INDEX NUMBER THEORY AND MEASUREMENT ECONOMICS

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CHAPTER 6: Problems with the Theory of the Cost of Living Index

1. Introduction

In this chapter,[1] we discuss some of the criticisms that have been directed toward the economic approach to the consumer price index.

In section 2 below, we discuss how to modify the theory when the consumer faces prices that are dependent on the amount of the commodity that is purchased.

Section 3 deals with the complications that occur when the number of households changes over time or the composition of households changes over time so that we are not comparing like with like in the economic approach.

Section 4 discusses how the theory can be modified if household preferences and environmental variables are not constant over time.

Section 5 discusses whether it is realistic to assume that the household has well defined preferences over all possible commodities. For economists, this assumption is made routinely but many price statisticians find this assumption to be rather far fetched.

Section 6 addresses some of the problems that are posed by household production.

Section 7 asks how valid is the theory if commodities can only be purchased in integer amounts rather than in continuous units.

Finally, section 8 asks whether economic approaches to the CPI can deal adequately with the problem of seasonal commodities.

2. Do Households Face Prices that are Independent of the Quantity Purchased?

The economic approach to the cost of living index assumes that households face prices that are independent of the quantity purchased by the individual household. This assumption is often satisfied but there are some situations where this assumption is not warranted:

- Price discounts for bulk purchases or frequent buyer discounts;
- The after tax price of leisure (the after tax wage rate) changes as more hours are worked due to a progressive income tax;
- Frequent buyer discounts that are contingent on paying a membership fee;
- Discounts for "tied" purchases of other commodities.

[1] This chapter is largely based on Diewert (2001).

In all of the above situations, the price that the household faces for a commodity is not completely independent of the quantities supplied or demanded by the household. Frisch (1936; 14-15) showed how, in theory, the economic approach can be adapted to deal with prices that are dependent on quantities purchased or supplied. What is required to implement his approach is a knowledge of the nonlinear budget set that a household actually faces, taking into account the dependence of prices on quantities. If these nonlinear budget sets are known and differentiable at the point where the household ends up in each period[2], then the nonlinear budget set can be linearized and the coefficient associated with each quantity can be used in place of its price and the analysis can proceed along the same lines as in the subsection above, where households faced different prices for the same commodity. However, in addition to this "general" solution to the monopsony problem, some specific solution techniques can be tried for each of the above four problems.

The first problem can be solved by treating purchases of the same commodity of different sizes as separate goods.[3]

The second and third problems can be solved by using a rough and ready *unit value approach*: simply calculate an estimate of total after tax earnings from working in the period under consideration and divide by the total number of hours worked (or calculate the total period cost of purchases after applicable discounts, add the period fixed cost to these variable costs and divide by the total number of homogeneous units purchased during the period) to get unit value prices.

The fourth problem can be solved by treating the tied purchase as a separate commodity.[4]

3. Household Composition is not Constant over Time

Our theory of the plutocratic cost of living presented in chapter 5 assumed that *exactly* the same households are present in the two periods being compared. In reality, marriages take place, children become adults, there are deaths and there is in and out migration. Thus the households that are present in period 0 will not be *exactly* identical to the households that are present in period 1.

This is indeed a problem with the cost of living index theory that we have presented in chapter 5. We can only suggest two solutions to this problem:

- Ignore the problem. Typically, when making comparisons over periods that are reasonably close in time, household composition will not change very much.
- Try to make adjustments to the aggregate data to exclude households that were present in one period but not in the other. Obviously, if complete micro data on each household were

[2] See Diewert (1993; 171). The technique can be adapted to the nondifferentiable case as well; see Wales and Woodland (1976) (1979). The same linearization techniques can be applied to the theory of the producer price index when the firm has market power; see Diewert (1993; 172) and Paul (1999; 149-160).

[3] Alternatively, a hedonic regression approach can be taken that will "quality adjust" purchases of differing sizes into constant quality comparable units. We will study the hedonic regression approach in a later chapter.

[4] Unfortunately, often the product ties are not stable from period to period so that like cannot always be compared with like. In this case, it may be useful to try a hedonic regression approach.

available, this would not be a problem and we could restrict our comparison to households that were more or less unchanged and present in both periods.

We pass on to the next set of criticisms of the economic approach to the consumer price index.

4. Household Preferences and Environmental Variables are not Constant over Time

There are many specific criticisms that fall under this general heading, including:

- Tastes are not constant. In particular, education and general life experience systematically changes ones tastes and preferences from period to period.
- Biological aging also systematically changes one tastes. In particular, as one becomes very old, choice sets become more restricted due to deterioration in physical skills and in mental acuity.
- Accidents and illness also impact upon choice sets that are feasible for consumers going from period to period; i.e., a severe accident effectively changes ones feasible preferences going from one period to the next.
- Exogenous environmental variables (such as the weather or temperature) could be different in the two periods and these differences could change consumer preferences.

However, in response to criticisms of this type, we presented a theory of the cost of living index in chapter 5 that could accommodates changing environmental variables. One of the environmental variables could be time t, which could be used as a variable to map the preferences of period 0 into the preferences of period 1 in a continuous manner; i.e., household h's utility function could be defined as $f^h(q,t)$, a continuous function, with $f^h(q,0)$ representing the preferences of period 0 and $f^h(q,1)$ representing the preferences of period 1. Thus taste changes and gradual aging could be accommodated using the model of consumer behavior presented in chapter 5. However, the model cannot readily accommodate discontinuous or discrete changes in tastes or environmental variables. This is an area that requires further thought and research.

5. Is it Realistic to Assume that Households have Preferences over all Possible Commodities?

"As the Boskin Report expressed it, such an index 'is a comparison of the minimum expenditure required to achieve the same level of well-being (also known as welfare, utility, standard-of-living) across two different sets of prices.' This concept has been expounded by a number of authors, notably Robert A. Pollak and Erwin Diewert in papers notable for their intellectual rigour, formality of expression and minimal reference to the actual behaviour of individual consumers. … Two questions about this theory lack an answer: Whose preferences are concerned, what is a consumer? Is it a household? How can this static, timeless, theory be applied to a period of time? What is its appropriate length? Presumably it must be short enough for prices to remain constant throughout, but long enough for a consumer to buy the set of items." Ralph Turvey (1999; 1-2).

There is some considerable merit in Turvey's criticisms. Think first of a multiple adult household. How are consistent household preferences to be formed from individual preferences? This is not a trivial problem. Even in the case of a single person household, how is the individual to even know about all of the millions of possible consumer goods and services that are out there somewhere, let alone form consistent preferences over these commodities? However, if commodity n is not consumed by household h in both periods under consideration, then it can be *dropped* from household h's utility function and the preference map can be restricted to the much smaller set of commodities that are actually consumed in at least one of the two periods. For commodities that are consumed repeatedly over many periods, it is at least plausible that consistent household preferences over these commodities might emerge.[5] However, consistency problems are likely to arise when we consider how an individual forms preferences over "new" commodities that were not tried in previous periods. Advertising, marketing of new products, the experience of friends, reading magazines that rate new products—all of these factors will influence preference formation over new goods and problems of inconsistency could arise. At least economists are willing to explore these problems of preference determination and how to measure the benefits of new products whereas in the past, the approach taken by many statistical agencies has been to simply ignore new commodities in their price indexes.[6]

6. Traditional Consumer Theory Ignores the Problems Posed by Household Production

Peter Hill (1999), in discussing the classic study by Nordhaus (1997) on the price of light, has raised the issue as to how should a cost of living index treat household production where consumers combine purchased market goods or "inputs" to produce finally demanded "commodities" that yield utility:

> "There is another area in which the definition of a COL requires further clarification and precision. From what is utility derived? Households do not consume many of the goods and services they purchase directly but use them to produce other goods or services from which they derive utility. In a recent stimulating and important paper, Nordhaus has used light as a case study. Households purchase items such as lamps, electric fixtures and fittings, light bulbs and electricity to produce light, which is the product they consume directly. … The light example is striking because Nordhaus provides a plausible case for arguing that the price of light, measured in lumens, has fallen absolutely (at least in US dollars) and dramatically over the last two centuries as a result of major inventions, discoveries and 'tectonic' improvements in the technology of producing light.
>
> The question that arises is whether goods and services that are essentially *inputs* into the production of other goods and services should be treated in a COL as if they provided utility directly. In principle, a COL should include the shadow, or imputed, prices, of the outputs from these processes of production and not the prices of the inputs. … There is a need to clarify exactly how this issue is to be dealt with in a COL index." Peter Hill (1999; 5).

We attempt to clarify the issues raised by Hill by using the model of household production of finally demanded commodities that was postulated by Becker (1965) many years ago. Becker's model illustrates not only how household production of the type mentioned by Hill can be integrated into a cost of living framework, but it also indicates the important role that the *allocation of household time* plays in a realistic model of household behavior. In Becker's model of consumer behavior, a household (consisting of a single individual for simplicity) purchases q_n units of *market commodity n* and combines it with a household input of time, t_n, to produce $z_n = f_n(q_n, t_n)$ units of a *finally demanded commodity* for n = 1,2,…,N say, where f_n is *the household production function* for the nth finally demanded commodity[7]. Some examples of such finally demanded commodities are:

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[5] For an economist, the assumption of consistent preferences for an individual who is repeatedly buying the same commodities is just an elaborate way of formalizing the empirical "fact" that when the price of a commodity goes up, the amount purchased goes down.

[6] Of course, the price statistician may be restricted by a lack of resources in trying to account for new goods and services. Also, due to the importance of the consumer price index, the price statistician must search for *reproducible* methods for dealing with the new goods problem whereas the armchair economist is not so constrained.

[7] More complicated household production functions could be introduced but the present assumptions will suffice to show how household production can be modeled in a COLI framework. For additional work on Becker's theory of the

- Making a meal; the inputs are the ingredients used, the use of utensils and possibly a stove and time required to make the meal and the output is the prepared meal.
- Eating a meal; the inputs are the prepared meal and time spent eating and the output is a consumed meal.
- Cleaning a house; the inputs are cleaning utensils, soapy water, polish and time and the output is a clean house.
- Gardening services; the inputs are tools used in the yard, fuel (if power tools are used) and time and the output is a beautiful yard.
- Reading a book; the inputs are computer services or a physical book and time and the output is a book which has been read.

During the period of time under consideration, the household also offers t_L hours of time on the labor market, earning an after tax wage of w per hour. The consumer-worker has preferences over different combinations of the finally demanded commodities, $z_1,\ldots,z_N$, and hours of work, t_L, that are summarized by the *utility function*, $U(z_1,\ldots,z_N,t_L)$.[8] In addition to the budget constraint, the household has to satisfy the *time constraint*, $\sum_{n=1}^{N} t_n + t_L = T$, where T is the number of hours available in the period under consideration. Rather than study the consumer's utility maximization problem subject to the budget and time constraints, we will study the equivalent consumer's cost or expenditure minimization problem subject to a utility constraint plus the time constraint. Thus we assume that the observable consumption vector $(q_1^0,\ldots,q_N^0) \equiv q^0$, time allocation vector $(t_1^0,\ldots,t_N^0) \equiv t^0$ and labor supply t_L^0 solve the following period 0 expenditure minimization problem:

$$(1)\quad \min_{q\text{'s and }t\text{'s}} \{\textstyle\sum_{n=1}^{N} p_n^0 q_n - w^0 t_L : U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N),t_L] = u^0; \sum_{n=1}^{N} t_n + t_L = T\}$$

where $u^0 \equiv U[f_1(q_1^0,t_1^0),\ldots, f_N(q_1^0,t_1^0),t_L^0]$ is the utility level actually attained by the household in period 0, $(p_1^0,\ldots,p_N^0) \equiv p^0$ is the vector of commodity prices that the household faces in period 0 and w^0 is the after tax wage rate faced by the consumer-worker in period 0.

If we use the time constraint in (1) to eliminate the hours worked variable t_L in both the objective function and the first constraint function in (1), we obtain an equivalent period 0 expenditure minimization problem and we find that under the above assumptions, q^0 and t^0 solve:

$$(2)\quad \min_{q\text{'s and }t\text{'s}} \{\textstyle\sum_{n=1}^{N} [p_n^0 q_n + w^0 t_n] - w^0 T : U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N),T - \sum_{n=1}^{N} t_n] = u^0\}.$$

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allocation of time and household production, see Pollak and Wachter (1975) (1977), Diewert (2001), Hill (2009) and Diewert and Schreyer (2013). The analysis in these notes follows the approach of Becker (1965), who used household market wage rates to value household time. Unfortunately, the situation is much more complicated than indicated by Becker's analysis. Suppose that all members of the household are retired or unemployed; then there is no market opportunity wage rate to value household time. Furthermore, even if there are market wage rates of household members to value time, it is not clear whether this opportunity cost wage rate should be used to value household work. Instead one could use market wage rates for hiring workers to do household work. This second approach is the more commonly used approach these days. A third problem is: how exactly should household leisure time be valued? These issues are discussed more fully in Diewert and Schreyer (2013).

[8] The utility function is assumed to be quasiconcave and nondecreasing in the $z_1,\ldots,z_N$ and nonincreasing in t_L.

Now we introduce a simpler notation for the utility function, treating the vector of time allocation variables $t = (t_1,\ldots,t_N)$ as a vector of environmental variables:[9]

(3) $f(q,t) = f(q_1,\ldots,q_N,t_1,\ldots,t_N) \equiv U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N),T-\sum_{n=1}^{N} t_n]$.

Thus q^0 and t^0 also solve:

(4) $\min_{q\text{'s and }t\text{'s}} \{\sum_{n=1}^{N} [p_n^0 q_n+w^0 t_n] - w^0 T : f(q,t) = u^0\}$.

Now if we *condition* on the optimal time allocation variables, $t^0 \equiv (t_1^0,\ldots,t_N^0)$, it can be seen that $q^0 \equiv (q_1^0,\ldots,q_N^0)$ solves:

(5) $\min_q \{\sum_{n=1}^{N} p_n^0 q_n : f(q,t^0) = u^0 \} \equiv C(u^0,p^0,t^0)$

where $C(u,p,t)$ is the conditional cost function that corresponds to the utility function $f(q,t)$. In a similar fashion, letting $t^1 \equiv (t_1^1,\ldots,t_N^1)$ be the optimal vector of time allocation variables for the consumer-worker's period 1 expenditure minimization problem that is analogous to (1), we can show that the consumer's observed period 1 consumption vector $q^1 \equiv (q_1^1,\ldots,q_N^1)$ solves:

(6) $\min_q \{\sum_{n=1}^{N} p_n^1 q_n : f(q,t^1) = u^1 \} \equiv C(u^1,p^1,t^1)$

where $u^1 \equiv f(q^1,t^1)$. Now we can more or less repeat the analysis presented in chapter 5, with the time variables in the vector t replacing the environmental variables in the vector e. Thus we can define a *theoretical family of cost of living indexes*,

(7) $P(p^0,p^1,u,t) \equiv C(u,p^1,t)/C(u,p^0,t)$

that is indexed by the utility level u and the vector of time variables $t \equiv (t_1,\ldots,t_N)$. As usual, we can specialize u and t to equal the period 0 utility level u^0 and the vector of period 0 time allocations, t^0, and we can derive the Laspeyres upper bound:

(8) $P(p^0,p^1,u^0,t^0) \leq p^1\cdot q^0 / p^0\cdot q^0 \equiv P_L$.

We can also specialize u to equal the period 1 utility level u^1 and t to equal the period vector of period 1 time allocations, t^1, and we can derive the usual Paasche lower bound:

(9) $P(p^0,p^1,u^1,t^1) \geq p^1\cdot q^1 / p^0\cdot q^1 \equiv P_P$.

Finally, we can adapt the proof of Proposition 1 in chapter 5 and show that there exists a reference utility level u^* that lies between the period 0 and 1 utility levels, u^0 and u^1, and a reference time allocation vector t^* whose components lie between the period 0 and 1 time allocation vectors, t^0 and

[9] Diewert (2001; 236) called the new utility function f the "home production utility function" since it absorbed the home production functions f_n into the preference function. A shorter name for the new utility function is the "blended utility function" since it is a blend of the original U and the f_n.

t^1, such that $P(p^0,p^1,u^*,t^*)$ lies between the observable Laspeyres and Paasche indexes for our consumer-worker, P_L and P_P.

Thus a theory of the cost of living index that is based on a model where consumers buy market goods and combine them, along with time inputs, to yield (unobservable) finally demanded commodities is completely isomorphic to the theory of the conditional cost of living index, where time variables take the place of the environmental variables. There is no need to estimate shadow prices for these finally demanded commodities.

Some points of interest emerge from the above analysis:

(i) If we want to base our theory for the consumer price index on *an unconditional cost function*, say $C^*(u,p,w)$ where $C^*(u^0,p^0,w^0)$ is the optimized objective function for (1), then it will be necessary to collect information on the household's allocation of time.
(ii) The utility function $f(q,t)$ defined by (3) above is a *blend* of the consumer's *utility function* U defined over finally demanded commodities z_n and labor supply t_L, $U(z_1,\ldots,z_N,t_L)$, and the *household production functions*, $f_n(q_n,t_n)$, $n = 1,\ldots,N$. Thus the *blended utility function* $f(q,t)$ will not remain constant over time due to *technological progress* in the production of finally demanded commodities, as in the Nordhaus price of light analysis. Hence shifts in the blended utility function f over time could be due to taste changes or to production innovations.
(iii) All of the time inputs into the various household production activities are valued at the consumer's after tax market wage rate, w^t in period t. But suppose that the consumer is retired or chooses to be unemployed or cannot vary his or her hours of work? In these cases, the above Beckerian model breaks down.[10]

We now suppose that information on the consumer's allocation of time is available and we will now spell out how a cost of living index for the consumer can be worked out that does not condition on the consumer's time allocation in each period.

The period t utility maximization problem that corresponds to the period 0 expenditure minimization problem defined by (1) above is:

$$(10)\ \max_{q\text{'s},\, t\text{'s}} \{U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N),t_L] : \textstyle\sum_{n=1}^N p_n^t q_n = w^t t_L + Y^t\ ;\ \sum_{n=1}^N t_n + t_L = T\}\ ; \qquad t = 0,1$$

where Y^t is nonlabour income that is spent in period t on market goods and services.[11] We can use the time constraint to eliminate t_L from the objective function and from the budget constraint in (10). After eliminating the time constraint, (10) becomes the following constrained utility maximization problem with a single constraint:

$$\begin{aligned}(11)\ &\max_{q\text{'s},\, t\text{'s}} \{U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N),T-\textstyle\sum_{n=1}^N t_n] : \sum_{n=1}^N p_n^t q_n + \sum_{n=1}^N w^t t_n = Y^t + w^t T\}\ ; \quad t = 0,1\\ &= \max_{q\text{'s},\, t\text{'s}} \{f(q_1,\ldots,q_N,t_1,\ldots,t_N) : \textstyle\sum_{n=1}^N p_n^t q_n + \sum_{n=1}^N w^t t_n = F^t\}\end{aligned}$$

[10] See Diewert and Schreyer (2011) for an approach that deals with this problem.
[11] If the consumer supplies a sufficient amount of labour services during period t, then Y^t could be negative but the sum of labour services and nonlabour income that is spent must be positive.

where the second line above follows using definition (3) for the blended utility function f and *Becker's full income* in period t, F^t, is defined as the sum of nonlabour income expenditures Y^t and the value of time in period t, w^tT; i.e., we have

$$(12)\ F^t \equiv Y^t + w^tT\ ; \qquad t = 0,1.$$

Now define the *unconditional cost function*, C^*, that is dual to f for $u > 0$, $p_n > 0$ and $w_n > 0$ for n = 1,...,N as follows:

$$(13)\ C^*(u,p_1,...,p_N,w_1,...,w_N) \equiv \min_{q's,t's} \{\sum_{n=1}^N p_nq_n + \sum_{n=1}^N w_nt_n : f(q_1,\ldots,q_N,t_1,\ldots,t_N) \geq u\}.$$

We can use the unconditional cost function C^* in order to define the usual family of (unconditional) Konüs true cost of living indexes as follows:

$$(14)\ P^*(p^0,w^0,p^1,w^1,u) \equiv C^*(u,p^1,w^1 1_N)/C^*(u,p^0,w^0 1_N).$$

As usual, we can specialize u to equal the period 0 utility level u^0 and we can derive the following Laspeyres upper bound:

$$(15)\ \ P^*(p^0,w^0,p^1,w^1,u^0) \leq [p^1\cdot q^0 + w^1 1_N{}^T t^0] / [p^0\cdot q^0 + w^0 1_N{}^T t^0] \equiv P_L{}^*.$$

We can also specialize u to equal the period 1 utility level u^1 and we can derive the following Paasche lower bound:

$$(16)\ \ P^*(p^0,w^0,p^1,w^1,u^1) \geq [p^1\cdot q^1 + w^1 1_N{}^T t^1] / [p^0\cdot q^1 + w^0 1_N{}^T t^1] \equiv P_P{}^*.$$

Finally, we can adapt the proof of Proposition 1 in chapter 5 and show that there exists a reference utility level u^* that lies between the period 0 and 1 utility levels, u^0 and u^1 such that $P^*(p^0,w^0,p^1,w^1,u^*)$ lies between the observable Laspeyres and Paasche indexes for our consumer-worker, $P_L{}^*$ and $P_P{}^*$.

However, this last unconditional approach to true consumer cost of living indexes when there is household production is not completely satisfactory because it ignores the *special structure* of the consumer's blended utility function, $U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N),T-\sum_{n=1}^N t_n]$. It is possible to take this special structure into account and we will show how this can be done for a special case where the consumer's utility function, $U(z_1,...,z_N,t_L)$, does not depend on t_L and the household production functions $f_n(q_n,t_n)$ are linearly homogeneous in q_n,t_n so that there are constant returns to scale in household production. With the constant returns to scale assumption on f_n, it can be seen that the problem of minimizing the cost of producing z_n has the following structure:

$$(17)\ \min_{q,t} \{p_nq_n + wt_n : f_n(q_n,t_n) \geq z_n\} = c_n(p_n,w)z_n\ ; \qquad n = 1,...,N$$

where the *unit cost function* c_n dual to the household production function f_n is defined as follows:

$$(18)\ c_n(p_n,w) \equiv \min_{q,t} \{p_nq_n + wt_n : f_n(q_n,t_n) \geq 1\}\ ; \qquad n = 1,...,N.$$

If $U(z_1,...,z_N,t_L)$ does not depend on t_L, then the consumer's utility function becomes $U(z_1,...,z_N)$. Now suppose that the consumer faces the prices P_n for purchasing one unit of z_n for n = 1,...,N. The *consumer's unconditional cost function* C^{**} can be defined as follows:

$$(19)\ C^{**}(u,P_1,...,P_N) \equiv \min_{z\text{'s}} \{\textstyle\sum_{n=1}^{N} P_n z_n : U(z_1,...,z_N) \geq u\}.$$

The *consumer's actual period t cost minimization problem* is the following one:

$$\begin{aligned}(20)\ &\min_{q\text{'s},t\text{'s}} \{\textstyle\sum_{n=1}^{N} p_n^t q_n + \sum_{n=1}^{N} w^t t_n: U[f_1(q_1,t_1),\ldots, f_N(q_N,t_N)] \geq u\} && t = 0,1\\ &= \min_{q\text{'s},t\text{'s},z\text{'s}} \{\textstyle\sum_{n=1}^{N} p_n^t q_n + \sum_{n=1}^{N} w^t t_n: U[z_1,\ldots, z_N] \geq u;\ f_n(q_n,t_n) \geq z_n\ ,\ n = 1,...,N\} &&\\ &= \min_{z\text{'s}} \{\textstyle\sum_{n=1}^{N} c_n(p_n^t,w^t) z_n: U[z_1,\ldots, z_N] \geq u\} && \text{using (17) and (18)}\\ &= C^{**}[u,c_1(p_1^t,w^t),...,c_N(p_N^t,w^t)] && \text{using definition (19).}\end{aligned}$$

Thus the unit cost of production for the nth household production function, $c_n(p_n^t,w^t)$, acts as the price for the nth finally demanded commodity z_n for n = 1,...,N.

We can use the unconditional cost function C^{**} in order to define the usual family of (unconditional) Konüs true cost of living indexes as follows:

$$(21)\ P^{**}(P^0,P^1,u) \equiv C^{**}(u,P^1)/C^{**}(u,P^0)$$

where the P_n^t are defined as $c_n(p_n^t,w^t)$ for t = 0,1 and n = 1,...,N. When we specialize (21) to the usual Laspeyres and Paasche cases, we obtain the same bounds as were derived in (15) and (16).[12] This approach to the household's time allocation problem is essentially due to Becker (1965), except that he used Leontief (no substitution) production functions for his household production functions.

Becker's theory of the allocation of time and the generalization of it given by (17)-(21) rest on the assumption that t_L does not enter the consumer's utility function. This is not very realistic but to relax this assumption will require more advanced techniques.

We turn to the next criticism of the economic approach to consumer price indexes.

7. The Problem of Integer Purchases rather than Continuous Unit Purchases

The economic approach to the cost of living index assumes that commodities can be purchased in fractional units instead of integral amounts. Thus gasoline can be purchased can be purchased in fractional units of liters but television sets can only be purchased in discrete units. However, the economic model that the cost of living index is based on assumes that all commodities can be purchased in fractional units.[13] Diewert noted this problem and proposed a solution:

[12] The advantage of the $P^{**}(P^0,P^1,u)$ cost of living framework is that we can see the role of the consumer's time allocation more clearly in defining the "full price" (which includes the time cost) of consuming the z_n in each period. The prices P_n^t and quantities z_n^t can be approximated fairly closely using superlative index number techniques.

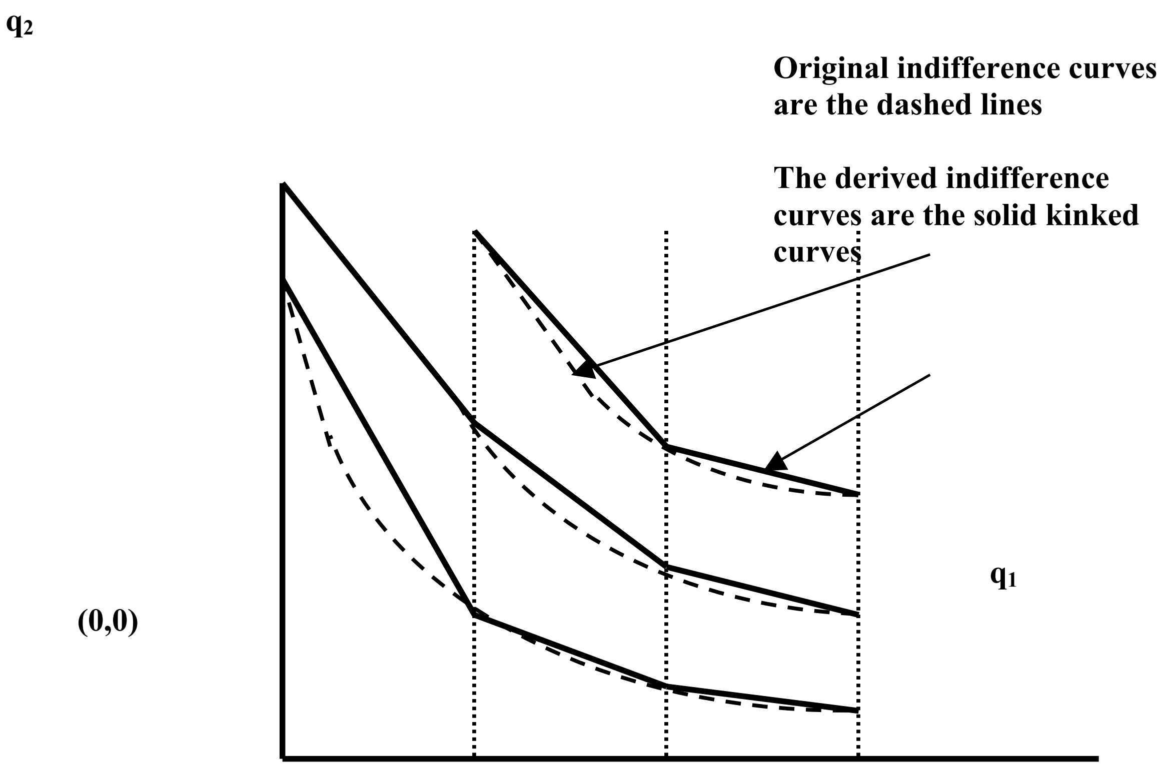
[13] In fact, the economic model assumes that any real number can be purchased, not only integer or fractional amounts.

"Most goods can only be purchased in integral numbers, and for most goods, this does not cause major problems. However, some durable goods such as cars and houses may be purchased only in integer units, and such purchases would form a large share of the consumer's total expenditure. Hence we cannot neglect the lumpiness problem for such classes of durables. How may we apply traditional 'continuous' utility and index number theory to this situation? … For all practical purposes, we can replace the original preferences defined over integer combinations of TV sets by continuous preferences with 'kinks'. The resulting preference function F(x) may be treated in the normal manner as far as index number theory is concerned. Note that the economic effect of the 'kinks' will be to make the consumer change his durable holdings only after relatively large changes in the rental prices of the durables relative to nondurable goods; i.e., responses will be 'sticky'. This point should be taken into account in econometric work, but it need not concern us from the viewpoint of index number theory." W.E. Diewert (1983; 212-213).

Figure 1 illustrates how this problem can be treated assuming that a continuous utility function, $f(q_1,q_2)$, exists that represents the consumer's preferences over all nonnegative combinations of the two commodities. Three indifference curves using the original preferences are drawn in Figure 1. These are the dashed curves and represent indifference curves of the form $\{(q_1,q_2): f(q_1,q_2) = u\}$ for three different levels of utility u. We assume that commodity 2 can be purchased in arbitrary nonnegative amounts, $q_2 \geq 0$, but commodity 1 can only be purchased in positive integer amounts, 0,1,2,… . In order to obtain preferences that are consistent with this setup, we intersect each indifference curve with the line segments, $q_1 = 0$, $q_1 = 1$, $q_1 = 2$, and so on. Then we simply draw straight lines connecting up these points of intersection to obtain a *derived indifference curve*. We can do this operation with each original indifference curve, obtaining a derived indifference curve for each utility level. These derived indifference curves are the solid line kinked curves in Figure 1. Now simply assume that the consumer's preferences are representable by this family of derived indifference curves. It can be seen that these derived preferences will be very close to the original preferences but the new derived preferences have the property that solutions to the consumer's cost minimization problem will always be consistent with the integer constraints on q_1.[14] Thus normal consumer theory can be applied to this integer model and hence the theory of the cost of living index will still be valid.

[14] In order to construct the derived preferences mathematically, define for each feasible utility level u, the upper level set of the original preferences by $L(u) \equiv \{(q_1,q_2): f(q_1,q_2) \geq u\}$. We assume that these sets are closed. Define the feasible set of q_1's as $F \equiv \{(q_1,q_2): q_2 \geq 0;\ q_1 = 0,1,2,3,\ldots\}$. Now define the family of sets M(u) for each u by intersecting L(u) with F; i.e., $M(u) \equiv L(u) \cap F$. Finally, define the family of sets K(u) by taking the convex hull of each M(u); i.e., $K(u) \equiv \{q: q = \lambda q^1 + (1-\lambda)q^2;\ 0 \leq \lambda \leq 1;\ q^1 \in M(u) \text{ and } q^2 \in M(u)\}$. The upper level set for the derived preferences for the utility level u is the set K(u) and the corresponding (kinked) indifference curve is the lower boundary of this set.

Figure 1: The Problem of Integer Purchases and Consumer Preferences



Thus the problem of integer purchases does not present a major challenge to the economic approach to index number theory and so we pass on to our last criticism of the economic approach.

8. Economic Approaches to the CPI do not Deal Adequately with the Problem of Seasonal Commodities

The above criticism is actually a criticism that applies equally well to *all* approaches to index number theory. The problem is this: a seasonal commodity can be present in one month or quarter and then be absent from the marketplace in the following month or quarter. How then are we to calculate the price change pertaining to the commodity over the two periods when the commodity is simply not present in one of the periods? It is simply a mission impossible to do this!

Turvey (1979) conducted an ingenious experiment to see if the presence of seasonal commodities in a CPI could be a problem empirically. He constructed an artificial data set giving fictitious monthly price and quantity data for 5 types of fruit for 4 years. He sent this data set to every statistical agency in the world with the instructions to construct a monthly price index using this data and using their normal seasonal adjustment procedures. Needless to say, the answers varied tremendously.

The problems raised by Turvey remain with us today. Diewert (1998) (1999) has recently taken a new look at this very old problem from the perspective of the economic approach to index number theory. Diewert concluded that in the presence of seasonal commodities, there is a need for at least *three* separate consumer price indexes. The *first index* should be a short term month to month index defined over *nonseasonal commodities*.[15] This index should be useful for the purpose of monitoring short run inflationary trends in the economy. The *second index* should be a year over year index, where the prices in January are compared to the January prices of a base year, the prices in February are compared to the February prices of a base year, etc. This index should give an accurate measure of year over year inflation, which is free from seasonal influences. The *third index* should be an annual one[16], which compares a moving total of 12 months with 12 base year months. This type of annual index can serve as a substitute for the present classes of seasonally adjusted price indexes that rely on "black box" time series methods for seasonal adjustment.

In any case, the topic of seasonal adjustment deserves a lot more attention in the CPI literature than it has received in recent years by price statisticians. We will pursue this topic in more depth in a later chapter.

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[15] Since harmonized indexes are supposed to exclude new commodities from their domains of definition due to the difficulties in making objective and reproducible comparisons, it would seem that harmonized indexes should also exclude seasonal commodities on the same grounds.

[16] This index can be built up from the second class of year over year indexes.

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