Decompositions of Productivity Growth into Sectoral Effects

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March 6, 2013

Abstract

The paper provides some new decompositions of labour productivity growth and Total Factor Productivity (TFP) growth into sectoral effects. These new decompositions draw on the earlier work of Tang and Wang (2004). The economy wide labour productivity growth rate turns out to depend on the sectoral productivity growth rates, output price effects and changes in sectoral labour input shares. The economy wide TFP growth decomposition is similar but some extra terms due to input price inflation make their appearance in the decomposition.

Journal of Economic Literature Classification Numbers

C43, C82, D24.

Key Words

Total Factor Productivity, labour productivity, index numbers, sectoral contributions to growth.

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1. Introduction

Jianmin Tang and Weimin Wang (2004; 426) provided an interesting decomposition for economy wide labour productivity into sectoral contribution effects. However, the interpretation of the individual terms in their decomposition is not completely clear and so in section 2, we rework their methodology in order to provide a more transparent and simple decomposition. In section 3, we present some alternative interpretations of the basic decomposition. In section 4, we generalize the results in section 2 in order to provide a decomposition of economy wide Total Factor Productivity growth into industry explanatory factors. Section 5 concludes.

2. The Tang and Wang Methodology Reworked

Let there be N sectors or industries in the economy.[2] Suppose that for period t = 0,1, the *output* (or real value added or volume) of sector n is Y_n^t with corresponding period t *price* P_n^t [3] and *labour input* L_n^t for n = 1,...,N. We assume that these labour inputs can be added across sectors and that the *economy wide labour input* in period t is L^t defined as follows:

$$(1)\ L^t \equiv \sum_{n=1}^N L_n^t\ ; \qquad t = 0,1.$$

Industry n labour productivity in period t, X_n^t, is defined as industry n real output divided by industry n labour input:

$$(2)\ X_n^t \equiv Y_n^t/L_n^t\ ; \qquad t = 0,1\ ;\ n = 1,...,N.$$

It is not entirely clear how aggregate labour productivity should be defined since the outputs produced by the various industries are measured in heterogeneous, noncomparable units. Thus we need to weight these heterogeneous outputs by their prices, sum the resulting period t values and then divide by a *general output price index*, say P^t for period t, in order to make the economy wide nominal value of aggregate output comparable in real terms across periods. Thus with an appropriate choice for the aggregate output price index P^t, the period t *economy wide labour productivity*, X^t, is defined as follows:[4]

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[2] The material in this section follows Diewert (2004).

[3] These industry real output aggregates Y_n^t and the corresponding prices P_n^t are *indexes* of the underlying micro net outputs produced by industry n. The exact functional form for these indexes does not matter for our analysis but we assume the indexes satisfy the property that for each t and n, $P_n^tY_n^t$ equals the industry n nominal value added for period t.

[4] This follows the methodological approach taken by Tang and Wang (2004; 425). In particular, the aggregate output price index P^t can be formed by applying an index number formula to the industry output prices (or value added deflators) for period t, $(P_1^t,...,P_N^t)$, and the corresponding real output quantities (or industry real value added estimates) for period t, $(Y_1^t,...,Y_N^t)$. The application of a superlative index number formula would be appropriate in this context but again, the exact form of index does not matter for our analysis. Note that the approach suggested by Tang and Wang is equivalent to deflating the aggregate value of output (or nominal aggregate value added) by the chosen output price index.

$$(3)\ X^t \equiv \sum_{n=1}^N P_n^t Y_n^t / P^t L^t = \sum_{n=1}^N (P_n^t/P^t) Y_n^t / L^t = \sum_{n=1}^N p_n^t Y_n^t / L^t\ ; \qquad t = 0,1$$

where the *period t industry n real output price*, p_n^t, is defined as the industry t output price P_n^t, divided by the aggregate output price index for period t, P^t; i.e., we have the following definitions:[5]

$$(4)\ p_n^t \equiv P_n^t/P^t\ ; \qquad n = 1,\ldots,N\ ;\ t = 0,1.$$

Using definitions (2) and (3), it is possible to relate the period t aggregate productivity level X^t to the industry productivity levels X_n^t as follows:[6]

$$\begin{aligned}(5)\ X^t &\equiv \sum_{n=1}^N P_n^t Y_n^t / P^t L^t \\ &= \sum_{n=1}^N p_n^t [Y_n^t/L_n^t][L_n^t/L^t] \\ &= \sum_{n=1}^N p_n^t s_{Ln}^t X_n^t \qquad \text{using definitions (2)}\end{aligned}$$

where the *share of labour used by industry n in period t*, s_{Ln}^t, is defined in the obvious way as follows:

$$(6)\ s_{Ln}^t \equiv L_n^t/L^t\ ; \qquad n = 1,\ldots,N\ ;\ t = 0,1.$$

Thus aggregate labour productivity for the economy in period t is a weighted sum of the sectoral labour productivities where the weight for industry n is p_n^t, the real output price for industry n in period t, times s_{Ln}^t, the share of labour used by industry n in period t.

Up to this point, our analysis follows that of Tang and Wang (2004; 425-426) but now our analysis will diverge from theirs.[7]

First, we define the *value added or output share of industry n in total value added for period t*, s_{Yn}^t, as follows:

$$\begin{aligned}(7)\ s_{Yn}^t &\equiv P_n^t Y_n^t / \sum_{i=1}^N P_i^t Y_i^t \qquad t = 0,1\ ;\ n = 1,\ldots,N \\ &= p_n^t Y_n^t / \sum_{i=1}^N p_i^t Y_i^t \qquad \text{using definitions (4).}\end{aligned}$$

Note that the product of the sector n real output price times its labour share in period t, $p_n^t s_{Ln}^t$, with the sector n labour productivity in period t, X_n^t, equals the following expression:

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[5] These definitions follow those of Tang and Wang (2004; 425).

[6] Equation (5) corresponds to equation (2) in Tang and Wang (2004; 426).

[7] Tang and Wang (2004; 425-426) *combined* the effects of the real price for industry n for period t, p_n^t, with the industry n labour share s_{Ln}^t for period t by defining the relative size of industry n in period t, s_n^t, as the product of p_n^t and s_{Ln}^t; i.e., they defined the industry n *weight* in period t as $s_n^t \equiv p_n^t s_{Ln}^t$. They then rewrote equation (5), $X^t = \sum_{n=1}^N p_n^t s_{Ln}^t X_n^t$, as $X^t = \sum_{n=1}^N s_n^t X_n^t$. Thus their analysis of the effects of the changes in the weights s_n^t did not isolate the separate effects of changes in industry real output prices and industry labour input shares.

$$(8)\ p_n^t s_{Ln}^t X_n^t = p_n^t [L_n^t/L^t][Y_n^t/L_n^t] \,; \qquad t = 0,1 \,;\ n = 1,...,N$$
$$= p_n^t Y_n^t/L^t \,.$$

Now we are ready to develop an expression for the rate of growth of economy wide labour productivity. Using definition (3) and equation (5), *aggregate labour productivity growth* (plus 1) going from period 0 to 1, X^1/X^0, is equal to:

$$(9)\ X^1/X^0 = \sum_{n=1}^N p_n^1 s_{Ln}^1 X_n^1 / \sum_{n=1}^N p_n^0 s_{Ln}^0 X_n^0$$
$$= \sum_{n=1}^N [p_n^1/p_n^0][s_{Ln}^1/s_{Ln}^0][X_n^1/X_n^0][p_n^0 Y_n^0/L^0] / \sum_{i=1}^N [p_i^0 Y_i^0/L^0] \qquad \text{using (8)}$$
$$= \sum_{n=1}^N [p_n^1/p_n^0][s_{Ln}^1/s_{Ln}^0][X_n^1/X_n^0]\, s_{Yn}^0 \qquad \text{using definitions (7).}$$

Thus overall economy wide labour productivity growth, X^1/X^0, is an output share weighted average of three growth factors associated with industry n. The three growth factors are:

- X_n^1/X_n^0, (one plus) the rate of growth in the labour productivity of industry n;
- s_{Ln}^1/s_{Ln}^0, (one plus) the rate of growth in the share of labour being utilized by industry n and
- $p_n^1/p_n^0 = [P_n^1/P_n^0]/[P^1/P^0]$ which is (one plus) the rate of growth in the real output price of industry n.

Thus in looking at the contribution of industry n to overall (one plus) labour productivity growth, we start with a straightforward share weighted contribution factor, $s_{Yn}^0[X_n^1/X_n^0]$, which is the period 0 output or value added share of industry n in period 0, s_{Yn}^0, times the industry n rate of labour productivity growth (plus one), X_n^1/X_n^0. This straightforward contribution factor will be augmented if real output price growth is positive (if p_n^1/p_n^0 is greater than one) and if the share of labour used by industry n is growing (if s_{Ln}^1/s_{Ln}^0 is greater than one). The decomposition of overall labour productivity growth given by the last line of (13) seems to be intuitively reasonable and fairly simple as opposed to the decomposition obtained by Tang and Wang (2004; 426) which does not separately distinguish the effects of real output price change from changes in the industry's labour share.

3. Alternative Expressions and Discussion

The literature on aggregate labour productivity decompositions[8] has focused on decompositions that decompose aggregate labour productivity growth into explanatory factors that are functions of growth rates (percentage changes in variables) rather than growth factors (one plus the growth rates). Thus in this section, we will rewrite (9) in growth rate form as opposed to its present contribution factor form.

[8] See Tang and Wang (2004) and de Avillez (2012) for references to this literature.

Define the *aggregate labour productivity growth rate* Γ, the *sectoral labour productivity growth rates* γ_n, the *sectoral real output price growth rates* ρ_n and the *sectoral labour input share growth rates* σ_n between periods 0 and 1 as follows:

$$(10)\ \Gamma \equiv (X^1/X^0) - 1\ ;$$
$$(11)\ \gamma_n \equiv (X_n^{\,1}/X_n^{\,0}) - 1\ ; \qquad n = 1,\ldots,N;$$
$$(12)\ \rho_n \equiv (p_n^{\,1}/p_n^{\,0}) - 1\ ; \qquad n = 1,\ldots,N;$$
$$(13)\ \sigma_n \equiv (s_{Ln}^{\,1}/s_{Ln}^{\,0}) - 1\ ; \qquad n = 1,\ldots,N.$$

Now substitute definitions (10)-(13) into (9) and we obtain the following decomposition for the *aggregate labour productivity growth rate* Γ:[9]

$$\begin{aligned}(14)\ \Gamma &= \textstyle\sum_{n=1}^{N} s_{Yn}^{\,0}\,\{[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1\} \\ &= \textstyle\sum_{n=1}^{N} s_{Yn}^{\,0}\,\{\gamma_n + \rho_n + \sigma_n + \gamma_n\rho_n + \gamma_n\sigma_n + \rho_n\sigma_n + \gamma_n\rho_n\sigma_n\} \\ &= \textstyle\sum_{n=1}^{N} s_{Yn}^{\,0}\gamma_n + \sum_{n=1}^{N} s_{Yn}^{\,0}\rho_n + \sum_{n=1}^{N} s_{Yn}^{\,0}\sigma_n \\ &\quad + \textstyle\sum_{n=1}^{N} s_{Yn}^{\,0}\gamma_n\rho_n + \sum_{n=1}^{N} s_{Yn}^{\,0}\gamma_n\sigma_n + \sum_{n=1}^{N} s_{Yn}^{\,0}\rho_n\sigma_n + \sum_{n=1}^{N} s_{Yn}^{\,0}\gamma_n\rho_n\sigma_n.\end{aligned}$$

The above *exact* expressions for aggregate labour productivity growth tell us that Γ is a *quadratic function* in the industry growth rates for labour productivity γ_n, real output price growth rates ρ_n and industry labour input share growth rates σ_n. The *total contribution to the overall growth rate Γ from industry n* is the nth term in (14), $s_{Yn}^{\,0}\{[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1\}$. This expression is relatively easy to interpret. If industry n's real output price and labour input share remained constant, then the growth rates ρ_n and σ_n would be 0 and the contribution of industry n to economy wide labour productivity growth would be its output share in period 0, $s_{Yn}^{\,0}$, times $[1+\gamma_n][1+0][1+0] - 1$ which is equal to γ_n. Thus under these conditions, the contribution of industry n to economy wide labour productivity growth is equal to the industry n labour productivity growth rate γ_n times its period 0 value added share in economy wide value added, $s_{Yn}^{\,0}$. This is an entirely sensible result. In the case where ρ_n and σ_n are positive, one plus the industry n labour productivity growth, $1+\gamma_n$, is *augmented* by the industry n real output price growth factor $1+\rho_n$ and further *augmented* by the industry n labour input share growth factor $1+\sigma_n$ and then 1 is subtracted from the product of these factors to give us the *industry n augmented labour productivity growth factor*, $[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1$. Augmenting $1+\gamma_n$ by $1+\sigma_n$ is reasonable since the increased labour share for industry n in period 1 will indicate a relative increase in labour input into the industry and increase the importance of industry n in overall labour productivity. Augmenting $1+\gamma_n$ by $1+\rho_n$ is more difficult to explain but the increase in the real price of industry n's output will increase the importance of the output of industry n in the economy wide aggregate output index and evidently, multiplying $1+\gamma_n$ by $1+\rho_n$ will reflect this increased importance.

The last equation in (14) tells us that Γ is equal to an output share weighted average of the industry productivity growth rates, $\sum_{n=1}^{N} s_{Yn}^{\,0}\gamma_n$, plus a share weighted average of the

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[9] We use the fact that the industry period 0 value added output shares $s_{Yn}^{\,0}$ sum to one.

industry real output price growth rates, $\sum_{n=1}^{N} s_{Yn}^{0}\rho_n$, plus a share weighted average of the industry labour input share growth rates, $\sum_{n=1}^{N} s_{Yn}^{0}\sigma_n$, plus the quadratic terms in the industry growth rates, $\sum_{n=1}^{N} s_{Yn}^{0}\gamma_n\rho_n + \sum_{n=1}^{N} s_{Yn}^{0}\gamma_n\sigma_n + \sum_{n=1}^{N} s_{Yn}^{0}\rho_n\sigma_n$, plus the cubic terms in the industry growth rates, $\sum_{n=1}^{N} s_{Yn}^{0}\gamma_n\rho_n\sigma_n$. Since growth rates are generally small, the first three sets of terms on the right hand side of (15) will generally be the dominant ones. The last four sets of terms represent second and third order *interaction terms*.

It is possible to give interpretations for the first three sets of terms on the right hand side of the last equation in (14). The first set of terms, $\sum_{n=1}^{N} s_{Yn}^{0}\gamma_n$, can be interpreted as economy wide labour productivity growth provided that all real output price growth rates ρ_n are equal to zero and all labour input share growth rates σ_n are equal to zero. This set of terms is just the straightforward aggregation of industry productivity growth rates and could be called the *direct effect*. The second set of terms, $\sum_{n=1}^{N} s_{Yn}^{0}\rho_n$, can be interpreted as economy wide labour productivity growth provided that all industry labour productivity growth rates γ_n are equal to zero and all labour input share growth rates σ_n are equal to zero. Thus even if all industry labour productivity levels remain constant and all labour input shares remain constant, economy wide labour productivity growth can change due to changes in industry real output prices. As mentioned above, this effect is due to the changes in output prices leading to changes in the price weights for the industry output growth rates, which in turn affects aggregate labour productivity growth. This effect could be called the *output price weighting effect*. The third set of terms, $\sum_{n=1}^{N} s_{Yn}^{0}\sigma_n$, can be interpreted as economy wide labour productivity growth provided that all industry labour productivity growth rates γ_n are equal to zero and all real output growth rates ρ_n are equal to zero. Thus even if all industry labour productivity levels remain constant and all industry real output prices remain constant, economy wide labour productivity growth can change due to changes in industry labour input shares. This effect could be called the *labour input reallocation effect*.[10] Note that it is not possible to see this reallocation effect in the industry n contribution term, $s_{Yn}^{0}\sigma_n$. This term correctly gives the contribution to economy wide labour productivity growth of an increase in industry n's labour share (so that σ_n is greater than 0 in this case) but the overall effect of this increase in n's labour share is offset by a decrease in other industry labour shares and it is the net effect of the change in n's labour share that gives rise to the reallocation effect. However, if N is greater than two, it is not possible to determine precisely how the increase in labour share for industry n is offset by decreases in shares for the other industries. However, it is possible to determine the overall labour input reallocation effect. Similarly, although the overall output price weighting effect can be determined as the weighted sum $\sum_{n=1}^{N} s_{Yn}^{0}\rho_n$ of the industry output price changes ρ_n, one cannot interpret the industry n contribution term $s_{Yn}^{0}\rho_n$ as the *independent* effect of a change in industry n's real output price because an increase in industry n's nominal price P_n^{1} will affect the economy wide price index P^t and thus the industry real prices $p_n^{t} = P_n^{t}/P^t$ cannot

[10] This type of effect was first noticed by Denison (1962). It is called the *reallocation level effect* by de Avillez (2012).

vary independently, just as the industry labour input shares s_{Ln}^{t} cannot vary independently.[11]

The general decomposition formula (14) can be specialized to give Denison's (1962) decomposition formula. Suppose that the real output price growth rates ρ_n are all equal to 0. Then (14) reduces to the following decomposition of aggregate labour productivity growth:

$$(15)\ \Gamma = \sum_{n=1}^{N} s_{Yn}^{0}\{[1+\gamma_n][1+\sigma_n] - 1\}$$
$$= \sum_{n=1}^{N} s_{Yn}^{0}\{\gamma_n + \sigma_n + \gamma_n\sigma_n\}$$
$$= \sum_{n=1}^{N} s_{Yn}^{0}\gamma_n + \sum_{n=1}^{N} s_{Yn}^{0}\sigma_n + \sum_{n=1}^{N} s_{Yn}^{0}\gamma_n\sigma_n.$$

De Avillez (2012) calls the decomposition given by (15) the *traditional labour productivity decomposition*. However, it can only be justified under the restrictive assumption that all $\rho_n = 0$.[12]

The reader may have noticed that there seems to be a slight asymmetry in the labour productivity growth formula (9) in that the output value added shares for the *base period*, s_{Yn}^{0}, are used as weights for the symmetric growth factors p_n^{1}/p_n^{0}, s_{Ln}^{1}/s_{Ln}^{0} and X_n^{1}/X_n^{0}. However, it is possible to develop a decomposition for the reciprocal of aggregate productivity growth where the period 1 value added shares, s_{Yn}^{1}, are used as weights in this alternative decomposition. Thus using the same notation and steps that were used to establish (9), we can establish the following decomposition:[13]

$$(16)\ X^0/X^1 = \sum_{n=1}^{N} p_n^{0} s_{Ln}^{0} X_n^{0} / \sum_{n=1}^{N} p_n^{1} s_{Ln}^{1} X_n^{1}$$
$$= \sum_{n=1}^{N} [p_n^{0}/p_n^{1}][s_{Ln}^{0}/s_{Ln}^{1}][X_n^{0}/X_n^{1}][p_n^{1}Y_n^{1}/L^{1}] / \sum_{i=1}^{N} [p_i^{1}Y_i^{1}/L^{1}] \qquad \text{using (8)}$$
$$= \sum_{n=1}^{N} [p_n^{0}/p_n^{1}][s_{Ln}^{0}/s_{Ln}^{1}][X_n^{0}/X_n^{1}]\, s_{Yn}^{1}.$$

Now take reciprocals of both sides of (16) and using definitions (12)-(15), we obtain the following *alternative decomposition for aggregate productivity growth* Γ:

$$(17)\ \Gamma = [\sum_{n=1}^{N} s_{Yn}^{1}\{(1+\gamma_n)(1+\rho_n)(1+\sigma_n)\}^{-1}]^{-1} - 1$$

Both (14) and (17) provide exact decompositions of aggregate labour productivity growth using the industry growth rates γ_n, ρ_n and σ_n and the industry value added shares s_{Yn}^{t} as explanatory variables. But it can be seen that the decomposition given by (14) is much easier to interpret so we will not discuss (17) further.

In the following section, we will show how the analysis presented in section 2 can be generalized to provide a decomposition of economy wide Total Factor Productivity growth.

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[11] On the other hand, the industry productivity growth rates γ_n can vary independently.

[12] De Avillez (2012) called the first term in the last line of (15) the *within effect*, the second term the *reallocation level effect* and the third term the *reallocation growth effect*.

[13] This derivation of (16) is due to Balk (2008) who noticed the asymmetry in (9).

4. An Extension to a Decomposition of Aggregate Total Factor Productivity Growth

Again, let there be N sectors or industries in the economy and again suppose that for period t = 0,1, the *output* (or real value added or volume) of sector n is Y_n^t with corresponding period t *price* P_n^t. However, we now assume that each sector uses many inputs and index number techniques are used to form industry input aggregates Z_n^t with corresponding aggregate industry input prices W_n^t for n = 1,...,N and t = 0,1.

Industry n Total Factor Productivity (*TFP*) *in period t*, X_n^t, is defined as industry n real output divided by industry n real input:

$$(18)\ X_n^t \equiv Y_n^t/Z_n^t\ ; \qquad t = 0,1\ ;\ n = 1,...,N.$$

As in section 2, *economy wide real output in period t*, Y^t, is defined as total value added divided by the economy wide output price index P^t. Thus we have[14]

$$(19)\ Y^t = \sum_{n=1}^N P_n^tY_n^t/P^t = \sum_{n=1}^N p_n^tY_n^t\ ; \qquad t = 0,1$$

where the *period t industry n real output price* is defined as $p_n^t \equiv P_n^t/P^t$ for n = 1,...,N and t = 0,1.

We define economy wide real input in an analogous manner. Thus we form an *economy wide input price index* for period t, W^t, by aggregating the industry input prices W_n^t (with corresponding input quantities or volumes Z_n^t) into the aggregate period t input price index W^t using an appropriate index number formula. *Economy wide real input in period t*, Z^t, is defined as economy wide input cost divided by the economy wide input price index W^t:

$$(20)\ Z^t = \sum_{n=1}^N W_n^tZ_n^t/W^t = \sum_{n=1}^N w_n^tZ_n^t\ ; \qquad t = 0,1$$

where the *period t industry n real input price* is defined as $w_n^t \equiv W_n^t/W^t$ for n = 1,...,N and t = 0,1.

The *economy wide level of TFP in period t*, X^t, is defined as aggregate real output divided by aggregate real input:

$$(21)\ X^t \equiv Y^t/Z^t\ ; \qquad t = 0,1.$$

We the output share of industry n in period t, s_{Yn}^t, by (7) again and we define the *input share of industry n in economy wide cost in period t*, s_{Zn}^t, as follows:

$$(22)\ s_{Zn}^t \equiv W_n^tZ_n^t/\sum_{i=1}^N W_i^tZ_i^t \qquad t = 0,1\ ;\ n = 1,...,N$$

[14] Note that $P_n^tY_n^t$ is nominal industry n value added in period t so that Y_n^t is deflated (by the industry n value added price index P_n^t) industry n value added.

$= w_n^t Z_n^t / \sum_{i=1}^N w_i^t Z_i^t$

where the second equation in (22) follows from the definitions $w_n^t \equiv W_n^t / W^t$.

Substitute (19) and (20) into definition (21) and we obtain the following expression for the economy wide level of TFP in period t:

$$
\begin{aligned}
(23)\ X^t &= \sum_{n=1}^N p_n^t Y_n^t / \sum_{n=1}^N w_n^t Z_n^t && t = 0,1 \\
&= \sum_{n=1}^N p_n^t (Y_n^t / Z_n^t) Z_n^t / \sum_{n=1}^N w_n^t Z_n^t \\
&= \sum_{n=1}^N (p_n^t / w_n^t) X_n^t w_n^t Z_n^t / \sum_{n=1}^N w_n^t Z_n^t && \text{using (18)} \\
&= \sum_{n=1}^N (p_n^t / w_n^t) X_n^t s_{Zn}^t && \text{using (22).}
\end{aligned}
$$

Now we are ready to develop an expression for the rate of *growth* of economy wide Total Factor Productivity. Using (23), *aggregate TFP growth* (plus 1) going from period 0 to 1, X^1/X^0, is equal to:

$$
\begin{aligned}
(24)\ X^1/X^0 &= \sum_{n=1}^N (p_n^1/w_n^1) X_n^1 s_{Zn}^1 / \sum_{n=1}^N (p_n^0/w_n^0) X_n^0 s_{Zn}^0 \\
&= \sum_{n=1}^N (p_n^1/p_n^0)(w_n^0/w_n^1)(X_n^1/X_n^0)(s_{Zn}^1/s_{Zn}^0)(p_n^0/w_n^0) X_n^0 s_{Zn}^0 / \sum_{n=1}^N (p_n^0/w_n^0) X_n^0 s_{Zn}^0 \\
&= \sum_{n=1}^N s_{Yn}^0 (p_n^1/p_n^0)(w_n^0/w_n^1)(s_{Zn}^1/s_{Zn}^0)(X_n^1/X_n^0).
\end{aligned}
$$

The last equation in (24) follows from the following equations for n = 1,...,N:

$$
\begin{aligned}
(25)\ (p_n^0/w_n^0) X_n^0 s_{Zn}^0 &= (p_n^0/w_n^0)(Y_n^0/Z_n^0)(w_n^0 Z_n^0 / \sum_{i=1}^N w_i^0 Z_i^0) && \text{using (18) and (22)} \\
&= p_n^0 Y_n^0 / \sum_{i=1}^N w_i^0 Z_i^0.
\end{aligned}
$$

Thus one plus economy wide TFP growth, X^1/X^0, is equal to an output share weighted average (with the base period weights s_{Yn}^0) of one plus the industry TFP growth rates, times an augmentation factor, which is the product $(p_n^1/p_n^0)(w_n^0/w_n^1)(s_{Zn}^1/s_{Zn}^0)$. Thus formula (24) is very similar to our previous labour productivity growth formula (9) except we have an additional multiplicative contribution factor, which is w_n^0/w_n^1, the *reciprocal* of one plus the rate of growth of real input prices for sector n.

We can use definitions (10)-(13) in section 3 to rewrite the decomposition (24), which is in contribution factor form, into a growth rate form. We need to add the following definition:

$$(26)\ \omega_n \equiv (w_n^0/w_n^1) - 1\ ; \qquad n = 1,\ldots,N.$$

Note that $1 + \omega_n$ equals w_n^0/w_n^1 so that ω_n is a *reciprocal growth rate of real input prices for sector n*. Now substitute (10)-(13) and (26) into (24) and we obtain the following decomposition for economy wide TFP growth $\Gamma \equiv (X^1/X^0) - 1$:

$$(27)\ \Gamma = \sum_{n=1}^N s_{Yn}^0 \{[1+\gamma_n][1+\rho_n][1+\omega_n][1+\sigma_n] - 1\}.$$

Thus the contribution term for industry n is the nth term in the above summation, which depends only on industry n TFP growth γ_n, the output share s_{Yn}^0, the real output price growth ρ_n, the real reciprocal input price growth ω_n and industry n input share growth σ_n.

5. Conclusion

If one wishes to decompose aggregate labour productivity growth into explanatory factors that depend on the industries in the aggregate, then the decompositions given by (9) and the first line in (14) appear to be the simplest ones in the literature to date. These decompositions have the advantage that only four industry variables need to be reported in order to explain each industry contribution term: the industry shares of aggregate value added in the base period s_{Yn}^0, the industry labour productivity growth rates γ_n, the industry real output price growth rates ρ_n and the industry labour input share growth rates σ_n between the two periods under consideration. The overall industry n contribution term is $s_{Yn}^0\,\{[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1\}$.

If a decomposition of aggregate TFP growth is desired, then the decompositions given by (24) and (27) are also very simple. In this framework, the overall industry n contribution term is $s_{Yn}^0\,\{[1+\gamma_n][1+\rho_n][1+\omega_n][1+\sigma_n] - 1\}$ where $1+\omega_n$ equals w_n^0/w_n^1, which is turn is the reciprocal of one plus the real input price growth for industry n.

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