# Comment on Estimating Capital Input for Measuring Canadian Multifactor Productivity Growth

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## Abstract

The paper lays out the algebra behind the Diewert-Yu capital services methodology and the corresponding methodology used by the Statistics Canada Canadian Productivity Program. The large differences in the estimates can mainly be explained by different treatments of the expected capital gains term in user costs and the use of sectoral balancing rates of return versus economy wide balancing rates of return.

**Journal of Economic Literature Classification Numbers**

C43, C67, C82, D24, E22, E43.

**Keywords**

Measurement of capital, measurement of inventory change, user costs, real interest rates.

## Introduction

Wulong Gu (2012) has provided a very thoughtful analysis of the differences in methodology for deriving business sector Multifactor Productivity Growth using the Statistics Canada Canadian Productivity Program (CPP) methodology (a bottom up approach) versus the top down approach used by Diewert and Yu (2012). He attributes the differences in the overall productivity estimates for the Canadian Business Sector to the following three main factors:

- The use of the top down approach involves some aggregation (over industries) bias which the bottom up approach does not have.

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- The Diewert-Yu approach estimates a balancing or endogenous real rate of return whereas the CPP approach estimates a balancing or endogenous nominal rate of return.
- The Diewert-Yu approach is subject to some aggregation (over assets) bias that the CPP approach avoids.

I actually agree with Gu on points 1 and 3 above but what Gu does not mention is that there are some problems with the Statistics Canada methodology which the Diewert-Yu methodology largely avoids. In section 2 below, I will lay out the algebra behind the two approaches and show that both approaches involve approximations to the "truth". Section 3 looks at the sectoral balancing rates of return used by Statistics Canada and concludes that the variability and size of these Internal Rates of Return are the main explanation for the differences in the estimates of Multifactor Productivity. Section 4 will address some of the less important issues that Gu raises and section 5 will conclude.

## 2. The Algebra Behind User Costs and Endogenous Rates of Return

In order to explain the differences between the Diewert-Yu methodology and the CPP methodology, it will be necessary to introduce some notation.

We want to work out what the value of capital services would be for a business sector that consists of I industries and where there are N assets that are used by these industries. Let $P_{ni}^t$ denote the price of asset n in industry i at the beginning of period t and let $K_{ni}^t$ denote the corresponding beginning of period t capital stock for n = 1,...,N and i = 1,...,I. Define the following variables:

$r_i^t \equiv$ the nominal cost of capital that industry i faces at the beginning of period t;
$\rho_{ni}^t \equiv$ the *expected* rate of price change in asset n for industry i during period t;
$\delta_{ni}^t \equiv$ the depreciation rate for asset n in industry i during period t;
$\tau_{ni}^t \equiv$ the rate of taxation (as a fraction of asset value) on asset n in industry i during period t.

Using the above definitions, the *user cost of asset n in industry i during period t*, $U_{ni}^t$, is defined as follows:

$$(1)\ U_{ni}^t \equiv [r_i^t - \rho_{ni}^t + \delta_{ni}^t + \tau_{ni}^t]P_{ni}^t\ ; \qquad n = 1,...,N;\ i = 1,...,I .$$

Both Diewert and Yu and the CPP would like to construct estimates of the *total value of capital services over all assets and all industries*, $\sum_{n=1}^N \sum_{i=1}^I U_{ni}^t K_{ni}^t$, for period t. I believe that the above approach to measuring capital services is a fairly good approximation to what is international best practice methodology for the determination of the price and quantity of capital services.[2]

[2] The basic methodology is due to Jorgenson and Griliches (1967). See also Diewert (1980) and Schreyer (2009). There will be minor variations on the form of the user cost formula from different experts.

The two main problems with this methodology are as follows:

- It is difficult to determine precisely what the correct opportunity cost of financial capital, $r_i^t$, is for each industry and
- It is difficult to determine exactly how to measure the anticipated capital gains term, $\rho_{ni}^t$.

Diewert and Yu and the CPP solve the above two problems in different ways as we shall explain below. We first need to introduce some additional notation. Let $GOS_i^t$ denote the Gross Operating Surplus of industry i in period t[3] for i = 1,...,I. We can now explain the Diewert-Yu methodology. We start with the value of business sector capital services, make various simplifying assumptions about the variables defined above and set the resulting approximate value of capital services equal to the value of business sector Gross Operating Surplus. Our key simplifying assumptions are that the nominal cost of capital is the same across industries (so that $r_i^t = r^t$ for i = 1,...,I) and that expected asset appreciation rates are the same across assets and industries (so that $\rho_{ni}^t = \rho^t$ for i = 1,...,I and n = 1,...,N). These are strong assumptions. Some additional simplifying assumptions that we will make are: asset depreciation rates, asset prices and asset tax rates are constant across industries so that $\delta_{ni}^t = \delta_n^t$ , $P_{ni}^t = P_n^t$ and $\tau_{ni}^t = \tau_n^t$ for i = 1,...,I and n = 1,...,N. Thus the Diewert-Yu value of capital services is equal to:

$$
\begin{aligned}
(2)\ \sum_{n=1}^{N} \sum_{i=1}^{I} U_{ni}^t K_{ni}^t &= \sum_{n=1}^{N} \sum_{i=1}^{I} [r_i^t - \rho_{ni}^t + \delta_{ni}^t + \tau_{ni}^t] P_{ni}^t K_{ni}^t && \text{using definitions (1)} \\
&\approx \sum_{n=1}^{N} \sum_{i=1}^{I} [r^t - \rho^t + \delta_n^t + \tau_n^t] P_n^t K_{ni}^t && \text{using our simplifying assumptions} \\
&= \sum_{n=1}^{N} [r^t - \rho^t + \delta_n^t + \tau_n^t] P_n^t [\sum_{i=1}^{I} K_{ni}^t] \\
&= \sum_{n=1}^{N} [r^t - \rho^t + \delta_n^t + \tau_n^t] P_n^t K_n^t && \text{where } K_n^t \equiv \sum_{i=1}^{I} K_{ni}^t \\
&= \sum_{i=1}^{I} GOS_i^t.
\end{aligned}
$$

Thus the approximate value of capital services is set equal to the business sector sum of the industry Gross Operating Surpluses, which is an equation which can be solved for the *overall business sector real rate of return*, $r^t - \rho^t \equiv r^{t*}$. In our empirical work, these balancing real rates of return ranged between 2.2% and 9.9% with the average rate equal to 6.13%. Note that the Diewert-Yu methodology has solved the problem of generating estimates for $r_i^t$ and the expected capital gains terms $\rho_{ni}^t$. However, it is clear that the Diewert-Yu methodology is subject to a kind of generalized unit value bias of the type that was described by Gu (2012).[4]

I will now explain how the CPP methodology works. This methodology works on a sectoral level and makes fewer simplifying assumptions. The key assumption that the CPP methodology is that *expected capital gains*, $\rho_{ni}^t$, can be approximated by ex post *actual capital gains*, $\pi_{ni}^t$ for n = 1,...,N and i = 1,...,I.[5] With this simplifying assumption in

[3] This is the value of industry i's revenues, less intermediate input cost, less labour cost.
[4] See Diewert and von der Lippe (2010) for an analysis of the magnitude of "regular" unit value bias.
[5] Another not so important simplifying assumption that the CPP methodology makes is to assume that depreciation rates are constant across time and industries so that $\delta_{ni}^t = \delta_n$ for n = 1,...,N, i = 1,...,I and t =

hand, we can start with the industry i value of capital services, make the CPP assumption about expected capital gains and set the resulting approximate value of industry i capital services equal to the industry i Gross Operating Surplus:

$$(3)\ \sum_{n=1}^{N} U_{ni}^{t}K_{ni}^{t} = \sum_{n=1}^{N} [r_i^t - \rho_{ni}^t + \delta_{ni}^t + \tau_{ni}^t]\, P_{ni}^{t}K_{ni}^{t}\ ; \qquad i = 1,...,I$$
$$\approx \sum_{n=1}^{N} [r_i^t - \pi_{ni}^t + \delta_n + \tau_{ni}^t]\, P_{ni}^{t}K_{ni}^{t} \qquad \text{using the CPP simplifications}$$
$$= GOS_i^t.$$

The I equations in (3) can be solved for $r_i^{t*}$, the industry balancing nominal rates of return.

Comparing the two methodological approaches, it would appear that the CPP bottom up approach has a clear advantage over the top down approach, since it makes fewer simplifying assumptions.[6] However, I have two problems with the CPP approach:

- Assuming that expected capital gains $\rho_{ni}^t$ can be set equal to actual ex post capital gains $\pi_{ni}^t$ leads to volatile user costs and sometimes negative user costs, which are not plausible. We would like user costs to approximately follow market rents and leasing rates (when available) and these actual rental rates are fairly stable and never negative.
- The balancing industry nominal rates of return $r_i^{t*}$ generated by solving equations (3) are frequently rather extreme; i.e., negative over prolonged periods or very large indeed. Abnormal rates of return cannot be regarded as good approximations to the industry's ex ante cost of capital.[7]

It turns out that at least some of the Statistics Canada balancing rates of return seem to be published as part of the World KLEMS accounts.[8] We will look at these Internal Rate of Return estimates in the following section.

It is easy to rework our results using an exogenous real rate of return rather than the endogenous ones we used. Thus if we simply assumed that the real rate of return faced by the aggregate business sector was equal to 6.13% (our sample average real rate of return), then our geometric average rate of MFP growth turns out to be 1.03% per year, the resulting capital services aggregate average geometric growth rate becomes 2.97% per year (compared to the CPP rate of 4.81% per year) and our capital stock aggregate

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1,...T. Diewert and Yu use another Statistics Canada source (CANSIM Table 310003) for depreciation rates and this alternative source does not assume that asset depreciation rates are constant over time.

[6] In fact, many years ago, I advocated the type of bottom up approach that was described above; see Diewert (1980).

[7] The problem is that all of the measurement errors that are associated with the construction of industry accounts show up in the balancing rate of return. Measurement errors are much larger at the industry level than at the national or aggregate business sector level. At the national level, most intermediate input transactions cancel out and hence it is not so important that these intermediate input flows are not measured very accurately. Similar problems occur with the allocation of labour effort and capital purchases to industries: it is much easier to do these allocations at the national level than at the industry level.

[8] If these rates of return are reasonably accurate, these statistics would be quite important for many policy purposes. Note that our balancing rates of return were quite reasonable but of course, they cover only the entire business sector.

growth rate becomes 2.33% per year (compared to the CPP rate of 3.11% per year). Thus the huge difference in results is not changed by assuming an exogenous real rate of return for the Diewert-Yu top down methodology.

My summary of the issues raised above is as follows: I agree with Gu that the bottom up methodology is methodologically best but I disagree with the use of actual capital gains to approximate expected capital gains. As we shall see in the next section, a more serious problem is the substantial sectoral measurement errors that seem to show up in the Statistics Canada estimates of the sectoral balancing nominal rates of return.

## 3. The Statistics Canada Estimates of Sectoral Nominal Internal Rates of Return

World KLEMS (2012) has a table for the balancing nominal rates of return for 30 Statistics Canada industries for the years 1961-2008. I have downloaded these data and reproduced them in Charts 1-3 below. These rates of return are labelled as IRRs; i.e., as Internal Rates of Return. Chart 1 shows the IRRs for Industries 1-10 and the arithmetic average by year of the 30 rates of return while Charts 2 and 3 show the IRRs for Industries 11-20 and 21-30 respectively. Conceptually, these IRRs are (approximately) equal to the industry balancing nominal rates of return $r_i^t$ that appeared in equations (3) above.

The 30 industries are as follows:

1 = Agriculture, hunting, forestry and fishing;
2 = Mining and quarrying;
3 = Food products, beverages and tobacco;
4 = Textiles, textile products, leather and footwear;
5 = Wood and products of wood and cork;
6 = Pulp, paper, paper products, printing and publishing;
7 = Coke, refined petroleum products and nuclear fuel;
8 = Chemicals and chemical products;
9 = Rubber and plastics products;
10 = Other non-metallic mineral products;
11 = Basic metals and fabricated metal products;
12 = Machinery, NEC;
13 = Electrical and optical equipment;
14 = Transport equipment;
15 = Manufacturing NEC; recycling;
16 = Electricity, gas and water supply;
17 = Construction;
18 = Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel;
19 = Wholesale trade and commission trade, except of motor vehicles and motorcycles;
20 = Retail trade, except of motor vehicles and motorcycles; repair of household goods;
21 = Hotels and restaurants;
22 = Transport and storage;
23 = Post and telecommunications;

24 = Financial intermediation;
25 = Real estate activities;
26 = Renting of M&E and other business activities;
27 = Public administration and defence; compulsory social security;
28 = Education;
29 = Health and social work;
30 = Other community, social and personal services.

**Chart 1: Business Sector Average IRR and IRRs for Industries 1-10, Canada, 1961-2008**

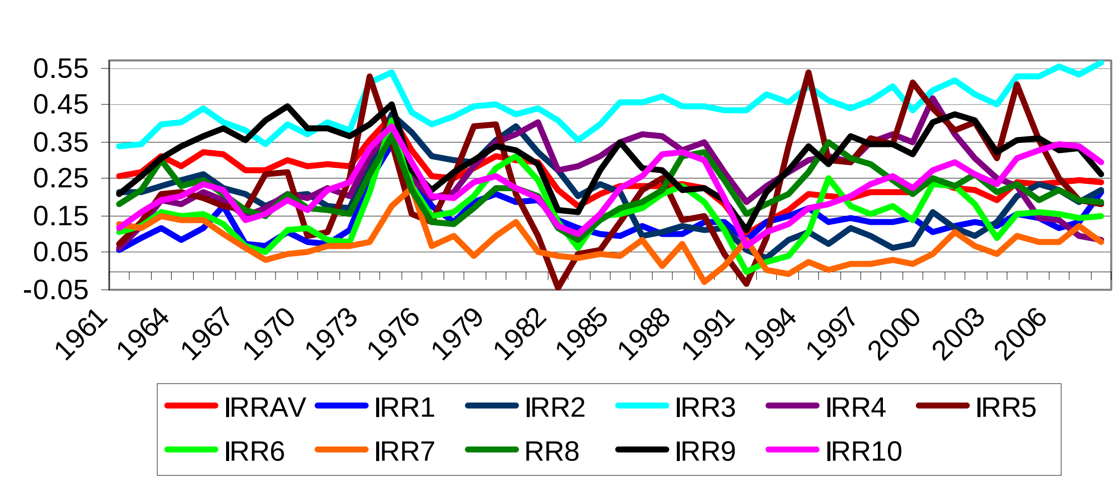
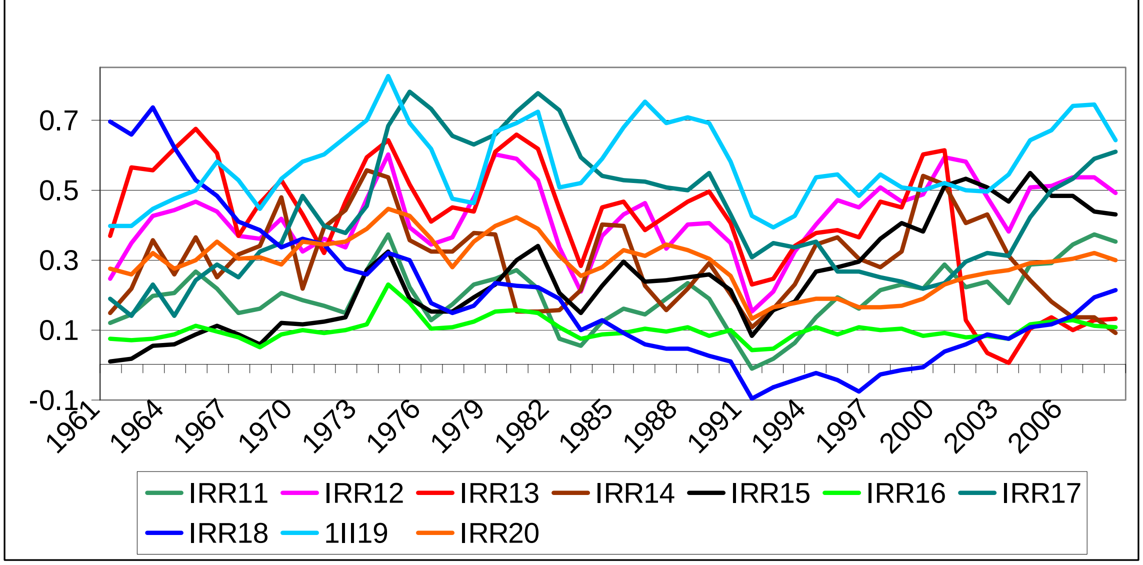
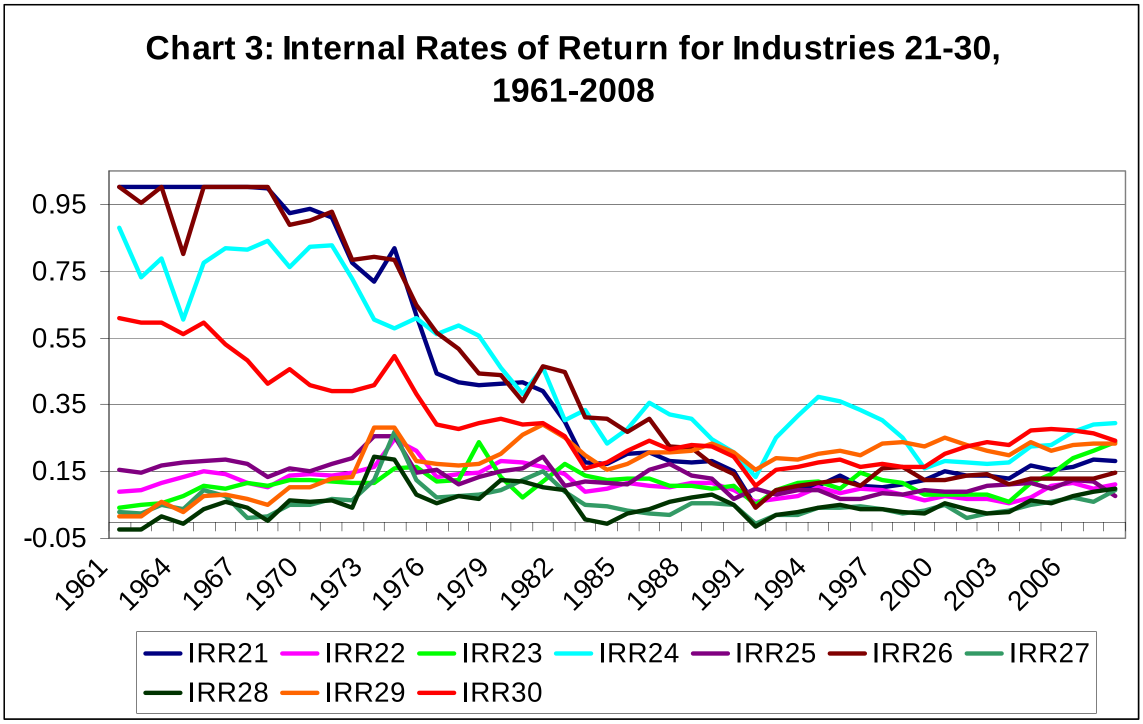



**Chart 2: Internal Rates of Return for Industries 11-20, 1961-2008**






It can be seen that these IRRs are generally too large[9] (the overall average rate of return over all industries and all years was 25.6%) and they are too variable to represent credible nominal opportunity costs of capital for the various industries. I believe it is these very high and variable internal rates of return that explain most of the differences between the Diewert-Yu MFP estimates and the Statistics Canada estimates. It is very likely that the official sectoral MFP estimates are subject to a considerable amount of measurement error.[10]

## 4. Other Points

*Depreciation rates*: Gu (2012) noted that the CPP program uses geometric depreciation rates that were constant over the entire sample period. This contradicts what other divisions in Statistic Canada are assuming; see CANSIM Table 310003 which assumes variable depreciation rates (which we used).[11] I think that it is quite sensible to have gradually changing depreciation rates. However, this difference in methodology is not big

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[9] The industry average rates of return over the entire period 1961-2008 were as follows: 0.138, 0.195, 0.445, 0.261, 0.238, 0.156, 0.069, 0.215, 0.313, 0.226, 0.195, 0.423, 0.408, 0.298, 0.251, 0.102, 0.433, 0.192, 0.574, 0.291, 0.423, 0.116, 0.117, 0.444, 0.133, 0.435, 0.059, 0.053, 0.179, 0.305. If industry rates of return were really this high, Canada would be one of the richest countries in the world!

[10] A word of caution: it is likely that the official Statistics Canada MFP estimates are based on a more detailed industry classification than was reported to the World KLEMS data base. However, it is also very likely that the more detailed IRRs are generally large and variable.

[11] For example, the variable depreciation rates for computers that we obtained from CANSIM Table 310003 started at 28% in 1961, moved up to 60% in 1998 and drifted down to finish at 54% per year.

enough to explain the differences in results.[12] A medium term goal for Statistics Canada should be to harmonize their information on capital stocks across Divisions.[13]

*Different scopes*: Diewert-Yu exclude all housing, including rental housing whereas the CPP business aggregate, quite properly includes it. Our reason for excluding rental housing is that there is no accurate information on the structure and land components for rental housing and so it seemed best to us to simply exclude this sector. We did not make an adjustment for the labour input into the rental sector (it will be small) so our MFP growth rate will have a small downward bias from this omission.

*The Degree of Disaggregation*: Gu notes that Diewert and Yu have only 14 types of reproducible asset whereas the CPP has 28. It seems to me that the CPP program could put information on all 28 assets into a CANSIM Table, at least for the entire business sector, and then all researchers would have the advantages associated with the use of this expanded data base. Right now, the CPP makes available information on only 5 assets in CANSIM Table 3830025 which ends in 2008! I realize that there might be confidentiality problems in releasing data on 28 assets (plus land and inventories) by industry but there can be no problem at the business sector level.

*The Land Problem*: The price and quantity of business land is poorly measured both by the CPP program and by Diewert and Yu. Improvement of measurement in this area should be a priority of Statistics Canada; i.e., there is no accurate breakdown of the price and quantity of land used for business and residential housing use (the price and quantity of agricultural land is relatively well determined). Another problem with land is that approximating anticipated capital gains for the price of land by the actual ex post capital gains (as happens using the Statistics Canada methodology) leads to small or negative user costs for land and leads to a land weight in overall capital services that is too small. This will tend to (incorrectly in my view) increase the growth of capital services and decrease the rate of growth in MFP.[14]

*The Computer Price Problem*: Statistical agencies frequently assume very big geometric depreciation rates for computers. In the case of computers, if the anticipated capital gains term is included in the user cost of computers, this term will frequently be quite large and will add a fairly large positive term (which essentially reflects obsolescence) into the user cost of computers. When this is combined with a large depreciation rate, there is a chance that the combined effects of depreciation and obsolescence are overdone; i.e., the user

[12] One related point about depreciation that is important enough to change the results is that the Statistics Canada depreciation rates are unusually large by international standards; e.g., US and Australian depreciation rates are generally much smaller. A careful audit of the Statistics Canada procedures for estimating depreciation rates seems called for.

[13] The Australian Bureau of Statistics has succeeded in harmonizing their information on capital stocks, flows and depreciation back to 1960. So it can be done!

[14] Even using anticipated land price change in the user cost formula can lead to a negative user cost for land services. My solution to this problem is to apply an *opportunity cost principle* here: the value of land services should be equal to the *maximum* of the user cost and the price at which the land could be rented. See Diewert (2009), Diewert and Nakamura (2009) and Diewert, Nakamura and Nakamura (2009) for more on this opportunity cost approach to estimating capital services.

cost of computers may become too large using the geometric model of depreciation with either anticipated or actual ex post capital gains in the user cost formula. A method for checking the reasonableness of the assumed geometric depreciation is to calculate computer capital services using the one hoss shay model. In this model, the computer is assumed to deliver the same physical capital services for all years of use until it is retired.[15] The price, quantity and value of capital services can be calculated using this one hoss shay model of depreciation and then the geometric model of capital services can be calibrated to at least roughly match the one hoss shay counterpart prices, quantities and values of capital services.[16] This measurement of computer services problem is not big enough to explain the differences in the two sets of MFP results for Canada.

*International Guidelines and Practices*: Does the top down methodology meet international guidelines for best practice in this area? I have conceded that the top down approach is not the best approach and a bottom up approach that had *accurate* sectoral data would be preferable.[17] However, the estimates of MFP growth generated by the CPP program are so small and the Statistics Canada industry balancing nominal rates of return are so large and variable that I have trouble believing that the sectoral data are accurate. Gu mentions Australia as a country that follows the same methodology as Statistics Canada. However, the Australian statisticians are far from happy with this methodology: they recognize that using ex post capital gains as a proxy for expected capital gains is not the final answer and they also encountered the abnormal sectoral rates of return problem.[18] The Australian Statistician, Brian Pink, in a public meeting on November 20, 2012 in Canberra told an audience that there is no universal consensus on user cost methodology and he was open to alternative ideas.

## 5. Conclusion

I think that the article by Wulong Gu, explaining the differences between the Diewert-Yu methodology and the Statistics Canada methodology is very useful. It made clear that our methodology is not the final answer to the measurement of a nation's Multifactor Productivity. However, I hope that in this response that I have made clear that there are some problems with the Statistics Canada methodology, taking into account the accuracy of the underlying sectoral data. In order to address these problems, I believe that Statistics Canada will need more resources in order to improve the quality of their sectoral estimates of inputs and outputs.

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[15] The algebra of the one hoss shay model is laid out in Diewert (2005).

[16] The calibration parameter would be the geometric depreciation rate; i.e., we would choose this rate so that the geometric model approximated the one hoss shay model.

[17] Note that the CPP estimates of MFP for an industry could be subject to same criticisms made by Gu of the top down method as applied to the entire business sector. Thus when Statistics Canada forms industry estimates of capital inputs, to do the job according to best practice methodology, it would need firm by firm estimates of the cost of capital and it is unlikely that this information would be available to Statistics Canada.

[18] But the Australian IRRs are better behaved than the Statistics Canada sectoral IRRs.

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