Information, Misallocation and Aggregate Productivity*

Joel M. David†  Hugo A. Hopenhayn‡  Venky Venkateswaran§
USC  UCLA  NYU Stern

October 16, 2013

Abstract

We propose a theory linking imperfect information to resource misallocation and hence to aggregate productivity and output. In our setup, firms learn from both private sources and imperfectly informative stock market prices. We devise a novel calibration strategy that uses a combination of firm-level production and stock market data to pin down the information structure in the economy. Applying this methodology to data from the US, China, and India reveals substantial losses in productivity and output due to informational frictions - even when only one factor, namely capital, is subject to the friction. Our estimates for these losses range from 5-19% for productivity and 8-28% for output in China and India, and are smaller, though still significant, in the US. Losses are substantially higher when labor decisions are also made under imperfect information. Private learning plays a significant role in mitigating uncertainty and improving aggregate outcomes; learning from financial markets contributes little, even in the US.

*We thank Jaroslav Borovicka, Virgiliu Midrigan, and Yongs Shin for helpful comments, Cynthia Yang for excellent research assistance, and seminar participants at NYU, Brown, the 2013 North American Summer Meeting of the Econometric Society, the 2013 SED Annual Meeting in Seoul, and the 4th Advances in Macroe-Finance Tepper-LAEF conference.
†joeldavi@usc.edu.
‡hopen@econ.ucla.edu
§vvenkate@stern.nyu.edu
1 Introduction

The optimal allocation of factor inputs across productive units requires the equalization of marginal products. Deviations from this outcome represent a misallocation of such resources and translate into sub-optimal aggregate outcomes, specifically, depressed levels of productivity and output. A recent literature empirically documents the presence of substantial misallocation and points out its potentially important role in accounting for large observed cross-country differences in productivity and income per-capita. With some notable exceptions, however, the literature has remained largely silent about the underlying factors driving misallocation, that is, the primitive frictions that prevent marginal product equalization.

In this paper, we propose just such a theory, linking imperfect information to resource misallocation and hence to aggregate productivity and output. Our point of departure is a standard general equilibrium model of firm dynamics along the lines of Hopenhayn (1992). The key modification here is that firms choose productive inputs under imperfect information about fundamentals, i.e., regarding their productive efficiency or demand characteristics. This lack of information leads to a misallocation of factors across firms, that is, to dispersion in marginal products, the extent of which is a function of the firm’s residual uncertainty at the time of its input choice decisions. In turn, the degree of micro-level uncertainty depends on the volatility of the fundamental shocks the firm faces and the quality of the information to which it has access, a subject to which we turn in a moment. The parsimonious nature of our analytical framework enables a sharp characterization of these relationships and yields simple closed-form expressions linking informational parameters at the micro-level to aggregate outcomes. Our general equilibrium framework also embeds an amplification mechanism - informational frictions and the resulting misallocation reduce economy-wide incentives to accumulate capital. As a result, allocative inefficiencies translate into even greater declines in output above and beyond those in aggregate productivity.

Bearing in mind these connections between firm-level uncertainty and aggregate outcomes, the second piece of our theoretical framework puts focus squarely on the firm’s learning problem. Here, we develop a rich yet flexible information structure in which firms learn not only from their own private sources of information, but also from their own stock market prices. This latter feature captures the idea that financial markets aggregate dispersed information across investors to generate informative prices, which in turn guide real activity.\footnote{Note that this does not imply that investors have better information than firms; only that they are privy to different information that may also be relevant for firm decisions.} That markets may play such an informational role in the economy is a notion dating back at least to Tobin (1982) and continues to be the subject of much study - a recent body of work finds evidence
of the so-called “feedback effect” of market prices on real economic activity working through an informational channel. In this light, we explicitly model the informational role of prices with a fully specified financial market in the spirit of the noisy rational expectations paradigm of Grossman and Stiglitz (1980), in which imperfectly informed investors and noise traders buy and sell shares of the firm’s stock. Equilibrium stock prices aggregate information across investors, albeit imperfectly due to the presence of noise traders, with the result that prices provide firms with a noisy signal of fundamentals, which is combined with their own private information to guide input decisions.

The presence of learning from financial markets serves two purposes in our analysis: first, we are able to quantitatively evaluate the contribution of financial markets to allocative efficiency through an informational channel, i.e., by providing higher quality information to decision-makers within firms and thus reducing uncertainty at the micro-level. Our analysis is, to the best of our knowledge, the first to measure and shed light on the aggregate consequences of this channel in a standard macroeconomic framework. Second, as we describe next, the informational content of observed market prices is at the core of our empirical strategy and allows us to identify the severity of otherwise unobservable informational frictions in the economy.

One of the contributions of this paper is a novel empirical strategy that combines firm-level production and financial market data to infer the quality of firm-level information, allowing us to quantify the effects of this friction. This is a challenging task in that firm-level information is not directly observable (to the econometrician). Our key insight is that in the presence of learning from markets, we observe a subset of that information (namely, stock prices). Combining this with firm-level production data (specifically, input choice, i.e., investment, and realized outcomes), we are able to measure both the noisiness of this signal (by measuring the correlation of returns with future fundamentals) and the responsiveness of firm decisions to it (by measuring the correlation of returns with investment). Intuitively, the variability of stock returns and their ability to forecast fundamentals provides information on the magnitude of noise in market prices. Fixing the amount of noise in this signal, the extent to which firms adapt their decisions to it then allows us to infer the quality of information available to them from other (unobserved) sources. The less precise is the latter, the greater the reliance on financial markets for information and therefore, the higher will be the correlation between investment decisions and stock market returns. It is worth emphasizing that none of these moments (the variability of stock returns or their correlation with fundamentals and investment) is by itself a sufficient statistic for the informational role of markets - our explicit modeling of both production and

2In the interest of brevity, we discuss a few particularly relevant examples of such work below and refer the reader to Bond et al. (2012) for an excellent survey.
3We rely particularly on recent work by Albagli et al. (2011b) for our specific modeling structure.
financial markets allows us to use them jointly to pin down the informational parameters of the economy.\textsuperscript{4}

We apply our empirical methodology to data from 3 countries - the US, China and India. Our results point to substantial uncertainty at the micro level, particularly in China and India. Even in the US, which has the highest degree of learning, our lowest estimate for the posterior variance of the firm is 37\% of the total uncertainty (measured by the ex-ante, or prior, variance). The corresponding value for the other two countries ranges from about 60-95\%, with the implication firms in China and India seem to know very little about contemporaneous changes to fundamentals when making investment decisions.\textsuperscript{5} The associated implications for aggregate productivity and output are then quite significant. In China and India, TFP losses (relative to the first best) are between about 5\% and almost 20\%, while losses in steady state output (again, relative to the first best) range from about 10\% to almost 30\%. The corresponding values in the US are noticeably smaller but still significant - 2-12\% for productivity and 4-17\% for output. Importantly, these baseline calculations assume that only capital investment decisions are made under imperfect information, while labor is free to adjust perfectly to contemporaneous conditions. In this sense, they are conservative estimates of the total impact of informational frictions. Assuming that the friction affects labor inputs to the same degree as capital leads to losses that are substantially higher. For example, in this case, losses in China and India range from about 50\% to 150\% in productivity and from 75\% to over 200\% in output. We interpret this as an upper bound on the total effect of the friction, with reality likely falling somewhere in between this case and the baseline version where only the capital choice was distorted.

As mentioned earlier, our framework also enables us to quantify the role of learning, and of particular interest, assess the extent to which financial markets ameliorate informational frictions. Here, we arrive at a striking conclusion - learning from stock prices is a very small part of total learning at the firm level, even in a relatively well-functioning financial market like the US. Thus, the impact of this channel on overall allocative efficiency and hence aggregate performance of the economy is negligible.\textsuperscript{6} We show that this is primarily due to the high levels of noise in market prices, making them relatively poor signals of fundamentals, even without

\textsuperscript{4}To give a simple example, the correlation between returns and investment can be high either because firms and investors are both perfectly informed, in which case all firm-level variables are functions of a single fundamental shock, or alternatively, because firms are poorly informed and therefore, learn much from market prices.

\textsuperscript{5}This is not to say that there is no learning in these countries. Since fundamentals are persistent, firms learn from history and therefore, their decisions reflect past innovations to fundamentals almost perfectly.

\textsuperscript{6}Of course, our findings do not rule out other channels through which well-functioning stock markets can play an important role in the real economy - e.g., to provide better incentives to managers, takeover risk, or in the IPO market. However, it appears that the additional actionable information they provide to decision-makers within the firm is not very significant.
taking into account the relatively better quality of firm-level private information.\textsuperscript{7} A counterfactual experiment delivering access to US-quality financial markets (in an informational sense) to firms in China and India generates only small improvements in allocative efficiency. In contrast, we find that private learning is the primary channel through which firms mitigate the uncertainty that they face, significantly improving resource allocations and aggregate outcomes. Moreover, the quality of information from this source for US firms is substantially better than in China and India. Our results suggest then that to the extent the losses from informational frictions vary significantly across countries, it is primarily disparities in the ability of firms to learn from private sources that are to blame, not a lack of access to well-functioning financial markets. Finally, we show that differences in the volatility of the shocks to firm-level fundamentals also play a role in generating cross-country differences in the severity of informational frictions. Specifically, firms in China and India are subject to larger shocks to fundamentals than firms in the US, making the inference problem more difficult in those countries even without the effect of differences in signal qualities.

Our paper relates to several existing branches of literature. We bear a direct connection to recent studies on the aggregate implications of misallocated resources, for example, Hsieh and Klenow (2009), Restuccia and Rogerson (2008), and Bartelsman et al. (2013). Indeed, we will see that the measure of informational frictions generated in our model maps directly into the measures of misallocation studied in these papers, i.e., into the dispersion in marginal products and the covariance between firm-level fundamentals (productivity, for example) and activity (i.e, factor use or output). We differ from these papers in our explicit modeling of a specific friction as the source of misallocation, a feature we share with Midrigan and Xu (2013) and Asker et al. (2012), who study the role of financial frictions and capital adjustment costs, respectively. Our focus on the role of imperfect information is related to that of Jovanovic (2013), who studies an overlapping generations model where informational frictions impede the efficient matching of entrepreneurs and workers.

Our structure of firm learning holds some similarity with Jovanovic (1982) and our linking of financial markets, information transmission, and real outcomes is reminiscent of Greenwood and Jovanovic (1990). As mentioned earlier, the informational role of stock markets is the subject of a large body of work in empirical finance. One strand of this literature focuses on measuring the information content of stock prices. Durnev et al. (2003) show that firm-specific variation in stock returns, i.e., “price-nonsynchronicity,” is useful in forecasting future earnings and Morck et al. (2000) find that this measure of price informativeness is higher in

\textsuperscript{7}This distinction is related to the concepts of forecasting price efficiency (i.e., to what extent do prices reflect and predict fundamentals) versus revelatory price efficiency (i.e., to what extent do prices promote real efficiency by revealing new information to the firm) as put forth in Bond et al. (2012). We explore the connections between our findings and these concepts in more depth when discussing our results.
richer countries. A related body of work closer to our own and recently surveyed by Bond et al. (2012) looks directly at the feedback from stock prices to investment and other decisions. Chen et al. (2007), Luo (2005), and Bakke and Whited (2010) are examples of studies that find evidence of managers learning from markets while making investment decisions. Bai et al. (2013) combines a simple investment model with a noisy rational expectations framework to assess whether US stock markets have become more informative over time. Our analysis complements these papers by placing information aggregation through financial markets into a standard aggregative setting, measuring the importance of this channel for information transmission, and quantifying the macroeconomic implications of the “feedback effect,” i.e., the implications for aggregate outcomes. Our results - that stock market informativeness has a negligible effect on aggregate allocative efficiency - are similar in spirit to the conclusions reached by Morck et al. (1990).

The remainder of the paper is organized as follows. Section 2 describes our model of production and financial market activity under imperfect information. Section 3 outlines our calibration strategy and presents our quantitative results, while we summarize our findings and discuss directions for future research in Section 4. Details of derivations and data work are provided in the Appendix.

2 The Model

In this section, we develop our model of production and financial market activity under imperfect information. We turn first to the production side of the economy and derive sharp relationships linking the extent of uncertainty at the micro level to aggregate outcomes. Next, we flesh out the information environment and in particular, lay out a fully specified financial market in which dispersed private information of investors and noise trading interact to generate imperfectly informative price signals.

2.1 Production

We consider an infinite-horizon economy set in discrete time. The economy is populated by a representative large family endowed with a fixed quantity of labor that is supplied inelastically to firms. The aggregate labor endowment is denoted by $N$. The household has preferences over consumption of a final good and accumulates capital that is then rented to firms. We purposely keep households simple as they play a limited role in our analysis.
Technology. A continuum of firms of fixed measure one, indexed by \( i \), produce intermediate goods using capital and labor according to

\[
Y_{it} = K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2}, \quad \hat{\alpha}_1 + \hat{\alpha}_2 \leq 1
\]

Intermediate goods are bundled to produce the single final good using a standard CES aggregator

\[
Y_t = \left( \int A_{it}^{\theta - 1} d\tilde{\theta} \right)^{\frac{\theta}{\theta - 1}}
\]

where \( A_{it} \) is the idiosyncratic quality or productivity component of good \( i \) and represents the only source of uncertainty in the economy (i.e., we abstract from aggregate risk). We assume that \( A_{it} \) follows an AR(1) process in logs:

\[
a_{it} = (1 - \rho) \bar{a} + \rho a_{it-1} + \mu_{it}, \quad \mu_{it} \sim N \left( 0, \sigma^2_\mu \right)
\]

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., \( a_{it} = \log A_{it} \). In this specification, \( \bar{a} \) represents the unconditional mean of \( a_{it} \), \( \rho \) the persistence, and \( \mu_{it} \) an i.i.d. innovation with variance \( \sigma^2_\mu \).

Market structure and revenue. The final good is produced by a competitive firm under perfect information. This yields a standard demand function for intermediate good \( i \)

\[
Y_{it} = P_{it}^{-\theta} A_{it}^\theta Y_t \quad \Rightarrow \quad P_{it} = \left( \frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\theta}} A_{it}
\]

where \( P_{it} \) denotes the relative price of good \( i \) in terms of the final good, which serves as numeraire.

The elasticity of substitution \( \theta \) indexes the market power of intermediate good producers. Our specification nests various market structures. In the limiting case of \( \theta = \infty \), we have perfect competition, i.e., all firms produce a homogeneous intermediate good. In this case, the survival of heterogenous firms requires decreasing returns to scale in production to limit firm size, that is, \( \hat{\alpha}_1 + \hat{\alpha}_2 < 1 \). When \( \theta < \infty \), we have monopolistic competition, with constant or decreasing returns to scale. No matter the assumption here, however, firm revenue can be expressed as

\[
P_{it} Y_{it} = Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2}
\]

where

\[
\alpha_j = \left( 1 - \frac{1}{\theta} \right) \hat{\alpha}_j
\]
Our framework accommodates two alternative interpretations of the idiosyncratic component $A_{it}$, either as a firm-specific level of demand or productive efficiency. The analysis is identical under both interpretations, though one could argue that learning from markets may be more plausible for demand-side factors. Neither the theory nor our empirical strategy requires us to differentiate between the two, so we will simply refer to $A_{it}$ as a firm-specific fundamental.

**Factor markets.** In our theory, the key decision affected by imperfect information is the firm’s choice of factor inputs, that is, capital and labor, which are modeled as static and otherwise frictionless decisions. Specifically, firms rent capital and/or hire labor period-by-period but with potentially imperfect knowledge of its fundamental $A_{it}$. Clearly, the impact of the friction will vary depending on whether it affects both inputs or just one. Rather than take a particular stand on this important issue regarding the fundamental nature of the production process, we present results for two cases: in case 1, both factors of production are chosen simultaneously under the same (imperfect) information set; in case 2, only capital is chosen under imperfect information whereas labor is freely adjusted after the firm perfectly learns the current state.

**Case 1: Both factors chosen under imperfect information.** In this case, the firm’s profit-maximization problem is given by

$$\max_{K_{it}, N_{it}} \quad Y_t^{\frac{1}{\alpha_1}} E_{it} [A_{it}] K_{it}^{\alpha_2} N_{it}^{\alpha_2} - W_t N_{it} - R_t K_{it}$$

where $E_{it} [A_{it}]$ denotes the firm’s expectation of fundamentals conditional on its information set $I_{it}$, which we make explicit below. Standard optimality and market clearing conditions imply

$$\frac{N_{it}}{K_{it}} = \frac{\alpha_2 R}{\alpha_1 W} = \frac{N}{K_t}$$

i.e., the capital-labor ratio is constant across firms.

Our empirical analysis relies on moments of firm-level investment data and with this in mind, we use the optimality conditions characterized in (4) to rewrite (3) simply as a capital input choice problem:

$$\max_{K_{it}} \quad \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} E_{it} [A_{it}] K_{it}^{\alpha} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) RK_{it}$$

---

8With a more general production function, the impact will also depend on the substitutability of the inputs. Intuitively, the impact is lower the greater the substitutability between inputs that are chosen under imperfect information and those that can be perfectly adjusted once the state is revealed.
where
\[ \alpha = \alpha_1 + \alpha_2 = (\tilde{\alpha}_1 + \tilde{\alpha}_2) \left( \frac{\theta - 1}{\theta} \right) \]

Notice that the firm’s expected revenues depend only on the aggregate capital-labor ratio, its conditional expectation of \( A_{it} \), and the chosen level of its capital input. The curvature parameter \( \alpha \) depends both on the returns to scale in production as well as on the elasticity of demand, and will play an important role in our quantitative analysis below. Solving this problem and imposing capital market clearing gives the following expressions for the firm’s capital choice and realized revenue (the labor choice exactly parallels that of capital),

\[
K_{it} = \frac{(E_{it}[A_{it}])^{1-\alpha}}{\int (E_{it}[A_{id}])^{1-\alpha} \, di} K_t
\]

\[
P_{it} Y_{it} = K_{it}^{\alpha_1} N_{it}^{\alpha_2} Y_{it}^{1-\theta} A_{it} \left( \frac{(E_{it}[A_{it}])^{1-\alpha}}{\int (E_{it}[A_{it}])^{1-\alpha} \, di} \right)^{\alpha}
\]

**Case 2: Only capital chosen under imperfect information.** The firm’s problem now is

\[
\max_{K_{it}} E_{it} \left[ \max_{N_{it}} Y_{it}^{1-\theta} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} \right] - R_t K_{it}
\]

and optimizing over \( N_{it} \) gives

\[
N_{it} = \left( \frac{\alpha_2}{W} Y_{it}^{\frac{1}{\alpha_2}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1-\alpha_2}}
\]

Note that in contrast to (4), capital-labor ratios are now functions of the firm’s fundamental \( A_{it} \) and chosen level of capital \( K_{it} \), the former fully observed when making the labor choice and the latter fixed. Imposing labor market clearing and substituting, we can write the firm’s capital choice problem as:

\[
\max_{K_{it}} (1 - \alpha_2) \left( \frac{\alpha_2}{W} \right)^{\frac{\alpha_2}{1-\alpha_2}} Y_{it}^{\frac{1}{\alpha_2}} E_{it} \left[ \tilde{A}_{it} \right] - R_t K_{it}
\]

where

\[\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}}, \quad \tilde{\alpha} = \frac{\alpha_1}{1-\alpha_2}\]

Thus, the firm’s capital choice problem here has the same structure as in case 1 (compare equations (5) and (8)), but with a slightly modified fundamental and overall curvature. This will make the two cases qualitatively very similar, though, as we will see, the quantitative implications will be quite different. We mark with a \( \sim \) the transformed objects that are relevant in case 2, a convention we will carry throughout the remainder of the analysis. Solving for input
choices and imposing capital market clearing gives the firm’s capital choice, labor choice, and realized revenue as:

\[
K_{it} = \frac{\left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{\alpha}}}{\int \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{\alpha}} \, di} K_t
\]

\[
N_{it} = \frac{\tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\alpha}{1 - \alpha}}}{\int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\alpha}{1 - \alpha}} \, di} N
\]

\[
P_{it} Y_{it} = Y_t^{\frac{1}{\beta}} K_t^{\alpha_1} N^{\alpha_2} \frac{\tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\alpha}{1 - \alpha}}}{\left\{ \int \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{\alpha}} \, di \right\}^{\alpha_1} \left\{ \int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\alpha}{1 - \alpha}} \, di \right\}^{\alpha_2}}
\]

While the capital choice looks quite similar to case 1, the labor choice now depends on the joint distribution of \( \tilde{A}_{it} \) and \( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \). Despite this, the analysis remains quite tractable and we will derive simple expressions for the economic aggregates as functions of the underlying uncertainty.

To complete our characterization of the firm’s problem and therefore of the production-side equilibrium in the economy, we must explicitly spell out the information set \( \mathcal{I}_{it} \) on which the firm relies to form its expectations. We defer this discussion to the following section and for now directly make conjectures about firm beliefs, which we will later verify under the particular information structure we put forth. Specifically, we assume the conditional distribution of the fundamental to be log-normal in both case 1 and 2, i.e.,

\[
a_{it} | \mathcal{I}_{it} \sim \mathcal{N} \left( \mathbb{E}_{it} [a_{it}], \mathcal{V} \right)
\]

\[
\tilde{a}_{it} | \mathcal{I}_{it} \sim \mathcal{N} \left( \mathbb{E}_{it} [\tilde{a}_{it}], \tilde{\mathcal{V}} \right)
\]

where \( \mathbb{E}_{it} [a_{it}] \) and \( \mathcal{V} \) denote the posterior mean and variance of \( a_{it} \) in case 1, respectively, and similarly \( \mathbb{E}_{it} [\tilde{a}_{it}] \) and \( \tilde{\mathcal{V}} \) in case 2. Further, we will show that cross-sectional distribution of the posterior mean \( \mathbb{E}_{it} [a_{it}] \) is also normal, centered around the true mean \( \bar{a} \) with associated variance \( \sigma_a^2 - \mathcal{V} \). Focusing on case 1 for the moment, the variance \( \mathcal{V} \) indexes the severity of informational frictions in the economy and will turn out to be a sufficient statistic for misallocation and the associated productivity/output losses. It is straightforward to show that \( \mathcal{V} \) is closely related to commonly used measures of allocative efficiency. For example, it maps exactly into the dispersion of the marginal product of capital, measured along the lines of Hsieh and Klenow (2009), i.e., \( \sigma_{MPK}^2 = \mathcal{V} \). Similarly, it has a negative effect on the covariance between
fundamentals and firm activity as examined in Bartelsman et al. (2013) and Olley and Pakes (1996). For example, the covariance between $a_{it}$ and $k_{it}$ satisfies $\sigma_{ak} = \frac{\sigma^2_{a} - \nu}{1 - \alpha}$. In this sense, our measure of informational frictions is easily related to measures of misallocation studied in the literature. An analogous correspondence holds for case 2.

**Aggregation.** We now turn to the aggregate economy, and in particular, measures of total factor productivity (TFP) and output. Given our focus on misallocation, we abstract from aggregate risk and restrict our attention to the economy’s stationary equilibrium, in which all aggregate variables remain constant through time. Relegating the rather lengthy details to the Appendix, we use (6) and (9) along with the fact that $Y = \int P_{it}Y_{id}di$ and properties of the log-normal distributions to derive the following simple representation for aggregate output, where the reader should recall that lower-case denotes natural logs:

$$y = a + \tilde{\alpha}_1 k + \tilde{\alpha}_2 n$$

(10)

Aggregate TFP, denoted $a$, is endogenous and is given by

$$\text{Case 1:} \quad a = a^* - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\alpha}{1 - \alpha} \nu$$

(11)

$$\text{Case 2:} \quad a = a^* - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{(1 - \alpha_2) \alpha_1 \tilde{\nu}}{1 - \alpha}$$

(12)

where

$$a^* = \frac{\theta}{\theta - 1} \pi + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma^2_a}{1 - \alpha}$$

is aggregate TFP under full information, which is clearly identical in the two cases. Thus, these expressions reveal a sharp connection between the micro-level uncertainty summarized by $\nu$ (or $\tilde{\nu}$) and aggregate productivity:

$$\text{Case 1 : } \quad \frac{da}{d\nu} = -\frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\alpha}{1 - \alpha} < 0$$

(13)

$$\text{Case 2 : } \quad \frac{da}{d\tilde{\nu}} = -\frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\alpha_1 (1 - \alpha_2)}{1 - \alpha} < 0$$

(14)

i.e., aggregate productivity monotonically decreases in uncertainty as measured by $\nu$ (or $\tilde{\nu}$), with the magnitude of the effect depending on the aggregate curvature parameter (and in case 2, on the relative shares of capital and labor in the production function). The higher is $\alpha$, that is, the closer we are to constant returns/perfect competition, the more severe are the losses from misallocated resources. Similarly, in case 2, the greater the share of capital, i.e., the
misallocated resource, in production, the higher are the losses from misallocation.

Holding the aggregate factor stocks fixed, the effect of informational frictions on aggregate productivity \( a \) is also the effect on aggregate output \( y \). However, the aggregate capital stock in the steady state is not invariant to the severity of the friction. Uncertainty has two effects on capital demand - at the micro level, greater uncertainty induces firms to hold more capital. To see this, note that \( \mathbb{E}_{it} [A_{it}] = \exp (\mathbb{E}_{it} [a_{it}] + \frac{1}{2} V) \) is increasing in \( V \). However, there is a stronger offsetting effect operating in general equilibrium: uncertainty and the resulting misallocation drives down the marginal product of inputs and therefore the aggregate capital stock. To see this in case 1, for example, note that in the steady state, the rental rate of capital is pinned down by the discount rate and depreciation, i.e., \( R = \frac{1}{\beta} - 1 + \delta \). Along with the expression for the aggregate capital-labor ratio derived in (4), this implies that the aggregate capital stock is proportional to the wage. It is straighforward to show that \( V \) depresses the wage, and by the above reasoning, the aggregate capital stock. A similar logic holds in case 2.

Incorporating this additional effect of uncertainty amplifies the impact of allocative inefficiencies on output and leads to the following simple relationships in our two cases, where we again leave the rather lengthy details to the Appendix:

\[
\frac{dy}{dV} = \frac{da}{dV} \left( \frac{1}{1 - \hat{\alpha}_1} \right) \tag{15}
\]

\[
\frac{dy}{\tilde{dV}} = \frac{da}{\tilde{dV}} \left( \frac{1}{1 - \tilde{\alpha}_1} \right) \tag{16}
\]

The impact of informational frictions on output is thus subject to a “multiplier,” which depends only on the capital share in the production function.\(^9\)

Thus far, we have derived sharp relationships between the severity of informational frictions, the degree of misallocation and aggregate productivity and output. Uncertainty at the firm level is summarized by \( V \) (or in case 2, by \( \tilde{V} \)), i.e., the variance of the firm’s posterior beliefs, and the parsimonious nature of our framework has enabled an analytic characterization of the degree of misallocation and the impact on the economic aggregates as a function of this simple object. We now make explicit the information structure in the economy, that is, the elements of the information set \( I_{it} \) on which the firm relies to guide its decisions, which in turn will allow us to characterize the extent of uncertainty in terms of the primitives of the economy - specifically, the variances of shocks and signal noises.

\(^9\)This is analogous to the total effect on steady state output from a change in TFP in a standard neoclassical model with fixed labor supply.
2.2 Information

The firm’s information set $\mathcal{I}_t$ is composed of three elements. First, it contains the entire history of its fundamental shock realizations, i.e., $\{a_{t-1}\}_{s=1}^{\infty}$. Obviously, given the AR(1) specification of the shock process, only the most recent realization, $a_{t-1}$, is relevant for the firm’s forecasting problem. Second, the firm also observes a noisy private signal of its contemporaneous fundamental

$$s_{it} = a_{it} + e_{it}, \quad e_{it} \sim \mathcal{N}(0, \sigma^2_e)$$

where $e_{it}$ is an i.i.d. mean-zero and normally distributed noise term. The third and last element of the information available to the firm is the price of its own stock. The final piece of our theory then is to outline how the stock price $p_{it}$ is determined and to characterize its informational content.

The stock market. Our formulation of the stock market and its informational properties follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz (1980). For our specific model structure, we draw heavily from recent work by Albagli et al. (2011a) and Albagli et al. (2011b). For each firm $i$, there is a unit measure of outstanding stock or equity, representing a claim on the firm’s profits. These claims are traded by two groups of agents - imperfectly informed investors and noise traders. As we will see, the presence of noise traders will prevent prices from perfectly aggregating investors’ information.

There is a unit measure of risk-neutral investors for each stock. Every period, each investor decides whether or not to purchase up to a single unit of equity in firm $i$ at the current market price $p_{it}$. The market is also populated by noise traders who purchase a random quantity $\Phi(z_{it})$ of stock $i$ each period, where $z_{it} \sim \mathcal{N}(0, \sigma^2_z)$ and $\Phi$ denotes the standard normal CDF.

Like firms, investors also observe the entire history of fundamental realizations, and in particular, observe $a_{it-1}$ at time $t$. They also see the current stock price $p_{it}$. Finally, each investor $j$ is endowed with a noisy private signal about the firm’s contemporaneous fundamental $a_{it}$ (it is in this sense that they are imperfectly informed):

$$s_{ijt} = a_{it} + v_{ijt}, \quad v_{ijt} \sim \mathcal{N}(0, \sigma^2_v)$$

The total demand of investors for stock $i$ is then given by

$$D(a_{it-1}, a_{it}, p_{it}) = \int d(a_{it-1}, s_{ijt}, p_{it}) dF(s_{ijt}|a_{it})$$

One interpretation of these financial market participants are as intermediaries investing on behalf of households. The details of such intermediation are not of particular importance for our analysis so we do not discuss them in greater depth here.
where \( d(a_{it-1}, s_{ijt}, p_{it}) \in [0, 1] \) is the demand of investor \( j \) and \( F \) is the conditional distribution of investors’ private signals. The expected payoff to investor \( j \) from purchasing the stock is given by

\[
\mathbb{E}_{ijt} [\Pi_{it}] = \int [\pi(a_{it-1}, a_{it}, p_{it}) + \beta\mathbb{P}(a_{it})] dH(a_{it-1}, s_{ijt}, p_{it})
\]

where \( H(a_{it}|a_{it-1}, s_{ijt}, p_{it}) \) is the investor’s posterior belief over \( a_{it} \) and \( \mathbb{P}(a_{it}) \) is the expected future (period \( t+1 \)) stock price of a firm with realized fundamental \( a_{it} \), defined by

\[
\mathbb{P}(a_{it}) = \int p(a_{it}, a_{it+1}, z_{it+1}) dG(a_{it+1}, z_{it+1}|a_{it})
\]

where \( G \) is the joint distribution of \((a_{it+1}, z_{it+1})\), conditional on \( a_{it} \). The term \( \pi(a_{it-1}, a_{it}, p_{it}) \) denotes the expected profit of the firm from the investor’s perspective, which is a function of the elements of the firm’s information set that are observable to investors, namely \( a_{it-1} \) and \( p_{it} \). Clearly, optimality implies:

\[
d(a_{it-1}, s_{ijt}, p_{it}) = \begin{cases} 
1 & \text{if } \mathbb{E}_{ijt} [\Pi_{it}] > p_{it} \\
0 & \text{if } \mathbb{E}_{ijt} [\Pi_{it}] < p_{it} 
\end{cases}
\]

that is, the investor purchases a share when the expected payoff exceeds the price, does not purchase any shares when the expected payoff is less than the price, and is different when the two are equalized.

We conjecture that both \( p(\cdot) \) and \( \pi(\cdot) \) are monotonically increasing in \( a_{it} \). Combined with the fact that investors’ posterior beliefs are first-order stochastically increasing in \( s_{ijt} \), we can then show that the trading decisions of investors are characterized by a threshold rule: in equilibrium, there is a signal \( \hat{s}_{it} \) such that only investors observing signals higher than \( \hat{s}_{it} \) choose to buy a share.\(^{11}\) Aggregating the demand decisions of all investors, market clearing implies

\[
1 - \Phi \left( \frac{\hat{s}_{it} - a_{it}}{\sigma_v} \right) + \Phi(z_{it}) = 1
\]

which leads to a simple characterization of the threshold signal

\[
\hat{s}_{it} = a_{it} + \sigma_v z_{it}
\]

Next, note that the marginal investor, i.e., the investor satisfying \( s_{ijt} = \hat{s}_{it} \), is exactly indifferent between buying or not buying the stock. It follows then that the price \( p_{it} \) must

\(^{11}\)See Albagli et al. (2011a) for more details.
equal her expected payoff from holding the stock:

\[ p_{it} = \int \left[ \pi \left( a_{it-1}, a_{it}, p_{it} \right) + \beta \tilde{p} \left( a_{it} \right) \right] dH \left( a_{it} | a_{it-1}, \hat{s}_{it}, p_{it} \right) \]

Since \( H \) is first-order stochastically increasing in \( \hat{s}_{it} \), this translates into a monotonic relationship between \( p_{it} \) and \( \hat{s}_{it} \), with the implication that observing the stock price is informationally equivalent to observing \( \hat{s}_{it} \): in other words, the stock price serves as an additional noisy signal of firm fundamentals as defined in (17). The precision of this signal is \( \frac{1}{\sigma^2} \), i.e., it is jointly determined and multiplicative in the variance of the noise in investors’ private information and the size of the noise trader shock and decreasing in both.

This simple expression for price informativeness is the key payoff of our modeling approach here: with this in hand, we have a complete characterization of the firm’s information set and hence the posterior variance \( \mathbb{V} \), even without an explicit solution for the price function. Formally, the firm’s information set is then defined by \( \mathcal{I}_{it} = (a_{it-1}, s_{it}, \hat{s}_{it}) \), where \( a_{it-1} \) is the relevant history, \( s_{it} \) the firm’s private signal, and \( \hat{s}_{it} \) the market signal defined in (17), which in turn yields a simple expression for \( \mathbb{V} \):

\[ \mathbb{V} = \left( \frac{1}{\sigma^2} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_v \sigma^2_z} \right)^{-1} \]

In the absence of any learning, \( \mathbb{V} = \sigma^2_\mu \), that is, all fundamental uncertainty remains unresolved at the time of the firm’s input choice. At the other extreme, under perfect information, \( \mathbb{V} = 0 \). More generally, \( \mathbb{V} \) is increasing in the noisiness of the firm’s private signal, i.e., \( \sigma^2_e \) and of stock market prices \( \sigma^2_v \sigma^2_z \). An analogous expression and interpretation holds for \( \tilde{\mathbb{V}} \).

Finally, we characterize the price function, which we will require for our quantitative analysis. With a slight abuse of notation, we can express the indifference condition of the marginal investor in recursive form, yielding a fixed-point characterization of the price function:

\[ p \left( a_{-1}, a, z \right) = \int \pi \left( a_{-1}, a, a + \sigma_v z \right) dH \left( a | a_{-1}, a + \sigma_v z, a + \sigma_v z \right) \]

\[ + \beta \int \left[ \int p \left( a_{-1}, a, z \right) dG \left( a, z | a_{-1} \right) \right] dH \left( a | a_{-1}, a + \sigma_v z, a + \sigma_v z \right) \]

Given a profit function \( \pi \left( \cdot \right) \), which is readily obtained from the firm’s problem characterized above, this functional equation can be solved numerically using a standard iterative procedure.

\(^{12}\)It is straightforward to show that the conditional and cross-sectional distributions are log-normal under this information set, exactly as conjectured.
3 Quantitative Analysis

We now use the theory laid out in the previous sections to quantify the extent of informational frictions, the degree of resulting misallocation, and the impact on aggregate outcomes, e.g., aggregate productivity and output across a number of countries. Our analysis will also shed light on the role of learning from the various channels present in the model. Of some particular interest is the importance (or as we will see, lack thereof) of financial markets in improving allocative efficiency by delivering useful information to firms. We turn first to our calibration approach, in which we combine firm-level production and stock market data to infer the informational parameters of the model. We then present our results - the estimated parameters and their effect on aggregate outcomes. Lastly, we perform a number of counterfactual experiments to decompose the uncertainty we uncover and explore the quantitative role of various sources of information.

3.1 Calibration

A key hurdle in quantifying the effects of imperfect information is imposing discipline on the information structure in the economy, given that we do not directly observe signals at the firm level. We devise a novel calibration strategy that overcomes this difficulty by using moments of both firm-level production and stock market data to pin down the informational parameters of our model. The presence of learning from financial markets is key to this approach and thus its dual role in our analysis: first, as an important piece of the learning environment in which firms operate, enabling us to quantify the informational role of these markets in improving resource allocation; and second, as an identification device, allowing us to infer the severity of otherwise unobservable informational frictions.

Before describing our approach and the resulting parameter estimates in greater detail, we begin by assigning values to the more standard parameters in our model - specifically, those governing the preference and production structure of the economy. Throughout our analysis, we maintain the assumption that these are constant across countries, that is, the only differences across countries will come from the parameters governing the stochastic processes on firm fundamentals and learning. Although a simplification, we feel that this approach is a natural starting point and allows us to focus on the role of imperfect information and the cross-country differences arising from this channel.

An important issue is the choice of period length in the model. We feel that our modeling strategy and focus on investment decisions naturally pushes us towards larger period lengths. The empirical evidence on investment lags suggests that they can be quite long, with a mean duration time between the planning stage and project completion ranging in the vicinity of
between 2 and 3 years.\textsuperscript{13} In this light, it seems reasonable to assume that firms are required to forecast fundamentals over a relatively long horizon (2-3 years) when making large investment decisions and have only limited flexibility to adjust capital choices ex-post in response to additional information. One approach would be to explicitly model these lags as well as other features (such as adjustment costs/irreversibilities) which are likely relevant for investment decisions over short horizons. We take a different tact and model capital input as a static decision, but interpret a period in the model as spanning a relatively long horizon, making the omission of explicit lags/irreversibilities, etc. somewhat less problematic.\textsuperscript{14} This approach allows us to preserve the analytical tractability on the production side, particularly the simple expressions sharply relating aggregates to uncertainty. Specifically then, we use a period length of 3 years.

In line with our choice of period length, we set the discount rate $\beta$ equal to 0.9. We assume constant returns to scale in production. Firm scale is then limited only by the curvature in demand, captured by the elasticity of substitution $\theta$. This will be a key parameter in governing the quantitative impact of the informational frictions we infer. The literature contains a wide range of estimates for this parameter, and in this light, we report our results for two values of $\theta$, 4 and 10. Under case 1, in which all inputs are chosen under imperfect information, it is not necessary to calibrate the individual production parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$, as only the aggregate returns to scale plays a role. In contrast, under case 2, in which only capital is subject to the informational friction, the relative magnitudes of these parameters is important, and we set them to standard values of 0.33 and 0.67, respectively.\textsuperscript{15}

Next, we turn to the country-specific parameters. We begin with those governing the process on firm fundamentals $a_{it}$, that is, the persistence $\rho$ and variance of the innovations $\sigma^2_{\mu}$. Notice that in both of our cases, we can directly construct the fundamental for each firm (up to an additive constant) as the log of revenue (which maps into $y$) less the relevant $\alpha$ (depending on the case and the choice of $\theta$) multiplied by $k$.\textsuperscript{16} We then estimate the parameters of the fundamental process by performing the autoregression implied by (1), additionally including a time fixed-effect in order to isolate the idiosyncratic component of the innovations. The

\textsuperscript{13}The classic reference here is Mayer (1960), who uses survey data on new industrial plants and additions to existing plants to find a mean gestation lag between the drawing of plans and the completion of construction of about 22 months. More recently, Koeva (2000) finds the average length of time-to-build lags to be about 2 years in most industries, defined as the period between the announcement of new construction and the ensuing date of completion. Given that this excludes the planning period prior to the announcement date, the total gestation lag is likely somewhat longer.

\textsuperscript{14}Morck et al. (1990) make a similar argument and perform their baseline empirical analysis using 3-year spans. They also point out that the explanatory power of investment growth regressions at shorter horizons (e.g., 1 year) are quite low.

\textsuperscript{15}Although it should be noted that a value for $\hat{\alpha}_1$ is needed in both cases to compute the output effects in (15) and (16).

\textsuperscript{16}Substitute either (4) or (7) into (2).
resulting coefficient delivers an estimate for $\rho$ and the variance of the residuals for $\sigma^2_\mu$. We report the results in Table 3 below.

Finally, it remains to pin down the three informational parameters of our model, i.e., the variances of the error terms in firm and investor signals $\sigma^2_\varepsilon$ and $\sigma^2_\nu$, and the variance of the noise trader shock $\sigma^2_z$. The moments we target here are the correlations of investment growth and revenue growth with stock returns (lagged by one period), and the variance of stock returns. Before discussing this approach in greater depth, we summarize our calibration strategy in Table 1 (note that we use $\Delta p$ as shorthand for returns, a convention we follow hereafter).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.90</td>
</tr>
<tr>
<td>$\hat{\alpha}_1$</td>
<td>Capital share</td>
<td>0.33</td>
</tr>
<tr>
<td>$\hat{\alpha}_2$</td>
<td>Labor share</td>
<td>0.67</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution</td>
<td>4, 10</td>
</tr>
</tbody>
</table>

Table 1: Calibration Summary

Country-specific

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Persistence of fundamentals</td>
<td>Estimates of (1): $a_{it} = (1 - \rho) \bar{a} + \rho a_{it-1} + \mu_{it}$</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>Shocks to fundamentals</td>
<td>$\rho (\Delta i, \Delta p)$</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>Firm private info</td>
<td>$\rho (\Delta y, \Delta p)$</td>
</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>Investor private info</td>
<td>$\sigma (\Delta p)$</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Noise trading</td>
<td>$\sigma (\Delta p)$</td>
</tr>
</tbody>
</table>

The combination of firm-level production and financial market data is a key feature of our approach and it makes sense here to give a clear sense of how our model parameters map into these moments. Of course, the parameters are calibrated jointly and so there is not a one-to-one mapping between moments and parameters. However, it is possible to make a heuristic argument in order to gain some intuition about the connection between specific moments and parameters.

To illustrate this intuition, Table 2 reports our target moments for the US and the model-implied values under various informational assumptions. Specifically, stock returns in the

---

17 We lag stock returns by one period to avoid feedback from investment and sales to returns, the reverse of the relationship in which we are interested. We follow Morck et al. (1990) by focusing on first-differences in variables, i.e., sales and investment growth, rather than levels, in an effort to cleanse the data of firm fixed-effects, which tend to be the dominant influence in cross-sectional differences in these variable across firms.

18 We describe the sources and procedure for computing these moments in greater detail below. For illustration purposes, we display results from case 2 with $\theta = 4$ and hence $\hat{\alpha} = 0.50$. 
US show a variance of 0.27 and correlations with investment growth and revenue growth of 0.20 and 0.34 respectively. A full information version of our model would imply substantially lower return volatility and much higher correlations, with that between revenue growth and returns approaching 1. The next row shows the implied moments when firms are assumed to have perfect information but the stock market is noisy.\textsuperscript{19} Under this scenario, the volatility of returns increases substantially (though still falling short of its empirical counterpart), pointing to the strong connection between imperfectly informed investors and noise trading on the one hand and return volatility on the other. Perhaps more strikingly, the correlations of returns with both investment and revenue growth fall drastically, a pattern to which we return in a moment. Finally, the last row shows the model-implied moments when firms have no private information and markets are noisy, so that all learning takes place from imperfectly revealing market prices. While return volatility changes only slightly relative to the previous row, both correlations rise substantially, though remaining well below the full information case. Intuitively, the last two cases hold fixed the noisiness of markets and vary the precision of the firm’s private information. When the firm has full information, it pays no attention to the market and these correlations fall very close to zero. At the other extreme, when the firm has no information, it relies solely on the market and these correlations rise. In sum then, we can heuristically map the volatility of returns into market noise, and for a given level of noise, the correlations of returns with investment decisions and revenues allow us to infer the quality of firm private information.\textsuperscript{20}

Table 2: Implied Moments from Various Informational Scenarios

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^2(\Delta p)$</th>
<th>$\rho(\Delta i, \Delta p)$</th>
<th>$\rho(\Delta y, \Delta p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Data</td>
<td>0.27</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>Firm Info</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full</td>
<td>0.12</td>
<td>0.63</td>
<td>0.97</td>
</tr>
<tr>
<td>Full</td>
<td>0.23</td>
<td>0.01</td>
<td>0.19</td>
</tr>
<tr>
<td>None</td>
<td>0.21</td>
<td>0.53</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Table 2 also highlights an important insight: a high correlation between returns and investment is not necessarily indicative of significant learning from markets. This is perhaps best illustrated by the full information case, where markets provide no additional information to

\textsuperscript{19}More precisely, the precision of investor’s private signals and the noise trading shock are fixed at our baseline estimates.

\textsuperscript{20}One possible criticism of this approach is that returns and investment might be correlated for reasons other than learning - e.g., managerial incentive contracts. To address this concern, we have evaluated the robustness of our quantitative findings to significant changes in the correlation targets. The parameter estimates of course change with the targets, but our overall conclusions regarding the severity of informational frictions and the role of financial markets are not significantly altered.
firms (which are assumed to have perfect information from alternative sources), yet this correlation is quite high. Next, consider the case where firms are perfectly informed but markets are noisy: the correlation drops to zero, even though markets continue to play no role in firm learning. Finally, consider the last case, where firms learn only from markets, and compare it to the full information benchmark: the correlation is lower, despite the fact that firms now rely heavily on markets for information, where they did not do so at all in the prior case. These examples spell out a more general message - a single moment, e.g., the correlation between returns and investment growth, is not sufficient to identify the informational role of markets or the extent of uncertainty at the micro level; rather, it is the joint pattern of the three moments that matters and our explicit modeling of both production and financial market activity is precisely what allows us to tease this out.

With this intuition in hand, we use a simulated method of moments approach to assign values to the informational parameters. Formally, we search over the parameter vector \((\sigma^2_e, \sigma^2_v, \sigma^2_z)\) to find the combination that minimizes the (unweighted) sum of squared deviations of the model-implied moments from the targets, and it is to the construction of these latter that we now turn.

### 3.2 Data and Targets

We construct the target moments using data on firm-level production variables and stock returns from Compustat North America for the US and Compustat Global for China and India. We focus on a single cross-section of firms in each country for the year 2012. This is the period with the largest number of observations, particularly so for China and India.\(^{21}\) Note that our data requirements are quite stringent: due to our focus on 3 year horizons, we require at least 6 consecutive years of data for a firm in order to construct two 3 year periods and include it in our sample, with 2012 representing the final year of the second period.

We compute the firm’s capital stock in each period as the average of its property, plant and equipment (PP&E) over the relevant 3 years, and investment as the change in the capital stock relative to the preceding 3 year period. We compute sales analogously as the average over the 3 year period. Taking first-differences of these measures gives investment and sales growth between the two 3 year periods. Stock returns are constructed as the change in the firm’s stock price over each 3 year period, adjusted for splits and dividend distributions. In order to be comparable to the unlevered returns in our model, stock returns in the data need to be adjusted for financial leverage. To do so, we assume that claims to firm profits are sold

\(^{21}\)This is less of an issue for the US, and so as a robustness exercise, we recomputed the moments using a larger sample with more years. While there is some time variation, the results are quite similar to those from the single cross-section.
to investors in the form of debt and equity in a constant proportion (within each country). Observed return variances must then be multiplied by a constant factor in order to make them comparable to returns in the model, where the factor depends on the ratio of debt to total assets (or alternatively, debt to equity). Computing the debt-asset ratio from our sample gives average values of about 0.30 in both the US and India, and about 0.16 in China, with corresponding adjustment factors of about 0.5 and 0.7, respectively (the figures we report below, and above in Table 2 for the US, reflect this adjustment).

To isolate the firm-specific variation in our data series, we extract a time fixed-effect from each and utilize the residual as the component that is idiosyncratic to the firm. This is equivalent to demeaning each series from the unweighted average in each time period. Finally, the target moments are computed using returns lagged by one period. This avoids the simultaneity problems associated with comparing price movements to contemporaneous investment/revenue numbers. The Appendix provides further details on how we build our sample and construct the variables.

We report the moments for all 3 countries in Table 3. The moments exhibit significant cross-country variation. The US and India show similar levels of return variances but quite different comovements - returns in the US are more correlated with future revenue and investment growth than in India. China has a return variance that is almost half that of the other two countries, along with the lowest correlations with investment and revenues. As we will see next, these patterns will lead to very different estimates for the severity of informational frictions in the 3 countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\sigma^2(\Delta p)$</th>
<th>$\rho(\Delta i, \Delta p)$</th>
<th>$\rho(\Delta y, \Delta p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.27</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>China</td>
<td>0.15</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>India</td>
<td>0.28</td>
<td>0.18</td>
<td>0.21</td>
</tr>
</tbody>
</table>

### 3.3 Results

We begin by presenting results for case 2, in which only capital is chosen under imperfect information. This is the more conservative scenario (in the sense that it leads to lower TFP/output losses), but may well be closer to reality than case 1, particularly given our 3-year planning

---

22 In brief, letting $d$ denote the debt-asset ratio, the observed return variances must be multiplied by $(1 - d)^2$ to obtain the variance of unlevered returns. We describe the details of the calculations in the Appendix.

23 An alternative is to use CAPM $\beta$'s to remove the aggregate component from individual stock returns. This approach yields very similar results.

24 For example, $\rho(\Delta i, \Delta p)$ is the correlation between 2006-09 returns and investment growth during 2009-12.
horizon. Table 4 presents our parameter estimates, the implied value for \( \tilde{V} \), both in absolute terms and as a percentage of the underlying uncertainty \( (\sigma_u^2) \) and the total dispersion in fundamentals \( (\sigma_a^2) \), as well as the implied aggregate TFP and output losses relative to the full information benchmark. As mentioned above, we report results for two values of the elasticity of substitution \( \theta \), namely 4 and 10, and note that our results depend quantitatively to a great extent on this value. As we would expect from the cross-country variation in the calibration targets in Table 3, the parameter estimates also vary markedly across countries. The US has less volatile fundamental shocks, better private information at the firm level (lower \( \sigma_e \)), and lower levels of noise in the stock market (lower \( \sigma_v \) and \( \sigma_z \)). This is reflected in the relatively low levels of \( V \), especially in the \( \theta = 4 \) case, where total learning eliminates almost two-thirds of the uncertainty \( (V/\sigma_u^2 = 0.37) \). Firms in India are the least informed, while Chinese firms display an intermediate level of learning. There remains, however, a substantial degree of uncertainty across the board. The resulting productivity and output losses are significant: the former range from 2% to 12% in the US and from 5% to 19% in China and India; the latter are even higher, reflecting the “multiplier” effect discussed above, ranging from 4% to 17% in the US and from 8% to 28% in China and India.\(^{26,27}\) The impact of informational frictions varies significantly across the three countries, leading to substantial differences in productivity and output between the US and the two emerging markets ranging from 3% to 7% for the former and 4% to 11% for the latter (subtract \( a^* - a \) and \( y^* - y \) for the US from the corresponding values for China and India). These remain somewhat modest, however, in comparison to standard measures of cross-country differences in aggregate TFP and income per-capita.

Table 5 reports the results from case 1, in which both capital and labor are chosen under imperfect information. The general patterns in fundamentals and learning found in case 2 continue to hold in case 1, but the quantitative impact of informational frictions is now much larger. Productivity losses compared to the full information benchmark range between 35% and 150% and output losses between 52% and well over 200%. Additionally, differences in the severity of informational frictions account for large disparities in TFP and income across countries, ranging from 15% to about 50% for the former and about 25% to about 75% for the latter.

\(^{25}\)However, because \( a_{it-1} \) is perfectly known, uncertainty about total fundamentals is much lower.

\(^{26}\)Recall that productivity losses stem solely from the misallocation of resources while output losses take into account the additional effect on the steady state capital stock.

\(^{27}\)Note that the impact of the friction increases dramatically in \( \theta \). This occurs for 2 reasons. First, as can be seen in (14), for a given \( \tilde{V} \) the aggregate consequences are larger for higher values of \( \theta \) (i.e., higher \( \tilde{\alpha} \)). Second, the estimated \( \tilde{V} \) itself increases with \( \theta \) even while the target moments remain the same. To understand why, recall from (9) that the elasticity of revenue with respect to expectations is increasing in \( \tilde{\alpha} \) and so in \( \theta \). As revenue becomes more responsive to signals, holding all else constant, the correlation between revenue and returns should increase. Because the data moments are held fixed, that this correlation does not rise implies that uncertainty must be greater.
Table 4: Results - Case 2

<table>
<thead>
<tr>
<th></th>
<th>ρ</th>
<th>σμ</th>
<th>σε</th>
<th>σν</th>
<th>σz</th>
<th>V̂</th>
<th>V̂  σ̂μ</th>
<th>V̂  σ̂ν</th>
<th>a∗ − a</th>
<th>y∗ − y</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.94</td>
<td>0.45</td>
<td>0.36</td>
<td>0.41</td>
<td>2.95</td>
<td>0.07</td>
<td>0.37</td>
<td>0.04</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>China</td>
<td>0.80</td>
<td>0.50</td>
<td>0.66</td>
<td>0.89</td>
<td>4.51</td>
<td>0.16</td>
<td>0.63</td>
<td>0.23</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>India</td>
<td>0.97</td>
<td>0.53</td>
<td>1.03</td>
<td>0.55</td>
<td>6.66</td>
<td>0.22</td>
<td>0.78</td>
<td>0.05</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>θ = 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.89</td>
<td>0.45</td>
<td>1.12</td>
<td>1.48</td>
<td>5.32</td>
<td>0.17</td>
<td>0.86</td>
<td>0.18</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>China</td>
<td>0.76</td>
<td>0.52</td>
<td>1.95</td>
<td>2.60</td>
<td>12.64</td>
<td>0.25</td>
<td>0.93</td>
<td>0.39</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>India</td>
<td>0.90</td>
<td>0.54</td>
<td>2.63</td>
<td>1.98</td>
<td>9.31</td>
<td>0.28</td>
<td>0.96</td>
<td>0.18</td>
<td>0.19</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 5: Results - Case 1

<table>
<thead>
<tr>
<th></th>
<th>ρ</th>
<th>σμ</th>
<th>σε</th>
<th>σν</th>
<th>σz</th>
<th>V̂</th>
<th>V̂  σ̂μ</th>
<th>V̂  σ̂ν</th>
<th>a∗ − a</th>
<th>y∗ − y</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.89</td>
<td>0.45</td>
<td>1.12</td>
<td>1.48</td>
<td>5.32</td>
<td>0.17</td>
<td>0.86</td>
<td>0.18</td>
<td>0.35</td>
<td>0.52</td>
</tr>
<tr>
<td>China</td>
<td>0.76</td>
<td>0.52</td>
<td>1.95</td>
<td>2.60</td>
<td>12.64</td>
<td>0.25</td>
<td>0.93</td>
<td>0.39</td>
<td>0.50</td>
<td>0.76</td>
</tr>
<tr>
<td>India</td>
<td>0.90</td>
<td>0.54</td>
<td>2.63</td>
<td>1.98</td>
<td>9.31</td>
<td>0.28</td>
<td>0.96</td>
<td>0.18</td>
<td>0.56</td>
<td>0.84</td>
</tr>
<tr>
<td>θ = 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.87</td>
<td>0.47</td>
<td>1.46</td>
<td>1.76</td>
<td>6.36</td>
<td>0.20</td>
<td>0.90</td>
<td>0.22</td>
<td>1.00</td>
<td>1.51</td>
</tr>
<tr>
<td>China</td>
<td>0.75</td>
<td>0.55</td>
<td>2.30</td>
<td>2.52</td>
<td>12.77</td>
<td>0.29</td>
<td>0.95</td>
<td>0.41</td>
<td>1.43</td>
<td>2.15</td>
</tr>
<tr>
<td>India</td>
<td>0.87</td>
<td>0.56</td>
<td>2.80</td>
<td>2.28</td>
<td>9.96</td>
<td>0.30</td>
<td>0.96</td>
<td>0.23</td>
<td>1.51</td>
<td>2.26</td>
</tr>
</tbody>
</table>

Under either scenario then, informational frictions have a significant detrimental impact on aggregate performance, both when comparing the calibrated economies to their full information benchmarks and when comparing the consequences of such frictions in emerging markets to the US. The magnitude of these effects depend to a great extent on the nature of the firm’s input decisions, that is, whether all inputs are chosen subject to the friction or only traditionally quasi-fixed inputs like capital. The truth likely lies in between the two extreme cases we have examined here, so that we think of the two scenarios as providing bounds, although admittedly fairly wide ones, on the adverse consequences of imperfect information for aggregate performance. An important direction for future work is to shed light on which of these cases is empirically more relevant.28

28One possible approach to disentangle the nature of input decisions would be to perform a similar analysis using employment data. Unfortunately, data on labor input (e.g., wage bill) is available only for a small set of firms in our dataset and even for these, is not very reliable.
3.4 Exploring the Sources of Learning

Decomposing $\mathbb{V}$. Tables 4 and 5 show that at least under some parameterizations, firm learning is quite significant and mitigates a substantial portion of the underlying uncertainty. Here, we quantitatively explore the relative importance of the two sources of learning present in our model, i.e., private information versus market prices. We begin by reporting in the left-hand panel of Table 6 the total extent of learning and the aggregate consequences. To do so, we compute the reduction in $\mathbb{V}$ both in absolute and percentage terms due to learning from both channels, i.e. $\Delta \mathbb{V} = \mathbb{V} - \sigma_\mu^2$, and the resulting effects on aggregate productivity and output.\(^{29}\) The table shows that total learning can be quite important and translates into significant improvements in TFP and output, ranging from 2% to 11% for the US for the former and from 3% to over 15% for the latter. The corresponding figures in China and India are noticeably smaller, an observation to which we return below, but are still significant.\(^{30}\)

To break down the sources of learning, it is easier to work with the inverse of $\mathbb{V}$, i.e. the total precision of the firm’s information, which, given our information structure, lends itself to a simple linear decomposition, i.e.,

$$
\frac{1}{\mathbb{V}} = \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}
$$

We focus on the latter two terms, which capture the contributions of private and market learning to overall precision and compute their relative import as $\frac{1}{\sigma_v^2\sigma_z^2} + \frac{1}{\sigma_e^2 + \sigma_\mu^2}$ and $\frac{1}{\sigma_v^2\sigma_z^2}$, respectively. In other words, we are calculating the fraction of the total increase in precision due to each source. We report the results in the second panel of Table 6. Strikingly, learning is due overwhelmingly to private sources: at best, markets account for 8% of the increase in precision in the US and 7% in India. In most cases, the contribution of markets is negligible, ranging from practically zero to only about 3% of total learning. As we will see, the limited informational role of markets will prove to be a robust finding throughout our investigation.

The role of financial markets. To explore in greater depth the importance of new information in stock prices, we recompute $\mathbb{V}$ under the assumption that firms learn nothing from these prices, i.e., that they contain no information. This simply involves sending the noise in markets, $\sigma_v^2\sigma_z^2$, to infinity. The change in $\mathbb{V}$ is a measure of the contribution of stock market

\(^{29}\)Strictly speaking, the calculations for case 2 pertain to $\tilde{\mathbb{V}}$, but for the remainder of the analysis, we economize on notation and use $\Delta \mathbb{V}$ for both cases.

\(^{30}\)Note the differences between these calculations and the losses reported in Tables (4) and (5). There, we compute losses compared to a full-information benchmark. Here, we compute losses when moving to a no-information (about innovations to fundamentals) benchmark.
### Table 6: The Importance and Sources of Learning

<table>
<thead>
<tr>
<th>Source</th>
<th>Share from source</th>
<th>Case 2, $\theta = 4$</th>
<th>Case 2, $\theta = 10$</th>
<th>Case 1, $\theta = 4$</th>
<th>Case 1, $\theta = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\Delta V$</td>
<td>$\frac{\Delta V}{\sigma_{\mu}^2}$</td>
<td>$\Delta a$</td>
<td>$\Delta y$</td>
</tr>
<tr>
<td>US</td>
<td></td>
<td>-0.13</td>
<td>-0.63</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>China</td>
<td></td>
<td>-0.09</td>
<td>-0.37</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>India</td>
<td></td>
<td>-0.06</td>
<td>-0.22</td>
<td>0.02</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The left hand panel shows the total effect of learning (i.e., from all sources). The right hand panel shows the relative shares of private and market sources in total learning.

The information to firm learning and satisfies

$$\Delta V = V - \left(\frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_{e}^2}\right)^{-1}$$

We report this value in the first columns of Table 7, both in absolute value and as a percentage of the underlying uncertainty $\sigma_{\mu}^2$, along with the associated aggregate consequences. Even for the US, which has the highest degree of investment-return correlation, market information reduces uncertainty only modestly and leads to aggregate output gains of only 0.2% (at a maximum). This also serves to reinforce our earlier point - the need to analyze these moments jointly through the lens of a structural model. For China and India, the contribution of market-produced information is virtually zero.

Next, we ask whether the negligible informational role of markets is due to the level of noise in prices or to the fact that firms already have considerable information about fundamentals, limiting the incremental contribution of market information. This distinction is related to the concepts of forecasting price efficiency (FPE) versus revelatory price efficiency (RPE) as put
### Table 7: The Contribution of Market Information

<table>
<thead>
<tr>
<th>Case 2, $\theta = 4$</th>
<th>With both sources</th>
<th>With only market learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td>US</td>
<td>-0.004</td>
<td>-0.020</td>
</tr>
<tr>
<td>China</td>
<td>-0.002</td>
<td>-0.006</td>
</tr>
<tr>
<td>India</td>
<td>-0.004</td>
<td>-0.013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2, $\theta = 10$</th>
<th>With both sources</th>
<th>With only market learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td>US</td>
<td>-0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td>China</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>India</td>
<td>-0.000</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1, $\theta = 4$</th>
<th>With both sources</th>
<th>With only market learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td>US</td>
<td>-0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td>China</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>India</td>
<td>-0.000</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1, $\theta = 10$</th>
<th>With both sources</th>
<th>With only market learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \bar{V}$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td>US</td>
<td>-0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>China</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>India</td>
<td>-0.000</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

The table shows the incremental effect of market information on $\bar{V}$, both in the presence of private learning (the first set of columns) and when markets are the only source of information (the second set).

The former captures the extent to which prices reflect and predict fundamentals, i.e., the absolute level of information in prices; the latter, the extent to which prices promote real efficiency by revealing new information to the firm. In our framework, $\Delta \bar{V}$ in the first columns of Table 7 is the natural measure of RPE, since it is the marginal impact of the information in prices on uncertainty, given the other sources of firm information. As the table shows, at the estimated parameter values, markets in all 3 countries exhibit very low RPE. To measure FPE, we compute the reduction in $\bar{V}$ from the information in stock prices alone, i.e.,

$$\Delta \bar{V} = \left( \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_v^2\sigma_z^2} \right)^{-1} \sigma_\mu^2$$

This measures the contribution of market information under the assumption that it is the only source of learning for firms. We report the results in the last columns of Table 7. With the exception of the US in the first row, stock markets are rather weak predictors of fundamentals - even as the only source of learning, the information they provide leads to negligible reductions in uncertainty. In the most optimistic scenario in the first row, market information reduces
fundamental uncertainty by only about 12%. The associated TFP and output effects are 0.8% and 1.2%, respectively for this case and negligible for all other cases. Even in a forecasting sense then, the efficiency of stock markets is very low. Comparing these values to the corresponding ones in the first set of columns suggests that the limited role of market information is largely due to its poor quality, i.e., the poor RPE of stock markets here can be attributed to their low FPE.

**The role of private information.** Next, we explore the contribution of firm private learning to aggregate performance by performing experiments analogous to those above: specifically, we calculate the marginal contribution of private information to allocative efficiency both in the presence of market information and when it is the only source of learning for firms.

We report the results in Table 8. In stark contrast to the effect of market learning, firm private information plays a significant role in reducing uncertainty and improving aggregate performance. Turning to the first set of columns, private information eliminates anywhere from 9-50% of the underlying uncertainty \( \sigma_\mu^2 \) in the US. The associated implications for the aggregates are substantial, with TFP gains ranging from 2-10% and output gains from 3-16%. The corresponding results for India and China are smaller, indicating the worse quality of firm private information in those countries. Having said that, they are still quite significant, and of course, are noticeably larger than the effects of market information.

The second set of columns in Table 8 reports the contribution of firms private information under the assumption that it is the only source of learning. A comparison of the first and second set of columns reveals that the values are quite similar - the presence of market information does not alter the importance of private information for overall learning.

### 3.5 Cross-Country Differences

In the last piece of our analysis, we explore in greater depth the cross-country patterns uncovered by our estimates. Specifically, we assess the potential gains to China and India from having access to US quality information, whether through more informative financial markets or from better firm-level private information. In a first experiment, we compute the change in \( V \) in China and India under the assumption that the informativeness of prices - summarized by \( \sigma_p^2/\sigma_z^2 \) - is equal to that in the US, leaving all other country-specific parameters fixed. In other words, we quantify the improvements in aggregate performance were China and India to have financial markets that were as informative as that in the US. Second, we perform the same exercise for firm private information, that is, compute the change in \( V \) and aggregate improvements under the assumption that firms in China and India have the same \( \sigma_e^2 \) as their US counterparts (again leaving the other country-specific parameters fixed).
Table 8: The Contribution of Private Information

<table>
<thead>
<tr>
<th>Case 2, $\theta = 4$</th>
<th>With both sources</th>
<th>With only private learning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta V$</td>
<td>$\frac{\Delta V}{\sigma_{\mu}}$</td>
</tr>
<tr>
<td>US</td>
<td>-0.10</td>
<td>-0.51</td>
</tr>
<tr>
<td>China</td>
<td>-0.09</td>
<td>-0.35</td>
</tr>
<tr>
<td>India</td>
<td>-0.06</td>
<td>-0.20</td>
</tr>
<tr>
<td>Case 2, $\theta = 10$</td>
<td>-0.03</td>
<td>-0.14</td>
</tr>
<tr>
<td>US</td>
<td>-0.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>China</td>
<td>-0.01</td>
<td>-0.04</td>
</tr>
<tr>
<td>Case 1, $\theta = 4$</td>
<td>-0.03</td>
<td>-0.14</td>
</tr>
<tr>
<td>US</td>
<td>-0.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>China</td>
<td>-0.01</td>
<td>-0.04</td>
</tr>
<tr>
<td>Case 1, $\theta = 10$</td>
<td>-0.02</td>
<td>-0.09</td>
</tr>
<tr>
<td>US</td>
<td>-0.02</td>
<td>-0.05</td>
</tr>
<tr>
<td>China</td>
<td>-0.01</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

The table shows the incremental effect of private information on $V$, both in the presence of market learning (the first set of columns) and when private signals are the only source of information (the second set).

We report the results from these experiments in the first two sets of columns of Table 9. Delivering US-quality markets to emerging economies reduces uncertainty, but only slightly, with only a minor impact on the aggregate economy (the maximum output gain is just over 1%). This is yet another instance of a result we have now seen several times: examined in a variety of ways, financial markets play a very limited role in promoting aggregate efficiency. In contrast, the middle set of columns shows that access to US-quality private information would have a significant effect in these markets, reducing $V$ by as much as 50% of the underlying uncertainty in the most optimistic case, and generally between 10% and 20% throughout. The resulting gains in TFP and output are substantial, ranging from 2% to almost 15% for the former and from 3% to over 20% for the latter. Clearly, to the extent that differences in learning lead to cross-country variation in economic aggregates, these disparities are primarily due to a lack of high quality firm private information, rather than to a lack of well-functioning (in an informational sense) financial markets in emerging markets.
Table 9: The Effects of a US Information Structure

<table>
<thead>
<tr>
<th>Case</th>
<th>Market information</th>
<th>Private information</th>
<th>Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta V$</td>
<td>$\Delta V/\sigma_\mu$</td>
<td>$\Delta a$</td>
</tr>
<tr>
<td>Case 2, $\theta = 4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.014</td>
<td>-0.056</td>
<td>0.005</td>
</tr>
<tr>
<td>India</td>
<td>-0.026</td>
<td>-0.092</td>
<td>0.009</td>
</tr>
<tr>
<td>Case 2, $\theta = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.001</td>
<td>-0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>India</td>
<td>-0.001</td>
<td>-0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>Case 1, $\theta = 4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.001</td>
<td>-0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>India</td>
<td>-0.001</td>
<td>-0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>Case 1, $\theta = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>-0.001</td>
<td>-0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>India</td>
<td>-0.001</td>
<td>-0.002</td>
<td>0.003</td>
</tr>
</tbody>
</table>

The table shows the effects in India and China of having access to US-quality financial market information (the first set of columns), private information (the second set) and fundamental shocks (the last set).
Finally, we turn away from learning for a moment and study the role of differences in fundamentals in leading to a differential impact of informational frictions across countries. In this last exercise then, we recompute \( V \) in China and India under the assumption that firms in these countries face the same fundamental shocks as do US firms, that is, have a \( \sigma^2_\mu \) equal to that of the US. We report the results in the last set of columns in Table 9, which shows that disparities in the volatility of shocks generate substantial differences in the impact of informational frictions. Exposure to a fundamental process such as that in the US reduces \( V \) between about 10% and 30% of the underlying uncertainty, leading to TFP and output gains that are potentially substantial, ranging from 1-43% and 1-65% respectively.\(^{31}\) Clearly, in addition to differences in learning, differences in fundamentals also play a significant role in explaining cross-country disparities in the aggregate consequences of informational frictions.\(^{32}\)

4 Conclusion

We have laid out a theory of informational frictions that distort the allocation of factors across heterogeneous firms, leading to reduced aggregate productivity and output. A central element of our framework is a rich information structure in which firms learn from both private sources and imperfectly revealing stock market prices, a feature which allows us to jointly analyze moments of firm-level production variables and stock market returns to infer the severity of informational frictions.

Our approach reveals substantial micro-level uncertainty, particularly in China and India. Perhaps more interestingly, we find that to the extent firm learning serves to mitigate some portion of this uncertainty, firms turn overwhelmingly to private sources to obtain such information. Further, we find significant cross-country variation in the impact of information frictions, much of which is attributable to differences in the quality of firms’ own information sources (and to some extent, the volatility of the underlying shocks). Learning from financial markets seems to contribute little to aggregate allocative efficiency, even in a relatively developed market such as that in the US.

Our results suggest then that policies aimed at directly improving firm-level information would be more fruitful in improving emerging economy performance than those intended to develop financial markets to a level closer to that in the US. Because our modeling of information is rather abstract, we are left to speculate on the exact form of such policies. As an example, one interpretation of cross-country differences in private information is as the result of

\(^{31}\)These numbers correspond to \( \Delta (a - a^*) \) and \( \Delta (y - y^*) \). Recall that \( a^* \), the full information productivity level, is also affected by the size of fundamental shocks, \( \sigma^2_\mu \).

\(^{32}\)Asker et al. (2012) also highlight the role of different firm-specific shock processes in generating misallocation in a model with capital adjustment costs.
better information collection/processing systems within firms, and/or the skill of managers.³³ A thorough investigation of these issues is an important direction for future research.

Finally, we conclude with a note of caution about learning from financial markets. Our results point to a limited role for markets when it comes to learning about firm-specific fundamentals for large, well-established firms (which make up most of Compustat). However, they are largely silent regarding the importance of financial market information when it comes to mitigating uncertainty about aggregate (or industry) conditions, particularly for so-called “outsiders” (i.e., potential entrants, creditors, regulators, etc.). Explicitly modeling these features in a fully-fledged general equilibrium environment is a challenging task, but may well be essential for a comprehensive evaluation of the informational role of well-functioning financial markets.

References


³³This is reminiscent of Bloom et al. (2013), who highlight the role of better management practices and/or manager skill in explaining cross-country differences in performance.


Appendix

A Detailed Derivations

A.1 Case 1: Both factors chosen under imperfect information

As we show in equation (5) in the text, the firm’s capital choice problem can be written as

$$\max_{K_{it}} \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha'}} \mathbb{E}_{it} [A_{it}] K_{it}^{\alpha} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R_t K_{it}$$

and optimality requires

$$\alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha'}} \mathbb{E}_{it} [A_{it}] K_{it}^{-1} = R_t$$

$$\Rightarrow \left[ \alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha'}} \mathbb{E}_{it} [A_{it}] \right]^{\frac{1}{1-\alpha}} = K_{it}$$

Capital market clearing then implies

$$\int K_{it} di = \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} \alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) Y_t^{\frac{1}{\alpha'}} \mathbb{E}_{it} [A_{it}] \right]^{\frac{1}{1-\alpha}} \int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di = K_t$$

$$\Rightarrow \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} \alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) Y_t^{\frac{1}{\alpha'}} \mathbb{E}_{it} [A_{it}] \right]^{\frac{1}{1-\alpha}} = \frac{K_t}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di}$$

from which we can solve for

$$K_{it} = \frac{\left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} K_t$$
From here, it is straightforward to express firm revenue as in equation (6) in the text:

\[ P_{it}Y_{it} = K_t^{\alpha_1} N^{\alpha_2} Y_{t}^{\frac{1}{\bar{\alpha}}} A_{it} \left( \frac{(E_{it}[A_{it}])^{\frac{1}{\bar{\alpha}}}}{\int (E_{it}[A_{it}])^{\frac{1}{\bar{\alpha}}} \, di} \right)^{\alpha} \]

and noting that aggregate revenue must equal aggregate output, we have

\[ Y_t = \int P_{it} Y_{it} \, di = K_t^{\alpha_1} N^{\alpha_2} Y_{t}^{\frac{1}{\bar{\alpha}}} \int A_{it} (E_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} \, di \]

or in logs,

\[ y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \log \int A_{it} (E_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} \, di - \alpha \log \int (E_{it}[A_{it}])^{\frac{1}{1-\alpha}} \, di \]

Now, note that under conditional log-normality,

\[ a_{it} | I_{it} \sim \mathcal{N}(E_{it}[a_{it}], \sigma^2_a) \Rightarrow E_{it}[A_{it}] = \exp \left( E_{it}[a_{it}] + \frac{1}{2} \sigma^2_a \right) \]

The true fundamental \( a_{it} \) and its conditional expectation \( E_{it}[a_{it}] \) are also jointly normal, i.e.

\[ \begin{bmatrix} a_{it} \\ E_{it}[a_{it}] \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \bar{a} \\ \bar{\bar{a}} \end{bmatrix}, \begin{bmatrix} \sigma^2_a & \sigma^2_a - \sigma^2_a \\ \sigma^2_a - \sigma^2_a & \sigma^2_a - \sigma^2_a \end{bmatrix} \right) \]

We then have that

\[ \log \int A_{it} (E_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} \, di = \log \int \exp \left( a_{it} + \frac{\alpha}{1-\alpha} E_{it}[a_{it}] + \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma^2_a \right) \, di \]

\[ = \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \sigma^2_a + \frac{1}{2} \left( \frac{\alpha}{1-\alpha} \right)^2 \left( \sigma^2_a - \sigma^2_a - \sigma^2_a \right) + \frac{\alpha}{1-\alpha} \sigma^2_a - \bar{\bar{a}} + \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma^2_a \]

\[ = \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \sigma^2_a + \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma^2_a - \frac{1}{2} \left( \frac{\alpha}{1-\alpha} \right)^2 \left( \sigma^2_a - \sigma^2_a \right) + \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma^2_a \]

and

\[ \log \int (E_{it}[A_{it}])^{\frac{1}{1-\alpha}} \, di = \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \left( \frac{1}{1-\alpha} \right)^2 \left( \sigma^2_a - \sigma^2_a \right) + \frac{1}{2} \frac{1}{1-\alpha} \sigma^2_a \]
Combining these,

\[ \log \int A_{it} \left( \mathbb{E}_{it} [A_{it}] \right)^{\alpha_1} \, di - \alpha \log \int \left( \mathbb{E}_{it} [A_{it}] \right)^{1/\alpha} \, di = \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V} \]

Substituting and rearranging, we obtain the expressions in (10) and (11) in the text,

\[ y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{\theta - 1} - \frac{1}{2} \frac{\alpha}{2 - \alpha} \mathbb{V} \]

\[ = \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + \frac{\theta}{\theta - 1} \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{\theta - 1} - \frac{1}{2} \frac{\alpha}{2 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \mathbb{V} \]

with aggregate productivity given by

\[ a = \frac{\theta}{\theta - 1} \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{\theta - 1} - \frac{1}{2} \frac{\alpha}{2 - \alpha} \left( \frac{\theta}{\theta - 1} \right) \mathbb{V} \]

It now remains to endogenize \( K_t \). The rental rate in steady state satisfies

\[ R = \frac{1}{\beta} - 1 + \delta \]

Then, from the optimality and market clearing conditions, we have from (4) that

\[ \frac{\alpha_1 N}{\alpha_2 K} = \frac{R}{W} \quad \Rightarrow \quad K \propto W \]

i.e., the aggregate capital stock is proportional to the wage. To characterize wages, we return to the firm’s profit maximization problem

\[ \max_{K_{it}, N_{it}} Y_t^{1/\alpha} \mathbb{E}_{it} [A_{it}] \left( K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W N_{it} - R K_{it} \right) \]

which, after maximizing over capital, can be written as

\[ \max_{N_{it}} \left( 1 - \alpha_1 \right) \left( \frac{\alpha_1}{R} \right)^{1 - \alpha_1} \left( Y_t^{1/\alpha} \mathbb{E}_{it} [A_{it}] N_{it}^{\alpha_2} \right)^{1 - \alpha_1} = W N_{it} \]

Optimality and labor market clearing imply

\[ \left( \frac{\alpha_2}{W} \right)^{1 - \alpha_1 - \alpha_2} \left( \frac{\alpha_1}{R} \right)^{1 - \alpha_1 - \alpha_2} \left( Y_t^{1/\alpha} \mathbb{E}_{it} [A_{it}] \right)^{1 - \alpha_1 - \alpha_2} = \frac{1}{1 - \alpha_1 - \alpha_2} \]

\[ \frac{\alpha_2}{W} \int \left( Y_t^{1/\alpha} \mathbb{E}_{it} [A_{it}] \right)^{1 - \alpha_1 - \alpha_2} \, di = N \]

35
As before, letting $\alpha = \alpha_1 + \alpha_2$, we see that

$$W \propto \left( \int \mathbb{E}_{it} \left[ A_{it} \right] \right)^{\frac{1-\alpha}{1-\alpha_1}} Y_t^{\frac{1}{2} \frac{1-\alpha}{1-\alpha_1}}$$

$$= \left[ \left( \int \left( \exp \left( \mathbb{E}_{it} a_{it} + \frac{1}{2} V \right) \right) \right)^{\frac{1-\alpha}{1-\alpha_1}} Y_t^{\frac{1}{2} \frac{1-\alpha}{1-\alpha_1}} \right]^{\frac{1}{1-\alpha_1}} = \left[ \exp \left( \bar{a} + \frac{1}{2} \left( \frac{\sigma^2_a - \alpha V}{1-\alpha} \right) \right) Y_t^{\frac{1}{2} \frac{1-\alpha}{1-\alpha_1}} \right]^{\frac{1}{1-\alpha_1}}$$

or in logs,

$$w \propto \left( \frac{1}{1-\alpha_1} \right) \bar{a} + \frac{1}{1-\alpha_1} \frac{1}{2} \left( \frac{\sigma^2_a - \alpha V}{1-\alpha} \right) + \frac{1}{\theta} \frac{1}{\bar{a} - \alpha V}$$

Recalling that

$$K \propto W \implies \frac{dk}{dV} = \frac{dw}{dV}$$

which, in conjunction with (10) and (11), implies

$$\frac{dy}{dV} = \hat{\alpha}_1 \frac{dk}{dV} - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\alpha}{1-\alpha}$$

$$= \frac{\hat{\alpha}_1}{1-\alpha_1} \left[ \frac{1}{2} \frac{\alpha}{1-\alpha} + \frac{1}{\theta} \frac{dy}{dV} \right] - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\alpha}{1-\alpha}$$

and finally, collecting terms and rearranging, and using the fact that $\hat{\alpha}_1 = \frac{\theta}{\theta - 1} \alpha_1$, we obtain

$$\frac{dy}{dV} = -\frac{1}{2} \frac{\alpha}{1-\alpha} \left( \frac{\theta}{\theta - 1} \right) \frac{1}{1-\hat{\alpha}_1} = \frac{da}{dV} \frac{1}{1-\hat{\alpha}_1}$$

### A.2 Case 2: Only capital chosen under imperfect information

The firm’s labor choice problem can be written as

$$\max_{N_{it}} N_{it}^\alpha Y_t^{\frac{1}{2} \frac{1}{2} A_{it} K_{it}^\alpha} - W_t N_{it}$$

and optimality requires

$$N_{it} = \left( \frac{\alpha_2 Y_t^{\frac{1}{2} A_{it} K_{it}^\alpha}}{W_t} \right)^{\frac{1}{1-\alpha_2}}$$
Labor market clearing then implies

\[ \int N_{it} \, di = \int \left( \frac{\alpha_2^2 Y_t^\beta A_{it} K_{it}^{\alpha_1}}{W} \right) \frac{1}{1-\alpha_2} \, di = N \]

\[ \Rightarrow N_{it} = \frac{(A_{it} K_{it}^{\alpha_1})^{\frac{1}{1-\alpha_2}}}{\int (A_{it} K_{it}^{\alpha_1})^{\frac{1}{1-\alpha_2}} \, di} N \]

Letting \( \tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}} \) and \( \tilde{\alpha} = \frac{\alpha_1}{1-\alpha_2} \), we have

\[ N_{it} = \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di} N \]

which implies

\[ W = Y_t^\beta A_{it} K_{it}^{\alpha_1} \left( \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di} N \right) \]

\[ = \frac{\alpha_2}{N^{1-\alpha_2}} Y_t^\beta \left( \int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \, di \right)^{1-\alpha_2} A_{it} K_{it}^{\alpha_1} \left( \tilde{A}_{it} K_{it}^{\tilde{\alpha}} \right) \]

From here, it is straightforward to express the firm’s capital choice problem as in (8):

\[ \max_{K_{it}} (1 - \alpha_2) \left( \frac{\alpha_2^2}{W} \right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^\beta \frac{1}{1-\alpha_2} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] K_{it}^{\tilde{\alpha}} - R_t K_{it} \]

Optimality requires

\[ R_t = (1 - \alpha_2) \left( \frac{\alpha_2^2}{W} \right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^\beta \frac{1}{1-\alpha_2} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \tilde{\alpha} K_{it}^{\tilde{\alpha}-1} \]

\[ \Rightarrow K_{it} = \left[ \frac{(1 - \alpha_2)\tilde{\alpha}}{R} \right]^{\frac{1}{1-\alpha}} \left( \frac{\alpha_2^2}{W} \right)^{\frac{\alpha_2}{1-\alpha_2}} \frac{1}{1-\alpha} Y_t^{\frac{\alpha_2}{1-\alpha_2} \frac{1}{1-\alpha}} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{1-\alpha}} \]

Capital market clearing then implies

\[ \int K_{it} \, di = \left[ \frac{(1 - \alpha_2)\tilde{\alpha}}{R} \right]^{\frac{1}{1-\alpha}} \left( \frac{\alpha_2^2}{W} \right)^{\frac{\alpha_2}{1-\alpha_2}} \frac{1}{1-\alpha} Y_t^{\frac{\alpha_2}{1-\alpha_2} \frac{1}{1-\alpha}} \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{1-\alpha}} \int \, di = K_t \]

\[ \Rightarrow K_{it} = \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{1-\alpha}} K_t \]

\[ \frac{1}{\int \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{1-\alpha}} \, di} \]
and we can rewrite the labor choice as

\[ N_{it} = \frac{\bar{A}_{it} K^\tilde{\alpha}_{it}}{\int \bar{A}_{it} K^\tilde{\alpha}_{it} di} N = \frac{\bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di} N \]

Combining the solutions for capital and labor, we can express firm revenue as

\[ P_{it} Y_{it} = Y_t^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} \bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} \int \mathbb{E}_{it} \left[ \bar{A}_{it} \right]^{1-\tilde{\alpha}} di \left( \int \bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \right)^{\alpha_1} \left( \int \bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \right)^{\alpha_2} \]

and again using the fact that \( y_t = \log \int P_{it} Y_{it} di \), we can write

\[ y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n - \alpha_1 \log \int \mathbb{E}_{it} \left[ \bar{A}_{it} \right]^{1-\tilde{\alpha}} di + (1 - \alpha_2) \log \int \bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \]

Again, we exploit log-normality to obtain

\[ \log \int \bar{A}_{it} \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di = \log \int \exp \left( \tilde{a}_{it} + \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \mathbb{E}_{it} \tilde{a}_{it} + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{V} \right) di \]

\[ = \frac{1}{1-\tilde{\alpha}} \tilde{a} + \frac{1}{2} \tilde{\alpha} + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \left( \tilde{\sigma}_a^2 - \tilde{V} \right) + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{V} \]

and similarly,

\[ \log \int \left( \mathbb{E}_{it} \left[ \bar{A}_{it} \right] \right)^{\frac{1}{1-\tilde{\alpha}}} di = \log \int \exp \left( \mathbb{E}_{it} \tilde{a}_{it} + \frac{1}{2} \tilde{V} \right) \left( \frac{1}{1-\tilde{\alpha}} \right) \]

\[ = \frac{1}{1-\tilde{\alpha}} \tilde{a} + \frac{1}{2} \tilde{\alpha} + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \left( \tilde{\sigma}_a^2 - \tilde{V} \right) + \frac{1}{2} \frac{1}{1-\tilde{\alpha}} \tilde{V} \]
Combining, and using the fact that $\tilde{a}_{it} = \frac{a_{it}}{1-\alpha_2}$:

$$
-\alpha_1 \log \int \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{1}{1-\alpha}} \, di + \left( 1 - \alpha_2 \right) \log \int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\hat{\alpha}}{1-\alpha}} \, di
$$

$$
= \frac{1 - \alpha_1}{1 - \hat{\alpha}} \tilde{a} + \frac{(1 - \alpha_2)}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \left( \frac{(1 - \alpha_2)(2\tilde{\alpha} - \	ilde{\alpha}^2)}{(1 - \hat{\alpha})^2} \right) \left( \sigma_{\tilde{a}}^2 - \tilde{V} \right)
$$

$$
= \frac{1 - \alpha}{1 - \hat{\alpha}} \tilde{a} \frac{1}{1 - \alpha} \sigma_{\tilde{a}}^2 + \frac{1}{2} \left( \frac{\alpha_1}{2} \right) \left( \sigma_{\tilde{a}}^2 - \tilde{V} \right)
$$

$$
= \tilde{a} + \frac{1}{2} \left[ \frac{\sigma_{\tilde{a}}^2}{1 - \alpha} - \frac{1}{1 - \alpha} (1 - \alpha_2) \tilde{V} \right]
$$

Substituting and collecting terms, we obtain expressions (10) and (12) in the text,

$$
y_t = \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + \frac{\theta}{\theta - 1} \tilde{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \left( \sigma_{\tilde{a}}^2 \right) \left( \frac{\alpha_1}{1 - \alpha} \right) \left( 1 - \alpha_2 \right) \tilde{V}
$$

with aggregate productivity given by

$$
a = \frac{\theta}{\theta - 1} \tilde{a} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \left( \sigma_{\tilde{a}}^2 \right) \left( \frac{\alpha_1}{1 - \alpha} \right) \left( 1 - \alpha_2 \right) \tilde{V}
$$

To endogenize $K_t$, we begin by characterizing the wage $W$ in terms of $\int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\hat{\alpha}}{1-\alpha}} \, di$ and $Y^{\frac{1}{2}}$:

$$
W = \frac{\alpha_2}{N^{1-\alpha_2}} Y^{\frac{1}{2}} \left( \int \tilde{A}_{it} K^\alpha_{it} \, di \right)^{1-\alpha_2}
$$

$$
= \frac{\alpha_2}{N^{1-\alpha_2}} Y^{\frac{1}{2}} \left\{ \int \tilde{A}_{it} \left( \frac{(1 - \alpha_2)}{\hat{\alpha}} \frac{\alpha_2}{\hat{\alpha}} \left( \frac{1}{1-\alpha} \right) Y^{\frac{1}{2}} \left( \frac{1}{1-\alpha} \right) \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\hat{\alpha}}{1-\alpha}} \, di \right\}
$$

$$
= \frac{\alpha_2}{N^{1-\alpha_2}} Y^{\frac{1}{2}} \left( \frac{(1 - \alpha_2)}{\hat{\alpha}} \frac{\alpha_2}{\hat{\alpha}} \left( \frac{1}{1-\alpha} \right) Y^{\frac{1}{2}} \left( \frac{1}{1-\alpha} \right) \left\{ \int \tilde{A}_{it} \left( \mathbb{E}_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\hat{\alpha}}{1-\alpha}} \, di \right\} \right)
$$

39
and rearranging,

\[ W = \left( \frac{\alpha_2}{N^{1-\alpha_2}} \right)^{1-\alpha} \left[ \frac{(1-\alpha_2)\hat{\alpha}}{R} \right]^{\alpha_1} \frac{\alpha_2^\alpha}{\alpha_1^{1-\alpha_1}} Y^{\frac{1}{\theta}} 1^{\frac{1}{1-\alpha_1}} \left\{ \int \tilde{A}_{it} \left( E_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} di \right\}^{\frac{(1-\alpha_2)(1-\alpha)}{\alpha_1^{1-\alpha_1}}}
\]

\[ \propto \left\{ \int \tilde{A}_{it} \left( E_{it} \left[ \tilde{A}_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} di \right\}^{\frac{1}{1-\alpha_1}} Y^{\frac{1}{\theta}} \]

\[ = \left\{ \exp \left( \frac{1}{1-\alpha} \hat{\alpha} + \frac{1}{2} \sigma^2_{\tilde{\alpha}} + \frac{1}{2} \frac{\tilde{\alpha}^2}{(1-\alpha)^2} \left( \sigma^2_{\tilde{\alpha}} - \tilde{\nu} \right) + \frac{1}{2} \frac{\hat{\alpha}}{1-\alpha} \tilde{\nu} \right) \right\}^{\frac{1}{1-\alpha_1}} Y^{\frac{1}{\theta}} \]

or in logs,

\[ w \propto \left( \frac{1}{1-\alpha_1} \right) \hat{\alpha} + \frac{1}{2} \frac{1}{(1-\alpha_1)(1-\alpha)} \sigma^2_{\tilde{\alpha}} - \frac{1}{2} \frac{\tilde{\alpha}(1-\alpha_2)}{1-\alpha} \tilde{\nu} + \frac{1}{\theta} \frac{1}{1-\alpha_1} y_t \]

As before,

\[ K \propto W \Rightarrow \frac{dk}{d\tilde{\nu}} = \frac{dw}{d\tilde{\nu}} = -\frac{1}{2} \frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha)(1-\alpha)} + \frac{1}{\theta} \frac{1}{1-\alpha_1} \frac{dy}{d\tilde{\nu}} \]

and substituting into the derivative of aggregate output,

\[ \frac{dy}{d\tilde{\nu}} = \hat{\alpha}_1 \left( \frac{dk}{d\tilde{\nu}} \right) - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \left( \frac{\alpha_1}{1-\alpha} \right) \left( 1-\alpha_2 \right) \]

\[ = \hat{\alpha}_1 \left[ -\frac{1}{2} \frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha)(1-\alpha)} + \frac{1}{\theta} \frac{1}{1-\alpha_1} \frac{dy}{d\tilde{\nu}} \right] - \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \left( \frac{\alpha_1}{1-\alpha} \right) \left( 1-\alpha_2 \right) \]

Finally, collecting terms and rearranging, and using the facts that \( 1 - \alpha = (1 - \tilde{\alpha}) (1 - \alpha_2) \) and \( \frac{\theta}{\theta - 1} \alpha_1 = \tilde{\alpha}_1 \), we obtain

\[ \frac{dy}{d\tilde{\nu}} = -\frac{1}{2} (1-\alpha_2) \left( \frac{\theta}{\theta - 1} \right) \left( \frac{\alpha_1}{1-\alpha} \right) \frac{1}{1-\alpha_1} = \frac{da}{d\tilde{\nu}} \left( \frac{1}{1-\tilde{\alpha}_1} \right) \]

**B Data**

We obtain annual data on firm-level production variables and stock market returns from Compustat North America (for the US) and Compustat Global (for China and India). For each country, we exclude duplicate observations (firms with multiple observations within a single year), firms not incorporated within that country, and firms not reporting in local currency.
We build three year production periods as the average of firm sales and capital stock over non-overlapping 3-year horizons (i.e., $K_{2012} = \frac{K_{2010} + K_{2011} + K_{2012}}{3}$, and analogously for sales). We measure the capital stock using gross property, plant and equipment (PPEGT in Compustat terminology), defined as the “valuation of tangible fixed assets used in the production of revenue.” We then calculate investment (in logs) as $i_{it} = \Delta k_{it}$, i.e., as the change in the firm’s capital stock relative to the preceding period (for example, $i_{2012} = k_{2012} - k_{2009}$, where $k$ is as just described. Finally, we first-difference the investment and sales series to compute revenue and investment growth as $\Delta y_{it} = y_{it} - y_{it-1}$ and $\Delta i_{it} = i_{it} - i_{it-1}$.

Stock returns are constructed as the change in the firm’s stock price over the three year period, adjusted for splits and dividend distributions. We follow the procedure outlined in the Compustat manual. In Compustat terminology, and as in the text, using $\Delta p_{it}$ as shorthand for log returns, returns for the US are computed as

$$\Delta p_{it} = \log \left( \frac{PRCCM_{it} \times TRFM_{it}}{AJEXM_{it}} \right) - \log \left( \frac{PRCCM_{it-1} \times TRFM_{it-1}}{AJEXM_{it-1}} \right)$$

where periods denote 3 year spans (i.e., returns for 2012 are calculated as the adjusted change in price between 2009 and 2012), PRCCM is the firm’s stock price, and TRFM and AJEXM adjustment factors needed to translate prices to returns from the Compustat monthly securities file. Data are for the last trading day of the firm’s fiscal year so that the timing lines up with the production variables just described. The calculation is analogous for China and India, with the small caveat that global securities data come daily, so that the Compustat variables are PRCCD, TRFD, and AJEXDI, where “D” denotes days. Again, the data are for the last trading day of the firm’s fiscal year.

To extract the firm-specific variation in our variables, $\Delta y_{it}$, $\Delta i_{it}$, and $\Delta p_{it}$, we regress each on a time fixed-effect and work with the residual. This eliminates the component of each series common to all firms in a time period and leaves only the idiosyncratic variation. As described in the text, we limit our sample to a single cross-section, namely 2012, and finally, we trim the 1% tails of this sample using the following algorithm: for each series, we identify if an observation falls within the 1% tails of that particular series. We then drop observations that fall into the 1% tail on any dimension. This simultaneous procedure is more appropriate than a sequential one since our calibration approach relies heavily on the correlations between the series. It is then straightforward to compute our calibration targets, i.e., $\sigma^2(\Delta p)$, $\rho(\Delta i, \Delta p)$, and $\rho(\Delta y, \Delta p)$. As described in the text, we lag returns by one period, so that, e.g. $\rho(\Delta i, \Delta p)$ is the correlation of 2006-09 returns with investment growth from 2009-12.

Our leverage adjustment is as follows: we assume that claims to firm profits are sold to investors in the form of both debt and equity in a constant proportion (within each country).
This implies that the payoff from an equity claim is \( S_{it} = V_{it} - D_{it-1} \), where \( V_{it} \) is the value of the unlevered firm and \( D_{it-1} = d \mathbb{E}_t [V_{it}] \), where \( d \in (0, 1) \) represents the share of expected firm value in the hands of debt-holders. In other words, firm value is allocated to investors as a debt claim that pays off a constant fraction of its ex-ante expected value and as a residual claim to equity holders. The change in value of an equity claim is then equal to \( \Delta S_{it} = \Delta V_{it} \) and dividing both sides by the ex-ante expected value of the claim (i.e., the price) \( \overline{S} = \overline{V} - d \overline{V} \), where \( \overline{V} = \mathbb{E}_t [V_{it}] \), gives returns as \( \frac{\Delta S_{it}}{\overline{S}} = \frac{\Delta V_{it}}{(1-d)\overline{V}} \). Taking logs and computing variances shows \( \sigma_{\epsilon_{it}}^2 = (1-d)^2 \sigma_{s_{it}}^2 \), i.e., the volatility in (unlevered) firm value is a fraction \( (1-d)^2 \) of the volatility in (levered) equity returns. To assign values to \( d \) in each country, we examine the debt-asset and debt-equity ratios of the set of firms in Compustat over the period 2006-2009. Because these vary to some degree from year to year and depend to some extent on the precise approach taken (i.e., whether we use debt-assets or debt-equity and whether we compute average ratios or totals), we simply take the approximate midpoints of the ranges for each country, which are about 0.30 for the US and India and 0.16 for China, leading to adjustment factors of about 0.5 and 0.7, respectively.

Lastly, as described in the text, we construct the firm fundamental \( a_{it} \) annually as the log of firm revenue less the relevant \( \alpha \) (which depends on our various cases) multiplied by the log of its capital stock. In line with our focus on the three year period ending in 2012, we then estimate the parameters of the fundamental process by running the autoregression of \( a_{i2012} \) on \( a_{i2009} \) and a time fixed-effect, in order to isolate the idiosyncratic components of changes in \( a \). Differences in firm fiscal years means that different firms within the same calendar year are reporting data over different time periods, and so the time fixed-effect incorporates both the reporting year and month. The coefficient from this regression delivers an estimate for \( \rho \) and the variance of the residuals for \( \sigma_{\mu}^2 \), where we again trim the 1% tails.