The Reconciliation of Industry Productivity Measures with National Measures: An Exact Translog Approach[1]

Erwin Diewert,
Department of Economics,
University of British Columbia,
Vancouver, B.C.,
Canada, V6T 1Z1.

Email: diewert@econ.ubc.ca
Website: http://www.econ.ubc.ca/diewert/hmpgdie.htm

December 8, 2006

Denis Lawrence,
Meyrick and Associates Pty Ltd,
6 Kurundi Place, Hawker, ACT 2614, AUSTRALIA
Email: denis@meyrick.com.au
Website: www.meyrick.com.au

CONTENTS

[1] The authors thank Industry Canada and the Australian Research Council for financial support. Neither institution is responsible for any opinions expressed in this paper.

1 INTRODUCTION

Improvements in a country's terms of trade have effects that are similar to an improvement in a country's productivity growth. This is because an increase in export prices relative to import prices allows a larger quantity of imports to be purchased for a given quantity of exports, thus improving domestic welfare.

Calculating the exact magnitude of each source of welfare gain is not straightforward. Diewert (1983), Diewert and Morrison (1986), Morrison and Diewert (1990) and Kohli (1990) (1991) (2003) (2004a) (2004b) developed a production theory methodology that enables one to obtain index number estimates of the contribution of each type of gain. In section 2, we adapt this methodology and show how it can be used to measure the determinants of growth in an economy's real income. We show how this theoretical approach can be implemented in sections 3-5 using techniques that are used in index number theory. Sections 3 and 4 assume that the market sector of the economy can be represented by a translog GDP function whereas section 5 pursues a first order approximation approach to implementing the theoretical indexes defined in section 2.[2]

The main determinants of growth in real income generated by the market sector of the economy are:

- Technical progress or improvements in Total Factor Productivity;
- Growth in domestic output prices or the prices of internationally traded goods and services relative to the price of consumption; and
- Growth in primary inputs.

However, GDP is a measure of productive potential, not welfare. For welfare measurement purposes, it is generally conceded that Net Domestic Product (NDP) is a better measure of output, since investment that just meets depreciation means that society is not made any better off from the viewpoint of sustainable final consumption possibilities; see, for example, Weitzman (1976), (1997) and Oulton (2002). Hence, in the second part of the report, we propose to subtract depreciation from gross investment and use consumption plus sales to the government sector plus *net investment* plus the

---

[2] The material in sections 2-6 is drawn from Diewert and Lawrence (2006).

trade balance as our output concept. Thus, depreciation will be treated as an intermediate input in this model of production. Section 6 explains this real net product approach and adapts the translog model of sections 3 and 4 to this new model of market sector real income generation.

Researchers are interested in not only the performance of the aggregate market sector of an economy but they are also interested in how the various industries in the market sector contribute to the overall performance. Thus in section 7, we show how our methodology can be extended to cover the case where industry data on inputs and outputs are available and the separate contributions of each industry to overall real income growth can be assessed in a more comprehensive theoretical framework.

2 THE PRODUCTION THEORY FRAMEWORK

In this section, we present the production theory framework that will be used in the remainder of the report. The main reference is Diewert and Morrison (1986)[3] but we also draw on the theory of the output price index, which was developed by Fisher and Shell (1972) and Archibald (1977). This theory is the producer theory counterpart to the theory of the cost of living index for a single consumer (or household) that was first developed by the Russian economist, A. A. Konüs (1924). These economic approaches to price indexes rely on the assumption of (competitive) *optimizing behaviour* on the part of economic agents (consumers or producers). Thus, we consider only the market sector of the economy in what follows; ie that part of the economy that is motivated by profit maximizing behaviour. In our empirical work, we define the market sector to be the entire production sector of the economy as defined in the System of National Accounts, less the general government sector and the owner occupied housing sector.[4]

Initially, we assume that the market sector of the economy produces quantities of M (net)[5] outputs, $y \equiv [y_1,...,y_M]$, which are sold at the positive producer prices $P \equiv [P_1,...,P_M]$.

[3] The theory also draws on Samuelson (1953), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Fox and Kohli (1998), Kohli (1978) (1990) (1991) (2003) (2004a) (2004b), Morrison and Diewert (1990), Samuelson (1953) and Sato (1976).

[4] For both of these sectors, output is equal to input and hence no productivity improvements can be generated by these two sectors according to SNA conventions.

[5] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are

We further assume that the market sector of the economy uses positive quantities of N primary inputs, $x \equiv [x_1,...,x_N]$ which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors y that can be produced by the market sector if the vector of primary inputs x is utilised by the market sector of the economy; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.[6]

Given a vector of output prices P and a vector of available primary inputs x, we define *the period t market sector GDP function*, $g^t(P,x)$, as follows:[7]

$$(1)\ g^t(P,x) \equiv \max_y \{P\cdot y : (y,x) \text{ belongs to } S^t\} ; \qquad t = 0,1,2, ...$$

Thus market sector GDP depends on t (which represents the period t technology set S^t), on the vector of output prices P that the market sector faces and on x, the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and x^t is the vector of inputs used by the market sector during period t and if the GDP function is differentiable with respect to the components of P at the point P^t,x^t, then the period t vector of market sector outputs y^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of P; ie we will have the following equations for each period t:[8]

$$(2)\ y^t = \nabla_P\, g^t(P^t,x^t) ; \qquad t = 0,1,2, ...$$

"transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

[6] For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function g^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978)(1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974) (1993).

[7] The function $g^t(P,x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x; see Diewert (1973) (1974; 136). Notation: $P\cdot y \equiv \sum_{m=1}^M P_m y_m$.

[8] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P\, g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial P_1, ...,\partial g^t(P^t,x^t)/\partial P_M]$.

Thus the period t market sector supply vector y^t can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t output price vector P^t.

If the GDP function is differentiable with respect to the components of x at the point P^t,x^t, then the period t vector of input prices W^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of x; ie we will have the following equations for each period t:[9]

$$(3)\ W^t = \nabla_x\, g^t(P^t,x^t)\ ; \qquad\qquad t = 0,1,2, \ldots .$$

Thus, the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t input quantity vector x^t.

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t; ie we have the following relationships:

$$(4)\ g^t(P^t,x^t) = P^t\cdot y^t = W^t\cdot x^t\ ; \qquad\qquad t = 0,1,2, \ldots .$$

The above material will be useful in what follows but of course, our focus is not on GDP; instead our focus is on the income generated by the market sector or more precisely, on *the real income generated by the market sector*. However, since market sector GDP (the value of market sector production) is distributed to the factors of production used by the market sector, nominal market sector GDP will be equal to nominal market sector income; ie from (4), we have $g^t(P^t,x^t) = P^t\cdot y^t = W^t\cdot x^t$. As an approximate welfare measure that can be associated with market sector production,[10] we will choose to measure the *real income generated by the market sector in period t*, r^t, in terms of the number of consumption bundles that the nominal income could purchase in period t; ie define ρ^t as follows:

---

[9] These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla x\ g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial x_1, \ldots,\partial g^t(P^t,x^t)/\partial x_N]$.

[10] Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of *domestic* welfare generated by the market production sector is only an approximate one. Moreover, our suggested welfare measure is not sensitive to the distribution of the income that is generated by the market sector.

$$(5)\ \rho^t \equiv W^t \cdot x^t / P_C{}^t \,; \qquad t = 0,1,2, \ldots$$
$$= w^t \cdot x^t$$
$$= p^t \cdot y^t$$
$$= g^t(p^t, x^t)$$

where $P_C{}^t > 0$ is the *period t consumption expenditures deflator* and the market sector period t *real output price* p^t and *real input price* w^t vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures price index; ie we have the following definitions:[11]

$$(6)\ p^t \equiv P^t/P_C{}^t \,;\ w^t \equiv W^t/P_C{}^t \,; \qquad t = 0,1,2, \ldots .$$

The first and last equality in (5) imply that period t real income, ρ^t, is equal to the period t GDP function, evaluated at the period t real output price vector p^t and the period t input vector x^t, $g^t(p^t,x^t)$. Thus *the growth in real income over time can be explained by three main factors: t (Technical Progress or Total Factor Productivity growth), growth in real output prices and the growth of primary inputs*. We will shortly give formal definitions for these three growth factors.

Using the linear homogeneity properties of the GDP functions $g^t(P,x)$ in P and x separately, we can show that the following counterparts to the relations (2) and (3) hold using the deflated prices p and w:[12]

$$(7)\ y^t = \nabla_p\, g^t(p^t,x^t) \,; \qquad t = 0,1,2, \ldots$$
$$(8)\ w^t = \nabla_x\, g^t(p^t,x^t) \,; \qquad t = 0,1,2, \ldots .$$

Now we are ready to define a family of *period t productivity growth factors or technical*

[11] Our approach is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; ie he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

[12] If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,x)$, which uses the original output prices P, then they will also solve the profit maximisation problem that uses the normalised output prices $p \equiv P/P_C$; ie they will also solve the problem defined by $g^t(p,x)$.

progress shift factors $\tau(p,x,t)$:[13]

(9) $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$; $t = 1,2, \ldots$.

Thus $\tau(p,x,t)$ measures the proportional change in the real income produced by the market sector at the reference real output prices p and reference input quantities used by the market sector x where the numerator in (9) uses the period t technology and the denominator in (9) uses the period t−1 technology. Thus, each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t−1 to period t. Note that we are using the chain system to measure the shift in technology.

It is natural to choose special reference vectors for the measure of technical progress defined by (9): a *Laspeyres type measure* $\tau_L^{\,t}$ that chooses the period t−1 reference vectors p^{t-1} and x^{t-1} and a *Paasche type measure* $\tau_P^{\,t}$ that chooses the period t reference vectors p^t and x^t:

(10) $\tau_L^{\,t} \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$; $t = 1,2, \ldots$;
(11) $\tau_P^{\,t} \equiv \tau(p^t,x^t,t) = g^t(p^t,x^t)/g^{t-1}(p^t,x^t)$; $t = 1,2, \ldots$.

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from index number theory[14] (so that the estimate going backwards is equal to the reciprocal of the estimate going forwards), then the geometric mean will be the best simple average to take.[15] Thus, we define the geometric mean of (10) and (11) as follows:[16]

(12) $\tau^t \equiv [\tau_L^{\,t}\tau_P^{\,t}]^{1/2}$; $t = 1,2, \ldots$.

At this point, it is not clear how we will obtain empirical estimates for the theoretical

[13] This family of measures of technical progress is due to Diewert (1983; 1063) and Diewert and Morrison (1986; 662).
[14] See Fisher (1922; 64).
[15] See the discussion in Diewert (1997) on choosing the "best" symmetric average of Laspeyres and Paasche indexes that will lead to the satisfaction of the time reversal test by the resulting average index.
[16] The specific theoretical productivity change indexes defined by (10)-(12) were first defined by Diewert and Morrison (1968; 662-663) following Diewert's (1983; 1063) more general definition. See Diewert (1993) for properties of symmetric means.

productivity growth indexes defined by (10)–(12). One obvious way would be to assume a functional form for the GDP function $g^t(p,x)$, collect data on output and input prices and quantities for the market sector for a number of years (and for the consumption expenditures deflator), add error terms to equations (7) and (8) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different econometricians will make different stochastic specifications and will choose different functional forms.[17] Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus, we will suggest methods for implementing measures like (12) in this report that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income due to changes in real output prices. Define a family of *period t real output price growth factors* $\alpha(p^{t-1},p^t,x,s)$:[18]

$$(13)\ \alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x)/g^s(p^{t-1},x)\ ; \qquad s = 1,2, \ldots .$$

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in real output prices going from period t−1 to t, using the technology that is available during period s and using the reference input quantities x. Thus, each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income of a change in real output prices going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (13): a *Laspeyres type measure* $\alpha_L{}^t$ that chooses the period t−1 reference technology and reference input vector x^{t-1} and a *Paasche type measure* $\alpha_P{}^t$ that chooses the period t reference technology and reference input vector x^t:

---

[17] "The estimation of GDP functions such as (19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).

[18] This measure of real output price change was essentially defined by Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89). Readers who are familiar with the theory of the true cost of living index will note that the real output price index defined by (13) is analogous to the Konüs (1924) *true cost of living index* which is a ratio of cost functions, say $C(u,p^t)/C(u,p^{t-1})$ where u is a reference utility level: g^s replaces C and the reference utility level u is replaced by the vector of reference variables x.

(14) $\alpha_L{}^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$; $t = 1,2,...$;
(15) $\alpha_P{}^t \equiv \alpha(p^{t-1},p^t,x^t,t) \quad = g^t(p^t,x^t)/g^t(p^{t-1},x^t)$; $t = 1,2,...$.

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in real output prices:[19]

(16) $\alpha^t \equiv [\alpha_L{}^t \alpha_P{}^t]^{1/2}$; $t = 1,2,...$.

Finally, we look at the problem of defining theoretical indexes for the effects on real income due to changes in input quantities. Define a family of *period t real input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:[20]

(17) $\beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t)/g^s(p,x^{t-1})$; $s = 1,2,...$.

Thus $\beta(x^{t-1},x^t,p,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in input quantities used by the market sector going from period t−1 to t, using the technology that is available during period s and using the reference real output prices p. Thus, each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in input quantities going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (17): a *Laspeyres type measure* $\beta_L{}^t$ that chooses the period t−1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* $\beta_P{}^t$ that chooses the period t reference technology and reference real output price vector p^t:

(18) $\beta_L{}^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})$; $t = 1,2,...$;
(19) $\beta_P{}^t \equiv \beta(x^{t-1},x^t,p^t,t) \quad = g^t(p^t,x^t)/g^t(p^t,x^{t-1})$; $t = 1,2,...$.

Since both measures of real input growth are equally valid, it is natural to average them

[19] The indexes defined by (13)-(16) were defined by Diewert and Morrison (1986; 664) in the nominal GDP function context.
[20] This type of index was defined as a true index of value added by Sato (1976; 438) and as a real input index by Diewert (1980; 456).

to obtain an overall measure of the effects of input growth on real income:[21]

$$(20)\ \beta^t \equiv [\beta_L{}^t \beta_P{}^t]^{1/2}\ ; \qquad t = 1,2, \ldots .$$

Recall that market sector real income for period t was defined by (5) as ρ^t equal to nominal period t factor payments $W^t \cdot x^t$ deflated by the household consumption price deflator $P_C{}^t$. It is convenient to define γ^t as the *period t chain rate of growth factor for real income*:

$$(21)\ \gamma^t \equiv \rho^t/\rho^{t-1}\ ; \qquad t = 1,2, \ldots .$$

It turns out that the definitions for γ^t and the technology, output price and input quantity growth factors $\tau(p,x,t)$, $\alpha(p^{t-1},p^t,x,s)$, $\beta(x^{t-1},x^t,p,s)$ defined by (9), (13) and (17), respectively, satisfy some interesting identities, which we will now develop. We have:

$$\begin{aligned}(22)\ \gamma^t &\equiv \rho^t/\rho^{t-1}\ ; && t = 1,2, \ldots . \\ &= g^t(p^t,x^t)/g^{t-1}(p^{t-1},x^{t-1}) && \text{using definitions (4) and (5)} \\ &= [g^t(p^t,x^t)/g^{t-1}(p^t,x^t)][g^{t-1}(p^t,x^t)/g^{t-1}(p^{t-1},x^t)][g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})] && \\ &= \tau_P{}^t\ \alpha(p^{t-1},p^t,x^t,t-1)\ \beta_L{}^t && \text{using definitions (11), (13) and (18).}\end{aligned}$$

In a similar fashion, we can establish the following companion identity:

$$(23)\ \gamma^t \equiv \tau_L{}^t\ \alpha(p^{t-1},p^t,x^{t-1},t)\ \beta_P{}^t \qquad \text{using definitions (10), (13) and (19).}$$

Thus, multiplying (22) and (23) together and taking positive square roots of both sides of the resulting identity and using definitions (12) and (20), we obtain the following identity:

$$(24)\ \gamma^t \equiv \tau^t\ [\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}\ \beta^t\ ; \qquad t = 1,2, \ldots .$$

In a similar fashion, we can derive the following alternative decomposition for γ^t into growth factors:

[21] The theoretical indexes defined by (17)-(20) were defined in Diewert and Morrison (1986; 665) in the nominal GDP context.

(25) $\gamma^t \equiv \tau^t\, \alpha^t\, [\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}$; $t = 1,2, \ldots$.

It is quite likely that the real output price growth factor $[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}$ is fairly close to α^t defined by (16) and it is quite likely that the input growth factor $[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}$ is quite close to β^t defined by (20); ie we have the following approximate equalities:

(26) $[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2} \approx \alpha^t$; $t = 1,2, \ldots$;
(27) $[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2} \approx \beta^t$; $t = 1,2, \ldots$.

Substituting (26) and (27) into (24) and (25), respectively, leads to the following approximate decompositions for the growth of real income into explanatory factors:

(28) $\gamma^t \approx \tau^t\, \alpha^t\, \beta^t$; $t = 1,2, \ldots$

where τ^t is a technology growth factor, α^t is a growth in real output prices factor and β^t is a growth in primary inputs factor.

Rather than look at explanatory factors for the growth in real market sector income, it is sometimes convenient to express the level of real income in period t in terms of an *index of the technology level* or of Total Factor Productivity in period t, T^t, of the *level of real output prices* in period t, A^t, and of the *level of primary input quantities* in period t, B^t.[22] Thus, we use the growth factors τ^t, α^t and β^t as follows to define the levels T^t, A^t and B^t:

(29) $T^0 \equiv 1$; $T^t \equiv T^{t-1}\, \tau^t$; $t = 1,2, \ldots$;
(30) $A^0 \equiv 1$; $A^t \equiv A^{t-1}\alpha^t$; $t = 1,2, \ldots$;
(31) $B^0 \equiv 1$; $B^t \equiv B^{t-1}\beta^t$; $t = 1,2, \ldots$.

Using the approximate equalities (28) for the chain links that appear in (29)-(31), we can establish the following approximate relationship for the level of real income in period t, ρ^t, and the period t levels for technology, real output prices and input quantities:

(32) $\rho^t/\rho^0 \approx T^t\, A^t\, B^t$; $t = 0,1,2, \ldots$.

[22] This type of levels presentation of the data is quite instructive when presented in graphical form. It was suggested by Kohli (1990) and used extensively by him; see Kohli (1991), (2003) (2004a) (2004b) and Fox and Kohli (1998).

In the following section, we note a set of assumptions on the technology sets that will ensure that the approximate real income growth decompositions (28) and (32) hold as exact equalities.

3 THE TRANSLOG GDP FUNCTION APPROACH

We now follow the example of Diewert and Morrison (1986; 663) and assume that the log of the period t (deflated) GDP function, $g^t(p,x)$, has the following translog functional form:[23]

$$(33)\ \ln g^t(p,x) \equiv a_0^t + \sum_{m=1}^M a_m^t \ln p_m^t + (1/2) \sum_{m=1}^M \sum_{k=1}^M a_{mk} \ln p_m^t \ln p_k^t + \sum_{n=1}^N b_n^t \ln x_n^t + (1/2) \sum_{n=1}^N \sum_{j=1}^N b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^M \sum_{n=1}^M c_{mn} \ln p_m^t \ln x_n^t ;\quad t = 0,1,2, \ldots .$$

Note that the coefficients for the quadratic terms are assumed to be constant over time. The coefficients must satisfy the following restrictions in order for g^t to satisfy the linear homogeneity properties that we have assumed in section 2 above:[24]

(34) $\sum_{m=1}^M a_m^t = 1$ for $t = 0,1,2, \ldots$;
(35) $\sum_{n=1}^N b_n^t = 1$ for $t = 0,1,2, \ldots$;
(36) $a_{mk} = a_{km}$ for all k,m ;
(37) $b_{nj} = b_{jn}$ for all n,j ;
(38) $\sum_{k=1}^M a_{mk} = 0$ for $m = 1,\ldots,M$;
(39) $\sum_{j=1}^N b_{nj} = 0$ for $n = 1,\ldots,N$;
(40) $\sum_{n=1}^N c_{mn} = 0$ for $m = 1,\ldots,M$;
(41) $\sum_{m=1}^M c_{mn} = 0$ for $n = 1,\ldots,N$.

Recall the approximate decomposition of real income growth going from period $t-1$ to t given by (28) above, $\gamma^t \approx \tau^t \alpha^t \beta^t$. Diewert and Morrison (1986; 663) showed that[25] if g^{t-1}

[23] This functional form was first suggested by Diewert (1974; 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson and Lau (1971). Diewert (1974; 139) indicated that this functional form was flexible.

[24] There are additional restrictions on the parameters which are necessary to ensure that $g^t(p,x)$ is convex in p and concave in x.

[25] Diewert and Morrison established their proof using the nominal GDP function $g^t(P,x)$. However, it is

and g^t are defined by (33)-(41) above and there is competitive profit maximizing behavior on the part of all market sector producers for all periods t, then (28) holds as an exact equality; ie we have

(42) $\gamma^t = \tau^t\, \alpha^t\, \beta^t$; $\qquad t = 1,2, \ldots$.

In addition, Diewert and Morrison (1986; 663-665) showed that τ^t, α^t and β^t could be calculated using empirically observable price and quantity data for periods t−1 and t as follows:

(43) $\ln\alpha^t = \sum_{m=1}^{M} (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^t y_m^t/p^t\cdot y^t)] \ln(p_m^t/p_m^{t-1})$
$= \ln P_T(p^{t-1},p^t,y^{t-1},y^t)$;
(44) $\ln\beta^t = \sum_{n=1}^{N} (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1})$
$= \ln Q_T(w^{t-1},w^t,x^{t-1},x^t)$;
(45) $\tau^t = \gamma^t/\alpha^t\, \beta^t$

where $P_T(p^{t-1},p^t,y^{t-1},y^t)$ is the Törnqvist (1936) and Törnqvist and Törnqvist (1937) output price index and $Q_T(w^{t-1},w^t,x^{t-1},x^t)$ is the Törnqvist input quantity index.

Since equations (42) now hold as exact identities under our present assumptions, equations (32), the cumulated counterparts to equations (28), will also hold as exact decompositions; ie under our present assumptions, we have

(46) $\rho^t/\rho^0 = T^t\, A^t\, B^t$; $\qquad t = 1,2, \ldots$.

4 THE TRANSLOG GDP FUNCTION APPROACH AND CHANGES IN THE TERMS OF TRADE

For some purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices α^t into separate effects for each change in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all market sector primary input quantities β^t into separate effects for each change in each input quantity. In this section, we indicate

---

easy to rework their proof using the deflated GDP function $g^t(p,x)$ using the fact that $g^t(p,x) = g^t(P/P_C,x) = g^t(P,x)/P_C$ using the linear homogeneity property of $g^t(P,x)$ in P.

how this can be done, making the same assumptions on the technology that were made in the previous section.

We first model the effects of a change in a single (deflated) output price, say p_m, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type price indexes defined by (14) and (15) above for changes in all (deflated) output prices are the following *Laspeyres type measure* $\alpha_{Lm}{}^t$ that chooses the period t−1 reference technology and holds constant other output prices at their period t−1 levels and holds inputs constant at their period t−1 levels x^{t-1} and a *Paasche type measure* $\alpha_{Pm}{}^t$ that chooses the period t reference technology and reference input vector x^t and holds constant other output prices at their period t levels:

$$(47)\ \alpha_{Lm}{}^t \equiv g^{t-1}(p_1{}^{t-1},...,p_{m-1}{}^{t-1},p_m{}^t,p_{m+1}{}^{t-1},..., p_M{}^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad m = 1,...,M;\ t = 1,2, ...\ ;$$

$$(48)\ \alpha_{Pm}{}^t \equiv g^t(p^t,x^t)/g^t(p_1{}^t,...,p_{m-1}{}^t,p_m{}^{t-1},p_{m+1}{}^t,..., p_M{}^t,x^t)\ ; \qquad m = 1,...,M;\ t = 1,2, ...$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on real income of the change in the real price of output m*:[26]

$$(49)\ \alpha_m{}^t \equiv [\alpha_{Lm}{}^t\alpha_{Pm}{}^t]^{1/2}\ ; \qquad m = 1,...,M\ ;\ t = 1,2, ...$$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 666) can be adapted to give us the following result:

$$(50)\ \ln\alpha_m{}^t = (1/2)[(p_m{}^{t-1}y_m{}^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m{}^t y_m{}^t/p^t\cdot y^t)]\ \ln(p_m{}^t/p_m{}^{t-1})\ ; \qquad m = 1,...,M\ ;\ t = 1,2, ...$$

Note that $\ln\alpha_m{}^t$ is equal to the mth term in the summation of the terms on the right hand side of (43). This observation means that we have the following exact decomposition of the period t aggregate real output price contribution factor α^t into a product of separate

[26] The indexes defined by (47)-(49) were defined by Diewert and Morrison (1986; 666) in the nominal GDP function context.

price contribution factors; ie we have under present assumptions:

(51) $\alpha^t = \alpha_1{}^t\alpha_2{}^t\ldots\ \alpha_M{}^t$; $\qquad t = 1,2, \ldots$.

The above decomposition is useful for analysing how real changes in the price of exports (ie a change in the price of exports relative to the price of domestic consumption) and in the price of imports impact on the real income generated by the market sector. In the empirical illustration which follows later, we let M equal three. The three net outputs are:

- Domestic sales (C+I+G);
- Exports (X) and
- Imports (M).

Since commodities 1 and 2 are outputs, y_1 and y_2 will be positive but since commodity 3 is an input into the market sector, y_3 will be negative. Hence an increase in the real price of exports will *increase* real income but an increase in the real price of imports will *decrease* the real income generated by the market sector, as is evident by looking at the contribution terms defined by (50) for m = 2 (where $y_m{}^t > 0$) and for m = 3 (where $y_m{}^t < 0$).

As mentioned above, it is also useful to have a decomposition of the aggregate contribution of input growth to the growth of real income into separate contributions for each important class of primary input that is used by the market sector. We now model the effects of a change in a single input quantity, say x_n, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type quantity indexes defined by (18) and (19) above for changes in input n are the following *Laspeyres type measure* $\beta_{Ln}{}^t$ that chooses the period t−1 reference technology and holds constant other input quantities at their period t−1 levels and holds real output prices at their period t−1 levels p^{t-1} and a *Paasche type measure* $\beta_{Pn}{}^t$ that chooses the period t reference technology and reference real output price vector p^t and holds constant other input quantities at their period t levels:

(52) $\beta_{Ln}{}^t \equiv g^{t-1}(p^{t-1},x_1{}^{t-1},\ldots,x_{n-1}{}^{t-1},x_n{}^t,x_{n+1}{}^{t-1},\ldots,\ x_N{}^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$; $\qquad n = 1,\ldots,N;\ t = 1,2, \ldots$;

(53) $\beta_{Pn}{}^t \equiv g^t(p^t,x^t)/g^t(p^t,x_1{}^t,\ldots,x_{n-1}{}^t,x_n{}^{t-1},x_{n+1}{}^t,\ldots,\ p_N{}^t)$; $\qquad m = 1,\ldots,M;\ t = 1,2, \ldots$.

Since both measures of input change are equally valid, as usual, we average them to obtain *an overall measure of the effects on real income of the change in the quantity of input n*:[27]

(54) $\beta_n^t \equiv [\beta_{Pn}^t \beta_{Pn}^t]^{1/2}$; $\qquad n = 1,\ldots,N$; $t = 1,2, \ldots$.

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 667) can be adapted to give us the following result:

(55) $\ln\beta_n^t = (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1})$;
$n = 1,\ldots,N$; $t = 1,2, \ldots$.

Note that $\ln\beta_n^t$ is equal to the nth term in the summation of the terms on the right hand side of (44). This observation means that we have the following exact decomposition of the period t aggregate input growth contribution factor β^t into a product of separate input quantity contribution factors; ie we have under present assumptions:

(56) $\beta^t = \beta_1^t\beta_2^t \ldots \beta_N^t$; $\qquad t = 1,2, \ldots$.

There is another approach to contribution analysis that was suggested in Diewert (1983; 1096) and Diewert and Morrison (1986; 674-676) and we outline that approach in the following section.

5 THE FIRST ORDER APPROXIMATION NONPARAMETRIC APPROACH

Recall definitions (10) and (11) in section 2 that defined the Laspeyres productivity growth contribution factor, $\tau_L^t \equiv g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$ and the Paasche contribution factor, $\tau_P^t \equiv g^t(p^t,x^t)/g^{t-1}(p^t,x^t)$. The denominator in the definition of τ_L^t is $g^{t-1}(p^{t-1},x^{t-1})$ and this is equal to the observable (in principle) deflated market sector GDP in period $t-1$, $p^{t-1}\cdot y^{t-1}$, which in turn is equal to deflated factor payments, $w^{t-1}\cdot x^{t-1}$. The numerator

[27] The indexes defined by (52)-(54) were defined by Diewert and Morrison (1986; 667) in the nominal GDP function context.

in the expression for τ_L^t is $g^t(p^{t-1},x^{t-1})$ and this is not directly observable. However, we can use (7) and (8) in order to form the following first order Taylor series approximation to $g^t(p^{t-1},x^{t-1})$:

$$(57)\ g^t(p^{t-1},x^{t-1}) \approx g^t(p^t,x^t) + \nabla_p\, g^t(p^t,x^t)\cdot[p^{t-1} - p^t] + \nabla_x\, g^t(p^t,x^t)\cdot[x^{t-1} - x^t]$$
$$= p^t\cdot y^t + y^t\cdot[p^{t-1} - p^t] + w^t\cdot[x^{t-1} - x^t] \qquad \text{using (7) and (8)}$$
$$= p^{t-1}\cdot y^t + w^t\cdot[x^{t-1} - x^t].$$

The numerator in the definition of τ_P^t is $g^t(p^t,x^t)$ and this is equal to the observable (in principle) deflated market sector GDP in period t, $p^t\cdot y^t$, which in turn is equal to deflated factor payments, $w^t\cdot x^t$. The denominator in the expression for τ_P^t is $g^{t-1}(p^t,x^t)$ and this is not directly observable. Again, we can use (7) and (8) in order to form the following first order Taylor series approximation to $g^{t-1}(p^t,x^t)$:

$$(58)\ g^{t-1}(p^t,x^t) \approx g^{t-1}(p^{t-1},x^{t-1}) + \nabla_p\, g^{t-1}(p^{t-1},x^{t-1})\cdot[p^t - p^{t-1}] + \nabla_x\, g^{t-1}(p^{t-1},x^{t-1})\cdot[x^t - x^{t-1}]$$
$$= p^{t-1}\cdot y^{t-1} + y^{t-1}\cdot[p^t - p^{t-1}] + w^{t-1}\cdot[x^t - x^{t-1}] \qquad \text{using (7) and (8)}$$
$$= p^t\cdot y^{t-1} + w^{t-1}\cdot[x^t - x^{t-1}].$$

Using (57) and (58), we have the following (observable) *first order approximations to the theoretical productivity growth factors* $\tau_L^t \equiv g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$ and $\tau_P^t \equiv g^t(p^t,x^t)/g^{t-1}(p^t,x^t)$:

$$(59)\ \tau_L^t \approx \{p^{t-1}\cdot y^t + w^t\cdot[x^{t-1} - x^t]\}/p^{t-1}\cdot y^{t-1}\ ;$$
$$(60)\ \tau_P^t \approx p^t\cdot y^t/\{p^t\cdot y^{t-1} + w^{t-1}\cdot[x^t - x^{t-1}]\}.$$

Now use the right hand sides of (59) and (60) in order to form the following approximation to $\tau^t \equiv [\tau_L^t\, \tau_P^t]^{1/2}$:

$$(61)\ \tau^t \approx (p^t\cdot y^t\{p^{t-1}\cdot y^t + w^t\cdot[x^{t-1} - x^t]\}/p^{t-1}\cdot y^{t-1}\{p^t\cdot y^{t-1} + w^{t-1}\cdot[x^t - x^{t-1}]\})^{1/2}; \qquad t = 1,2, \ldots .$$

In a similar fashion, we can form *first order Taylor series approximations to the Laspeyres and Paasche real output price contribution factors* $\alpha_L^t \equiv g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$ and $\alpha_P^t \equiv g^t(p^t,x^t)/g^t(p^{t-1},x^t)$, respectively, and then use these approximations to form an approximation to $\alpha^t \equiv [\alpha_L^t\, \alpha_P^t]^{1/2}$. The resulting observable approximations are:

$$(62)\ \alpha_L{}^t \approx p^t \cdot y^{t-1}/p^{t-1} \cdot y^{t-1} \qquad = P_L(p^{t-1},p^t,y^{t-1},y^t)\ ;$$
$$(63)\ \alpha_P{}^t \approx p^t \cdot y^t/p^{t-1} \cdot y^t \qquad = P_P(p^{t-1},p^t,y^{t-1},y^t)\ ;$$
$$(64)\ \alpha^t \approx [\{p^t \cdot y^{t-1}/p^{t-1} \cdot y^{t-1}\}\{p^t \cdot y^t/p^{t-1} \cdot y^t\}]^{1/2} = P_F(p^{t-1},p^t,y^{t-1},y^t)$$

where P_L, P_P and P_F are the Laspeyres, Paasche and Fisher (1922) price index formulae. Thus, the average of the first order approximations to $\alpha_L{}^t$ and $\alpha_P{}^t$ turns out to be the superlative Fisher price index for deflated output prices, going from period t−1 to period t, and so it is likely that the Fisher approximation to α^t is a very good one.

We can also form *first order Taylor series approximations to the Laspeyres and Paasche input quantity contribution factors* $\beta_L{}^t \equiv g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})$ and $\beta_P{}^t \equiv g^t(p^t,x^t)/g^t(p^t,x^{t-1})$, respectively, and then use these approximations to form an approximation to $\beta^t \equiv [\beta_L{}^t\, \beta_P{}^t]^{1/2}$. The resulting observable approximations are:

$$(65)\ \beta_L{}^t \approx w^{t-1} \cdot x^t/w^{t-1} \cdot x^{t-1} \qquad = Q_L(w^{t-1},w^t,x^{t-1},x^t)\ ;$$
$$(66)\ \beta_P{}^t \approx w^t \cdot x^t/w^t \cdot x^{t-1} \qquad = Q_P(w^{t-1},w^t,x^{t-1},x^t)\ ;$$
$$(67)\ \beta^t \approx [\{w^{t-1} \cdot x^t/w^{t-1} \cdot x^{t-1}\}\{w^t \cdot x^t/w^t \cdot x^{t-1}\}]^{1/2} = Q_F(w^{t-1},w^t,x^{t-1},x^t)$$

where P_L, P_P and P_F are the Laspeyres, Paasche and Fisher (1922) quantity index formulae. Thus the average of the first order approximations to $\beta_L{}^t$ and $\beta_P{}^t$ turns out to be the superlative Fisher input quantity index, going from period t−1 to period t, and so it is likely that the Fisher approximation to the aggregate input growth contribution factor β^t is a very good one.

The same methodological approach can be applied to the problem of approximating the *contribution to real market sector income growth of the change in the real price of output m* defined by (49) above, $\alpha_m{}^t \equiv [\alpha_{Lm}{}^t\, \alpha_{Pm}{}^t]^{1/2}$. The resulting first order approximations are:

$$(68)\ \alpha_{Lm}{}^t \approx \{p^{t-1} \cdot y^{t-1} + y_m{}^{t-1}[p_m{}^t - p_m{}^{t-1}]\}/p^{t-1} \cdot y^{t-1}\ ; \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots\ ;$$
$$(69)\ \alpha_{Pm}{}^t \approx p^t \cdot y^t/\{p^t \cdot y^t + y_m{}^t[p_m{}^{t-1} - p_m{}^t]\}\ ;$$
$$(70)\ \alpha_m{}^t \approx [\alpha_{Lm}{}^t\, \alpha_{Pm}{}^t]^{1/2}.$$

Finally, the first order approximation approach can be applied to the individual input n contribution terms defined by (54) above, $\beta_n{}^t \equiv [\beta_{Pn}{}^t\, \beta_{Pn}{}^t]^{1/2}$. The resulting first order approximations are:

$$(71)\ \beta_{Ln}^{\ t} \approx \{w^{t-1}\cdot x^{t-1} + w_n^{\ t-1}[x_n^{\ t} - x_n^{\ t-1}]\}/w^{t-1}\cdot x^{t-1}\ ; \qquad n = 1,\ldots,N\ ;\ t = 1,2,\ldots\ ;$$
$$(72)\ \beta_{Pn}^{\ t} \approx w^t\cdot x^t/\{w^t\cdot x^t + w_n^{\ t}[x_n^{\ t-1} - x_n^{\ t}]\}\ ;$$
$$(73)\ \beta_n^{\ t} \approx [\beta_{Ln}^{\ t}\beta_{Pn}^{\ t}]^{1/2}.$$

Why is the approach explained in this section necessary, given that we have a perfectly good translog approach that was outlined in the previous section? There are two answers to this very reasonable question:

- The present approach is a completely nonparametric approach (at first glance) whereas the translog approach rests on very specific assumptions about the technology;
- The present approach may be preferred by statistical agencies that use Fisher ideal indexes to do their basic aggregation, since it appears that this approach is more consistent with the use of Fisher indexes.

6 THE DEFLATED NDP TRANSLOG APPROACH

There is a potential shortcoming with the analysis presented in the previous sections. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence our real income measure defined by (5) above is overstated.

To see why Gross Domestic Product overstates income, consider the model of production that is described by the following quotations:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).[28]

All of the "flow" inputs that are purchased during the period and all of the "flow" outputs that are sold during the period are the inputs and outputs that appear in the usual definition of cash flow. These are the flow inputs and outputs that are very familiar to national income accountants. But this is not the end of the story: the firm inherits an endowment of assets at the beginning of the production period and at the end of the period, the firm will have the net profit or loss that has occurred due to its sales of outputs and its purchases of inputs during the period. As well, *it will have a stock of assets that it can use when it starts production in the following period*. Just focusing on the flow transactions that occur within the production period will not give a complete picture of the firm's productive activities. Hence, to get a complete picture of the firm's production activities over the course of a period, it is necessary to add the value of the closing stock of assets less the beginning of the period stock of assets to the cash flow that accrued to the firm from its sales and purchases of market goods and services during the accounting period.

We illustrate the above theory by considering a very simple two output, two input model of the market sector. One of the outputs is output in period t, Y^t and the other output is

[28] For more on this model of production and additional references to the literature, see the Appendices in Diewert (1977) (1980). The usual user cost of capital can be derived from this framework if depreciation is independent of use.

an investment good, I^t. One of the inputs is the flow of noncapital primary input X^t and the other input is K^t, capital services. Suppose that the average prices during period t of a unit of Y^t, X^t and I^t are P_Y^t, P_X^t and P_I^t , respectively. Suppose further that the interest rate prevailing at the beginning of period t is r^t. The value of the beginning of period t capital stock is assumed to be P_I^t, the investment price for period t. In order to induce households to let the business sector use the initial stock of capital, firms have to pay households interest equal to $r^t P_I^t K^t$. Then neglecting balance sheet items, the market sector's period t *cash flow* is:[29]

(74) $CF^t \equiv P_Y^t Y^t + P_I^t I^t - P_X^t X^t - r^t P_I^t K^t$.

K^t is interpreted as the firm's beginning of period t stock of capital it has at its disposal and its end of period stock of capital is defined to be K^{t+1}. These capital stocks are valued at the balance sheet prices prevailing at the beginning and end of period t, P_I^t and P_I^{t+1} , respectively.

The market sector period t *pure profit* is defined as its cash flow plus the value of its end of period t capital stock less the value of its beginning of period t capital stock:

(75) $\prod^t \equiv CF^t + P_I^{t+1} K^{t+1} - P_I^t K^t$.

Now, the end of period depreciated stock of capital is related to the beginning of the period stock by the following equation:

(76) $K^{t+1} = (1 - \delta)K^t$

where $0 < \delta < 1$ denotes the depreciation rate.

Now substitute (74) and (76) into the definition of pure profits (75) and we obtain the following expression:

$$\begin{aligned}(77)\ \textstyle\prod^t &\equiv P_Y^t Y^t + P_I^t I^t - P_X^t X^t - r^t P_I^t K^t + P_I^{t+1}(1 - \delta)K^t - P_I^t K^t \\ &= P_Y^t Y^t + P_I^t I^t - P_X^t X^t - \{r^t P_I^t + \delta P_I^{t+1} - (P_I^{t+1} - P_I^t)\}K^t.\end{aligned}$$

[29] For equity financed firms, we need to include an imputed return for equity capital.

The expression that precedes the capital stock K^t, $\{r^t P_I^t + \delta P_I^{t+1} - (P_I^{t+1} - P_I^t)\}$, can be recognised as the *user cost of capital*;[30] it is the gross rental price that must be paid to a capitalist in order to induce him or her to loan the services of a unit of the capital stock to the production sector.

Some simplifications for (77) occur if we make two additional assumptions:

- Assume that producers and households expect price level stability so that the end of the period price for a new unit of capital P_I^{t+1} is expected to be equal to the beginning of the period price for a new unit of capital P_I^t; in this case, we can interpret r^t as the period t real interest rate;
- Assume that pure profits are zero so that Π^t equals zero.

Substituting these two assumptions into equation (77) leads to the following expression:

$$(78)\ \prod{}^t = P_Y^t Y^t + P_I^t I^t - P_X^t X^t - \{r^t P_I^t + \delta P_I^t\}K^t = 0.$$

Equation (78) can be rearranged to yield the following value of output equals value of input equation:

$$(79)\ P_Y^t Y^t + P_I^t I^t = P_X^t X^t + \{r^t P_I^t + \delta P_I^t\}K^t.$$

Equation (79) is essentially the closed economy counterpart to the (gross) value of outputs equals (gross) value of primary inputs equation (4), $P^t \cdot y^t = W^t \cdot x^t$, that we have been using thus far in this study. We now come to the point of this rather long digression: *the (gross) payments to primary inputs that is defined by the right hand side of (79) is not income*, in the sense of Hicks.[31] The owner of a unit of capital cannot spend the entire period t gross rental income $\{r^t P_I^t + \delta P_I^t\}$ on consumption during period t because the depreciation portion of the rental, δP_I^t, is required in order to keep his or her capital intact. Thus, the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $\{r^t P_I^t + \delta P_I^t\}$ at the end of the

[30] See Christensen and Jorgenson (1969) for a derivation in continuous time and Diewert (1980; 471) for a derivation in discrete time.

[31] We will use Hicks' third concept of income here: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount *in real terms* in each ensuing week." J.R. Hicks (1946; 174).

period plus the depreciated unit of the initial capital stock, which is worth only $(1 - \delta)P_I^t$. Thus δP_I^t of this gross return must be set aside in order to restore the lender of the capital services to his or her original wealth position at the beginning of period t. This means that *period t Hicksian market sector income* is not the value of payments to primary inputs, $P_X^t X^t + \{r^t P_I^t + \delta P_I^t\}K^t$; instead it is the value of payments to labour $P_X^t X^t$ plus the reward for waiting, $r^t P_I^t K^t$. Using this definition of market sector (net) Hicksian income, we can rearrange equation (79) as follows:

(80) Hicksian market sector income $\equiv P_X^t X^t + r^t P_I^t K^t$
$= P_Y^t Y^t + P_I^t I^t - \delta P_I^t K^t$
= Value of consumption + value of gross investment − value of depreciation.

Thus, in this Hicksian net income framework, our new output concept is equal to our old output concept less the value of depreciation. We take the price of depreciation to be the corresponding investment price P_I^t and the quantity of depreciation is taken to be the depreciation rate times the beginning of the period stock, δK^t.

Hence, the overstatement of income problem that is implicit in the approaches used in previous sections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t K^t$, and
- The remaining part that is the reward for postponing consumption, $r^t P_I^t K^t$.

In productivity studies using the gross output framework developed in the earlier sections of this report, the user costs employed have the following form:

(81) $u^t = (r^t + \delta^t + \tau^t)P_I^t$

where r^t is the balancing period t real rate of interest, δ^t is a geometric depreciation rate for period t, τ^t is an average capital taxation rate on the asset and P_I^t is the period t investment price for the asset. Thus, in this section we split up each user cost times the beginning of the period stock K^t into the depreciation component $\delta^t P_I^t K^t$ and the remaining term $(r^t + \tau^t)P_I^t K^t$ and we regard the second term as a genuine income component but the first term is treated as an intermediate input cost for the market sector and is an offset to gross investment made by the market sector during the period under consideration. *Thus, in this section, we use a net product approach instead of a gross product approach*. In this section, our investment aggregate I is a *net investment aggregate* (gross investment components are indexed with a positive sign in the aggregate and depreciation components are indexed with a negative sign in the aggregate). Our capital services aggregate is now a "reward for waiting" capital services aggregate rather than the gross return aggregate that was used in the previous section.[32] Using chained Törnqvist price indexes to do the aggregation, our old gross investment price index P_I is supplemented with the new price for "waiting" capital services P_{KW}, the price of the depreciation aggregate P_{DEP} and the price of the new net investment aggregate P_{NI}.

Using the net output framework developed in this section we can again apply the translog decomposition methodology developed in the earlier sections to examine the relative contributions of productivity and terms of trade changes to output growth – although this time it is to net rather than gross output growth. We can also again use the average of the first order approximations approach explained in section 5 above.

7 INDUSTRY STRUCTURE

The above theory applied to the market sector as a whole. However, *it is of considerable interest to determine which separate industries contributed the most to the overall growth of real income generated by the market sector of the economy*. Hence, in this section, we outline how this can be done if industry data on outputs, inputs and the corresponding prices are available.[33]

We assume that there are I industries in the market sector of the economy. As in section 2, we assume that there is a common list of M (net) outputs which each industry produces or uses as intermediate inputs. The net output vector for industry i in period t is $y^{it} \equiv [y_1^{it},...,y_M^{it}]$, which are sold at the positive producer prices for industry i in period t, $P^{it} \equiv [P_1^{it},...,P_M^{it}]$, for i = 1,...,I. There is also a common list of N primary inputs used by each industry. In period t, we assume that industry i uses nonnegative quantities of N primary

[32] This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: "Second, one can consider the 'waiting' or 'abstinence' associated with the net returns to capital as the nonlabour primary input." T.K. Rymes (1968; 362). Denison (1974) also advocated a net product approach to productivity measurement.

[33] In Canada, such data are available from the Input-Output and Productivity Divisions of Statistics Canada.

inputs, $x^{it} \equiv [x_1^{it},...,x_N^{it}]$ which are purchased at the positive primary input prices $W^{it} \equiv [W_1^{it},...,W_N^{it}]$ for i = 1,...,I. In each period t, we assume that there is a feasible set of net output vectors y^i that can be produced by industry i if the vector of primary inputs x^i is utilised by that industry; denote this period t production possibilities set by S^{it}. We assume that S^{it} is a closed convex cone that exhibits a free disposal property. We shall take the net product point of view developed in the previous section for each industry in what follows.

Given a vector of industry i net output prices P^{it} and a vector of available primary inputs x^{it} for that industry, we define *the industry i period t net product function*, $g^{it}(P^{it},x^{it})$, as follows:

(81) $g^{it}(P^{it},x^{it}) \equiv \max_y \{P^{it}\cdot y : (y,x^{it}) \text{ belongs to } S^{it}\} = P^{it}\cdot y^{it}$; i = 1,...,I ; t = 0,1,2, ...

Since we have assumed constant returns to scale for each industry, it is natural to assume that the income generated by industry i in period t, $W^{it}\cdot x^{it}$, is equal to the corresponding value of net product, $P^{it}\cdot y^{it}$; i.e., we have:

(82) $P^{it}\cdot y^{it} = W^{it}\cdot x^{it}$; i = 1,...,I ; t = 0,1,2, ...

Define the *period t, industry i real input and output price vectors*, w^{it} and p^{it} respectively, as follows:

(83) $w^{it} \equiv W^{it}/P_C^{\,t}$; $p^{it} \equiv P^{it}/P_C^{\,t}$; i = 1,...,I ; t = 0,1,2, ...

As in section 2, we can define the *real income generated by industry i in period t*, ρ^{it}, as the nominal income generated by industry i in period t, $W^{it}\cdot x^{it}$, divided by the consumption price deflator for period t, $P_C^{\,t}$. Using (81)-(83), we have:

(84) $\rho^{it} \equiv W^{it}\cdot x^{it}/P_C^{\,t}$ i = 1,...,I ; t = 0,1,2, ...

$= w^{it}\cdot x^{it}$

$= P^{it}\cdot y^{it}/P_C^{\,t}$

$$= p^{it} \cdot y^{it}$$
$$= g^{it}(p^{it},x^{it})$$

where the last equality follows using (81), (83) and the linear homogeneity of $g^{it}(P^{it},x^{it})$ in P^{it}.

We now rework the theoretical analysis presented in sections 2-4 above, except we apply it at the industry level instead of the economy wide market sector level. Thus define γ^{it} as the *period t chain link rate of growth factor for the real income generated by industry i*:

$$(85)\ \gamma^{it} \equiv \rho^{it}/\rho^{it-1}\ ; \qquad i = 1,...,I\ ;\ t = 1,2,... .$$

Now assume that the industry i, period t (deflated) GDP function, $g^{it}(p,x)$, has a translog functional form analogous to that defined above by (33)-(41). Repeat the analysis at the national level that led up to equation (42), except now apply it at the industry level. We can derive the following industry counterparts to the national equation (42):

$$(86)\ p^{it} \cdot y^{it}/p^{it-1} \cdot y^{it-1} = \rho^{it}/\rho^{it-1} = \gamma^{it} = \tau^{it}\ \alpha^{it}\ \beta^{it}\ ; \qquad i = 1,...,I\ ; t = 0,1,2,\ ...$$

where the *period t, industry i chain link technical progress growth rate* τ^{it}, *output price growth rate* α^{it} and *input quantity growth rate* β^{it} can be calculated using the period t and t−1 price and quantity data for industry i as follows, for i = 1,...,I ; t = 0,1,2, ... :

$$(87)\ \ln\alpha^{it} \equiv \sum_{m=1}^{M} (1/2)[(p_m^{it-1}y_m^{it-1}/p^{it-1} \cdot y^{it-1}) + (p_m^{it}y_m^{it}/p^{it} \cdot y^{it})] \ln(p_m^{it}/p_m^{it-1})$$
$$= \ln P_T(p^{it-1},p^{it},y^{it-1},y^{it});$$
$$(88)\ \ln\beta^{it} \equiv \sum_{n=1}^{N} (1/2)[(w_n^{it-1}x_n^{it-1}/w^{it-1} \cdot x^{it-1}) + (w_n^{it}x_n^{it}/w^{it} \cdot x^{it})] \ln(x_n^{it}/x_n^{it-1})$$
$$= \ln Q_T(w^{it-1},w^{it},x^{it-1},x^{it});$$
$$(89)\quad \tau^{it} \equiv \gamma^{it}/\alpha^{it}\ \beta^{it}$$

where $P_T(p^{it-1},p^{it},y^{it-1},y^{it})$ is the period t, industry i Törnqvist output price index and $Q_T(w^{it-1},w^{it},x^{it-1},x^{it})$ is the period t, industry i Törnqvist input quantity index.

Recall that in section 2, we defined cumulated counterparts to the chain link equations (42). We can do the same type of operation for the industry data. Thus define the industry i *level of total factor productivity* in period t relative to period 0 as T^{it}, the industry i *level of real output prices* in period t relative to period 0 as A^{it} and the industry i level of *primary input* in period t relative to period 0 as B^{it}. These industry levels can be defined in terms of the corresponding industry chain link factors, τ^{it},α^{it} and β^{it} as follows:

(90) $T^{i0} \equiv 1$; $T^{it} \equiv T^{it-1}\tau^{it}$; $t = 1,2,\ldots$;
(91) $A^{i0} \equiv 1$; $A^{it} \equiv A^{it-1}\alpha^{it}$; $t = 1,2,\ldots$;
(92) $B^{i0} \equiv 1$; $B^{it} \equiv B^{it-1}\beta^{it}$; $t = 1,2,\ldots$.

Since equations (86) hold as exact identities under our present assumptions, the following cumulated counterparts to these equations will also hold as exact decompositions:

(93) $p^{it}\cdot y^{it}/p^{i0}\cdot y^{i0} = \rho^{it}/\rho^{i0} = T^{it} A^{it} B^{it}$; $\qquad i = 1,\ldots,I$; $t = 1,2,\ldots$.

Thus three factors contribute to the period t level of real income generated by industry i relative to the period 0 level: the level of period t total factor productivity of industry i in period t (relative to period 0), T^{it}, the growth in real output prices for industry i going from period 0 to t, A^{it}, and the growth in primary inputs utilized by industry i going from period 0 to t, B^{it}.

The nominal value of market sector output in period t is the corresponding sum of industry nominal values, $\sum_{i=1}^{I} P^{it}\cdot y^{it}$, which can be converted into the *period t real income generated by the market sector*, ρ^t, by dividing this sum by the period t consumption price deflator, P_C^t:

(94) $\rho^t \equiv \sum_{i=1}^{I} P^{it}\cdot y^{it}/P_C^t = \sum_{i=1}^{I} p^{it}\cdot y^{it} = \sum_{i=1}^{I} \rho^{it}$; $\qquad t = 0,1,\ldots$

where the last equality follows using (84). Define industry i's share of market sector nominal (or real) net output in period 0 as

(95) $s_i^0 \equiv \rho^{i0}/\rho^0$; $\qquad i = 1,\ldots,I.$

Using the above definitions, we can decompose the growth in market sector real income, going from period 0 to t, as follows:

(96) $\rho^t/\rho^0 = [\sum_{i=1}^{I} \rho^{it}]/\rho^0$ $\qquad$ using (94)
$\qquad = \sum_{i=1}^{I} [\rho^{it}/\rho^{i0}][\rho^{i0}/\rho^{i0}]$
$\qquad = \sum_{i=1}^{I} s_i^0 [\rho^{it}/\rho^{i0}]$ $\qquad$ using (95)

$= \sum_{i=1}^{I} s_i^0 T^{it} A^{it} B^{it}$ using (93).

Equation (96) shows the factors that determine the evolution of market sector real income growth over time. There are four sets of factors at work:

- The industrial structure of net product in the base period; i.e., the base period industry shares of market sector net output, s_i^0;
- The total factor productivity performance of industry i cumulated from the base period to the current period; i.e., the industry productivity factors, T^{it};
- The growth in industry output prices (deflated by the price of the consumption aggregate) going from period 0 to t; i.e., the industry real output price factors, A^{it} and
- The growth in primary inputs utilized by industry i going from period 0 to t; i.e., the industry primary input growth factors, B^{it}.

Note that if high productivity industries absorb more primary inputs over time relative to low productivity industries, this composition effect will make a positive contribution to overall real income growth. Note also that if an industry i experiences growth in its (net) output prices relative to the price of consumption, then the corresponding real output price factor A^{it} will be greater than one and this effect will contribute to overall real income growth. It is this type of factor that is missing in traditional Total Factor Productivity decompositions; i.e., the traditional analysis ignores favourable (or unfavourable) output price effects.[34]

REFERENCES

Archibald, R.B. (1977), "On the Theory of Industrial Price Measurement: Output Price Indexes", *Annals of Economic and Social Measurement* 6, 57-72.

Balk, B.M. (1998), *Industrial Price, Quantity and Productivity Indices*, Boston: Kluwer Academic Publishers.

[34] Improvements in the country's terms of trade are also ignored by the traditional methodology. This does not mean that the traditional emphasis on pure efficiency improvements is "wrong"; it just does not answer the question that we are focusing on, which is: what is the rate of growth in consumption equivalents that the market sector of the economy is generating?

Christensen, L.R. and D.W. Jorgenson (1969), "The Measurement of U.S. Real Capital Input, 1929-1967", *Review of Income and Wealth* 15, 293–320.

Christensen, L.R., D.W. Jorgenson and L.J. Lau (1971), "Conjugate Duality and the Transcendental Logarithmic Production Function", *Econometrica* 39, 255-256.

Denison, Edward F. (1974); *Accounting for United States Economic Growth 1929-69*, Washington D.C.: The Brookings Institution.

Diewert, W.E. (1973), "Functional Forms for Profit and Transformation Functions", *Journal of Economic Theory* 6, 284-316.

Diewert, W.E., (1974), "Applications of Duality Theory," pp. 106-171 in M.D. Intriligator and D.A. Kendrick (ed.), *Frontiers of Quantitative Economics*, Vol. II, Amsterdam: North-Holland.

Diewert, W.E. (1977), "Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium", pp. 73-126 in *Equilibrium and Disequilibrium in Economic Theory*, E. Schwödiauer (ed.), Reidel Publishing Company.

Diewert, W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Diewert, W.E. (1980), "Aggregation Problems in the Measurement of Capital", pp. 433-528 in *The Measurement of Capital*, D. Usher (ed.), Chicago: The University of Chicago Press.

Diewert, W.E. (1983), "The Theory of the Output Price Index and the Measurement of Real Output Change", pp. 1049-1113 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada.

Diewert, W.E. (1993), "Symmetric Means and Choice Under Uncertainty", pp. 355-433 in *Essays in Index Number Theory, Volume I,* Contributions to Economic Analysis 217, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North Holland.

Diewert, W.E. (1997), "Commentary" on Mathew D. Shapiro and David W. Wilcox, "Alternative Strategies for Aggregating Price in the CPI", *The Federal Reserve Bank of St. Louis Review*, 79:3, 127-137.

Diewert, W.E. (2004), "Measuring Capital", Discussion Paper 04–10, Department of Economics, University of British Columbia, Vancouver, BC, Canada, V6T 1Z1.

Diewert, W.E. (2005), "On Measuring Inventory Change in Current and Constant

Dollars", Discussion Paper 05-12, Department of Economics, The University of British Columbia, Vancouver, Canada, V6T 1Z1. Available at http://www.econ.ubc.ca/diewert/disc.htm

Diewert, W.E. and C.J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade", *The Economic Journal* 96, 659-679.

Diewert, W.E. and D. Lawrence (1999), *Measuring New Zealand's Productivity*, The Treasury Working Paper 99/5, Wellington.

Diewert, W.E. and D.A. Lawrence (2000), "Progress in Measuring the Price and Quantity of Capital", pp. 273–326 in *Econometrics and the Cost of Capital: Essays in Honor of Dale W. Jorgenson*, L.J. Lau (ed.), Cambridge MA: The MIT Press.

Diewert, W.E. and D. Lawrence (2002), "The Deadweight Costs of Capital Taxation in Australia", pp. 103–167 in *Efficiency in the Public Sector*, Kevin J. Fox (ed.), Boston: Kluwer Academic Publishers.

Diewert, W.E. and D. Lawrence (2005), "Australia's Productivity Growth and the Role of Information and communications Technology: 1960-2004", Report prepared by Meyrick and Associates for the Department of Communications, Information Technology and the Arts, Canberra.

Diewert, W.E. and D. Lawrence (2006), *Measuring the Contributions of Productivity and Terms of Trade to Australia's Economic Welfare*, Report by Meyrick and Associates to the Australian Government, Productivity Commission, Canberra, Australia, 106 pp.
http://www.oecd.org/dataoecd/7/19/37503743.pdf

Diewert, W. E. and A. M. Smith (1994), "Productivity Measurement for a Distribution Firm", *The Journal of Productivity Analysis* 5, 335–347.

Edwards, E.O. and P.W. Bell (1961), *The Theory and Measurement of Business Income*, Berkeley: University of California Press.

Feenstra, R.C. (2004), *Advanced International Trade: Theory and Evidence*, Princeton N.J.: Princeton University Press.

Fisher, F.M. and K. Shell (1972), "The Pure Theory of the National Output Deflator", pp. 49-113 in *The Economic Theory of Price Indexes*, New York: Academic Press.

Fisher, I. (1922), *The Making of Index Numbers*, Houghton-Mifflin, Boston.

Fox, K.J. and U. Kohli (1998), "GDP Growth, Terms of Trade Effects and Total Factor Productivity", *Journal of International Trade and Economic Development* 7, 87-

110.

Gorman, W.M. (1968), “Measuring the Quantities of Fixed Factors”, pp. 141-172 in *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, J.N Wolfe (ed.), Chicago: Aldine Press.

Hicks, J.R. (1946), *Value and Capital*, Second Edition, Oxford: The Clarendon Press.

Hicks, J.R. (1961), “The Measurement of Capital in Relation to the Measurement of Other Economic Aggregates”, pp. 18-31 in *The Theory of Capital*, F.A. Lutz and D.C. Hague (eds.), London: Macmillan.

Hotelling, H. (1932), “Edgeworth’s Taxation Paradox and the Nature of Demand and Supply Functions”, *Journal of Political Economy* 40, 577-616.

Hulten, C.R. (1990), “The Measurement of Capital”, pp. 119–152 in *Fifty Years of Economic Measurement*, E.R. Berndt and J.E. Triplett (eds.), Studies in Income and Wealth, Volume 54, The National Bureau of Economic Research, Chicago: The University of Chicago Press.

Hulten, C.R. (1996), “Capital and Wealth in the Revised SNA”, pp. 149–181 in *The New System of National Accounts*, J.W. Kendrick (ed.), New York: Kluwer Academic Publishers.

Jorgenson, D.W. (1989), “Capital as a Factor of Production”, pp. 1–35 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge MA: The MIT Press.

Jorgenson, D.W. (1996a), “Empirical Studies of Depreciation”, *Economic Inquiry* 34, 24-42.

Jorgenson, D.W. (1996b), *Investment: Volume 2; Tax Policy and the Cost of Capital*, Cambridge, Massachusetts: The MIT Press.

Jorgenson, D.W. and Z. Griliches (1967), The Explanation of Productivity Change”, *The Review of Economic Studies* 34, 249–283.

Jorgenson, D.W. and Z. Griliches (1972), “Issues in Growth Accounting: A Reply to Edward F. Denison”, *Survey of Current Business* 52:4, Part II (May), 65–94.

Kohli, U. (1978), “A Gross National Product Function and the Derived Demand for Imports and Supply of Exports”, *Canadian Journal of Economics* 11, 167-182.

Kohli, U. (1990), “Growth Accounting in the Open Economy: Parametric and

Nonparametric Estimates", *Journal of Economic and Social Measurement* 16, 125-136.

Kohli, U. (1991), *Technology, Duality and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports*, Ann Arbor: University of Michigan Press.

Kohli, U. (2003), "Growth Accounting in the Open Economy: International Comparisons", *International Review of Economics and Finance* 12, 417-435.

Kohli, U. (2004a), "An Implicit Törnqvist Index of Real GDP", *Journal of Productivity Analysis* 21, 337-353.

Kohli, U. (2004b), "Real GDP, Real Domestic Income and Terms of Trade Changes", *Journal of International Economics* 62, 83-106.

Konüs, A.A. (1924), "The Problem of the True Index of the Cost of Living", translated in *Econometrica* 7, (1939), 10-29.

Lau, L. (1976), "A Characterization of the Normalized Restricted Profit Function", *Journal of Economic Theory*, 12:1, 131-163.

McFadden, D. (1978), "Cost, Revenue and Profit Functions", pp. 3-109 in *Production Economics: A Dual Approach to Theory and Applications*. Volume 1, M. Fuss and D. McFadden (eds.), Amsterdam: North-Holland.

Morrison, C.J. and W.E. Diewert (1990), "Productivity Growth and Changes in the Terms of Trade in Japan and the United States", pp. 201-227 in *Productivity Growth in Japan and the United States*, Chicago: University of Chicago Press.

Oulton, N. (2002), *Productivity versus Welfare: or, GDP versus Weitzman's NDP*, Working Paper, Bank of England, London.

Rymes, T.K. (1968), "Professor Read and the Measurement of Total Factor Productivity", *The Canadian Journal of Economics* 1, 359-367.

Rymes, T.K. (1983), "More on the Measurement of Total Factor Productivity", *The Review of Income and Wealth* 29 (September), 297-316.

Samuelson, P.A. (1953), "Prices of Factors and Goods in General Equilibrium', *Review of Economic Studies* 21, 1-20.

Samuelson, P.A. and S. Swamy (1974), "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis", *American Economic Review* 64, 566-593.

Sato, K. (1976), "The Meaning and Measurement of the Real Value Added Index", *Review of Economics and Statistics* 58, 434-442.

Törnqvist, L. (1936), "The Bank of Finland's Consumption Price Index", *Bank of Finland Monthly Bulletin* 10: 1-8.

Törnqvist, L. and E. Törnqvist (1937), "Vilket är förhällandet mellan finska markens och svenska kronans köpkraft?", *Ekonomiska Samfundets Tidskrift* 39, 1-39 reprinted as pp. 121-160 in *Collected Scientific Papers of Leo Törnqvist*, Helsinki: The Research Institute of the Finnish Economy, 1981.

Weitzman, M.L. (1976), "On the welfare significance of national product in a dynamic economy", *Quarterly Journal of Economics* XC, 156–62.

Weitzman, M.L. (1997), "Sustainability and technical progress", *Scandinavian Journal of Economics* 99, 1–14.

Woodland, A.D. (1982), *International Trade and Resource Allocation*, Amsterdam: North-Holland.