The Measurement of Inflation After Tax Reform

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Abstract

It is suggested that instead of attempting to adjust the consumer price index (CPI) after tax reform it is better to measure changes in after-tax income.

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1 Introduction

The immediate motivation for this paper is the recent trend of governments to increase indirect taxes (i.e., taxes on purchases of commodities) and reduce direct taxes. For example, New Zealand, Japan and Canada have recently introduced a goods and services tax. New Zealand raised its Goods and Services Tax (GST) in 1989 to 12.5%, from its initial (1986) level of 10%. Japan increased its consumption tax (*shōhizei*) in 1997 to 5%, from its initial (1989) level of 3%. Such a tax has also been debated during the last two election campaigns in Australia. An increase in indirect taxation will cause an increase in the consumer price index (CPI), and for a variety of indexation purposes, it would be desirable to be able to measure the magnitude of this tax-reform-induced increase in the CPI. For example, pressures to target inflation have lead central banks to produce adjusted-CPI measures which aim to reveal "underlying" inflation. These measures attempt to exclude (among other things) the effects of changes in indirect taxes on the CPI. Such adjusted measures are also regarded as important due to the indexation of welfare payments in many countries.

A straightforward, but rather crude method for adjusting the CPI is as follows. We can represent the difference between a consumer's purchase price of a good and a producer's selling price of the same good as indirect taxes less subsidies on the good. Assuming that the producer's selling price remains unaffected by the tax reform, a hypothetical consumer's purchase price can be computed using the indirect tax rates from prior to the tax reform and then a hypothetical CPI can be constructed using these hypothetical consumer prices.

The above approach neglects tax-reform-induced changes in the structure of producers' selling prices. Another approach takes these structural changes into account, but it requires the rather restrictive assumption that the structure of production and intermediate input utilization remains constant going from the period prior to a tax reform to the period after the tax reform; i.e., it assumes that the input-output structure of the economy remains constant.

These CPI adjustment methods correspond roughly to the methods used by agencies in e.g., five European countries (Denmark, Finland, Ireland, The Netherlands and Sweden; see

Donkers, Jensen, Hyrkkö, Lehtinen, Murphy, Stolpe and Turvey (1983)), New Zealand (see Roger, 1995) and Canada (see Bank of Canada, 1991).

Recognizing the inadequacies of the above measures of CPI adjustment, we suggest a rather different approach to this problem. We note that the tax reform will lead to an *increase* in indirect taxes and hence an *increase* in the CPI, but at the same time, the tax reform will lead to a *decrease* in tax rates on sources of income and hence the tax reform should also lead to an *increase* in the after-tax income of consumers. So instead of attempting to construct a hypothetical CPI that is based on tax rates prior to the tax reform, we suggest that we simply measure after-tax income, since for many purposes, what we are interested in is the ratio of after-tax income deflated by the relevant CPI. We feel that this approach is preferable. However, after-tax income divided by the CPI is not a pure price measure, since after-tax income includes the effects of economic growth, technological progress, increased hours of work and changes in the country's international asset position.

Thus in section 2 we introduce the concept of a *constant utility income deflator* for a single consumer. A Laspeyres-type after-tax income deflator is subject to the same kind of substitution bias (except in the opposite direction) that the CPI has when it is regarded as an approximation to a consumer's true cost-of-living index. The constant utility income deflator is the income counterpart to the true cost-of-living index. We show how a reasonable approximation to a constant utility income deflator can be obtained by taking the geometric mean of the Laspeyres and Paasche after-tax income deflators. We extend this initial single-household theory to the entire economy.

Section 3 concludes. Proofs of theorems are given in the appendix.

2 Income Deflators and Tax Reform

An increase in indirect taxation will increase the CPI, but similtaneous reductions in direct taxation will increase after-tax income. This scenario is typical of tax reforms in an increasing number of countries. Therefore, examining the impact of tax reforms on the CPI only tells half of the story. For the purposes of welfare indexation, for example, the income side of the story is also of great interest. We therefore suggest the following index:

$$W = I/C, \tag{1}$$

where C is the consumer price index and I is an income deflator, i.e., a price index for income rather than consumption. If $W > 1$, then $I > C$ which means that the effect of the tax reform has been to increase income prices by more than consumption prices. In this case the tax reform has been favourable to the consumer, and the reverse if $W < 1$. Note that using this index, W, we do not have to attempt any difficult adjustments to C or I after a tax reform, yet obtain information of interest about the effects of the reform.

It remains to determine appropriate forms for C and I. Diewert (1983), using a result due to Konüs (1939), found that the Fisher Ideal price index (Fisher, 1922) provided a reasonably close approximation to the conditional (on the utility level) cost-of-living index (Pollak, 1975). We therefore suggest that an appropriate formula for C is as follows:

$$C = \{(p^1 \cdot c^0/p^0 \cdot c^0)(p^1 \cdot c^1/p^0 \cdot c^1)\}^{(1/2)}. \tag{2}$$

where p^i and c^i are respectively the period i consumer price and consumption quantity vectors.

Now we try to determine an appropriate form for I. Suppose that the consumer's one period preferences over combinations of the N consumer goods and the J types of labour services can be represented by the utility function f where $u = f(c, h)$, $c \equiv [c_1, ..., c_N]$ is a consumption vector, $h \equiv [h_1, ..., h_J]$ is a labour supply vector and u is a utility or real income level. Let $w \equiv [w_1, ..., w_J]$ be a positive after-tax vector of wage rates for the J types of labour services, let $r \equiv [r_1, ..., r_K]$ be a vector of one period rates of return for the K non-labour assets, and let $a \equiv [a_1, ..., a_K]$ be a reference vector of asset holdings. Then the consumer's *constant utility income* (or revenue) *function* R is defined as follows.

$$R(w, r, u, c, a) \equiv \max_h \{w \cdot h + r \cdot a : f(c, h) \geq u\}. \tag{3}$$

If the consumer-worker is not qualified to supply the jth type of labour service, we need to add the constraint $h_j = 0$ to (3) for each such labour service j. Assuming that $f(c, h)$

is nondecreasing with respect to the components of c and nonincreasing with repect to the components of h, then R will be nonincreasing in the components of c and nonincreasing with respect to u. R will also be linearly homogeneous and convex with respect to the components of w and r. In the definition of R we also hold consumption constant. We are assuming that the consumer derives no direct utility or disutility from the holding of the financial assets $a \equiv [a_1, ..., a_K]$.

Suppose that the vectors of after-tax wage rates and asset returns were w^i and r^i respectively in periods $i = 0, 1$. Given the reference level of utility u, the reference consumption vector c and the reference asset holdings vector a, we define the consumer's *constant utility* (and endowment) *income deflator* by

$$I(w^0, r^0, w^1, r^1, u, c, a) \equiv R(w^1, r^1, u, c, a)/R(w^0, r^0, u, c, a). \tag{4}$$

It can be seen that $I(w^0, r^0, w^1, r^1, u, c, a)$ is an index of after-tax wage rates and asset returns in period 1 relative to after-tax wage rates and asset returns in period 0. This theoretical index depends on the unobservable revenue function, R, that is dual to the consumer's unobservable utility function, f. Thus in order to operationalize this theoretical index, we need to develop bounds or approximations to it.

Suppose that the consumer's observed period i consumption vector is c^i, the labour supply vector is h^i and the vector of asset holdings is a^i for $i = 0, 1$. Define the period i utility levels by $u^i \equiv f(c^i, h^i)$, for $i = 0, 1$. Assuming maximizing behaviour with respect to labour supply decisions in each period, we have

$$R(w^i, r^i, u^i, c^i, a^i) = w^i \cdot h^i + r^i \cdot a^i, \quad i = 0, 1. \tag{5}$$

Using definition (4) with $(u, c, a) = (u^0, c^0, a^0)$, and using (3) and (5), we have

$$\begin{aligned} I(w^0, r^0, w^1, r^1, u^0, c^0, a^0) &\equiv R(w^1, r^1, u^0, c^0, a^0)/R(w^0, r^0, u^0, c^0, a^0) \\ &= \max_h \{w^1 \cdot h + r^1 \cdot a^0 : f(c^0, h) \geq u^0\}/(w^0 \cdot h^0 + r \cdot a^0) \\ &\geq (w^1 \cdot h^0 + r^1 \cdot a^0)/(w^0 \cdot h^0 + r^0 \cdot a^0) \qquad (6) \end{aligned}$$

where the inequality follows since h^0 is feasible for the maximization problem but it is not necessarily optimal. Similarly, if $w^1 \cdot h^1 + r^1 \cdot a^1 > 0$, we can show that

$$I(w^0, r^0, w^1, r^1, u^1, c^1, a^1) \leq (w^1 \cdot h^1 + r^1 \cdot a^1)/(w^0 \cdot + r^0 \cdot a^1). \tag{7}$$

The lower bound in (6) is the Laspeyres income deflator, and the upper bound in (7) is the corresponding Paasche income deflator.

Theorem 1 *There exists a* λ^* *such that* $0 \leq \lambda^* \leq 1$, $u^* \equiv \lambda^* u^0 + (1-\lambda^*)u^1$, $c^* \equiv \lambda^* c^0 + (1-\lambda^*)c^1$, $a^* \equiv \lambda^* a^0 + (1-\lambda^*)a^1$ *and the theoretical income deflator* $I(w^0, r^0, w^1, r^1, u^*, c^*, a^*)$ *lies between the bounds in (6) and (7).*

Thus a reasonably close approximation to this theoretical index may be obtained by taking the geometric mean of the two bounds, i.e.,

$$\begin{aligned} &I(w^0, r^0, w^1, r^1, u^*, c^*, a^*) \\ &= \{(w^1 \cdot h^0 + r^1 \cdot a^0)(w^1 \cdot h^1 + r^1 \cdot a^1)/(w^0 \cdot h^0 + r^0 \cdot a^0)(w^0 \cdot h^1 + r^0 \cdot a^1)\}^{1/2}. \end{aligned} \tag{8}$$

Thus we have derived, for a single consumer, methods for measuring the pure price effects of tax reform. In order to reduce the bias due to substitution effects, on the consumer expenditure side we suggest the use of expression (2) and on the income side the use of expression (8).

We now suppose that there are H households or consumers in the economy and the preference function for household i is $f^i(c, h)$ for $i = 1, ..., H$, where c and h represent consumption and labour supply vectors respectively.

For each household i, define the constant utility income function R^i as in definition (3), except f^i replaces f.

Define the aggregate constant utility income deflation function I_A as

$$\begin{aligned} &I_A(w^0, r^0, w^1, r^1, u_1, ..., u_H, c_1, ..., c_H, a_1, ..., a_H) \\ &\equiv \textstyle\sum_{i=1}^H R^i(w^1, r^1, u_i, c_i, a_i) / \sum_{i=1}^H R^i(w^0, r^0, u_i, c_i, a_i) \end{aligned} \tag{9}$$

where w^t and r^t are the after-tax period t wage rate and asset return vectors for $t = 0, 1$, u_i is a reference utility level for household i, c_i is a reference consumption vector for household i and

a_i is reference nonlabour asset holdings vector for household i, $i = 1, ..., H$. This definition is similar to the Prais (1959) "plutocratic" price index, or the Pollak (1981: 328) Scitovsky-Laspeyres cost-of-living index, since the importance of each household in the aggregate index is proportional to the size of the household's income.

Assuming income maximization behaviour between periods 0 and 1, then the following bounds are valid:

$$\begin{aligned} &I_L \equiv (w^1 \cdot h^0 + r^1 \cdot a^0)/(w^0 \cdot h^0 + r^0 \cdot a^0) \\ &\leq I_A(w^0, r^0, w^1, r^1, u_1^0, ..., u_H^0, c_1^0, ..., c_H^0, a_1^0, ..., a_H^0); \end{aligned} \tag{10}$$

$$\begin{aligned} &I_A(w^0, r^0, w^1, r^1, u_1^1, ..., u_H^1, c_1^1, ..., c_H^1, a_1^1, ..., a_H^1) \\ &\leq (w^1 \cdot h^1 + r^1 \cdot a^1)/(w^0 \cdot h^1 + r^0 \cdot a^1) \equiv I_P. \end{aligned} \tag{11}$$

where the inequalities follow from equations (6) and (7).

Finally, the aggregate counterpart to Theorem 1 is as follows.

Theorem 2 *There exists a* λ^* *such that* $0 \leq \lambda^* \leq 1$, $u_i^* \equiv \lambda^* u_i^0 + (1 - \lambda^*) u_i^1$, $c_i^* \equiv \lambda^* c_i^0 + (1 - \lambda^*) c_i^1$, $a_i^* \equiv \lambda^* a_i^0 + (1 - \lambda^*) a_i^1$ *and the theoretical income deflator* $I_A(w^0, r^0, w^1, r^1, u_1^*, ..., u_H^*, c_1^*, ..., c_H^*, a_1^*, ..., a_H^*)$ *lies between the bounds in (10) and (11).*

Thus a reasonably close approximation to this theoretical index may be obtained by taking the geometric mean of the two bounds, i.e.,

$$I_A(w^0, r^0, w^1, r^1, u_1^*, ..., u_H^*, c_1^*, ..., c_H^*, a_1^*, ..., a_H^*) \simeq (I_L I_P)^{1/2}, \tag{12}$$

which is a Fisher Ideal income-price index counterpart to the consumer-price index of (2).

The approximation defined by (12) leads to the following "practical" observations. The Laspeyres-type after-tax income deflator I_L is a useful measure of inflation. It has the advantage that it can be calculated using only base period prices and quantities and current period prices. However, this index is subject to a certain amount of substitution bias (recall the inequality in (10)). This substitution bias can be minimized if current period quantities are also available so that the appropriate Paasche and Fisher Ideal indexes can be constructed. Hence we suggest that when new quantities become available, these Paasche and Fisher

Ideal indexes should be constructed as special supplementary historical series for the periods between the two periods for which quantities are available. Thus, we recommend the use of the right-hand-side of (12) as I in calculating W, as defined by (1), with C given by (2).

3 Conclusion

The inadequacies of CPI adjustment methods led us to suggest an alternative approach to the problem of measuring the effects of a tax reform; we suggested that the CPI could be left unchanged and that instead, changes in after-tax income should be measured. To reduce the bias due to substitution effects, we suggested that Fisher indexes should be calculated when the required information becomes available.

We note that there would be significant practical measurement problems in order to construct indexes of after-tax income. However, we believe that it would be very worthwhile to attempt to implement an appropriate income and wealth survey. We believe that the construction of at least a Laspeyres-type after-tax income deflator would prove to be as useful as the current CPI.

Appendix

Proof of Theorem 1

Let $\xi(0) \equiv I(w^0, r^0, w^1, r^1, (1-\lambda)u^0 + \lambda u^1, (1-\lambda)c^0 + \lambda c^1, (1-\lambda)a^0 + \lambda a^1)$ for $0 \leq \lambda \leq 1$, and note that $\xi(0) = I(w^0, r^0, w^1, r^1, u^0, c^0, a^0)$, $\xi(1) = P(w^0, r^0, w^1, r^1, u^1, c^1, a^1)$. There are 24 possible *a priori* inequality relations between the four numbers $\xi(0), \xi(1), I_L = (w^1 \cdot h^0 + r^1 \cdot a^0)/(w^0 \cdot h^0 + r^0 \cdot a^0)$ and $I_P = (w^1 \cdot h^1 + r^1 \cdot a^1)/(w^0 \cdot h^1 + r^0 \cdot a^1)$. However, from equations (6) and (7), $\xi(0) \geq I_L$ and $\xi(1) \leq I_P$, which implies that we have only six possible inequalities between the four numbers: (i) $\xi(0) \geq I_L \geq I_P \geq \xi(1)$, (ii) $\xi(0) \geq I_P \geq I_L \geq \xi(1)$, (iii) $\xi(0) \geq I_P \geq \xi(1) \geq I_L$, (iv) $I_P \geq \xi(0) \geq I_L \geq \xi(1)$, (v) $I_P \geq \xi(1) \geq \xi(0) \geq I_L$, (vi) $I_P \geq \xi(0) \geq \xi(1) \geq I_L$. As the income functions $R(w^i, r^i, u, c, a), i = 0, 1$, are continuous in u, c and a, $I(w^0, r^0, w^1, r^1, u, c, a)$ defined by (4) is also continuous in u, c and a. Hence $\xi(\lambda)$ is a continuous function for $0 \leq \lambda \leq 1$ and assumes all intermediate values between $\xi(0)$ and $\xi(1)$. From the inequalities (i)–(vi), there exists a λ^*, $0 \leq \lambda^* \leq 1$, so that $I_L \geq \xi(\lambda^*) \geq I_P$ for inequality (i), or so that $I_P \geq \xi(\lambda^*) \geq I_L$ for inequalities (ii)–(vi). Now define $u^* \equiv (1-\lambda^*)u^0 + \lambda^* u^1$, $c^* \equiv (1-\lambda^*)c^0 + \lambda c^1$ and $a^* \equiv (1-\lambda^*)a^0 + \lambda^* a^1$, which completes the proof.

Proof of Theorem 2

Analogous to the proof of Theorem 1.

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