The Measurement of Capital

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1. INTRODUCTION.

Capital measures are constructed for two main purposes: (1) to measure wealth (the market value of assets) and (2) to analyze the role of capital in production. Because capital is durable, the value of using it in any given year is not the same as the value of owning it. There are thus different measures of capital depending on the purpose of accounting. However, these different measures should be consistently derived from a single framework.

The scope of the discussion below is restricted to fixed assets and land, i.e., we do not deal with financial or intangible assets, inventories or environmental assets.

2. FUNDAMENTAL RELATIONS BETWEEN STOCKS AND FLOWS OF CAPITAL.

In equilibrium, the *stock value* of an asset is equal to the discounted stream of future rental payments for *capital services* that the asset is expected to yield, an insight that goes at least back to Walras (1874) and Böhm-Bawerk (1891).

Let the price of an n-period old asset purchased at the beginning of period t be P_n^t. When prices change over time, it is necessary to distinguish between the observable rental prices for the asset at different ages in period t and future expected rental prices. Let f_n^t be the rental price of an n-period old asset at the beginning of period t. Then the fundamental equation relating the stock value of an asset, P_n^t, to the sequence of rental prices by age, $\{f_n^t : n = 0,1,2,\ldots\}$ is:

$$(1)\ P_n^t = f_n^t + [(1+i^t)/(1+r^t)]\, f_{n+1}^t + [(1+i^t)/(1+r^t)]^2\, f_{n+2}^t + [(1+i^t)/(1+r^t)]^3\, f_{n+3}^t + \ldots \qquad n = 0, 1, 2,\ldots$$

where the i^t are expected rates of change of rental prices that are formed at the beginning of period t. For simplicity, it has been assumed that i^t does not depend on the asset's age. The term $1+r^t$ is the discount factor that makes a dollar received at the beginning of period t equivalent to a dollar received at the beginning of period t+1. Thus the r_n^t are one-period *nominal interest rates* where the assumption has been made that the term structure of interest rates is constant. However, as the period t changes, r^t and i^t can change. The sequence of stock prices $\{P_n^t\}$ is not affected by general inflation provided that it affects the expected asset inflation rates i_n^t and the nominal interest rates r_n^t in a proportional manner.

The rental prices $\{f_n^t\}$ are potentially observable. In producer equilibrium, the ratio of any pair of rental prices equals the relative marginal productivity of the corresponding capital goods; see Hulten (1990).

By successive insertion for different P_n^t, (1) can be transformed into

(2) $P_n^t = f_n^t + [(1+i^t)/(1+r^t)]\, P_{n+1}^t$ or
(3) $f_n^t = (1+r^t)^{-1}\, [P_n^t\,(1+r^t) - (1+i^t)\, P_{n+1}^t] = [P_n^t - (1+i^t)\, P_{n+1}^t/(1+r^t)]$; $\qquad n = 0,1,2,\ldots$

Christensen and Jorgenson (1969) derived a version of (3) for the geometric depreciation model and end of period rental payments. Other variants are due to Christensen and Jorgenson (1973), Diewert (1980) (2005), Jorgenson (1989), Hulten (1990) and Diewert and Lawrence (2000).

(3) represents the *rental price* or *user cost* of an n-year old asset: the cost of using it during a period is given by the difference between the purchase price at the beginning of the period P_n^t and the value of the depreciated asset $(1+i^t)\, P_{n+1}^t = P_{n+1}^{t+1}$ at the end of period t. Since this offset to the initial expense will only be received by the end of the period, it must be divided by the discount factor $(1+r^t)$.

3. DEPRECIATION, ASSET PRICES AND USER COSTS.

Depreciation is typically defined as the decline in asset value as one goes from an asset of a particular age to the next oldest at the same point in time; see Hicks (1939), Hulten and Wykoff (1981a) (1981b), Hulten (1990), Jorgenson (1996) and Triplett (1996). Define the *depreciation rates* δ_n^t for an asset that is n periods old at the start of period t as:

(4) $\delta_n^t \equiv 1 - [P_{n+1}^t/P_n^t]$; $\qquad n = 0,1,2,\ldots$

Thus, given $\{P_n^t\}$, the period t sequence of $\{\delta_n^t\}$ is determined. Conversely, given $\{\delta_n^t\}$ and the price of a new asset in period t, $\{P_n^t\}$ is determined.

(5) $P_n^t = (1 - \delta_0^t)(1 - \delta_1^t)\ldots(1 - \delta_{n-1}^t)\, P_0^t$; $\qquad n = 0,1,2,\ldots$

With expressions (5) and (3), the sequence of user costs $\{f_n^t\}$ can be expressed in terms of the price of a new asset at the beginning of period t, P_0^t, and $\{\delta_n^t\}$:

(6) $f_n^t = (1+r^t)^{-1}(1 - \delta_0^t)\ldots(1 - \delta_{n-1}^t)[(1+r^t) - (1+i^t)(1 - \delta_n^t)]\, P_0^t$
$\quad = (1+r^t)^{-1}[r^t + \delta_n^t\,(1+i^t) - i^t]\, P_n^t \qquad n = 0,1,2,\ldots$

Thus given any one of these sequences, all of the other sequences are completely determined. This means that assumptions about depreciation rates, the pattern of user costs by age or the pattern of asset prices by age cannot be made independently of each other. This point was first explicitly made by Jorgenson and Griliches (1967) (1972).

4. AGGREGATION.

Asset prices are relevant for the construction of *wealth measures* of capital and the user costs are relevant for the construction of *capital services measures*. Let there be N different types of assets and let the quantity of period t investment in asset i be I_i^t with a sequence of asset prices $\{P_{n,i}^t\}$. Then the value of the period t wealth stock is

$$(7)\ W_i^t \equiv P_{0,i}^t I_i^{t-1} + P_{1,i}^t I_i^{t-2} + P_{2,i}^t I_i^{t-3} + \ldots \qquad i = 1,2,\ldots N.$$

Turning to capital services, and neglecting issues of capital utilization, the flow of services that an asset of a particular age delivers is proportional to the corresponding quantity of past investment. The value of capital services for all ages of a given asset class i during period t using the sequence of user costs $\{f_{n,i}^t\}$ is

$$(8)\ S_i^t \equiv f_{0,i}^t I_i^{t-1} + f_{1,i}^t I_i^{t-2} + f_{2,i}^t I_i^{t-3} + \ldots \qquad i = 1,2,\ldots N.$$

The value aggregates W_i^t and S_i^t can be decomposed into separate price and quantity components by standard index number methods, if each new unit of capital lasts only a finite number of periods, L. Define the period t price, user cost and quantity vectors, P_i^t, f_i^t and K_i^t respectively, as follows:

$$(9)\ P_i^t \equiv [P_{0,i}^t,P_{1,i}^t,\ldots,P_{L-1,i}^t];\ \ f_i^t \equiv [f_{0,i}^t,f_{1,i}^t,\ldots,f_{L-1,i}^t];\ \ K_i^t \equiv [I_i^{t-1},I_i^{t-2},\ldots,I_i^{t-L-1}];\ i = 1,2\ldots,N.$$

Fixed base or chain indexes may be used to decompose value ratios into price change and quantity change components. The values of W_i^t and S_i^t relative to their values in the preceding period, W_i^{t-1}, S_i^{t-1} have the following index number decomposition:

$$(10)\ W_i^t / W_i^{t-1} = P_i^W (P_i^{t-1},P_i^t,K_i^{t-1},K_i^t)\ Q_i^W(P_i^{t-1},P_i^t,K_i^{t-1},K_i^t)\ ; \qquad i = 1,2,\ldots,N;$$

$$(11)\quad S_i^t / S_i^{t-1} = P_i^S (f_i^{t-1},f_i^t,K_i^{t-1},K_i^t)\ Q_i^S (f_i^{t-1},f_i^t,K_i^{t-1},K_i^t)\ ; \qquad i = 1,2,\ldots,N;$$

where P_i^W, P_i^S and Q_i^W, Q_i^S are bilateral price and quantity indexes respectively. In particular, Q_i^S measures the service flow of type i assets into production. It is thus an appropriate measure of capital input.

A functional form has to be chosen. For empirical work, Diewert (1976) (1992) has shown that the *Fisher* (1922) *ideal price and quantity indexes* appear to be "best" from the axiomatic viewpoint and can also be given strong economic justifications. The above index number approach to aggregating over vintages of capital was first suggested by Diewert and Lawrence (2000) and it is more general than the usual aggregation procedures for homogenous assets, which essentially assume that the different ages of the same capital good are perfectly substitutable so that linear aggregation techniques can be used.

However, most researchers use an index number approach to form price and quantity aggregates across *different* types of assets. The overall values of the period t wealth stock and capital services are respectively

(12) $W^t \equiv P_1^{W,t}Q_1^{W,t} + P_2^{W,t}Q_2^{W,t} + P_3^{W,t}Q_3^{W,t} + \ldots$
(13) $S^t \equiv P_1^{S,t}Q_1^{S,t} + P_2^{S,t}Q_2^{S,t} + P_3^{S,t}Q_3^{S,t} + \ldots$

Akin to (10)-(12), the value aggregates W^t and S^t can be decomposed into separate price and quantity components. Define the period t price and quantity vectors, $P^{W,t}$, $P^{S,t}$ and $K^{W,t}$, $K^{S,t}$ respectively, as follows:

(14) $P^{W,t} \equiv [P_1^{W,t},P_2^{W,t},\ldots,P_N^{W,t}]$; $P^{S,t} \equiv [P_1^{S,t},P_2^{S,t},\ldots,P_N^{S,t}]$; $K^{W,t} \equiv [K_1^{W,t},K_2^{W,t},\ldots,K_N^{W,t}]$;
$K^{S,t} \equiv [K_1^{S,t},K_2^{S,t},\ldots,K_N^{S,t}]$.

The values of W^t and S^t relative to their values in the preceding period, $W^{t-1}$and S^{t-1}, have the following index number decomposition:

(15) $W^t / W^{t-1} = P^W(P^{W,t-1},P^{W,t},K^{W,t-1},K^{W,t})\ Q^W(P^{W,t-1},P^{W,t},K^{W,t-1},K^{W,t})$;
(16) $S^t / S^{t-1} = P^S(P^{S,t-1},P^{S,t},K^{S,t-1},K^{S,t})\ Q^S(P^{S,t-1},P^{S,t},K^{S,t-1},K^{S,t})$

where P^W, P^S and Q^W, Q^S are bilateral price and quantity indexes respectively. In particular, Q^S measures the overall service flow of capital into production.

5. EMPIRICAL DETERMINATION OF RATES OF RETURN AND ASSET PRICE CHANGES.

Rates of return r^t can either be based on a balancing procedure or they can be based on market interest rates. The balancing procedure postulates that the value of capital services is equal to the value of gross operating surplus as shown by the national accounts plus the capital income of the self-employed. A rate of return is then chosen so that this equality holds. If market interest rates are used, there is still a choice between ex-ante and ex-post rates. The majority of empirical work on capital services has relied on an ex-post, balancing procedure based on Jorgenson and Griliches (1967) (1972) and Christensen and Jorgenson (1969). However, empirical problems arise when these methods yield highly volatile and sometimes negative user costs of capital. The debate has therefore continued – see Harper, Berndt and Wood (1989), Diewert (1980)(2005) and Schreyer (2006).

Possibilities for the choice of the asset inflation rates i^t include using the ex-post asset price changes (consistent with the ex-post, balancing procedure for rates of return), forecasting ex-ante rates on the basis of ex-post rates and assuming that expected asset price changes equal to general inflation. The latter implies that the term r^t-i^t in the user cost expression (6) becomes a real rate of return that is simple to measure and typically not too volatile. At the same time, the procedure may induce a bias in user costs and capital measures if the prices of different assets move with different trends and/or if asset prices move very differently from general inflation.

6. EMPIRICAL DETERMINATION OF RATES OF DEPRECIATION.

Possibilities for determining depreciation rates include: first, information on market prices of assets of different age at the same point in time can be used to derive measures of depreciation. Empirical studies include Hall (1971), Beidleman (1973), Hulten and Wykoff (1981a) (1981b) and Oliner (1996). The literature has been reviewed by Hulten and Wykoff (1996) and Jorgenson (1996). The second approach uses rental prices for assets where they exist, along with information on the rate of return and on asset prices to solve the user cost equation (6) for the rate of depreciation; for a review see Jorgenson (1996). The third approach is based on production function estimation where output is regressed on non-durable inputs and past investment. The estimated coefficients of the investment variable can be used to identify a constant rate of depreciation. Empirical studies using this approach include Epstein and Denny (1980), Pakes and Griliches (1984), Nadiri and Prucha (1996) and Doms (1996). Method four relies on insurance and other expert appraisals.

The fifth method makes assumptions about the relative efficiency sequence $\{f_n^t / f_0^t\}$ and the service life of assets and then derives, via (1) and (5), a consistent measure of the rate of depreciation. For example, the *One Hoss Shay* model of efficiency states that an asset yields a constant level of services throughout its useful life of L years: $f_n^t / f_0^t = 1$ for $n = 0,1,2,...,L-1$ and zero for $n = L, L+1, L+2,...$. Another example is a *Model of Linear Efficiency Decline* where the sequence $\{f_n^t / f_0^t\}$ is given by $f_n^t / f_0^t \equiv [L - n]/L$ for $n = 0,1,2,...,L - 1$ and zero for $n = L, L+1, L+2,...$.

The sixth method makes direct assumptions about the depreciation sequence $\{P_n^t / P_0^t\}$. The most frequent approaches are the *Straight Line Depreciation Model* and the *Geometric or Declining Balance Model*. Under the former, there is a constant amount of depreciation between every vintage: $P_n^t/P_0^t = [L - n]/L$ for $n = 0,1,2,...,L$ and zero for $n>L$. Under the latter, which dates back to Matheson (1910), there is a constant rate of depreciation $\delta_n^t = \delta$ for $n = 0,1,2,...$ The geometric model greatly simplifies the algebra of capital measurement and has been supported empirically through studies on used asset markets; see Hulten and Wykoff (1981a) (1981b). When there is only information on the average asset life L, the double declining balance method determines the rate of depreciation as $\delta = 2/(L+1)$.

BIBLIOGRAPHY

Beidelman, C. (1973), *Valuation of Used Capital Assets*, Sarasota Florida: American Accounting Association.

Böhm-Bawerk, E. V. (1891), *The Positive Theory of Capital*, W. Smart (translator of the original German book published in 1888), New York: G.E. Stechert.

Christensen, L.R. and D.W. Jorgenson (1969), "The Measurement of U.S. Real Capital Input, 1929-1967", *Review of Income and Wealth* 15, 293-320.

Christensen, L.R. and D.W. Jorgenson (1973), "Measuring the Performance of the Private Sector of the U.S. Economy, 1929-1969", pp. 233-351 in *Measuring Economic and Social Performance*, M. Moss (ed.), New York: Columbia University Press.

Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 115-145.

Diewert, W.E. (1980), "Aggregation Problems in the Measurement of Capital", pp. 433-528 in *The Measurement of Capital*, D. Usher (ed.), Chicago: The University of Chicago Press.

Diewert, W.E. (1992), "Fisher Ideal Output, Input and Productivity Indexes Revisited", *The Journal of Productivity Analysis* 3, 211-248.

Diewert, W.E. (2005), "Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes and Interest Rates", pp. 479-542 in *Measuring Capital in the New Economy*, C. Corrado, J. Haltiwanger and D. Sichel (eds.), Chicago: University of Chicago Press.

Diewert, W.E. and D.A. Lawrence (2000), "Progress in Measuring the Price and Quantity of Capital", pp. 273-326 in *Econometrics and the Cost of Capital: Essays in Honor of Dale W. Jorgenson*, L.J. Lau (ed.), Cambridge MA: The MIT Press.

Doms, M.E. (1996), "Estimating Capital Efficiency Schedules within Production Functions", *Economic Inquiry* 34, 78-92.

Epstein, L.G. and M. Denny (1980), "Endogenous Capital Utilization in a Short Run Production Model: Theory and Empirical Application", *Journal of Econometrics* 12, 189-207.

Fisher, I. (1922), *The Making of Index Numbers*, London: Macmillan and Company.

Hall, R.E. (1971), "The Measurement of Quality Change from Vintage Price Data", pp. 240-271 in *Price Indexes and Quality Change*, Z. Griliches (ed.), Cambridge Massachusetts: Harvard University Press.

Harper, M.J., E.R. Berndt and D.O. Wood (1989), "Rates of Return and Capital Aggregation Using Alternative Rental Prices", pp. 331-372 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge MA: The MIT Press.

Hicks, J.R. (1939), *Value and Capital*, Oxford: The Clarendon Press.

Hulten, C.R. (1990), "The Measurement of Capital", pp. 119-152 in *Fifty Years of Economic Measurement*, E.R. Berndt and J.E. Triplett (eds.), Studies in Income and Wealth, Volume 54, The National Bureau of Economic Research, Chicago: The University of Chicago Press.

Hulten, C.R. (1996), "Capital and Wealth in the Revised SNA", pp. 149-181 in *The New System of National Accounts*, J.W. Kendrick (ed.), New York: Kluwer Academic Publishers.

Hulten, C.R. and F.C. Wykoff (1981a), "The Estimation of Economic Depreciation using Vintage Asset Prices", *Journal of Econometrics* 15, 367-396.

Hulten, C.R. and F.C. Wykoff (1981b), "The Measurement of Economic Depreciation", pp. 81-125 in *Depreciation, Inflation and the Taxation of Income from Capital*, C.R. Hulten (ed.), Washington D.C.: The Urban Institute Press.

Hulten, C.R. and F.C. Wykoff (1996), "Issues in the Measurement of Economic Depreciation: Introductory Remarks", *Economic Inquiry* 34, 10-23.

Jorgenson, D.W. (1989), "Capital as a Factor of Production", pp. 1-35 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge MA: The MIT Press.

Jorgenson, D.W. (1996), "Empirical Studies of Depreciation", *Economic Inquiry* 34, 24-42.

Jorgenson, D.W. and Z. Griliches (1967), The Explanation of Productivity Change", *The Review of Economic Studies* 34, 249-283.

Jorgenson, D.W. and Z. Griliches (1972), "Issues in Growth Accounting: A Reply to Edward F. Denison", *Survey of Current Business* 52:4, Part II (May), 65-94.

Matheson, E. (1910), *Depreciation of Factories, Mines and Industrial Undertakings and their Valuations*, Fourth Edition, (First Edition 1884), London: Spon.

Nadiri, M.I. and I.R. Prucha (1996), "Estimation of the Depreciation Rate of Physical and R and D Capital in the U.S. Total Manufacturing Sector", *Economic Inquiry* 34, 43-56.

Oliner, S.D. (1996), "New Evidence on the Retirement and Depreciation of Machine Tools", *Economic Inquiry* 34, 57-77.

Pakes, A. and Z. Griliches (1984), "Estimating Distributed Lags in Short Panels with an Application to the Specification of Depreciation Patterns and Capital Stock Constructs", *Review of Economic Studies* 51, 243-262.

Schreyer, P. (2006); "Measuring Multi-factor Productivity when Rates of Return are Exogenous"; in: W. Erwin Diewert, Bert M. Balk, Dennis Fixler, Kevin J. Fox and Alice O. Nakamura (eds.), *Price and Productivity Measurement* Volumes 1 and 2, Trafford Press.

Triplett, J.E. (1996), "Depreciation in Production Analysis and in Income and Wealth Accounts: Resolution of an Old Debate", *Economic Inquiry* 34, 93-115.

Walras, L. (1954), *Elements of Pure Economics*, a translation by W. Jaffé of the Edition Définitive (1926) of the *Eléments d'économie pure*, first edition published in 1874, Homewood, Illinois: Richard D. Irwin.