~~31.~~1. An additional conclusion from Chapter 22 was that chained indices would usually reduce the spread between the Laspeyres (P_L) and Paasche (P_P) indices. In **Table 3** below we compare the spread between these two formulae before and after chaining.

Table 3: Mean Annual Ratio Between Year over Year Monthly Laspeyres and Paasche Indices (P_L/P_P)

Year	1997	1998	1999	2000	2001	2002
Fixed Base	1.00	0.97	0.974	1.03	0.94	0.93
Chained	1.00	0.97	0.972	0.98	0.93	0.94

~~32.~~2. When inserting real CPI data with strong seasonality, the average spread between the Paasche and Laspeyres did not change substantially for the chained indices, compared to the fixed base indices. The total period average spread for the fixed base indices is to a small extent less, than the spread for the chained indices (0.973 versus 0.965, respectively). These findings seem to *contradict* the conclusion from Chapter 22.

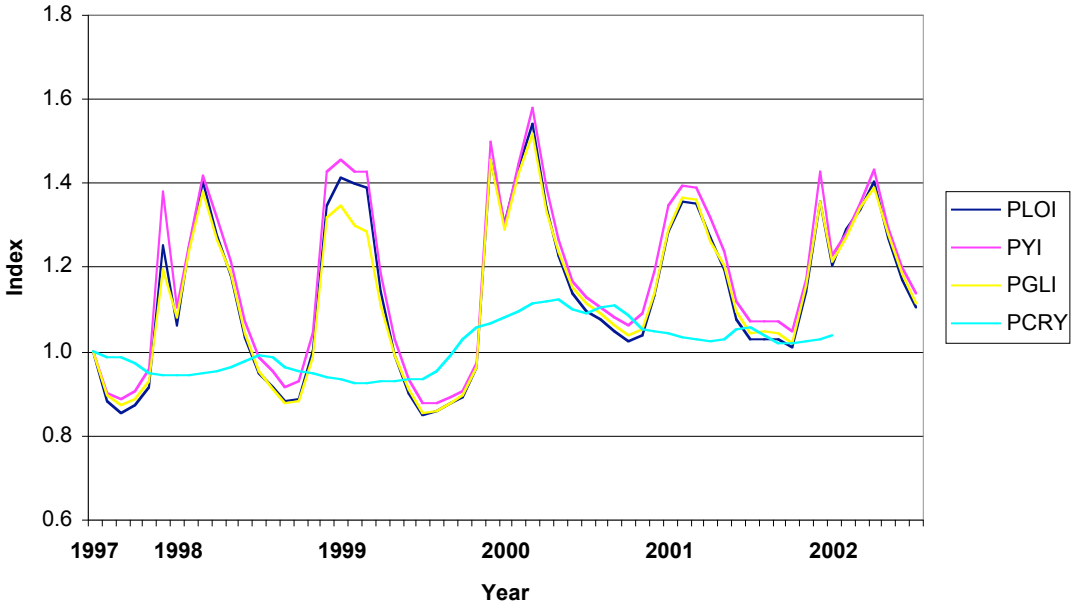
~~33.~~3. We remember that the traditional month-to-month calculation methods all suffer from large fluctuation. We also mentioned that the maximum overlap method had tremendous downward bias. Our findings were consistent with these conclusions: The Laspeyres, Paasche and Fisher indices were all near to zero at the end of the 5 year period.

~~34.~~4. In Figure 5 below, extreme (yet consistent) seasonal fluctuations are found when comparing the Lowe, Young and Geometric Laspeyres indices with the target centered year rolling index[1] (using imputed prices[2]).

[1] This series was normalized to equal 1 in December 1997 so that it would be comparable to the other month-to-month indices. Also, the centered rolling year indices cannot be calculated for the last 6 months, since the data set doesn't extend 6 months into 2003.

[2] The basic idea of *the imputation method* was to take the last available price and impute prices for the missing periods that trend with another index. This other index was taken to be the price index of the general category of fresh fruits. For each month m we imputed the price of the previous month (m-1) multiplied with the average of indices for three adjacent months (m-1, m and m+1).

Figure 5: Lowe, Young, Geometric Laspeyres, and Centered Rolling Year Indices with Imputed Prices.

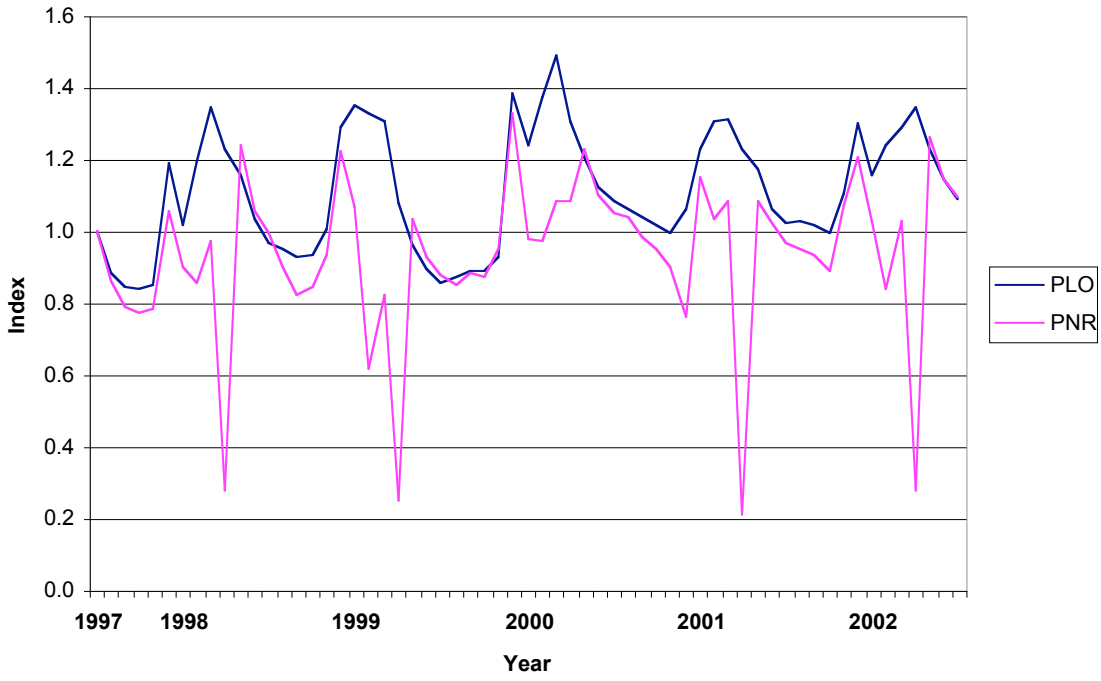


~~35.~~5. It can be seen that the Lowe, Young and Geometric Laspeyres indices have a considerable amount of seasonality in them and do not at all approximate their rolling year counterparts.[3]

~~36.~~6. Another finding from Chapter 22 was that the Rothwell indices exhibited smaller seasonal movements than the Lowe index and were less volatile in general. The findings in the Israeli CPI were quite different as can be seen in Figure 6 below:

[3] The sample means of the four indices are 1.1220 (Lowe), 1.1586 (Young), 1.1190 (Geometric Laspeyres) and 1.0072 (rolling year). Of course, the geometric Laspeyres indices will always be equal to or less than their Young counterparts since a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

Figure 6: Lowe and Normalized Rothwell Indices[4]



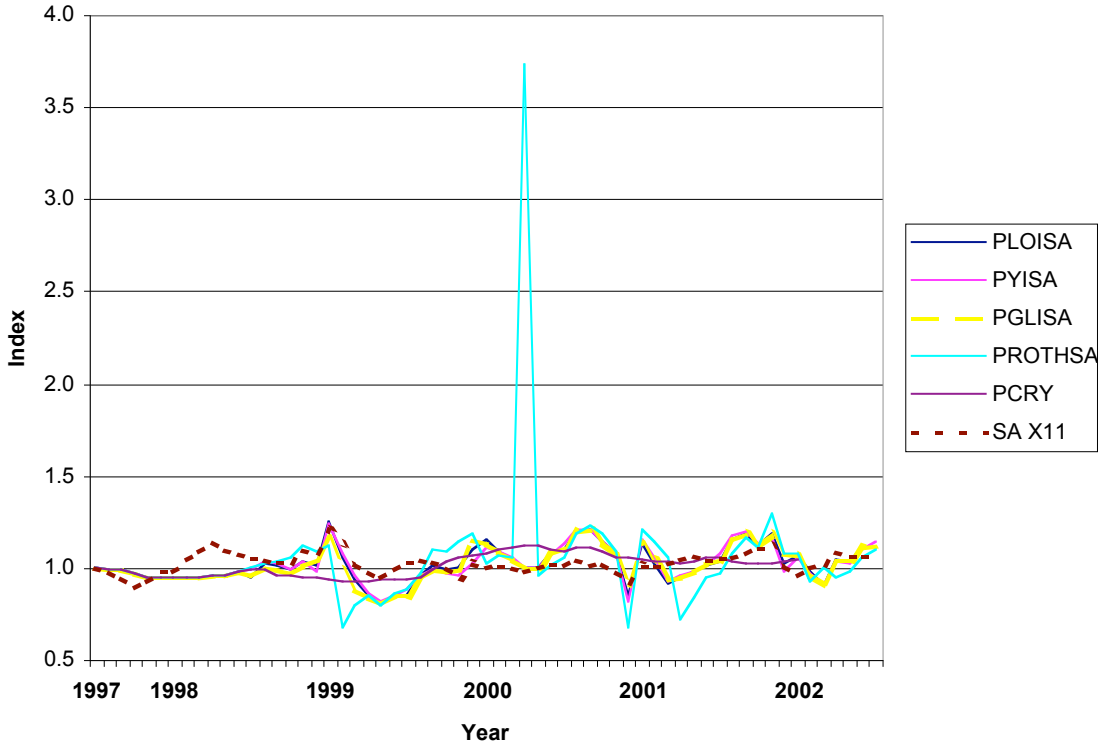
~~37.~~7. The normalized Rothwell index is much more volatile than the annual basket month-to-month Lowe index[5]. This seems to be consistent with the decision made in the Israeli CPI to move over to the Lowe index. However, both indices suffer from large seasonal fluctuations.

~~38.~~8. In Section K of Chapter 22, presented in the summary above, an attempt to forecast rolling year indices using month-to-month annual basket indices was exhibited. A seasonal adjustment factor was computed for the Lowe, Young and geometric Laspeyres indices with imputed prices and Rothwell index based on the centered rolling year index. In addition, we introduce the actual Israeli CPI "fresh fruits" price index that has been seasonally adjusted using an Arima X-11 multiplicative model. This series enables us to compare whether the pattern of price change for the chosen set repeats the trend of the general category of the product, or higher-level component of the CPI. To make the series more comparable, it was normalized by dividing the original indices by the first observation. Figure 6 below shows that the predicted values of these "seasonally adjusted" indices, **in the Israeli CPI like in the artificial data set from Chapter 22**, are fairly close to the corresponding target index values[6]. Though, the centered rolling year index for the chosen data set fails to represent reliably the overall

[4] The Rothwell index is compared to the Lowe index with carry forward of missing prices (i.e. for the prices that are not available in the current month, the last available price is carried forward).
[5] The Lowe index has a mean of 1.1087 and a standard deviation of .1677 while the normalized Rothwell has a mean of .9368 and a standard deviation of .2282.
[6] For observations 13 through 66, we regressed the seasonally adjusted series on the centered rolling year series. For the seasonally adjusted Lowe index, an R^2 of .1916 is obtained; for the seasonally adjusted Young index, an R^2 of .1707 is obtained and for the seasonally adjusted Geometric Laspeyres index, an R^2 of .3050 is obtained. For the seasonally adjusted Rothwell index, an R^2 of .1298 is obtained, which is lower than the other three fits.

price index of fresh fruits with X-11 seasonal adjustment.[7] It should be stated, that only the Seasonally-Adjusted Rothwell had some outliers compared to the target indices. This again seems to weaken the argument for using this index in the Israeli CPI.

Figure 7: Seasonally Adjusted Lowe, Young and Geometric Laspeyres Indices with Imputed Prices, Seasonally Adjusted Rothwell and Central Rolling Year Indices and Normalized Price Index of Fresh Fruits with X-11 Adjustment.



V. Modification of the Dataset

~~39.~~9. Comparison of the real CPI dataset used in this study, with the artificial one presented in the price index manuals, reveals several structural differences, which may account for some of the contradictory findings in reality in contrast to the theory. These differences are summarized in the following table.

Theory	Reality
Annual price change of an item is always in one direction. An increase (decrease) in price in one year, compared to the previous one, cannot be followed by a decrease (increase) in price in the next year.	The trend of price change may change direction from year to year.
There is one seasonal cycle a year for	Two seasonal cycles are possible.

[7] For the X-11 seasonally adjusted series, regressed on the centered rolling year series, for observations 1 through 66, the R^2 value is .0410.

each item.	
An item is available (unavailable) in the same months each year.	Seasonal fluctuations are not completely synchronized with the calendar months for the items with strong seasonality. Thus, an item may appear/disappear a month before/later than in the previous year.
There is constant consumption behavior, in comparison with the same month in previous year, independent of price behavior.	Erratic consumption patterns.

~~40.~~10. Further we concentrate on the inconsistency between years, regarding the month when the item appears/disappears. On the one hand, the treatment of this problem is simple and does not require major changes in the dataset. On the other hand, a brief analysis of our calculations showed that this fact might be the ultimate culprit of the contradiction between our findings and those presented in Chapter 22.

~~41.~~11. The dataset of fresh fruits [8] was modified slightly, in order to align the availability of items with strong seasonality between years. The total of 16 observations out of 504 were modified (3.2%). Of these, 8 observations were omitted (an item appears in the month, when it is usually unavailable in other years), and 8 observations were imputed by carrying forward/back[9] the price of the adjacent month (an item is unavailable in the month, when it is usually available in other years).

~~42.~~12. The modification led to substantial changes in several findings described above. In Table 4 below we compare once again the year over year "current month" fixed base Fisher index (PF) with its approximate (PAF). The approximate Fisher indices provide adequate approximations to their true Fisher counterparts. The number of cases in which the differential between indices is 5 percent or more decreased by more than 50% (5 cases in contrast to 11). Besides, in none of these cases the differential is really extreme (as it was with 6 cases in Table 1).

Table 4: Ratio Between Year over Year Monthly Fixed Base Fisher "Current" and Approximate Indices (P_{AF}/P_F) in the Modified Data Set

Month/Year	1997	1998	1999	2000	2001	2002
1	1.00	1.01	0.99	0.99	1.02	0.98
2	1.00	1.00	0.99	1.00	1.00	1.00
3	1.00	1.00	0.99	1.00	1.00	1.01
4	1.00	1.00	0.98	1.01	1.00	1.02
5	1.00	0.99	1.04	**1.05**	**1.05**	1.02
6	1.00	1.01	1.00	1.01	1.01	1.03

[8] We modified the dataset of fruits only, since there are no items with strong seasonality in the dataset of vegetables.

[9] Imputation by carrying back the price of the following month was preferred only in cases when there was no price in the previous months of the same year.

7	1.00	1.01	1.02	1.01	1.04	1.00
8	1.00	1.01	1.02	1.01	1.04	1.03
9	1.00	1.00	1.03	1.01	1.00	**0.89**
10	1.00	1.01	1.00	1.01	0.99	0.96
11	1.00	0.99	0.98	0.99	0.98	**0.94**
12	1.00	1.01	0.99	1.00	1.00	**0.93**

43. In **Table 5** below we return to the comparison of year over year monthly chained Fisher indices. Once again, the decrease of about 50% is achieved in the number of cases in which the differential between indices is 5 percent or more (7 in contrast to 16 cases). September is no more an "outlier" month.

Table 5: Ratio Between Year over Year Monthly Chained Fisher "Current" and Approximate Indices (P_{AF}/P_F) in the Modified Data Set

Month/Year	1997	1998	1999	2000	2001	2002
1	1.00	1.01	0.98	0.99	**1.05**	1.01
2	1.00	1.00	0.98	1.00	1.01	1.01
3	1.00	1.00	1.00	1.02	1.02	1.03
4	1.00	1.00	0.98	1.03	1.01	**1.07**
5	1.00	0.99	**1.09**	**1.12**	**1.09**	**1.15**
6	1.00	1.01	1.00	1.00	1.00	**1.06**
7	1.00	1.01	1.01	1.01	1.04	1.00
8	1.00	1.01	1.01	1.01	1.04	1.02
9	1.00	1.00	1.02	1.00	0.98	0.98
10	1.00	1.01	1.02	1.01	1.00	**1.05**
11	1.00	0.99	0.99	1.02	1.01	0.99
12	1.00	1.01	1.01	1.01	1.01	0.99

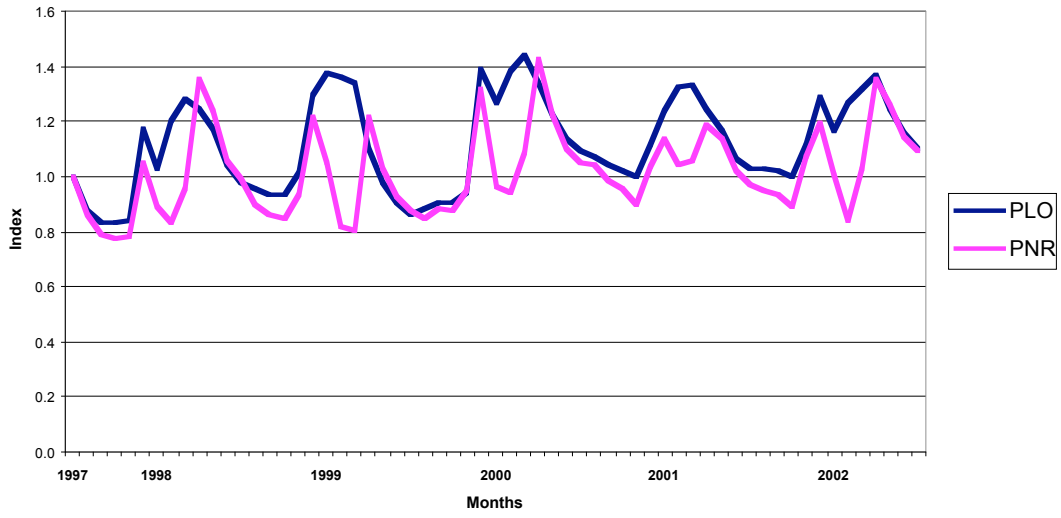
44. In **Table 6** below we compare the spread between the Laspeyres (P_L) and Paasche (P_P) indices before and after chaining. When inserting real CPI data with strong seasonality, the average spread between the Paasche and Laspeyres did not change substantially for the chained indices, compared to the fixed base indices. The use of chained indices still increased the spread between Paasche and Laspeyres indices compared to their fixed base counterparts: the total period average spread is 1.03 for the fixed base indices, and is 1.1 for the chained indices. These findings still *contradict* the conclusion from Chapter 22.

Table 6: Mean Annual Ratio Between Year over Year Monthly Laspeyres and Paasche Indices (P_L/P_P) in the Modified Data Set

Year	1997	1998	1999	2000	2001	2002
Fixed Base	1.00	1.03	1.06	1.03	1.05	.99
Chained	1.00	1.03	1.06	1.12	1.15	1.21

45. Another major change in findings occurred with Lowe and Rothwell indices. Rothwell indices exhibited smaller seasonal movements than the Lowe indices (mean of 1.01 and 1.12, respectively) and were less volatile (standard deviation of 0.15 and 0.17 respectively; see Figure 6).

Figure 8: Lowe and Normalized Rothwell Indices in the Modified Data Set

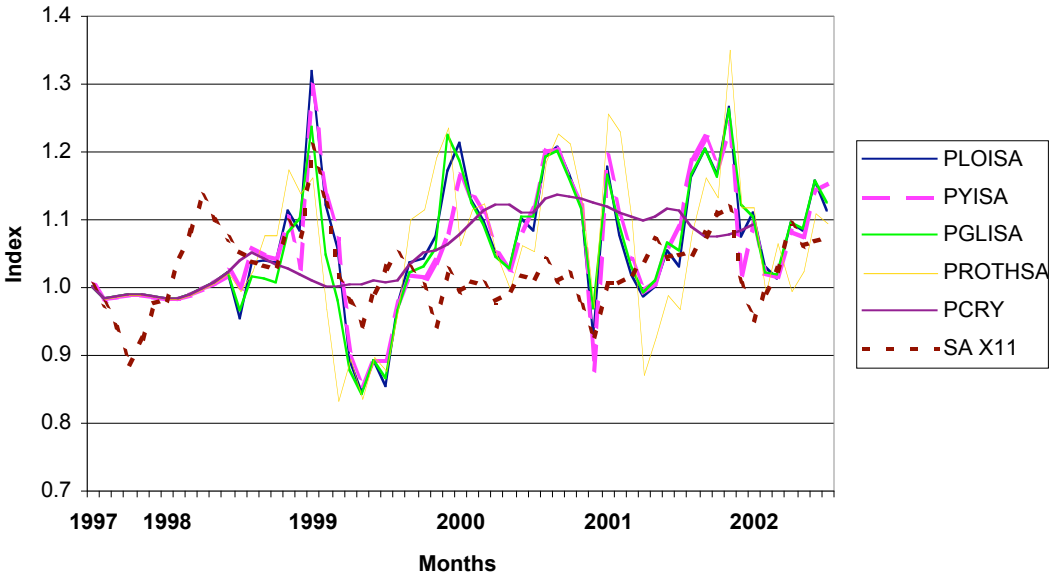


46. Finally, the seasonally adjusted series of the Lowe, Young and geometric Laspeyres indices with imputed prices and Rothwell index seem to perform less well to repeat the trend of target index values (the centered rolling year index)[10]. The centered rolling year index for the chosen data set still fails to represent reliably the overall price index of fresh fruits with X-11 seasonal adjustment.[11] The extreme outliers found in the Seasonally-Adjusted Rothwell compared to the target indices disappeared after having modified the original dataset (see Figure 9 below).

Figure 9: Seasonally Adjusted Lowe, Young and Geometric Laspeyres Indices with Imputed Prices, Seasonally Adjusted Rothwell and Central Rolling Year Indices and Normalized Price Index of Fresh Fruits with X-11 Adjustment in the Modified Data Set.

[10] For the seasonally adjusted Lowe index, an R^2 of .1094 is obtained for regression on the centered rolling year series (the lowest fit); for the seasonally adjusted Young index, an R^2 of .1274 is obtained and for the seasonally adjusted Geometric Laspeyres index, an R^2 of .2197 is obtained. For the seasonally adjusted Rothwell index, an R^2 of .1477 is obtained..

[11] The R^2 value is .0026.



I. Summary

47. This paper has three objectives: (1) to summarize Chapter 22 on the treatment of seasonal products from the new international manuals on CPI/PPI (2) to describe some of the methods used in the Israeli CPI to overcome seasonal fluctuations (and bias) in a month to month index (3) examine some of the conclusions from Chapter 22 by simulating the methods with "real" Israeli CPI data.

48. Chapter 22 introduces methods for computing year over year monthly indices in order to overcome the challenges of strong seasonality. The month-to-month methods seem to fail due to large fluctuations in prices and quantities. The yearly indices are quite important for trend analysis but seem to miss the monthly trends. However, month-to-month indices can be transformed into seasonally adjusted ones (and forecast the trend) by using seasonal adjustment factors derived from rolling year over year monthly indices. Therefore, these additional annual and monthly indices should be used, at least as an analytical series to go along side the traditional ones.

49. Using real data from the Israeli CPI led to some interesting, and at time contradicting, results compared to the findings in Chapter 22. No doubt that the methods presented in Chapter 22 should be conducted using fuller data sets and in many countries around the world. This kind of empirical research can improve the methods for treatment of seasonal products in the updated versions of the international manual.

50. Finally, analytical series are very important to understand what is right or wrong with the "official" one. Statistical agencies introduce multiple modifications in observations, in order to treat various problems, in particular the problem of seasonality in CPI. Computing analytical series on a current basis will enable comparisons that can lead to enhanced procedures in the

production of price indices. Setting rules for modifications in a way that will approach the results obtained by the theoretical dataset may help to treat these problems in a more effective manner. In addition, the investment on analytical series is once only – spread sheets enable the improvement of the index on a current basis.

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