed base price index formulae are used in this study with $b = 0$. The resulting fixed based annual index numbers are shown in panel 1, table 2.

³ See Stone (1956, pp. 77-88) and Allen (1975, pp. 169-176). Kuiper (1978) provides a survey of these methods.

⁴ Diewert (1980, pp. 501-503) outlines an econometric procedure in the context of the new goods problem that could work well in the seasonal commodities context too.

⁵ Allen (1975, pp. 88-191) and Diewert (1980, p. 507) advocate this principle.

⁶ See Balk (1980, 1981) for additional approaches.

⁷ Stone (1956, p. 75) and Diewert (1980, p. 508) suggested constructing annual indexes with seasonal data this way.

⁸ In the original 1983 paper, a “dummy year” of 1969 is inserted and used at various points. That is a non essential detail of how to begin calculating an index from the first year data were collected. Hence the dummy year and all table entries involving the dummy year are dropped in this condensed version of the paper.

⁹ In the full Diewert (1983) paper, results are shown too for the translog formula.

Table 2. Annual Price Levels Using Alternative Index Number Formulae

Year	t	P _L	P _P	P _F	P _{CL}	P _{CP}	P _{CF}
		panel 1			panel 2		
1971	1	1.11945	1.11956	1.11950	1.11945	1.11956	1.11950
1972	2	1.25263	1.25238	1.25200	1.25253	1.25174	1.25214
1973	3	1.40296	1.40050	1.40173	1.40259	1.40154	1.40207

Source: Based on tables 2 and 3 in Diewert (1983).

The chain index counterparts to the fixed base indexes are given by $P_C^{0,1} = P^{0,1}$, $P_C^{0,2} = P^{0,1} \times P^{1,2}$ and $P_C^{0,3} = P^{0,1} \times P^{1,2} \times P^{2,3}$, where P denotes any one of the index formulae.¹⁰ The chain index values are shown in panel 2 of table 2. Comparing the panel 1 and panel 2 values in the last two rows of table 2, we see that the Turvey data yield essentially the same annual indexes using the fixed base and chained methods.

Where seasonal products are concerned, price (and quantity) indexes that are for a yearly period can potentially include every seasonal product traded in at least one month. Indeed, for any of the consecutive 12 month periods, it will be possible to include any product traded in at least one month. There is no reason why we must always make January-to-December yearly comparisons. Why not compare the year of, say, February 1971 through January 1972 with February 1970 through January 1971, or the year of December 1972 through November 1973 with December 1970 through November 1971? Why not make rolling year instead of fixed calendar year comparisons?

To represent the rolling year price vectors, we need notation for the monthly price vectors. Let $p_i^{t:m}$ and $x_i^{t:m}$ denote the price and quantity vectors for product i ($i = 1, \dots, n_{t,m}$) in sub period m (a month here) in unit time period t (a year here). As noted above, for the Turvey data, $n_{t:1} = \dots = n_{t:4} = 3$, $n_{t:5} = 4$, $n_{t:6} = n_{t:7} = 5$, $n_{t:8} = n_{t:9} = n_{t:10} = 4$, and $n_{t:11} = n_{t:12} = 3$. The fixed base conventional calendar year indexes (for which values are shown in panel 1, table 2) can be represented using this new notation as:

$$\begin{aligned}
 (2) \quad P^{0,1} &= P(p^{0:1}, p^{0:2}, \dots, p^{0:11}, p^{0:12}; p^{1:1}, p^{1:2}, \dots, p^{1:11}, p^{1:12}; \\
 &\quad x^{0:1}, x^{0:2}, \dots, x^{0:11}, x^{0:12}; x^{1:1}, x^{1:2}, \dots, x^{1:11}, x^{1:12}) \\
 P^{0,2} &= P(p^{0:1}, p^{0:2}, \dots, p^{0:11}, p^{0:12}; p^{2:1}, p^{2:2}, \dots, p^{2:11}, p^{2:12}; \\
 &\quad x^{0:1}, x^{0:2}, \dots, x^{0:11}, x^{0:12}; x^{2:1}, x^{2:2}, \dots, x^{2:11}, x^{2:12}) \\
 P^{0,3} &= P(p^{0:1}, p^{0:2}, \dots, p^{0:11}, p^{0:12}; p^{3:1}, p^{3:2}, \dots, p^{3:11}, p^{3:12}; \\
 &\quad x^{0:1}, x^{0:2}, \dots, x^{0:11}, x^{0:12}; x^{3:1}, x^{3:2}, \dots, x^{3:11}, x^{3:12}),
 \end{aligned}$$

¹⁰ For example, the chained Laspeyres index for 1973 ($t=3$) versus 1970 ($t=0$) is:

$$P_{CL}^{0,3} = P_L(p^0, p^1, x^0, x^1)P_L(p^1, p^2, x^1, x^2)P_L(p^2, p^3, x^2, x^3) = P_L^{0,1} \times P_L^{1,2} \times P_L^{2,3}.$$

The full set of fixed base rolling year index comparisons for the Turvey data, using 1970 as the base year, can be represented as follows:

(3)

$$\begin{aligned}
 p^{1:1-1:12,2:1-2:12} &= P(p^{1:1}, p^{1:2}, \dots, p^{1:11}, p^{1:12}; p^{2:1}, p^{2:2}, \dots, p^{2:11}, p^{2:12}; \\
 &\quad x^{1:1}, x^{1:2}, \dots, x^{1:11}, x^{1:12}, x^{2:1}, x^{2:2}, \dots, x^{2:11}, x^{2:12}) \\
 p^{1:2-2:1,2:2-3:1} &= P(p^{1:2}, p^{1:3}, \dots, p^{1:12}, p^{2:1}, p^{2:2}, p^{2:3}, \dots, p^{2:12}, p^{3:1}; \\
 &\quad x^{1:2}, x^{1:3}, \dots, x^{1:12}, x^{2:1}, x^{2:2}, x^{2:3}, \dots, x^{2:12}, x^{3:1}) \\
 &\quad \vdots \\
 p^{1:12-2:11,3:12-4:11} &= P(p^{1:12}, p^{2:1}, \dots, p^{2:10}, p^{2:11}; p^{3:12}, p^{4:1}, \dots, p^{4:10}, p^{4:11}; \\
 &\quad x^{1:12}, x^{2:1}, \dots, x^{2:10}, x^{2:11}, x^{3:12}, x^{4:1}, \dots, x^{4:10}, x^{4:11}) \\
 p^{2:1-2:12,4:1-4:12} &= P(p^{2:1}, p^{2:2}, \dots, p^{2:11}, p^{2:12}; p^{4:1}, p^{4:2}, \dots, p^{4:11}, p^{4:12}; \\
 &\quad x^{2:1}, x^{2:2}, \dots, x^{2:11}, x^{2:12}, x^{4:1}, x^{4:2}, \dots, x^{4:11}, x^{4:12})
 \end{aligned}$$

Alternatively, we could compute index values using the chain principle. The chained index numbers are tabled in panel 2 of table 3.

The values of the annual inflation rates from panel 1 and panel 2 of table 2 can be found in panel 1 and panel 2 of table 3 in rows 1, 13 and 25: the cases where the specified 12 months for the rolling index comprise a calendar year. Hence, it seems that our suggested method does, in fact, remove the seasonal fluctuations in the original Turvey data set.¹¹

When implemented on the Turvey data, the Laspeyres, Paasche and Fisher formulae all yield very similar figures, and this is true for both the fixed base and the chain principle methods. However, for longer periods than the four years covered by the Turvey data and for real data that may differ in other important ways from the Turvey artificial data, we would expect the differences among the different types of index numbers to be more substantial.

Our proposed method of seasonal adjustment should be useful in the producer context too.

¹¹ Treating each seasonal product as a separate product in each sub-annual seasonal period (e.g., in each month here) when it is sold will greatly increase the dimensionality of the product space. Thus, in the original paper, Diewert suggests a two-stage procedure based on the two-stage aggregation theorem in Diewert (1978, pp. 889-890) that could be used to obtain an approximation to the results for his full new method.

Table 3. Year-to-Year Price Levels by Month

Row	Base period	Current period	P_L	P_P	P_F	P_{CL}	P_{CP}	P_{CF}
			panel 1			panel 2		
1	0:1-0:12	1:1-1:12	1.11945	1.11956	1.11950	1.11945	1.11956	1.11950
2	0:2-1:1	1:2-2:1	1.13164	1.13228	1.13196	1.13179	1.13224	1.13201
3	0:3-1:2	1:3-2:2	1.14193	1.14319	1.14256	1.14204	1.14304	1.14254
4	0:4-1:3	1:4-2:3	1.15240	1.15479	1.15360	1.15260	1.15439	1.15349
5	0:5-1:4	1:5-2:4	1.15999	1.16302	1.16150	1.16015	1.16242	1.16128
6	0:6-1:5	1:6-2:5	1.16755	1.17100	1.16927	1.16743	1.17008	1.16875
7	0:7-1:6	1:7-2:6	1.17742	1.18090	1.17916	1.17674	1.17951	1.17812
8	0:8-1:7	1:8-2:7	1.18917	1.19326	1.19121	1.18839	1.19123	1.18981
9	0:9-1:8	1:9-2:8	1.20447	1.20806	1.20627	1.20355	1.20594	1.20474
10	0:10-1:9	1:10-2:9	1.21534	1.21830	1.21677	1.21414	1.21596	1.21505
11	0:11-1:10	1:11-2:10	1.22757	1.23011	1.22884	1.22653	1.22776	1.22714
12	0:12-1:11	1:12-2:11	1.24235	1.24148	1.24191	1.24215	1.24172	1.24193
13	0:1-0:12	2:1-2:12	1.25263	1.25138	1.25200	1.25253	1.25174	1.25214
14	0:2-1:1	2:2-3:1	1.26151	1.26039	1.26095	1.26155	1.26141	1.26148
15	0:3-1:2	2:3-3:2	1.27157	1.27077	1.27117	1.27163	1.27220	1.27191
16	0:4-1:3	2:4-3:3	1.28195	1.28206	1.28201	1.28210	1.28359	1.28284
17	0:5-1:4	2:5-3:4	1.29175	1.29242	1.29208	1.29198	1.29397	1.29297
18	0:6-1:5	2:6-3:5	1.30185	1.30272	1.30229	1.30171	1.30409	1.30290
19	0:7-1:6	2:7-3:6	1.31400	1.31462	1.31431	1.31292	1.31550	1.31421
20	0:8-1:7	2:8-3:7	1.32715	1.32845	1.32780	1.32601	1.32867	1.32734
21	0:9-1:8	2:9-3:8	1.34791	1.34794	1.34793	1.34664	1.34872	1.34768
22	0:10-1:9	2:10-3:9	1.36044	1.35919	1.35981	1.35873	1.36022	1.35947
23	0:11-1:10	2:11-3:10	1.37419	1.37230	1.37325	1.37286	1.27272	1.37329
24	0:12-1:11	2:12-3:11	1.39192	1.39002	1.39097	1.39136	1.39084	1.39110
25	0:1-0:12	3:1-3:12	1.40296	1.40050	1.40173	1.40259	1.40154	1.40207

Source: Based on tables 4 and 5 in Diewert (1983).

References

- Allen, R.G.D. (1975), *Index Numbers in Theory and Practice*, MacMillan.
- Balk, B.M. (1980), "A Method for Constructing Price Indices for Seasonal Commodities," *Journal of the Royal Statistical Society Series A* 143, 68-75.
- Balk, B.M. (1981), "A Simple Method for Constructing Price Indices for Seasonal Commodities," *Statistische Hefte* 22, 72-78.
- Dagum, E.B. (1983), "The X-11 ARIMA Seasonal Adjustment Method," Statistics Canada Catalogue No. 12-564E.
- Diewert, W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation," *Econometrica* 46, 883-900. and reprinted as Chapter 9 in Diewert and Nakamura (1993, pp. 223-252).
- Diewert, W.E. (1980), "Aggregation Problems in the Measurement of Capital," in *The Measurement of Capital*, ed. by Dan Usher, NBER Studies in Income and Wealth, Vol. 45, University of Chicago Press, 433-528. <http://www.econ.ubc.ca/diewert/1capital.pdf> and <http://www.econ.ubc.ca/diewert/2capital.pdf>
- Diewert, W.E. (1983), "The Treatment of Seasonality in a Cost of Living Index", in W.E. Diewert and C. Montmarquette (eds.), *Price Level Measurement*, Statistics Canada, 1019-1045. <http://www.econ.ubc.ca/diewert/living.pdf>.
- Kuiper, J. (1978), "A Survey and Comparative Analysis of Various Methods of Seasonal Adjustment," in A. Zellner (ed.), *Seasonal Analysis of Economic Time Series, Proceedings of the Conference on the Seasonal Analysis of Economic Time Series*, Washington, D.C., September 9-10, U.S. Government Printing Office, 59-76.
- Stone, R. (1956), *Quantity and Price Indexes in the National Accounts*, Paris: OECD.
- Turvey, R. (1979), "The Treatment of Seasonal Items in Consumer Price Indices," *Bulletin of Labour Statistics, Fourth Quarter*, Geneva: ILO, 13-33.