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Concepts and Measures of Productivity: An Introduction

INTRODUCTION

THIS VOLUME IS FILLED with estimates and analyses of productivity. But what is productivity? It seems to be like love in that everyone knows they want it, but few have a good definition of it. As the following quotations demonstrate, several different types of productivity measures are used in the studies in this volume:

> "Even more striking is the growth of labour productivity in telecommunications services…."
>
> (Chen, chapter 12)

> "The factor driving Canada's superior business sector services labour productivity growth has been better multifactor productivity growth…."
>
> (Rao, Sharpe and Tang, chapter 14)

> "[Information communications technology (ICT)] contributes to economy-wide total factor productivity growth."
>
> (Wernerheim and Sharpe, chapter 13)

This study defines different types of productivity measures and draws distinctions among them. A production process can be thought of as a black box with purchased inputs taken in on one side and outputs sold out the other. Measures of productivity assess how well the black box is doing at turning quantities of inputs into quantities of outputs. Different productivity measures standardize for and provide a basis for different types of comparisons. In this study, we demonstrate the importance of distinguishing between measures of productivity level and productivity growth.

Also, some authors flip between discussing productivity and remarks about prices and price indexes. We explain the connection. We address these and other issues while introducing the reader to the language and formulas of productivity measurement. In other areas of life, everyone recognizes the difference between "levels" and "growth," the latter being a comparative assessment. "I love you" is a "level" type of declaration. The declaration itself is unconditional and neither limits nor recommends standards of comparison for the declaration. The recipient of the declaration, however, can choose to compare it with things the other person said the day or year before, or is reported to have said to others, or things others have said to them. In contrast, statements such as "I love you more than anyone before" or "I've grown to love you" specify a basis of comparison. Similar considerations hold in differentiating between measures of productivity level and productivity growth.

In this volume, there are discussions of various aspects of production and the circumstances that may affect productivity. For instance, mention is made of "resource allocation improvements" (Whalley, chapter 7); of "poor R&D performance" linked to "Canada's productivity gap" (Hejazi, chapter 8); of "agglomeration economies that offer productivity enhancing opportunities" (Globerman, Shapiro and Vining, chapter 6); of how "ICT contributes to economy-wide total factor productivity growth" (Wernerheim and Sharpe, chapter 13); and of how "innovativeness has improved… operating efficiency" (Neave, chapter 11). It is important to keep in mind that these are not alternative forms or definitions of productivity. Lipsey is right in the keynote address that is included in this volume when he cautions that the usual productivity indexes such as total factor productivity (TFP) are not measures of technological change:

> "[A]s it is measured in practice, changes in TFP emphatically do not measure changes in technology, in spite of the common belief that they do."
>
> (Lipsey, chapter 3)

Historically, industries with strong productivity growth have often had rising wages. Interest in understanding the interrelationships between productivity growth and wage-rate changes is reflected in many of the studies in this volume such as Acharya's:

> "[W]e combine the insights on output and employment and discuss productivity growth and the wage distribution."
>
> (Acharya, chapter 4)

Indeed, some researchers (Wölfl, chapter 9) imply that observed relative wages or wage trends might be used to support or question reported productivity results for specific industries. However, results in other studies in this volume point to the fact that productivity growth, employment growth and wage growth do not always go together:

"Our main finding is that even though employment grew much faster in high-knowledge industries than in other sectors during the last two decades, trends in relative wages and real wages of university and high school graduates have displayed remarkably similar patterns across industries. In other words, the acceleration of employment growth in high-knowledge industries has not been accompanied by an acceleration of real and relative wages of university graduates in this sector (relative to other sectors) …"

<div align="right">(Picot, Morissette and Ostrovsky, chapter 5)</div>

Examinations of relative wages or wage trends cannot substitute for productivity analysis.

Yet for many industries we lack the price and quantity information needed for productivity measurement. This reality is driven home in several of the studies:

"[T]he work presented in this conference volume is a useful reminder of how little we still know about the services economy — how poor and too highly-aggregated (if nonetheless improving) services sector data continues to be relative to manufacturing; … how difficult it is to measure labour and total factor productivity in fields where output takes intangible forms, such as in health care and education."

<div align="right">(Sauvé, chapter 16)</div>

"Without proper price indexes, it will not be possible to measure the real output of these new [National American Industrial Classification System (NAICS)] industries with any degree of accuracy. This in turn implies that it will not be possible to measure the productivity … with any degree of accuracy."

<div align="right">(Diewert, chapter 15</div>

"Diewert and Fox (1999)… argue that the proliferation of new products and new processes could have led to a systematic underestimation of productivity growth. This measurement problem could be the reason that we even see negative productivity growth in some services industries for a long period of time!"

<div align="right">(Acharya, chapter 4)</div>

This study is a methodological introduction to the studies in this volume. It constitutes a crash course on the measures of productivity level and growth used in these research studies. There is an emphasis on measures of total factor productivity (TFP) and total factor productivity growth (TFPG) in part because the other measures commonly used can be viewed as special cases of these two fundamental indicators. We use them to describe the production scenario under consideration in relationship to a comparison scenario ("s"). The comparison scenario could represent an earlier time period for the same production unit or a different production unit for the same time period.

Basic definitions are introduced in the following section.

Formulas for the productivity measures are first introduced in the simplest possible context of activities embodying one input and one output. Of course, most production units have multiple outputs, and virtually all use multiple inputs. Nevertheless, it helps to begin with a 1–1 process before moving on to a general production process with N inputs and M outputs. That is because in the 1–1 case, there is no need to add up the quantities of different types of inputs or outputs to form total input and output variables.

This study then proceeds to present an analysis that is broadened to include two inputs that are used to produce one output. This introduces some of the problems that must be faced with multiple inputs or outputs.

There are different sorts of formulas that can be used for adding up the quantities of different inputs and outputs. All of the common ones involve using price information (or value share, which embodies price information) to calculate weightings for the quantities to be added. This includes the Paasche, Laspeyres and Fisher formulas introduced later. The Paasche and Laspeyres formulas are the ones most commonly mentioned in general economics, business statistics and accounting textbooks. We demonstrate by example how a Laspeyres-type productivity index controls for price change and, by analogy, how the Paasche productivity index does this as well. This is followed by a demonstration of how the Fisher formula relates to the formulas of Paasche and Laspeyres.[1] An appendix describes the Törnqvist formula which is widely used by productivity researchers including a number of the authors in this volume. The Törnqvist formula approximates Fisher's.[2]

The study concludes with a summary of key points for understanding productivity measures.

DIFFERENT TYPES OF PRODUCTIVITY MEASURES

THIS VOLUME CONTAINS REFERENCES to the following productivity level indexes:

- Single factor productivity (SFP) defined as the ratio of a measure of output quantity to the quantity of a single input used.

- Labour productivity (LP) defined as the ratio of a measure of output quantity to some measure of the quantity of labour used, such as total hours worked.

- Multifactor productivity (MFP) defined as the ratio of a measure of output quantity to a measure of the quantity of a bundle of inputs often intended to approximate total input.

- Total factor productivity (TFP) defined as the ratio of a measure of total

output quantity to a measure of the quantity of total input.[3]

Most of the usual productivity growth measures can be defined in terms of the growth[4] or change from s to t in an associated productivity level measure, where t denotes the production scenario of interest and s denotes the comparison scenario.[5] Thus, we usually have

(1) $SFPG^{s,t} = SFP^t / SFP^s$,

(2) $LPG^{s,t} = LP^t / LP^s$,

(3) $MFPG^{s,t} = MFP^t / MFP^s$, and

(4) $TFPG^{s,t} = TFP^t / TFP^s$.

All of the productivity indexes we consider have some measure of output quantity or change in the numerator and some measure of input quantity or change in the denominator. A key issue in the construction of variables of input and output quantity is that they should only change in response to changes in *quantity*. If a factory produces a constant 10 widgets a day as its output, the output quantity measure should reflect this constancy in output quantity, even if the price for the widgets and the revenues generated change daily. If only one good is under consideration, quantity data can be used directly, without any price or value share information. In contrast, "constant" relative price or value share information is needed when multiple inputs or outputs are involved. In the section on the general N input and M output case below, we demonstrate how this adding up problem is handled in productivity measurement.

PRODUCTIVITY MEASURES FOR THE
ONE INPUT–ONE OUTPUT CASE

MOST PEOPLE WOULD PREFER that mathematical notation, like taxes, be kept to the minimum needed to accomplish the objectives desired. Hence our notation for the 1–1 case is chosen so that we can continue using the same conventions with multiple inputs and outputs. The quantity of input 1 for production scenario t is x_1^t . Following the same conventions, the price for input 1 is w_1^t , and the quantity and price of output 1 are y_1^t and p_1^t .

When labour is the only input, the whole collection of productivity level measures — *SFP, LP, MFP* and *TFP* — are the same. We have:

(5) $SFP = LP = MFP = TFP = (y_1^t / x_1^t)$.

For this 1–1 case, the productivity growth measures are also the same. We have *SFPG=LPG=MFPG=TFPG*, which is the case dealt with in this section.

It is a convenient starting point for establishing some productivity measurement basics.

Even when labour is the only input — so that the single factor, labour, multifactor and total factor measures are all the same — it turns out that there are still several ways of thinking about productivity growth. These different concepts lead to measures that can be shown to be rearrangements of the same thing. Such different concepts, however, are useful when thinking about different sorts of policy problems.

Examples can be helpful for understanding the meaning of formulas. We have set up some hypothetical car-wash production scenarios to clear up misunderstandings about productivity measurement.

In the first scenario, we use the following small-town hand car wash operation:

> Two new operators were hired at $8 per hour for 8-hour days. The first day, they each washed 1 car per hour. They did 2 an hour on days 2 and 3. Customers paid $10 for a car wash. The specifics of the scenario are summarized in rows 1-4 of Table 1.

Labour productivity level values are shown in row 6 of Table 1. Labour is the only input, so these are also *TFP* values. Measured productivity rose from day 1 to day 2, but there was no technological change. The new operators simply got faster at doing a job that has been carried out in much the same way since the days of the Model T. This illustrates Lipsey's point that these indexes should not be viewed as measures of technological change.

Productivity level measures do not dictate standards of comparison. It is up to those using the results of these measures to be sensible about the comparisons they choose to make. In contrast, productivity growth measures build in a standard of comparison. This is the key difference between productivity level and growth measures. Suppose some standard of comparison — comparison scenario s — has been selected. Then, there are several ways that a productivity growth index can be conceptualized. The first is as the rate of growth for the corresponding productivity level index. *TFPG*, defined conceptually as the rate of growth over time for *TFP* and denoted here by *TFPG*(1), can be represented for the 1-1 case as:

$$(6) \qquad TFPG(1) = \left(\frac{y_1^t}{x_1^t}\right) \Big/ \left(\frac{y_1^s}{x_1^s}\right).$$

Alternatively, *TFPG* could be conceptualized in terms of how the growth in output compares with the growth in input. *TFPG* could be defined as the ratio of the output growth rate, y_1^t / y_1^s, and the input growth rate, x_1^t / x_1^s. Thus, for this second concept of *TFPG* we have:

(7) $$TFPG(2) = \left(\frac{y_1^t}{y_1^s}\right) \Big/ \left(\frac{x_1^t}{x_1^s}\right).$$

Expressions for revenue and cost are needed to implement a third concept of *TFPG*: the ratio of the growth rates for real revenue and real cost. For the 1-1 case, revenue and cost are given, respectively, by

(8) $$R^t = p_1^t y_1^t \text{ and } C^t = w_1^t x_1^t.$$

Thus, the third concept of *TFPG* can be represented as

(9) $$TFPG(3) = \left[\frac{R^t / R^s}{p_1^t / p_1^s}\right] \Big/ \left[\frac{C^t / C^s}{w_1^t / w_1^s}\right].$$

Diewert and Nakamura (2003, 2005) have shown that the formulas for *TFPG*(1), *TFPG*(2) and *TFPG*(3) are equal even for the general case of N inputs and M outputs when they are applied to the types of functional forms introduced below in the discussion of this general case. Hence the same productivity numbers will result no matter which of these three concepts of *TFPG* is adopted. In contrast, the nature of a *TFPG* measure will differ greatly depending on the choice of a comparison scenario s. This is even so in the simple 1-1 case.

Past performance can be used as a standard of comparison. Comparisons to the previous period are common in applied research, with the previous period often being the previous year.[6] In our car-wash example, if we let $s=t–1$, then the *TFPG* values are the ratios for the current to the previous day's productivity. These productivity growth values are shown in row 7 of Table 1.[7]

Alternatively, we could compare the performance in period t with the performance for some fixed choice for the comparison scenario s. For instance, a series of productivity comparisons could be made with some base year. In our car-wash example, we might use a fixed day — say, day 1 — as the standard of comparison. Then we would get the *TFPG* values in row 8 of Table 1.

The *TFP* figures in Table 1, row 6, which are also the labour productivity figures for this example, and the *TFPG* figures in Table 1, rows 7 and 8 all confirm that productivity rose from day 1 to 2.[8] However, from day 2 to 3, the figures in Table 1, rows 6 and 8 stay the same but those in row 7 fall. Depending on the selected basis of comparison, the *TFPG* values move differently. A value of 1 in row 7 means no change in productivity from the previous day, in accordance with the results in rows 6 and 8.[9] The choice of a standard of comparison has implications for addressing different sorts of questions about productivity.

	DAY (T)		
	$T=1$	$T=2$	$T=3$
1. Operator hours: x_1^t	16 hours	16 hours	16 hours
2. Operator wage: w_1^t	$8	$8	$8
3. Cars washed per day: y_1^t	16 cars	32 cars	32 cars
4. Price per car wash: p_1^t	$10	$10	$10
5. Revenue/cost: R^t/C^t	$160/$128=1.25	$320/$128=2.5	$320/$128=2.5
6. LP=TFP: y_1^t/x_1^t	16 cars/16 hours=1	32 cars/16 hours=2	32 cars/16 hours=2
7. TFPG with s=day t-1	—		
8. TFPG with s=day 1			$\dfrac{32\,\text{cars/16 hours}}{16\,\text{cars/16 hours}}=2$

TABLE 1

SMALL-TOWN HAND CAR WASH

Interest in productivity often stems from an interest in maintaining or improving the revenue return on cost expenditures. The third concept of productivity growth is useful for examining this. Equation (9) representing the third concept of *TFPG* can be rewritten as a formula that breaks down growth in the revenue/cost ratio into two terms: a productivity growth term which is the growth in the rate of conversion of input into output, and a term for the output versus input price growth:

$$(10) \qquad \frac{(R^t/C^t)}{(R^s/C^s)} = \left[\frac{y_1^t/x_1^t}{y_1^s/x_1^s}\right]\left[\frac{p_1^t/p_1^s}{w_1^t/w_1^s}\right] = TFPG^{s,t}\left[\frac{p_1^t/p_1^s}{w_1^t/w_1^s}\right].$$

Suppose we would also like to compare the productivity of the small town car wash with a hypothetical larger-volume city operation that has the following specifics, which are also shown in rows 1–4 of Table 2:

> On days 1, 2 and 3, the city car wash has 4, 5 and 6 operators, respectively, working at $12 per hour for 8-hour days. They washed an average of 3, 2.5 and 2 cars per day over the period of days 1–3. Customers paid $20 a car wash.

The figures in row 5 of Tables 1 and 2 show that the city operation earns more per dollar of cost expenditure. Also, the figures in row 6 show that the daily labour productivity levels — the cars washed per operator-hour — are as high or higher on all days for the city operation. Yet, the *TFPG* figures in rows 7 and 8 are lower for the city operation.

TABLE 2

CITY CAR WASH

	DAY (T)		
	$T=1$	$T=2$	$T=3$
1. Operator hours: x_1^1	32 hours	40 hours	48 hours
2. Operator wage: w_1^1	$12	$12	$12
3. Cars washed per day: y_1^1	96 cars	100 cars	96 cars
4. Price per car wash: p_1^1	$20	$20	$20
5. Revenue/cost ratio: R^1/C^1	$1920/$384=5	$2000/$480=4.2	$1920/$576=3.3
6. $LP=TFP$: y_1^1/x_1^1	96 cars/32 hours=3	100 cars/40 hours=2.5	96 cars/48 hours=2
7. TFPG for s=day t-1	—	2.5/3=.83	2/2.5=.8
8. TFPG with s=day 1	—	2.5/3=.83	2/3=.67
9. TFPG with the small town car wash figures used as the standard of comparison	3/1 = 3	2.5/2 = 1.25	2/2 = 1

The figures in row 9 of Table 3 were computed using the small-town car wash as the standard of comparison for the city car wash. These show that the city car wash was more productive on days 1 and 2, and equally so on day 3. This information could not be gleaned from just the figures in rows 7 and 8 for productivity growth over time for the two different production units.

The figures in rows 7-9 of Table 2 illustrate that estimates of productivity growth over time cannot be used to examine the *relative* productivity levels for different production units.[10] When there is interest in making comparisons for different productive units such as different industries, then productivity level measures must be used or two-way comparisons must be made using one production unit in each pair as a standard of comparison for the other one. This is why Industry Canada often produces and often focuses on measures of productivity levels.[11]

THE TWO INPUT, ONE OUTPUT CASE

WE NEXT USE A SLIGHTLY MORE COMPLEX production process as the context for introducing choices that must be faced with multiple inputs or outputs.

Our small-town car wash company rents a car-wash machine for $100 per day, with a first day introductory rate of $50. Suppose this machine can handle up to 100 cars per 8-hour day with 1 operator. Hence, operator hours are 8 per day less than before.

Input costs at current prices are higher than the costs without the machine (days 1–3 shown in row 3 of Table 1). The machine rental is more than double the cost of the operator who was fired, and the remaining single operator pushed for and got a raise to $12 per hour on day 6. However, the owner plans on being able to increase volume, so the machine may save money over time.[12] This illustrates Lipsey's point that a change in technology will not necessarily increase the measured productivity at that time.

> Suppose 32 cars are washed on day $t=4$, which is the first day for the new machine. On day $t=5$, the car wash has a half price sale which brings in 40 cars to wash. On day $t=6$, there are also 40 cars to wash even though the sale has ended.[13]

Notice that the labour productivity numbers (cars washed per operator hour) in Table 3, row 7 are higher than the old figures for the small-town car wash (Table 1, row 6). Give a worker a machine and that worker will produce more! However, the operation is not more profitable. The revenue/cost figures in row 8 of Table 3 are mostly lower than the Table 1 figures.

A common reason given for using labour productivity measures is that the data are lacking to compute a more comprehensive productivity measure. But this is not a good reason for making inappropriate comparisons that could yield misleading results and wrong choices.[14]

This example, however, also makes it clear why looking just at the profit rate, or the revenue/cost ratio, is not satisfactory either. The revenue/cost ratio figures in row 8, Table 3 change greatly from day to day. This effect could be attributable to either a productivity change or a price change. To find out which, we need a way of measuring productivity that takes account of both inputs — operator time and machine time — but controls for the effects of price change.

TABLE 3

LABOUR AND TOTAL FACTOR PRODUCTIVITY FOR A SMALL-TOWN AUTOMATIC CAR WASH

	DAY (T)		
	$T=4$	$T=5$	$T=6$
1. Cars washed per day	32	40	40
2. Price per car washed	$10	$5	$10
3. Operator hours: x_1^1	8 hours	8 hours	8 hours
4. Operator hourly wage: w_1^t	$8	$8	$12
5. Car wash machine: x_2^t	1 machine	1 machine	1 machine
6. Car wash machine daily rental: w_2^t	$50	$100	$100
7. Cars washed per operator hour: y_1^t/x_1^t	32/8 = 4	40/8 = 5	40/8 = 5
8. Revenue/cost ratio: R^t/C^t	$320/$114 = 2.81	$200/$164 = 1.22	$400/$196 = 2.04
9. Sales/total input evaluated at day 4 prices: $p_1^4 y_1^t/(w_1^4 x_1^t + w_2^4 x_2^t)$	$320/$114 = 2.81	$400/$114 = 3.51	$400/$114 = 3.51
10. $TFPG_L^{4,t}$	—	3.51/2.81 = 1.25	3.51/2.81 = 1.25

One way to form a total input quantity measure when there are two inputs is to use current-period price weights for the quantities. An advantage of current-period price weights is that they represent the current opportunity cost of using one more unit of each associated input. Notice that the numerator and denominator of the revenue/cost ratio are current price-weighted sums of the quantities of the outputs (1 in this case) and the inputs (2 in this case). However, as is clear from our example, the revenue/cost ratio also reflects the *price changes* from period to period. It can change even when there are no changes in input or output quantities. For example, from day 5 to 6 in our example in Table 3, there is no change in either the output or the input quantities. Hence, there should be no change in the productivity level measure. But from Table 3, row 8 we see that the revenue/cost ratio almost doubles because of the price changes.

To deal with the problem of changing price weights, we could instead use the prices from some fixed comparison scenario such as a previous time period for the same production unit. In row 9 of Table 3, we use day 4 as the comparison scenario; that is, we let $s=4$. This embeds the relative price values of that particular time period into the resulting productivity measures: the time period was one when the relative prices were similar but not the same as in period t. In row 9 of Table 3 we show values for the ratio of output to input, all evaluated at day 4 prices. That is, we show values for the following type of productivity level expression that we will refer to as a Laspeyres-type measure since Laspeyres indexes use comparison scenario weights:

(11) $p_1^s y_1^t/(w_1^s x_1^t + w_2^s x_2^t)$.

For our Table 3 example, if we divide the day 5 row 9 value by the day 4 value, this gives the value of the Laspeyres productivity growth index for $t = 5$ and $s = 4$. And if we divide the day 6 row 9 value by the day 5 one, this gives the value of the Laspeyres productivity growth index for $t = 6$ and $s = 4$. These are the values shown in row 10 of Table 3.

If we chose some other comparison period — such as $s = 6$ — then the resulting productivity and productivity growth measures would embed the relative prices of that period. In particular, they would embed the opportunity costs/gains or changes in the relative amounts used or produced for the inputs and outputs. These choices are made in different ways in the productivity index formulas introduced in the next section. It is necessary first to define the time period over which productivity levels comparisons are to be made, or for which productivity growth measures are to be computed. Once this is selected, the Laspeyres approach is to use the price weights from the start of that time interval. By contrast, the Paasche approach is to use the price weights from the end of the period. The Fisher productivity index uses a geometric average of the Laspeyres and Paasche results.

THE GENERAL N INPUT, M OUTPUT CASE

THE SIMPLEST SORT OF PRODUCTION PROCESS is one with a single input and single output. In that simple context, we were able to introduce the distinction between level and growth (or comparison) measures of productivity as well as three different concepts of *TFPG* that can be useful in policy analysis and that all can be evaluated using the same computational formula. We also discussed the significance of the choice of the comparison scenario for productivity growth measures. Next we added one more input. This introduced the adding up issues that must be confronted as soon as there is more than one input or output.

It can be seen from the material in the previous section that the weights for the input and output quantity aggregates can greatly affect the computed productivity measures.

For a general production process involving N inputs and M outputs, the Laspeyres, Paasche and Fisher productivity measures can be defined using eight price-weighted sums of quantity data for the production scenario of interest (t) and the one used as the base line comparison (s). The first four of these sums are the total costs and revenue for t (C^t and R^t) and for s (C^s and R^s):

(12) $$C^t = \sum_{n=1}^{N} w_n^t x_n^t \ , \ R^t = \sum_{m=1}^{M} p_m^t y_m^t \ ,$$

(13) $$C^s = \sum_{n=1}^{N} w_n^s x_n^s \ \text{ and } \ R^s = \sum_{m=1}^{M} p_m^s y_m^s \ .$$

Four hypothetical quantity aggregates are also needed.[15] The first two result from evaluating period t quantities using period s price weights:

(14) $\qquad \sum_{n=1}^{N} w_n^s x_n^t$ and $\sum_{m=1}^{M} p_m^s y_m^t$

These sums are what the cost and revenue would have been if the period t inputs had been purchased and the period t outputs had been sold at period s prices. In contrast, the third and fourth aggregates are sums of period s quantities evaluated using period t prices:

(15) $\qquad \sum_{n=1}^{N} w_n^t x_n^s$ and $\sum_{m=1}^{M} p_m^t y_m^s$.

These are what the cost and revenue would have been if the period s inputs had been purchased and the period s outputs had been sold at period t prices.

A Laspeyres-type *TFP* index can be defined as:

(16) $\qquad TFP_L^{t|s} = \sum_{m=1}^{M} p_m^s y_m^t / \sum_{n=1}^{N} w_n^s x_n^t$.

Equation (11) in the previous section is a special case of this formula. Values for this productivity level index can be meaningfully compared over the time interval of period s to t provided that relative prices have not shifted too much over that time interval.

The corresponding productivity growth measure is given by:

(17) $\qquad TFPG_L^{s,t} = \left[\dfrac{\sum_{m=1}^{M} p_m^s y_m^t}{\sum_{n=1}^{N} w_n^s x_n^t} \right] \Big/ \left[\dfrac{\sum_{m=1}^{M} p_m^s y_m^s}{\sum_{n=1}^{N} w_n^s x_n^s} \right]$.

Suppose that values for the Laspeyres type productivity level index defined in (16) are computed for period $t = s, \ldots, T$. The measure embeds period s relative prices over the entire time interval of s through T. The longer this time interval is and the greater the amount of relative price change there was over this interval, the less satisfactory the productivity level index given in Equation (16) will be. This is why it is common to use $s=t–1$ for the Laspeyres productivity growth index, so that the price weights are only being held fixed for a two-period stretch. For a longer time interval, a series of period-to-period productivity growth estimates can be computed.

Along the lines of the concept 3 form of the *TFPG* index for the 1-1 case given in Equation (9), it has been shown that the Laspeyres productivity growth index given in Equation (17) can also be defined in terms of revenue and cost totals converted to period s dollar terms using the Paasche output and input price indexes.[16] Thus we have:

(18) $$TFPG_L^{s,t} = \frac{(R^t / R^s) / P_P^{s,t}}{(C^t / C^s) / P_P^{s,t}} \, .$$

The output and input price indexes are given, respectively, by:

(19) $$P_P \equiv \sum_{i=1}^{M} p_i^t y_i^t \, / \, \sum_{j=1}^{M} p_j^s y_j^t \quad \text{and}$$

(20) $$P_P^* \equiv \sum_{i=1}^{N} w_i^t x_i^t \, / \, \sum_{j=1}^{N} w_j^s x_j^t \, .$$

There is no satisfactory Paasche-type counterpart of the Laspeyres-type productivity level index.[17] However, the Paasche *TFP* growth measure controls for price change by fixing the price weights at their period t values. That is, we have:

(21) $$TFPG_P^{s,t} = \left(\frac{\sum_{m=1}^{M} p_m^t y_m^t}{\sum_{n=1}^{N} w_n^t x_n^t} \right) \Big/ \left(\frac{\sum_{m=1}^{M} p_m^t y_m^s}{\sum_{n=1}^{N} w_n^t x_n^s} \right) \, .$$

A Paasche productivity growth measure embeds period t relative prices for both periods s and t. As with the Laspeyres productivity growth index, when there is a need to assess productivity growth over a longer time span, say from $t = s, \ldots, T$, it is common to compute the productivity growth measure for each successive value of t taking the comparison period for that "chain link" productivity estimate to be period t–1. Hence the price weights for each productivity growth calculation are just held fixed over a two-period time span.

It has been shown that this same Paasche productivity growth index given in Equation (17) can also be defined in terms of revenue and cost totals, converted to period s dollar terms using the Laspeyres output and input price indexes.[18] This alternative formulation of the Paasche productivity growth index is given by:

(22) $$TFPG_P^{s,t} = \frac{(R^t / R^s) / P_L^{s,t}}{(C^t / C^s) / P_L^{*s,t}} \, .$$

The Laspeyres output and input price indexes are given by:

(23) $$P_L \equiv \sum_{i=1}^{M} p_i^t y_i^s \, / \, \sum_{j=1}^{M} p_j^s y_j^s \quad \text{and}$$

(24) $$P_L^* \equiv \sum_{i=1}^{N} w_i^t x_i^s \, / \, \sum_{j=1}^{N} w_j^s x_j^s \, .$$

A Paasche-type productivity measure embeds period t relative prices for both periods s and t. Rather than choosing between the Laspeyres and Paasche

productivity growth indexes, Diewert (1992b) recommends using a geometric average of the two. This is the Fisher index and it is given by:

(25) $\qquad TFPG_F^t = (TFPG_P^t \times TFPG_L^t)^{1/2}$.

CONCLUSIONS

OUR FINDINGS can be summarized as follows:

- Most production processes involve multiple outputs and virtually all involve multiple inputs, in which case the choice of the productivity measure matters. Indeed, even with just 1 input and 1 output, it matters whether a productivity level or growth index is used.

- Productivity growth indexes build in a standard of comparison but productivity level indexes do not. With productivity growth measures, it is important to notice whether the standard of comparison is suitable for the intended uses of the productivity estimates. For instance, if a comparison over time is built into a productivity growth measure, it will not usually be appropriate to compare the resulting estimates with figures for other production units. Productivity level index values can be compared in whatever ways are deemed sensible. In this respect, they can be used more flexibly than the productivity growth figures.

- The fact that the value of productivity *growth* is higher for one production unit than for another (e.g. for a particular industry or sector or nation as compared with another industry, sector or nation) says nothing about which one has the higher productivity level.

- For a productivity growth index, a value of 1 means that, relative to the standard of comparison built into the productivity growth index, productivity is unchanged, whereas a value greater than (less then) 1 means that, relative to the standard of comparison scenario, productivity has increased (decreased).

- A productivity growth index can take on a value different from 1 with, or without, any change in technology over the time interval for which the index is calculated.

- Productivity level measures that embed relative price information from some given comparison period should not be used for computing productivity level or growth estimates in production scenarios where the actual relative prices are very different from those in the selected comparison period.

APPENDIX

THE TÖRNQVIST (OR TRANSLOG) INDEXES

TÖRNQVIST INDEXES ARE WEIGHTED GEOMETRIC AVERAGES of growth rates for micro-economic data (the quantity or price relatives).[19] These indexes have been widely used by national statistical agencies and in the economics literature. The formula for the natural logarithm of a Törnqvist index is the one that is usually shown. For the output quantity index, this is

$$(\text{A-1}) \quad \ln Q_T = (1/2) \sum_{m=1}^{M} [(p_m^s y_m^s / \sum_{i=1}^{M} p_i^s y_i^s) + (p_m^t y_m^t / \sum_{j=1}^{M} p_j^t y_j^t)] \ln(y_m^t / y_m^s).$$

The Törnqvist input quantity index Q_T^* is defined analogously, with input quantities and prices substituted for the output quantities and prices in Equation (12).

Reversing the role of the prices and quantities in the formula for the Törnqvist output quantity index yields the Törnqvist output price index, P_T, defined by

$$(\text{A-2}) \quad \ln P_T = (1/2) \sum_{m=1}^{M} [(p_m^s y_m^s / \sum_{i=1}^{M} p_i^s y_i^s) + (p_m^t y_m^t / \sum_{j=1}^{M} p_j^t y_j^t)] \ln(p_m^t / p_m^s).$$

The input price index P_T^* is defined in a similar manner.

The implicit Törnqvist output quantity index, $Q_{\widetilde{T}}$, is defined by $(R^t/R^s)/P_T \equiv Q_{\widetilde{T}}$,[20] and the implicit Törnqvist input quantity index, $Q_{\widetilde{T}}^*$, is defined analogously using the cost ratio and P_T^*. The implicit Törnqvist output price index, $P_{\widetilde{T}}$, is given by $(R^t/R^s)/Q_T \equiv P_{\widetilde{T}}$, and the implicit Törnqvist input price index, $P_{\widetilde{T}}^*$, is defined analogously.

Diewert coined the term "superlative" to describe an index number functional form that is "exact" in that it can be derived algebraically from a producer or consumer behavioural equation that satisfies Diewert's flexibility criterion: it can provide a second-order approximation to an arbitrary twice continuously differentiable linearly homogeneous function. Diewert (1976, 1978) and Hill (2000) established that all of the commonly-used superlative index number formulas, including the Fisher, the Törnqvist and implicit Törnqvist, approximate each other to the second order when evaluated at an equal price and quantity point. This is a numerical analysis approximation result that does not rely on any assumptions of economic theory.

ENDNOTES

1   The Fisher formula is increasingly being used for official statistics purposes in Canada and the United States. Diewert (1992b) provides an analysis of the properties of the Fisher index.

2   For example, in the data Appendix to their study in this volume, Rao, Sharpe and Tang write: "The data source for the U.S. data is Jorgenson, Ho and Stiroh (2002). For their study, they have developed such a dataset for 44 industries, which are collapsed into 34 common industries using Törnqvist aggregation indexes. The Canadian data are obtained from the Canadian Productivity Accounts that provide a consistent set of detailed industry (122 industries) and aggregated data on inputs and outputs (current prices and chained Fisher indexes) for productivity measurement and related economic performance analysis."

3   It is almost never the case that *all* inputs are included in a productivity study. This is why official agencies tend to prefer the terms multifactor productivity (MFP) and multifactor productivity growth (MFPG) instead of total factor productivity (TFP) and total factor productivity growth (TFPG). However, the TFP and TFPG terminology has caught on in the economics literature and the popular press. Also, there are useful relationships between TFPG and the total revenue and cost. Thus we focus on MFPG and TFPG. To the extent that the MFPG indexes are approximations of TFPG ones, the properties developed for the latter are also relevant to the former.

4   A "G" added to the name of a productivity level index denotes the corresponding growth index.

5   This is not the case for the Törnqvist formula, as explained in Diewert and Nakamura (2005).

6   In fact, indexes with $s=t–1$ are used so much, there is a special name for them: chain indexes.

7   The interested reader can verify that formulas (6) and (7) yield the same TFPG values as formula (9): the Table 1, row 7 values when s is taken to be the previous day, and the Table 1, row 8 values when s is day 1.

8   Perhaps the workers learned on the job. Or the station manager might have made suggestions, in which case there is one more factor of production that is not being accounted for. Moreover, either way, the knowledge of how to do the job faster becomes embodied in the workers; they become "experienced" and this change in their status could be thought of as another output of this production process. These more complex issues are outside the scope of this technical introduction, but some of these issues are taken up in studies in this volume.

9   In general, a value of 1 means that the rate of conversion of input into output was the same in period t as in s, whereas a value greater than 1 (less than 1) means the rate of conversion was greater (less) in period t than in s.

10　This is also why there is literature on the proper methods of international and intersectoral or industry comparisons. See Diewert (1987); Caves, Christensen and Diewert (1982a); and Diewert and Nakamura (1999) for an introduction to some alternative approaches for making multilateral comparisons among production units as well as the presentation of additional references on that topic.

11　Statistical agencies and researchers often prefer the productivity growth indexes to the levels ones because it seems likely the growth measures can be estimated more accurately. However, when confronting policy questions, productivity growth measures are of little help, however accurately measured, if levels measures are needed.

12　Also, the machine will not threaten to go on strike for higher wages at peak business times the way the operators did sometimes, and it could be operated by the owner if need be without a loss of business.

13　So perhaps the sale was an investment in more business for the future. This complication, having to do with the proper treatment of advertising services, is also ignored in this technical introduction. But advertising services are one of the service industries in need of improved price and quantity measurement.

14　We are not trying to argue that labour productivity indexes are never useful. They can be used for monitoring the productive performance of labour for the same productive unit over periods when it is known that there was little change in the use of other factors of production. For an individual production line, office or plant, or even a firm, management would know when there were changes in capital equipment. Also, comparisons of labour productivity may make sense between production units with similar production processes, plant and equipment.

15　Formally, the first two of these can be shown to result from deflating the period t cost and revenue by a Paasche price index. The second two result from deflating the period t cost and revenue by a Laspeyres price index.

16　See Diewert and Nakamura (2003, 2005).

17　The Paasche counterpart of the Laspeyres-type measure in Equation (16) is just the revenue/cost ratio, and it is not a good productivity measure because the values from period to period will reflect relative price changes as well as the changes in the rate at which input quantities are being transformed into output quantities.

18　See Diewert and Nakamura (2003, 2005).

19　Törnqvist indexes are also known as translog indexes following Jorgenson and Nishimizu (1978) who introduced this terminology because Diewert (1976) related Q_T^* to a translog production function. For a study of the properties see Balk and Diewert (2001).

20　See Diewert (1992a).

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