A Note on the Effects of a Real Interest Rate Increase on Asset Lives

Erwin Diewert,
Department of Economics,
University of British Columbia,
Vancouver, Canada, V6T 1Z1.
Email: diewert@econ.ubc.ca

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Anne Harrison in an email raised the issue of whether an increase in the real interest rate would increase or decrease asset lives. The answer to this question requires that we have a model where the length of life that an asset is used is *endogenously* determined by the model.[1] In this note, we consider two models and attempt to answer Harrison's question. In Model 1, which assumes constant real revenues for the asset and geometric real growth rates for complementary inputs, the answer is no. In Model 2, which assumes constant real gross revenues and a profile of increasing maintenance expenditures, the answer is also no. However, other endogenous asset life models may well give a different answer to the question.

Model 1: Geometrically Increasing Complementary Asset Expenditures

Let R^t be the gross revenue that can be attributed to the use of an asset in period t and let C^t be the corresponding period t costs that are complementary to the use of the asset or required in order to maintain it in working condition. If the interest rate that the user of the capital faces is r (constant over time for simplicity), then the asset value from the perspective of period 0 if it is used t periods is A(t) defined as follows:

$$(1)\ A(t) \equiv R^0 + R^1/(1+r) + R^2/(1+r)^2 + \ldots + R^{t-1}/(1+r)^{t-1}$$
$$- [C^0 + C^1/(1+r) + C^2/(1+r)^2 + \ldots + C^{t-1}/(1+r)^{t-1}].$$

In order to further simplify the model, we assume that $r > 0$ is a real interest rate and gross revenues are constant in real terms over all periods but that costs accelerate over time at the (real) rate of cost inflation, $i > 0$, so that

$$(2)\ R^t = R^0\ ; \qquad t = 1,2, \ldots$$
$$(3)\ C^t = C^0(1+i)^t\ ; \qquad t = 1,2, \ldots$$

where

$$(4)\ 0 < C^0 < R^0 \ \text{ and } 0 < i \ \text{ and } 0 < r.$$

Under these assumptions, A(t) becomes:

$$(5)\ A(t) = R^0 - C^0 + (1+r)^{-1}[R^0 - (1+i)C^0] + (1+r)^{-2}[R^0 - (1+i)^2C^0] + \ldots$$
$$+ (1+r)^{-(t-1)}[R^0 - (1+i)^{t-1}C^0]\ .$$

[1] For examples of endogenous life models, see Diewert's (2001) linearly increasing maintenance expenditures model or Harper's (2002) generalized Solow vintage model.

Define g(t) as follows:

(6) $g(t) \equiv [R^0 - (1+i)^{t-1}C^0]$; $\quad\quad t = 1,2,\ldots$.

Note that $g(1) = R^0 - C^0$ is positive and that g(t) monotonically decreases as t increases, becoming negative eventually, since i is assumed to be positive. The beginning of period 0 asset value, A^0, can be defined as the maximum A(t) where t ranges over all positive integers; i.e.,

(7) $A^0(r,i) \equiv \max_t \{A(t): t = 1,2,\ldots\}$.

Let t* be the unique positive integer such that:

(8) $g(t^*) \geq 0$ but $g(t^*+1) < 0$.

Using (5) and (6), it can be seen that this t* solves the integer programming problem (7) so that the asset value A^0 is equal to:

(9) $A^0 \equiv A^0(r,i) = A(t^*)$

where A(t) is defined by (5) and t* is defined by (8). Note that if we increase the real interest rate r, then g(t) defined by (6) does not change and hence t* does not change as r changes. Thus *in this model, an increase in the real interest rate does not lead to a change in the length of time that the asset is used*.

By examining (5) for t = t* and noting that each term in square brackets is nonnegative and independent of r, it can be seen that *as the real interest rate increases, the asset value will decrease*; i.e., we have:

(10) $A^0(r',i) < A^0(r,i)$ for all $r' > r$.

Now consider the case where i increases to i' (but i' is still less than r). In this case, g(t*) defined by (6) when t = t* will decrease and if the increase in i' is big enough, g(t*) will become negative. Hence *as the inflation rate for complementary inputs increases, the optimal length of life t* will decrease* (or at least not increase). It can also be seen that *as the inflation rate for complementary inputs increases, the asset value will decrease*; i.e., we have:

(10) $A^0(r,i') < A^0(r,i)$ for all $i' > i$.

All four of the above comparative statics results are intuitively plausible, except perhaps for the first result, that asserts that a change in the real interest rate will *not* affect the asset life.

Model 2: Time Increasing Complementary Asset Expenditures

As in the previous model, we assume that $r > 0$ is the real interest rate and gross revenues are constant in real terms over all periods so that assumption (2) holds. However, we now assume that complementary costs are equal to a nonnegative fixed cost, $C^0 \geq 0$, that is constant over time but that costs accelerate over time according to the general function $M(t)$, so that period t complementary costs, C^t, are given by:

$$(11)\ C^t = C^0 + M(t)\ ; \qquad t = 0,1,2, \ldots$$

where $M(t)$ is a monotonically increasing function in t that in addition, satisfies the following conditions:

(12) $M(0) = 0$ and $M(t)$ tends to plus infinity as t tends to plus infinity.

Thus in plain language, we are simply assuming that complementary costs increase over time. We assume that:

(13) $0 \leq C^0 < R^0$ and $0 < r$.

Under these assumptions, the asset value if it is used t periods, $A(t)$ defined by (5), becomes:

$$\begin{aligned}(14)\ A(t) = {} & R^0 - C^0 + (1+r)^{-1}[R^0 - C^0 - M(1)] + (1+r)^{-2}[R^0 - C^0 - M(2)] + \ldots \\ & + (1+r)^{-(t-1)}[R^0 - C^0 - M(t-1)].\end{aligned}$$

Define $h(t)$ as follows:

$$(15)\ h(t) \equiv [R^0 - C^0 - M(t-1)]\ ; \qquad t = 1,2,\ldots\ .$$

Note that $h(1) = R^0 - C^0$ is positive and that $h(t)$ monotonically decreases as t increases, becoming negative eventually under our monotonicity assumptions on $M(t)$. The beginning of period 0 asset value, A^0, can be defined by (7), where $A(t)$ is now defined by (13).

Let t^* be the unique positive integer such that:

(16) $h(t^*) \geq 0$ but $h(t^*+1) < 0$.

Using (14) and (15), it can be seen that this t^* solves the integer programming problem (7) so that the optimal asset value A^0 is again defined by (9).

The properties of Model 2 with respect to the real interest rate r are exactly the same as the properties of Model 1 above: if we increase the real interest rate r, then $h(t)$ defined by (15) does not change and hence t^* does not change as r changes. Thus *in this model, an increase in the real interest rate does not lead to a change in the length of time that the asset is used.*

By examining (14) for $t = t^*$ and noting that each term in square brackets is nonnegative and independent of r, it can be seen that *as the real interest rate increases, the asset value will decrease*; i.e., we have:

(17) $A^0(r',i) < A^0(r,i)$ for all $r' > r$.

Now consider the case where the marginal complementary cost functions shift upwards[2] to $M^*(t)$ so that

(18) $M^*(t) > M(t)$; $t = 1,2, \ldots$.

Let A(t) be defined by (14) and h(t) by (15), except that the functions $M^*(t)$ replace the functions M(t) for $t = 1,2, \ldots$. As usual, we define the optimal asset value by solving the integer programming problem (7) with our new definitions for the functions A(t). It can be seen that the new t^* defined by (16) will always be equal to or less than our previous t^* under our new assumptions. Hence *as the costs for complementary inputs increase, the optimal length of life t^* will decrease* (or at least not increase). It can also be seen that *as the costs for complementary inputs increase, the asset value will decrease.*

Conclusion

For the two classes of endogenous life of asset models that we have considered, a change in the real interest rate did not change the optimal asset life. Furthermore, an increase in the real interest rate led to a decrease in the asset value in both models. However, an increase in the profile of intertemporal complementary costs will lead to a shortened asset life and a decreased asset value.

References

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[2] It can be seen that Model 1 is a special case of Model 2.