SHOULD CANADA'S RATE OF CAPITAL GAINS TAXATION BE REDUCED?

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Abstract

The paper lays out a simple model of business income taxation that includes the taxation of asset capital gains. The concepts of tax distorted and undistorted user costs of capital are also defined. The tax distorted user cost differs from its undistorted counterpart by two distortion terms. The first distortion term is associated with the fact that the imputed interest cost of equity capital cannot be deducted from taxable income and the second distortion term is associated with the present 75% inclusion rate for capital gains. Rather than further reducing the inclusion rate, the paper argues for a more fundamental tax reform that would reduce both distortion terms to zero. A technical appendix relates the size of the distortion terms and the efficiency costs to the economy of the tax distortions.

Keywords: Tax reform, capital gains, user cost of capital, deadweight loss, marginal excess burdens

Journal of Economic Literature Classification Numbers: D24, D58, D61, D92, E62, H25, M41

Should Canada's Rate of Capital Gains Taxation be Reduced?

By Erwin Diewert and Denis Lawrence.[1]

1. Introduction.

Since most of the participants at this conference will be arguing for a *lower* rate of capital gains taxation, we would like to make the case for a *higher* rate of capital gains taxation. However, at the same time, we advocate at least indexing equity capital for inflation to help offset the effects of the higher rate of capital gains taxation.

Our recommendations cannot be discussed in a scientific manner without some sort of a model of how business income taxation works. Thus in section 2 below, we lay out a simple model of business taxation that includes the effects of capital gains taxation of a capital asset. The key concepts are the user cost of capital and the tax distorted user cost of capital. In section 3 below, we note that the tax distorted user cost of capital is equal to the undistorted user cost plus two tax distortion terms. The first distortion term is associated with the fact that the imputed interest cost of equity capital cannot be deducted from taxable income. The second distortion term is associated with the present favourable tax treatment of realised capital gains. In section 4 below, we propose a tax reform that would eliminate both distortion terms.

A technical appendix makes the link between tax distorted user costs and the efficiency cost to the economy of these tax distortions.

2. User Costs and a Model of Business Taxation.

In order to focus on the issues involved in taxing business income, we consider a highly simplified model. A firm produces output Y during an accounting period using variable inputs L

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and a quantity of capital K. The average price of output during the accounting period is p and the average price of the variable input is w. Thus the firm's cash flow during the period is $pY - wL$. What price should the firm charge for its use of the capital input?

We assume that the firm can buy a unit of the capital input at price P at the beginning of the accounting period. The economic depreciation rate for the period is δ (a number between 0 and 1). At the end of the accounting period, the depreciated asset sells for the price $(1 - \delta)(1 + i)P$, where the rate of asset inflation for the period is i. Hence, the cost of buying one unit of the asset at the beginning of the period, using it during the accounting period and then selling it at the end of the period is the purchase price P less the value at the end of the period, $(1 - \delta)(1 + i)P$. However, this net cost neglects the fact that financial capital is tied up during the period. Thus if the firm faces the nominal interest rate (or cost of equity) of r, then rP has to be added to the above net cost of using the asset for the accounting period. The sum of these costs is U, the *user cost of capital*:[2]

(1) $U \equiv P(1+r) - (1 - \delta)(1 + i)P$

(2) $\quad = [r + \delta - (1 - \delta)\, i\,]P$

where (2) follows from (1) after simplification. In words, the user cost formula (2) says that U is equal to the interest cost rP plus the depreciation cost δP *less* the realised capital gains on the asset if sold or the unrealised capital gains if not sold, $(1 - \delta)\, i\, P$.

Once the user cost of capital has been defined, then the *economic income* E or pure profits that the firm earns during the accounting period can be defined as cash flow less total user cost:

(3) $E \equiv pY - wL - UK$.

However, *accounting income* A (for tax purposes) is not generally defined by (3) above. Assuming that the asset has been sold at the end of the accounting period, *accounting income* A is defined as follows:

(4) $A \equiv pY - wL - [\varphi r + d]PK + f[(1+i)(1 - \delta) - (1 - d)]PK$

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2 Formula (2) was derived by Christensen and Jorgenson(1969) but user cost formulae similar to (2) date back to the economist Walras (1954; 269) and the early industrial engineer Church (1901; 907-908). The user cost formula defined by (2) can be viewed as translating the beginning of the period cost into its equivalent end of period cost. A more traditional user cost would discount the end of the period revenues back to the beginning of the period using the discount factor 1/(1+r); i.e., the traditional user cost would be $P - (1+r)^{-1}(1 - \delta)(1 + i)P$. The user cost (2) is simply (1+r) times the traditional user cost.

where d is the depreciation rate that Revenue Canada sets for the asset, φ is the fraction of the capital purchase that is financed by *debt* (and hence the associated interest is tax deductible) and f is the fraction of realised capital gains that must be included as taxable income (currently equal to .75 in Canada). Note that $[(1+i)(1-\delta)]P$ is the selling price of one unit of the depreciated asset while $(1-d)P$ is the depreciated value of one unit of the asset for tax purposes.

Since our focus is on capital gains taxation and not on the adequacy of depreciation provisions, we will assume that the economic depreciation rate δ equals the tax purposes depreciation rate d; i.e., we assume that

$$(5)\quad \delta = d.$$

Substituting (5) into (4) yields the following expression for accounting income for tax purposes:

$$(6)\quad A = pY - wL - [\varphi r + \delta]PK + f\,i(1-\delta)PK.$$

Having defined income for tax purposes, A, business tax liability can be defined as accounting income A times the relevant rate of business income tax, τ say. Finally, *after tax income* ATI can be defined as economic income E less tax paid τA:

$$(7)\quad ATI \equiv E - \tau A$$

$$= (1-\tau)E - \tau UK + \tau[\,(\varphi r + \delta) - f\,i\,(1-\delta)]PK$$

using definitions (3) and(6)

$$= (1-\tau)\{E - (1-\tau)^{-1}\tau[\,(1-\varphi)r - (1-f\,)\,i\,(1-\delta)]PK\}$$

$$(8)\qquad = (1-\tau)\{pY - wL - U^*K\}$$

where the *tax distorted user cost* U* in (8) above is defined as

$$(9)\quad U^* \equiv U + D_1 + D_2.$$

Note that U is the *undistorted user cost of capital* defined earlier by (1) or (2) above and the two *distortion terms* D_1 and D_2 are defined as

$$(10)\quad D_1 \equiv (1-\tau)^{-1}\tau(1-\varphi)rP \text{ and}$$

$$(11)\quad D_2 \equiv -(1-\tau)^{-1}\tau(1-f\,)\,i\,(1-\delta)P.$$

We summarize the above algebra as follows. If there is no business income taxation, then the producer will choose Y, L and K so as to maximise economic income $pY - wL - UK$ defined by (3) above where the undistorted user cost of capital is U defined by (1) above. However, with business taxation, the firm will be induced to choose Y, L and K to maximise $pY - wL - U^*K$

(see (8) above) where U* is the *tax distorted user cost of capital* defined by (9) to (11) above. If the sum of the distortion terms $D_1 + D_2$ is positive, then the tax distorted user cost of capital will exceed the undistorted cost and the firm will tend to use too little capital and there will be an overall loss of economy wide productive efficiency. On the other hand, if the sum of the distortion terms $D_1 + D_2$ is negative, then the tax distorted user cost of capital will be less than the undistorted user cost and the firm will tend to use too much capital but there will still be an overall loss of economy wide productive efficiency. In the technical appendix, we indicate how the tax distorted user cost leads to a loss of productive efficiency in a highly simplified model.

3. Discussion of the Distortion Terms.

Consider the distortion term D_1 defined by (10) above. All of the terms which multiply together to define D_1 are nonnegative and hence D_1 must be nonnegative as well. If the firm's purchases of capital are entirely finance by debt, then φ is unity and D_1 becomes zero. If the firm's purchases of physical capital are entirely finance by equity capital, then φ is zero and the first distortion term D_1 attains a maximum value. Thus the first distortion term can be associated with the fact that the current system of business taxation *does not allow a deduction from income for the cost of equity capital.* If such a deduction were to be allowed, the first distortion term *would be identically zero*. We note also that as inflation increases, the nominal rate of return r on equity and debt financial capital would tend to increase so that D_1 will tend to increase as inflation increases.

Now consider the distortion term D_2 defined by (11) above. Assuming that the inflation rate i is positive, then all of the terms which multiply together to define D_2 are nonnegative, except for the negative sign at the beginning. Hence D_2 must be nonpositive. If capital gains are treated in the same way as regular income, then the parameter f is equal to 1 and D_2 collapses down to zero. Hence the second distortion parameter can be associated with the *nonneutral treatment of capital gains income*. Treating capital gains in the same manner as regular income means that the second distortion term *would be identically zero*.

Since the first distortion term will generally be positive due to the fact that the cost of equity capital is not deductible and since the second distortion term will be negative if the parameter f is less than one (which it is in Canada), *there is the possibility that the two distortions will offset each other* and an efficient system of business taxation would result. This is the core of a scientific argument for reducing the rate of capital gains taxation in Canada; such a reduction would help offset the present massive distortion due to the nondeductibility of the cost of equity capital. However, it can be seen that the offsetting distortions case could occur only by chance and it would be *impossible* to reduce the tax distortion terms down to zero for all firms without resorting to firm specific tax parameters.

4. Suggestions for Business Tax Reform.

In view of the analysis in the previous section, we would like to make the following concrete suggestions for reform of the current Canadian system of business income taxation:

1. Each quarter, Revenue Canada should announce an interest rate r which each firm could apply to its beginning of the quarter equity capital base B say so that the firm would get an automatic deduction rB from its taxible income for that quarter. Any tax losses could be carried forward at the announced rate of interest.

2. There should be no special treatment of income received as a result of capital gains (so that the tax parameter f should be *increased* from .75 to 1). We are not persuaded that it makes sense to treat income from different sources differently; a level playing field will reduce the incentive to convert income that is taxed at a higher rate into income that is taxed at a lower rate.

3. If there is a big loss of business tax revenues after implementing recommendations 1 and 2 above, and the Government wishes to undertake a revenue neutral tax reform, then serious consideration should be given to *increasing* the rate of business taxation τ in order to make up the revenue shortfall. If our analysis of the nature of the system of business taxation is correct, then implementing recommendations 1 and 2 above should lead to a *neutral tax system* and the burden of the resulting tax system should fall on *pure profits or rents*. Taxing away a higher proportion of these rents should not have an adverse effect on efficiency.

We are willing to be persuaded away from recommendation 3 above because our very highly simplified model does not cover all aspects of economic reality. For example, if our rate of business income taxation is very much higher than that of our major trading partners, then in this age of globalisation we will have trouble getting multinational firms to declare their income in Canada; i.e., they will engage in transfer pricing behavior and Canadian profits will end up in lower tax jurisdictions. There is a related problem with nonmarginal investments. In the present era of much lower trade barriers between countries, footloose industries will set up shop in countries that offer the lowest rate of business taxation, taking into account other aspects of comparative advantage. Thus Ireland has done tremendously well in attracting foreign investment in recent years by offering a business tax rate of 10% for foreign manufacturers or foreign providers of international business services. (However, domestic Irish firms pay a 20% tax rate, which is going to be reduced to 12.5% by the year 2003).

Thus if the loss of revenue that would occur due to the implementation of recommendation 1 above were too great and could not be covered by increasing other taxes, we strongly recommend that firms be given at least an inflation deduction on their equity capital base: instead of getting the full imputed interest deduction of rB as in 1 above, they could still get the deduction iB, where i is an appropriate quarterly inflation rate. If this watered down recommendation were implemented at a time when inflation is low, obviously the revenue opportunity cost would be low and easily affordable. However, this modest tax reform would protect the economy from enormous harm should inflation ever take off again.

We conclude with a note on some of the limitations of our analysis:

- We have not modeled the business tax system in all of its complexity. In particular, we have not modeled the extra layer of taxation that occurs at the personal level and the Canadian tax code provisions that mitigate this double taxation of business income. In general, taxation of capital income at the personal level will tend to increase the nominal interest rate r that firms face above its true opportunity cost, creating additional deadweight losses. However, the tax treatment of foreign lenders and investors must also be considered.
- We have not modeled the interaction of nonneutral depreciation provisions (recall that we assumed that the tax code depreciation rate d was equal to the economic depreciation rate δ) and this will add additional complexities to our user cost formula.
- We have not modeled the arguments for reduced capital gains taxation that rely on the stimulus for increased risk taking.

However, inspite of the above limitations, we hope that our analysis has caused the proponents of reduced capital gains taxation to carefully consider their position. In our view, instead of reducing the capital gains inclusion rate below the current 75%, the inclusion rate should be increased to 100% but at the same time, equity capital should at least be indexed for the safe rate of return if possible or, at a minimum, for inflation. The resulting move towards a more neutral tax system should lead to increased investment in Canada, a higher steady state capital stock, higher growth rates for investment[3] and an increase in real wages.

3 Romer (1987; 193) presents some persuasive evidence that higher growth rates for investment are associated with higher output growth rates and hence higher rates of productivity growth.

Technical Appendix: Efficiency Losses in a Simple Production Function Model

We illustrate the efficiency costs of taxing capital by considering a very simple model of a closed economy (i.e., we neglect the effects of international trade in goods). We suppose that units of private sector reproducible capital are combined with factors that are held fixed during the short run to produce units of aggregate output that can be used for either consumption C or investment I. Letting L denote the number of units of labour and other factors that are fixed in the short run we have:

(1) $Y = C + I = f(K, L)$

where f is the production function, Y denotes output and K denotes the beginning of the period capital stock. Note that we are assuming that units of the investment good I are perfectly substitutable with units of the consumption good C. We also assume that investment goods produced during the current period are added to the reproducible capital stock at the beginning of the following period. Thus, investment goods can be viewed as intertemporal intermediate inputs into the private production sector: I is produced this period so that it can be used as capital input next period and offset this period's depreciation of the capital stock.

We assume that each unit of the capital stock has a physical decline in its efficiency over the period at the rate δ ; ie if K units of the capital stock are in place at the beginning of the period, only $(1-\delta)K$ units are available for further use at the end of the current period.

We consider a steady state capital optimisation problem where investment is set equal to depreciation; ie we replace I in (1) by δK and maximise $C = f(K, L) - I = f(K, L) - \delta K$ with respect to K. Another way of viewing depreciation in this formulation is to regard it as a cost of production; the capital used at the beginning of the period, K, should be assessed a charge equal to the decline in value of the capital stock due to deterioration and a shorter life. Another charge that should be assessed against the starting capital stock is the opportunity cost of capital; ie the interest cost which will be just sufficient to induce owners of the capital stock to hold the capital stock through the period. Thus, if the real interest rate is r^*, then the optimal long run capital stock $K°$ is the solution to the following maximisation problem:

(2) $\max_K \{f(K,L) - (r^*+\delta)K\}$.

Since we are regarding *L* as fixed, write the production function *f*(*K*,*L*) as *f*(*K*). Then the first order necessary condition for *K*° to solve (2) is:

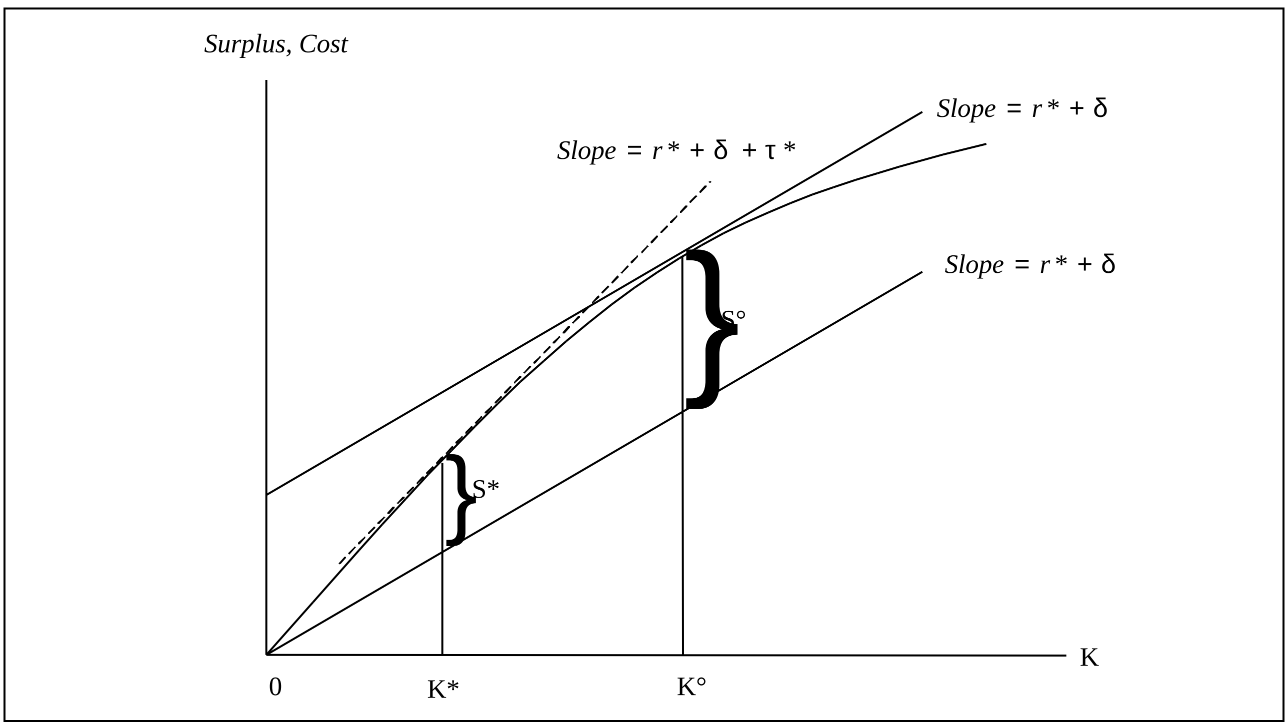
(3) $f'(K^{o}) = r^*+\delta$

where f' denotes the first derivative of *f*. We assume that the following second order sufficient condition is also satisfied:

(4) $f''(K^{o}) < 0$.

The geometry of the unconstrained maximisation problem (2) is illustrated in Figure 1 below. The curved line through the origin represents the production function constraint, *C*+*I* = *f*(*K*), while the straight line through the origin represents the depreciation and interest cost of capital. The difference between the two lines represents sustainable consumption (after interest payments) or surplus as a function of the beginning of the period capital stock *K*. The maximum sustainable surplus *S*° is achieved at the capital stock *K*° where the slope of the production function equals the slope of the capital cost function.

Figure 1: Stylised Loss from Capital Taxation



When capital is taxed, private producers will face the price $r^*+\delta+\tau^*$ per unit of capital used, where τ^* is the capital tax rate expressed as a fraction of the asset value of capital.[4] Thus, instead of solving (2) in the long run, private producers will be induced to choose the capital stock K^* which solves:

(5) $$\max_K \{ f(K) - (r^*+\delta+\tau^*)K \}.$$

We may regard the K^* which solves (5) as a function of the tax rate τ^*; ie $K^* = K(\tau^*)$. This solution to (5) will satisfy the following first order necessary condition:

(6) $$f'[K(\tau^*)] \;\; = r^*+\delta+\tau^*.$$

The fact that producers must pay capital taxes to the government increases the cost of using reproducible capital as an input and the resulting steady state capital stock $K(\tau^*)$ is smaller than the optimal capital stock, $K^o = K(0)$, which solved (2). The tax distorted surplus, $S^* \equiv f(K^*) - (r^* + \delta)K^*$ is smaller than the optimal surplus $S^o \quad f(K^o) - (r^*+\delta)K^o$; (see Figure 1).

Figure 1 illustrates *qualitatively* the effects of taxing reproducible capital — the higher the level of taxation, the lower will be the long run level of capital utilised and the corresponding surplus. In what follows, we indicate how a *quantitative* estimate of the decline in the sustainable surplus can be obtained.

First, differentiate equation (6) with respect to τ^*. We obtain the following equation for the change in capital due to a small increase in the tax rate, $K'(\tau^*)$:

(7) $$K'(\tau^*) = 1/f''[K(\tau^*)],$$

where f'' is the second derivative of the production function and will be negative under the usual assumptions on the production function. Now define producer surplus (or sustainable consumption after interest payments) as a function of the capital tax rate τ^* as follows:

(8) $$S(\tau^*) \quad f[K(\tau^*)] - (r^*+\delta)\; K(\tau^*).$$

Differentiating (8) with respect to τ^* and using (6) yields the following formula for the rate of change of surplus with respect to the level of capital taxation:

(9) $$S'(\tau^*) = [f'[K(\tau^*)] - (r^*+\delta)]\; K'(\tau^*) = \tau^* K'(\tau^*).$$

Evaluating (9) at $\tau^* = 0$ yields

(10) $$S'(\tau^*) = 0.$$

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4 We use r* to distinguish the real interest rate from the nominal interest rate r and we use τ* to distinguish the capital asset tax rate from the capital income tax rate τ.

Differentiating (9) with respect to $\tau *$ and evaluating the resulting derivative at $\tau * = 0$ yields

(11) $S''(0) = K'(0) = 1/ f''[K(0)] < 0$

where the second equality in (11) follows from (7) evaluated at $\tau * = 0$ and the inequality follows from $K^{o} = K(0)$ and (4). We now use (10) and (11) to form the following second order Taylor series approximation to $S(\tau *)$:

(12) $S(\tau *) \quad S(0) + S'(0)\tau * + (1/2)S''(0)\tau *^{2}$

$= S(0) + (1/2)\tau *^{2} \,/\, f''[K(0)].$

Define $L(\tau *)$ as the loss of sustainable output as a fraction of optimal output $Y(0)$:

(13) $L(\tau *) \quad [S(0) - S(\tau *)] \,/\, Y(0).$

Using (12), a second order approximation to $L(\tau *)$ is

(14) $A(\tau *) \quad -(1/2)\tau *^{2} \,/\, Y(0)\, f''[K(0)].$

We need to provide an economic interpretation for the second derivative of the production function, $f''[K(0)]$, evaluated at the optimal capital stock $K(0)$. The first derivative of the production function, $f'[K(0)]$, is the optimal return to one unit of the capital stock or the rental price of capital, $P_{K} = r* + \delta$. Thus, the second derivative can be interpreted as the change in the rental price due to a small change in the use of capital, $dp_{K}(K) / dK$:

(15) $dp_{K}[K(0)] / dK = f''[K(0)] = f''[K^{o}] < 0$

where the inequality follows from (4). We convert dp_{K} / dK into a non-negative (inverse) elasticity of demand for capital ε by changing the sign of $f''[K(0)]$ and multiplying by $K(0) / p_{K}[K(0)] = K(0) / f'[K(0)]$:

(16) $\varepsilon \quad - f''[K(0)]K(0) / f'[K(0)] = - f''[K(0)]K(0) / (r + \delta)$

where the second equality in (16) follows from (3).

It is also useful to define the economy's optimal capital output ratio γ as the ratio of the optimal capital stock *K*(*0*) to the optimal gross output $Y(0) = f[K(0)]$:

(17) $\gamma \quad K(0)/Y(0).$

Substitution of (16) and (17) into (14) yields the following formula for the approximate loss of producer surplus as a fraction of optimal output:

(18) $A(\tau *) = (1/2)\tau *^{2} \gamma \,/\, \varepsilon(r* + \delta) > 0.$

For advanced industrial economies, a typical range for the capital tax rate τ^* is 0.01 to 0.03 (ie one percent to three percent of the asset value of capital), for the capital output ratio γ is two to four, for the inverse elasticity of demand for capital ε is 0.5 to 1.0, for the real after tax rate of return r^* is 0.01 to 0.05 and for the depreciation rate δ is 0.04 to 0.08. If we substitute the midrange values for these parameters into the right hand side of (18), we find that the approximate output loss due to capital taxation at the rate $\tau^* = 0.02$ is $A(0.02) = 0.0089$ or 0.89 percent of gross domestic product. Table 1 below indicates how the approximate loss of output $A(\tau^*)$ varies as each parameter varies between the low and high values of its assumed range while letting the remaining parameters equal their midrange values.

Table 1: Percentage Loss of Output

Parameter	*Midrange*	τ^*		r^*		γ		δ		ε	
Value	*Values*	0.01	0.03	0.01	0.05	2.00	4.00	0.04	0.08	0.50	1.00
% loss	0.89	0.22	2.00	1.14	0.73	0.59	1.19	1.14	0.73	1.33	0.67

Note that the approximate loss of output increases as the square of the capital tax rate τ^* so if τ^* increases from 0.02 to 0.03 and the other parameters remain at their midpoint values, the loss of output due to capital taxation increases from 0.89 percent of GDP to two percent of GDP. These output losses persist year after year so that the present value of these annual output losses is substantial.

It should be emphasised that the above efficiency losses induced by the taxation of capital are entirely avoidable: equivalent amounts of revenue could be raised by taxing the final outputs of the private production sector or by taxing primary inputs. We note that the latter two forms of taxation do not involve a loss of productive efficiency for the economy whereas taxing an intermediate input like capital invariably involves a loss of productive efficiency.[5]

The efficiency losses listed above in Table 1 are likely to underestimate substantially the actual losses that are induced by capital taxation in an industrialised economy. The above model assumes only one capital stock with an average tax rate of τ^* which is applied to the asset value of reproducible capital. In actual economies, the system of business income taxation invariably taxes lightly some components of the capital stock and taxes other components very heavily. The efficiency losses associated with the differential taxation of each type of capital will grow approximately as the square of the tax rate. Thus, the large losses associated with the heavily

5 See Diewert (1983a) (1983b) for the productive efficiency approach to tax policy. Although capital taxation cannot be justified on productivity or efficiency grounds, it still can be justified on equity grounds.

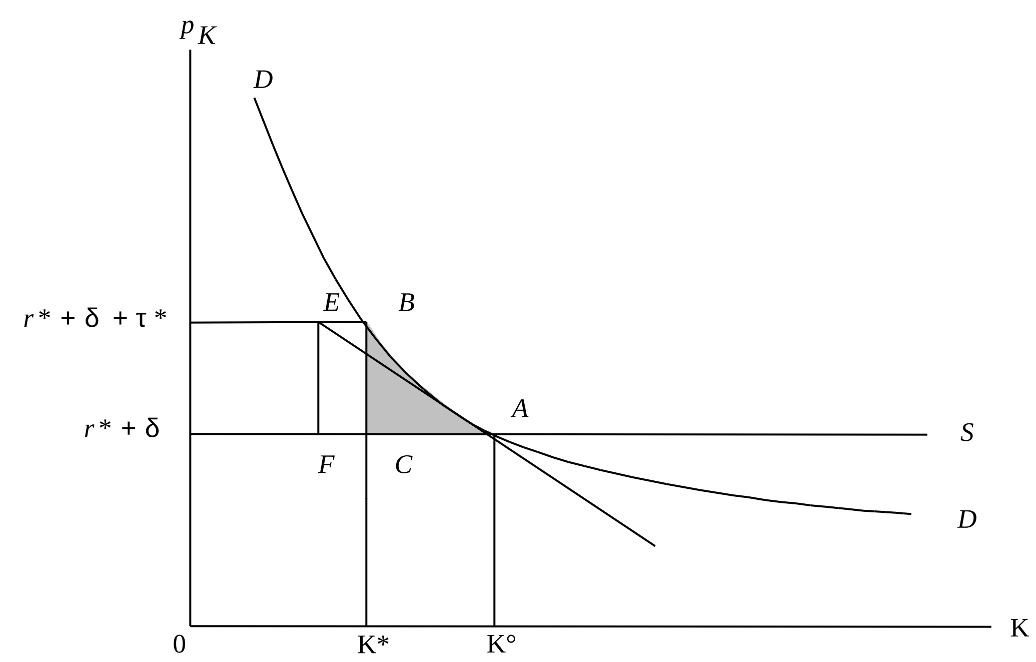
taxed components will not be balanced by the small losses associated with the lightly taxed components and the total loss will be much larger than the loss obtained by applying an average tax rate to the total reproducible capital stock.[6]

Another diagram may be helpful in illustrating the efficiency costs of capital taxation. Note that $K(\tau^*)$, the capital stock solution to equation (6), can be regarded as the long run demand for reproducible capital as a function of the tax rate τ^*. Now rewrite equation (6) as follows:

$$(19) \quad f'[K(r^*+\delta+\tau^*)] = r^*+\delta+\tau^*$$

ie the demand for capital $K(r^*+\delta+\tau^*)$ which solves (19) can be written as a function of the tax distorted rental price of capital, $r^*+\delta+\tau^*$. In Figure 2 below, the inverse of this demand for capital function is graphed as the curve *DD*. If there were no capital taxes, capital would be supplied to the private production sector at the rental price $r^*+\delta$ which would just cover the real interest and depreciation costs of using a unit of capital for the period. This horizontal supply of capital curve intersects the demand curve at the point *A*. The imposition of the capital tax τ^* shifts the supply of capital curve up and this tax distorted supply curve intersects the demand curve at *B*. Note that the equilibrium level of capital used has decreased from *K*° to *K**.

Figure 2: Alternative Representation of Capital Tax Loss



6 Reproducible capital stocks are stocks produced by the production sector. There are no efficiency losses associated with taxing capital stock components that are fixed (such as land).

Equation (19) can be integrated to obtain an expression for the gross change in output; ie the change in gross output produced due to capital taxation before deducting depreciation and interest costs; ie we have

$$(20)\quad f(K^{o}) - f(K^{*}) = \int_{K^{*}}^{K^{o}} f'(K)dK = Area\ BAK^{o}K^{*}.$$

The efficiency cost of capital taxation is defined to be the net change in output after deducting depreciation and interest payments:

$$(21)\quad S(0) - S(\tau^{*}) \quad [f(K^{o}) - (r^{*}+\delta)K^{o}] - [f(K^{*}) - (r^{*}+\delta)K^{*}]$$

$$= f(K^{o}) - f(K^{*}) - (r^{*}+\delta)(K^{o} - K^{*})$$

$$= Area\ BAK^{o}K^{*} - Area\ CAK^{o}K^{*}$$

$$= Area\ ABC.$$

Thus, the area of the shaded triangular region under the demand curve is a measure of the efficiency costs of capital taxation. This is a producer surplus measure of deadweight loss.[7]

We now linearise the demand curve around the undistorted equilibrium point *A* and use the triangle *AEF* as an approximation to the exact deadweight loss *ABC*. From (15) and (16), it can be verified that the absolute value of the slope of the linear approximation to *DD* at *A* is

$\varepsilon(r^{*}+\delta)/K^{o}$. The vertical distance *EF* in Figure 2 is equal to τ^{*} so the horizontal distance of the triangle *AEF*, *AF*, will equal the vertical distance τ^{*} divided by the slope $\varepsilon(r^{*}+\delta)/K^{o}$. Thus,

$$(22)\quad Area\ AEF = (1/2)[\tau^{*}/\{\varepsilon(r^{*}+\delta)/K^{o}\}]\tau^{*}$$

$$= (1/2)\tau^{*2}K^{o}/\varepsilon(r^{*}+\delta)$$

$$= Y(0)\,A(\tau^{*})$$

where we have used (18) to derive the last equality in (22). Thus, the area *AEF* in Figure 2 is equal to the approximate loss $A(\tau^{*})$, defined earlier by (18), times optimal GDP, $Y(0)$. We note that for small tax rates τ^{*}, the approximate loss measure *AEF* should be quite close to the exact loss measure *ABC*.

The approximate total efficiency loss or excess burden of capital taxation defined by (22) is not the most interesting number from the viewpoint of economic policy. A more interesting concept, initiated by Browning (1976) (1987), is the marginal excess burden of capital taxation. This

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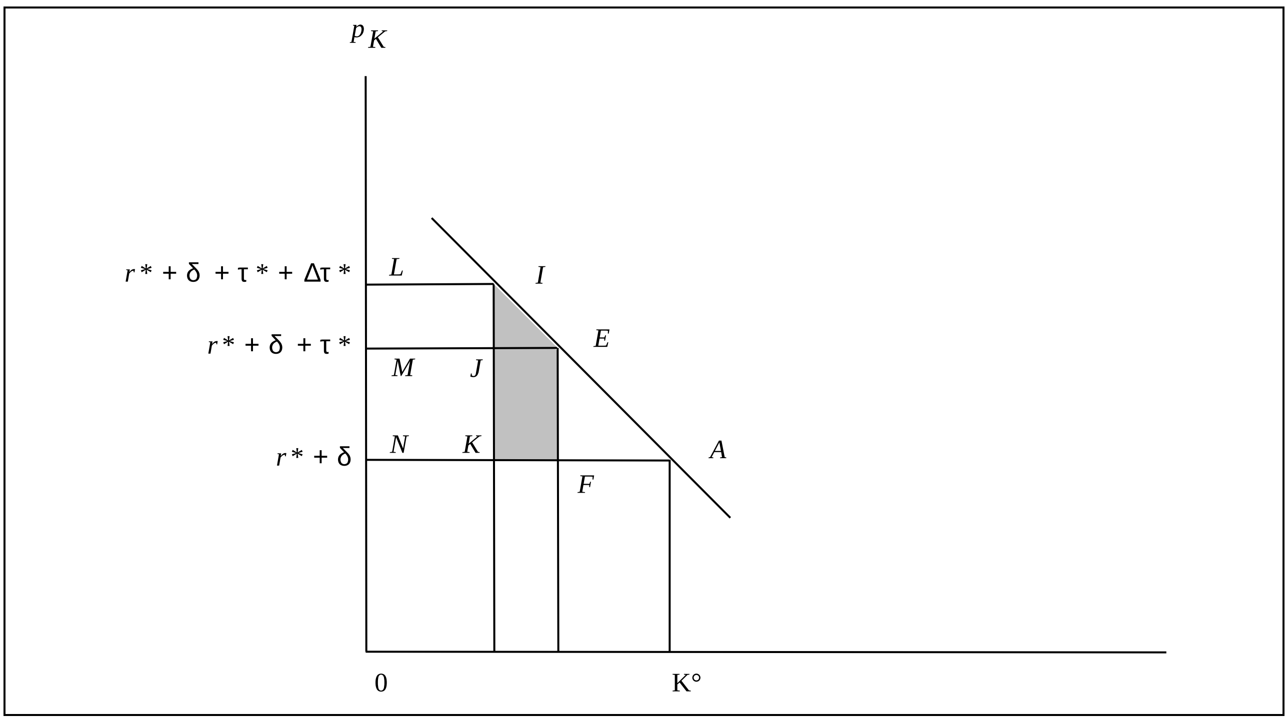
7 Note that our loss measure does not contain a consumer surplus term. For analogous consumer surplus measures of deadweight loss, see Browning (1976) (1987) and Findlay and Jones (1982).

concept compares the increase in efficiency loss due to a small increase in the level of capital taxation to the increase in tax revenue that can be attributed to the tax increase. Another diagram may be helpful in explaining the concept.

In Figure 3 below, we have reproduced the approximate deadweight loss triangle *AEF* as in Figure 2 and this triangle corresponds to the efficiency loss when the capital tax rate is τ^*. We now increase the capital taxation rate by a small amount $\Delta\tau^*$ and we note that the increase in efficiency loss is equal to the triangle *EIJ* plus the rectangle *EFKJ*. The initial tax revenue is equal to the area of the rectangle *EFNM* and the new tax revenue is equal to the area of the rectangle *IKNL*. Thus, the change in tax revenue is equal to the area of *IJML* minus the area of *EFKJ*. This change in tax revenue (the incremental benefits of the tax increase) can be compared to the increased efficiency loss, *EFKI*, (the incremental costs of the tax increase). Note that if the initial level of taxation τ^* is very high, then the incremental tax revenue can be negative; ie the induced reduction in the use of capital can outweigh the increased tax revenue per unit of capital used by private producers.

Figure 3: Marginal Excess Burden of Capital Taxation



We now provide an analytic formulation that corresponds to the marginal excess burden measure described by Figure 4.3. Equation (18) describes the approximate efficiency loss $A(\tau^*)$ as a fraction of optimal output $Y(0)$. Differentiating this function with respect to τ^* gives us the following formula for the marginal efficiency loss (as a fraction of $Y(0)$):

(23) $A'(\tau^*) = \tau^*\gamma \,/\, \varepsilon(r^* + \delta) > 0$.

Define tax revenue as a function of the capital tax rate τ^*, $T(\tau^*)$ as follows:

(24) $T(\tau^*) \quad \tau^* K(\tau^*)$.

Note that $T(0) = 0$ and the first and second order derivatives of $T(\tau^*)$ evaluated at the no distortion point $\tau^* = 0$ are:

(25) $T'(0) = K(0)$

(26) $T''(0) = 2K'(0)$.

Thus a second order Taylor series approximation to $T(\tau^*)$ is

(27) $T(\tau^*) \quad K(0)\tau^* + K'(0)\tau^{*2}$

Define the approximate benefit function $B(\tau^*)$ as the right hand side of (27), divided by the optimal output $Y(0)$:

(28)

$$
\begin{aligned}
B(\tau^*) &\quad [K(0)\tau^* + K'(0)\tau^{*2}] \,/\, Y(0) \\
&= \gamma\tau^* + K'(0)\tau^{*2} \,/\, Y(0) && \text{using (17)} \\
&= \gamma\tau^* - \gamma\tau^{*2} \,/\, \varepsilon(r^* + \delta) && \text{using (7) and (16).}
\end{aligned}
$$

Now differentiate (28) with respect to τ^* which gives us a formula for the marginal benefit of increasing capital taxes $B'(\tau^*)$ (as a fraction of optimal output $Y(0)$):

(29) $B'(\tau^*) = \gamma[1 - 2\tau^* / \varepsilon(r^* + \delta)]$.

Finally, define the (approximate) marginal excess burden of capital taxation $MEB(\tau^*)$ as the ratio of the marginal efficiency cost $A'(\tau^*)$ defined by (23) to the marginal benefit $B'(\tau^*)$ defined by (29):

(30) $MEB(\tau^*) = A'(\tau^*) \,/\, B'(\tau^*) = \tau^* \,/\, [\varepsilon(r^* + \delta) - 2\tau^*]$.

Note that $MEB(\tau^*)$ depends not only on the rate of capital taxation τ^* but it also depends on the inverse elasticity of demand for capital ε, the real interest rate r^* and the depreciation rate δ. However, $MEB(\tau^*)$ does not depend on the capital output ratio γ, in contrast to our earlier formula for the approximate total efficiency loss (as a fraction of optimal output), $A(\tau^*)$, defined by (18).

In Table 2 below, we evaluate $MEB(\tau^*)$ defined by (30) at our midrange estimates for the capital tax rate $(\tau^* = 0.02)$, the real interest rate $(r^* = 0.04)$, the depreciation rate $(\delta = 0.06)$ and the (inverse) elasticity of demand for capital $(\varepsilon = 0.75)$. We also table $MEB(\tau^*)$ as each parameter varies between the low and high values of its assumed range, while letting the other parameter values equal their assumed midrange values.

Table 2: Marginal Excess Burdens of Capital Taxation

Parameter	Midrange	$\tau*$		$r*$		δ		ε	
Values	*Values*	0.01	0.03	0.01	0.05	0.04	0.08	0.50	1.00
MEB	0.727	0.211	4.000	1.600	0.471	1.600	0.471	4.000	0.400

From Table 2, the marginal excess burden of capital taxation when $\tau * = 0.02$, $r* = 0.03$, $\delta = 0.06$ and $\varepsilon = 0.75$ is 72.7 percent. This means that if the government is contemplating financing a new recurring program expenditure by increasing capital taxation, then for each dollar of tax revenue spent on the program, its benefits should exceed 1.727 dollars; ie the loss of productive efficiency that is induced by a tax increase that yields an extra dollar of revenue is 72.7 cents. In contrast to the rather small numbers in Table 1, the numbers in Table 2 are rather large. For example, if the level of capital taxation increases from two percent to three percent, then the marginal excess burden increases from 72.7 percent to 400 percent; i.e., the marginal benefits that accrue to an incremental program that is financed by the increased level of capital taxation should exceed five dollars for each dollar of revenue raised. Of that five dollars, one dollar of benefits is required to make up for the one dollar of tax revenue that is diverted from private uses and the other four dollars of benefits are required to offset the loss of output that the increased level of capital taxation induces in the private production sector.

Table 2 indicates that the marginal excess burden of capital taxation is very sensitive to the parameter values that were inserted into formula (30). This is unfortunate, because it is difficult to determine $\tau*$, $r*$, δ and ε with great precision in actual economies. Hence relatively small errors in these parameters can translate into relatively large errors in the associated excess burdens. However, our qualitative assessment of the numbers presented in Table 2 is that the marginal excess burdens generated by the taxation of reproducible capital are likely to be considerably larger than the marginal excess burdens generated by taxing consumption or labour.[8] Our reason for this a priori expectation is that even though $\tau*$ is a relatively small fraction, it is a relatively large *proportion* of the undistorted rental price of capital $r*+\delta$ and hence has a relatively large effect on the allocation of resources.

We conclude this section by reworking Tables 1 and 2 in the case where the production function has the Cobb-Douglas functional form. In this case, the production function is

8 Diewert and Lawrence (1994) (1996) found that the marginal excess burdens for labour and consumption taxes were in the 10 percent to 20 percent range for the New Zealand economy.

(31) $\quad Y = f\left(K,L\right) \quad K^{\beta}L^{1-\beta}$

where β is a parameter between 0 and 1. In the Cobb-Douglas case, the capital output ratio γ and the inverse elasticity of demand ε are no longer independent parameters: both are determined by β, the real interest rate r^* and the depreciation rate δ. To see this, note that when the production function is defined by (31), the first order condition (3) can be rewritten as

(32) $\quad \beta Y\left(0\right) / K\left(0\right) = r^* + \delta$

$$\text{or } \beta\gamma = r^* + \delta$$

where the second equality in (32) follows from the line above using definition (17). Rearranging (32) yields

(33) $\quad \gamma = (r^* + \delta) / \beta$.

Using definition (16) and performing the differentiation for the Cobb-Douglas case, we find that the (inverse) demand elasticity for capital ε is the following function of β:

(34) $\quad \varepsilon = 1 - \beta$.

Thus, in the Cobb-Douglas case, γ and ε are determined by (33) and (34) once β, r^* and δ have been specified.

When we use (33) and (34) to eliminate γ and ε in our approximate loss formula (18), we obtain the following expression for the (approximate) percentage of optimal output lost due to capital taxation at the rate τ^* in the Cobb-Douglas case:

(35) $\quad A(\tau^*) = \left(1/2\right)\tau^{*2} \; \beta \; / \left(1-\beta\right)(r^* + \delta)^2$

Table 3 below lists the percentage losses of output $A(\tau^*)$ defined by (35) for various values of τ^*, r^*, δ and β .

Table 3: Percentage loss of output in the Cobb-Douglas case

Parameter	Midrange	τ^*		r^*		δ		β	
Values	*Values*	0.01	0.03	0.01	0.05	0.04	0.08	0.10	0.40
% Loss	0.82	0.21	1.85	1.36	0.55	1.36	0.55	0.27	1.65

From Table 3, the loss at our midrange values for the parameters when $\tau^* = 0.02$, $r^* = 0.03$, $\delta = 0.06$ and $\beta = 0.25$ is 0.82 percent of GDP. If τ is reduced to 0.01 (a one percent capital tax rate), the loss drops to a small 0.21 percent of GDP. On the other hand, if τ^* increases to 0.03 (a three percent capital tax rate), then the loss of productive efficiency increases dramatically to

1.85 percent of GDP. As in Table 2, the magnitude of the efficiency loss is quite sensitive to variations in each of the parameters τ^*, r^*, δ and β .

Since γ does not appear in our earlier more general formula for the marginal excess burden $MEB(\tau^*)$, we can simply replace ε in (30) by 1-β to find the marginal excess burdens for the Cobb-Douglas case. However, there is no need for us to re-compute Table 2 to give marginal excess burdens for the Cobb-Douglas case since our midrange estimate for ε in Table 2 is 0.75 which corresponds to our midrange estimate for β equal to 0.25.

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