Export Import Price Index Manual

Chapter 19 Price Indices Using an Artificial Data Set

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A. Introduction

19.1 In order to give the reader some idea of how much the various index numbers might differ using a "real" data set, in section C below, all of the major indices defined in the previous chapters are computed using an artificial data set consisting of prices and quantities for 6 commodities over 5 periods. The period can be thought of as somewhere between a year and 5 years. The trends in the data are generally more pronounced than one would see in the course of a year. The 6 commodities can be thought of as the deliveries to the domestic final demand sector of all industries in the economy.

19.2 The data on domestic final demand deliveries are embedded in a model of production. There are three industries in the economy and in principle, each industry could produce and use combinations of the 6 final demand commodities plus an additional imported "pure" intermediate input that is not delivered to the domestic final demand sector. In section B below, the basic industry data are listed in the input output framework that was explained in Chapter 17; i.e., there are separate Supply and Use matrices for domestically produced and used commodities and for internationally traded commodities.

19.4 To summarize: price and quantity data for three industrial sectors of the economy are presented in section B. This industrial data set is consistent with the domestic final demand data set listed in section C. A wide variety of indices are computed in section C using this final demand data set.

19.5 Section D constructs domestic gross output, export, domestic intermediate input and import price indices for the aggregate production sector. Only the Laspeyres, Paasche, Fisher and Törnqvist fixed base and chained formulas are considered in section D and subsequent sections since these are the formulas that are likely to be used in practice. The data used in sections D, E and F are at producer prices; this means that basic prices are used for domestic outputs and exports and purchasers' prices are used for imports and domestic intermediate inputs.

19.6 In sections E.1-E.3, value added price deflators are constructed for each of the three industries. A national value added deflator is constructed in section E.4.

19.7 Section F compares alternative two stage methods for constructing the national value added deflator. This deflator can be constructed in a single stage by aggregating the detailed industry data (and this will be done in section E.4) or it can be constructed in two stages by either aggregating up the three industry value added deflators (see section F.1) or by aggregating up the gross output, export, intermediate input and import price indices that were constructed in section D (see section F.2). These two stage national value added deflators are compared with each other and their single stage counterpart.

19.8 Finally, in section G, final demand purchasers' prices are used in order to construct domestic final demand price indices (section G.1), export price indices (section G.2) and import price indices (section G.3). In section G.4, national GDP price deflators are constructed using final demand prices. Finally, in section G.5, the national value added deflator, which is constructed using producer prices, is compared to the national GDP deflator, which is constructed using final demand prices. This section also shows how these two national deflators can be reconciled with each other, provided that detailed industry by commodity data on commodity taxes and subsidies are available.

B. The Artificial Data Set

B.1 The Artificial Data Set Framework: Real Supply and Use Matrices

19.9 Recall the Supply and Use Tables that were defined in Chapter 17. It is useful to expand the commodity classification from one good G to four goods, G1, G2, G3 and G4, and from one service to two services, S1 and S2. The four goods are:

- G1, agricultural products or food good;
- G2, crude oil or more generally, energy products;
- G3, an imported pure intermediate good that is used by the domestic goods producing industry and
- G4, a general consumption non-energy, non-food good.

The two services are:

- S1, traditional services and
- S2, high technology services such as telecommunications and internet access.

The remaining commodity in the commodity classification is T, transportation services.

19.10 The constant dollar table counterparts to Tables 17.8-17.11 are now modified into Tables 19.1-19.4 below. The counterpart to Table 17.8 is Table 19.1. This matrix shows the production by commodity and by industry that is delivered to domestic demanders. Thus y_{G4}^{GS} denotes the quantity of good G4 that is delivered by the goods producing industry G to the services industry S, y_{G4}^{GT} denotes the quantity of good G4 that is delivered by the goods producing industry G to the transportation industry T, y_{G4}^{GF} denotes the quantity of good G4 that is delivered by the goods producing industry G to the domestic final demand sector F, y_{G1}^{SF} denotes the quantity of good G1 (food imports) delivered by the services industry S (which includes retailing and wholesaling) to the domestic final demand sector F, y_{G2}^{SF} denotes the quantity of good G2 (energy imports) delivered by the services industry S (which includes retailing and wholesaling) to the domestic final demand sector F, y_{S1}^{SG} denotes the quantity of traditional services S1 that is delivered by the services industry S to the goods producing industry G, y_{S2}^{SG} denotes the quantity of high tech services S2 that is delivered by the services industry S to the goods producing industry G, y_T^{TG} denotes the quantity of transportation services T that is delivered by the transportation industry T to the goods producing industry G, and so on.

Table 19.1: Real Domestic Supply Matrix

	Industry G	Industry S	Industry T
G1	0	y_{G1}^{SF}	0
G2	0	y_{G2}^{SF}	0
G3	0	0	0
G4	$y_{G4}^{GS}+y_{G4}^{GT}+y_{G4}^{GF}$	0	0
S1	0	$y_{S1}^{SG}+y_{S1}^{ST}+y_{S1}^{SF}$	0
S2	0	$y_{S2}^{SG}+y_{S2}^{ST}+y_{S2}^{SF}$	0
T	0	0	$y_T^{TG}+y_T^{TS}+y_T^{TF}$

19.11 Looking at the entries in Table 19.2, it can be seen that there is no domestic production of goods G1 (agricultural products) and G2 (crude oil) by Industries G and T and no domestic production of G3 (the imported intermediate good used by the goods producing industry G) by any of the industries. Industry G produces good G4 and delivers y_{G4}^{GS} units of this good to the service industry S to be used as an intermediate input there, delivers y_{G4}^{GT} units of this good to the transportation industry T to be used as an intermediate input there and delivers y_{G4}^{GF} units of this good to the domestic final demand sector F. Similarly, Industry S produces the general service commodity S1 and delivers y_{S1}^{SG} units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers y_{S1}^{ST} units of this service to the transportation industry T to be used as an intermediate input there and delivers y_{S1}^{SF} units of this service to the domestic final demand sector F. Industry S also produces the high technology service commodity S2 and delivers y_{S2}^{SG} units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers y_{S2}^{ST} units of this service to the transportation industry T to be used as an intermediate input there and delivers y_{S2}^{SF} units of this service to the domestic final demand sector F. It is also assumed that the service industry imports G1 (agricultural produce) and G2 (crude oil) and stores and distributes these imports to the household sector; these are the deliveries y_{SG1}^{SF} and y_{SG2}^{SF}.[1] Finally, Industry T produces the transportation services commodity T and delivers y_T^{TG} units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers y_T^{TS} units of these transport services to the service industry S to be used as an intermediate input there and delivers y_T^{TF} units of transport services directly to the domestic final demand sector F.

19.12 The counterpart to Table 17.9 in Chapter 17 is now Table 19.2. This matrix lists the industry demands for commodities that originate from domestic sources; i.e., it shows

[1] This is not the only way the accounts could be set up. Note that the distribution services (in distributing G1 and G2) that the domestic service industry provides in this accounting framework is on a gross basis whereas the treatment of transportation services in Industry T is on a net basis; i.e., the present setup treats transportation services as a margin industry whereas the services associated with the direct distribution of imports to households is not treated in this way. This treatment of imports makes reconciliation of the production accounts with the final demand accounts fairly straightforward.

the industry by commodity intermediate input demands for commodities that are supplied from domestic sources.

Table 19.2: Real Domestic Use Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	0	y_{G4}^{GS}	y_{G4}^{GT}
S1	y_{S1}^{SG}	0	y_{S1}^{ST}
S2	y_{S2}^{SG}	0	y_{S2}^{ST}
T	y_T^{TG}	y_T^{TS}	0

19.13 Since there is no domestic production of goods G1-G3, the rows that correspond to these commodities in Table 19.2 all have 0 entries. The remainder of the Table is the same as Table 17.9. Note that the domestic intersectoral transfers of goods and services in Tables 19.1 and 19.2 match up exactly; i.e., the 8 nonzero quantities in Table 19.2 are exactly equal to the corresponding entries in Table 19.1.

19.14 The counterpart to Table 17.10 is now Table 19.3.

Table 19.3: Real ROW Supply or Export by Industry and Commodity Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	x_{G4}^{GR}	0	0
S1	0	x_{S1}^{SR}	0
S2	0	0	0
T	0	0	x_T^{TR}

19.15 Since there is no exportation of goods G1-G3, the rows that correspond to these commodities in Table 19.3 all have 0 entries. The remainder of the Table is the same as Table 17.10. Thus Industry G exports x_{G4}^{GR} units of Good G4, Industry S exports x_{S1}^{SR} units of traditional services S1 and no units of high tech services and Industry T exports x_T^{TR} units of transportation services to the Rest of the World.

19.16 The counterpart to Table 17.11 is now Table 19.4. This matrix lists the industry demands for commodities that originate from foreign sources; i.e., it shows the industry by commodity intermediate input demands for intermediate inputs from foreign sources.

Table 19.4: Real ROW Use or Import by Industry and Commodity Matrix

	Industry G	**Industry S**	**Industry T**

G1	m_{G1}^{GR}	m_{G1}^{SR}	0
G2	m_{G2}^{GR}	m_{G2}^{SR}	m_{G2}^{TR}
G3	m_{G3}^{GR}	0	0
G4	0	0	0
S1	m_{S1}^{GR}	m_{S1}^{SR}	0
S2	0	0	0
T	0	m_T^{SR}	m_T^{TR}

19.17 From Table 19.4, it can be seen that the goods producing industry uses m_{G1}^{GR} units of agricultural imports, m_{G2}^{GR} units of crude oil imports, m_{G3}^{GR} units of a pure imported intermediate good and m_{S1}^{GR} units of imported service inputs. Industry G does not import the domestically produced good, G4, nor does it import transportation services in this simplified example. Industry S imports m_{G1}^{SR} units of agricultural goods (for distribution to domestic households), m_{G2}^{sR} units of crude oil (for distribution to households and own use), m_{S1}^{GR} units of foreign general services and m_T^{SR} units of foreign transportation services. Industry T imports m_{G2}^{TR} units of crude oil and m_T^{TR} units of foreign sourced transportation services.

B.2 The Artificial Data Set Framework: Value Supply and Use Matrices

19.18 The value matrix counterparts to the two Supply and two Use matrices listed in section B.1 above will now be listed in the present section. Table 19.5 listed below is the counterpart to Table 17.5 listed in Chapter 17.

Table 19.5: Nominal Value Domestic Supply Matrix with Commodity Taxes

	Industry G	**Industry S**	**Industry T**
G1	0	$(p_{G1}^{SF}-t_{G1}^{SF})y_{G1}^{SF}$	0
G2	0	$(p_{G2}^{SF}-t_{G2}^{SF})y_{G2}^{SF}$	0
G3	0	0	0
G4	$(p_{G4}^{GS}-t_{G4}^{GS})y_{G4}^{GS}$ $+(p_{G4}^{GT}-t_{G4}^{GT})y_{G4}^{GT}$ $+(p_{G4}^{GF}-t_{G4}^{GF})y_{G4}^{GF}$	0	0
S1	0	$(p_{S1}^{SG}-t_{S1}^{SG})y_{S1}^{SG}$ $+(p_{S1}^{ST}-t_{S1}^{ST})y_{S1}^{ST}$ $+(p_{S1}^{SF}-t_{S1}^{SF})y_{S1}^{SF}$	0
S2	0	$(p_{S2}^{SG}-t_{S2}^{SG})y_{S2}^{SG}$ $+(p_{S2}^{ST}-t_{S2}^{ST})y_{S2}^{ST}$ $+(p_{S2}^{SF}-t_{S2}^{SF})y_{S2}^{SF}$	0
T	0	0	$(p_T^{TG}-t_T^{TG})y_T^{TG}$ $+(p_T^{TS}-t_T^{TS})y_T^{TS}$ $+(p_T^{TF}-t_T^{TF})y_T^{TF}$

19.19 All of the prices which begin with the letter p are the prices that *domestic final demanders* pay for a unit of the commodity (except for minor complications with respect to the treatment of export prices). In the above Table, these prices correspond to *purchasers' prices* in the *System of National Accounts 1993*.[2] However, the industry sellers of these commodities do not generally receive the full final demand price: commodity taxes less commodity subsidies must be subtracted from these final demand prices in order to obtain the net prices that are listed in the above Table. These *net selling prices* are the prices that the industrial producers actually receive for their sales of outputs to domestic demanders. In the above Table, these prices correspond to *basic prices* in *SNA 1993*.[3] The notation used for prices in Table 19.5 matches the notation used for quantities in Table 19.1.

19.20 The reader should note that in this chapter, for convenience, the p prices will be referred to as *final demand prices* and the p−t prices will be referred to as *producer prices*. Conceptually, the *final demand prices* are the prices that domestic final demanders pay per unit for their purchases of commodities delivered to final demand categories. However, for an exported commodity, the final demand price is not the total purchase price (including transportation services provided by foreign establishments, import duties and other applicable commodity taxes) that the foreign importer pays for the commodity; rather, in this case, the final demand price is only the payment made to the domestic producer by the foreign importer. Conceptually, *producer prices* are the prices that domestic producers receive per unit of output produced that is sold or the prices that domestic producers pay per unit of input that is purchased (including applicable commodity taxes and all transportation margins).[4]

19.21 Table 19.6 listed below is the counterpart to Table 17.2 listed in Chapter 17. It is also the value counterpart to Table 19.2 listed above.

Table 19.6: Nominal Value Domestic Use Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	0	$p_{G4}^{GS}y_{G4}^{GS}$	$p_{G4}^{GT}y_{G4}^{GT}$
S1	$p_{S1}^{SG}y_{S1}^{SG}$	0	$p_{S1}^{ST}y_{S1}^{ST}$

[2] See part (a) of paragraph 15.28 in *SNA 1993*.

[3] See paragraphs 15.28-15.33 in *SNA 1993*. Note that the tax terms in Tables 19.5-19.8 are equal to per unit (or specific) commodity taxes less per unit commodity subsidies. These two effects could be distinguished separately at the cost of additional notational complexity.

[4] If the production accounts are to be used in order to measure Total Factor Productivity growth using the economic approach suggested by Jorgenson and Griliches (1967) (1972), it is important to use the prices that producers face in the accounting framework. The treatment of commodity taxes suggested in this Manual is consistent with the treatment suggested by Jorgenson and Griliches who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output." Dale W. Jorgenson and Zvi Griliches (1972; 85).

S2	$p_{S2}^{SG}y_{S2}^{SG}$	0	$p_{S2}^{ST}y_{S2}^{ST}$
T	$p_T^{TG}y_T^{TG}$	$p_T^{TS}y_T^{TS}$	0

19.22 Note that in Table 19.5, Industry G receives only the revenue $(p_{G4}^{GS}-t_{G4}^{GS})y_{G4}^{GS}$ for its sales of Commodity G4 to Industry S, whereas in Table 19.6, Industry S pays the amount $p_{G4}^{GS}y_{G4}^{GS}$ for these purchases of intermediate inputs from Industry G. The difference between these two intersectoral value flows is $t_{G4}^{GS}y_{G4}^{GS}$, the tax (less subsidy) payments made by Industry G to the government on this intersectoral value flow. Thus the values of domestic intersectoral transfers of goods and services in Tables 19.5 and 19.6 do not match up exactly unless the commodity tax less subsidy terms t_{S1}^{SG}, t_{G4}^{GS} and so on are all zero.

19.23 The counterpart to Table 17.6 in Chapter 17 is now Table 19.7, which in turn is the value counterpart to the real Table 19.3.

Table 19.7: Value ROW Supply or Export by Industry and Commodity Matrix

	Industry G	Industry S	Industry T
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	$(p_{G4x}^{GR}-t_{G4x}^{GR})x_{G4}^{GR}$	0	0
S1	0	$p_{S1x}^{SR}x_{S1}^{SR}$	0
S2	0	0	0
T	0	0	$p_{Tx}^{TR}x_T^{TR}$

19.24 Since there is no exportation of goods G1-G3, the rows that correspond to these commodities in Table 19.7 all have 0 entries. The remainder of the Table is straightforward. Thus Industry G exports x_{G4}^{GR} units of Good G4,, the foreign final demander pays the price p_{G4x}^{GR} per unit but the exporting industry gets only the amount $p_{G4x}^{GR}-t_{G4x}^{GR}$ per unit; i.e., the government gets the per unit (net) revenue t_{G4x}^{GR} on these sales if it imposes a (net) export tax equal to t_{G4x}^{GR}. Similarly, net export taxes (if applicable) must be subtracted from the final demand prices for the other industries. In the numerical example which follows, it will be assumed that the net export tax in Industry G is negative (so that exports are subsidized in industry G) and it will be assumed that taxes in Industries S and T are zero.

19.25 The counterpart to Table 17.7 in Chapter 17 is now Table 19.8, which in turn is the value counterpart to the real Table 19.4.

Table 19.8: Value ROW Use or Import by Industry and Commodity Matrix

	Industry G	Industry S	Industry T
G1	$(p_{G1m}^{GR}+t_{G1m}^{GR})m_{G1}^{GR}$	$(p_{G1m}^{SR}+t_{G1m}^{SR})m_{G1}^{SR}$	0
G2	$(p_{G2m}^{GR}+t_{G2m}^{GR})m_{G2}^{GR}$	$(p_{G2m}^{SR}+t_{G2m}^{SR})m_{G2}^{SR}$	$(p_{G2m}^{TR}+t_{G2m}^{TR})m_{G2}^{TR}$
G3	$(p_{G3m}^{GR}+t_{G3m}^{GR})m_{G3}^{GR}$	0	0

G4	0	0	0
S1	$(p_{S1m}{}^{GR}+t_{S1m}{}^{GR})m_{S1}{}^{GR}$	$(p_{S1m}{}^{SR}+t_{S1m}{}^{SR})m_{S1}{}^{SR}$	0
S2	0	0	0
T	0	$(p_{Tm}{}^{SR}+t_{Tm}{}^{SR})m_T{}^{SR}$	$(p_{Tm}{}^{TR}+t_{Tm}{}^{TR})m_T{}^{TR}$

19.26 It should be straightforward for the reader to interpret the final demand prices (these terms begin with p) and the accompanying import duties, excise duties and other commodity taxes on imports (these terms begin with t). The quantities of imports (these terms begin with an m) are the same as the quantity terms in the corresponding real table, Table 19.4. From a practical point of view, governments have a tendency to tax imports (so that the tax terms in this table will tend to be positive) and to subsidize exports (so that the tax terms in the previous table will tend to be 0 or negative).

B.4 Industry G Prices and Quantities

19.27 All of the price and quantity series that will be used in this chapter are listed in the four nominal value Supply and Use matrices that are listed in Tables 19.5-19.8. The eleven final demand price series that form part of the Industry G data in these matrices are listed for 5 periods in Table 19.9. The commodity that the price refers to is listed in the first row of the Table.

Table 19.9 Industry G Final Demand Prices for All Transactions

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
	$p_{G4}{}^{GS}$	$p_{G4}{}^{GT}$	$p_{G4}{}^{GF}$	$p_{S1}{}^{SG}$	$p_{S2}{}^{SG}$	$p_T{}^{TG}$	$p_{G4x}{}^{GR}$	$p_{G1m}{}^{GR}$	$p_{G2m}{}^{GR}$	$p_{G3m}{}^{GR}$	$p_{S1m}{}^{GR}$
1	0.9	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	1.2	1.3	1.5	0.8	1.6	1.3	1.4	2.0	0.8	1.4
3	1.5	1.5	1.6	1.8	0.6	1.5	1.6	0.9	1.5	0.6	1.7
4	1.55	1.55	1.65	1.9	0.4	1.3	1.5	1.3	0.9	0.4	1.8
5	1.6	1.6	1.7	2.0	0.2	1.8	1.4	1.5	2.1	0.3	1.9

19.28 Some points to note about the price entries in Table 19.9 are as follows. The Industry G final demand prices that it faces for deliveries of Commodity G4 to the service industry, $p_{G4}{}^{GS}$, to the transportation services industry, $p_{G4}{}^{GT}$, and for exports, $p_{G4x}{}^{GR}$, are all much the same: prices trend up fairly rapidly for periods 2 and 3 and then level off for periods 4 and 5. However, the final demand price for deliveries of G4 to the domestic final demand sector F, $p_{G4}{}^{GF}$, is somewhat higher than the corresponding prices for deliveries of G4 to the service sector S, $p_{G4}{}^{GS}$, and to the transportation sector T, $p_{G4}{}^{GT}$, due to higher commodity taxes on deliveries to sector F. The price of traditional domestic services used as an intermediate input by Industry G, $p_{S1}{}^{SG}$, also increases rapidly initially and then levels off for the last two periods. However, the price of high tech domestic services used as an intermediate input by Industry G, $p_{S2}{}^{SG}$, drops rapidly throughout the sample period. The price of transportation services used as an intermediate input by Industry G, $p_T{}^{TG}$, increases dramatically in period 2 due to the increase the price of imported oil and then decreases for the next two periods as the price of oil drops before increasing again in period 5. The price of agricultural imports into

Industry G, p_{G1m}^{GR}, fluctuates considerably from period to period but overall, agricultural import prices do not increase as rapidly as many other prices. The price of oil imports into Industry G, p_{G2m}^{GR}, fluctuates violently, doubling in period 2, then falling so that by period 4, the price is below the period 1 price but then the price more than doubles for period 5. The price of the imported intermediate good, p_{G3m}^{GR}, steadily drops at a rapid pace over the 5 periods.[5] Finally, the price of the imported services commodity, p_{Sm}^{GR}, increases rapidly over periods 2 and 3 but then the rate of price increase slows down. Over the entire period, the price of services tends to increase somewhat more rapidly than the price of manufactured output, G4.

19.29 The eleven commodity tax series that form part of the Industry G taxes listed in Tables 19.5-19.8 are listed for 5 periods in Table 19.10. Recall that by convention, the selling industry pays all commodity taxes so the taxes on Industry G's purchases of intermediate inputs from Industries S and T, t_{S1}^{SG}, t_{S2}^{SG} and t_T^{TG}, are all identically equal to zero. However, in Table 19.10, these tax rates are listed (with 0 entries) so that Table 19.10 is dimensionally comparable to Table 19.9.

Table 19.10 Industry G Commodity Taxes

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
Period	t_{G4}^{GS}	t_{G4}^{GT}	t_{G4}^{GF}	t_{S1}^{SG}	t_{S2}^{SG}	t_T^{TG}	t_{G4x}^{GR}	t_{G1m}^{GR}	t_{G2m}^{GR}	t_{G3m}^{GR}	t_{S1m}^{GR}
1	0.05	0.05	0.10	0	0	0	−0.10	0.11	0.15	0.10	0.05
2	0.07	0.07	0.15	0	0	0	−0.13	0.14	0.20	0.08	0.07
3	0.08	0.08	0.20	0	0	0	−0.16	0.09	0.25	0.06	0.08
4	0.08	0.08	0.22	0	0	0	−0.15	0.13	0.20	0.04	0.09
5	0.09	0.09	0.23	0	0	0	−0.05	0.15	0.25	0.03	0.10

19.30 Note that the taxes listed above are all positive or zero except that the exports of good G4 by Industry G are subsidized so the taxes t_{G4x}^{GR} have a negative sign attached to them instead of the usual positive sign.

19.31 The eleven quantity series that form part of the Industry G data in Tables 19.5-19.8 are listed for 5 periods in Table 19.11.

Table 19.11 Industry G Quantities of Outputs and Intermediate Inputs

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
	y_{G4}^{GS}	y_{G4}^{GT}	y_{G4}^{GF}	y_{S1}^{SG}	y_{S2}^{SG}	y_T^{TG}	x_{G4x}^{GR}	m_{G1m}^{GR}	m_{G2m}^{GR}	m_{G3m}^{GR}	m_{S1m}^{GR}
1	5	2	35	4	2	3	25	5	10	10	2
2	6	2.5	40	5	4	3.5	28	6	12	13	2
3	7	3	45	6	8	4	32	7	15	19	3
4	7	3.5	49	8	14	5	40	7.5	18	25	4
5	8	4	55	10	20	6	54	8	15	35	6

[5] Think of this pure imported intermediate as being high tech equipment, which has been dropping in price due to the computer chip revolution.

19.32 The quantities of good G4 produced by Industry G, y_{G4}^{GS}, y_{G4}^{GT}, y_{G4}^{GF}, which are deliveries to the domestic services industry, the domestic transportation industry and the domestic final demand sector respectively, all grow at roughly the same rate. However, the quantities of G4 exported by Industry G, x_{G4x}^{GR}, grow a bit more rapidly, particularly during the final two periods. The quantity of traditional domestic services used as an intermediate input by Industry G, y_{S1}^{SG}, more than doubles over the 5 periods but the quantity of high tech services used as an intermediate input, y_{S1}^{SG}, grows tenfold due to the rapid price drop in this commodity. The quantity of domestic transportation services used as an intermediate input by Industry G, y_{T}^{TG}, exactly doubles over the 5 periods. The quantity of agricultural imports used by Industry G, m_{G1m}^{GR}, increases steadily from 5 units to 8 units while the quantity of oil imports increases from 10 in period 1 to 15 in period 3 but then the growth rate slows over the final two periods. Imports of the high technology pure intermediate imported good, m_{G3m}^{GR}, increase rapidly from 10 to 35 units, reflecting the real world tendency towards globalization and international outsourcing. Finally, imports of service inputs into Industry G increase rapidly, growing from 2 units in period 1 to 6 units in period 5.

B.5 Industry S Prices and Quantities

19.33 The fifteen final demand price series that form part of the Industry S data in Tables 19.5-19.8 are listed for 5 periods in Table 19.9.

Table 19.12 Industry S Final Demand Prices

	G1	G2	S1	S1	S1	S2	S2	S2
Period	p_{G1}^{SF}	p_{G2}^{SF}	p_{S1}^{SG}	p_{S1}^{ST}	p_{S1}^{SF}	p_{S2}^{SG}	p_{S2}^{ST}	p_{S2}^{SF}
1	1.2	1.4	1.0	1.0	1.3	1.0	1.0	1.15
2	1.5	2.8	1.5	1.4	1.8	0.8	0.8	0.94
3	1.2	2.2	1.8	1.7	2.2	0.6	0.6	0.72
4	1.6	1.5	1.9	1.8	2.4	0.4	0.4	0.45
5	1.7	3.0	2.0	1.9	2.6	0.2	0.2	0.23

	G4	T	S1	G1	G2	S1	T
Period	p_{G4}^{GS}	p_{T}^{TS}	p_{S1x}^{SR}	p_{G1m}^{SR}	p_{G2m}^{SR}	p_{S1m}^{SR}	p_{Tm}^{SR}
1	0.9	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	1.6	1.3	1.3	2.1	1.3	1.6
3	1.5	1.5	1.6	1.0	1.6	1.6	1.5
4	1.55	1.3	1.5	1.4	1.1	1.7	1.3
5	1.6	1.8	1.4	1.5	2.2	1.7	1.8

19.34 Some points to note about the price entries in Table 19.12 are as follows. The prices of service sector deliveries to Industry G, p_{S1}^{SG} and p_{S2}^{SG}, and the prices of deliveries of good G4 from Industry G to Industry S, p_{G4}^{GS}, are exactly the same in Tables 19.9 and 19.12. This reflects the bilateral nature of transactions between sectors. Industry S sells Commodities S1 and S2 to Industries G and T (these are the prices p_{S1}^{SG}

and p_{S2}^{SG} for sales to Industry G and p_{S1}^{ST} and p_{S2}^{ST} for sales to Industry T) and it sells commodities S1 and S2 to the domestic Final Demand sector F at prices p_{S1}^{SF} and p_{S2}^{SF} and it sells S1 to the Rest of the World R as an export at the price p_{S1x}^{SR}. The Industry S final demand selling prices are much the same over these 4 destinations, except that export price for S1 falls off somewhat and the selling prices to the domestic Final Demand Sector for the high technology service S2 are somewhat higher, reflecting a higher level of final demand taxation. Industry S also imports G1 (agricultural or food imports for resale to domestic households), G2 (oil imports for resale to domestic households) and it also imports some foreign general services S1 and some foreign transportation services T. These import prices are p_{G1m}^{SR}, p_{G2m}^{SR}, p_{S1m}^{SR} and p_{Tm}^{SR} respectively. The import prices for these first 3 classes of imports are much the same as the corresponding import prices that applied to the imports of these commodities by Industry G. The price of imported transportation services, p_{Tm}^{SR}, is the same as the price of domestic transportation services provided to Industry S, p_T^{TS}. Note that the service sector selling prices of Goods G1 and G2 to the domestic final demand sector, p_{G1}^{SF} and p_{G2}^{SF}, are somewhat higher than the corresponding import purchase prices for these good, p_{G1m}^{SR} and p_{G2}^{SF}, but this is natural: the service sector must make a positive margin on its trading in these commodities in order to cover the costs of storage and distribution.

19.35 The Service Industry obviously contains elements of the traditional storage, wholesaling and retailing industries. The treatment of these industries that is followed in the artificial data example is a *gross output treatment* as opposed to a *margin industry treatment*. In the gross output treatment, goods for resale are purchased and the full purchase price times the amount purchased appears as an intermediate input cost and then the goods are sold subsequently at a higher price and this selling price times the amount sold appears as a contribution to gross output. In the margin treatment, it is assumed that the amount sold during the accounting period is at least roughly equal to the amount purchased, and the difference between the selling price and the purchase price (the margin) is multiplied by the amount bought and sold and is treated as a gross output with no corresponding intermediate input cost. Thus for the case of an imported good, if the margin treatment of wholesaling/retailing/storage (WRS) output is used, the margin would be credited to this WRS industry and the full import price plus the margin would appear as an intermediate input by the purchasing industry (or final demand sector). Thus the margin treatment of the WRS industry would be similar to the margin treatment that has been accorded to the transportation industry. However, there is a difference between the WRS industry and the transportation industry: for the transportation industry, one can be fairly certain that the goods "purchased" by the transport industry are equal to the goods "sold" by the industry and the margin treatment is perfectly justified. This is not necessarily the case for the WRS industry: sales are not necessarily equal to purchases in any given accounting period. Thus it seems preferable to use the gross output treatment for these distributive industries over the margin approach, although individual countries may feel that sales are sufficiently close to purchases so that the margin approach is a reasonable approximation to the actual situation and hence can be used in their national accounts.[6]

19.36 The fifteen commodity tax series that form part of the Industry S taxes listed in Tables 19.5-19.8 are listed for 5 periods in Table 19.13.

Table 19.13 Industry S Commodity Taxes

Commodity	G1	G2	S1	S1	S1	S2	S2	S2
	t_{G1}^{SF}	t_{G2}^{SF}	t_{S1}^{SG}	t_{S1}^{ST}	t_{S1}^{SF}	t_{S2}^{SG}	t_{S2}^{ST}	t_{S2}^{SF}
1	0.02	0.17	0.01	0.01	0.10	0.15	0.15	0.30
2	0.05	0.23	0.02	0.02	0.15	0.11	0.11	0.25
3	0.02	0.19	0.03	0.03	0.18	0.08	0.08	0.20
4	0.06	0.17	0.03	0.03	0.19	0.05	0.05	0.10
5	0.07	0.24	0.03	0.02	0.20	0.02	0.02	0.05

G4	T	S1	G1	G2	S1	T
t_{G4}^{GS}	t_{T}^{TS}	t_{S1x}^{SR}	t_{G1m}^{SR}	t_{G2m}^{SR}	t_{S1m}^{SR}	t_{Tm}^{SR}
0	0	0	0.02	0.15	0.05	0.03
0	0	0	0.03	0.20	0.06	0.04
0	0	0	0.04	0.25	0.09	0.04
0	0	0	0.04	0.20	0.09	0.03
0	0	0	0.04	0.25	0.10	0.03

19.37 Note that the tax rates on domestic intermediate inputs used by Industry S are all set equal to zero under the convention used in this chapter that the selling industry pays any applicable commodity taxes.[7]

19.38 The fifteen quantity series that form part of the Industry S data in Tables 19.5-19.8 are listed for 5 periods in Table 19.14.

Table 19.14 Industry S Quantities of Outputs and Inputs

	G1	G2	S1	S1	S1	S2	S2	S2
	y_{G1}^{SF}	y_{G2}^{SF}	y_{S1}^{SG}	y_{S1}^{ST}	y_{S1}^{SF}	y_{S2}^{SG}	y_{S2}^{ST}	y_{S2}^{SF}
1	10	8	4	2.0	15	2	1.1	3.0
2	11	9	5	2.5	20	4	1.5	4.3
3	12	9	6	3.0	25	8	2.1	6.5
4	13	10	8	3.5	33	14	3.5	10.5

[6] For theoretical treatments of the accounting problems associated with measuring the contribution of inventories to retailing and wholesaling production, see paragraphs 6.57-6.79 of *SNA 1993*, Diewert and Smith (1994), Ehemann (2005), Diewert (2005) and Hill (2005).

[7] The selling industry also receives any applicable commodity subsidies.

5	14	11	10	3.5	40	20	5.0	15.0

G4	T	S1	G1	G2	S1	T
$y_{G4}{}^{GS}$	$y_T{}^{TS}$	$x_{S1x}{}^{SR}$	$m_{G1m}{}^{SR}$	$m_{G2m}{}^{SR}$	$m_{S1m}{}^{SR}$	$m_{Tm}{}^{SR}$
5	1.0	14	10	10	3	1.0
6	1.1	19	11	11	4	1.5
7	1.2	24	12	11	6	1.7
7	1.3	31	13	12	9	1.9
8	1.3	42	14	13	13	2.0

19.39 The quantities of Industry S deliveries to Industry G, $y_{S1}{}^{SG}$ and $y_{S2}{}^{SG}$, and the quantities of deliveries of good G4 from Industry G to Industry S, $y_{G4}{}^{GS}$, are exactly the same in Tables 19.11 and 19.14.

19.40 Note that $y_{G1}{}^{SF}$, the quantity of imported food G1 sold by Industry S to domestic final demanders F, is exactly equal to $m_{G1m}{}^{SR}$, imports of food into Industry S. However, $y_{G2}{}^{SF}$, the quantity of imported energy products G2 sold by Industry S to domestic final demanders, is less than the quantity of energy imported by Industry S, $m_{G2m}{}^{SR}$. The reason for this difference is that Industry S uses some of the imported energy for heat and other purposes as it supplies services to other sectors of the economy. Sales by Industry S of traditional services S1 to Industry G, Industry T, domestic final demand F and to the rest of the world R, $y_{S1}{}^{SG}$, $y_{S1}{}^{ST}$, $y_{S1}{}^{SF}$ and $x_{S1x}{}^{SR}$ respectively, all increase quite rapidly, doubling or tripling over the 5 periods. Imports of traditional services S1 into Industry S, $m_{S1m}{}^{SR}$, increase even more rapidly, growing from 3 to 13 over the 5 periods. The sales of high tech services by Industry S to Industries G and T and to the domestic final demand sector F, $y_{S2}{}^{SG}$, $y_{S2}{}^{ST}$ and $y_{S2}{}^{SF}$ respectively, all increase very rapidly, growing between 5 and 10 fold over the 5 periods. The quantities of domestic intermediate inputs of good G4, $y_{G4}{}^{GS}$, and of the transportation service, $y_T{}^{TS}$, used by Industry S grew fairly slowly over the 5 periods. Imported transportation services, $m_{Tm}{}^{SR}$, associated with the importation of G1 and G2 by Industry S, doubled over the 5 periods.

B.6 Industry T Prices and Quantities

19.41 The nine final demand price series that form part of the Industry T data in Tables 19.5-19.8 are listed for 5 periods in Table 19.15.

Table 19.15 Industry T Final Demand Prices

Period	T	T	T	G4	S1	S2	T	G2	T
	$p_T{}^{TG}$	$p_T{}^{TS}$	$p_T{}^{TF}$	$p_{G4}{}^{GT}$	$p_{S1}{}^{ST}$	$p_{S2}{}^{ST}$	$p_{Tx}{}^{TR}$	$p_{G2m}{}^{TR}$	$p_{Tm}{}^{TR}$
1	1.0	1.0	1.2	0.9	1.0	1.0	1.1	1.0	1.0
2	1.6	1.6	1.8	1.2	1.4	0.8	1.7	2.1	1.6
3	1.5	1.5	1.7	1.5	1.7	0.6	1.5	1.6	1.4
4	1.3	1.3	1.6	1.55	1.8	0.4	1.3	1.1	1.2
5	1.8	1.8	2.2	1.6	1.9	0.2	1.8	2.2	1.8

The entries for p_T^{TS}, p_{S1}^{ST} and p_{S2}^{ST} in Tables 19.15 and 19.12 are the same series as are the entries for p_T^{TG} and p_{G4}^{GT} in Tables 19.15 and 19.9. Again, this reflects the fact that the sellers and purchasers of domestic intermediate inputs pay and receive the same amounts of money for their cross industry purchases and sales.

19.42 The industry selling prices for transportation services shows much the same trends across all destinations. The selling prices of transportation services to domestic final demand, p_T^{TF}, are higher than the other selling prices due to higher commodity taxation for deliveries to final demand.

19.43 The commodity tax series that form part of the Industry T taxes listed in Tables 19.5-19.8 are listed for 5 periods in Table 19.16.

Table 19.16 Industry T Commodity Taxes

	T	T	T	G4	S1	S2	T	G	T
Period	t_T^{TG}	t_T^{TS}	t_T^{TF}	t_{G4}^{GT}	t_{S1}^{ST}	t_{S2}^{ST}	t_{Tx}^{TR}	t_{G2m}^{TR}	t_{Tm}^{TR}
1	0.01	0.01	0.10	0	0	0	0	0.15	0.03
2	0.02	0.02	0.15	0	0	0	0	0.20	0.04
3	0.03	0.03	0.18	0	0	0	0	0.25	0.04
4	0.03	0.03	0.19	0	0	0	0	0.20	0.03
5	0.03	0.03	0.20	0	0	0	0	0.25	0.03

19.44 The commodity taxes on deliveries of transportation services to Industries G and S, t_T^{TG} and t_T^{TS} respectively, are small but the taxes on deliveries to the final demand sector, t_T^{TF}, are fairly substantial, as are the taxes on the transportation sector's imports of energy products, t_{G2m}^{TR}. By convention, any taxes on Industry T's use of domestic intermediate inputs are paid by the selling industry so t_{G4}^{GT}, t_{S1}^{ST} and t_{S2}^{ST} are all 0. There are no taxes on the export of transportation services in this economy so that t_{Tx}^{TR} is 0 as well. There are small taxes on Industry T's importation of transport services, t_{Tm}^{TR}.

19.45 The nine final demand quantity series that form part of the Industry T data in Tables 19.5-19.8 are listed for 5 periods in Table 19.17.

Table 19.17 Industry T Quantities of Outputs and Inputs

	T	T	T	G4	S1	S2	T	G2	T
Period	y_T^{TG}	y_T^{TS}	y_T^{TF}	y_{G4}^{GT}	y_{S1}^{ST}	y_{S2}^{ST}	x_{Tx}^{TR}	m_{G2m}^{TR}	m_{Tm}^{TR}
1	3	1.0	5	2	2.0	1.1	3	3	1.5
2	3.5	1.1	5	2.5	2.5	1.5	4	3	1.7
3	4	1.2	6	3	3.0	2.1	5	3.5	2.2
4	5	1.3	7	3.5	3.5	3.5	5.5	4	2.4
5	6	1.3	7	4	3.5	5.0	6	4.5	2.5

19.46 The entries for y_T^{TS}, y_{S1}^{ST} and y_{S2}^{ST} in Tables 19.17 and 19.14 are the same series as are the entries for y_T^{TG} and y_{G4}^{GT} in Tables 19.17 and 19.11. All transportation service inputs and outputs grow relatively smoothly and roughly double over the 5 periods.

19.47 This completes the listing of the basic price, tax and quantity data that will be used in subsequent sections of this chapter in order to illustrate how various index number formulae differ and how consistent sets of producer price indices can be formed in a set of production accounts that are roughly equivalent to the production accounts that are described in Chapter 15 of *SNA 1993*.

C. The Artificial Data Set for Domestic Final Demand

C.1 The Final Demand Data Set

19.48 In order to illustrate what kind of differences can result from the choice of different index number formulae, the price and quantity data that correspond to domestic deliveries to final demand that were listed in the previous section are used as a test data set in this section. The 6 final demand price series are listed in Table 19.18 and the corresponding quantity series are listed in Table 19.19.

Table 19.18 Prices for Six Domestic Final Demand Commodities

	G1	G2	G4	S1	S2	T
	Food	Energy	Goods	Services	High Tech Ser	Transport
Period t	p_1^t	p_2^t	p_3^t	p_4^t	p_5^t	p_6^t
1	1.2	1.4	1.0	1.3	1.15	1.2
2	1.5	2.8	1.3	1.8	0.94	1.8
3	1.2	2.2	1.6	2.2	0.72	1.7
4	1.6	1.5	1.65	2.4	0.45	1.6
5	1.7	3.0	1.7	2.6	0.23	2.2

19.49 The prices p_1^t, p_2^t, p_3^t, p_4^t, p_5^t and p_6^t in Table 19.19 correspond to the final demand prices p_{G1}^{SF}, p_{G2}^{SF}, p_{G4}^{GF}, p_{S1}^{SF}, p_{S2}^{SF} and p_T^{TF} respectively, which are listed in Tables 19.9, 19.12 and 19.15.

Table 19.19 Quantities for Six Domestic Final Demand Commodities

	G1	G2	G4	S1	S2	T
	Food	Energy	Goods	Services	High Tech Ser	Transport
Period t	q_1^t	q_2^t	q_3^t	q_4^t	q_5^t	q_6^t
1	10	8	35	15	3.0	5
2	11	9	40	20	4.3	5
3	12	9	45	25	6.5	6
4	13	10	49	33	10.5	7
5	14	11	55	40	15.0	7

19.50 The quantities q_1^t, q_2^t, q_3^t, q_4^t, q_5^t and q_6^t in Table 19.19 correspond to the final demand quantities y_{G1}^{SF}, y_{G2}^{SF}, y_{G4}^{GF}, y_{S1}^{SF}, y_{S2}^{SF} and y_T^{TF} respectively, which are listed in Tables 19.11, 19.14 and 19.17.

19.51 It is useful to also list the period t expenditures on all six domestic finally demanded commodities, $p^t \cdot q^t$, along with the corresponding expenditure shares, s_1^t, ... , s_6^t; see Table 19.20.

Table 19.20 Total Expenditures and Expenditure Shares for Six Domestic Final Demand Commodities

	Expenditures	**Food**	**Energy**	**Goods**	**Services**	**H.T. Services**	**Transport Services**
Period t	$p^t \cdot q^t$	s_1^t	s_2^t	s_3^t	s_4^t	s_5^t	s_6^t
1	87.150	0.1377	0.1285	0.4016	0.2238	0.0396	0.0688
2	142.742	0.1156	0.1765	0.3643	0.2522	0.0283	0.0631
3	176.080	0.0818	0.1124	0.4089	0.3124	0.0266	0.0579
4	211.775	0.0982	0.0708	0.3818	0.3740	0.0223	0.0529
5	273.150	0.0871	0.1208	0.3423	0.3807	0.0126	0.0564

19.52 The expenditure shares for food, goods and high tech services decrease markedly over the 5 periods, the share for transport services decreases somewhat, the share of energy stays roughly constant but with substantial period to period fluctuations and the share of general services increases substantially.

19.53 Note that the price of food and energy fluctuates considerably from period to period but the quantities demanded tend to trend upwards at a fairly smooth rate, reflecting the low elasticity of price demand for these products. The fluctuations in energy prices tends to produce similar fluctuations in the price of domestic transportation services but the fluctuations in price are more damped in the case of transport services. The price of goods tends up fairly rapidly in periods 2 and 3 but then the rate of increase falls off. The corresponding quantity trends upwards fairly steadily. The price of traditional services, p_4^t, increases rapidly in periods 2 and 3 and then increases more slowly. Overall, the price of traditional services increases more rapidly than the price of goods but the quantity of services demanded q_4^t increases more rapidly than the quantity of goods, q_3^t. The price of high technology services, p_5^t, decreases rapidly over the 5 periods, falling to about 1/5 of the initial price level. The corresponding quantities demanded, q_5^t, increase rapidly, increasing five fold over the sample period. Thus overall, the data set exhibits a rather wide variety of trends in prices and quantities but these trends are not unrealistic in today's world economy. The movements of prices and quantities in this artificial data set are more pronounced than the year to year movements that would be encountered in a typical country but they do illustrate the problem that is facing compilers of producer and consumer price indices: namely, *year to year price and quantity movements are far from being proportional across commodities so the choice of index number formula will matter*.

C.2 Some Familiar Index Number Formulae

19.54 Every price statistician is familiar with the *Laspeyres index*, P_L, and the *Paasche index*, P_P, defined in Chapter 15 above. These indices are listed in Table 19.21 along with the two main unweighted indices that were considered in Chapters 15 and 16: the *Carli index* and the *Jevons index*. The indices in Table 19.20 compare the prices in period *t* with the prices in period 1; i.e., they are *fixed base indices*. Thus the period *t* entry for the Carli index, P_C, is simply the arithmetic mean of the 8 price relatives, $\sum_{i=1}^{6} (1/6)(p_i^t/p_i^1)$, while the period *t* entry for the Jevons index, P_J, is the geometric mean of the 6 long term price relatives, $\prod_{i=1}^{6} (p_i^t/p_i^1)^{1/6}$.

Table 19.21 The Fixed Base Laspeyres, Paasche, Carli and Jevons Indices

Period t	P_L^t	P_P^t	P_C^t	P_J^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3753	1.3293
3	1.4832	1.4775	1.3177	1.2478
4	1.5043	1.4916	1.2709	1.1464
5	1.7348	1.6570	1.5488	1.2483

19.55 Note that by period 5, the spread between the fixed base Laspeyres and Paasche price indices is not negligible: P_L is equal to 1.7348 while P_P is 1.6570*, a spread of about 4.7 percent*. Since both of these indices have exactly the same *theoretical* justification, it can be seen that the choice of index number formula matters. There is also a substantial spread between the two unweighted indices by period 5: the fixed base Carli index is equal to 1.5488, while the fixed base Jevons index is 1.2483, *a spread of about 24 percent*. However, more troublesome than this spread is the fact that *the unweighted indices are well below both the Paasche and Laspeyres indices* by period 5. Thus when there are divergent trends in both prices and quantities, it will usually be the case that unweighted price indices will give very different answers than their weighted counterparts. Since none of the index number theories considered in previous chapters supported the use of unweighted indices, their use is not recommended for aggregation at the "higher level," that is, when data on weights are available. However, in Chapter 20 aggregation at the "lower level" is considered for which weights are unavailable and the use of unweighted index number formulas will be revisited.. Finally, note that the Jevons index is always considerably below the corresponding Carli index. This will always be the case (unless prices are proportional in the two periods under consideration) because a geometric mean is always equal to or less than the corresponding arithmetic mean.[8]

19.56 It is of interest to recalculate the four indices listed in Table 19.21 above using *the chain principle* rather than the *fixed base principle*. Our expectation is that the spread between the Paasche and Laspeyres indices will be reduced by using the chain principle. These chained indices are listed in Table 19.22.

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[8]This is the Theorem of the Arithmetic and Geometric Mean; see Hardy, Littlewood and Polyá (1934) and Chapter 20.

Table 19.22 The Chained Laspeyres, Paasche, Carli and Jevons Indices

Period t	P_L^t	P_P^t	P_C^t	P_J^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3753	1.3293
3	1.4931	1.4952	1.3178	1.2478
4	1.5219	1.5219	1.2527	1.1464
5	1.7176	1.7065	1.4745	1.2483

19.57 It can be seen comparing Tables 19.21 and 19.22 that chaining eliminated most of the spread between the fixed base Paasche and Laspeyres indices for period 5; i.e., the spread between the chained Laspeyres and Paasche indices has dropped from 4.7% to 0.6%. Note that chaining did not affect the Jevons index. This is an advantage of the index but the lack of weighting is a fatal flaw. The "truth" would be expected to lie between the Paasche and Laspeyres indices but from Tables 19.21 and 19.22, the unweighted Jevons index is far below this acceptable range. The fixed base and chained Carli indices also lie outside this acceptable range.

C.3 Asymmetrically Weighted Index Number Formulae

19.58 A systematic comparison of all of *the asymmetrically weighted price indices* is now undertaken. The *fixed base indices* are listed in Table 19.23. The fixed base *Laspeyres* and *Paasche indices*, P_L and P_P, are the same as those indices listed in Table 19.21 above. The *Palgrave index*, P_{PAL}, is defined by equation (16.55). The indices denoted by P_{GL} and P_{GP} are *the geometric Laspeyres and geometric Paasche indices*[9] which were defined in paragraph 16.82 of Chapter 16. For *the geometric Laspeyres index*, P_{GL}, the weights for the price relatives are the *base period expenditure shares*, s_i^1. This index should be considered an alternative to the fixed base Laspeyres index since each of these indices makes use of the same information set. For *the geometric Paasche index*, P_{GP}, the weights for the price relatives are the *current period expenditure shares*, s_i^t. Finally, the index P_{HL} *is the harmonic Laspeyres index* that was defined by (16.59).

Table 19.23 Asymmetrically Weighted Fixed Base Indices

Period t	P_{PAL}^t	P_{GP}^t	P_L^t	P_{GL}^t	P_P^t	P_{HL}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4381	1.4129	1.3967	1.3743	1.3893	1.3527
3	1.5400	1.5145	1.4832	1.4477	1.4775	1.3995
4	1.6064	1.5650	1.5043	1.4469	1.4916	1.3502
5	1.8316	1.7893	1.7348	1.6358	1.6570	1.3499

[9]Vartia (1978, p. 272) used the terms *logarithmic Laspeyres* and *logarithmic Paasche*, respectively.

19.59 By looking at the period 5 entries in Table 19.23, it can be seen that the spread between all of these fixed base asymmetrically weighted indices has increased to be much larger than our earlier spread of 4.7 percent between the fixed base Paasche and Laspeyres indices. In Table 19.23, the period 5 Palgrave index is about 1.36 times as big as the period 5 harmonic Laspeyres index, P_{HL}. Again, *this illustrates the point that due to the non-proportional growth of prices and quantities in most economies today, the choice of index number formula is very important*.[10]

19.60 It is possible to explain why certain of the indices in Table 19.23 are bigger than others. When all weights are positive, it can be shown that a *weighted arithmetic mean* of N numbers is equal to or greater than the corresponding *weighted geometric mean* of the same N numbers which in turn is equal to or greater than the corresponding *weighted harmonic mean* of the same N numbers.[11] It can be seen that the three indices P_{PAL}, P_{GP}, and P_P all use the current period expenditure shares s_i^t to weight the price relatives (p_i^t/p_i^1) but P_{PAL} is a weighted *arithmetic* mean of these price relatives, P_{GP} is a weighted *geometric* mean of these price relatives and P_P is a weighted *harmonic* mean of these price relatives. Thus since there are no negative components in final demand, by Schlömilch's inequality,:[12]

(19.1) $P_{PAL} \geq P_{GP} \geq P_P$.

19.61 Viewing Table 19.23, it can be seen that the inequalities (19.1) hold for all periods. It can also be verified that the three indices P_L, $P_{GL,}$ and P_{HL} all use the base period expenditure shares s_i^1 to weight the price relatives (p_i^t/p_i^1) but P_L is a weighted *arithmetic* mean of these price relatives, P_{GL} is a weighted *geometric* mean of these price relatives, and P_{HL} is a weighted *harmonic* mean of these price relatives. Since all of the expenditure shares are positive, then by Schlömilch's inequality,:[13]

(19.2) $P_L \geq P_{GL} \geq P_{HL}$.

Viewing Table 19.23, it can be seen that the inequalities (19.2) hold for all periods.

19.62 Now continue with the systematic comparison of all of *the asymmetrically weighted price indices*. These indices using the *chain principle* are listed in Table 19.24.

Table 19.24 Asymmetrically Weighted Chained Indices

[10] Allen and Diewert (1981) showed that the Paasche, Laspeyres and Fisher indices will all be equal if either prices or quantities move in a proportional manner over time. Thus in order to get a spread between the Paasche and Laspeyres indices, it is required that *both* prices and quantities move in a nonproportional manner.

[11] This follows from Schlömilch's (1858) inequality; see Hardy, Littlewood and Polyá (1934, chapter 11).

[12] These inequalities were noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

[13] These inequalities were also noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

Period t	P_{PAL}^t	P_{GP}^t	P_L^t	P_{GL}^t	P_P^t	P_{HL}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4381	1.4129	1.3967	1.3743	1.3893	1.3527
3	1.6019	1.5488	1.4931	1.4400	1.4952	1.3870
4	1.6734	1.5987	1.5219	1.4461	1.5219	1.3690
5	1.9802	1.8375	1.7176	1.5954	1.7065	1.4788

19.63 Viewing Table 19.24, it can be seen that the use of the chain principle only marginally reduced the spread between all of the asymmetrically weighted indices compared to their fixed base counterparts in Table 19.23. For period 5, the spread between the smallest and largest asymmetrically weighted fixed base index was 35.7 percent but for the period 5 chained indices, this spread was marginally reduced to 33.9 percent.

C.4 Symmetrically Weighted Index Number Formulae

19.64 Symmetrically weighted indices can be decomposed into two classes: *superlative indices* and *other symmetrically weighted indices*. Superlative indices have a close connection to economic theory; i.e., as was seen in Chapter 17, a superlative index is exact for a representation of the producer's production function or the corresponding unit revenue function that can provide a second order approximation to arbitrary technologies that satisfy certain regularity conditions. In Chapter 17 four primary superlative indices were considered:

- the *Fisher ideal price index, P_F*, defined by (17.9);
- the *Walsh price index, P_W*, defined by (15.19) (this price index also corresponds to the quantity index Q^1 defined by (17.26) in Chapter 17);
- the *Törnqvist-Theil price index, P_T*, defined by (17.10) and
- the *implicit Walsh price index, P_{IW}*, that corresponds to the Walsh quantity index Q_W defined by (16.34) (this is also the index P^1 defined by (17.31)).

These four symmetrically weighted superlative price indices are listed in Table 19.8 using the fixed base principle. Also listed in this table are two symmetrically weighted price indices:[14]

- the Marshall Edgeworth price index, P_{ME}, defined by (15.18) and
- the Drobisch price index, P_D, the arithmetic average of the Paasche and Laspeyres price indices.

Table 19.25 Symmetrically Weighted Fixed Base Indices

[14]Diewert (1978; 897) showed that the Drobisch Sidgwick Bowley price index approximates any superlative index to the second order around an equal price and quantity point; i.e., P_{SB} is a *pseudo-superlative index*. Straightforward computations show that the Marshall Edgeworth index P_{ME} is also pseudo-superlative.

Period t	P_T^t	P_{IW}^t	P_W^t	P_F^t	P_D^t	P_{ME}^t
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.39347	1.39312	1.39307	1.39297	1.39298	1.39267
3	1.48073	1.48219	1.48129	1.48034	1.48034	1.47990
4	1.50481	1.50627	1.50216	1.49796	1.49797	1.49645
5	1.71081	1.72041	1.70612	1.69545	1.69589	1.68389

19.65 Note that the Drobisch index P_D is always equal to or greater than the corresponding Fisher index P_F. This follows from the facts that the Fisher index is the geometric mean of the Paasche and Laspeyres indices while the Drobisch index is the arithmetic mean of the Paasche and Laspeyres indices and an arithmetic mean is always equal to or greater than the corresponding geometric mean. Comparing the fixed base asymmetrically weighted indices, Table 19.23, with the symmetrically weighted indices, Table 19.25, *it can be seen that the spread between the lowest and highest index in period 5 is much less for the symmetrically weighted indices*. The spread was 1.8316/1.3499 = 1.357 for the asymmetrically weighted indices but only 1.72041/1.68389 = 1.022 for the symmetrically weighted indices. If the analysis is restricted to the superlative indices listed for period 5 in Table 19.8, then this spread is further reduced to 1.72041/1.69545 = 1.015; i.e., the spread between the fixed base superlative indices is only 1.5 percent compared to the fixed base spread between the Palgrave and Harmonic Laspeyres indices of 35.7 percent (1.8316/1.3499 = 1.357). The spread between the superlative indices can be expected to be further reduced by using the chain principle.

19.66 The symmetrically weighted indices are recomputed using the chain principle. The results may be found in Table 19.26. A quick glance at Table 19.9 shows that *the combined effect of using both the chain principle as well as symmetrically weighted indices is to dramatically reduce the spread between all indices constructed using these two principles*. The spread between all of the symmetrically weighted indices in period 5 is only 1.7127/1.7116 = 1.0006 or 0.06 percent, which is negligible.

Table 19.26 Symmetrically Weighted Chained Indices

Period t	P_T^t	P_{IW}^t	P_W^t	P_F^t	P_D^t	P_{ME}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3935	1.3931	1.3931	1.3930	1.3930	1.3927
3	1.4934	1.4941	1.4945	1.4941	1.4941	1.4942
4	1.5205	1.5219	1.5224	1.5219	1.5219	1.5218
5	1.7122	1.7122	1.7127	1.7120	1.7121	1.7116

19.67 The results listed in Table 19.26 reinforce the numerical results tabled in Hill (2006) and Diewert (1978, p. 894): *the most commonly used chained superlative indices will generally give approximately the same numerical results*.[15] This numerical

[15]More precisely, the superlative quadratic mean of order *r* price indices P^r defined by (17.84) and the implicit quadratic mean of order *r* price indices P^{r*} defined by (17.81) will generally closely approximate each other provided that r is in the interval $0 \le r \le 2$.

approximation property holds in spite of the erratic nature of the fluctuations in the data in Tables 19.18-19.20. In particular, the chained Fisher, Törnqvist and Walsh indices will generally approximate each other very closely.

C.5 Superlative Indices and Two Stage Aggregation

19.68 Attention is now turned to the differences between superlative indices and their counterparts that are constructed in two stages of aggregation; see section D.6 of Chapter 17 for a discussion of the issues and a listing of the formulas used. In the artificial data set for domestic final demand, the first three commodities are aggregated into a *goods aggregate* and the final three commodities are aggregated into a *services aggregate*. In the second stage of aggregation, the good and services components will be aggregated into a domestic final demand price index.

19.69 The results of single stage and two stage aggregation are reported in Table 19.27 using period 1 as the *fixed base* for the Fisher index P_F, the Törnqvist index P_T and the Walsh and implicit Walsh indexes, P_W and P_{IW}.

Table 19.27 Single Stage and Two Stage Fixed Base Superlative Indices

Period t	P_F^t	P_{F2S}^t	P_T^t	P_{T2S}^t	P_W^t	P_{W2S}^t	P_{IW}^t	P_{IW2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3930	1.3931	1.3935	1.3935	1.3931	1.3931	1.3931	1.3932
3	1.4803	1.4808	1.4807	1.4800	1.4813	1.4813	1.4822	1.4821
4	1.4980	1.4998	1.5048	1.5003	1.5022	1.5021	1.5063	1.5051
5	1.6954	1.7012	1.7108	1.7007	1.7061	1.7063	1.7204	1.7176

19.70 Viewing Table 19.127, it can be seen that the fixed base single stage superlative indices generally approximate their fixed base two stage counterparts fairly closely. The divergence between the single stage Törnqvist index P_T and its two stage counterpart P_{T2S} in period 5 is 1.7108/1.7007 = 1.006 or 0.6 percent. The other divergences are even less.

19.71 Using *chained indices*, the results are reported in Table 19.28 for the two stage aggregation procedure. Again, the single stage and their two stage counterparts are listed for the Fisher index P_F, the Törnqvist index P_T and the Walsh and implicit Walsh indexes, P_W and P_{IW}.

Table 19.28 Single Stage and Two Stage Chained Superlative Indices

Period t	P_F^t	P_{F2S}^t	P_T^t	P_{T2S}^t	P_W^t	P_{W2S}^t	P_{IW}^t	P_{IW2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3930	1.3931	1.3935	1.3935	1.3931	1.3931	1.3931	1.3932
3	1.4941	1.4943	1.4934	1.4942	1.4945	1.4944	1.4941	1.4945
4	1.5219	1.5221	1.5205	1.5218	1.5224	1.5223	1.5219	1.5226
5	1.7120	1.7125	1.7122	1.7136	1.7127	1.7127	1.7122	1.7132

19.72 Viewing Table 19.28, it can be seen that the chained single stage superlative indices generally approximate their fixed base two stage counterparts quite closely. The divergence between the chained Törnqvist index P_T and its two stage counterpart P_{T2S} in period 5 is 1.7136/1.7122 = 1.0008 or 0.08 percent. The other divergences are all less than this. Given the large dispersion in period to period price movements, these two stage aggregation errors are not large. However, the important point that emerges from Table 19.28 is that *the use of the chain principle has reduced the spread between all 8 single stage and two stage superlative indices* compared to their fixed base counterparts in Table 19.27. The maximum spread for the period 5 chained index values is 0.09 percent while the maximum spread for the period 5 fixed base index values is 1.5 percent.

C.6 Additive Percentage Change Decompositions for the Fisher Index

19.73 The final formulas that is illustrated using the artificial final expenditures data set are the *additive percentage change decompositions* for the Fisher ideal index that were discussed in section B.8 of Chapter 16. The *chain links* for the Fisher price index will first be decomposed using the Diewert (2002) decomposition formulas (16.41) to (16.43). The results of the decomposition are listed in Table 19.29. Thus $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t - 1$ to t and *the decomposition factor* $v_{Fi}\Delta p_i = v_{Fi}\ (p_i^t - p_i^{t-1})$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1,2,\ldots,6$.

Table 19.29 The Diewert Additive Percentage Change Decomposition of the Fisher Index

Period t	$P_F^t - 1$	$v_{F1}^t\Delta p_1^t$	$v_{F2}^t\Delta p_2^t$	$v_{F3}^t\Delta p_3^t$	$v_{F4}^t\Delta p_4^t$	$v_{F5}^t\Delta p_5^t$	$v_{F6}^t\Delta p_6^t$
2	0.3930	0.0331	0.1253	0.1185	0.0928	-0.0082	0.0314
3	0.0726	-0.0225	-0.0353	0.0831	0.0586	-0.0077	-0.0036
4	0.0186	0.0261	-0.0347	0.0123	0.0301	-0.0118	-0.0034
5	0.1250	0.0059	0.0693	0.0114	0.0321	-0.0123	0.0185

19.74 Viewing Table 19.29, it can be seen that the price index going from period 1 to 2 grew 39.30 percent and the contributors to this change were the increases in the price of commodity 1, finally demanded agricultural products (3.31 percentage points); commodity 2, finally demanded energy (12.53 percentage points); commodity 3, finally demanded goods (11.85 percentage points); commodity 4, traditional services (9.28 percentage points) and commodity 6, transportation services (3.14 percentage points). High technology services, commodity 5, decreased in price and this fall in prices subtracted 0.82 percentage points from the overall Fisher price index going from period 1 to 2. The sum of the last six entries for period 2 in Table 19.29 is equal to .3930, the percentage increase in the Fisher price index going from period 1 to 2. It can be seen that a big price change in a particular component i combined with a big expenditure share in the two periods under consideration will lead to a big decomposition factor, $v_{Fi}\Delta p_i$.

19.75 Our final set of computations illustrate the *additive percentage change decomposition* for the Fisher ideal index that is due to Van IJzeren (1987, p. 6) that was mentioned in section C.8 of Chapter 16.[16] First, the Fisher price index going from period t−1 to t is written in the following form:

$$(19.3)\ P_F(p^{t-1},p^t,q^{t-1},q^t) = \frac{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t}}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}}$$

where the reference quantities need to be defined somehow. Van IJzeren (1987; 6) showed that the following reference weights provided an *exact additive representation for the Fisher ideal price index*:

$$(19.4)\ q_{Fi}^{*} \equiv (1/2)q_i^{t-1} + [(1/2)Q_F(p^{t-1},p^t,q^{t-1},q^t)]\ ; \qquad i = 1,2,...,6$$

where Q_F is the overall Fisher quantity index. Thus using the Van IJzeren quantity weights q_{Fi}^{*}, the *following Van IJzeren additive percentage change decomposition for the Fisher price index* is obtained:

$$(19.5)\ P_F(p^0,p^1,q^0,q^1) - 1 = \frac{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t}}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}} - 1$$
$$= \sum_{i=1}^{6} v_{Fi}^{t*}(p_i^t - p_i^{t-1})$$

where the *Van IJzeren weight* for commodity i, v_{Fi}^{t*}, is defined as

$$(19.6)\ v_{Fi}^{t*} \equiv \frac{\sum_{i=1}^{6} q_{Fi}^{*}}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}}\ ; \qquad i = 1,2,...,6.$$

The *chain links* for the Fisher price index will be decomposed using the formulas (19.3) to (19.6) listed above. The results of the decomposition are listed in Table 19.30. Thus P_F − 1 is *the percentage change in the Fisher ideal chain link* going from period t − 1 to t and *the Van IJzeren decomposition factor* $v_{Fi}^{t*}\Delta p_i^t$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for i = 1,2,…,6.

Table 19.30 The Van IJzeren Additive Percentage Change Decomposition of the Fisher Index

[16] It was also independently derived by Dikhanov (1997) and used by Ehemann, Katz, and Moulton (2002).

Period t	$P_F^t - 1$	$v_{F1}^{t*}\Delta p_1^t$	$v_{F2}^{t*}\Delta p_2^t$	$v_{F3}^{t*}\Delta p_3^t$	$v_{F4}^{t*}\Delta p_4^t$	$v_{F5}^{t*}\Delta p_5^t$	$v_{F6}^{t*}\Delta p_6^t$
2	0.3930	0.0333	0.1256	0.1186	0.0917	-0.0080	0.0318
3	0.0726	-0.0226	-0.0354	0.0833	0.0586	-0.0077	-0.0036
4	0.0186	0.0261	-0.0347	0.0123	0.0301	-0.0118	-0.0034
5	0.1250	0.0059	0.0693	0.0114	0.0320	-0.0122	0.0185

19.76 Comparing the entries in Tables 19.29 and 19.30, it can be seen that the differences between the Diewert and Van IJzeren decompositions of the Fisher price index are *very small*.[17] This is somewhat surprising given the very different nature of the two decompositions.[18] As was mentioned in section C.8 of Chapter 16, the Van IJzeren decomposition of the chain Fisher *quantity* index is used by the Bureau of Economic Analysis in the U.S.[19]

D. National Producer Price Indices

D.1 The National Gross Domestic Output Price Index at Producer Prices

19.77 In this subsection and the following 3 subsections, national domestic gross output, export, domestic intermediate input and import price indices at producer prices (i.e., at basic prices for outputs and purchaser's prices for intermediate inputs) will be calculated using the data for each of the 3 industrial sectors listed in section B above. Only fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist indices will be computed since these are the ones most likely to be used in practice.

19.78 It should be noted that the price indices computed in this section are appropriate ones to use for the calculation of business sector labour or multifactor productivity purposes.

19.79 The data listed in Tables 19.9-19.17 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for domestic outputs (at producer prices or basic prices in this case) for periods t equal 1 to 5, P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. Producer prices are used in these computations (as opposed to final demand prices). There are 3 domestic output deliveries from Industry G, 8 domestic output deliveries from Industry S and 3 domestic output deliveries from Industry T so that each index is an aggregate of 14 separate series. The fixed base results are listed in Table 19.31.

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[17] The maximum difference between the two tables occurs in period 2 for the p_4 contribution factor, which is 0.0928 in Table 19.29 and 0.0917 in Table 19.30.

[18]The terms in Diewert's decomposition can be given economic interpretations whereas the terms in the other decomposition are more difficult to interpret from the economic perspective. However, Reinsdorf, Diewert and Ehemann (2002) show that the terms in the two decompositions approximate each other to the second order around any point where the two price vectors are equal and where the two quantity vectors are equal.

[19]See Ehemann, Katz and Moulton (2002).

Table 19.31 Fixed Base National Domestic Gross Output Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3865	1.3735	1.3800	1.3810
3	1.4762	1.4459	1.4610	1.4650
4	1.4826	1.4203	1.4511	1.4683
5	1.7017	1.5424	1.6201	1.6581

19.80 By period 5, the spread between the fixed base Laspeyres and Paasche national domestic output price indices is 1.7017/1.5424 = 1.103 or 10.3% and the spread between the Fisher and Törnqvist indices is 1.6581/1.6201 = 1.023 or 2.3%. In Table 19.32, the four indexes are recomputed using the chain principle. It is expected that the use of the chain principle will narrow the spreads between the various indices.

Table 19.32 Chained National Domestic Gross Output Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3865	1.3735	1.3800	1.3810
3	1.4832	1.4728	1.4780	1.4783
4	1.4919	1.4759	1.4839	1.4839
5	1.6644	1.6328	1.6485	1.6500

19.81 An examination of the entries in Table 19.32 shows that chaining did indeed reduce the spread between the various index numbers. In period 5, the spread between the chained Laspeyres and Paasche national domestic output price indices is 1.6644/1.6328 = 1.019 or 1.9% and the spread between the chained Fisher and Törnqvist indices is 1.6500/1.6485 = 1.0009 or 0.09%, which is negligible considering the variation in the underlying data.

D.2 The National Export Price Index at Producer Prices

19.82 The data listed in Tables 19.9-19.17 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for all exported outputs (at producer prices or basic prices in this case), P_L^t, P_P^t, P_F^t and P_T^t, respectively. There is one exported good from each of the three industries so that each export price index is an aggregate of 3 separate series. The fixed base results are listed in Table 19.33.

Table 19.33 National Fixed Base Export Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3181	1.3199	1.3190	1.3191
3	1.5826	1.5799	1.5812	1.5813
4	1.4766	1.4762	1.4764	1.4763

5	1.3672	1.3694	1.3683	1.3682

19.83 There is very little difference in any of the fixed base series listed in Table 19.33. The corresponding chained indices are listed below and are also very close to each other.

Table 19.34 National Chained Export Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3181	1.3199	1.3190	1.3191
3	1.5786	1.5788	1.5787	1.5786
4	1.4717	1.4729	1.4723	1.4723
5	1.3690	1.3624	1.3657	1.3654

D.3 The National Domestic Intermediate Input Price Index at Producer Prices

19.84 The data listed in Tables 19.9-19.17 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for all domestic intermediate inputs (at producer prices or purchase prices in this case), P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. There are 3 domestic intermediate inputs used in each of Industries G, S and T so that each domestic intermediate input price index is an aggregate of 8 separate series. The fixed base results are listed in Table 19.35.

Table 19.35 Fixed Base National Domestic Intermediate Input Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3443	1.3053	1.3247	1.3265
3	1.4928	1.3441	1.4165	1.4324
4	1.4686	1.1836	1.3184	1.3619
5	1.5887	1.1306	1.3402	1.4268

19.85 The spread between the Laspeyres and Paasche fixed base indices is very large by period 5, equaling 1.5887/1.1306 = 1.405 or 40.5%. The spread between the Fisher and Törnqvist fixed base indices is not negligible either, equaling 1.4268/1.3402 = 1.065 or 6.5% in period 5. These relatively large spreads are due to the fact that the price of high tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As usual, we expect these spreads to diminish if the chained indices are used.

19.86 The corresponding chained indices are listed in Table 19.36.

Table 19.36 Chained National Domestic Intermediate Input Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000

2	1.3443	1.3053	1.3247	1.3265
3	1.4765	1.4045	1.4400	1.4435
4	1.4217	1.3272	1.3736	1.3782
5	1.4573	1.3398	1.3973	1.4015

19.89 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.4573/1.3398 = 1.088 or 8.8% and between the Fisher and Törnqvist to 1.4015/1.3973 = 1.003 or 0.3%, which is an acceptable degree of divergence considering the volatility of the underlying data.

D.4 The National Import Price Index at Producer Prices

19.90 The data listed in Tables 19.9-19.17 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for all imported intermediate inputs (at producer prices or purchase prices in this case), P_L^t, P_P^t, P_F^t and P_T^t, respectively. There are 4 imported intermediate inputs used in Industry G, 4 imported intermediate inputs used in Industry S and 2 imported intermediate inputs used in Industry T so that each import input price index is an aggregate of 10 separate series. The fixed base results are listed in Table 19.37.

Table 19.37 Fixed Base National Import Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.5210	1.5003	1.5106	1.5089
3	1.2426	1.2037	1.2230	1.2241
4	1.0844	1.0370	1.0604	1.0669
5	1.5776	1.3596	1.4645	1.4736

19.91 The spread between the Laspeyres and Paasche fixed base import price indices is fairly large by period 5, equaling 1.5776/1.3596 = 1.160 or 16.0%. The spread between the Fisher and Törnqvist fixed base indices is much smaller, equaling 1.4736/1.4645 = 1.006 or 0.6% in period 5. Note that each import price index has relatively large period to period fluctuations due to the large fluctuations in the price of imported energy. As usual, we expect the fixed base spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 19.38.

Table 19.38 Chained National Import Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.5210	1.5003	1.5106	1.5089
3	1.2438	1.2384	1.2411	1.2415
4	1.0810	1.0723	1.0766	1.0773
5	1.5128	1.4236	1.4675	1.4680

19.92 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.5128/1.4236 = 1.063 or 6.3% and between the Fisher and Törnqvist to 1.4680/1.4675 = 1.0003 or 0.03%, a negligible amount.

19.93 The domestic output price index and the domestic export index can be regarded as subindexes of an overall gross output price index of the type that was described in the *PPI Manual*. Similarly, the domestic intermediate input price index and the import price index can be regarded as subindexes of the overall intermediate input price index that was described in the *PPI Manual*. All of these subindices can be thought of as aggregations of the same commodity (or group of commodities) across industries. At a second stage of aggregation, it is possible to aggregate over the domestic output price index and the export price index and to also aggregate over the domestic intermediate input price index and the import price index (with quantities indexed with negative signs) in order to form an economy wide *value added price index*. In the following section, the first stage of aggregation will be across commodities within an industry; i.e., in the following section, industry value added price indices will be constructed. A national value added price index will also be constructed in section E. In section F, the industry value added deflators constructed in section E will be aggregated in order to form a two stage economy wide value added price index. This two stage aggregate value added deflator will be compared with the two stage aggregation method that aggregates over the domestic output price index, the export price index, the domestic intermediate input price index and the import price index. These two methods of two stage aggregation will be compared in section F along with the corresponding single stage national value added deflator.

E. Value Added Price Deflators

E.1 Value Added Price Deflators for the Goods Producing Industry

19.94 The data listed in Tables 19.9-19.11 for Industry G are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist value added price indices or deflators at producer prices. This means that basic prices are used for domestic outputs and exports and purchasers' prices are used for imports and domestic intermediate inputs. The quantities of domestic intermediate inputs and imports are indexed with negative signs. Fixed base and chained value added Laspeyres, Paasche, Fisher and Törnqvist price indices will be constructed, P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. There are 3 domestic outputs and one export produced by Industry G, and 3 domestic intermediate inputs and 4 imported commodities used as inputs by Industry G so that each value added price index is an aggregate of 11 separate series. The fixed base results are listed in Table 19.39.

Table 19.39 Fixed Base Value Added Price Deflators for Industry G

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1655	1.1889	1.1772	1.1535
3	2.2260	3.5528	2.8122	2.5489
4	2.4403	8.0774	4.4398	3.0649

5	1.7605	5.7905	3.1928	2.1276

19.95 The spread between the Laspeyres and Paasche fixed base value added price indices is enormous by period 5, equaling 5.7905/1.7605 = 3.289 or 328.9%. The spread between the Fisher and Törnqvist fixed base indices is large as well, equaling 3.1928/2.1276 = 1.501 or 50.1% in period 5. These very large spreads are due to the fact that the price of high tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As well, because quantities have positive and negative weights in value added price indices, the divergences between various index number formulae can become very large. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 19.40.

Table 19.40 Chained Value Added Price Deflators for Industry G

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1655	1.1889	1.1772	1.1535
3	2.4490	3.2741	2.8317	2.7527
4	2.8776	4.0277	3.4044	3.3096
5	1.8066	2.9594	2.3122	2.2720

19.96 Chaining reduces the period 5 spread in period 5 between Laspeyres and Paasche to 2.9594/1.8066 = 1.638 or 63.8% and between the Fisher and Törnqvist to 2.3122/2.2720 = 1.018 or 1.8%, which is an acceptable degree of divergence considering the volatility of the underlying data. However, note that using the chained Laspeyres or Paasche value added price indices for this industry will give rise to estimates of price change that are very far from the corresponding superlative index estimates. *Thus the corresponding Laspeyres or Paasche estimates of real value added may be rather inaccurate, giving rise to inaccurate estimates of industry productivity growth*.

E.2 Value Added Price Deflators for the Services Industry

19.97 The data listed in Tables 19.12-19.14 for Industry S are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist value added price indices at producer prices. There are 8 domestic outputs and one export produced by Industry S, and 2 domestic intermediate inputs and 4 imported commodities used as inputs by Industry S so that each value added price index is an aggregate of 15 separate series. The fixed base results are listed in Table 19.41. Producer prices are used in these computations.

Table 19.41 Fixed Base Value Added Price Deflators for Industry S

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2365	1.2337	1.2351	1.2360
3	1.4876	1.4160	1.4514	1.4537
4	1.5035	1.3531	1.4264	1.4380
5	1.4913	1.2797	1.3814	1.3942

19.98 The spread between the Laspeyres and Paasche fixed base value added price indices for Industry S is 1.4913/1.2797 = 1.165 or 16.5%, which is a substantial gap. The spread between the Fisher and Törnqvist fixed base indices is fairly small, equaling 1.3942/1.3814 = 1.009 or 0.9% in period 5. Note that the gap between the fixed base Paasche and Laspeyres value added price indices for the services industry is very much less than the corresponding gap for the fixed base Paasche and Laspeyres value added price indices for the goods producing industry. An explanation for this narrowing of the Paasche and Laspeyres gap is that while the services industry was subject to some very large fluctuations in the prices it faced, since most of the big fluctuations occurred for the food and energy imports which are margin goods for the industry, these fluctuations were passed on to final demanders, leaving industry distribution margins largely intact. Thus the fluctuations in the value added price indices for Industry S turned out to be less severe than for Industry G. As usual, the spreads between the Paasche and Laspeyres price indices should narrow when the chain principle is used; see Table 19.42 below.

Table 19.42 Chained Value Added Price Deflators for Industry S

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2365	1.2337	1.2351	1.2360
3	1.4700	1.4411	1.4555	1.4579
4	1.4620	1.4201	1.4409	1.4432
5	1.4363	1.3863	1.4111	1.4145

19.99 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.4363/1.3863 = 1.036 or 3.6% in period 5 and between the Fisher and Törnqvist to 1.4145/1.4111 = 1.002, which is negligible.

E.3 Value Added Price Deflators for the Transportation Industry

19.100 The data listed in Tables 19.15-19.17 for Industry T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist value added price indices at producer prices. There are 3 domestic outputs and one export produced by Industry T, and 3 domestic intermediate inputs and 2 imported commodities used as inputs by Industry T so that each value added price index is an aggregate of 9 separate series. The fixed base results are listed in Table 19.43.

Table 19.43 Fixed Base Value Added Price Deflators for Industry T

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4764	1.6417	1.5569	1.5572
3	1.1204	1.1913	1.1553	1.1173
4	1.0977	1.3541	1.2192	1.0679
5	1.8028	4.8128	2.9456	2.2114

19.101 The spread between the Laspeyres and Paasche fixed base value added price indices is enormous by period 5, equaling 4.8128/1.8028 = 2.670 or 267.0%. The spread between the Fisher and Törnqvist fixed base indices is fairly large as well, equaling 2.9456/2.2114 = 1.332 or 33.2% in period 5. These very large spreads are due to the fact that the price of high tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 19.44.

Table 19.44 Chained Value Added Price Deflators for Industry T

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4764	1.6417	1.5569	1.5572
3	1.0374	1.1271	1.0813	1.0509
4	0.9428	1.0563	0.9979	0.9667
5	1.9916	2.4248	2.1975	2.2389

19.102 Chaining reduces the period 5 spread in period 5 between Laspeyres and Paasche to 2.4248/1.9916 = 1.218 or 21.8% and between the Fisher and Törnqvist to 2.2389/2.1975 = 1.019 or 1.9%, which is an acceptable degree of divergence considering the volatility of the underlying data. However, note that using the chained Laspeyres or Paasche value added price indices for this industry will give rise to estimates of price change that are fairly far from the corresponding chained superlative index estimates, a situation that is similar to what occurred for the Industry G data. Thus whenever possible, it seems preferable to use chained superlative indices when constructing annual industry value added deflators as opposed to using fixed base or chained Paasche or Laspeyres indices.

19.103 In the following section, all of the industry data are aggregated to form a national value added deflator.

E.4 The National Value Added Price Deflator

19.104 The data listed in Tables 19.9-19.17 for Industries G, S and T are used to calculate national Laspeyres, Paasche, Fisher and Törnqvist value added price indices at producer prices; i.e., in this subsection, the national value added deflator is constructed. Fixed base and chained value added Laspeyres, Paasche, Fisher and Törnqvist price indices will be constructed, P_L^t, P_P^t, P_F^t and P_T^t, respectively. There are 14 domestic outputs, 3 exported commodities, 8 domestic intermediate inputs and 10 imported commodities so that each national value added deflator is an aggregate of 35 separate series. The fixed base results are listed in Table 19.45.

Table 19.45 Fixed Base National Value Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000

2	1.2180	1.2353	1.2267	1.2261
3	1.7776	1.8533	1.8151	1.8173
4	1.8743	1.9822	1.9275	1.9455
5	1.6176	1.7555	1.6851	1.6970

19.105 The spread between the national Laspeyres and Paasche fixed base value added price indices is fairly large by period 5, equaling 1.7555/1.6176 = 1.085 or 8.5%. The spread between the Fisher and Törnqvist fixed base indices is small, equaling 1.6970/1.6851 = 1.007 or 0.7% in period 5. The corresponding chained indices are listed in Table 19.46.

Table 19.46 Chained National Value Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261
3	1.7711	1.8336	1.8021	1.8098
4	1.8855	1.9530	1.9190	1.9315
5	1.6380	1.7612	1.6985	1.7156

19.106 The spread in period 5 between the national Laspeyres and Paasche chained value added price indices equals 1.7612/1.6380 = 1.075 or 7.5% which is slightly smaller than the corresponding 8.5% spread for the fixed base Laspeyres and Paasche indices. The spread between the Fisher and Törnqvist chained indices in period 5 is 1.7156/1.6985 = 1.010 or 1.0%, which is slightly larger than the corresponding fixed base spread of 0.7%. At the national level, the fixed base and chained Fisher and Törnqvist indices all give much the same answer.

F. Two Stage Value Added Price Deflators

F.1 Two Stage National Value Added Price Deflators: Aggregation over Industries

19.107 In section D.6 of chapter 17, methods for constructing a price index by aggregating in two stages were discussed. It was pointed out that if a Laspeyres index is constructed in two stages of aggregation and the Laspeyres formula is used in each stage of aggregation, then the two stage index will necessarily coincide with the corresponding single stage index. A similar consistency in aggregation property holds if the Paasche formula is used at each stage of aggregation. Unfortunately, this consistency in aggregation property does not hold for superlative indices but it was pointed out in chapter 17, that superlative indices should be approximately consistent in aggregation. In this section, the artificial data set will be used in order to evaluate this approximate consistency in aggregation property of the Fisher and Törnqvist indices.

19.108 In the present context, there are two natural ways of aggregating in two stages. In Method 1, the first stage of aggregation is the construction of a value added deflator for each industry (along with the corresponding quantity indices) and in the second stage, the three industry value added deflators are aggregated into a national value added deflator. In Method 2, the first stage of aggregation is the construction of national domestic output, domestic intermediate input, export and import price indices (along with the corresponding quantity indices) and in the second stage, these four price indices are aggregated into a national value added deflator.[20] The results for Method 1 will be listed in this subsection while the results for Method 2 will be listed in section F.2 below.

19.109 In Table 19.47, the fixed base single stage Laspeyres, Paasche, Fisher and Törnqvist indices are listed in the first 4 columns of the table[21] and the corresponding Method 1 fixed base two stage indices are listed in the last 4 columns of the table.

Table 19.47 Fixed Base Single Stage and Two Stage National Value Added Deflators: Aggregation over Industries Method

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2276	1.2190
3	1.7776	1.8533	1.8151	1.8173	1.7776	1.8533	1.8915	1.8110
4	1.8743	1.9822	1.9275	1.9455	1.8743	1.9822	2.2616	1.9254
5	1.6176	1.7555	1.6851	1.6970	1.6176	1.7555	1.9488	1.6579

19.110 As is expected from the theory in Chapter 17, the single stage Laspeyres and Paasche indices coincide exactly with their two stage counterparts. What was not expected is how far the two stage Fisher index, P_{F2S}^t, is from its single stage counterpart, P_F^t, for periods 3-5. Obviously, the period to period changes in the Fisher industry value added indices are so large that the two stage approximation results discussed in Chapter 17 break down for this artificial data set. The spread between the fixed base single stage Fisher and Törnqvist indices in period 5 is 1.6970/1.6851 = 1.007 or 0.7% but the spread between the two stage Fisher and Törnqvist indices in period 5 is 1.9488/1.6579 = 1.175 or 17.5%, a rather large deviation.

19.111 In Table 19.48, the chained single stage Laspeyres, Paasche, Fisher and Törnqvist indices are listed in the first 4 columns of the table[22] and the corresponding Method 1 chained two stage indices are listed in the last 4 columns of the table.

Table 19.48 Chained Single Stage and Two Stage National Value Added Deflators: Aggregation over Industries Method

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t

[20] The domestic output and export quantities are positive numbers in this second stage of aggregation but the domestic intermediate input and import quantities are negative numbers in the second stage of aggregation.

[21] These indices are the same as those listed in Table 19.45.

[22] These indices are the same as those listed in Table 19.46.

1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2276	1.2190
3	1.7711	1.8336	1.8021	1.8098	1.7711	1.8336	1.8365	1.8124
4	1.8855	1.9530	1.9190	1.9315	1.8855	1.9530	1.9587	1.9326
5	1.6380	1.7612	1.6985	1.7156	1.6380	1.7612	1.7270	1.7137

19.112 It can be seen that chaining has reduced the spread between the two stage superlative indices. The spread between the chained single stage Fisher and Törnqvist indices in period 5 is 1.7156/1.6985 = 1.007 or 1.0% and the spread between the chained two stage Fisher and Törnqvist indices in period 5 is 1.7270/1.7137 = 1.008 or 0.8%, a rather modest deviation. As is expected from the theory in Chapter 17, the single stage chained Laspeyres and Paasche indices coincide exactly with their two stage counterparts.

F.2 Two Stage National Value Added Price Deflators: Aggregation over Commodities

19.113 In this subsection, the national value added price index is formed by an alternative two stage aggregation procedure. In the first stage aggregation, national domestic output, export, domestic intermediate input and import price indices are calculated along with the corresponding quantity indexes as was done in section D above. In the second stage of aggregation, the sign of the quantity indices that correspond to the domestic intermediate input and import indices is changed from positive to negative and the four price and quantity series are aggregated together to form an estimate for the national value added deflator. The resulting two stage fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices, $P_{L2S}{}^t$, $P_{P2S}{}^t$, $P_{F2S}{}^t$ and $P_{T2S}{}^t$, are listed in the last 4 columns of Table 19.49 along with their fixed base single stage counterparts, $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$.

Table 19.49 Fixed Base Single Stage and Two Stage National Value Added Deflators: Aggregation over Commodities Method

Period t	$P_L{}^t$	$P_P{}^t$	$P_F{}^t$	$P_T{}^t$	$P_{L2S}{}^t$	$P_{P2S}{}^t$	$P_{F2S}{}^t$	$P_{T2S}{}^t$
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2272	1.2294
3	1.7776	1.8533	1.8151	1.8173	1.7776	1.8533	1.8067	1.8094
4	1.8743	1.9822	1.9275	1.9455	1.8743	1.9822	1.9066	1.9269
5	1.6176	1.7555	1.6851	1.6970	1.6176	1.7555	1.6641	1.6822

19.114 Note that the single stage fixed base indices, $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$, listed in Table 19.49 coincide with the single stage fixed base indices $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$ listed in Table 19.47. Note also that the single stage Paasche and Laspeyres indices coincide with their two stage counterparts in Table 19.49 as is expected from index number theory. Finally, note that the two stage superlative indices, $P_{F2S}{}^t$ and $P_{T2S}{}^t$, are reasonably close to their single stage counterparts, $P_F{}^t$ and $P_T{}^t$. The spread between the four superlative indices is 1.6970/1.6641 = 1.054 or 5.4%. It seems that the Method 2 (aggregation over commodities method) two stage aggregation procedure works more smoothly than the

Method 1 (aggregation over industry value added method) two stage aggregation procedure, leading to a reasonably close approximation between the single stage and two stage estimators for the national value added deflator in the case of Method 2.

19.115 In the following Table 19.50, the Method 2 two stage chained Laspeyres, Paasche, Fisher and Törnqvist price indices, $P_{L2S}{}^t$, $P_{P2S}{}^t$, $P_{F2S}{}^t$ and $P_{T2S}{}^t$, are listed in the last 4 columns of along with their fixed base single stage counterparts, $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$.

Table 19.50 Chained Single Stage and Two Stage National Value Added Deflators: Aggregation over Commodities Method

Period t	$P_L{}^t$	$P_P{}^t$	$P_F{}^t$	$P_T{}^t$	$P_{L2S}{}^t$	$P_{P2S}{}^t$	$P_{F2S}{}^t$	$P_{T2S}{}^t$
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2272	1.2294
3	1.7711	1.8336	1.8021	1.8098	1.7711	1.8336	1.8037	1.8150
4	1.8855	1.9530	1.9190	1.9315	1.8855	1.9530	1.9202	1.9318
5	1.6380	1.7612	1.6985	1.7156	1.6380	1.7612	1.7069	1.7186

19.116 As expected, chaining reduces the spread between the superlative indices. The spread between the four superlative indices is now 1.7186/1.6985 = 1.012 or 1.2%. Note also that the single stage Paasche and Laspeyres chained indices coincide with their two stage counterparts in Table 19.50.

19.117 In the following section, the focus shifts from industry price indices to final demand price indices.

G. Final Demand Price Indices

G.1 Domestic Final Demand Price Indices

19.118 In this section, the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are listed for deliveries of commodities to the domestic final demand sector; see Table 19.51 below. Each index is an aggregate of 6 separate final demand series.

Table 19.51 Fixed Base and Chained Domestic Final Demand Deflators

	Fixed	Base	Indices		Chained		Indices	
Period t	$P_L{}^t$	$P_P{}^t$	$P_F{}^t$	$P_T{}^t$	$P_L{}^t$	$P_P{}^t$	$P_F{}^t$	$P_T{}^t$
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3930	1.3935	1.3967	1.3893	1.3930	1.3935
3	1.4832	1.4775	1.4803	1.4807	1.4931	1.4952	1.4941	1.4934
4	1.5043	1.4916	1.4980	1.5048	1.5219	1.5219	1.5219	1.5205
5	1.7348	1.6570	1.6954	1.7108	1.7176	1.7065	1.7120	1.7122

19.119 The indices listed in Table 19.51 have already been listed in various tables in section C above but for convenience, they are tabled again. Since the above indices have been discussed in section C, the discussion will not be repeated here.

G.2 Export Price Indices at Final Demand Prices

19.120 In this subsection, the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are calculated for the 3 export series that are listed in section B above. Final demand prices are used when calculating the indices listed in Table 19.52 below.

Table 19.52 Fixed Base and Chained Export Price Indices at Final Demand Prices

	Fixed	Base	Indices		Chained		Indices	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3191	1.321	1.3201	1.3202	1.3191	1.3210	1.3201	1.3202
3	1.5816	1.5789	1.5802	1.5803	1.5775	1.5777	1.5776	1.5776
4	1.4752	1.4750	1.4751	1.4750	1.4703	1.4716	1.4709	1.4709
5	1.4184	1.4152	1.4168	1.4167	1.4140	1.4076	1.4108	1.4105

19.121 Since the 3 export price and quantity series have fairly smooth trends that are roughly proportional to each other, all of the indices listed above in Table 19.52 are quite close to each other.

G.3 Import Price Indices at Final Demand Prices

19.122 In this subsection, the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are calculated for the 10 import series that are listed in section B above. Final demand prices are used when calculating the indices listed in Table 19.53 below.

Table 19.53 Fixed Base and Chained Import Price Indices at Final Demand Prices

	Fixed	Base	Indices		Chained		Indices	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.5495	1.5279	1.5387	1.5369	1.5495	1.5279	1.5387	1.5369
3	1.2270	1.1907	1.2087	1.2099	1.2293	1.2261	1.2277	1.2285
4	1.0739	1.0289	1.0512	1.0580	1.0709	1.0642	1.0676	1.0682
5	1.5946	1.3726	1.4794	1.4873	1.5257	1.4321	1.4782	1.4785

19.123 Since price and quantity trends for imports are far from being proportional, there are substantial differences between the Paasche and Laspeyres price indices. The spread between the fixed base Paasche and Laspeyres is 1.5946/1.3726 = 1.162 or 16.2% while the spread between the chained Paasche and Laspeyres is 1.5257/1.4321 = 1.065 or 6.5% so that as usual, chaining reduces the spread. All of the superlative indices are close to each other.

G.4 GDP Deflators

19.124 In this subsection, various GDP deflators are calculated; i.e., the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are calculated for the 19 final demand series that are listed in section B above. Final demand prices are used when calculating the indices listed in Table 19.54 below.

Table 19.54 Fixed Base and Chained GDP Deflators

	Fixed	Base	Indices		Chained		Indices	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2417
3	1.7317	1.7696	1.7506	1.7546	1.7252	1.7632	1.7441	1.7499
4	1.8107	1.8476	1.8291	1.8488	1.8139	1.8507	1.8322	1.8420
5	1.6591	1.7044	1.6816	1.6995	1.6581	1.7391	1.6981	1.7099

19.125 The spread between the Paasche and Laspeyres fixed base GDP deflators in period 5 is 1.7044/1.6591 = 1.027 or 2.7% while the spread between the Paasche and Laspeyres chained GDP deflators in period 5 is 1.7044/1.6591 = 1.048 or 4.8%. Thus in this case, chaining did not reduce the spread between the Paasche and Laspeyres indices. The superlative indices are all rather close to each other; in period 5, the spread between the 4 superlative indices was 1.7099/1.6816 = 1.017 or 1.7% and the spread between the two chained superlative indices was only 1.7099/1.6981 = 1.007 or 0.7%.

G.5 The Reconciliation of the GDP Deflator with the Value Added Deflator

19.126 The final set of tables for this chapter draws on the theory developed in section B.3 of Chapter 17. In that section, it was shown how volume estimates for GDP at final demand prices, GDP_F, could be reconciled with volume estimates for GDP at producer prices, GDP_P, using equation (17.27). Equation (17.27) said that GDP_F equals GDP_P plus a sum of tax terms, T. In Chapter 17, it was shown that two stage price and quantity indices for GDP_F could be constructed by aggregating over the 35 separate price and quantity series that are used to construct price and quantity indices for GDP_P plus aggregating over all of the tax series that make up the T aggregate. It was shown in Chapter 17 that the resulting price and volume estimates for GDP_F and GDP_P + T will coincide if the Laspeyres, Paasche or Fisher formulae are used. This methodology is tested out on the artificial data set for both fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices in Tables 19.55 (fixed base indices) and 19.56 (chained indices) below. The P_L^t, P_P^t, P_F^t and P_T^t indices reported in Table 19.55 are the fixed base single stage GDP deflators (for GDP_F) that were listed in the first 4 columns of Table 19.54 while the P_{L2S}^t, P_{P2S}^t, P_{F2S}^t and P_{T2S}^t indices reported in Table 19.55 are the two stage fixed base price indices that result when we aggregate over the 35 component price and quantity series that make up GDP at producer prices, GDP_P, plus the nonzero tax series that are listed in section B above and make up the tax aggregate T.

Table 19.55 Fixed Base GDP Deflators Calculated in Two Stages

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2428
3	1.7317	1.7696	1.7506	1.7546	1.7317	1.7696	1.7506	1.7538
4	1.8107	1.8476	1.8291	1.8488	1.8107	1.8476	1.8291	1.8470
5	1.6591	1.7044	1.6816	1.6995	1.6591	1.7044	1.6816	1.7020

19.127 As predicted by the theory presented in Chapter 17, the Laspeyres, Paasche and Fisher single stage estimates for the GDP deflator (the first 3 columns in Table 19.55) coincide exactly with the corresponding two stage estimates that are built up by aggregating over GDP at producer prices plus aggregating over the tax series. The single stage Törnqvist GDP deflator, P_T^t, does not coincide with its two stage counterpart, P_{T2S}^t, but the correspondence is fairly close.

19.128 The P_L^t, P_P^t, P_F^t and P_T^t indices reported in Table 19.56 are the chained single stage GDP deflators (for GDP_F) that were listed in the last 4 columns of Table 19.54 while the P_{L2S}^t, P_{P2S}^t, P_{F2S}^t and P_{T2S}^t indices reported in Table 19.56 are the two stage chained price indices that result when we aggregate over the 35 component price and quantity series that make up GDP at producer prices, GDP_P, plus the nonzero tax series that are listed in section B above and make up the tax aggregate T.

Table 19.56 Chained GDP Deflators Calculated in Two Stages

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2428
3	1.7252	1.7632	1.7441	1.7499	1.7252	1.7632	1.7441	1.7488
4	1.8139	1.8507	1.8322	1.8420	1.8139	1.8507	1.8322	1.8405
5	1.6581	1.7391	1.6981	1.7099	1.6581	1.7391	1.6981	1.7120

19.129 Again as predicted by the theory presented in Chapter 17, the Laspeyres, Paasche and Fisher single stage estimates for the GDP deflator (the first 3 columns in Table 19.56) coincide exactly with the corresponding two stage estimates that are built up by aggregating over GDP at producer prices plus aggregating over the tax series. The single stage Törnqvist GDP deflator, P_T^t, does not coincide with its two stage counterpart, P_{T2S}^t, but again, the correspondence is fairly close.

19.130 The equality of the single stage and two stage Laspeyres, Paasche and Fisher GDP_F deflators in Tables 19.55 and 19.56 provides a very good check on the correctness of all of the various index number calculations that are associated with PPI programs and the production of GDP volume estimates.

H. Conclusion

19.131 Some tentative conclusions that can be drawn from the various indices that have been computed using the artificial data set are as follows:

- It is risky to use fixed base Paasche or Laspeyres indices in the sense that they can be rather far from the theoretically preferred superlative indices.
- Chained indices seem preferable to the use of fixed base indices in the sense that chaining generally reduces the spread between the Paasche and Laspeyres indices.
- Chained Paasche and Laspeyres indices can be close to the theoretically preferred superlative indices, except in the value added context; i.e., chained Paasche and Laspeyres indices are often fairly close to each other (and the corresponding chained superlative indices) when constructing output, export, intermediate input and import price indices. However, when constructing value added indices, it seems preferable to use chained superlative indices.

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