US TFP Growth and the Contribution of Changes in Export and Import Prices to Real Income Growth

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Abstract

Using recent data from the Bureau of Economic Analysis (BEA), the Bureau of Labour Statistics (BLS), the Board of Governors of the Federal Reserve System and the US Department of Agriculture, the paper uses a data set that covers the outputs produced and inputs used by an Extended Business Sector of the US economy for the years 1987-2011. The Extended Business Sector (EBS) consists of the entire US economy less the inputs used and outputs produced by the Public Administration sector and less the US housing sector. The study found that Multifactor Productivity (MFP) growth in the EBS averaged 1.157% per year, somewhat higher than BLS MFP growth in the Private Sector which averaged 0.962% per year. The study also produced estimates of MFP where a net output concept was used.

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1. Introduction

This special issue of the *Journal of Productivity Analysis* is devoted to honor the memory of Catherine Morrison-Paul, who died June 30, 2010 from complications resulting from leukemia. She was a leading expert in the measurement of productivity, production theory and resource economics, with over 60 journal articles to her credit. She was a long time contributor to this *Journal*. My own connection to Cathy dates back to working with her while she did her Ph.D degree at the University of British Columbia, which was completed in 1982. While I was not her thesis advisor (who was Keizo Nagatani), I worked closely with her because her thesis was on cost functions and duality theory. Cathy and I went on to work on research of mutual interest over the years with three published papers to show for our joint efforts. The first published paper was Diewert and Morrison (1986) and the present paper merely adapts the theory developed there in a relatively straightforward way. This first paper attempted to develop a production theoretic framework where the contributions of Total Factor Productivity (TFP) growth and changes in the terms of trade could be jointly evaluated in a welfare oriented framework. In this first paper, we used exact index number techniques in the context of a translog technology. In our second paper, Morrison and Diewert (1990a), we addressed the same issues of TFP growth and the effects of changes in the terms of trade but we developed a nonparametric approach to these measurement issues that involved the use of averages of first order approximations to changes in various exogenous variables. In this second paper, we applied our first and second approaches to these measurement questions using Japanese data and showed that the two approaches agreed reasonably well. Finally, in our third paper, Morrison and Diewert (1990b) which appeared in the first volume of the *Journal of Productivity Analysis*, Cathy laid out a general methodological approach that she used in many of her previous and subsequent papers. The traditional index number approach to the measurement of TFP growth assumes competitive price taking behavior, no fixed inputs and constant returns to scale. Cathy showed how this traditional index number approach to TFP measurement could be modified to deal with the above complications, provided that one could use additional information obtained by the use of econometric methods to modify the traditional index number estimates.

Many observers have noted that an improvement in a country's terms of trade has effects that are similar to an improvement in a country's productivity growth. However, it is not straightforward to work out the exact magnitude of each source of gain. Diewert (1983), Diewert and Morrison (1986), Morrison and Diewert (1990a) and Kohli (1990) (2004) (2006) developed production theory methodologies which enable one to obtain index number estimates of the contribution of each type of gain. In particular, Kohli (2006) criticized the Diewert and Morrison (1986) approach to this problem if trade was not balanced. Methodologies that could deal with unbalanced trade were developed by Diewert, Mizobuchi and Nomura (2005) and Kohli (2004) (2006) and this approach will be followed in the present paper. Thus sections 2 and 3 below present the details of this exact index number methodology.

In Diewert (2013), an annual data base for an *Extended Business Sector* (EBS) was developed for the U.S. economy for the years 1987-2011. This data base is described and it is used to calculate the Total Factor Productivity (TFP) growth[2] of the U.S. EBS for the years 1988-2011. A traditional user cost approach to the measurement of capital services was used in this section. Section 6 returns to the theoretical model developed in sections 2 and 3 and implements it using U.S. data; i.e., the real (gross) income generated by the U.S. business sector is decomposed into the product of 6 explanatory factors. Section 7 moves to a net income framework; i.e., depreciation is subtracted from gross output to form net output and real net income generated by the U.S. Extended Business Sector is decomposed into explanatory factors. Section 7 concludes.

2. The Production Theory Framework

In this section, we present the production theory framework which will be used in this paper.[3] The main reference is Diewert and Morrison (1986).[4]

Assume that the business sector of the economy produces quantities of M (net)[5] outputs, $y \equiv [y_1,...,y_M]$, which are sold at the positive producer prices $P \equiv [P_1,...,P_M]$. We further assume that the business sector of the economy uses positive quantities of N primary inputs, $x \equiv [x_1,...,x_N]$ which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors y that can be produced by the business sector if the vector of primary inputs x is utilized by the business sector of the economy; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.[6]

Given a vector of output prices P and a vector of available primary inputs x, define *the period t business sector GDP function*, $g^t(P,x)$, as follows:[7]

$$(1)\ g^t(P,x) \equiv \max_y \{P\cdot y : (y,x)\in S^t\}.$$

[2] This is the term used by Jorgenson and Griliches (1967) used to describe a concept that is essentially output growth divided by input growth. The Bureau of Labor Statistics (1983) (2012) uses the equivalent term Multifactor Productivity (MFP) growth.

[3] Diewert, Mizobuchi and Nomura (2005), Diewert and Lawrence (2006), Diewert and Yu (2012) and Cho, Kim and Schreyer (2012) have used variants of this methodology for Japan, Australia, Canada and Korea.

[4] The theory also draws on Diewert (1983; 1077-1100), Kohli (1990) (2003) (2004) (2006), Morrison and Diewert (1990a), Fox and Kohli (1998) and Chapter 24 in the IMF, ILO, OECD, Eurostat, UNECE and the World Bank (2009).

[5] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

[6] The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function g^t is known as the *GDP function* or the *national product function* in the international trade literature; see Kohli (1978) (1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953).

[7] The function $g^t(P,x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x; see Diewert (1974; 136). Notation: $P\cdot y \equiv \sum_{m=1}^{M} P_m y_m$.

Thus business sector GDP depends on t (which represents the period t technology set S^t), on the vector of output prices P that the business sector faces and on x, the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and x^t is the vector of inputs used by the business sector during period t and if the GDP function is differentiable with respect to the components of P at the point P^t, x^t, then the period t vector of business sector outputs y^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of P:[8]

$$(2)\ y^t = \nabla_P\, g^t(P^t,x^t).$$

Thus the period t market sector supply vector y^t can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t output price vector P^t.

If the GDP function is differentiable with respect to the components of x at the point P^t,x^t, then the period t vector of input prices W^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of x:[9]

$$(3)\ W^t = \nabla_x\, g^t(P^t,x^t).$$

Thus the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t input quantity vector x^t.

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t:

$$(4)\ g^t(P^t,x^t) = P^t\cdot y^t = W^t\cdot x^t\ .$$

The above material will be useful in what follows but of course, our focus is not on GDP; instead our focus is on the income generated by the business sector or more precisely, on *the real income generated by the business sector*. However, since business sector GDP is distributed to the factors of production used by the business sector, nominal business sector GDP will be equal to nominal business sector income; i.e., from (4), we have $g^t(P^t,x^t) = P^t\cdot y^t = W^t\cdot x^t$. An approximate welfare measure that can be associated with business sector production is the *real income generated by the business sector in period t*, ρ^t, which is defined as the number of consumption bundles that the nominal income could purchase in period t:

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[8] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P\, g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial P_1, ...,\partial g^t(P^t,x^t)/\partial P_M]$.

[9] These relationships are due to Samuelson (1953) and Diewert (1974; 140).

(5) $\rho^t \equiv W^t \cdot x^t / P_C{}^t = w^t \cdot x^t = p^t \cdot y^t = g^t(p^t, x^t)$

where $P_C{}^t > 0$ is the *period t consumption expenditures deflator* and the business sector period t *real output price* p^t and *real input price* w^t vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures price index:[10]

(6) $p^t \equiv P^t/P_C{}^t$; $w^t \equiv W^t/P_C{}^t$;

The first and last equality in (5) imply that the period t real income generated by the business sector, ρ^t, is equal to the period t GDP function, evaluated at the period t real output price vector p^t and the period t input vector x^t, $g^t(p^t,x^t)$. Thus *the growth in real income over time generated by the business sector can be explained by three main factors: t (Technical Progress or Total Factor Productivity growth), growth in real output prices and the growth of primary inputs*. Formal definitions for these three growth factors will be given below.

Using the linear homogeneity properties of the GDP functions $g^t(P,x)$ in P and x separately, it can be shown that the following counterparts to the relations (2) and (3) hold using the deflated prices p and w:

(7) $y^t = \nabla_p g^t(p^t,x^t)$;
(8) $w^t = \nabla_x g^t(p^t,x^t)$. ;

Define a family of *period t productivity growth factors or technical progress shift factors* $\tau(p,x,t)$:[11]

(9) $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$.

Thus $\tau(p,x,t)$ measures the proportional change in the real income produced by the business sector at the reference real output prices p and reference input quantities used by the business sector x where the numerator in (9) uses the period t technology and the denominator in (9) uses the period t−1 technology. Thus each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t−1 to period t.

It is natural to choose special reference vectors for the measure of technical progress defined by (9): a *Laspeyres type measure* $\tau_L{}^t$ that chooses the period t−1 reference vectors

[10] Our approach is similar to the approach advocated by Kohli (2004; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP. Our treatment of the balance of trade surplus or deficit is also different. [Deal with this earlier]
[11] This measure of technical progress is due to Diewert and Morrison (1986; 662). A special case of it was defined earlier by Diewert (1983; 1063).

p^{t-1} and x^{t-1} and a *Paasche type measure* $\tau_P{}^t$ that chooses the period t reference vectors p^t and x^t:[12]

$$(10)\ \tau_L{}^t \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ;$$
$$(11)\ \tau_P{}^t \equiv \tau(p^t,x^t,t) \quad = g^t(p^t,x^t)/g^{t-1}(p^t,x^t).$$

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from index number theory[13] (so that the estimate going backwards is equal to the reciprocal of the estimate going forwards), then the geometric mean will be the best simple average to take.[14] Thus we define the geometric mean of (10) and (11) as follows:[15]

$$(12)\ \tau^t \equiv [\tau_L{}^t \tau_P{}^t]^{1/2}\ ;$$

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (10)-(12); this problem will be addressed shortly.

We turn now to the problem of defining theoretical indexes for the effects on real income generated by the business sector due to changes in real output prices. Define a family of *period s real output price growth factors* $\alpha(p^{t-1},p^t,x,s)$:[16]

$$(13)\ \alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x)/g^s(p^{t-1},x).$$

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the proportional change in the real income produced by the business sector that is induced by the change in real output prices going from period t−1 to t, using the technology that is available during period s and using the reference input quantities x. Each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income of a change in real output prices going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (13): a *Laspeyres type measure* $\alpha_L{}^t$ that chooses the period t−1 reference technology and reference input vector x^{t-1} and a *Paasche type measure* $\alpha_P{}^t$ that chooses the period t reference technology and reference input vector x^t:

[12] Salter (1960; 30-31) defined similar Laspeyres and Paasche measures of technical change using cost functions instead of GDP functions.
[13] See Fisher (1922; 64).
[14] See the discussion in Diewert (1997) on choosing the "best" symmetric average of Laspeyres and Paasche indexes that will lead to the satisfaction of the time reversal test by the resulting average index.
[15] The theoretical productivity change indexes defined by (10)-(12) were first defined by Diewert and Morrison (1986; 662-663) in the nominal GDP context.
[16] This measure of real output price change was essentially defined by Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89).

$$(14)\ \alpha_L{}^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ;$$
$$(15)\ \alpha_P{}^t \equiv \alpha(p^{t-1},p^t,x^t,t) \qquad = g^t(p^t,x^t)/g^t(p^{t-1},x^t).$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in real output prices:[17]

$$(16)\ \alpha^t \equiv [\alpha_L{}^t\,\alpha_P{}^t]^{1/2}.$$

Finally, we look at the problem of defining theoretical indexes for the effects on real income due to changes in input quantities. Define a family of *period t real input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:[18]

$$(17)\ \beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t)/g^s(p,x^{t-1}).$$

Thus $\beta(x^{t-1},x^t,p,s)$ measures the proportional change in the real income produced by the business sector that is induced by the change in input quantities used by the business sector going from period t−1 to t, using the technology that is available during period s and using the reference real output prices p. Each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in input quantities going from period t−1 to period t.

As usual, it is natural to choose special reference vectors for the measures defined by (17): a *Laspeyres type measure* $\beta_L{}^t$ that chooses the period t−1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* $\beta_P{}^t$ that chooses the period t reference technology and reference real output price vector p^t:

$$(18)\ \beta_L{}^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})\ ;$$
$$(19)\ \beta_P{}^t \equiv \beta(x^{t-1},x^t,p^t,t) \qquad = g^t(p^t,x^t)/g^t(p^t,x^{t-1}).$$

Since both measures of real input growth are equally valid, it is natural to average them to obtain an overall measure of the effects of input growth on real income:[19]

$$(20)\ \beta^t \equiv [\beta_L{}^t\,\beta_P{}^t]^{1/2}.$$

Recall that business sector real income for period t was defined by (5) as ρ^t equal to nominal period t factor payments $W^t{\cdot}x^t$ deflated by the household consumption price deflator $P_C{}^t$. It is convenient to define γ^t as the *period t chain rate of growth factor for real income*:

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[17] The indexes defined by (13)-(16) were defined by Diewert and Morrison (1986; 664) in the nominal GDP function context.
[18] This type of index was defined as a true index of value added by Sato (1976; 438) and as a real input index by Diewert (1980; 456).
[19] The theoretical indexes defined by (17)-(20) were defined in Diewert and Morrison (1986; 665) in the nominal GDP context.

(21) $\gamma^t \equiv \rho^t/\rho^{t-1}$.

3. The Translog GDP Function and Exact Index Numbers

We now follow the example of Diewert and Morrison (1986; 663) and assume that the log of the period t (deflated) GDP function, $g^t(p,x)$, has the following translog functional form:[20]

$$(22)\ \ln g^t(p,x) \equiv a_0^t + \sum_{m=1}^M a_m^t \ln p_m^t + (1/2) \sum_{m=1}^M \sum_{k=1}^M a_{mk} \ln p_m^t \ln p_k^t + \sum_{n=1}^N b_n^t \ln x_n^t + (1/2)\sum_{n=1}^N \sum_{j=1}^N b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^M \sum_{n=1}^M c_{mn} \ln p_m^t \ln x_n^t .$$

Note that the coefficients for the quadratic terms are assumed to be constant over time. The coefficients of the above translog functional form must satisfy certain restrictions in order for g^t to satisfy the appropriate regularity conditions that were listed earlier.[21]

Diewert and Morrison (1986; 663) showed that[22] if g^{t-1} and g^t are defined by (22) above and there is competitive profit maximizing behavior on the part of all market sector producers for all periods t, then

$$(23)\ \gamma^t = \tau^t \alpha^t \beta^t ; \qquad t = 1,2, ...$$

where γ^t, τ^t, α^t and β^t are defined above by (21), (12), (16) and (20) respectively. In addition, Diewert and Morrison (1986; 663-665) showed that τ^t, α^t and β^t could be calculated using empirically observable price and quantity data for periods t−1 and t as follows:

$$(24)\ \ln\alpha^t = \sum_{m=1}^M (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^t y_m^t/p^t\cdot y^t)] \ln(p_m^t/p_m^{t-1}) = \ln P_T(p^{t-1},p^t,y^{t-1},y^t);$$

$$(25)\ \ln\beta^t = \sum_{n=1}^N (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1}) = \ln Q_T(w^{t-1},w^t,x^{t-1},x^t);$$

$$(26)\ \tau^t = \gamma^t/\alpha^t \beta^t$$

where $P_T(p^{t-1},p^t,y^{t-1},y^t)$ is the Törnqvist (1936) output price index and $Q_T(w^{t-1},w^t,x^{t-1},x^t)$ is the Törnqvist input quantity index.

Equations (23) are in rates of growth. It is possible to obtain counterparts to these equations in a levels form as follows. Thus we can express the level of real income generated by the business sector in period t in terms of an *index of the technology level* or

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[20] This functional form was first suggested by Diewert (1974; 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson and Lau (1971). Diewert (1974; 139) indicated that this functional form was flexible.

[21] These restrictions may be found in Diewert (1974; 139).

[22] Diewert and Morrison established their proof using the nominal GDP function $g^t(P,x)$. However, it is easy to rework their proof using the deflated GDP function $g^t(p,x)$ using the fact that $g^t(p,x) = g^t(P/P_C,x) = g^t(P,x)/P_C$ using the linear homogeneity property of $g^t(P,x)$ in P.

of Total Factor Productivity in period t T^t, of the *level of real output prices* in period t A^t, and of the *level of primary input quantities* in period t, B^t.[23] Thus we use the growth factors τ^t, α^t and β^t as follows to define the levels T^t, A^t and B^t:

(27) $T^0 \equiv 1$; $T^t \equiv T^{t-1}\tau^t$; t = 1,2, ... ;
(28) $A^0 \equiv 1$; $A^t \equiv A^{t-1}\alpha^t$; t = 1,2, ... ;
(29) $B^0 \equiv 1$; $B^t \equiv B^{t-1}\beta^t$; t = 1,2, ...

Using the above definitions and the exact equations (26), we can establish the following exact relationship for the level of real income generated by the business sector in period t, ρ^t, and the period t levels for technology, real output prices and input quantities:

(30) $\rho^t/\rho^0 = T^t A^t B^t$; t = 1,2, ...

For some purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices α^t into separate effects for each change in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all market sector primary input quantities β^t into separate effects for each change in each input quantity.

We first model the effects of a change in a single (deflated) output price, say p_m, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type price indexes defined by (14) and (15) above for changes in all (deflated) output prices are the following *Laspeyres type measure* α_{Lm}^t that chooses the period t−1 reference technology and holds constant other output prices at their period t−1 levels and holds inputs constant at their period t−1 levels x^{t-1} and a *Paasche type measure* α_{Pm}^t that chooses the period t reference technology and reference input vector x^t and holds constant other output prices at their period t levels:

(31) $\alpha_{Lm}^t \equiv g^{t-1}(p_1^{t-1},...,p_{m-1}^{t-1},p_m^t,p_{m+1}^{t-1},..., p_M^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$; m = 1,...,M;
(32) $\alpha_{Pm}^t \equiv g^t(p^t,x^t)/g^t(p_1^t,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,..., p_M^t,x^t)$; m = 1,...,M.

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on real income of the change in the real price of output m*:[24]

(33) $\alpha_m^t \equiv [\alpha_{Lm}^t\alpha_{Pm}^t]^{1/2}$; m = 1,...,M.

[23] This type of levels presentation of the data is quite instructive when presented in graphical form. It was suggested by Kohli (1990) and used extensively by him; see Kohli (2003) (2004) and Fox and Kohli (1998).
[24] The indexes defined by (31)-(33) were defined by Diewert and Morrison (1986; 666) in the nominal GDP function context.

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (22) in the previous section, the arguments of Diewert and Morrison (1986; 666) can be adapted to give us the following result:

$$(34)\ \ln\alpha_m{}^t = (1/2)[(p_m{}^{t-1}y_m{}^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m{}^t y_m{}^t/p^t\cdot y^t)]\ \ln(p_m{}^t/p_m{}^{t-1})\ ; \qquad m = 1,\ldots,M.$$

Note that $\ln\alpha_m{}^t$ is equal to the mth term in the summation of the terms on the right hand side of (24). This observation means that we have the following exact decomposition of the period t aggregate real output price contribution factor α^t into a product of separate price contribution factors; i.e., we have under present assumptions:

$$(35)\ \alpha^t = \alpha_1{}^t\alpha_2{}^t\ldots\ \alpha_M{}^t.$$

The above decomposition is useful for analyzing how real changes in the price of exports (i.e., a change in the price of exports relative to the price of domestic consumption) and in the price of imports impact on the real income generated by the market sector. In the empirical illustration which follows later, we let M equal three. The three net outputs are:

- Domestic sales (C+I+G);
- Exports (X) and
- Imports (M).

Since commodities 1 and 2 are outputs, y_1 and y_2 will be positive but since commodity 3 is an input into the market sector, y_3 will be negative. Hence an increase in the real price of exports will *increase* real income but an increase in the real price of imports will *decrease* the real income generated by the market sector, as is evident by looking at the contribution terms defined by (34) for m = 2 (where $y_m{}^t > 0$) and for m = 3 (where $y_m{}^t < 0$).

As mentioned above, it is also useful to have a decomposition of the aggregate contribution of input growth to the growth of real income into separate contributions for each important class of primary input that is used by the market sector. We now model the effects of a change in a single input quantity, say x_n, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type quantity indexes defined by (18) and (19) above for changes in input n are the following *Laspeyres type measure* $\beta_{Ln}{}^t$ that chooses the period t−1 reference technology and holds constant other input quantities at their period t−1 levels and holds real output prices at their period t−1 levels p^{t-1} and a *Paasche type measure* $\beta_{Pn}{}^t$ that chooses the period t reference technology and reference real output price vector p^t and holds constant other input quantities at their period t levels:

$$(36)\ \beta_{Ln}{}^t \equiv g^{t-1}(p^{t-1},x_1{}^{t-1},\ldots,x_{n-1}{}^{t-1},x_n{}^t,x_{n+1}{}^{t-1},\ldots,\ x_N{}^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad n = 1,\ldots,N;$$
$$(37)\ \beta_{Pn}{}^t \equiv g^t(p^t,x^t)/g^t(p^t,x_1{}^t,\ldots,x_{n-1}{}^t,x_n{}^{t-1},x_{n+1}{}^t,\ldots,\ p_N{}^t)\ ; \qquad n = 1,\ldots,N.$$

Since both measures of input change are equally valid, as usual, we average them to obtain *an overall measure of the effects on real income of the change in the quantity of input n*:[25]

$$(38)\ \beta_n^t \equiv [\beta_{Pn}^t\,\beta_{Pn}^t]^{1/2}\ ; \qquad n = 1,...,N.$$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (22) in the previous section, the arguments of Diewert and Morrison (1986; 667) can be adapted to give us the following result:

$$(39)\ \ln\beta_n^t = (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)]\ \ln(x_n^t/x_n^{t-1}); \qquad n = 1,...,N.$$

Note that $\ln\beta_n^t$ is equal to the nth term in the summation of the terms on the right hand side of (25). This observation means that we have the following exact decomposition of the period t aggregate input growth contribution factor β^t into a product of separate input quantity contribution factors; i.e., we have under present assumptions:

$$(40)\ \beta^t = \beta_1^t\beta_2^t \ldots \beta_N^t.$$

4. Output and Input Aggregates and Conventional Productivity Growth for the U.S.

In Diewert (2013), an annual data base for an *Extended Business Sector* (EBS) was developed for the U.S. economy for the years 1987-2011. The output concept for this EBS was defined to be the GDP of the entire economy in year t (at producer prices) less the gross value of housing services less general government value added. It is obvious that the general government sector does not engage in competitive profit maximizing behavior so it is natural to exclude this sector from the business sector. It is also natural to exclude the services of Owner Occupied Housing (OOH) since these services are imputed and are not based on profit maximizing behavior. However, the EBS concept also excluded the services of rented housing. The reason for this exclusion is that accurate information on the stock of rental housing and the value of the land that the rental stock sits on is not available so it was decided to exclude all residential housing services from the business sector concept.

Diewert (2013) lists price and quantity index data for the following net outputs of the ECB: C (personal consumption less housing services), I (investment), G (deliveries of goods and services to the general government sector t by the EBS), 8 classes of exports of goods, 9 classes of imports of goods, exports of services and imports of services. The source for these series was the Bureau of Economic Analysis (2012) (BEA). Diewert also constructed a price and quantity series for aggregate labor input L into the Extended Business Sector based on BEA information and information from the Bureau of Labor Statistics (2012) Productivity Program. Information from the BEA (2012) was used to construct series for 27 types of reproducible capital and inventory stocks used by the

[25] The indexes defined by (36)-(38) were defined by Diewert and Morrison (1986; 667) in the nominal GDP function context.

Extended Business Sector. Finally, information from the USDA (2012a) (2012b) and from the Board of Governors of the Federal Reserve (2012) was used to construct estimates for the price and quantity of agricultural and non-agricultural land used by the ECB. Thus in total, asset stocks used by the EBS for 29 different asset classes were constructed.

Denote the price indexes for *consumption*, *investment* and *net deliveries of the EBS to the general government sector* as P_C^t, P_I^t and P_G^t and denote the corresponding quantity indexes as Q_C^t, Q_I^t and Q_G^t. Define P_D^t, the *year t price of domestic EBS production*, as the chained Törnqvist price index of P_C^t, P_I^t and P_G^t. P_D^t is listed in Table 1 below and the corresponding implicit Törnqvist quantity index Q_D^t is listed in Table 2 below.

Define P_X^t as the *chained export Törnqvist price index* of the 9 export price series listed in Diewert (2013), P_{X1}^t,...,P_{X9}^t. This aggregate export price series is listed in Table 1 below and the corresponding implicit Törnqvist quantity index Q_X^t is listed in Table 2 below. Similarly, define P_M^t as the *chained Törnqvist import price index* of the 10 import price series (adjusted for tariffs) listed in Diewert (2013), P_{M1}^t,...,P_{M10}^t. This aggregate import price series is listed in Table 1 below and the corresponding implicit Törnqvist quantity indexes Q_M^t is listed in Table2 below.

We define an EBS *aggregate output price index* for year t, P_Y^t, as a chained Törnqvist price index of the price indexes P_D^t, P_X^t, P_M^t with quantity weights Q_D^t, Q_X^t, $-Q_M^t$. Denote the corresponding implicit Törnqvist output index as Q_Y^t. The series P_Y^t and Q_Y^t are listed in Tables 1 and 2. Note that all price series have been normalized to equal unity in 1987. Thus the corresponding quantity units are in billions of constant 1987 dollars.[26]

Table 1: Price Indexes for EBS Output and Input Aggregates and TFP

Year t	P_C^t	P_D^t	P_X^t	P_M^t	P_L^t	P_K^t	P_{KW}^t	P_Y^t	P_Z^t	TFP^t
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.03914	1.03228	1.05178	1.04691	1.05387	1.02590	1.03808	1.03224	1.04564	1.01299
1989	1.08660	1.07156	1.06933	1.06993	1.08184	1.12253	1.08417	1.07134	1.09365	1.02083
1990	1.13572	1.11239	1.07676	1.09905	1.14336	1.13475	1.12646	1.10968	1.14071	1.02797
1991	1.17434	1.14331	1.09107	1.08888	1.18158	1.12223	1.15872	1.14456	1.16379	1.01680
1992	1.20916	1.16674	1.08663	1.08955	1.23841	1.17016	1.13006	1.16751	1.21794	1.04320
1993	1.23494	1.18815	1.08639	1.08098	1.26337	1.21965	1.13951	1.19059	1.25027	1.05013
1994	1.25540	1.20685	1.09857	1.08947	1.27999	1.28506	1.16439	1.21005	1.28139	1.05896
1995	1.28288	1.23098	1.08105	1.11446	1.30605	1.30854	1.19883	1.22741	1.30668	1.06458
1996	1.31084	1.25063	1.06691	1.09333	1.35197	1.37502	1.22906	1.24897	1.35872	1.08787
1997	1.33420	1.26608	1.04879	1.05391	1.39284	1.41856	1.24584	1.26930	1.40038	1.10327
1998	1.34314	1.26868	1.02476	0.99588	1.47226	1.37294	1.28736	1.27994	1.44205	1.12666
1999	1.36464	1.28413	1.01836	0.99947	1.53361	1.39349	1.31398	1.29409	1.49088	1.15206
2000	1.40124	1.31565	1.03637	1.04173	1.63566	1.36962	1.34133	1.32032	1.55341	1.17654
2001	1.42637	1.33580	1.03215	1.01669	1.70552	1.34185	1.43004	1.34705	1.59159	1.18153
2002	1.44127	1.34722	1.02792	1.00456	1.76674	1.38448	1.41251	1.36143	1.64684	1.20963
2003	1.46931	1.37169	1.04996	1.03921	1.83698	1.46922	1.45382	1.38267	1.72264	1.24588
2004	1.50887	1.41201	1.08655	1.08825	1.90377	1.59629	1.50354	1.41953	1.81117	1.27589
2005	1.55309	1.46335	1.12557	1.15516	1.96156	1.71258	1.68922	1.46371	1.89005	1.29127

[26] Note also that the producer price of the consumption aggregate, P_C^t, is also listed in Table 1. This is the price index that will be used to deflate nominal prices into real prices.

2006	1.59457	1.51066	1.16430	1.20223	2.03217	1.77158	1.84496	1.50877	1.95715	1.29718
2007	1.64043	1.55366	1.20332	1.24466	2.11251	1.78887	1.98908	1.55054	2.01585	1.30009
2008	1.69599	1.60482	1.26041	1.37655	2.17342	1.70822	2.06762	1.57976	2.02634	1.28269
2009	1.69592	1.59477	1.19252	1.23083	2.18900	1.67878	2.08388	1.59363	2.02526	1.27085
2010	1.73672	1.62461	1.24686	1.30486	2.23542	1.82502	1.92438	1.61688	2.11064	1.30538
2011	1.78075	1.66735	1.32699	1.40688	2.29767	1.89415	1.93947	1.65133	2.17640	1.31797
GAG	1.02433	1.02153	1.01186	1.01433	1.03527	1.02697	1.02799	1.02112	1.03293	1.01157

The last rows of Tables 1 and 2 list the Geometric Average Growth (GAG) rate for each of the series in the corresponding column.

Table 2: Quantity Indexes for EBS Output and Input Aggregates and Real Capital Output Ratios

Year t	Q_C^t	Q_D^t	Q_X^t	Q_M^t	Q_L^t	Q_K^t	Q_{KW}^t	Q_Y^t	Q_Z^t	Q_{KW}^t/Q_Y^t
1987	2505.9	3698.8	363.7	524.1	2492.8	1045.6	8378.7	3538.4	3538.4	2.37
1988	2611.0	3819.8	422.0	544.9	2580.9	1068.9	8540.6	3697.3	3649.9	2.31
1989	2686.9	3940.9	470.5	568.6	2670.0	1095.0	8696.4	3843.5	3765.1	2.26
1990	2739.9	3984.0	512.6	588.8	2677.8	1123.4	8857.9	3907.9	3801.6	2.27
1991	2731.6	3917.7	546.7	588.1	2663.6	1147.8	9002.0	3875.1	3811.1	2.32
1992	2826.7	4074.9	584.4	629.8	2696.9	1165.0	9094.8	4028.3	3861.5	2.26
1993	2935.1	4243.2	603.6	684.2	2779.2	1186.0	9200.0	4164.1	3965.3	2.21
1994	3045.9	4467.0	656.0	766.5	2901.7	1215.8	9348.7	4360.6	4117.8	2.14
1995	3126.6	4585.1	751.0	827.6	2981.2	1253.3	9545.1	4508.4	4234.9	2.12
1996	3244.6	4799.9	813.2	898.9	3036.7	1296.0	9752.5	4714.0	4333.2	2.07
1997	3367.9	5067.9	910.0	1020.2	3149.4	1345.6	9992.7	4959.8	4495.5	2.01
1998	3562.0	5398.3	930.8	1139.9	3216.1	1407.5	10292.1	5209.1	4623.5	1.98
1999	3761.3	5750.2	971.3	1271.2	3289.5	1476.7	10610.0	5488.5	4764.1	1.93
2000	3959.1	6059.4	1055.1	1436.7	3325.2	1555.2	10952.2	5732.6	4872.5	1.91
2001	4070.0	6119.2	995.6	1395.9	3277.2	1634.3	11298.9	5777.3	4889.7	1.96
2002	4199.9	6288.6	975.7	1443.6	3224.3	1681.8	11498.1	5894.5	4873.0	1.95
2003	4347.1	6515.6	991.2	1507.3	3208.7	1713.4	11646.3	6083.6	4883.0	1.91
2004	4485.8	6806.2	1086.4	1674.6	3248.8	1743.8	11790.9	6318.0	4951.8	1.87
2005	4630.4	7036.7	1159.6	1777.3	3313.2	1781.1	11963.8	6524.0	5052.4	1.83
2006	4753.8	7224.5	1263.7	1885.3	3391.8	1820.9	12134.9	6706.5	5170.0	1.81
2007	4858.1	7295.2	1381.1	1931.2	3430.3	1870.4	12360.0	6831.4	5254.6	1.81
2008	4796.4	7105.1	1465.0	1878.4	3398.2	1918.7	12579.8	6749.9	5262.3	1.86
2009	4690.4	6715.2	1331.1	1624.3	3205.9	1953.5	12738.0	6461.5	5084.4	1.97
2010	4779.6	6945.6	1479.4	1827.4	3215.3	1948.7	12704.2	6644.9	5090.4	1.91
2011	4914.1	7090.8	1578.3	1915.0	3267.4	1961.7	12781.7	6796.4	5156.8	1.88
GAG	1.02846	1.02749	1.06307	1.05547	1.01134	1.02656	1.01775	1.02757	1.01582	0.99045

We turn our attention to aggregate input series. Tables 1 and 2 above list Diewert's estimates for the price and quantity of EBS *labour services*, P_L^t and Q_L^t.

Denote P_{KW}^t as the beginning of the year t *chained Törnqvist price index* of the 29 EBS *capital stocks* listed in Diewert (2013) and Q_{KW}^t as the corresponding implicit quantity index. These indexes are also listed in Tables 1 and 2 respectively. The last column of Table 2 also lists the *real capital output ratios* for the Extended Business Sector, Q_{KW}^t/Q_Y^t, for the years 1987-2011. It can be seen that the capital output ratio declined from 2.37 in 1987 to 1.88 in 2011, an average rate of decline of about 1% per year.

It is necessary to explain how user costs of capital were constructed. Before doing this, it is useful to list various approximate tax rates that Diewert (2013) developed; see Table 3 below. The various taxes are defined as follows: τ_C^t is the year t *consumption tax*; τ_{MG}^t is the year t *tariff rate on imports of goods*; τ_L^t is the year t *tax rate on labor* (income taxes plus social security contributions); τ_P^t is the year t *property tax rate* on structures and business and agricultural land,[27] τ_K^t is the (*stock*) *rate of business income taxation* as a fraction of beginning of year t asset value and τ_{BI}^t is the *rate of business income taxation* as a fraction of net operating surplus in year t. The last column of Table 3 also lists r^t, the *balancing after tax Internal Rate of Return* in year t, which will be explained more fully below. The last row in Table 3 lists the arithmetic average of the various rates in each column.

Table 3: Tax Rates on Outputs and Inputs for the US Economy and the Balancing After Tax Rate of Return on Assets r^t: 1987-2011

Year t	τ_C^t	τ_{MG}^t	τ_L^t	τ_P^t	τ_K^t	τ_{BI}^t	r^t
1987	0.07583	0.03738	0.23226	0.00951	0.02323	0.31991	0.04938
1988	0.07551	0.03628	0.22879	0.00961	0.02354	0.32854	0.04811
1989	0.07347	0.03609	0.23585	0.00984	0.02411	0.30804	0.05416
1990	0.07326	0.03444	0.23396	0.00993	0.02327	0.30774	0.05233
1991	0.07609	0.03354	0.22983	0.01053	0.02142	0.30085	0.04978
1992	0.07582	0.03359	0.22748	0.01110	0.02287	0.29102	0.05573
1993	0.07550	0.03341	0.22881	0.01102	0.02461	0.29656	0.05837
1994	0.07820	0.03161	0.23173	0.01134	0.02685	0.30617	0.06085
1995	0.07719	0.02613	0.23635	0.01105	0.02840	0.32686	0.05849
1996	0.07588	0.02378	0.24396	0.01163	0.02960	0.32241	0.06221
1997	0.07579	0.02213	0.25037	0.01176	0.03121	0.32760	0.06406
1998	0.07518	0.02106	0.25444	0.01139	0.03080	0.35158	0.05680
1999	0.07424	0.01833	0.25639	0.01114	0.03049	0.34639	0.05754
2000	0.07237	0.01693	0.26078	0.01087	0.03100	0.37041	0.05269
2001	0.07032	0.01758	0.25660	0.01014	0.02363	0.30994	0.05260
2002	0.06970	0.01667	0.23071	0.01031	0.02155	0.26214	0.06066
2003	0.06971	0.01660	0.21952	0.01013	0.02261	0.25654	0.06551
2004	0.07050	0.01551	0.22015	0.00992	0.02567	0.26621	0.07077
2005	0.07200	0.01481	0.23164	0.00905	0.02854	0.30454	0.06517
2006	0.07221	0.01417	0.23897	0.00836	0.03007	0.34002	0.05837
2007	0.07028	0.01439	0.24427	0.00848	0.02700	0.32963	0.05491
2008	0.06881	0.01361	0.23665	0.00897	0.01981	0.27547	0.05210
2009	0.06674	0.01455	0.21560	0.01036	0.01564	0.22335	0.05438
2010	0.06667	0.01469	0.21732	0.01113	0.02174	0.24677	0.06636
2011	0.06678	0.01431	0.21998	0.01140	0.02299	0.25120	0.06852
Average	0.07272	0.02286	0.23530	0.01036	0.02523	0.30280	0.05799

Using the asset specific property tax rates τ_{Pn}^t, the (stock) general business tax rates τ_K^t, the depreciation rates δ_n^t for n = 1-29 that are listed in Diewert (2013) along with the corresponding asset prices and quantities, P_{Kn}^t and Q_{Kn}^t, *user costs* U_n^t for the 29 asset classes can be defined as follows:[28]

[27] In equations (41) and (42) below, $\tau_{Pn}^t \equiv 0$ if asset n is not a structure or a land asset and $\tau_{Pn}^t \equiv \tau_P^t$ is asset n is a structure or land asset.

[28] For additional material on user costs and many historical references, see Jorgenson (1963) (1989) (2012), Hall and Jorgenson (1967), Christensen and Jorgenson (1969), Harper, Berndt and Wood (1980), Diewert (1980) (2005a), Schreyer (2001) (2009), Gu (2012) and Diewert (2012).

(41) $U_n^t \equiv [r^t + \tau_K^t + \tau_{Pn}^t + \delta_n^t]P_{Kn}^t$; n = 1,...,29 ; t = 1987,...,2011

where r^t is suitable real after tax cost of capital that applies to the entire extended business sector in year t. We follow national income accounting conventions and define r^t to be the *balancing real rate of return*; i.e., it is the rate of return that is consistent with the year t value of business sector net output being equal to the value of primary inputs used by the business sector in year t, where the user costs defined by (41) are used as prices for the beginning of the year capital inputs. Thus r^t can be determined as the solution to the following linear in r^t equation for t = 1961,...,2011:

$$(42)\ P_C^tQ_C^t + P_I^tQ_I^t + P_G^tQ_G^t + P_X^tQ_X^t - P_M^tQ_M^t = P_L^tQ_L^t + \sum_{n=1}^{29} [r^t + \tau_K^t + \tau_{Pn}^t + \delta_n^t]P_{Kn}^tQ_{Kn}^t ; \quad t = 1987,...,2011$$

where the various price and quantity series were defined above. The resulting series of balancing real rates of return are listed in Table 3 above. It should be noted that r^t can be interpreted as a *real interest rate*; i.e., it is the income earned by the business sector in year t relative to the starting capital stock, valued at the average investment prices for the period. This explains why we have not included a capital gains term in the user cost formulae defined by (41).[29] Denote P_K^t as the year t *capital services price index*, which is the implicit chained Törnqvist price index of the 29 user costs U_n^t with the capital stocks Q_{Kn}^t as the corresponding quantities and let Q_K^t be the corresponding direct chained Törnqvist capital services quantity index.[30] The aggregate price and quantity indexes for capital services, P_K^t and Q_K^t, are listed in Tables 1 and 2. From the last row in Table 2, it can be seen that the Geometric Average Growth (GAG) rate of capital services over the sample period was 2.66% per year, about 0.88 percentage points per year bigger than the corresponding rate of growth of the EBS capital stock, which grew at 1.78% per year. The higher rate of growth of capital services is explained by the fact that machinery and equipment capital stock components tend to grow more rapidly than structures components but the higher depreciation rates for M&E inputs leads to a higher user cost weight for M&E inputs and relatively lower user cost weights for structures as compared to the corresponding stock weights. Thus overall capital services grew more rapidly than the corresponding aggregate capital stock due to this differential weighting effect.

Once the labour and capital aggregates have been constructed, we can construct a direct Törnqvist quantity *input aggregate* of Q_L^t and Q_K^t which we denote by Q_Z^t, which is listed in Table 2. The corresponding implicit *aggregate input price* index, P_Z^t, is listed in Table 1.

The *productivity level* in year t of the U.S. Extended Business Sector, TFP^t, is defined as the aggregate year t output, Q_Y^t divided by aggregate year t input, Q_Z^t:[31]

[29] We have essentially absorbed the capital gains (or losses) term into r^t.

[30] Note that output components of GDP were aggregated using direct chained Törnqvist price indexes whereas input components of GDP are aggregated using direct chained Törnqvist quantity indexes. The reason for this change is due to our desire to apply equations (24) and (25) later in the paper.

[31] This is also known as Multifactor Productivity or Total Factor Productivity; see the BLS (1983) or Schreyer (2001).

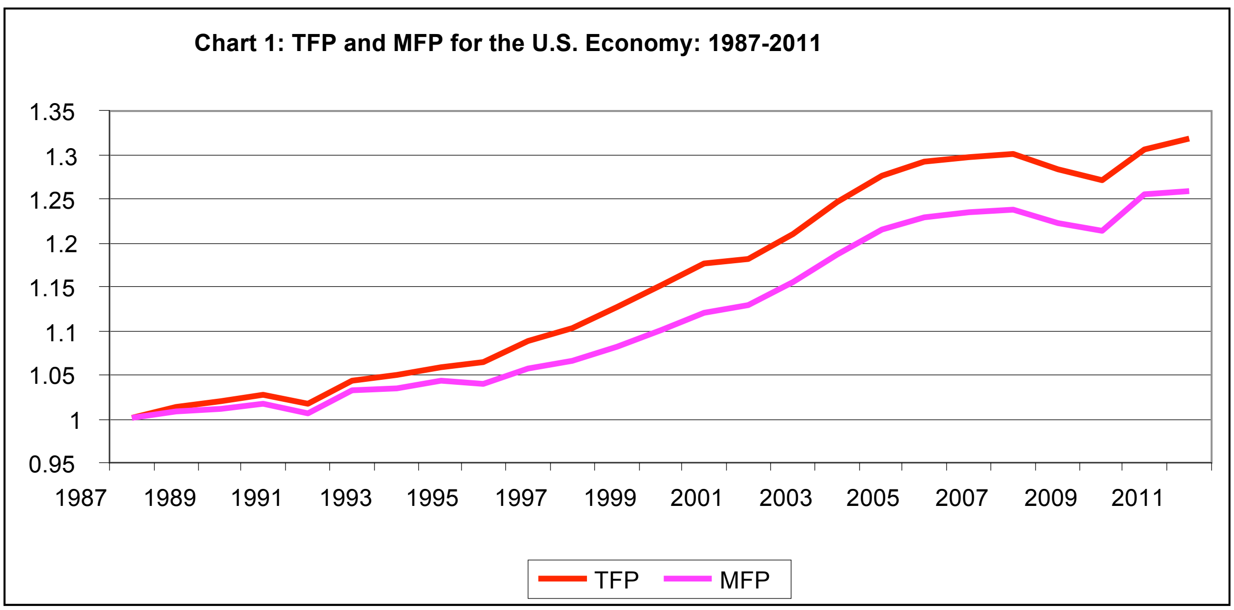
(43) $TFP^t \equiv Q_Y{}^t/Q_Z{}^t$; t = 1987, ..., 2011.

Productivity growth for year t, τ^t, is defined as the productivity level in year t divided by the previous year's productivity level:

(44) $\tau^t \equiv TFP^t/TFP^{t-1}$; t = 1987, ..., 2011.

The last column of Table 1 above lists the productivity levels, TFP^t, for the EBS for the years 1987-2011. The Geometric Average Growth rate for TFP over this period was 1.157% per year, which is quite satisfactory for an advanced country.[32] It can be seen that TFP for the EBS fell for only 3 years: 1991, 2008 and 2009.

It is possible to compare our TFP estimates for our EBS with the Bureau of Labor Statistics (2012) Multifactor Productivity (MFP) estimates for the U.S. Private Sector;[33] see Chart 1 below. Our TFP growth in the EBS averaged 1.157% per year, somewhat higher than BLS MFP growth in the Private Sector which averaged 0.962% per year, a rate that is about 20% lower. Determining the exact causes of the differences is a task for future research.[34]



[32] The arithmetic average of the TFP growth rates for the entire period was 1.16% per year. For the 1987-1999 period, the arithmetic average of the TFP growth rates was 1.19% per year and for the 2000-2011 period, the arithmetic average of the TFP growth rates was 1.14% per year.
[33] See the BLS (2012) or Table 32 in Diewert (2013) for the MFP^t series.
[34] The most likely sources of the differences are: (i) different scopes, in particular our exclusion of rented housing services (included in the BLS Private sector) and inclusion of the government enterprise sector (excluded in the BLS Private Sector) ; (ii) some unit value type aggregation bias in the aggregation of capital in the top down method (see the discussion in Jorgenson (2012; 12), Jorgenson and Schreyer (2012), Gu (2012) and Diewert (2012) on this source of bias); (iii) differing land series and (iv) different user cost formulae.

Productivity growth is not necessarily the entire story behind the growth in living standards: if the price of U.S. exports increases more rapidly than the price of Canadian imports, then the real income generated by the business sector should increase. This terms of trade effect is not taken into account in the above productivity computations. Thus in the following section, we implement the translog real income methodology explained in sections 2 and 3 above and this approach will enable us to assess the contribution to U.S. living standards of changes in the U.S. terms of trade.

5. Explaining Real Income Growth Generated by the Extended Business Sector: the Gross Output Approach

The basic methodology used in this section was explained in sections 2 and 3 above but can be summarized as follows. The business sector faces exogenous domestic and international prices for the net outputs it produces: domestic outputs, exports and (minus) imports. The business sector also utilizes inputs of labour and capital in order to produce its outputs. The value of outputs produced by the business sector less the value of imports used (value added) must eventually flow back to the labour and capital primary inputs that were used to produce value added. This is the (gross) income generated by the business sector. We divide this gross income in year t by the price of consumption in year t, P_C^t, in order to turn this nominal income into *real income equivalents* ρ^t;[35] this real income is the number of consumption bundles that *could* be purchased by the owners of the labour and capital inputs that were used in year t by the U.S. Extended Business Sector. Divide each of the prices P_D^t, P_X^t, P_M^t, P_L^t and P_K^t by the price of consumption, P_C^t, in order to form the corresponding real output and input prices facing the business sector in each year.

There are six quantitative factors that can be used to explain the real income ρ^t generated by the business sector in year t:

- The price of domestic production P_D^t (an aggregate of C+I+G) relative to the price of consumption in year t, P_C^t;
- The price of exports relative to the price of consumption in year t, P_X^t/P_C^t;
- The price of imports relative to the price of consumption in year t, P_M^t/P_C^t;
- The quantity of labour used by the business sector in year t, Q_L^t;
- The quantity of capital used by the business sector in year t, Q_K^t and
- The level of technology of the business sector in year t.

The formal model explained earlier, based on the work of Diewert and Morrison (1986), allows us to decompose the growth of real income from year t−1 to t, ρ^t/ρ^{t-1}, into multiplicative year to year contribution factors α_D^t, α_X^t, α_M^t, β_L^t, β_K^t and τ^t that describe the effects of changes in the six explanatory variables listed above going from year t−1 to t. Our translog model leads to the following equation which decomposes the year to year

[35] Formally, $\rho^t \equiv P_Y^tQ_Y^t/P_C^t = P_Z^tQ_Z^t/P_C^t$. Thus ρ^t can be defined for each year t using the information in Tables 1 and 2.

growth in real income generated by the business sector, ρ^t/ρ^{t-1}, into a product of six year to year explanatory contribution factors:

(45) $\rho^t/\rho^{t-1} = \tau^t \alpha_D{}^t \alpha_X{}^t \alpha_M{}^t \beta_L{}^t \beta_K{}^t$; t = 1987, 1963,...,2011.

Thus if $\alpha_D{}^t$ is greater than one, this means that the domestic price of output grew faster than the price of consumption going from year t−1 to t and $\alpha_D{}^t$ measures the contribution of rising real domestic output prices to the growth in real income. Similarly, if $\alpha_X{}^t$ is greater than one, this means that U.S. export prices grew faster than the price of consumption going from year t−1 to t and $\alpha_X{}^t$ measures the contribution of rising real export prices to the growth in real income generated by the EBS. However, if if $\alpha_M{}^t$ is greater than one, this means that U.S. import prices did not increase as quickly as the price of consumption going from year t−1 to t and $\alpha_M{}^t$ measures the contribution of falling real import prices to the growth in real income generated by the business sector. If $\beta_L{}^t$ is greater than one, then business sector labour input increased going from year t−1 to t and $\beta_L{}^t$ measures the contribution of the increase in labour input to the growth in real income generated by the EBS. Similarly, if $\beta_K{}^t$ is greater than one, then business sector (gross) capital services input increased going from year t−1 to t and $\beta_K{}^t$ measures the contribution of the increase in capital input to the growth in real income generated by the EBS. Finally, if τ^t is greater than one, then the efficiency of the EBS increased from year t−1 to t and τ^t measures the contribution of the efficiency increase to the growth in real income generated by the U.S. business sector. These year to year contribution factors are listed in Table 4 along with the (arithmetic) averages of these contribution factors in the last three rows of the Table.[36] The last column in Table 4 gives the product of the real export and real import price contribution factors, $\alpha_{XM}{}^t$, defined as $\alpha_{XM}{}^t \equiv \alpha_X{}^t \alpha_M{}^t$. Roughly speaking, $\alpha_{XM}{}^t$ is a *terms of trade contribution factor*; it gives the contribution to real income growth of the combined effects of real changes in the international prices facing the U.S. business sector.[37]

Table 4: Extended Business Sector Annual Growth in Real Income and Annual Contribution Factors

Year t	ρ^t/ρ^{t-1}	τ^t	$\alpha_D{}^t$	$\alpha_X{}^t$	$\alpha_M{}^t$	$\beta_L{}^t$	$\beta_K{}^t$	$\alpha_{XM}{}^t$
1988	1.03797	1.01299	0.99314	1.00133	0.99889	1.02492	1.00644	1.00022
1989	1.03179	1.00774	0.99250	0.99665	1.00341	1.02429	1.00709	1.00005
1990	1.00760	1.00699	0.99305	0.99536	1.00258	1.00206	1.00763	0.99793
1991	0.98915	0.98914	0.99390	0.99736	1.00629	0.99624	1.00628	1.00363
1992	1.02983	1.02596	0.99101	0.99552	1.00416	1.00886	1.00432	0.99966

[36] The third row from the bottom gives the arithmetic average over the years 1987-2011 and the remaining rows give the averages over the 12 years 1987-1999 and 2000-2011. Note that the contribution factors are one plus the growth rates.

[37] Ulrich Kohli has pointed out that this is a slight abuse of terminology. Strictly speaking, the terms of trade is the price of exports over the price of imports and hence involves only two prices. Our definition of $\alpha_{XM}{}^t$ involves three prices: the price of exports, the price of imports and the price of domestic consumption. Our terms of trade contribution factor is the rate of change counterpart to Kohli's (2006; 50) *trading gains factor*.

1993	**1.03213**	**1.00664**	**0.99706**	**0.99716**	**1.00429**	**1.02155**	**1.00522**	**1.00143**
1994	**1.04698**	**1.00841**	**0.99917**	**0.99929**	**1.00132**	**1.03092**	**1.00732**	**1.00061**
1995	**1.02625**	**1.00531**	**0.99811**	**0.99467**	**0.99983**	**1.01920**	**1.00904**	**0.99450**
1996	**1.04127**	**1.02187**	**0.99418**	**0.99491**	**1.00681**	**1.01300**	**1.01009**	**1.00169**
1997	**1.05055**	**1.01415**	**0.99452**	**0.99481**	**1.00923**	**1.02573**	**1.01144**	**1.00399**
1998	**1.05203**	**1.02120**	**0.99528**	**0.99561**	**1.01085**	**1.01485**	**1.01342**	**1.00642**
1999	**1.04850**	**1.02255**	**0.99611**	**0.99688**	**1.00215**	**1.01616**	**1.01401**	**0.99902**
2000	**1.03782**	**1.02125**	**0.99769**	**0.99873**	**0.99719**	**1.00774**	**1.01489**	**0.99592**
2001	**1.01008**	**1.00424**	**0.99729**	**0.99698**	**1.00803**	**0.98961**	**1.01407**	**1.00499**
2002	**1.02052**	**1.02378**	**0.99803**	**0.99814**	**1.00408**	**0.98845**	**1.00823**	**1.00221**
2003	**1.02818**	**1.02997**	**0.99865**	**1.00024**	**0.99732**	**0.99659**	**1.00549**	**0.99756**
2004	**1.03826**	**1.02409**	**1.00257**	**1.00098**	**0.99620**	**1.00867**	**1.00538**	**0.99718**
2005	**1.03443**	**1.01206**	**1.00737**	**1.00086**	**0.99358**	**1.01354**	**1.00668**	**0.99444**
2006	**1.03205**	**1.00458**	**1.00591**	**1.00105**	**0.99702**	**1.01609**	**1.00707**	**0.99807**
2007	**1.01757**	**1.00224**	**0.99969**	**1.00070**	**0.99857**	**1.00773**	**1.00855**	**0.99927**
2008	**0.97371**	**0.98661**	**0.99903**	**1.00216**	**0.98430**	**0.99355**	**1.00798**	**0.98643**
2009	**0.96571**	**0.99077**	**0.99344**	**0.99099**	**1.02472**	**0.96077**	**1.00564**	**1.01548**
2010	**1.01887**	**1.02717**	**0.99455**	**1.00339**	**0.99282**	**1.00198**	**0.99920**	**0.99619**
2011	**1.01878**	**1.00964**	**1.00098**	**1.00669**	**0.98846**	**1.01081**	**1.00221**	**0.99508**
Ave 88-11	**1.02460**	**1.01160**	**0.99722**	**0.99835**	**1.00130**	**1.00810**	**1.00780**	**0.99967**
Ave 88-99	**1.03280**	**1.01190**	**0.99483**	**0.99663**	**1.00420**	**1.01650**	**1.00850**	**1.00076**
Ave 00-11	**1.01630**	**1.01140**	**0.99960**	**1.00010**	**0.99853**	**0.99963**	**1.00710**	**0.99857**

Viewing Table 4, it can be seen that over the entire sample period, the real income generated by the Extended Business Sector grew at 2.46% per year. However, the average rates of growth were very different during the 12 years prior to 2000 and the 12 years subsequent to 2000. During the pre 2000 period, real income grew at a 3.28% per year rate and only at a 1.63% rate in the post 2000 period. During the pre 2000 period, the positive average growth factors were: labor growth (+1.65% per year), TFP growth (+1.19% per year), capital services growth (+0.85% per year) and falls in real import prices (+0.42% per year). The negative growth factors for the pre 2000 period were: falls in real domestic prices[38] (−0.52% per year) and falls in real export prices (−0.34% per year). During the post 2000 period, the positive average growth factors were: TFP growth (+1.14% per year), capital services growth (+0.71% per year) and a tiny increase in real export prices (+0.01% per year). The negative growth factors for the post 2000 period were: increases in real import prices[39] (−0.15% per year), falls in real domestic prices[40] (−0.04% per year) and decreases in labor input (−0.04% per year). Thus TFP growth and growth in capital services input were major positive contributors to real income growth in both periods but labor input growth changed from being a major positive contributor to a small negative contributor and changes in real export and import prices changed from a small positive contributor (+0.08% per year) in the pre 2000 period to a larger negative contributor (−0.14% per year) in the post 2000 period.

The annual change information in Table 4 can be converted into levels using equations (30), (35) and (40) in section 3 above. Thus let T^t, $A_D{}^t$, $A_X{}^t$, $A_M{}^t$, $B_L{}^t$, $B_K{}^t$ and $A_{XM}{}^t$ be the cumulated products of the annual link factors τ^t, $\alpha_D{}^t$, $\alpha_X{}^t$, $\alpha_M{}^t$, $\beta_L{}^t$, $\beta_K{}^t$ and $\alpha_{XM}{}^t$

[38] This fall in real domestic prices was caused by investment prices that increased much more slowly than consumption prices.
[39] The price of energy imports increased dramatically during this period.
[40] This fall in real domestic prices was caused by investment prices that increased much more slowly than consumption prices.

respectively. Using these definitions and cumulating equations (45) leads to the following equations, which explain the cumulative growth in real gross income generated by the Extended Business Sector relative to the base year 1987:

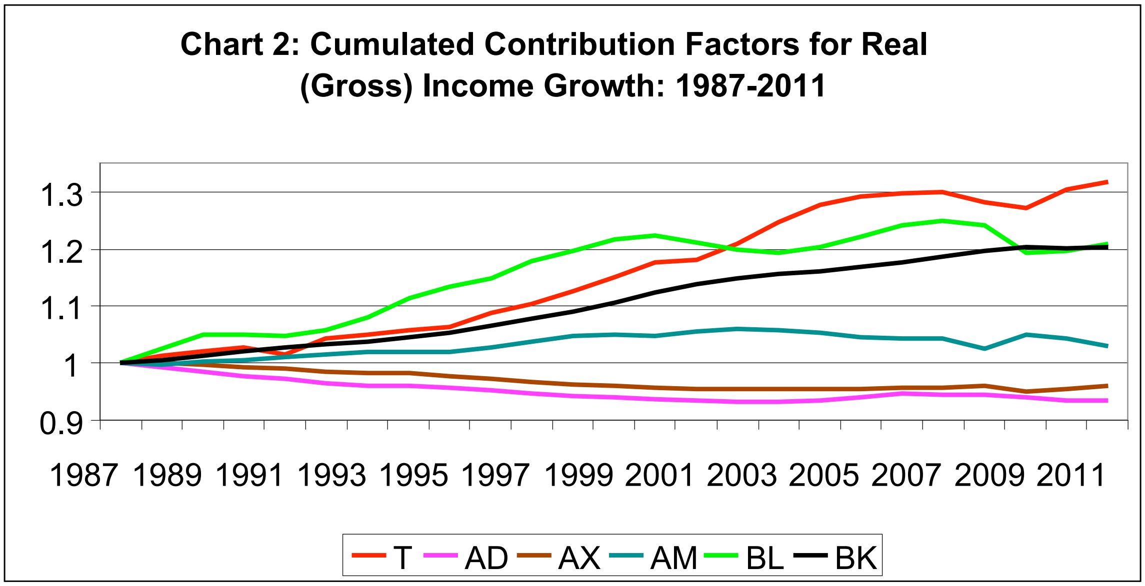
(46) $\rho^t/\rho^{1961} = T^t A_D{}^t A_X{}^t A_M{}^t B_L{}^t B_K{}^t$; t = 1988, 1989, ... , 2011.

The cumulated variables that appear in (46) above are reported in Table 5 below along with the cumulated terms of trade contribution factor, $A_{XM}{}^t$ defined to be the product of the two cumulated international price factors, $A_X{}^t$ and $A_M{}^t$.

Table 5: Extended Business Sector Cumulated Growth in Real Income and Cumulated Contribution Factors

Year t	ρ^t/ρ^{1987}	T^t	$A_D{}^t$	$A_X{}^t$	$A_M{}^t$	$B_L{}^t$	$B_K{}^t$	$A_{XM}{}^t$
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.03797	1.01299	0.99314	1.00133	0.99889	1.02492	1.00644	1.00022
1989	1.07096	1.02083	0.98569	0.99798	1.00230	1.04981	1.01358	1.00027
1990	1.07910	1.02797	0.97883	0.99334	1.00489	1.05197	1.02131	0.99820
1991	1.06739	1.01680	0.97287	0.99072	1.01121	1.04802	1.02772	1.00182
1992	1.09924	1.04320	0.96412	0.98628	1.01542	1.05730	1.03217	1.00149
1993	1.13456	1.05013	0.96128	0.98348	1.01977	1.08008	1.03756	1.00292
1994	1.18785	1.05896	0.96048	0.98278	1.02112	1.11348	1.04515	1.00353
1995	1.21904	1.06458	0.95866	0.97754	1.02095	1.13486	1.05460	0.99802
1996	1.26935	1.08787	0.95309	0.97256	1.02791	1.14962	1.06524	0.99970
1997	1.33352	1.10327	0.94786	0.96752	1.03739	1.17920	1.07743	1.00369
1998	1.40290	1.12666	0.94338	0.96327	1.04865	1.19671	1.09189	1.01013
1999	1.47094	1.15206	0.93971	0.96026	1.05090	1.21605	1.10719	1.00914
2000	1.52657	1.17654	0.93754	0.95905	1.04795	1.22546	1.12368	1.00503
2001	1.54195	1.18153	0.93500	0.95615	1.05637	1.21273	1.13949	1.01005
2002	1.57359	1.20963	0.93315	0.95437	1.06067	1.19872	1.14887	1.01227
2003	1.61793	1.24588	0.93190	0.95460	1.05783	1.19462	1.15517	1.00980
2004	1.67983	1.27589	0.93429	0.95554	1.05381	1.20498	1.16139	1.00696
2005	1.73767	1.29127	0.94117	0.95636	1.04705	1.22130	1.16915	1.00135
2006	1.79335	1.29718	0.94674	0.95737	1.04393	1.24095	1.17742	0.99943
2007	1.82486	1.30009	0.94644	0.95804	1.04244	1.25055	1.18749	0.99870
2008	1.77688	1.28269	0.94552	0.96010	1.02608	1.24248	1.19696	0.98514
2009	1.71596	1.27085	0.93931	0.95145	1.05144	1.19373	1.20372	1.00039
2010	1.74834	1.30538	0.93419	0.95468	1.04389	1.19610	1.20275	0.99658
2011	1.78117	1.31797	0.93511	0.96107	1.03185	1.20903	1.20541	0.99167

Thus the (gross) real income generated by the U.S. Extended Business Sector grew 1.78 fold over the period 1987-2011. This number factors into the product of the 6 numbers in columns 3-8 of the last row in Table 5. The biggest explanatory factors are TFP growth with a final cumulated contribution factor equal to 1.318, followed by labor input growth (1.209) and (gross) capital services growth (1.205). The 6 cumulative contribution factors are graphed in Chart 2. Perhaps the most interesting aspect of the Chart is it shows very clearly how growth in labor input went from being the most important contributor to real income growth in the period prior to 2000 to being essentially flat in the post 2000 period.



There is a problem with the income concept that we used up to this point. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence the measure of real income ρ^t that is used in this section is overstated. In the following section, we implement a more appropriate *net real income model*.

6. Explaining Real Income Growth Generated by the Extended Business Sector: the Net Output Approach

The overstatement of income problem that is implicit in the approach used in the previous section can readily be remedied: all we need to do is to take the user cost formula for an asset n that has asset price $P_{Kn}{}^t$ in year t and decompose the user cost into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_{Kn}{}^t$, and
- The remaining part that is the reward for postponing consumption, $r^t P_{Kn}{}^t$.

The depreciation part, $\delta P_{Kn}{}^t$, will be removed from the user cost and treated as an intermediate input and as an offset to gross investment.[41] We now explain this rather simple idea in more detail below.

Recall equations (41) where our user costs took the form $U_n{}^t \equiv [r^t + \tau_K{}^t + \tau_{Pn}{}^t + \delta_n{}^t]P_{Kn}{}^t$ where r^t was the balancing period t real rate of interest, $\delta_n{}^t$ is a geometric depreciation rate for period t, $\tau_K{}^t$ was an appropriate business taxation rate on the asset, $\tau_{Pn}{}^t$ was the property tax rate if applicable and $P_{Kn}{}^t$ was the beginning of year t stock price for the

[41] Some production theorists object to taking depreciation out of the user cost term, arguing that depreciation cannot be separated from waiting services. Diewert (2010) argued that depreciation can indeed be separated from waiting services.

asset. In the net product approach to the measurement of income,[42] we split up each (gross product) user cost into a depreciation component $\delta_n{}^t P_{Kn}{}^t$ and a waiting services component. The depreciation component is multiplied by the starting capital stock and is treated as an offset to gross investment made by the business sector during the year under consideration. Thus in the present section, our new aggregate for domestic output will aggregate the same C+I+G components as before, but now we subtract the depreciation series $\sum_{n=1}^{29} \delta_n{}^t P_{Kn}{}^t Q_{Kn}{}^t$ as negative outputs of the business sector.[43] Thus the new investment aggregate for year t, $Q_I{}^{t*}$, is a *net investment aggregate* (gross investment components are indexed with a positive signs in the aggregate and the depreciation components are indexed with negative signs in the aggregate) and the new capital services aggregate, $Q_K{}^{t*}$, is now a "reward for waiting" capital services aggregate.[44] Thus the new price of the investment aggregate $P_I{}^{t*}$ is a chained Törnqvist price index with component prices, $P_I{}^t, P_{K1}{}^t, ..., P_{K29}{}^t$ and corresponding quantities $Q_I{}^t, -\delta_1{}^t Q_{K1}{}^t, ..., -\delta_{29}{}^t Q_{K29}{}^t$ [45] and $Q_I{}^{t*}$ is the corresponding net investment quantity series. Define $P_D{}^{t*}$, the *year t price of net output of the EBS*, as the chained Törnqvist price index of $P_C{}^t$, $P_I{}^{t*}$ and $P_G{}^t$. $P_D{}^{t*}$ is listed in Table 6 below and the corresponding implicit Törnqvist quantity index $Q_D{}^{t*}$ is listed in Table 7 below. We define a new *aggregate output price index for net output* in year t, $P_Y{}^{t*}$, as a chained Törnqvist price index of the price indexes $P_D{}^{t*}$, $P_X{}^t$, $P_M{}^t$ with quantity weights $Q_D{}^{t*}$, $Q_X{}^t$, $-Q_M{}^t$. Denote the corresponding implicit Törnqvist net output quantity index as $Q_Y{}^{t*}$. The series $P_Y{}^{t*}$ and $Q_Y{}^{t*}$ are listed in Tables 6 and 7. Note that all price series have been normalized to equal unity in 1987. Thus the corresponding quantity units are in billions of constant 1987 dollars.[46]

Recall that $P_{Kn}{}^t$ and $Q_{Kn}{}^t$, n = 1,...,29, denoted the year t stock prices and quantities for the 29 asset classes in our model. The new *waiting services user costs* $U_n{}^{t*}$ for the 29 asset classes are defined as follows:[47]

$$(47)\ U_n{}^{t*} \equiv [r^t + \tau_K{}^t + \tau_{Pn}{}^t]P_{Kn}{}^t\ ; \qquad n = 1,...,29\ ;\ t = 1987,...,2011.$$

Denote $P_K{}^{t*}$ as the year t *waiting capital services price index*, which is the implicit chained Törnqvist price index of the 29 user costs $U_n{}^{t*}$ with the capital stocks $Q_{Kn}{}^t$ as the corresponding quantities and let $Q_K{}^{t*}$ be the corresponding direct chained Törnqvist waiting capital services quantity index. The aggregate price and quantity indexes for waiting capital services, $P_K{}^{t*}$ and $Q_K{}^{t*}$, are listed in Tables 6 and 7.

[42] See Diewert (2010) for a more detailed discussion of the net income approach to income measurement.
[43] Thus depreciation is now being treated as an intertemporal intermediate input into the production sector; i.e., depreciation expenses are now treated in the same way as imported inputs.
[44] This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: "Second, one can consider the 'waiting' or 'abstinence' associated with the net returns to capital as the nonlabour primary input." T.K. Rymes (1968; 362). Denison (1974) and Schreyer (2001) also considered a net product approach to productivity measurement.
[45] All of these component series are listed in Diewert (2013).
[46] The final rows of Tables 6 and 7 list the Geometric Average Growth (GAG) rate for each of the series in the corresponding column.
[47] The r^t are still defined by equations (42).

Once the new waiting services capital aggregates Q_K^{t*} have been constructed, we can construct a direct Törnqvist quantity *input aggregate* of Q_L^{t} and Q_K^{t*} which we denote by Q_Z^{t*}, which is listed in Table 7. The corresponding implicit *aggregate input price* index, P_Z^{t*}, is listed in Table 6.

The *net productivity level* in year t of the U.S. Extended Business Sector, TFP^{t*}, is defined as the aggregate year t net output, Q_Y^{t*} divided by aggregate year t input, Q_Z^{t*}:

(48) $TFP^{t*} \equiv Q_Y^{t*}/Q_Z^{t*}$; t = 1987, ..., 2011.

Net productivity growth for year t, τ^{t*}, is defined as the net productivity level in year t divided by the previous year's net productivity level:

(49) $\tau^{t*} \equiv T^{t*}/T^{t-1*}$; t = 1987, ..., 2011.

The last column of Table 6 above lists the net productivity levels, TFP^{t*}, for the EBS for the years 1987-2011 and the last column of Table 7 lists (one plus) the annual rates of net productivity growth. The Geometric Average Growth rate for TFP over this period was 1.157% per year, which is quite satisfactory for an advanced country.[48] It can be seen that net TFP for the EBS fell for only 3 years: 1991, 2008 and 2009.

Table 6: Price Indexes for Net EBS Output and Input Aggregates and Net TFP Levels

Year t	P_C^{t}	P_D^{t*}	P_X^{t}	P_M^{t}	P_L^{t}	P_K^{t*}	P_Y^{t*}	P_Z^{t*}	TFP^{t*}
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.03914	1.03294	1.05178	1.04691	1.05387	1.02506	1.03292	1.04814	1.01473
1989	1.08660	1.07356	1.06933	1.06993	1.08184	1.16806	1.07338	1.09870	1.02358
1990	1.13572	1.11675	1.07676	1.09905	1.14336	1.17438	1.11381	1.14938	1.03193
1991	1.17434	1.14836	1.09107	1.08888	1.18158	1.14142	1.14993	1.17342	1.02042
1992	1.20916	1.17504	1.08663	1.08955	1.23841	1.22400	1.17612	1.23541	1.05041
1993	1.23494	1.19872	1.08639	1.08098	1.26337	1.30010	1.20173	1.27054	1.05726
1994	1.25540	1.21869	1.09857	1.08947	1.27999	1.40289	1.22260	1.30399	1.06657
1995	1.28288	1.24453	1.08105	1.11446	1.30605	1.43067	1.24077	1.33039	1.07223
1996	1.31084	1.26577	1.06691	1.09333	1.35197	1.54992	1.26421	1.39043	1.09984
1997	1.33420	1.28432	1.04879	1.05391	1.39284	1.62937	1.28842	1.43871	1.11665
1998	1.34314	1.28854	1.02476	0.99588	1.47226	1.55271	1.30188	1.48830	1.14319
1999	1.36464	1.30767	1.01836	0.99947	1.53361	1.59304	1.31970	1.54563	1.17120
2000	1.40124	1.34388	1.03637	1.04173	1.63566	1.54700	1.35005	1.61889	1.19913
2001	1.42637	1.36560	1.03215	1.01669	1.70552	1.50912	1.37949	1.66727	1.20861
2002	1.44127	1.37879	1.02792	1.00456	1.76674	1.60056	1.39626	1.73480	1.24247
2003	1.46931	1.40628	1.04996	1.03921	1.83698	1.76424	1.42019	1.82484	1.28493
2004	1.50887	1.45197	1.08655	1.08825	1.90377	1.99544	1.46224	1.92660	1.31756
2005	1.55309	1.50597	1.12557	1.15516	1.96156	2.18437	1.50805	2.01224	1.33433
2006	1.59457	1.55361	1.16430	1.20223	2.03217	2.25429	1.55303	2.08284	1.34115
2007	1.64043	1.59614	1.20332	1.24466	2.11251	2.25838	1.59404	2.14732	1.34709
2008	1.69599	1.65311	1.26041	1.37655	2.17342	2.07227	1.62560	2.15576	1.32613
2009	1.69592	1.63501	1.19252	1.23083	2.18900	2.03757	1.63529	2.16025	1.32102

[48] The arithmetic average of the TFP growth rates for the entire period was 1.16% per year. For the 1987-1999 period, the arithmetic average of the TFP growth rates was 1.19% per year and for the 2000-2011 period, the arithmetic average of the TFP growth rates was 1.14% per year.

2010	**1.73672**	**1.67603**	**1.24686**	**1.30486**	**2.23542**	**2.34760**	**1.66907**	**2.26658**	**1.35799**
2011	**1.78075**	**1.72372**	**1.32699**	**1.40688**	**2.29767**	**2.45349**	**1.70721**	**2.33884**	**1.36998**
GAG	**1.02433**	**1.02295**	**1.01186**	**1.01433**	**1.03527**	**1.03810**	**1.02254**	**1.03604**	**1.01320**

The geometric average growth rates for the nominal prices of consumption, domestic production, exports and imports were 2.43%, 2.30%, 1.19%, 1.43% per year respectively. Thus the real prices of exports and imports fell substantially over the sample period, with the price of imports falling more rapidly than the price of exports. Wages increased 3.53% per year while the price of waiting services increased 3.81% per year. Overall, output prices increased 2.25% per year while input prices increased at 3.60% per year, giving rise to an average geometric rate of growth of Net TFP of 1.32% per year, about 0.16 percentage points higher than the corresponding Gross TFP growth rate which was 1.16% per year.

Table 7: Quantity Indexes for Net EBS Output and Input Aggregates and Net TFP Growth Rates

Year t	Q_C^t	Q_D^{t*}	Q_X^t	Q_M^t	Q_L^t	Q_K^{t*}	Q_Y^{t*}	Q_Z^{t*}	τ^{t*}
1987	2505.9	3277.7	363.7	524.1	2492.8	624.5	3117.3	3117.3	
1988	2611.0	3387.1	422.0	544.9	2580.9	636.2	3264.6	3217.2	1.01473
1989	2686.9	3493.1	470.5	568.6	2670.0	647.5	3395.6	3317.4	1.00872
1990	2739.9	3520.1	512.6	588.8	2677.8	659.2	3443.9	3337.3	1.00816
1991	2731.6	3444.3	546.7	588.1	2663.6	669.5	3401.5	3333.4	0.98884
1992	2826.7	3590.3	584.4	629.8	2696.9	676.2	3543.5	3373.4	1.02939
1993	2935.1	3740.8	603.6	684.2	2779.2	683.9	3661.6	3463.3	1.00652
1994	3045.9	3941.3	656.0	766.5	2901.7	694.7	3835.1	3595.7	1.00881
1995	3126.6	4032.5	751.0	827.6	2981.2	709.0	3955.6	3689.1	1.00530
1996	3244.6	4221.3	813.2	898.9	3036.7	724.1	4135.3	3759.9	1.02575
1997	3367.9	4450.6	910.0	1020.2	3149.4	741.7	4342.6	3889.0	1.01529
1998	3562.0	4735.4	930.8	1139.9	3216.1	763.5	4547.6	3977.9	1.02377
1999	3761.3	5031.2	971.3	1271.2	3289.5	786.5	4772.1	4074.5	1.02450
2000	3959.1	5281.1	1055.1	1436.7	3325.2	811.3	4958.4	4134.9	1.02385
2001	4070.0	5304.2	995.6	1395.9	3277.2	836.5	4966.8	4109.5	1.00790
2002	4199.9	5443.4	975.7	1443.6	3224.3	850.7	5055.0	4068.5	1.02802
2003	4347.1	5645.9	991.2	1507.3	3208.7	861.4	5220.5	4062.9	1.03417
2004	4485.8	5900.1	1086.4	1674.6	3248.8	871.9	5419.6	4113.4	1.02540
2005	4630.4	6095.3	1159.6	1777.3	3313.2	884.6	5590.9	4190.1	1.01272
2006	4753.8	6250.0	1263.7	1885.3	3391.8	897.0	5740.3	4280.1	1.00511
2007	4858.1	6297.0	1381.1	1931.2	3430.3	913.3	5839.9	4335.2	1.00443
2008	4796.4	6079.4	1465.0	1878.4	3398.2	929.0	5727.7	4319.1	0.98444
2009	4690.4	5715.9	1331.1	1624.3	3205.9	940.2	5463.0	4135.4	0.99615
2010	4779.6	5923.6	1479.4	1827.4	3215.3	937.4	5624.8	4142.0	1.02799
2011	4914.1	6045.5	1578.3	1915.0	3267.4	943.0	5752.7	4199.1	1.00883
GAG	1.02846	1.02584	1.06307	1.05547	1.01134	1.01732	1.02586	1.01249	

From Table 7, it can be seen that real consumption, Q_C^t, almost doubled over the sample period; it grew at the Geometric Average Growth rate of 2.85% per year. Domestic net output grew at 2.58% per year, exports at 6.31% per year and imports at 5.55% per year. Labor input grew at an average rate of 1.13% per year, with all of the growth occurring during the first half of the sample period. Waiting services grew at an average rate of 2.59% per year, a bit below the average rate of growth of capital services, which was 2.66% per year. Net output grew at the average rate of 2.59% per year while the corresponding aggregate input grew at 1.25% per year. As was the case for (gross) TFP, net TFP fell for only 3 years in the sample period: 1991, 2008 and 2009.

The same translog contributions methodology explained in sections 2 and 3 can be applied to the net output model used in the present section. Our translog model leads to the following counterpart to equation (45) which decomposes the year to year growth in *net real income generated by the business sector*, ρ^{t*}/ρ^{t-1*},[49] into a product of six year to year explanatory contribution factors:

(50) $\rho^{t*}/\rho^{t-1*} = \tau^{t*}\, \alpha_D^{\ t*}\, \alpha_X^{\ t*}\, \alpha_M^{\ t*}\, \beta_L^{\ t*}\, \beta_K^{\ t*}$; t = 1987, 1963,...,2011.

Table 8 below is the net income counterpart to Table 4 in the previous section.

Table 8: Extended Business Sector Annual Growth in Net Real Income and Annual Contribution Factors

Year t	ρ^{t*}/ρ^{t-1*}	τ^{t*}	$\alpha_D^{\ t*}$	$\alpha_X^{\ t*}$	$\alpha_M^{\ t*}$	$\beta_L^{\ t*}$	$\beta_K^{\ t*}$	$\alpha_{XM}^{\ t*}$
1988	1.04099	1.01473	0.99377	1.00150	0.99874	1.02829	1.00366	1.00025
1989	1.03368	1.00872	0.99373	0.99622	1.00386	1.02751	1.00354	1.00006
1990	1.00688	1.00816	0.99511	0.99476	1.00292	1.00232	1.00367	0.99766
1991	0.98620	0.98884	0.99439	0.99701	1.00713	0.99575	1.00311	1.00411
1992	1.03479	1.02939	0.99370	0.99494	1.00471	1.01003	1.00196	0.99962
1993	1.03379	1.00652	0.99883	0.99679	1.00484	1.02433	1.00225	1.00161
1994	1.04821	1.00881	1.00009	0.99920	1.00149	1.03489	1.00324	1.00069
1995	1.02433	1.00530	0.99931	0.99400	0.99981	1.02166	1.00423	0.99381
1996	1.04246	1.02575	0.99527	0.99426	1.00768	1.01467	1.00445	1.00190
1997	1.05150	1.01529	0.99682	0.99416	1.01039	1.02901	1.00517	1.00449
1998	1.05110	1.02377	0.99652	0.99506	1.01223	1.01673	1.00604	1.00723
1999	1.04698	1.02450	0.99882	0.99648	1.00242	1.01823	1.00595	0.99890
2000	1.03517	1.02385	1.00089	0.99857	0.99683	1.00875	1.00603	0.99540
2001	1.00552	1.00790	0.99816	0.99658	1.00911	0.98823	1.00569	1.00566
2002	1.01948	1.02802	0.99918	0.99788	1.00463	0.98688	1.00318	1.00251
2003	1.03039	1.03417	1.00050	1.00027	0.99695	0.99612	1.00249	0.99723
2004	1.04087	1.02540	1.00583	1.00111	0.99570	1.00983	1.00258	0.99681
2005	1.03363	1.01272	1.00831	1.00097	0.99274	1.01534	1.00325	0.99371
2006	1.02983	1.00511	1.00523	1.00120	0.99663	1.01827	1.00317	0.99782
2007	1.01503	1.00443	0.99854	1.00079	0.99838	1.00879	1.00404	0.99917
2008	0.96743	0.98444	1.00190	1.00246	0.98210	0.99264	1.00367	0.98452
2009	0.95951	0.99615	0.98841	0.98965	1.02844	0.95506	1.00253	1.01780
2010	1.02621	1.02799	1.00106	1.00390	0.99176	1.00227	0.99933	0.99562
2011	1.02023	1.00883	1.00320	1.00766	0.98682	1.01237	1.00139	0.99438
Ave 88-11	1.02430	1.01330	0.99865	0.99814	1.00150	1.00910	1.00350	0.99962
Ave 88-99	1.03340	1.01330	0.99636	0.99620	1.00470	1.01860	1.00390	1.00090
Ave 00-11	1.01530	1.01330	1.00090	1.00010	0.99834	0.99955	1.00310	0.99838

The net real income generated by the U.S. business sector grew at an annual (arithmetic) rate of 2.43% on average over the 25 year period 1987-2011, which is a bit less than the gross real income growth rate of 2.46%. However, there was a big difference in the

[49] The net real income generated by the production sector in year t is defined as follows: $\rho^{t*} \equiv P_Y^{\ t*}Q_Y^{\ t*}/P_C^{\ t}$. The various contribution factors on the right hand side of (50) are defined in the same way as in the previous section, except that the definitions of aggregate output and input (and some components of these aggregates) have changed.

average rates of real income growth in the period 1988-1999 (3.34% per year) and in the period 2000-2011 (1.53% per year). The main explanatory factor behind this drop was the drop in labor input: an average rate of growth of 1.86% per year in the pre-2000 period and a rate of growth that was close to 0 in the post-2000 period. Productivity growth remained steady over both periods at 1.33% per year. The other important factor that explains real net income growth is capital accumulation: a positive average contribution factor of 0.39% per year in the pre-2000 period and of 0.31% in the post-2000 period. Not that the positive effect of capital accumulation is much lower in the net framework as compared to the gross framework. The effects of changes in real export and import prices are generally small.[50] Comparing these average contribution growth rates in the gross and net real income frameworks leads to the following important observations:

- The role of productivity improvements is *magnified* in the net income framework compared to the gross income framework;[51]
- The role of increases in labour input is also *magnified* in the net income framework and
- The role of increases in capital input is greatly *diminished* in the net income framework.

The annual change information in Table 8 can be converted into levels using equations (30), (35) and (40) in section 3 above. Thus let T^{t*}, A_D^{t*}, A_X^{t*}, A_M^{t*}, B_L^{t*}, B_K^{t*} and A_{XM}^{t*} be the cumulated products of the annual link factors τ^{t*}, α_D^{t*}, α_X^{t*}, α_M^{t*}, β_L^{t*}, β_K^{t*} and α_{XM}^{t*} respectively. Using these definitions and cumulating equations (50) leads to the following equations, which explain the cumulative growth in real gross income generated by the Extended Business Sector relative to the base year 1987:

$$(51)\ \rho^{t*}/\rho^{1961*} = T^{t*}\, A_D^{t*}\, A_X^{t*}\, A_M^{t*}\, B_L^{t*}\, B_K^{t*}\ ; \qquad t = 1988, 1989, \ldots , 2011.$$

The cumulated variables that appear in (51) above are reported in Table 9 below along with the cumulated terms of trade contribution factor, A_{XM}^{t*} defined to be the product of the two cumulated international price factors, A_X^{t*} and A_M^{t*}.

Table 9: Extended Business Sector Cumulated Growth in Net Real Income and Cumulated Contribution Factors

Year t	ρ^{t*}/ρ^{1987*}	T^{t*}	A_D^{t*}	A_X^{t*}	A_M^{t*}	B_L^{t*}	B_K^{t*}	A_{XM}^{t*}
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.04099	1.01473	0.99377	1.00150	0.99874	1.02829	1.00366	1.00025
1989	1.07605	1.02358	0.98754	0.99771	1.00260	1.05657	1.00721	1.00031
1990	1.08345	1.03193	0.98271	0.99248	1.00552	1.05903	1.01091	0.99796
1991	1.06850	1.02042	0.97719	0.98951	1.01269	1.05453	1.01405	1.00207
1992	1.10567	1.05041	0.97104	0.98450	1.01746	1.06510	1.01603	1.00169
1993	1.14303	1.05726	0.96990	0.98134	1.02238	1.09101	1.01832	1.00330

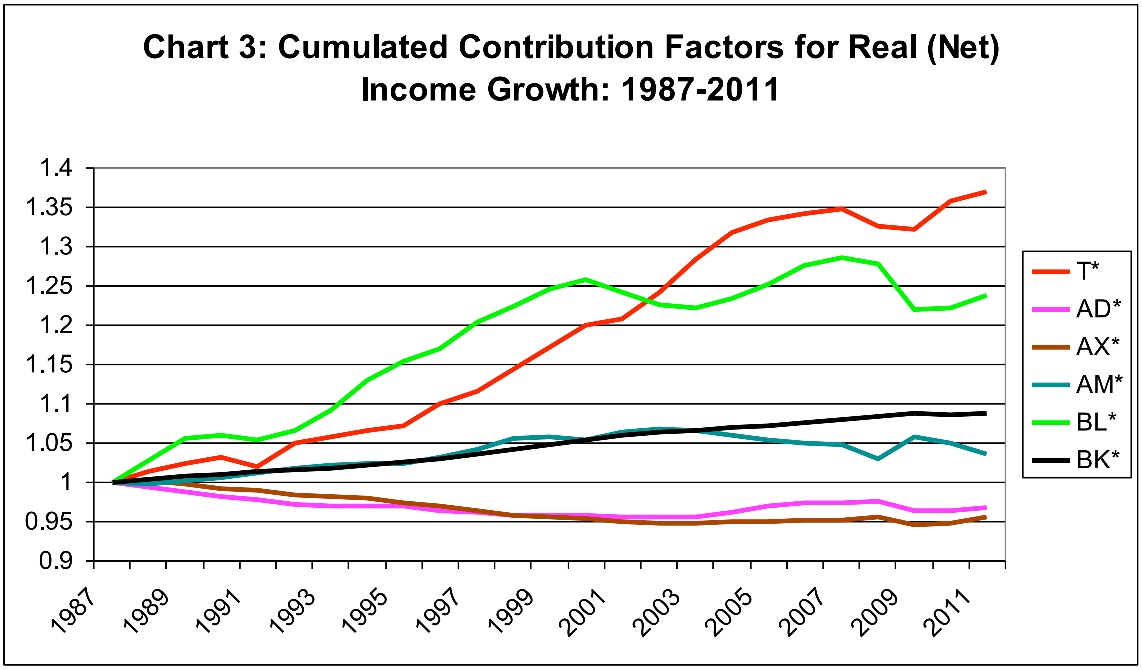
[50] This contrasts with the fairly large effects of changes in real export and import prices for Australia and Canada in the post 2000 period; see Diewert and Lawrence (2006) and Diewert and Yu (2012).

[51] This phenomenon is reasonably well known and was explained by Schreyer (2001): as the input denominator in a total factor productivity measure shrinks (by treating inputs as negative outputs and placing them in the net output numerator), the resulting measure of TFP will increase.

1994	**1.19813**	**1.06657**	**0.97000**	**0.98056**	**1.02390**	**1.12907**	**1.02162**	**1.00400**
1995	**1.22728**	**1.07223**	**0.96933**	**0.97467**	**1.02371**	**1.15353**	**1.02594**	**0.99778**
1996	**1.27938**	**1.09984**	**0.96474**	**0.96908**	**1.03157**	**1.17045**	**1.03051**	**0.99968**
1997	**1.34527**	**1.11665**	**0.96168**	**0.96343**	**1.04229**	**1.20440**	**1.03583**	**1.00417**
1998	**1.41402**	**1.14319**	**0.95833**	**0.95867**	**1.05503**	**1.22455**	**1.04209**	**1.01143**
1999	**1.48044**	**1.17120**	**0.95702**	**0.95530**	**1.05759**	**1.24687**	**1.04829**	**1.01031**
2000	**1.53250**	**1.19913**	**0.95805**	**0.95393**	**1.05423**	**1.25778**	**1.05461**	**1.00566**
2001	**1.54096**	**1.20861**	**0.95628**	**0.95067**	**1.06383**	**1.24297**	**1.06061**	**1.01135**
2002	**1.57097**	**1.24247**	**0.95550**	**0.94866**	**1.06876**	**1.22667**	**1.06399**	**1.01389**
2003	**1.61871**	**1.28493**	**0.95598**	**0.94892**	**1.06551**	**1.22191**	**1.06664**	**1.01108**
2004	**1.68486**	**1.31756**	**0.96155**	**0.94997**	**1.06092**	**1.23393**	**1.06939**	**1.00785**
2005	**1.74152**	**1.33433**	**0.96954**	**0.95090**	**1.05322**	**1.25286**	**1.07286**	**1.00150**
2006	**1.79346**	**1.34115**	**0.97461**	**0.95204**	**1.04966**	**1.27574**	**1.07626**	**0.99932**
2007	**1.82042**	**1.34709**	**0.97319**	**0.95279**	**1.04796**	**1.28696**	**1.08061**	**0.99849**
2008	**1.76113**	**1.32613**	**0.97504**	**0.95513**	**1.02920**	**1.27749**	**1.08457**	**0.98303**
2009	**1.68983**	**1.32102**	**0.96374**	**0.94525**	**1.05848**	**1.22007**	**1.08732**	**1.00053**
2010	**1.73412**	**1.35799**	**0.96476**	**0.94893**	**1.04975**	**1.22285**	**1.08659**	**0.99615**
2011	**1.76921**	**1.36998**	**0.96785**	**0.95620**	**1.03592**	**1.23797**	**1.08810**	**0.99054**

The net real income generated by the EBS grew 1.77 fold over the sample period, which is not that different from the gross income growth (1.78 fold). However, as noted earlier, the explanatory factors change considerably when we move from the gross income framework to the net framework: the effects of labor growth and TFP growth become more important and the importance of capital accumulation becomes less important.

The 6 cumulative contribution factors listed in Table 9 are graphed in Chart 3. Chart 3 again shows very clearly how growth in labor input went from being the most important contributor to real income growth in the period prior to 2000 to being essentially flat in the post 2000 period. Comparing Charts 2 and 3 shows the dramatic drop in the contribution of capital accumulation when we move from the gross output framework to the net framework. Chart 3 also shows that the effects of changes in real export or import prices can be important over subperiods; i.e., the contribution of lower import prices during the period 1987-2003 to real income growth is about the same as the contribution of capital growth in the net framework.



7. Conclusion

There are four major conclusions that we can draw from the above results.

First, using aggregate business sector data to construct top down estimates of Total Factor Productivity growth generates estimates that are above roughly comparable estimates of MFP that are produced by the BLS. However, since the scope of the two measures is not the same, the exact sources of the differences is a topic for further research.

Second, we have shown that the role of explanatory factors for growth in the real income generated by the business sector of the U.S. economy changes substantially when we shift from the standard gross product growth accounting model to a theoretically more appropriate net product growth accounting framework. In general, the main positive drivers of real income growth (growth in labour input, TFP growth and declining real import prices) are *magnified* but the effects of capital services input growth are greatly *diminished* when we switch to the net framework as compared to the gross product framework.[52]

Third, the results presented here show that over short periods of time, changes in the external price environment facing an economy can have substantial effects on living standards.

Finally, there are some data problems which should be addressed in future work on U.S. productivity performance. The BEA provides detailed (quarterly) information on outputs,

[52] Diewert, Mizobuchi and Nomura (2005), Diewert and Lawrence (2006) and Diewert and Yu (2012) found similar results for Japan, Australia and Canada using a similar net income framework.

intermediate inputs and reproducible capital stocks on a timely basis. The BLS provides detailed information on user costs of capital but on an annual basis and with some delay. The BLS also provides the best information on labor input but only at an aggregate level and with some delay. The Board of Governors of the Federal Reserve provides quarterly information on the value of business land but there is no decomposition into price and quantity components. It would be extremely useful if the BEA (perhaps in cooperation with the BLS and the Federal Reserve) could provide more accurate quarterly information on user costs, on labor (by type of labor and by industry) and on land (by industry). If this were done, then quarterly TFP (or MFP) estimates by industry could be produced on a timely basis. This would be of great benefit to governments and economic analysts.

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