A Note on the Treatment of Inventory Change

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We need a theoretical framework to measure the contribution of an inventory stock to production. We outline a possible framework taken from Diewert and Smith (1994).

Consider a firm that perhaps produces a noninventory output during period t, Y^t, uses a noninventory input X^t, sells the amount S^t of an inventory item during period t and makes purchases of the inventory item during period t in the amount B^t. Suppose that the average prices during period t of Y^t, X^t, S^t and B^t are $P_Y{}^t$, $P_X{}^t$, $P_S{}^t$ and $P_B{}^t$ respectively. Then neglecting balance sheet items, the firm's period t *cash flow* is:

$$(1)\quad CF^t \equiv P_Y{}^t Y^t - P_X{}^t X^t + P_S{}^t S^t - P_B{}^t B^t.$$

Let the firm's beginning of period t stock of inventory be K^t and let it's end of period stock of inventory be K^{t+1}. These inventory stocks are valued at the balance sheet prices prevailing at the beginning and end of period t, $P_K{}^t$ and $P_K{}^{t+1}$ respectively. Note that all 4 prices involving inventory items, $P_S{}^t$, $P_B{}^t$, $P_K{}^t$ and $P_K{}^{t+1}$ can be different.

The firm's period t *economic income* is defined as its cash flow plus the value of its end of period t stock of inventory items less $(1+r^t)$ times the value of its beginning of period t stock of inventory items:

$$(2)\quad EI^t \equiv CF^t + P_K{}^{t+1} K^{t+1} - (1+r^t) P_K{}^t K^t$$

where r^t is the nominal cost of capital that the firm faces at the beginning of period t. Thus in definition (2), we assume that the firm has to borrow financial capital or raise equity capital at the cost r^t in order to finance its initial holdings of inventory items. This cost could be real (in the case of a firm whose initial capital is funded by bonds) or it could be an opportunity cost (in the case of a firm entirely funded by equity capital).

Now the end of period stock of inventory is related to the beginning of the period stock by the following equation:

$$(3)\quad K^{t+1} = K^t + B^t - S^t - U^t$$

where U^t denotes inventory items that are lost, spoiled, damaged or are used internally by the firm. In the case of livestock inventories, there is a natural growth rate of inventories over the period so equation (3) is replaced by:

$$(4)\quad K^{t+1} = K^t + B^t - S^t + G^t$$

where G^t denotes the natural growth of the stock over period t.

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Define the change in inventory stocks over period t as:

(5) $\Delta K^t \equiv K^{t+1} - K^t$.

Using (5), both (3) and (4) can be written as:

(6) $K^{t+1} = K^t + \Delta K^t$.

Now substitute (6) into the definition of economic income (2) and we obtain the following expression:

(7) $EI^t \equiv CF^t + P_K^{t+1} [K^t + \Delta K^t] - (1+r^t) P_K^t K^t$
$= CF^t + P_K^{t+1} \Delta K^t - [r^t P_K^t - (P_K^{t+1} - P_K^t)] K^t$.

Thus economic income is equal to cash flow plus the value of the change in inventory (valued at end of period balance sheet prices) minus the user cost of inventories times the starting stocks of inventories where this period t user cost is defined as

(8) $P_I^t \equiv r^t P_K^t - (P_K^{t+1} - P_K^t)$.

Note that the above algebra works for both livestock and ordinary inventory items.

Of course, there can be two versions of the user cost:

- An ex post version where the actual end of period balance sheet price of inventories is used or
- An ex ante version where at the beginning of period t, we estimate a predicted value for the end of period balance sheet price.

In practice, beginning of period inventory stocks should be estimated along with appropriate balance sheet prices. When aggregating over a number of inventory stock items, normal index number theory can be used in order to decompose balance sheet values of inventory items into price and quantity components. Then the change in stocks can be obtained by differencing the stock volume series and the end of period balance sheet price (or a predicted version of it) could serve as the flow price for the change in inventories. Note that normal index number theory breaks down for value aggregates that can be either positive or negative over time.

References

Diewert, W. E. and A. M. Smith (1994), "Productivity Measurement for a Distribution Firm", *The Journal of Productivity Analysis* 5, 335-347.