The Measurement of Business Capital, Income and Performance

Tutorial presented at the University Autonoma of Barcelona, Spain, September 21-22, 2005; revised December 2005.

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V. Constructing a Capital Stock for Inventories and the Measurement of Inventory Change

1. Introduction
2. The SNA Treatment of Inventory Change
3. A Suggested Alternative Treatment of Inventory Change
4. Conclusion
Appendix: A Theoretical Treatment of Inventory Change

1. Introduction

The current System of National Accounts (SNA) treatment of inventory change in real terms is very confusing to users. The problem is that it can happen that the value of inventory change has a sign that is *opposite* to the sign of the corresponding constant dollar inventory change. This means that the corresponding implicit price deflator is meaningless. In this paper, the nature of the problem is explained and a solution to the problem is suggested. In the Appendix, a theoretical framework that provides a unified treatment for measuring inventory change and the user cost of inventories is explained.[1] Appendix 2 gives some background information on the origins of the theoretical framework used in Appendix 1.

In section 2, a simple 2 good, 4 period numerical example is introduced and it is explained how a "typical" SNA treatment of inventory change works in the context of this example. The example illustrates the problem described in the previous paragraph: the current dollar aggregate inventory change has a sign opposite to the corresponding constant dollar change.

In section 3, the same example is reworked using the methodological approach suggested in Appendix 1. The suggested solution involves treating inventory change in a manner that is symmetric to the current SNA treatment of exports and imports.

Section 4 concludes.

[1] This methodology is based on Diewert and Smith (1994) and Diewert (2004; 36). The initial accounting methodology can also be found in Diewert (2005; 21-23). Diewert and Lawrence (2005) used this framework as well. The underlying model of production that is used in this chapter is the Hicks (1961) and Edwards and Bell (1961) model explained in section 9.2 of chapter I.

2. The SNA Treatment of Inventory Change

Consider the following data on the end of period stocks of two inventory items for three periods, where p_n^t and q_n^t denote the price and quantity of stock n at the end of period t:

Table 1: Price and Quantity Data for Two Inventory Stocks

	p_1^t	p_2^t	q_1^t	q_2^t
Period 0	1.0	1.0	200	200
Period 1	.9	2.0	260	150
Period 2	.8	3.0	310	110
Period 3	.7	4.0	330	100

Thus the price of the first stock p_1^t is slowly declining while the corresponding end of period stock q_1^t grows from 200 to 330 over the three periods. On the other hand, the price of the second stock p_2^t quadruples over the three periods while the corresponding end of period stock q_2^t steadily falls from 200 to 100 over the three periods. These price changes are more violent than what is usually observed over the course of a year but they would not necessarily be unusual if we think of the first good as computer chip and the second good as crude oil.

The *end of period t SNA constant dollar stock of inventories*, K_{SNA}^t, using the end of period 0 as the base period, can be defined as the following Laspeyres type quantity aggregate:

(1) $K_{SNA}^t \equiv p_1^0 q_1^t + p_2^0 q_2^t$; $t = 0,1,2,3.$

Note that we use the inventory stocks q_n^t at the end of period t along with the prices of the stocks at the end of period 0, p_n^0, in the above definition of the period t constant dollar stock of inventories. Thus for period 0 (the beginning of period 1), the constant dollar stock coincides with the current dollar stock. The *value of the current dollar stock of inventories at the end of period t*, VK^t, is defined in the usual fashion as follows:

(2) $VK^t \equiv p_1^t q_1^t + p_2^t q_2^t$; $t = 0,1,2,3.$

If we divide VK^t by K_{SNA}^t, we obtain P_{SNA}^t, the *end of period t SNA implicit price index for the constant dollar stock of inventories*:

(3) $P_{SNA}^t \equiv VK^t/K_{SNA}^t = [p_1^t q_1^t + p_2^t q_2^t]/[p_1^0 q_1^t + p_2^0 q_2^t]$; $t = 0,1,2,3.$

Note that the SNA implicit price index for the inventory stock is a Paasche price index between period t and 0.

The *SNA constant dollar value of inventory change for period t*, ΔK_{SNA}^t, can be defined in a straightforward manner as the difference between the end of period t and beginning of period t constant dollar stocks defined above by (1):

$$
\begin{aligned}
(4)\ \Delta K_{SNA}^t &\equiv K_{SNA}^t - K_{SNA}^{t-1} && t = 1,2,3 \\
&= p_1^0 q_1^t + p_2^0 q_2^t - [p_1^0 q_1^{t-1} + p_2^0 q_2^{t-1}] && \text{using (1)} \\
&= p_1^0 [q_1^t - q_1^{t-1}] + p_2^0 [q_2^t - q_2^{t-1}] \\
&= p_1^0 \Delta q_1^t + p_2^0 \Delta q_2^t
\end{aligned}
$$

where $\Delta q_n^t \equiv q_n^t - q_n^{t-1}$ is the difference in the closing and opening stock of inventory item n over period t. Note that the last equation in (4) shows that the aggregate change in the constant dollar change in inventories is equal to the sum of the individual item changes, using the end of period 0 prices as weights.

The (approximate) *SNA current dollar value of inventory change for period t*, ΔVK_{SNA}^t, can be defined as the sum of the individual item changes, Δq_n^t, weighted by the average of the beginning and end of period prices, $(1/2)p_n^{t-1} + (1/2)p_n^t$:[2]

$$(5)\ \Delta VK_{SNA}^t \equiv [(1/2)p_1^{t-1} + (1/2)p_1^t]\Delta q_1^t + [(1/2)p_2^{t-1} + (1/2)p_2^t]\Delta q_2^t\ ; \qquad t = 1,2,3.$$

The corresponding *implicit price index for the SNA inventory change*, ΔP_{SNA}^t, is obtained by dividing the value series ΔVK_{SNA}^t defined by (5) by the constant dollar series ΔK_{SNA}^t defined by (4):

$$
\begin{aligned}
(6)\ \Delta P_{SNA}^t &\equiv \Delta VK_{SNA}^t/\Delta K_{SNA}^t && t = 1,2,3 \\
&= (1/2)\{[p_1^{t-1} + p_1^t]\Delta q_1^t + [p_2^{t-1} + p_2^t]\Delta q_2^t\}/\{p_1^0\,\Delta q_1^t + p_2^0\,\Delta q_2^t\}.
\end{aligned}
$$

The above definition for the change in stocks price index, ΔP_{SNA}^t, looks a bit strange at first sight but *if* the weights Δq_1^t and Δq_2^t are positive, it can be seen that it is a perfectly reasonable price index that compares an average of the beginning and end of period t prices with the base prices (which are the end of period 0 prices for the inventory components).[3]

The above definitions are used to construct the value, price and quantity of end of period inventory stocks (VK^t, P_{SNA}^t and K_{SNA}^t respectively) for periods 0,1,2 and 3 and the value, price and quantity of the change in inventory stocks (ΔVK_{SNA}^t, ΔP_{SNA}^t and ΔK_{SNA}^t respectively) for periods 1-3 using the data in Table 1. The results are listed in Table 2 below.

Table 2: Values, Prices and Quantities for Aggregate Inventories at Period 0 Prices

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[2] This is not quite the theoretically correct measure of inventory change that is suggested in the *System of National Accounts 1993* on pages 130-131 but is regarded as an approximation that is frequently used as the following quotation indicates: "This suggests that even when prices are changing a good approximation to the PIM may be obtained by taking the difference between the quantity of goods held in inventory at the beginning and at the end of the accounting period and valuing the difference at the average prices prevailing within the period. This measure, which may be described as the "quantity" measure, is widely used in practice and is sometimes mistakenly considered to be the theoretically appropriate measure under all circumstances." *SNA 1993,* page 131. For a more complete discussion of the SNA theoretically correct measure of inventory change, see Hill (2005). However, Hill (2005) notes that the theoretically correct method suffers from the same problems that arise using the approximate method.

[3] However, note that when we set t equal to zero, in general, ΔP_{SNA}^0 will not equal unity; i.e., the index does not satisfy the identity test.

	VK^t	$P_{SNA}{}^t$	$K_{SNA}{}^t$	$\Delta VK_{SNA}{}^t$	$\Delta P_{SNA}{}^t$	$\Delta K_{SNA}{}^t$
Period 0	400	1.000	400	_____	_____	_____
Period 1	534	1.302	410	−18.0	−1.800	10
Period 2	578	1.376	420	−57.5	−5.750	10
Period 3	631	1.467	430	−20.0	−2.000	10

At first glance, the values, prices and quantities for the aggregate inventory stock look reasonable, with the values growing fairly quickly due to rapid increases in the price of the second inventory good but the real stocks growing at the much slower rate of 10 units per year. Turning to the values, prices and quantities for the changes in the aggregate inventory stock, we see that the quantity growth, $\Delta K_{SNA}{}^t$, is equal to 10 in periods 1, 2 and 3, which is the difference in the corresponding beginning and end of period stocks, $K_{SNA}{}^t$. This is very satisfactory. However, when we calculate the change in the value of inventories at current prices, $\Delta VK_{SNA}{}^t$, a different picture emerges. Because the price of inventory item 2 increases much more rapidly than the price of item 1 declines, the steady decline in the quantity of item 2 when valued at current prices outweighs the steady increase in the quantity of item 1 at current prices so that overall, the change in the value of inventories at current prices turns out to be strongly negative (−18 over the course of period 1, −57.5 over the course of period 2 and −20 over the course of period 3). Thus the corresponding implicit price index for inventory change, $\Delta P_{SNA}{}^t$, turns out to be *negative* in all three periods (since the value change is negative and the corresponding constant dollar quantity change is positive). Looking at definition (6) above, it can be seen that the root of the problem is that the quantity weights in the price index number formula, $\Delta q_1{}^t$ and $\Delta q_2{}^t$, are of opposite signs and the value aggregates in the numerator and denominator of (6) are of opposite signs and fairly small. Index number theory breaks down under these circumstances and can frequently give rise to meaningless numbers as is the case in the present situation.[4]

The existence of negative implicit prices for an output component of the national accounts may not create any great conceptual problems for the compilers of the accounts (since the negative implicit prices are just a consequence of definitions that seem reasonable to accountants) but they do create problems for many macroeconomic modelers who base their models on microeconomic theory: negative prices create great difficulties for this class of user. Hence, in the following section, a different theoretical framework (based on microeconomic theory) is suggested that will avoid the negative implicit price problem.[5]

In addition to the negative implicit price problem, there is another problem with the above approximate SNA methodology: namely, it relies on a *fixed base Laspeyres type methodology*. Note that the inventory stock aggregate is a fixed base Laspeyres quantity index which uses the prices of period 0 as the weights for the individual stock components. Definitions (7)-(10) below redo definitions (1)-(6) above but instead of

[4] Hill (1971) noted this problem with traditional index number theory many years ago.

[5] However, there is a cost to the suggested solution: the single SNA output category, “change in inventories”, is replaced by the difference between two output categories: the “end of period stock of inventories” less the “beginning of the period stock of inventories”.

using the prices at the end of period 0 as the base prices, the prices at the end of period 3 are used as the base prices. Definitions (2) and (5) remain unchanged since they are values but the counterparts to (1), (3), (4) and (6) are listed below (the new stock and flow aggregates are denoted by $K_{SNA}^t(3)$, $P_{SNA}^t(3)$, $\Delta K_{SNA}^t(3)$ and $\Delta P_{SNA}^t(3)$):

(7) $K_{SNA}^t(3) \equiv p_1^3 q_1^t + p_2^3 q_2^t$; $\qquad t = 0,1,2,3;$

(8) $P_{SNA}^t(3) \equiv VK^t/K_{SNA}^t(3) = [p_1^t q_1^t + p_2^t q_2^t]/[p_1^3 q_1^t + p_2^3 q_2^t]$; $\qquad t = 0,1,2,3;$

$$
\begin{aligned}
(9)\ \Delta K_{SNA}^t(3) &\equiv K_{SNA}^t(3) - K_{SNA}^{t-1}(3) && t = 1,2,3 \\
&= p_1^3 q_1^t + p_2^3 q_2^t - [p_1^3 q_1^{t-1} + p_2^3 q_2^{t-1}] && \text{using (7)} \\
&= p_1^3 [q_1^t - q_1^{t-1}] + p_2^3 [q_2^t - q_2^{t-1}] \\
&= p_1^3 \Delta q_1^t + p_2^3 \Delta q_2^t ;
\end{aligned}
$$

$$
\begin{aligned}
(10)\ \Delta P_{SNA}^t(3) &\equiv \Delta VK_{SNA}^t/\Delta K_{SNA}^t(3) \qquad\qquad t = 1,2,3 \\
&= (1/2)\{[p_1^{t-1} + p_1^t]\Delta q_1^t + [p_2^{t-1} + p_2^t]\Delta q_2^t\}/\{p_1^3 \Delta q_1^t + p_2^3 \Delta q_2^t\}.
\end{aligned}
$$

The above definitions are used to construct the price and quantity of end of period inventory stocks ($P_{SNA}^t(3)$ and $K_{SNA}^t(3)$ respectively) for periods 0,1,2 and 3 and the price and quantity of the change in inventory stocks ($\Delta P_{SNA}^t(3)$ and $\Delta K_{SNA}^t(3)$ respectively) for periods 1,2 and 3 using the data in Table 1. The results are listed in Table 3 below.

Table 3: Values, Prices and Quantities for Aggregate Inventories at Period 3 Prices

	VK^t	$P_{SNA}^t(3)$	$K_{SNA}^t(3)$	ΔVK_{SNA}^t	$\Delta P_{SNA}^t(3)$	$\Delta K_{SNA}^t(3)$
Period 0	400	0.4255	940	______	______	______
Period 1	534	0.6829	782	−18.0	.1139	−158
Period 2	578	0.8798	657	−57.5	.4600	−125
Period 2	631	1.0000	631	−20.0	.7692	−26

From Table 2, it was seen that the constant dollar stock of inventories (using the end of period 0 prices as the weights) *grew* from 400 to 430 from the end of period 0 to the end of period 3 whereas from Table 3, it appears that the constant dollar stock of inventories *fell* from 940 to 631 over the three periods. Turning to the changes in the constant dollar stocks, Table 2 told us that the change in stocks at constant prices was *positive* (equal to 10 in each period) while Table 3 tells us that the change in stocks was *strongly negative* in each period (−158, −125 and −26). The corresponding implicit prices are all *negative* in Table 2 while they are all *positive* in Table 3. The lack of harmony in the two sets of results is due to the large change in relative prices (and the smaller but still significant change in relative quantities) over the three periods and the fact that a quantity index is being used that uses the price weights of only *one* of the two periods being compared. Chapter 16 in the *SNA 1993* recommends the use of symmetrically weighted index number formulae rather than the asymmetrically weighted Laspeyres formula. Thus in the next section, the Fisher (1922) price and quantity index (which is a symmetrically weighted formula) will be used in order to construct inventory stock aggregates. Since the price and quantity data move relatively smoothly over time, chained Fisher indexes will be used rather than fixed base Fisher indexes.[6]

3. A Suggested Alternative Treatment of Inventory Change

Following the advice given in Chapter 16 of *SNA93*, the Fisher price and quantity indexes, P_F^t and K_F^t, are adopted as the measure of the aggregate price and quantity (or volume) for the end of period stocks of inventories. Since the price and quantity data have relatively smooth trends over time, chained Fisher indexes were used.[7] The value of the end of period aggregate inventory stock, VK^t, is listed in Table 4 below along with P_F^t and K_F^t.[8]

Table 4: Values, Prices and Quantities for Aggregate Inventories using Chained Fisher Indexes

	VK^t	P_F^t	K_F^t	V_A^t	P_A^t	Q_A^t	V_I^t	$Error^t$
Period 0	400	1.000	400.0	_____	1.000	_____	_____	____
Period 1	534	1.374	388.6	−15.7	1.374	−11.4	−46.0	−30.3
Period 2	578	1.642	352.1	−60.0	1.642	−36.5	−80.0	−20.0
Period 3	631	1.851	340.8	−20.8	1.851	−11.2	−26.0	−5.2

Comparing P_F^t and K_F^t in Table 4 with P_{SNA}^t and K_{SNA}^t in Table 2, it can be seen that the Fisher price index grows more rapidly (from 1 to 1.851) than the SNA price index (from 1 to 1.467) and the corresponding Fisher volume index for the inventory stock grows more slowly. The SNA volume index uses the prices of period 0 as weights and the decreases in q_2 are just outweighed by the increases in q_1. However, when the Fisher chained volume index is used, the decreases in q_2 get a higher weight (due to the rapidly increasing price of q_2) than the weight accorded to the increases in q_1, leading to a decrease in the Fisher volume index compared to the increase in the fixed base SNA type index. Note that the differences are not insignificant.

The problem of determining the price and quantity of the change in the aggregate inventory stocks is now addressed. The methodology described in Appendix 1 below is used, which provides a consistent theoretical framework based on economic theory for not only the price and quantity of the change in inventories but also for the user cost of aggregate inventory stocks held at the beginning of each period. According to the model developed in Appendix 1, the *theoretically correct period t value aggregate for the value of inventory change*,[9] V_I^t, is given by (A10), which is rewritten using the notation in Table 1 as follows:

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[6] This is consistent with the advice given in Chapter 16 of the *SNA 1993*, pages 388-389, where fixed base symmetrically weighted indexes are recommended if the data fluctuate or bounce and chained indexes are recommended if the data have trends.

[7] The Bureau of Economic Analysis uses chained Fisher indexes to calculate inventory stocks; see Parker and Seskin (1996) and Ehemann (2005).

[8] The entries in the final 4 columns of Table 4 will be explained later.

[9] This is the theoretically correct value aggregate if it is desired to have a formula for the user cost for inventories that is completely symmetric to the user cost for reproducible capital. If this symmetry property is not regarded as important, then we need only use the first 3 columns in Table 4 in order to decompose the beginning and end of period values for the stock of inventories into Fisher ideal price and quantity

$$(11)\ V_I^t \equiv \sum_{j=1}^2 P_{Kj}^t\,\Delta K_j^t$$
$$= \sum_{j=1}^2 P_{Kj}^t\,[K_j^t - K_j^{t-1}] \qquad t = 1,2,3$$
$$= \sum_{j=1}^2 p_j^t\,[q_j^t - q_j^{t-1}]$$
$$= \sum_{j=1}^2 p_j^t\,q_j^t - \sum_{j=1}^2 p_j^t\,q_j^{t-1}$$
$$= V_E^t - V_B^t$$

where

$$(12)\ V_E^t \equiv \sum_{j=1}^2 p_j^t\,q_j^t$$

is the *end of period t inventory stock value aggregate* and

$$(13)\ V_B^t \equiv \sum_{j=1}^2 p_j^t\,q_j^{t-1}$$

is a (hypothetical) *beginning of period t inventory stock value aggregate* where the beginning of the period stocks are valued at the end of period prices. From the second line in (11) above, it would appear that the theoretical inventory change value aggregate for period t, V_I^t, has a straightforward decomposition into prices (the end of period t prices for the stocks p_j^t) times the quantity changes over period t, $q_j^t - q_j^{t-1}$. However, because the quantities in this value aggregate are really quantity differences and hence can be of either sign, index number theory may fail if the prices in the index number formula are taken to be the p_j^t and the quantities are taken to be the $q_j^t - q_j^{t-1}$. The suggested solution to this problem is to regard the inventory change value aggregate as the difference between the end of period value aggregate V_E^t and the hypothetical beginning of period value aggregate V_B^t and then use normal index number theory to decompose V_E^t into the product of the price and quantity components P_E^t and Q_E^t respectively and to decompose V_B^t into the product of the price and quantity components P_B^t and Q_B^t respectively. *In other words, it is suggested that the change in inventories value aggregate be treated in a manner that is symmetric to the treatment of the current trade balance as the difference between the value of exports less the value of imports.*[10] This trade balance aggregate has exactly the same type of problem as the inventory change aggregate: it could be positive in one period and negative in the following period. Index number theory cannot decompose this type of difference value aggregate into meaningful price and volume components, unless it is guaranteed that the value differences will remain well away from zero.

Before we illustrate our suggested treatment of inventory change using the data in Table 1, we will first use these data to illustrate a simple approach that is problematic. *An approximate approach to the treatment of inventory change* can be implemented as follows. First construct the chained Fisher price and quantity indexes for the end of period t inventory stocks, P_F^t and K_F^t respectively. Now define the *period t approximate price for inventory change*, P_A^t, to be the end of period t Fisher stock price for inventory components P_F^t and define the period t *approximate quantity or volume of inventory*

components. If this second approach is taken, then it is not necessary to perform the computations in the remainder of this section.

[10] This methodological approach was suggested in Diewert (2004; 36).

change Q_A^t to be the difference between the beginning and end of period t Fisher quantity indexes for the inventory stocks; i.e., we have the following definitions:[11]

(14) $P_A^t \equiv P_F^t$; t = 1,2,3;
(15) $Q_A^t \equiv [K_F^t - K_F^{t-1}]$; t = 1,2,3.

The corresponding *approximate value of inventory change in period t* is V_A^t defined as the product of the approximate price and quantity defined above:

(16) $V_A^t \equiv P_A^t Q_A^t$; t = 1,2,3.

We note that definitions (14)-(16) collapse down to the theoretical model presented in Appendix 1, *provided that there is only one inventory item in the aggregate*. Moreover, the use of these definitions makes the aggregate inventory stocks perfectly consistent with the aggregate value of inventory change; i.e., the stock and flow aggregates are perfectly consistent. V_A^t, P_A^t and Q_A^t are listed in Table 4 above for periods 1,2 and 3.

However, even though definitions (14)-(16) are perfectly consistent with the theoretical approach explained in Appendix 1 when there is only one inventory item in the aggregate, this correspondence does not hold in general when there are two or more inventory items in the aggregate.[12] When there are two or more inventory items in the aggregate, it is not necessarily the case that the value of the approximate change in the value of inventories V_A^t is equal to the theoretically correct value of inventory change, V_I^t, and so there will generally be an *aggregation error* between these two value aggregates defined for period t as follows:

(17) $\text{Error}^t \equiv V_I^t - V_A^t$; t = 1,2,3.

The "true" values of the inventory change aggregate, V_I^t, and the aggregation error between this value and the approximate value V_A^t are listed in the last two columns of Table 4. It can be seen that the aggregation errors are too large to be ignored in this case. Hence, we conclude that while under some circumstances, the approximate method for

[11] This approximate method for the treatment of inventory change is very close to the method presently in use by the BEA to calculate real estimates of inventory change. The BEA method uses the average of the beginning and end of period Fisher stock prices, $(1/2)P_F^{t-1} + (1/2)P_F^t$, in place of P_F^t on the right hand sides of (14) and (15); see Parker and Seskin (1996) and Ehemann (2005). However, if there is only one inventory item, then the use of our (14) and (15) will give the "right" answer if we use the user cost framework developed by Diewert and Smith (1994) whereas the BEA procedure will not. Ehemann (2005) developed a variant of the BEA procedure by constructing Fisher indexes of acquisitions and disposals and taking their difference, say $B_F^t - S_F^t$ using the notation in the Appendix, in place of the difference in Fisher stocks, $K_F^t - K_F^{t-1}$. In the case of only one inventory item, the Ehemann method will coincide with the BEA method, provided that U^t and G^t in the Appendix equations (A3) and (A4) are zero in the two periods being compared. In the many inventory item case, even if U^t and G^t are zero, the BEA and Ehemann methods will differ due to the different weights in the Fisher indexes K_F^t, B_F^t and S_F^t.

[12] If either end of period *prices* of inventory items vary in strict proportion over time or the *quantities* in the end of period inventory stocks vary in strict proportion over time, then the approximate approach will be perfectly consistent with the theoretical approach explained in Appendix 1. This is because the Fisher formula is consistent with both Hicks' (1946; 312-313) and Leontief's (1936; 54-57) Aggregation Theorems; see Allen and Diewert (1981).

calculating the price and quantity for inventory change *can* be satisfactory, in many cases it will *not* be satisfactory.

Recall the end of period value of inventories aggregate, V_E^t defined by (12) and the hypothetical beginning of period value of inventories aggregate V_B^t defined by (13). Using chained Fisher indexes for periods 1 to 3, the resulting price and quantity decompositions using the data in Table 1 are listed in Table 5.

Table 5: Values, Prices and Quantities for Beginning and End of Period Inventory Aggregates using Chained Fisher Indexes

	V_E^t	P_E^t	Q_E^t	V_B^t	P_B^t	Q_B^t
Period 1	534	1.0000	534.0	580	1.0000	580.0
Period 2	578	1.1947	483.8	658	1.2707	517.8
Period 3	631	1.3473	468.4	657	1.4769	444.9

Looking at Table 5, it can be seen that the volume of end of period inventories is decreasing more slowly (from 534.0 to 468.4) than the volume of beginning of period inventories (from 580.0 to 444.9). Hence the difference between the two volume aggregates is increasing. It can also be seen that the price of end of period inventories increases more slowly (from 1 to 1.3473) than the corresponding price of beginning of period inventories (from 1 to 1.4769). The relatively large discrepancy in these two rates of price increase explains why the approximate method for dealing with inventory change does not work well for this example.[13] Since the beginning of period inventory stock gets a negative weight when an inventory change aggregate is formed and it has a higher inflation rate than the end of period stock, it can be expected that the price of this net output aggregate will decrease.[14] Although this result is counterintuitive from the perspective of measuring general inflation, it is sensible from the perspective of production theory: the increase in the beginning of period price of inventories acts like an increase in the price of an intermediate input and so the net return to the producer of producing a unit of gross output less a unit of the intermediate has decreased; i.e., the price of net output has decreased.

[13] If the two rates of price increase were equal, then the aggregation errors associated with the approximate method would be zero.

[14] The problem is similar to an analogous problem that occurs when the price of imports increases faster than the price of exports and other output components of GDP. In this case, the *increase* in the price of imports will *reduce* the GDP deflator. Kohli (1982; 211) (1983) (2004; 91) noticed this problem with the GDP deflator many years ago: "Actually, it can easily be seen that any terms of trade change away from the base period price ratio results in a fall in real national product. This clearly reveals the weakness of this measure of real value added, the drawbacks of direct index numbers, and the dangers of aggregating positive with negative quantities." Ulrich Kohli (1983; 142). An example of this anomalous behavior of the GDP deflator just occurred in the advance release of gross domestic product for the third quarter of 2001 for the US national income and product accounts: the chain type price indexes for C, I, X and M *decreased* (at annual rates) over the previous quarter by 0.4%, 0.2%, 1.4% and 17.4% respectively but yet the overall GDP deflator *increased* by 2.1%. Thus there was general deflation in all sectors of the economy but yet the overall GDP deflator increased. See Table 4 in the Bureau of Economic Analysis (2001).

To indicate how further stages of aggregation might proceed, an *investment aggregate* is introduced, which has price p_5^t and quantity q_5^t in period t. It is assumed that the price and quantity of this investment aggregate is constant during periods 1 to 3 and in particular, it is assumed that:

(18) $p_5^t = 1; q_5^t = 1000$; $\quad t = 1,2,3.$

The task now is to construct chained Fisher aggregate prices and quantities for each year, P^t and Q^t (with corresponding value $V^t \equiv P^tQ^t$), that aggregate over end of period stocks, q_1^t and q_2^t (with corresponding prices p_1^t and p_2^t), beginning of year hypothetical stocks indexed with negative signs, $q_3^t \equiv -q_1^{t-1}$ and $q_4^t \equiv -q_2^{t-1}$ (with corresponding prices $p_3^t \equiv p_1^t$ and $p_4^t \equiv p_2^t$) and other investment flows, q_5^t (with corresponding price p_5^t). Thus there are 5 commodities in all that are being aggregated. The results for this *investment plus change in inventories Fisher aggregate*, V^t, P^t and Q^t, are listed in the first three columns of Table 6.

Table 6: Values, Prices and Quantities for Aggregate Investment plus Inventory Change using Chained Fisher Indexes

	V^t	P^t	Q^t	P_{2S}^t	Q_{2S}^t	P_{AA}^t	Q_{AA}^t	V_{AA}^t
Period 1	954	1.0000	954.0	1.0000	954.0	1.0000	984.3	984.3
Period 2	920	0.9473	971.2	0.9484	970.1	0.9933	946.4	940.0
Period 3	974	0.9182	1060.8	0.9217	1056.7	0.9881	991.0	979.2

As expected, the price of the investment plus inventory change aggregate, P^t, decreases over time (in a sensible manner) and the corresponding quantity or volume, Q^t, steadily increases. The columns in Table 5 that decompose the two inventory aggregates plus the first 3 columns in Table 6 are the core of the new suggested presentation of aggregate inventory change. *The key is to decompose the inventory change into two aggregates, show the price and quantity detail for those two aggregates and then move to the next stage of aggregation where the two inventory aggregates are aggregated with other flow aggregates.* All of the columns in Table 5 and the first 3 columns in Table 6 show sensible prices, quantities and values, which is not the case with the existing SNA method for dealing with inventory change.

In the first 3 columns of Table 6, we constructed the aggregate P^t and Q^t by using the Fisher chained formula over the 5 most finely disaggregated prices and quantities in the model.[15] It is also possible to construct this aggregate price and quantity in two stages. In the first stage, the end of period Fisher chained inventory aggregate price and quantities, P_E^t and Q_E^t, and the beginning of period hypothetical inventory aggregate price and quantities, P_B^t and Q_B^t are constructed: see the entries in Table 5. In the second stage of aggregation, chained Fisher indexes are calculated using P_E^t, P_B^t and p_5^t as the period t prices and Q_E^t, $-Q_B^t$ and q_5^t as the corresponding period t quantities. The results of this two stage aggregation procedure are listed in Table 6 under the columns with the headings P_{2S}^t and Q_{2S}^t (the corresponding two stage value aggregate equals V^t and so it is

[15] Diewert and Lawrence (2005) used this strategy to construct investment plus inventory change aggregates in their empirical work for Australia.

not listed). It can be seen that these two stage estimates are reasonably close to their one stage counterparts, P^t and Q^t.[16]

Finally, the approximate price and quantity for inventory change listed in Table 4, $P_A{}^t$ and $Q_A{}^t$, can be used, along with $p_5{}^t$ and $q_5{}^t$, in order to construct approximate investment and inventory change aggregate price, quantity and value for period t, $P_{AA}{}^t$, $Q_{AA}{}^t$ and $V_{AA}{}^t$ respectively, using the Fisher chain formula. The results are listed in the last 3 columns of Table 6. It can be seen that for this particular numerical example, the approximate method is not acceptable.[17] The errors in values, prices and quantities are large compared to the theoretically preferred measures, V^t, P^t and Q^t.

4. Conclusion

The SNA method for treating changes in inventories suffers from two major problems:

- Aggregate real inventory stocks and changes in stocks are evaluated at constant base period prices which leads to difficulties if the relative prices of inventory components are changing over time (and this method is not consistent with the use of symmetrically weighted or superlative indexes which is recommended in *SNA93*);
- The SNA implicit prices for inventory change can be negative and are extremely difficult for users to interpret.

Since the Canberra II Group has recommended that user costs for reproducible capital stocks be added to the SNA production accounts as a recommended decomposition of gross operating surplus and since the Group also recommended that inventory stocks be included as assets that should have user costs in these optional accounts, it is necessary to carefully specify the links between the user cost of inventories and the treatment of the change in inventories in the production accounts. The Appendix to this paper presents a coherent theoretical framework for the treatment of inventory change and for the construction of user costs for inventory items.

In addition to suggesting a consistent accounting framework for the user cost of inventories and the treatment of inventory change, the other main methodological suggestion in this chapter is to treat inventory in a manner that is symmetric to the treatment of the current trade balance as the difference between the value of exports less the value of imports. Although this suggested treatment of inventory leads to sensible price and volume estimates, it has the downside of being somewhat different than the current SNA treatment of inventory change, which is well established. Hence users may find our suggested solution to the problems associated with the current SNA treatment of

[16] This is in accordance with the experience of the U.S. Bureau of Economic Analysis in constructing two stage chained Fisher aggregates. Diewert (1978; 888) derived a theoretical result that showed that normally, the single stage and two stage estimates should approximate each other fairly closely.

[17] Lasky (1998; 106) and Ehemann (2005) essentially used this methodology to evaluate the adequacy of the BEA method for estimating inventory change, except that they used all components of GDP (C+G+I+X−M) in place of our use of just I as the outside commodity in the next stage of aggregation. Both Lasky and Ehemann found relatively large aggregation errors in using the BEA approximate method for estimating inventory change. Thus the problem that we are describing is not just a hypothetical one.

inventory change to be a bit strange at first.[18] However, if it is explained to users that the suggested treatment of inventory change is entirely analogous to the current SNA treatment of international trade, the suggested new treatment will eventually be regarded as being quite acceptable.

Appendix: A Theoretical Treatment of Inventory Change

A theoretical framework is needed to measure the contribution of the change inventory stock over a period to production. It is also necessary to work out the user cost of the beginning of the period stock of inventories. A framework to answer these questions is outlined, taken from Diewert and Smith (1994).[19]

First consider the theory for a single inventory stock item. Consider a firm that perhaps produces a noninventory output during period t, Y^t, uses a noninventory input X^t, sells the amount S^t of an inventory item during period t and makes purchases of the inventory item during period t in the amount B^t. Suppose that the average prices during period t of Y^t, X^t, S^t and B^t are $P_Y{}^t$, $P_X{}^t$, $P_S{}^t$ and $P_B{}^t$ respectively. Then neglecting balance sheet items, the firm's period t *cash flow* is:[20]

$$\text{(A1)}\ CF^t \equiv P_Y{}^t\, Y^t - P_X{}^t\, X^t + P_S{}^t\, S^t - P_B{}^t\, B^t.$$

Let the firm's beginning of period t stock of inventory be K^{t-1} and let its end of period stock of inventory be K^t. These inventory stocks are valued at the balance sheet prices prevailing at the beginning and end of period t, $P_K{}^{t-1}$ and $P_K{}^t$ respectively. Note that all 4 prices involving inventory items, $P_S{}^t$, $P_B{}^t$, $P_K{}^{t-1}$ and $P_K{}^t$ can be different.

The firm's end of period t *economic income* or *net profit* is defined as its cash flow plus the value of its end of period t stock of inventory items less $(1+r^t)$ times the value of its beginning of period t stock of inventory items:

$$\text{(A2)}\ EI^t \equiv CF^t + P_K{}^t\, K^t - (1+r^t)\, P_K{}^{t-1}\, K^{t-1}$$

where r^t is the nominal cost of capital that the firm faces at the beginning of period t. Thus in definition (A2), it is assumed that the firm has to borrow financial capital or raise equity capital at the cost r^t in order to finance its initial holdings of inventory items. This cost could be real (in the case of a firm whose initial capital is funded by bonds) or it could be an opportunity cost (in the case of a firm entirely funded by equity capital).

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[18] The problem is that most users are not aware that normal index number theory fails spectacularly as a value aggregate approaches zero.

[19] Their analysis is in turn based on the Austrian model of production explained by Böhm-Bawerk (1891), Hicks (1961) and Edwards and Bell (1961). For additional material on this model, see the Appendices in Diewert (1977) (1980) and section 9.2 of chapter I.

[20] Note that this framework is flexible enough to allow the firm to either purchase or produce internally inventory items. Note also that firm purchases of inventory items from other domestic firms would appear in the national accounts as intermediate input purchases and purchases from foreign suppliers would appear as imports. On the other hand, sales of inventory items by the firm to domestic producers, households or foreigners would appear in the national accounts as gross outputs, final household consumption or exports respectively.

The end of period stock of inventory is related to the beginning of the period stock by the following equation:

(A3) $K^t = K^{t-1} + B^t - S^t - U^t$

where U^t denotes inventory items that are lost, spoiled, damaged or are used internally by the firm. In the case of livestock inventories, there is a natural growth rate of inventories over the period so equation (3) is replaced by:

(A4) $K^t = K^{t-1} + B^t - S^t + G^t$

where G^t denotes the natural growth of the stock over period t.[21]

Define the *change in inventory stocks* over period t as:

(A5) $\Delta K^t \equiv K^t - K^{t-1}$.

Using (A5), both (A3) and (A4) can be written as:

(A6) $K^t = K^{t-1} + \Delta K^t$.

Now substitute (A6) into the definition of economic income (A2) and the following expression is obtained:

(A7) $EI^t \equiv CF^t + P_K{}^t [K^{t-1} + \Delta K^t] - (1+r^t) P_K{}^{t-1} K^{t-1}$
$= CF^t + P_K{}^t \Delta K^t - [r^t P_K{}^{t-1} - (P_K{}^t - P_K{}^{t-1})]K^{t-1}$.

Thus *economic income is equal to cash flow plus the value of the change in inventory (valued at end of period balance sheet prices) minus the user cost of inventories times the starting stocks of inventories* where this *period t user cost* is defined as

(A8) $P_U{}^t \equiv r^t P_K{}^{t-1} - (P_K{}^t - P_K{}^{t-1})$.

Note that the above algebra works for both livestock and ordinary inventory items.

Of course, there can be two versions of the user cost:

- An ex post version where the *actual* end of period balance sheet price of inventories is used or
- An ex ante version where at the beginning of period t, we estimate a *predicted value* for the end of period balance sheet price.

For the production accounts in the SNA, the ex ante version is the appropriate version, which means the national income accountant has some leeway in forming estimates of the end of period balance sheet price for the inventory item. Looking at (A7), it is

[21] If the firm is constructing inventory items either for direct sale or as an intermediate step in its production processes, then these produced additions to the stock would be included in the term G^t.

important to note that the change in inventories that occurred over period t, ΔK^t, should be valued at the end of period t price for the inventory item, $P_K{}^t$.[22]

If the firm is using or selling many inventory items, say J items, then equation (A7) becomes:

$$\text{(A9)}\; EI^t \equiv CF^t + \textstyle\sum_{j=1}^J P_{Kj}{}^t\, \Delta K_j{}^t - \sum_{j=1}^J [r^t P_{Kj}{}^{t-1} - (P_{Kj}{}^t - P_{Kj}{}^{t-1})\,]K_j{}^{t-1}$$

where the notation is obvious. The terms involving the value of the change in inventories over the period are the following ones:

$$\text{(A10)}\; \textstyle\sum_{j=1}^J P_{Kj}{}^t\, \Delta K_j{}^t = \sum_{j=1}^J P_{Kj}{}^t\, [K_j{}^t - K_j{}^{t-1}]$$
$$\text{(A11)}\; \qquad = \textstyle\sum_{j=1}^J P_{Kj}{}^t\, K_j{}^t - \sum_{j=1}^J P_{Kj}{}^t K_j{}^{t-1}.$$

Looking at (A10), it would appear that normal index number theory could be applied to the sum of terms in the value aggregate on the right hand side, with prices defined as the end of period t balance sheet prices $P_{Kj}{}^t$ and corresponding quantities defined as the inventory changes $K_j{}^t - K_j{}^{t-1}$ over period t. However, this value aggregate is not necessarily of one sign over time: it could be positive, negative or zero. *Normal index number theory breaks down for value aggregates that can be either positive or negative over time*.[23] Thus index number theory should not be applied to the value aggregate on the right hand side of (A10). Instead, it is recommended that index number theory be applied *separately* to the two value aggregates on the right hand side of (A11).[24] Thus $\sum_{j=1}^J P_{Kj}{}^t\, K_j{}^t$ should be decomposed (using normal index number theory) into $P_{KE}{}^t K_E{}^t$ where $P_{KE}{}^t$ is the scalar end of period t aggregate price of inventories and $K_E{}^t$ is the corresponding end of period t aggregate stock and $\sum_{j=1}^J P_{Kj}{}^t K_j{}^{t-1}$ should be decomposed into $P_{KB}{}^t K_B{}^t$ where $P_{KB}{}^t$ is the scalar beginning of period t aggregate price of inventories and $K_B{}^t$ is the corresponding beginning of period t aggregate stock. Then in place of the current single aggregate for inventory change that is reported in the current System of National Accounts, it is recommended that *inventory change be treated in a manner that is symmetric to the treatment of aggregate exports and imports in the accounts*; i.e., the end of period aggregates $P_{KE}{}^t$ and $K_E{}^t$ (the counterparts to the aggregate price of exports and the aggregate quantity of exports) and the beginning of period aggregates $P_{KB}{}^t$ and $K_B{}^t$ would be reported separately just as exports and imports are reported separately in the current SNA.

[22] However, the current SNA methodology requires that inventory change over the production period be evaluated at the average prices of the period. This requirement could be accommodated in our framework by replacing the end of period price of the inventory item, $P_K{}^t$, by an appropriate average inventory price for period t. If this is done, and if the actual end of period price of the inventory item is used for balance sheet purposes, then a reconciliation entry will be required in the Revaluation Accounts.

[23] To see why this breakdown occurs, consider a situation where the value aggregate just happens to be zero in the base period. Laspeyres price and quantity indexes will be undefined under these circumstances and nonsensical numbers will be obtained if the value aggregate is very close to zero in the base period. However, if the Laspeyres, Paasche or Fisher formula is used in forming a larger aggregate that is bounded well away from zero, then the right hand side of (A10) can be used when forming this larger aggregate and the same results will be obtained as using the right hand side of (A11) in forming the larger aggregate.

[24] This solution to the aggregation problem was suggested by Diewert (2004; 36).

There is another treatment of inventory change that could be used by statistical agencies that is much more straightforward. The definition of economic income, (A2) above, can be rewritten as follows:

$$\text{(A12) } EI^t \equiv CF^t + P_K{}^t K^t - P_K{}^{t-1} K^{t-1} - r^t P_K{}^{t-1} K^{t-1}.$$

Using (A12), *the value of inventory change for period t* is simply defined as the end of period t value of the stock, VK^t,[25] less the beginning of period t value of the stock, VK^{t-1}:

$$\text{(A13) } VK^t - VK^{t-1} = P_K{}^t K^t - P_K{}^{t-1} K^{t-1}.$$

Using this decomposition of economic income, the user cost value aggregate is defined as the last term on the right hand side of (A12) and so *the new user cost of inventories* is:

$$\text{(A14) } P_U{}^{t*} \equiv r^t P_K{}^{t-1}.$$

The new user cost of inventories, $P_U{}^{t*}$ defined by (A14), can be compared to the initial user cost of inventories, $P_U{}^t$ defined by (A8), and the new value of inventory change defined by (A13) can be compared to the earlier expression for the value of inventory change defined by (A11). Both the old and the new decomposition of economic income are theoretically valid. However, note that a nominal interest rate r^t appears in (A14) whereas a type of real interest rate appeared in (A8). Hence for a country experiencing high inflation, the new user cost of inventories will be higher than the old user cost and similarly, the new value of inventory change defined by (A13) will be higher than the old value of inventory change defined by (A11).[26] Thus nominal GDP will tend to be higher using the new decomposition compared to the initial one and it will be substantially higher under conditions of high inflation.

There are advantages and disadvantages of using the second decomposition of economic income compared to the first:

- The *main advantage* of the second decomposition is that it is much more straightforward and will be easier to explain to users. Also, it is much easier to reconcile quarterly changes in inventories to annual changes using the second decomposition.
- The *main disadvantage* of the second decomposition is that the resulting user cost of inventories is *different* from the user cost formula for reproducible capital and so an awkward asymmetry would be introduced into the SNA if a user cost approach to reproducible capital were introduced.[27]

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[25] See the first 3 columns of Table 4 for the Fisher chain decomposition of the end of period value of the stocks VK^t into price and quantity components for the numerical example.

[26] If the initial decomposition of economic income is used, then the beginning of the period inventory stocks are valued at the higher end of period prices but since this value aggregate is given a minus sign, this will reduce nominal GDP.

[27] The ex ante user cost for a reproducible capital asset contains an anticipated asset inflation rate in it similar to (A8), which offsets the nominal interest rate term. The ex ante user cost concept should be close to an actual rental or leasing price for the asset since it based on the same considerations that an owner would consider in setting a rental price. Hence, it seems desirable to have the user cost of inventories

Both decompositions of economic income involve a difference in two value aggregates where the sign of the difference cannot be bounded away from zero. Hence for both decompositions, it is recommended that the beginning and end of period values be separately deflated and shown as two items in the real accounts in a manner that is analogous to the present treatment of exports less imports.

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