Multiproduct Cost Functions and Subadditivity Tests: A Critique of the Evans and Heckman Research on the U.S. Bell Systems

by

W. E. Diewert & T.J. Wales

June 1991

Discussion Paper No.: 91-21

DEPARTMENT OF ECONOMICS
THE UNIVERSITY OF BRITISH COLUMBIA
VANCOUVER, CANADA V6T 1Z1

Multiproduct Cost Function Estimation and Subadditivity Tests: A Critique of the Evans and Heckman Research on the U.S. Bell System

by

W.E. Diewert and T.J. Wales
June 1991

Abstract

In this paper we examine the major finding of the Evans and Heckman research on the U.S. Bell System, namely that the Bell System cost function was not subadditive over the period 1958-1971, and hence that the system was not a natural monopoly. We find that although the multiproduct cost function estimated by Evans and Heckman satisfies many of the important requirements of theory, including both monotonicity and concavity in input prices, it fails to satisfy the requirement that it be nondecreasing in outputs. Indeed we find that all of the subadditivity calculations that form the basis for their conclusions involve a violation of this important condition, thus casting considerable doubt on their finding.

Multiproduct Cost Function Estimation and Subadditivity Tests: A Critique of the Evans and Heckman Research on the U.S. Bell System

by

W. E. Diewert and T. J. Wales
June 1991

1. Introduction

In a series of influential and important papers, David S. Evans and James J. Heckman (1983a, 1983b, 1984, 1986) develop an ingenious test of the subadditivity of industry cost functions, and apply the test to cost data from the U.S. Bell System. If the industry cost function is subadditive, then it is cheaper to have one firm produce a given level of output than to have the same total produced by two or more firms. Their major finding, that the Bell System cost function was not subadditive over the period 1958-1977, and hence that the System was not a natural monopoly, played an important role in the lengthy debate on whether or not there would be efficiency losses in breaking up the Bell System.

This major finding by Evans and Heckman (hereafter abbreviated E-H) has been subject to scrutiny over the years by a number of authors. It has been demonstrated that the finding rests rather precariously on, among other things, the variable used to measure technical change, the extent to which variables are transformed (in the Box-Cox sense), and the treatment of serial correlation.* Since the analysis involves estimating the complex

* Some of these issues, along with many others, are discussed in the excellent survey paper by Kiss and Lefebvre (1987).

multiproduct translog cost function using several data series that are highly collinear, in particular the two output levels and the technical change variable, this is not surprising. Indeed, in our view, it would be unlikely for the E-H results to remain robust in face of the long list of possible model variations suggested in the literature. However, our concern here is not with the robustness/nonrobustness of the E-H reported results, nor do we have any objection to the methodology underlying the subadditivity test.

Rather, our concern is with the conclusions reached by the authors in applying their estimated cost function to the Bell System. We find that their estimated cost function fails to satisfy one of the basic requirements of economic theory--that costs be nondecreasing in output levels--and thus we argue that its use in subadditivity calculations is clearly inappropriate. Although Roller (1990) appears to make this same basic point, our analysis differs to the extent that we focus on attempting to replicate the E-H subadditivity calculations as closely as possible. Using the corrected data series, we obtain an estimated cost function that is almost identical to the one that appears in the 1986 E-H paper. However, since this estimated cost function suffers from the criticism mentioned above, serious doubt is cast on the authors' major finding that the bell system was not a natural monopoly over the 1958-1977 period.

2. The Evans and Heckman Methodology

The natural monopoly test suggested by Evans and Heckman relies on the estimation of a multiproduct cost function for the Bell System. In particular, they assume that the Bell system uses three aggregate inputs, capital, labour, and materials, to produce two aggregate

outputs, local and long-distance telephone service. Following their notation, the cost function is given by

(1) $$C = f(L,T,r,m,w,t)$$

where L is local service output, T is long-distance service output, r is the capital rental rate, w is the wage rate, m is the price of materials and t is an index of technological change. The reasons for choosing this particular cost function are discussed in Appendix A of the 1983a E-H paper, and are not repeated here. The general cost function (1) is assumed to be approximated by the well-known translog cost function (again following the E-H notation)

(2) $$\ell n\, C = \alpha_0 + \sum_i \alpha_i \ell n p_i + \sum_i \beta_i \ell n q_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij}\, \ell n p_i \ell n p_j + \tfrac{1}{2} \sum_k \sum_j \delta_{kj} \ell n q_k \ell n q_j + \sum_i \sum_k \rho_{ik} \ell n p_i \ell n q_k + \sum_i \lambda_i \ell n p_i \ell n t + \sum_k \theta_k \ell n q_k \ell n t + \tau (\ell n t)^2 + \mu \ell n t$$

with $\sum_i \alpha_i = 1,\ \sum_j \gamma_{ij} = 0,\ \sum_i \rho_{ik} = 0,\ \sum_i \lambda_i = 0.\ \gamma_{ij} = \gamma_{ji},$
and $\delta_{kj} = \delta_{jk}$

where p denotes the vector of input prices (r,m,w) and q the vector of output quantities (L,T). The input cost share equations are obtained in the usual way by applying Shephard's Lemma to (2), and are (in share equation form)

$$(3) \qquad S_i \equiv \frac{P_i X_i}{C} = \alpha_i + \sum_j \gamma_{ij} \ell n p_j + \sum_k \rho_{ik} \ell n q_k + \lambda_i \ell n t \qquad i = 1, 2, 3,$$

where S_i is the cost share for input i. Although the authors experimented with some modifications of the translog, and with special cases, their major findings are for the basic translog form given by (2) and (3). For estimation purposes random disturbance terms are added to the cost and share equations. These disturbances are assumed to be multinormally distributed with a constant nondiagonal contemporaneous covariance matrix. Two alternative assumptions are made about the behaviour of these disturbances over time. First, they are assumed to be uncorrelated, and second, they are assumed to follow a first-order autoregressive process (AR(1)), with the disturbance in each equation related to the corresponding disturbance in the preceding period through a serial correlation coefficient. In both cases, since the input share equations and the observed shares add to one, the covariance matrix is singular and one equation may be dropped in the estimation. In order to ensure that the results are invariant to the equation deleted when the AR(1) process is assumed, the serial correlation coefficients for the share equation disturbances are restricted to be the same.

The equations are estimated by the method of maximum likelihood and are based on yearly data for the Bell System from 1947-1977. The data, originally tabled in Appendix C of the 1983a paper, are corrected in a subsequent paper by the authors (1986). This latter paper also contains revised parameter estimates of the translog cost function corrected for

first-order serial correlation. According to the authors, these corrected data make only small differences in the results.

Given the estimated translog cost function, prices of inputs and a value for the technical change index, the cost of producing any level of the two outputs q_1 and q_2 may easily be evaluated. Further, if it is assumed that these same levels of output are produced by two hypothetical firms rather than by one, then each firm's cost may be calculated in the same manner, and the total cost of production will be the sum of these costs. The two firms are treated symmetrically since they are assumed to have access to a common technology. The degree of subadditivity for this level of output of q_1 and q_2 is then defined to be the cost when one firm produces these outputs minus the sum of the costs when two firms produce the outputs, all expressed as a fraction of the cost when one firm produces. If this expression is negative, then the cost function is said to be subadditive at that point, which simply means that it is cheaper to produce with one firm than with two firms. Of course, this degree of subadditivity depends on how the two outputs are allocated to the two firms, and thus, for any q_1, q_2 combination, we have a matrix of subadditive measures that can be defined over some admissible region that describes how much of each output is produced by each firm. In order to avoid having to extrapolate to output regions well beyond sample observations, the authors define the admissible region to be the one in which (a) no firm produces less of either output than is observed in the data, and (b) both firms produce within the range of ratios of outputs actually observed in the data.

3. Cost Function Estimation Results and Implications

Clearly, the test for subadditivity of the cost function briefly outlined above depends critically on the appropriateness of the estimated cost function. In this regard, the authors appear at first glance to have been careful in their selection of the final translog function to use in their calculations. In particular, they select one for which the own-price elasticities of demand for capital, labour, and materials are negative in all years as required by producer theory, and *a priori* appear to be of reasonable magnitudes. This contrasts with some earlier studies involving one aggregate output, in which some of the own elasticities were found to be positive. Further, and again consistent with producer theory, their cost function is monotonically increasing and concave in input prices in all years. This is an important requirement of economic theory that is often not met in empirical cost function studies. Finally, the explanatory variables do an excellent job in explaining the generalized variance in the model, since all reported R squared values are above .97.

Unfortunately, however, the estimated cost function suffers from a flaw that casts serious doubt on the major conclusions of E-H. In particular, the derivative of the cost function with respect to one of the outputs, namely toll services, is negative at over one-half of the data points (yearly observations) in the sample. That is, the estimated marginal cost of toll production is negative for these years, implying that an increase (decrease) in toll output, *ceteris paribus,* would have resulted in a decrease (increase) in total cost. This is, of course, a nonsensical implication from a practical view, and indeed represents a violation of one of the basic properties of an economically viable cost function; namely, that it be nondecreasing in each of its outputs.

In some contexts, this property of the estimated cost function might not have serious practical implications. For example, if interest centres on the substitutability of inputs in the production process, then the implied effect on cost of changes in a particular output might not be of particular interest, provided the implied effect of changes in all outputs together was reasonable. Alternatively, extrapolations beyond the sample range might well result in negative marginal cost predictions, and this would not necessarily be of concern. However, in the present context, involving the evaluation of a cost function at different combinations of levels of the two outputs within the sample range, it is crucial that total cost as a function of each output separately be nondecreasing. That is, the fact that there is more than one output is an important feature of the analysis, and thus it is important that the functional relationship between cost and each output make economic sense. Unfortunately, this is not the case for the estimated cost function used by E-H in their subadditivity test calculations.

In order to study the E-H results in detail, we have attempted to duplicate their findings. Using the basic data tabled in Appendix C of their 1983a paper, together with the data revisions provided in footnote 1 of their 1986 paper, we estimate a translog cost function adjusted for first order serial correlation along the lines mentioned above. We focus on this particular model since it is the only one for which the authors provide parameter estimates that are based on their corrected data. These corrected data--the ones on which our estimates are based--appear in Appendix A of this paper. In Table 1 we present our parameter estimates together with those that appear in Table 1 of the 1986 E-H paper. The results are very close, with only minor differences in the parameter estimates; the largest difference (as a fraction of the E-H value) is only 4.4 percent, while the average over all parameters is 1.8 percent. These minor discrepancies may be due to differences in the nonlinear estimation algorithm, the convergence criterion, and/or the different methods of treating the first observation in the AR(1) process. In any event, since both sets of estimates yield virtually identical results for our diagnostic tests, we provide below a detailed account only of the results based on the E-H parameter estimates. These parameter estimates are, of course, the ones on which all of their conclusions are based.

In Table 2 we present some summary statistics that are implied by the estimated translog model (using the E-H parameter estimates). The first three columns contain the own price elasticities of demand for the inputs and, as mentioned above, these are of the correct sign and of reasonable magnitudes. The elasticity estimates for labour, capital, and materials of -.155, -.048, and -.608 in 1961 are very close to those of -.151, -.056, and -.590 reported by E-H in their 1983a paper. The fourth column contains the estimated effects of technical change on costs, and is defined as the percentage change in cost that results from one year to the next due to the changing technology. More specifically, it is calculated by taking the derivative of the log of the cost function with respect to the log of the technology variable, and multiplying by the percent change in the technology variable from the preceding year. The technology variable used by E-H is a cumulative measure of research and development expenditures by Bell Labs charged at AT&T, and is described in Appendix C of the authors' 1983a paper. As is evident from Table 2, the orders of magnitude of the technical change effect are similar to those found by others in the literature when using a time trend as a technical change indicator--up to one or two percent per year. However,

one would generally expect these effects to be negative, reflecting technical progress (i.e., cost reduction) rather than positive as they are in the last nine years of the sample.

The next two columns of the table contain the estimated elasticities of total cost with respect to the two outputs, and it is the second of these columns that indicates the major problem with the estimated model. The entries here are $\partial \ell n(C)/\partial \ell n(q_2) = (\partial C/\partial q_2)(q_2/C)$, and if these values are negative, then $\partial C/\partial q_2$ is also negative, since both q_2 and C are positive. But $\partial C/\partial q_2$ is just the marginal cost of toll service output; thus the negative entries in column 6 indicate that the implied marginal cost of toll services is negative evaluated at these observations, and this occurs for 21 of the 31 observations. Note that all but one of the positive entries are centred about the year 1961, which is the point of approximation of the translog cost function; that is, the observation for which the logarithms of all the right-hand explanatory variables (excepting the constant terms) are zero. As is well known, the translog can provide a second order approximation to an arbitrary cost function and thus has desirable properties about the point of approximation. One the other hand, it is also well known that second order flexible forms, such as the translog, do not always perform well away from this point. The results presented here appear to be a good illustration of this limitation of the translog model. Note that it is not estimates of the price effects that deteriorate away from the approximation point, but rather estimates of the output effects.

The final column contains a measure of returns to scale that is sometimes employed in conjunction with a multiproduct cost function. An entry here is defined to be the reciprocal of the sum of the elasticities of cost with respect to output. These entries essentially measure returns to scale when outputs change in the same proportion, with a value greater than one reflecting increasing returns, a value less than one decreasing returns, and a value of unity constant returns. In the case of one output this measure reduces to the inverse of the elasticity of cost with respect to output, and is a commonly used measure of returns to scale. Of course, this measure only makes sense if the cost elasticities used in its construction are positive and, as discussed above, this holds only for 10 observations near the approximation point. For these observations the values are greater than one, indicating increasing returns to scale.

4. Subadditivity Calculations

Calculation of the degree of subadditivity requires some additional notation and may be described as follows for the case of one firm versus two firms, which we denote as A and B. Let q_{1M} and q_{2M} be the minimum levels of outputs 1 and 2 observed in the sample, q_1 and q_2 the total observed levels of outputs in some year (where the time subscript is deleted for simplicity) and ϕ and ω two parameters satisfying the restrictions $0 \leq \phi \leq 1$ and $0 \leq \omega \leq 1$. Then, for any value of ϕ and ω we assume that firm A produces q_{1A} of output 1 and q_{2A} of output 2 where

(4)
$$q_{1A} = \phi(q_1 - 2q_{1M}) + q_{1M}$$
$$q_{2A} = \omega(q_2 - 2q_{2M}) + q_{2M}.$$

Firm B is assumed to produce the difference between the total observed amount produced and that produced by A:

(5)
$$q_{1B} = q_1 - q_{1A}$$
$$q_{2B} = q_2 - q_{1B}.$$

Thus, for various combinations of ϕ and ω, firms A and B produce varying fractions of the observed output levels q_1 and q_2. At one extreme $\phi = \omega = 0$, in which case firm A produces the minimum observed levels of the two outputs in the sample, and B produces the remainder. At the other extreme, $\phi = \omega = 1$, and in this case, it is easy to see (from (4) and (5)) that firm B produces the minimum observed levels of the two outputs, and firm A the remainder. To calculate the degree of subadditivity associated with each of the allocations implied by different values of ϕ and ω, we defined the following costs

(6)
$$C_A(\phi,\omega) = C(q_{1A},q_{2A})$$
$$C_B(\phi,\omega) = C(q_{1B},q_{2B})$$
$$C_T = C(q_1,q_2)$$

where $C(q_{1A}, q_{2A})$ is the cost to firm A of producing the outputs q_{1A} and q_{2A} given by (4), with the cost function C, $C(q_{1B},q_{2B})$ is defined analogously for B, and $C(q_1,q_2)$ is the cost when one firm produces the total amounts of each output. Following E-H, the degree of subadditivity is measured as

(7)
$$Sub(\phi,\omega) \equiv \frac{C_T - C_A(\phi,\omega) - C_B(\phi,\omega).}{C_T}$$

Thus, if Sub(ϕ,ω) is greater (less) than zero, the monopoly situation is less (more) efficient than the two-firm allocation described by that ϕ,ω combination. The value of Sub(ϕ,ω) represents the percent gain or loss from multifirm versus single firm production, and can be calculated for each sample observation.

In Table 3 we present our calculations of Sub(ϕ,ω) for 1961 using the E-H parameter values and reproduce those that appear in Table 10.10 of the E-H 1983a paper. We employ the same admissible region that restricts output levels to be greater than sample minimums, and to lie within the range of output ratios observed in the data. The pattern of results is the same, with the entries differing by either 1 or 2 percentage points. Presumably, these differences result because their estimates are based on their original (1983a) translog estimates, while ours are based on the corrected (1986) translog estimates. Our results appear to reinforce the E-H conclusions, since all of our entries are slightly larger positive numbers, indicating that multifirm production is even more efficient.

However, a closer analysis in the light of our earlier findings of negative marginal costs suggests that conclusions drawn from this table are invalid. Clearly, a particular Sub(ϕ,ω) value is economically meaningful only if marginal costs of both outputs are positive for all three cost function evaluations given in (6) that are used in the calculation. That is, marginal costs must be positive not only for the levels of outputs faced by the single firm, but also for the levels of outputs faced by the hypothetical firms A and B. The former requirement immediately rules out use of any observed sample point for which the predicted marginal cost of either output is less than zero. As discussed above, this occurs for 21 of the 31 sample points. Furthermore, our calculations show that the latter requirement rules out

the remainder of the E-H sample observations. The nature of the problem can be illustrated by considering the year 1961, which forms the basis for the results of Table 3. As is evident from Column 6 of Table 2, the marginal cost of producing each output is positive in 1961, thus the cost function evaluation is meaningful at this point. However, in allocating toll output to firms A and B, it is highly likely that a negative marginal cost will be encountered. As is evident from the data listing in Appendix A, the minimum toll output in the sample is about .35 when toll output is normalized to unity in 1961. Since no firm is permitted to produce less than the minimum in the subadditivity calculations, this implies that no firm can produce more than the total (1 unit) minus this minimum, which equals approximately .65 units. But for almost all observations with toll output levels below .65, the implied marginal cost is negative, as is evident from Column 6 of Table 2, and the data provided in Appendix A.

Of course, this is not a rigorous test since, in allocating 1961 output and imputing costs to firms A and B, we must also assume that 1961 prices and technology prevail. These prices and technology differ from those in earlier years (as does the level of the output whose marginal cost is not being considered) so that we cannot simply look at predicted marginal costs from earlier years, as given in Table 2. Thus, for each entry in Table 3, we calculate the implied marginal cost of local and toll production for firms A and B using 1961 price and technology values. We find that in no case is the marginal cost of toll production positive for both firms. Further, this conclusion holds not only for 1961, but also for all sample observations for which the marginal costs of producing (the total amounts of) both toll and local services are positive (1954 and 1957-65). Thus, not even the 10 sample observations for which both marginal costs are positive can form the basis for subadditivity calculations that are meaningful.

5. Conclusion

In summary, although the multiproduct cost function estimated by E-H satisfies many of the important requirements of economic theory, including both monotonicity and concavity in input prices, it fails to satisfy the requirement that it be nondecreasing in outputs. Violations of this condition occur at 21 of the 31 sample observations, thus ruling them out for use in subadditivity testing. For the remaining 10 observations including 1961, which is the translog approximation point, both marginal costs are positive; however, not even these observations can be used for testing since the required allocation of outputs to the two hypothetical firms always gives rise to at least one of them operating at a level for which the marginal cost of toll services is less than zero. Hence, we conclude that none of the subadditivity calculations forming the basis for the conclusions reported in the E-H papers are economically meaningful, since they all involve a violation of an important requirement of economic theory, and, indeed, one which it would seem imperative to have satisfied given the objective at hand.

Table 1

Parameter Estimates for Translog Cost Function
Corrected for Serial Correlation

Parameter	E-H Estimates	D-W Estimates
Constant	9.054	9.054
Capital (Price)	.536	.536
Labour (Price)	.354	.354
Local (Output)	.206	.212
Toll (Output)	.504	.496
Technology	-.201	-.196
Capital^2	.223	.221
Labour^2	.174	.174
Capital-Labour	-.183	-.182
Local^2	-16.646	-16.555
Toll^2	-8.969	-8.877
Local-Toll	12.167	12.074
Technology^2	-.180	-.172
Capital-Local	.343	.336
Labour-Local	-.362	-.349
Capital-Toll	-.180	-.176
Labour-Toll	.161	.156
Capital-Technology	.081	.079
Labour-Technology	-.052	-.054
Local-Technology	-1.553	-1.525
Toll-Technology	1.430	1.397
Autocorrelation Parameter in		
Cost equation	.186	.193
Share equations	.706	.684
R^2 in Cost Function	.9999	.9997
Capital Share	.9753	.979
Labour Share	.9834	.986

Notes:

1. The notation and E-H results are from their 1986 paper, Table 1, p. 857.

2. The Diewert-Wales (D-W) results are obtained from estimating (2) and (3) as discussed in the text.

Table 2

Summary Statistics for Translog Cost Function

	EP1	EP2	EP3	TECH	EQ1	EQ2	RS
1947	-0.1340	-0.0148	-0.5867	0.0000	2.9710	-1.5951	0.7288
1948	-0.1460	-0.0325	-0.5960	0.0086	2.0860	-0.9636	0.8909
1949	-0.1509	-0.0394	-0.5977	0.0009	1.4079	-0.4742	1.0710
1950	-0.1564	-0.0466	-0.5980	-0.0076	1.2861	-0.3800	1.1037
1951	-0.1610	-0.0519	-0.5973	-0.0090	1.5097	-0.5340	1.0248
1952	-0.1647	-0.0553	-0.5921	-0.0082	1.3246	-0.3888	1.0686
1953	-0.1655	-0.0555	-0.5944	-0.0068	0.8813	-0.0612	1.2193
1954	-0.1657	-0.0555	-0.5982	-0.0046	0.6247	0.1250	1.3339
1955	-0.1657	-0.0554	-0.6018	-0.0032	0.9120	-0.0902	1.2169
1956	-0.1650	-0.0548	-0.6062	-0.0047	0.8591	-0.0487	1.2341
1957	-0.1656	-0.0555	-0.6146	-0.0089	0.6969	0.0698	1.3044
1958	-0.1631	-0.0537	-0.6113	-0.0153	0.4086	0.3029	1.4055
1959	-0.1623	-0.0532	-0.6118	-0.0168	0.4967	0.2523	1.3352
1960	-0.1585	-0.0506	-0.6102	-0.0183	0.3196	0.4020	1.3858
1961	-0.1545	-0.0480	-0.6082	-0.0177	0.2060	0.5040	1.4085
1962	-0.1516	-0.0465	-0.6089	-0.0171	0.1564	0.5563	1.4029
1963	-0.1502	-0.0459	-0.6103	-0.0193	0.1234	0.5977	1.3867
1964	-0.1474	-0.0442	-0.6088	-0.0164	0.6426	0.2430	1.1292
1965	-0.1475	-0.0446	-0.6108	-0.0152	0.8432	0.1196	1.0387
1966	-0.1500	-0.0465	-0.6145	-0.0074	1.2523	-0.1631	0.9182
1967	-0.1492	-0.0464	-0.6159	-0.0085	1.1954	-0.1000	0.9130
1968	-0.1459	-0.0450	-0.6170	-0.0029	1.4283	-0.2553	0.8525
1969	-0.1543	-0.0503	-0.6236	0.0013	1.8085	-0.5343	0.7848
1970	-0.1568	-0.0520	-0.6269	0.0020	1.8718	-0.5775	0.7726
1971	-0.1604	-0.0541	-0.6300	0.0017	1.8014	-0.5259	0.7841
1972	-0.1623	-0.0550	-0.6317	0.0032	1.9606	-0.6443	0.7597
1973	-0.1655	-0.0549	-0.6373	0.0053	2.2001	-0.8263	0.7279
1974	-0.1657	-0.0527	-0.6403	0.0065	2.2800	-0.8824	0.7155
1975	-0.1655	-0.0546	-0.6389	0.0087	2.4999	-1.0290	0.6799
1976	-0.1653	-0.0550	-0.6385	0.0114	2.7800	-1.2267	0.6438
1977	-0.1646	-0.0554	-0.6389	0.0133	3.0122	-1.3865	0.6151

Notes:

1. EP1, EP2, and EP3 are the own price elasticities of demand for labour, capital, and materials, respectively.
2. EQ1 and EQ2 are the elasticities of cost with respect to local and toll output, respectively.
3. RS is defined as the reciprocal of (EQ1 + EQ2) and is a measure of returns to scale.
4. TECH is the percent change in cost due to the changing technology and should be negative if there is cost saving technical progress.

Table 3

Percentage of Gain or Loss from Multifirm Versus Single-firm Production for Alternative Industry Configurations, 1961

Evans-Heckman Estimates

ϕ

ω	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	8										
.1	8	8									
.2	9	8	8								
.3	12	10	9	9							
.4	15	13	10	9	9						
.5	20	16	13	11	9	9					
.6	25	21	17	14	11	10	9				
.7			23	18	15	12	10	9			
.8					20	16	12	10	8		
.9							17	13	10	8	
1.0										10	8

Diewert-Wales Estimates

ϕ

ω	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	10										
.1	9	10									
.2	10	10	10								
.3	13	11	10	10							
.4	17	14	12	10	10						
.5	21	18	15	12	11	10					
.6	27	23	19	15	13	11	10				
.7			25	20	16	13	11	10			
.8					22	18	14	11	10		
.9							19	15	12	10	
1.0										12	10

Notes:

1. The E-H results are from Table 10.10 of their 1983 paper, p. 267.
2. The D-W results are based on the revised translog parameter estimates that appear in the E-H 1986 paper, p. 857.

Appendix A -- Data

	Cost	Local Output	Toll Output	Capital Price	Labour Price	Materials Price	R & D Index	Capital Share	Labour Share
1947	2550.68	0.41014	0.34642	0.49948	0.53566	0.66952	0.57955	0.39552	0.49635
1948	2994.94	0.45783	0.37201	0.55879	0.58236	0.75117	0.55455	0.40430	0.48286
1949	3291.06	0.48703	0.38296	0.57440	0.60959	0.74530	0.55261	0.41936	0.47113
1950	3563.20	0.52004	0.41592	0.61810	0.63164	0.76525	0.56980	0.44096	0.45352
1951	4047.07	0.55560	0.46552	0.70031	0.66926	0.81572	0.59576	0.45338	0.44230
1952	4616.23	0.59149	0.50116	0.79500	0.70946	0.82863	0.62057	0.46670	0.43159
1953	4935.13	0.62452	0.52271	0.80853	0.73411	0.84389	0.63873	0.46436	0.43614
1954	5258.76	0.65669	0.55000	0.81269	0.76134	0.85563	0.65059	0.46596	0.42866
1955	5770.47	0.70289	0.61941	0.86056	0.80674	0.87558	0.66162	0.47840	0.41414
1956	6305.44	0.75645	0.68394	0.88033	0.81063	0.90493	0.68018	0.47642	0.41045
1957	6351.19	0.80355	0.74006	0.81997	0.84824	0.93896	0.71436	0.47138	0.41365
1958	6788.40	0.84224	0.77663	0.87304	0.85084	0.95305	0.76830	0.50754	0.38849
1959	7334.71	0.89657	0.86274	0.91051	0.91958	0.97417	0.83934	0.52030	0.37321
1960	7912.48	0.95314	0.93512	0.95733	0.95979	0.99061	0.91902	0.53120	0.36083
1961	8516.46	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.54381	0.34605
1962	9018.66	1.05411	1.08231	1.01457	1.03632	1.01995	1.08533	0.55077	0.33966
1963	9508.12	1.11068	1.17451	1.00832	1.07393	1.03404	1.18984	0.55139	0.33353
1964	10542.50	1.15909	1.31715	1.07804	1.12970	1.08451	1.32815	0.56240	0.32693
1965	11207.00	1.22822	1.47436	1.06139	1.17121	1.10681	1.49998	0.55286	0.32925
1966	11954.20	1.30609	1.68434	1.04475	1.22827	1.14085	1.66877	0.54302	0.33698
1967	12710.90	1.38312	1.84266	1.04058	1.29702	1.17371	1.86844	0.54079	0.34058
1968	13814.10	1.46568	2.05511	1.08325	1.36057	1.21948	2.02744	0.54614	0.33406
1969	14940.40	1.55869	2.33437	1.04579	1.49416	1.28286	2.16342	0.51402	0.35802
1970	16516.90	1.63899	2.53682	1.04891	1.62387	1.35211	2.28416	0.49799	0.37133
1971	17951.80	1.70956	2.69772	1.04058	1.80415	1.42019	2.40026	0.48313	0.38304
1972	20161.20	1.80454	2.96927	1.09157	2.06226	1.47653	2.52124	0.47953	0.39061
1973	21190.30	1.91210	3.31628	1.00312	2.26329	1.56221	2.65447	0.44558	0.41442
1974	23168.40	2.00785	3.60503	1.00104	2.51612	1.74061	2.80468	0.43407	0.42485
1975	27376.70	2.07532	3.86421	1.18939	2.85473	1.91315	2.97195	0.46178	0.40606
1976	31304.50	2.17307	4.24442	1.32778	3.21920	2.01408	3.15081	0.46977	0.39508
1977	34745.30	2.29155	4.68449	1.41935	3.40726	2.12911	3.33422	0.46712	0.39259

Sources:

Basic Data -- Evans and Heckman (1983a), Table 10.14, p. 216-17.

Corrections -- Evans and Heckman (1986), Footnote 1, p. 856.

References

Evans, D. S., and J. J. Heckman (1983a), "Multiproduct Cost Function Estimates and Natural Monopoly Tests for the Bell System," in D. Evans, ed., Breaking Up Bell, Amsterdam: North Holland, 1983.

______ (1983b), "Natural Monopoly," in D. Evans, ed., Breaking Up Bell, Amsterdam: North Holland, 1983.

______ (1984), "A Test for Subadditivity of the Cost Function with an Application to the Bell System," American Economic Review, September 1984, Volume 74, 615-623.

______ (1986), " A Test For Subadditivity of the Cost Function with an Application to the Bell System: Erratum," American Economic Review, September 1986, Volume 76, 856-58.

Kiss, F., and B. Lefebvre (1987), "Econometric Models of Telecommunications Firms," Reveu Economique, No. 2, March 1987, 307-374.

Roller, L. (1990), "Modelling Cost Structure - the Bell System Revisited," Applied Economics, 22, 1990, 1661-1674.