

## **New Methodology for Linking the Regions**

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### **Abstract**

The paper explains new methodology that was used in the 2005 International Comparison Program (ICP) that compared the relative price levels and GDP levels across 146 countries. In this round of the ICP, the world was divided into 6 regions: OECD, CIS, Africa, South America, Asia Pacific and West Asia. What is new in this round compared to previous rounds of the ICP is that each region was allowed to develop its own product list and collect prices on this list for countries in the region. The regions were then linked using another separate product list and 18 countries across the 6 regions collected prices for products on this list and this information was used to link prices and quantities across the regions. An additional complication was that the final linking of prices and volumes across regions had to respect the regional price and volume measures that were (separately) constructed by the regions.

### **Journal of Economic Literature Classification Numbers**

C43, C81, E31, O57.

### **Keywords**

Index numbers, multilateral comparison methods, GEKS, EKS, Geary-Khamis, Balk, Dikhanov, Iklé, Country Product Dummy (CPD) method, basic headings, Structured Product Descriptions, Purchasing Price Parities (PPPs), representative products, spatial chaining, fixity.

### **1. Introduction**

The final results for the 2005 International Comparison Program (ICP) have been released in February; for a tabulation of the results, see the World Bank (2008). The program compared the level of prices and the quantities or volumes of GDP (and its components) for 146 countries for the year 2005. International price statisticians

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developed Structured Product Descriptions (SPDs) for approximately 1000 products<sup>2</sup> and the individual countries collected price information on these products for the year 2005. The 1000 products were grouped into 155 Basic Heading (BH) categories. The price information collected in each country was then compared across countries, leading to a matrix of 155 basic heading prices by 146 countries. The precise way in which the individual product prices in each BH category were aggregated into a single country price for each BH heading is the topic which will be investigated in sections 2 and 3 below.

The 2005 ICP differed from previous ICP rounds.<sup>3</sup> In previous rounds, each country attempted to find prices in their country for a common product list. However, it is difficult to find products that are representative for *all* countries in the world and so the decision was made to break up the world into 6 regions and price statisticians developed *separate* product lists for each region. The 6 regions were: (1) Africa with 48 participating countries; (2) South America with 10 countries; (3) Asia Pacific with 23 countries; (4) The Commonwealth of Independent States (CIS) with 10 countries; (5) West Asia with 11 countries and (6) the OECD and other European countries covered by Eurostat plus Israel and Russia adding up to 46 countries in this region. This sums to 148 countries but Egypt appears in both the African and West Asia regions and Russia appears in both the OECD and CIS regions so there are 146 participating countries in all.

The fact that the product lists in each region were allowed to be different across regions means that without further information, prices and volumes could not be compared across regions. However, the World Bank, in cooperation with other national and international statistical agencies, developed an additional product list, which was priced out by 18 selected countries across the regions. These 18 countries were called *ring countries*. The prices that were collected by the ring countries using this final product list enabled price comparisons to be made across the 6 regions. We will indicate how this was done at the Basic Heading level in section 3 below and in section 5, we will indicate how comparisons at higher levels of aggregation between regions were made.

There was another methodological innovation made in this current ICP round in addition to having regional product lists: the price parities or Purchasing Power Parities (PPPs) and relative volumes for each country were determined using information on prices and GDP expenditure shares that pertained only to countries within the given region and these parities and relative volumes were preserved in the world comparison. Thus each region was independently allowed to determine its country PPPs and volume shares and the final linking of the regional results into a global world comparison left these regional relative parities undisturbed.<sup>4</sup>

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<sup>2</sup> Most of the products referred to are components of individual consumption: “There are about 830 SPDs that cover 100 Basic Headings for individual consumption. Each SPD contains price determining characteristics that will define unique products from any corner of the world.” Dennis Trewin (2008; 8). For an overview of the organization and methodology used in the 2005 ICP, see Trewin’s paper and the other contributions in the March 2008 issue of the *ICP Bulletin*, which was edited by Yonas Biru.

<sup>3</sup> For an overview of previous ICP rounds and an assessment of the current round, see Heston and Summers (2008).

<sup>4</sup> Egypt is an exception to this statement as will be explained below.

The final results from the 2005 International Comparison Program for the 146 participating countries are available on the World Bank website; see the World Bank (2008) for these results and explanations for various difficulties that were encountered. This publication explained the basic framework for the provision of the data as follows:

“The purchasing power parities and the derived indicators in this report are the product of a joint effort by national statistical offices, regional coordinators, and the ICP global office. PPPs cannot be computed in isolation by a single country. However, each country was responsible for submitting official estimates of 2005 gross domestic product and its components, population counts, and average exchange rates. The regional coordinators worked with the national statistical offices to review the national accounts data to ensure that they conformed to the standards of the 1993 System of National Accounts. Similar reviews were conducted for population and exchange rate data.” The World Bank (2008; 2)

The World Bank noted that the data provided by China were not quite complete and that the Tables broke China into 4 separate regions:

“China submitted prices for 11 administrative areas and the urban and rural components. The World Bank and the Asian Development Bank extrapolated these 11 city prices to the national level. The China data do not include Hong Kong, Macao, and Taiwan, China.” The World Bank (2008; 2).

The World Bank publication also explained how the ICP dealt with the fact that Egypt appeared in two regions (and priced out the product lists for both regions):

“Egypt participated in both the Africa and West Asia ICP programs by providing prices for the products included in each comparison. Therefore, it was possible to compute PPPs for Egypt separately for Africa and West Asia. Both regions included Egypt results in their regional reports. Egypt appears in the global report in both regions. The results for Egypt from each region were averaged by taking the geometric mean of the PPPs, allowing Egypt to be shown in each region with the same ranking in the world comparison.” The World Bank (2008; 2).

Finally, the World Bank explained how the CIS regional results were obtained:

“Russia participated in the price collection for both the CIS and OECD comparisons. As with Egypt, PPPs for Russia were computed separately for the OECD and CIS comparisons. However, the CIS region did not participate in the Ring. Therefore, following past practices the CIS region was linked to Eurostat-OECD using Russia as a link. For comparison purposes, Russia is shown in both regions in the report.” The World Bank (2008; 2).

Thus since Russia is the only country that belongs to both the OECD region and the CIS region, linking the two regions at both the Basic Heading level and higher levels of aggregation can be done through Russia. The same linking strategy could have been used to link the Africa and West Asia regions using Egypt as the linking country (or bridge country using ICP parlance) but a decision was made not to do this.<sup>5</sup>

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<sup>5</sup> The problems in the case of Egypt are more complicated than in the case of Russia since there were more than one ring countries in Africa and in West Asia. Hill (2007c; 13) listed the 18 ring countries as Brazil, Cameroon, Chile, Egypt, Estonia, Hong Kong, Japan, Jordan, Kenya, Malaysia, Oman, Philippines, Senegal, Slovenia, South Africa, Sri Lanka, United Kingdom and Zambia. Thus Cameroon, Jordan, Kenya, Oman, Senegal, South Africa and Zambia join Egypt as ring countries that are present in either the African or West Asian regions.

The above material presents a quick overview of the ICP. Our specific task in the present paper is to present some of the methodological details of the methods that were used to:

- Link the Basic Heading PPPs across the regions (sections 2 and 3) and
- Link the price levels and volumes for each country within a region across the regions in a way that preserves the regional relative price and volume measures (sections 4 and 5).

Thus sections 2 and 3 deal with the problems associated with the aggregation of price information at the lowest level of aggregation where information on expenditures or quantities is not available. Sections 4 and 5 deal with aggregation problems at higher levels of aggregation where expenditure information by category and country is available. It should be noted that the material to be covered in sections 2-5 below overlaps substantially with the material in the *ICP 2003-2006 Handbook*; see Hill (2007a) (2007b) (2007c) (2007d) (2007e). Also the material in sections 2 and 3 overlaps with Hill (2008) and the material in sections 3 and 5 overlaps substantially with Diewert (2004b).

Section 6 lists some of the methodological problems that require additional research before the next round of the ICP program, which is scheduled to take place in 2011.

Section 7 concludes.

## **2. The Comparison of Prices Across Countries Within a Region at the BH Level**

Three distinct methods for linking prices across countries within a region at the Basic Heading level were used by the regions in the 2005 ICP:

- The Country Product Dummy (CPD) method (used by the African, Asian Pacific and West Asian regions);
- The Extended Country Product Dummy (CPRD) method (used by South America) and
- The EKS\* method used by the OECD/Eurostat and CIS regions.

The most widely used statistical approach to the multilateral aggregation of prices at the first stage of aggregation is the *Country Product Dummy* (CPD) method, proposed by Robert Summers (1973). This method for making international comparisons of prices can be viewed as a very simple type of hedonic regression model where the only characteristic of the commodity is the commodity itself. The CPD method can also be viewed as an example of the stochastic approach<sup>6</sup> to index numbers. Since an extension of this method was used to link prices across regions, we will outline the algebra behind this approach.

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<sup>6</sup> See Selvanathan and Rao (1994) for examples of the stochastic approach to index number theory. A main advantage of the CPD method for comparing prices across countries over traditional index number methods is that we can obtain *standard errors* for the country price levels. This advantage of the stochastic approach to index number theory was stressed by Summers (1973).

Suppose that we are attempting to make an international comparison of prices between  $C$  countries over a reasonably homogeneous group of say  $N$  items.<sup>7</sup> In this section, we also assume that no expenditure weights are available for the price comparisons and for the sake of simplicity we assume that exactly  $K$  outlets are sampled for each of the  $N$  items in each of the  $C$  countries.<sup>8</sup> Thus there are  $CNK$  price quotes collected across all of the countries. These assumptions are not very realistic but it is useful to present this model as an introduction to more complex models.<sup>9</sup>

Let  $p_{cn}$  denote the average price of item  $n$ <sup>10</sup> in country  $c$  for  $c = 1, \dots, C$ ;  $n = 1, \dots, N$ . Each item  $n$  must be measured in the same quantity units across countries but the prices are collected in local currency units. The basic statistical model that is assumed is the following one:

$$(1) p_{cn} = a_c b_n e_{cn}; \quad c = 1, \dots, C; n = 1, \dots, N.$$

where the  $a_c$  and  $b_n$  are unknown parameters to be estimated and the  $e_{cn}$  are independently distributed error terms with means 1 and constant variances. The parameter  $a_c$  is to be interpreted as the *average level of prices* (over all items in this group of items) in country  $c$  relative to other countries and the parameter  $b_n$  is to be interpreted as the *average* (over all countries) *multiplicative premium* that item  $n$  is worth relative to an average item in this grouping of items. Thus the  $a_c$  are the basic heading country price levels that we want to determine while the  $b_n$  are item or individual product effects. The basic hypothesis is that the price of item  $n$  in country  $c$  is equal to a country price level  $a_c$  times an item commodity adjustment factor  $b_n$  times a random error that fluctuates around 1. Taking logarithms of both sides of (1) leads to the following model:

$$(2) y_{cn} = \alpha_c + \beta_n + \varepsilon_{cn}; \quad c = 1, \dots, C; n = 1, \dots, N$$

where  $y_{cn} \equiv \ln p_{cn}$ ,  $\alpha_c \equiv \ln a_c$ ,  $\beta_n \equiv \ln b_n$  and  $\varepsilon_{cn} \equiv \ln e_{cn}$ .

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<sup>7</sup> Using the language of the International Comparison of Prices (ICP) project, we are making a comparison of prices at the basic heading level. In ICP 2005 project, there are 155 basic headings. Thus each region using this method would have to run 155 regressions of the type described here.

<sup>8</sup> The case of unequal observations in each country for each commodity is discussed in Diewert (2004b) and Hill (2007a).

<sup>9</sup> A special case of the present model can be obtained by setting  $K$  equal to 1 and the price  $p_{cn1}$  can be set equal to the geometric mean of all of the outlet prices collected for product  $n$  in country  $c$ . The geometric mean is chosen over other methods for aggregating the outlet prices because, in the absence of weights, it seems to have the best axiomatic properties; e.g., see Diewert (2004a). (Note however, that when aggregating using geometric means, the micro prices should not approach zero). This is the “traditional” CPD model and it is discussed by Hill (2007a) and Rao (2004) in some detail. The problem with this model is that it neglects of the variability of the outlet prices *within* a country  $c$ , product  $n$ , cell. The advantage of the traditional CPD model is that the associated algebra is much simpler and hence, much easier to understand.

<sup>10</sup> In most cases, this item  $n$  price in country  $c$  was an unweighted arithmetic mean of prices collected over outlets and regions in the country during the reference year.

The model defined by (2) is obviously a linear regression model where the independent variables are dummy variables. The least squares estimators for the  $\alpha_c$  and  $\beta_n$  can be obtained by solving the following minimization problem:<sup>11</sup>

$$(3) \min_{\alpha_c, \beta_n} \left\{ \sum_{c=1}^C \sum_{n=1}^N [y_{cn} - \alpha_c - \beta_n]^2 \right\}.$$

However, it can be seen that the solution for the minimization problem (3) cannot be unique: if  $\alpha_c^*$  for  $c = 1, \dots, C$  and  $\beta_n^*$  for  $n = 1, \dots, N$  solve (3), then so does  $\alpha_c^* + \gamma$  for  $c = 1, \dots, C$  and  $\beta_n^* - \gamma$  for  $n = 1, \dots, N$ , for any arbitrary number  $\gamma$ . Thus it will be necessary to impose an additional restriction or normalization on the parameters  $\alpha_c$  and  $\beta_n$  in order to obtain a unique solution to the least squares minimization problem (3). The simplest normalization is:

$$(4) \alpha_1 = 0 \quad \text{or} \quad a_1 = 1 ;$$

The normalization (4) means that country 1 is chosen as the numeraire country and the parameter  $a_c$  for  $c = 2, \dots, C$  is the PPP (Purchasing Power Parity) of country  $c$  relative to country 1 for the class of commodity prices that are being compared across the  $C$  countries.<sup>12</sup>

Cuthbert and Cuthbert (1988; 57) introduced an interesting generalization of the Country Product Dummy method that can be used if information on *representativity* of the prices is collected by the countries in the comparison project along with the prices themselves. Hill (2007a) (2008) explains this method in some detail and he called the method the *extended CPD Method* or *CPRD Method* and he justified the method as follows:

“The reason for distinguishing between representative and unrepresentative products is that the relative prices of representative products in a country may be expected to be low compared with relative prices of the same products in countries in which they are not representative. Conversely, of course, the relative prices of unrepresentative products will tend to be high. This will tend to happen as result of normal substitution effects. Products will tend to be purchased in relatively large (small) quantities precisely because their relative prices are low (high). This conclusion is not merely a theoretical deduction, as there is ample empirical evidence of the substitution effect at work in both inter-temporal and inter-national comparisons.” Peter Hill (2007a; 3).

“The expected price depends on the interaction of three factors: the country, the product and its representativity. Given that the coefficient of a representative product is fixed at unity, the coefficient of an unrepresentative product may be expected to be greater than unity. The price of product is expected to be higher relatively to the reference product 1 in a country in which it is unrepresentative than in a country in which it is representative. The improvement over the traditional CPD method comes from the partial relaxation of the unrealistic assumption that the pattern of relative prices is the same in all countries. ...

The addition of the new variable, representativity, does not simply add another parameter to be estimated. It adds another dimension to the analysis. As there are three types of explanatory variables in the regression --

<sup>11</sup> Weighted (by expenditure shares) versions of the CPD model were considered by Prasada Rao (1990), (1995) (2001) (2002) (2004), Heston, Summers and Aten (2001), Sergueev (2001) (2003), Diewert (2004b) (2005) and Hill (2007a; 23-24).

<sup>12</sup> See Rao (2004) and Hill (2007a) for further analysis of this model. We note that Hill uses a different but equivalent normalization.

country, product and representativity -- the extended regression will be described as the CPRD method to distinguish it from the traditional CPD method.” Peter Hill (2007a; 26).

The basic idea is that representative products in a country should tend to be lower in price (and hence they should be more popular) compared to unrepresentative products; thus representativity becomes a price determining characteristic of the commodity.

The CPD method generalizes the model (2) above as follows. Define  $y_{cnu} = \ln p_{cnu}$  where  $p_{cnu}$  is the logarithm of the average product  $n$  price collected in country  $c$  and  $u$  is an index that denotes whether the collected price is unrepresentative (in which case  $u = 1$ ) or representative (in which case  $u = 2$ ). The basic (unweighted) statistical model that is assumed is the following one:

$$(5) y_{cnu} = \alpha_c + \beta_n + \delta_u + \varepsilon_{cnu}; \quad c = 1, \dots, C; n = 1, \dots, N; u = 1, 2$$

where the  $\alpha_c$  are the log country PPP's, the  $\beta_n$  are the log product price effects and the  $\delta_u$  are the two log representativity effects and the  $\varepsilon_{cnu}$  are independently distributed random variables with mean zero and constant variances. In order to identify the parameters, the following normalizations can be used:

$$(6) \alpha_1 = 0; \delta_1 = 0.$$

Thus the present model is much the same as the basic CPD model except that we have 3 classifications instead of 2. For additional discussion of this model, the reader is referred to Cuthbert and Cuthbert (1988), Diewert (2004b) and Hill (2007a) (2008).

We agree with Hill in endorsing the method in theory. However, in practice, it seems it was at times difficult for national price statisticians to agree on a workable definition of representativity that was uniform across countries and regions. Thus in the end, it appears that only the South American region used CPRD method to construct its 155 by 10 matrix of PPP's by Basic Heading and country. The other regions used the basic CPD method or the EKS\* method (which also used the representativity concept).

As mentioned at the beginning of this section, the EKS\* method was used to aggregate prices at the lowest level of aggregation in the OECD and CIS regions. This method is explained by Hill as follows:

“Eurostat abandoned EKS 1 in 1982 and replaced it by the method described in the present section, which will be called the asterisk method or EKS\*. A detailed exposition of EKS\* and its properties is given by Sergey Sergeev (2003). The EKS\* method is so called because it makes use of the distinction between representative and unrepresentative products, the representative products being identified in the product lists by an \*. The EKS\* method recognizes, and exploits, the fact that, as already explained, the prices of representative products are likely to be relatively low, whereas the prices of unrepresentative products are likely to be relatively high. The method proceeds by calculating two separate Jevons indices for each pair of countries. One Jevons index covers products that are representative in the first country, treated here as the base country. The other covers products that are representative in the second country. Of course, some products may be representative in both countries and included in both indices. The two indices may be described as *Jevons 1* and *Jevons 2* respectively.” Peter Hill (2007a; 9).

Thus two bilateral Jevons type indexes are calculated for any two countries. Jevons 1 (2) compares only the price relatives of products that are representative in country 1 (2). The final bilateral index of prices between the two countries under consideration is a geometric mean of the two Jevons indexes.<sup>13</sup> Once all of these bilateral parities have been constructed over each pair of countries in the region, they can be harmonized by using the EKS procedure.<sup>14</sup> For further details of this method, the reader is referred to Hill (2007a).

A majority of the members of the Technical Advisory Group who provided advice to ICP 2005 favoured the CPRD method described in the previous section over the EKS\* method described in this section for two reasons:

- The CPRD method used all of the available price information whereas EKS\* did not and
- The CPRD method gave straightforward measures of the statistical precision of the estimated parities.

However, it appears that Eurostat price statisticians are locked into the EKS\* method by legislation and thus the OECD/Eurostat region stuck by its EKS\* method in the current European Comparison Program. More research is required in order to determine how much difference there would be between CPRD and EKS\*. But without having this research in hand, I would certainly favor the use of CPRD over EKS\*, mainly because the EKS\* method throws away valuable information on some prices and this cannot be a statistically efficient procedure.

There is also the issue of choosing between the original CPD method and the enhanced CPRD method, which makes use of representativity information on the item prices. Hill (2007a) explains theoretically why the CPRD method should be preferred over the CPD method. However, in practice, national price collectors in all of the non OECD regions had great difficulty in deciding on which items were representative and which items were not. Thus when the CPRD regressions were run, the coefficients for the representative dummy variables had more or less random signs instead of the expected signs. This was the case even for the South American region, which used the CPRD method.<sup>15</sup> Thus at this stage of our knowledge of the various methods used to aggregate prices at the basic heading level, I would favor the use of the plain vanilla CPD method.

Having described the methods used to construct PPPs for the 155 basic headings for each country in a region, we now consider how to link these PPPs across regions.

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<sup>13</sup> Note that prices which are not representative in both countries but are collected in both countries do not appear in the final bilateral index of prices between the two countries. This means that the EKS\* procedure is not fully efficient in a statistical sense, whereas the CPRD procedure is fully efficient.

<sup>14</sup> The EKS method is explained in more detail by Balk (1996), Diewert (1999) and Hill (2007a) (2008). The method is due to Gini (1924) (1931) and independently rediscovered by Eltetö and Köves (1964) and Szulc(1964).

<sup>15</sup> Personal communication from Yuri Dikhanov.

### 3. The Comparison of Prices Across Regions at the Basic Heading Level

As noted in the introduction, a group of *ring countries* collected prices from a common list and this price information was used to link the regional basic heading prices across the 6 regions. However, since the CIS region was locked into the OECD/Eurostat region, in practice, there were only 5 regions to link, with the CIS, OECD and Eurostat countries forming a single region.

The methodology used to link basic heading prices across regions was developed by Diewert (2004b; 36-39) and we review that methodology here.<sup>16</sup> The model is basically an adaptation of the unweighted CPD model presented in section 2.1.

In order to set the stage for what was actually done in linking the regions, we first generalize the CPD model presented in section 2 to allow for a reorganization of the list of  $C$  countries into 5 regions and  $C(r)$  ring countries in each region  $r$ . Thus  $C(r)$  is not the total number of countries in region  $r$ ; it is only the number of ring countries in each region because only the ring countries collected data on prices from a common international product list. With these changes, the basic model becomes:

$$\begin{aligned} (7) \quad p_{rcn} &\approx a_r b_{rc} c_n ; & r = 1, \dots, 5; c = 1, \dots, C(r); n = 1, \dots, N; \\ (8) \quad a_1 &= 1; \\ (9) \quad b_{r1} &= 1; & r = 1, \dots, 5. \end{aligned}$$

The normalization (8) means that we have to choose a numeraire region. The normalizations (9) mean that within each region, we need to choose a numeraire country in order to identify all of the parameters uniquely. Thus the parameters  $a_r$  and  $b_{rc}$  replace our initial model parameters  $a_c$ . Note that the total number of parameters remains unchanged when we group all of the countries in the comparison into regions and countries within the regions.

Taking logarithms of both sides of (7) and then adding error terms  $\varepsilon_{rcn}$  (with means 0) leads to the following regression model:

$$\begin{aligned} (10) \quad \ln p_{rcn} &= \ln a_r + \ln b_{rc} + \ln c_n + \varepsilon_{rcn} ; & r = 1, \dots, 5; c = 1, \dots, C(r); n = 1, \dots, N; \\ &= \alpha_r + \beta_{rc} + \gamma_n + \varepsilon_{rcnk} \end{aligned}$$

where we impose the following normalizations on the parameters in order to uniquely identify them:

$$\begin{aligned} (11) \quad \alpha_1 &= 0 ; \\ (12) \quad \beta_{r1} &= 0 ; & r = 1, \dots, 5 \end{aligned}$$

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<sup>16</sup> The basic methodology is also described in Hill (2007d). However, Hill uses somewhat different normalizations than (28) and (29).

where  $\alpha_r \equiv \ln a_r$ ,  $\beta_{rc} \equiv \ln b_{rc}$ ,  $\gamma_n \equiv \ln c_n$ .

If all of the data collected for each regional comparison could be pooled and if there are product overlaps between the regions, then there will be 155 regressions of the form (10) to run, one for each basic heading category. In the above model, the interregional log parities (the  $\alpha_r$ ) are estimated along with the within region country log parities (the  $\beta_{rc}$ ) and the product log price premiums (the  $\gamma_n$ ). Call this the *first approach* to estimating the regional parities for each basic heading. It uses all of the available information in making comparisons between all of the countries.

However, the above one big regression approach (for each basic heading) is *not consistent* with approaches that used only the regional data to determine the within region parities, the  $\beta_{rc}$  parameters, holding  $r$  fixed. But a principle of the current ICP methodology was that regions should be allowed to determine their own parities, independently of other regions. However, the regression model (10) can be modified to deal with this problem. If the regional log parities  $\beta_{rc}$  are known, then the term  $\beta_{rc}$  (which is equal to  $\ln b_{rc}$ ) can be subtracted from both sides of (10), leading to the following regression model:

$$(13) \ln p_{rcn} - \ln b_{rc} = \ln a_r + \ln c_n + \varepsilon_{rcn} ; \quad r = 1, \dots, 5; c = 1, \dots, C(r); n = 1, \dots, N;$$

or

$$(14) \ln [p_{rcn}/b_{rc}] = \alpha_r + \gamma_n + \varepsilon_{rcn} ;$$

where the normalization (8) still holds. Thus if the within region parities are known, then prices in each region  $p_{rcn}$  can be divided by the appropriate regional parity for that country in that region  $b_{rc}$ , and these regionally adjusted prices can be used as inputs into the usual CPD model that has now only the regional log parities  $\alpha_r$  and the commodity adjustment factors  $\gamma_n$  as unknown parameters to be estimated.<sup>17</sup> Call the model defined by (11) and (14) the *second approach* to estimating the regional parities for each basic heading. This second approach respects the within region parities that have been constructed by the regional price administrators. It is this second approach that was used in ICP 2005.<sup>18</sup>

We now turn our attention to the problems associated with aggregating up the basic heading PPP information (along with country expenditure information) in order to form aggregate country price and volume comparisons within a region.

#### 4. Aggregate Price and Volume Comparisons Across Countries Within a Region

<sup>17</sup> Thus we have saved 144 degrees of freedom in this model compared to our previous example where we had 625 observations and 249 parameters to estimate.

<sup>18</sup> Yuri Dikhanov at the World Bank carried out the computations for the global linking.

Once the 155 BH price parities for each of the K countries in a region have been constructed, aggregate measures of country prices and relative volumes can be constructed using a wide variety of multilateral comparison methods that have been suggested over the years. These aggregate comparisons assume that in addition to BH price parities for each country, national statisticians have provided country expenditures (in their home currencies) for each of the 155 BH categories for the reference year 2005. Then the 155 by K matrices of Basic Heading price parities and country expenditures are used to form average price levels across all commodities and relative volume shares for each country.

There are a large number of methods that can be used to construct these aggregate Purchasing Power Parities and relative country volumes and Hill (2007b) surveys the main methods that have been used in previous rounds of the ICP and other methods that might be used.<sup>19</sup> Basically, only two multilateral methods have been used in previous rounds:

- The Gini-EKS (GEKS) method based on Fisher (1922) bilateral indexes and
- The Geary (1958) Khamis (1972) (GK) method, which is an additive method.

In the present ICP round, aggregate PPPs and relative country volumes for countries within each region were constructed for five of the six regions using the Gini-EKS method. However, the African region wanted to use an additive method and so this region used a relatively new additive method, the Iklé-Dikhanov-Balk (IDB) method, for constructing PPPs and relative volumes within the region.<sup>20</sup> We will not discuss these methods in detail since our focus is on new methodological developments for linking the regions. However, it may be appropriate to comment briefly on the relative merits of the GEKS, GK and IDB methods. The GK and IDB methods are additive methods; i.e., the real output of each country can be expressed as a sum of the country's individual outputs but each output is weighted by an international price which is constant across countries. This feature of an additive method is tremendously convenient for users and so for many purposes, it is useful to have available a set of additive international comparisons. In order to distinguish between the additive methods listed above, it is necessary to list the equations which define these methods. If we are making a comparison between K countries with N basic headings, then both of these methods involve K country price levels or PPPs,  $P^1, \dots, P^K$ , and N international commodity reference prices,  $\pi_1, \dots, \pi_N$ . The GK equations which determine these unknowns (up to a scalar multiple) are the following ones:

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<sup>19</sup> For additional methods, see Balk (1996), R.J. Hill (1997) (1999a) (1999b) (2001) (2004) and Diewert (1999).

<sup>20</sup> Iklé (1972; 203) proposed the equations for the method in a rather awkward manner and provided a proof for the existence of a solution for the case of two countries. Dikhanov (1994; 6-9) used the much more transparent equations (16) and (18), explained the advantages of the method over the GK method and illustrated the method with an extensive set of computations. Balk (1996; 207-208) used the Dikhanov equations and provided a proof of the existence of a solution to the system for an arbitrary number of countries. Van Ijzeren (1983; 42) also used Iklé's equations and provided an existence proof for the case of two countries.

$$(15) \pi_n = \sum_{k=1}^K [y_n^k / \sum_{j=1}^K y_n^j] [p_n^k / P^k]; \quad n = 1, \dots, N;$$

$$(16) P^k = \sum_{n=1}^N p_n^k y_n^k / \sum_{n=1}^N \pi_n y_n^k; \quad k = 1, \dots, K$$

where  $p_n^k$  is the  $n$ th basic heading PPP in country  $k$  and  $y_n^k$  is the corresponding volume for expenditure category  $n$  in country  $k$ . The  $\pi_n$  and  $P^k$  are only determined up to a scalar multiple and we require an additional normalization to completely determine the GK international commodity prices  $\pi_n$  and PPPs  $P^k$  such as:

$$(17) P^1 = 1.$$

For the IDB method, equations (15) are replaced by equations (18) while equations (16) continue to hold along with a normalization like (17):

$$(18) \pi_n = [\sum_{k=1}^K s_n^k [p_n^k / P^k]^{-1} / \sum_{j=1}^K s_n^j]^{-1} \quad n = 1, \dots, N$$

where  $s_n^k$  is the share of commodity group  $n$  in the GDP of country  $k$ . Thus for both of these methods, the  $n$ th international price  $\pi_n$  is a weighted average of the prices of commodity  $n$  in country  $k$ ,  $p_n^k$ , deflated by country  $k$ 's PPP,  $P^k$ , but for the GK method, we take an arithmetic share weighted average of these deflated prices whereas for the IDB method, we take a harmonic share weighted average. The weights for the two methods also differ. The GK method weights the  $n$ th deflated price of commodity group  $n$  in country  $k$ ,  $p_n^k / P^k$ , by country  $k$ 's share of commodity group  $n$  in the entire region,  $y_n^k / \sum_{j=1}^K y_n^j$ , so that large countries in the region will get a large weight in this average whereas the IDB method weights  $p_n^k / P^k$ , essentially by country  $k$ 's expenditure share of its own GDP on commodity group  $n$ ,  $s_n^k$  (these shares are then normalized to add up to one). Thus large countries do not get a very large weight in the formation of the international prices using the IDB method in contrast to the GK method, where large countries can have a very large influence. The GK method has been criticized in the literature for having large Gerschenkron effects: countries whose relative prices are rather far from the international prices will tend to have real output levels which are biased upwards using an additive method.<sup>21</sup> The IDB method will be less subject to Gerschenkron bias and hence I would recommend its use over the GK method but since both methods are additive, they will both be subject to bias.<sup>22</sup> Thus if an additive method must be used, then use the IDB method but if additivity is not a strict requirement, then use GEKS, which will give more accurate PPPs and relative volumes from the viewpoint of the economic approach to index number theory.<sup>23</sup>

<sup>21</sup> The Gerschenkron effect is due to Gini (1931; 14). For additional discussion on the relative merits of the GK and IDB indexes, see Balk (1996) (who took an axiomatic approach) and Cuthbert (2000) (who took a more statistical approach).

<sup>22</sup> See the explanation and diagram in Diewert (1999; 48-50) who explained why an additive method cannot work well (from the viewpoint of the economic approach to index number theory) if (i) there are more than two countries in the comparison; (ii) relative prices differ considerably across countries and (iii) final demanders substitute cheaper commodities for more expensive ones.

<sup>23</sup> See the discussion in Diewert (1999; 48-50) on this point.

We now turn our attention to the problem of linking the regions at higher levels of aggregation.

## 5. Aggregate Price and Volume Comparisons Across Regions

There are 146 countries in the ICP project and 155 basic headings. At this stage of the aggregation procedure, we assume that we have two 155 by 146 matrices of data: one matrix contains the PPPs for basic heading category  $n$  and country  $k$ ,  $p_n^k$ , and the other contains country expenditures in each country's currency,  $e_n^k$ , so that the notation is basically the same as in the previous section but now  $k$  runs over all 146 countries instead of just the countries in a given region. At this stage, we could use any suitable multilateral method to aggregate up these data into a set of 146 country PPP's and volumes, such as the EKS or IDB methods explained in the previous section. Call this *Approach 1*. However, the problem with this approach is that the multilateral method to be used would not necessarily respect the regional PPP's unless it was restricted in some manner.

Thus we consider *Approach 2*, which will link the regions, while respecting the within region overall PPP's that the regions deem best for their purposes.<sup>24</sup> The first step is to reorganize the countries into 5 regions (we regard the OECD/Eurostat/CIS countries as forming one region). Consider region  $r$  which has  $C(r)$  countries in it. Let  $p_n^{rc}$  denote the within region PPP for basic heading class  $n$  and country  $c$  in region  $r$ <sup>25</sup> and let  $e_n^{rc}$  denote the corresponding expenditure in local currency. The total regional expenditure on commodity group  $n$  in currency units of country 1 in each region,  $E_n^r$ , is defined as follows:

$$(19) E_n^r \equiv p_n^{r1} \sum_{c=1}^{C(r)} e_n^{rc}/p_n^{rc}; \quad r = 1, \dots, 5; n = 1, \dots, 155.$$

The corresponding regional PPPs by region and commodity,  $P_n^r$ , are defined to be the world BH parities for the numeraire country in each region:

$$(20) P_n^r \equiv p_n^{r1}; \quad r = 1, \dots, 5; n = 1, \dots, 155.$$

Now each region can be treated as if it were a single supercountry with supercountry expenditures  $E_n^r$  and basic heading PPPs  $P_n^r$  defined by (19) and (20) respectively for the 5 supercountries and any of the linking methods described in the previous section can be used to link the regions. Once the interregional price and volumes have been determined, the regional price and volume aggregates can be used to provide world wide price and volume comparisons for each individual country. This method necessarily preserves all regional relative parities. Moreover, Hill (2007e) shows that the overall procedure does

<sup>24</sup> This Approach was proposed by Diewert (2004b; 45-47). It is further described in much more detail by Hill (2007e).

<sup>25</sup> The parities  $p_n^{rc}$  are the interregionally consistent PPP's that were linked across regions as described in section 3 above; i.e., the  $p_n^{rc}$  are the estimated parameters  $a_r b_{rc} c_n$  which occur on the right hand side of equations (7). Assuming that country 1 is the numeraire country in each region, then the  $p_n^{r1}$  are the parities that link the numeraire countries in each region.

not depend on the choice of numeraire countries, either within regions or between regions; i.e., the relative country parities will be the same no matter what the choices are for the numeraire countries.

Approach 2 in conjunction with the EKS method was used to link the regions in the current ICP round; i.e., the EKS method was used to link the 5 supercountry regions.

Hill (2007e) discusses other possible methods that could be used to link the regions and these various alternative methods should part of the research agenda for the next round of comparisons. In particular, at higher levels of aggregation, we need to use the results of the present round to evaluate whether regional fixity is a good idea or not. The problem with regional fixity is that countries are not homogeneous within each region. In principle, it makes sense to compare countries whose (relative) price structures are similar and whose (absolute) quantity structures are similar: index number comparisons of price and volumes will work best under these conditions. Thus roughly speaking, it makes sense to compare directly countries who are at the same stage of development and build up a complete set of multilateral comparisons by linking (bilaterally) countries who are most structurally similar. R.J. Hill (1997) (1999a) (1999b) (2001) (2004) has developed methodology along these lines and it should be tested out using the detailed data generated by the present round.<sup>26</sup> It may well be that the fixity methodology developed in this round is not the most appropriate methodology for subsequent rounds.

## 6. Problem Areas and the Future Research Agenda

There are a number of problem areas associated with making international comparisons that require additional research and discussion before the next round of the ICP takes place:

- If a country experiences hyperinflation during the reference year, the average price concept may not be meaningful. A possible solution to this problem is to use within the year inflation rates to “discount” prices collected throughout the year to a single reference week or day.<sup>27</sup>
- The problem of pricing exports and imports.<sup>28</sup> At present, exchange rates are taken as the price of exports and imports. This is a reasonable approximation in some cases but the question is can we do anything better (that is not too costly)?

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<sup>26</sup> Another interesting issue is this: the present fixity imposed procedure is essentially a two stage GEKS procedure. At the first stage, countries are compared using GEKS within each region and then at the second stage, the five regions are linked together using another round of GEKS. Question: how does this two stage procedure compare to a single stage GEKS procedure using all 146 countries? The answer will probably be: they generate rather different parities. What then? What is the “truth”? We need criteria to determine “truth”. We could look at the axiomatic properties of two stage methods as compared to single stage methods but I am not sure that this would resolve the issues. At this point, I would fall back on the spatial linking methodology: it makes sense to build up a path of linked comparisons where we link together the countries which are most similar in structure.

<sup>27</sup> See Hill (1996) for a discussion of the accounting problems when there is high inflation.

<sup>28</sup> See Heston and Summers (2008; 4) for a discussion of this problem. A first approach to the problem would be to coordinate the calculation of national unit value export and import indexes across countries. This is a separate exercise that should be started well before the next ICP round. O’Connor (2008)

- The problem of negative expenditure categories. This problem arises with the net export category and the net additions to inventory category. Typically, there is not a problem provided that we do not attempt to provide PPPs for a single category that could be positive or negative across countries.<sup>29</sup> If it is necessary to provide PPPs across countries for such a category, the problems can be avoided by providing separate PPPs for exports and imports or for starting and finishing inventory stocks and users can difference the results.
- Inaccurate expenditure weights can cause grave difficulties. In the next ICP round, it would be very desirable to have more accurate information on expenditures by basic heading available from participating countries.
- Methodological difficulties with hard to measure areas of the accounts. There are particular problems with the treatment of housing<sup>30</sup>, financial services and nonmarket production.<sup>31</sup> These are problem areas for regular country accounts as well due to the lack of consensus on an appropriate methodology. Hopefully, international groups and academic economists interested in measurement problems will undertake additional research in these areas before the next ICP round.
- There is a very basic problem that makes international comparisons of prices and volumes very difficult and that is the lack of matching of products. The same problem occurs in the time series context due to the introduction of new products and the disappearance of “old” products but the lack of matching is much worse in the international context due to differences in tastes and big differences in the levels of development across countries, leading to very different consumption patterns. However, Structured Product Descriptions were introduced in the current ICP round and this does open up the possibility for undertaking hedonic regression exercises in the next round in order to improve the matching process. There are many problems to be addressed however, and it would be wise to undertake experimental hedonic studies well in advance of the next round.
- The fact that the ring list of commodities was somewhat different from the regional lists means that there is the possibility of anomalies in the final results; i.e., if different products are priced in the ring list, we cannot be sure the relative ring price levels really match up with the relative prices within the regions. The ring list of commodities was not determined completely independently from the country lists and this is all to the good.<sup>32</sup> But in the next round, this integration of

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mentions that the problems associated with calculating export and import price indexes is getting worse over time due to increasing trade in multinational intermediate goods and the transfer price problem.

<sup>29</sup> Index number theory tends to break down if a value aggregate crosses zero or is equal to zero!

<sup>30</sup> One area that we have not addressed is the impact of different procedures in different regions. For example, Asia and Africa used different methods for making productivity adjustments for government outputs index and they also used a different method for measuring housing output as compared to the CIS and South American regions. These problems need to be addressed well in advance of the next ICP round.

<sup>31</sup> See Heston and Summers (2008), Giovannini (2008) and Bevacqua, Fantin, Quintslr and Ruiz (2008) for a discussion of these problems. The fact that current System of National Accounts conventions do not allow an imputed interest charge for capital that is used in the nonmarket sector tends to understate the contribution of this sector and the degree of understatement will not be constant across rich and poor countries.

<sup>32</sup> Yonas Biru who was responsible for organizing the ring list describes how this was done as follows: “The Global ring list was developed in close consultation with regional and country experts in an iterative

- the ring product list with the regional product lists should be intensified with a best case scenario where the ring list becomes unnecessary.<sup>33</sup>
- It would be advisable to undertake some studies on alternative methods of aggregation at the higher levels of aggregation.<sup>34</sup> In particular, the program of making comparisons based on the degree of similarity of the price and quantity data being compared that was initiated by Robert Hill (1999a) (1999b) (2001) (2004) seems to be sensible but users have not embraced it, perhaps due to the instability of the method. In any case, the World Bank now has a considerable data set based on the current ICP round that could be used to experiment with alternative methods of aggregation.
  - Looking ahead into the more distant future, it would be desirable to integrate the ICP with the EU KLEMS project<sup>35</sup>, which is assembling data on the producer side of the economy as opposed to the final demand side, which is the focus of the ICP. Producer data are required in order to calculate relative productivity levels across economies, a topic of great interest to policy makers. Thus in addition to comparing components of final demand across countries, it would be desirable to compare outputs and inputs by industry across countries so that international comparisons of sectoral productivity levels could be undertaken.<sup>36</sup>

## 7. Conclusion

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processes. First, a consolidated global draft list was prepared that contained over 6,500 products from the five ICP regions and Eurostat-OECD comparison. Second, the list was then pruned by the Global Office to about 1,500 products, based on the country responses. The next step involved harmonizing product descriptions that originated from different regions and the list was sent back to the regions and a second round regional meetings were organized. A revised list was then created taking the second round comment from ring countries. The Global Office analyzed these second round country responses basic heading by basic heading to determine which products should be retained and which should be dropped. Key criteria for determining the final list included the number of regions and also the number of countries within each region where the product could be priced. A workshop was organized in Washington for regional coordinators and representatives of ring countries (one from each region) to go through the list and build consensus on a global list. The workshop modified some products, dropped some and came up with the final list containing about 1200 products. In doing so the Global Office made sure that at least one product was represented from each region for each basic heading.” Other World Bank researchers who were involved with the ring project were Yuri Dikhanov, Ramgopal Erabelly, Nada Hamadeh, Farah Hussain, Jinsook Lee and Amy Lee.

<sup>33</sup> Yonas Biru and Virginia Romand are currently undertaking a study to improve the SPD process. Yonas Biru describes the study as follows: “We are developing coding structures not only for products but also for product characteristics for the next generation SPDs. This would facilitate matching products across regions. This means we will be able to determine ring countries and ring products after data collection based on maximum overlap of products. Both the number and the mix of countries will be determined basic heading by basic heading. Potentially, linking can be done based on different criteria, including by ICP regions, consumption and price similarity indices, etc. The method would also facilitate hedonic type regression, particularly for equipment goods.”

<sup>34</sup> Alan Heston is currently undertaking such a study.

<sup>35</sup> See van Ark, Maddison and Timmer (2008) on this topic.

<sup>36</sup> See O’Connor (2008) for a similar long run proposal for the direction of the ICP. O’Connor also advocates making wealth comparisons across countries, which is feasible once we generate measures of capital input.

My overall conclusion is that the 2005 ICP round was a big success. The regions liked the idea that they could define their own list of products for international pricing and this improved the quality of the data. The new methodology to link prices across the regions using ring countries also seems to be a clear improvement over previous rounds. Finally, the use of hand held computers and the structured product description methodology led to improvements in the production of national price statistics in many cases.<sup>37</sup>

One issue that has not been entirely satisfactorily resolved is the issue of disclosure of the data; i.e., a great deal of effort has gone in to collecting PPPs for 155 categories for 146 countries but only data on 15 highly aggregated PPPs will be released. Why the reluctance to release the data? Probably because at lower level of aggregation, the results can be quite unreliable. Still one would think that more than 15 categories could be released.<sup>38</sup>

As indicated in the previous section, some challenges remain but hopefully, these problems will be addressed before the next round takes place.

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<sup>37</sup> See Trewin (2008) and Fenwick and Whitestone (2008) on the externalities created by the ICP program.

<sup>38</sup> Heston and Summers (2008; 5) and O'Connor (2008) discuss this issue.

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