On the Estimation of Returns to Scale, Technical Progress and Monopolistic Markups

by

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Abstract

This paper derives a number of theoretical results in the context of estimating returns to scale, technical progress and monopolistic markups when there are multiple outputs and/or multiple inputs. The choice between value added versus gross output in the estimation of returns to scale is also addressed, including consideration of problems that arise in aggregation across sectors of an economy. As an illustration, we use US data on manufacturing at the aggregate, sector and industry levels, and find evidence of strong increasing returns to scale across all levels of aggregation. Technical progress is typically found to be insignificant implying that, contrary to many previous results, US economic growth has been driven by increasing returns to scale rather than technical progress. Such findings have important implications for the macroeconomic modeling of economic fluctuations.

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1. Introduction

Under the simplifying assumptions of constant returns to scale and perfect competition, it is relatively easy to calculate estimates of technological change using econometric and index-number techniques. However, relaxing the simplifying assumptions may help us explain the observed procyclical nature of productivity growth, as has been suggested by Hall (1988)(1990) and Basu and Fernald (1997). In addition, evidence on returns to scale is important for assessing the relevance of theoretical macroeconomic and microeconomic models which assume increasing returns to scale; e.g., Romer (1986), Weil (1989), Baxter and King (1992), Beaudry and Devereux (1995), Benhabib and Farmer (1994).

This paper presents a number of theoretical results in the context of estimating returns to scale, technical progress and monopolistic markups when there are multiple outputs and/or multiple inputs. Our techniques build on the pioneering work of Nakajima, Nakamura and Yoshika (1998) and Nakajima, Nakamura and Nakamura (2002). The results are of particular practical use when there are a large number of outputs and/or inputs, implying a typical lack of degrees of freedom using standard econometric methods and aggregate annual data.

In deriving our results, a contribution is made to fundamental index-number theory. Specifically, it is shown that an index of total-factor-productivity growth can be derived from the economic approach to index numbers, without assuming perfect competition. Until now it has been thought that the assumption of perfect competition underlies the use of index-number techniques, leading to the qualification of empirical results in various applications. Hence, this result is of considerable relevance in considering the applicability of index-number techniques, and their interpretation, in many contexts.

The paper also addresses the issue of the choice between value added versus gross output in the estimation of returns to scale, including consideration of problems that arise in aggregation across sectors of an economy. This builds on the work of Basu and Fernald

(1997), who used a single-output production function framework. In our more general framework, we prove two propositions. The first is that a value-added framework for measuring the degree of returns to scale will lead to larger estimates than a gross-output framework. The second is that a value-added framework will tend to lead to larger estimates of monopolistic markups than a gross-output framework.

To illustrate the usefulness of the derived methods, we estimate a simple equation which allows us to obtain estimates of returns to scale and technical progress, starting with only an aggregate index of output and an aggregate index of input. Thus, the model that we estimate is no more complicated than a standard Cobb-Douglas production-function regression. However, our model requires the estimation of even fewer parameters. We use US data on manufacturing at the aggregate, sectoral and industry levels, and find evidence of strong increasing returns to scale across all sectors. Technical progress is typically found to be insignificant implying that, contrary to many previous results, US economic growth has been driven by increasing returns to scale rather than technical progress. Such findings have important implications for the macroeconomic modeling of economic fluctuations.

The rest of the paper is organized as follows. Section 2 introduces the translog joint cost function and derives expressions for the key theoretical concepts that are used in the following sections. Section 3 considers the simplifying case when there are constant monopolistic markups across outputs within each period. Two methods for empirically estimating the parameters of interest are introduced. Section 4 makes the alternative simplifying assumption of constant markups for each output across time periods. Again, two estimation procedures are introduced, with an assessment of their relative merits. Section 5 presents two propositions relating to the observations of Basu and Fernald (1997) on value added versus gross outputs estimates of returns to scale. Section 6 considers the related issue of aggregating returns to scale estimates over sectors. An empirical illustration using a simple estimating equation from Section 3 is presented in Section 7. The results are for U.S. manufacturing industries/sectors. Section 8 concludes.

2. Translog Cost Functions and the Firm's Profit Maximization Problem

We consider the case of a single firm or production unit that produces N outputs and uses M inputs for periods 0,1,...,T. Let $y \equiv [y_1,...,y_N]$ denote the vector of positive outputs that is produced by the positive vector of inputs, $x \equiv [x_1,...,x_M]$. Assume that in period t, the firm has a feasible set of inputs and outputs, S^t, and that it faces a positive vector of input prices, $w \equiv [w_1,...,w_M]$. The cost function is always defined as a dual characterization of the technology. The assumption that firms take input prices as given makes it possible to use this function in empirical work. Thus, assuming that the firm takes these input prices as fixed and beyond its control, the firm's *period t joint cost function*, C(w,y,t), conditional on a target set of outputs y that must be produced, is defined as follows:

$$(1)\ C(w,y,t) \equiv \min_x \{w \cdot x : (y,x) \text{ belongs to } S^t\}$$

where $w \cdot x \equiv \sum_{m=1}^{M} w_m x_m$ denotes the inner product between the vectors w and x. The joint cost function provides a characterization of the firm's technology.

A measure of the *local returns to scale* of a multiple output, multiple input firm can be defined as the percentage change in cost due to a one percent increase in all outputs. The technical definition is:[1]

$$(2)\ \rho(w,y,t) \equiv [C(w,y,t)]^{-1}\, dC(w,\lambda y,t)/d\lambda\, |_{\lambda=1}$$
$$= \sum_{n=1}^{N} C_n(w,y,t)y_n/C(w,y,t)$$
$$= \sum_{n=1}^{N} \partial \ln C(w,y,t)/\partial \ln y_n.$$

[1] This is the reciprocal of the usual returns to scale measure. Hence there are local decreasing costs (and increasing returns to scale) if $\rho(w,y,t) < 1$ and constant costs (and constant returns to scale) if $\rho(w,y,t) = 1$.

Thus our measure of (inverse) returns to scale is equal to the sum of the cost elasticities with respect to the N outputs.

We now assume that the logarithm of the firm's period t cost function is the following *non-constant returns to scale translog joint cost function*:[2]

$$(3)\ \ln C(w,y,t) \equiv -rt + \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln w_m + \sum_{n=1}^{N} \beta_n \ln y_n + (1/2)\sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln y_I \ln y_j$$
$$+ (1/2) \sum_{k=1}^{M} \sum_{m=1}^{M} \delta_{km} \ln w_k \ln w_m + \sum_{m=1}^{M} \sum_{n=1}^{N} \phi_{mn} \ln w_m \ln y_n$$

where the parameters on the right hand side of (3) satisfy the following restrictions:

(4) $\sum_{n=1}^{N} \beta_n = k > 0$;

(5) $\sum_{j=1}^{N} \gamma_{ij} = 0$ for i = 1,...,N;

(6) $\gamma_{ij} = \gamma_{ji}$ for all $1 \leq i < j \leq N$;

(7) $\sum_{m=1}^{M} \alpha_m = 1$;

(8) $\sum_{m=1}^{M} \delta_{km} = 0$ for k = 1,...,M;

(9) $\delta_{km} = \delta_{mk}$ for all $1 \leq k < m \leq M$;

(10) $\sum_{m=1}^{M} \phi_{mn} = 0$ for n = 1,...,N;

(11) $\sum_{n=1}^{N} \phi_{mn} = 0$ for m = 1,...,M.

The parameter r which occurs in (3) is a measure of technical progress, which in this case is expressed as exogenous cost reduction.[3] Usually, $r \geq 0$; if $r < 0$, then the technology

[2] The basic translog functional form was introduced by Christensen, Jorgenson and Lau (1971). This particular functional form was introduced by Diewert (1974; 139) as a joint revenue function, but the parameter k on the right hand side of (4) was set equal to 1 and the technical progress term, –rt was missing. The translog joint cost function was first introduced by Burgess (1974).

exhibits *technological regress*. We calculate the degree of local returns to scale, $\rho(w,y,t)$, using the $C(w,y,t)$ defined by (3)-(11) as follows:

$$(12)\ \rho(w,y,t) = \sum_{n=1}^{N} \partial \ln C(w,y,t)/\partial \ln y_n$$

$$= \sum_{n=1}^{N} \{\beta_n + \sum_{j=1}^{N} \gamma_{nj} \ln y_j + \sum_{m=1}^{M} \phi_{mn} \ln w_m\} \qquad \text{differentiating (3)}$$

$$= \sum_{n=1}^{N} \beta_n \qquad \text{using (5), (6) and (11)}$$

$$= k \qquad \text{using (4).}$$

Thus the translog cost function defined by (3)-(11) has returns to scale equal to the positive parameter k everywhere. In the case of a constant returns to scale technology, the parameter k on the right hand side of (4) is equal to 1.[4]

If there are increasing returns to scale or decreasing costs so that the parameter k is less than one, then it is well known that competitive profit maximization breaks down in this case. Hence, since we do not want to restrict k to be equal to or greater than one, it is necessary to allow for a monopolistic profit maximization problem in each period. Thus for period t, we assume that the firm faces the inverse demand function $P_n^t(y_n)$ which gives the market-clearing price for output n as a function of the amount of output y_n that the firm places on the market, for n = 1,…,N. Assuming that the firm faces the positive input price vector $w^t \equiv [w_1^t,\ldots,w_N^t]$, the *firm's period t monopolistic profit maximization problem* is the following unconstrained maximization problem involving the vector of period t output supplies $y \equiv [y_1,\ldots,y_N]$:

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[3] If we add the quadratic time trend term $-(1/2)st^2$ to the right hand side of (3) where s is a parameter, then our results in sections 3 and 4 are still valid with obvious modifications. In particular, $-r$ on the right hand side of (31), (35), (43) and (48) should be replaced by $- r - st$. This modification will allow the technical progress term to trend over time. With this more general specification, there will be technological regress in period t if $r + st < 0$.

[4] In this case, the period t production possibilities set S^t is a cone; i.e., if $(x,y) \in S^t$, then $(\lambda x,\lambda y) \in S^t$ for all $\lambda > 0$. In this constant returns to scale case, the translog cost function defined by (3)-(11) with $k = 1$ can provide a second order approximation to an arbitrary joint cost function that is consistent with a constant returns to scale technology. Thus our generalized functional form, where k is no longer restricted to equal 1, is the simplest possible extension of this constant-returns-to-scale type cost function to the case where returns to scale are equal to the arbitrary positive number k.

(13) $\max_y \{ \sum_{n=1}^{N} P_n^t(y_n)y_n - C(w^t,y,t)\}$.

Assuming that the inverse demand functions and the joint cost function are once differentiable, the observed period t output vector for the firm, y^t, should satisfy the following first-order necessary conditions for (13):

(14) $y_n^t \, dP_n^t(y_n^t)/dy_n + P_n^t(y_n^t) = \partial C(w^t,y^t,t)/\partial y_n$; $\qquad n = 1,\ldots,N$; $t = 0,1,\ldots,T$.

The observed period t price for output n will be:

(15) $p_n^t \equiv P_n^t(y_n^t)$; $\qquad n = 1,\ldots,N$; $t = 0,1,\ldots,T$.

Assuming that the demand derivatives $dP_n^t(y_n^t)/dy_n$ are nonpositive, the nonnegative *ad valorem monopolistic markup* m_n^t for the nth output in period t can be defined as follows:[5]

(16) $m_n^t \equiv -\,[dP_n^t(y_n^t)/dy_n]y_n^t/p_n^t \geq 0$; $\qquad n = 1,\ldots,N$; $t = 0,1,\ldots,T$.

Using definitions (15) and (16), the first-order conditions (14) can be rewritten as follows:

(17) $p_n^t \, [1 - m_n^t] = \partial C(w^t,y^t,t)/\partial y_n$; $\qquad n = 1,\ldots,N$; $t = 0,1,\ldots,T$.

In what follows, it will simplify the notation somewhat if we define one minus the markup for commodity n as the *markup factor* for output n in period t, M_n^t:[6]

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[5] In subsequent sections, we will assume that the markups m_n^t are constant. This is consistent with the inverse demand functions having the following constant elasticity form: $\ln P_n^t(y_n) \equiv a_n^t - c_n^t \ln y_n$ where a_n^t and c_n^t are positive constants. Basu and Fernald (2002; 976) note that constant markups can be justified from a variety of models. We are allowing for a wedge between price and marginal cost, so effectively we are considering any imperfect competition context.

(18) $0 < M_n{}^t \equiv 1 - m_n{}^t \leq 1$; $\qquad$ n = 1,…,N ; t = 0,1,…,T.

Using definitions (18), conditions (17) become:

(19) $p_n{}^t M_n{}^t = \partial C(w^t,y^t,t)/\partial y_n$; $\qquad$ n = 1,…,N ; t = 0,1,…,T.

Assuming differentiability of the period t cost function with respect to the input prices, using Shephard's (1953; 11) Lemma, the cost minimizing vector of input demands for the firm in period t, $x^t \equiv [x_1{}^t,\ldots,x_M{}^t]$, will be equal to the vector of first order partial derivatives of the cost function with respect to the components of the input price vector:

(20) $x^t \equiv \nabla_w C(w^t,y^t,t)$; $\qquad$ t = 0,1,…,T

and the period t observed total cost, $C(w^t,y^t,t)$, will be equal to the inner product of the period t input price and quantity vectors, w^t and x^t respectively:

(21) $C(w^t,y^t,t) = w^t \cdot x^t$; $\qquad$ t = 0,1,…,T.

We can rewrite (19) as follows:

(22) $p_n{}^t M_n{}^t = [C(w^t,y^t,t)/y_n{}^t]\, \partial \ln C(w^t,y^t,t)/\partial \ln y_n$; $\qquad$ n = 1,…,N ; t = 0,1,…,T

$= [w^t \cdot x^t/y_n{}^t]\, \partial \ln C(w^t,y^t,t)/\partial \ln y_n$ $\qquad$ using (21)

$= [w^t \cdot x^t/y_n{}^t][\beta_n + \sum_{j=1}^{N} \gamma_{nj} \ln y_j + \sum_{m=1}^{M} \phi_{mn} \ln w_m]$ $\qquad$ differentiating (3).

Rearranging (22) leads to the following equations, for n = 1,…,N and t = 0,1,…,T:

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[6] If there are constant or increasing costs so that the parameter $k \geq 1$, then this situation is consistent with the competitive pricing of outputs. To model this case in what follows, simply set each $M_n{}^t = 1$ and estimate the parameters k and r. In the production function literature on returns to scale and markups where there is only a single output, the markup factor is defined as price over marginal cost, which is the reciprocal of the markup factor $M_n{}^t$ which appears in (19); see Hall (1988) (1990) and Basu and Fernald (1997; 253) (2002; 975) for these single output production function approaches.

(23) $[p_n^t y_n^t M_n^t]/w^t \cdot x^t = \beta_n + \sum_{j=1}^{N} \gamma_{nj} \ln y_j + \sum_{m=1}^{M} \phi_{mn} \ln w_m$.

Now for t fixed, sum equations (23) over n in order to obtain the following equations:

(24) $\sum_{n=1}^{N} [p_n^t y_n^t M_n^t]/w^t \cdot x^t = \sum_{n=1}^{N} [\beta_n + \sum_{j=1}^{N} \gamma_{nj} \ln y_j + \sum_{m=1}^{M} \phi_{mn} \ln w_m]$; t = 0,1,…,T

$= k$ using (12).

Equations (24) can be rearranged to yield the following expressions for period t costs:

(25) $w^t \cdot x^t = k^{-1} \sum_{n=1}^{N} p_n^t y_n^t M_n^t$; t = 0,1,…,T.

Thus for each period t, an estimate of the firm's (reciprocal) returns to scale k can be obtained as the ratio of period t markup-adjusted revenues, $\sum_{n=1}^{N} p_n^t y_n^t M_n^t$, divided by period t total cost, $w^t \cdot x^t = \sum_{m=1}^{M} w_m^t x_m^t$.[7]

Rearranging the second equality in (22) leads to the following system of equations:

(26) $\partial \ln C(w^t,y^t,t)/\partial \ln y_n = p_n^t y_n^t M_n^t / w^t \cdot x^t$; n = 1,…,N ; t = 0,1,…,T

(27) $= k\, p_n^t y_n^t M_n^t / \sum_{j=1}^{N} p_j^t y_j^t M_j^t$ using (25).

Equations (25)-(27) will play key roles in the subsequent sections.

[7] If there is only one output so that N=1, then (25) can be rewritten as $k^{-1} = [M_1^t]^{-1} [w^t \cdot x^t / p_1^t y_1^t]$, which is a standard result in the one output production function literature on this topic: see Basu and Fernald (1997; 253) (2002; 976). The term $w^t \cdot x^t / p_1^t y_1^t$ is observed cost over observed revenue, which in turn is one minus the revenue share of pure profits.

3. Productivity Estimation Assuming Constant Markups within Each Period

It turns out that we cannot obtain simple estimating equations in a perfectly general situation where the markup factors M_n^t are allowed to be arbitrary for each output n and for each time period t. However, progress can be made if we assume that within each period all of the markup rates or markup factors are equal to each other, or if we assume that the markup factors are constant across periods for each output. In this section, we make the first assumption and in the following section, we shall make the second assumption.

Thus, we assume here that the markup factors within each period are constant across commodities so that

$$(28)\ M_n^t = M^t\ ; \qquad n = 1,\ldots,N\ ;\ t = 0,1,\ldots,T.$$

Substituting (28) into (27) leads to the following equations for the logarithmic derivatives of the period t cost function with respect to outputs:

$$(29)\ \partial \ln C(w^t,y^t,t)/\partial \ln y_n = k\, p_n^t y_n^t / \textstyle\sum_{j=1}^{N} p_j^t y_j^t \qquad n = 1,\ldots,N\ ;\ t = 0,1,\ldots,T$$
$$= k\, s_n^t$$

where $s_n^t \equiv p_n^t y_n^t / p^t \cdot y^t$ is the *observed revenue share* of output n in period t. Using (20) and (21), it can be seen that the logarithmic derivatives of the period t cost function with respect to input prices are equal to:

$$(30)\ \partial \ln C(w^t,y^t,t)/\partial \ln w_m = w_m^t x_m^t / w^t \cdot x^t \qquad m = 1,\ldots,M\ ;\ t = 0,1,\ldots,T$$
$$= S_m^t$$

where $S_m^t \equiv w_m^t x_m^t / w^t \cdot x^t$ is the *observed cost share* of input m in period t.

Since the right hand side of (3) is a quadratic function in the logarithms of output quantities, the logarithms of input prices and time, we can apply Diewert's (1976; 118) Quadratic Identity and obtain the following equations, relating the difference in the costs in periods t−1 and t, $w^{t-1} \cdot x^{t-1} = C(w^{t-1},y^{t-1},t-1)$ and $w^t \cdot x^t = C(w^t,y^t,t)$:

$$
\begin{aligned}
(31)\ & \ln C(w^t,y^t,t) - \ln C(w^{t-1},y^{t-1},t-1) \qquad\qquad t = 1,2,\ldots,T \\
&= (1/2)\{[\partial \ln C(w^{t-1},y^{t-1},t-1)/\partial t] + [\partial \ln C(w^t,y^t,t)/\partial t]\}[(t) - (t-1)] \\
&\quad +(1/2)\sum_{n=1}^{N} \{[\partial \ln C(w^{t-1},y^{t-1},t-1)/\partial \ln y_n] + [\partial \ln C(w^t,y^t,t)/\partial \ln y_n]\}[\ln y_n^t - \ln y_n^{t-1}] \\
&\quad +(1/2)\sum_{m=1}^{M} \{[\partial \ln C(w^{t-1},y^{t-1},t-1)/\partial \ln w_m] + [\partial \ln C(w^t,y^t,t)/\partial \ln w_m]\}[\ln w_m^t - \ln w_m^{t-1}] \\
&= (1/2)\{(-r) + (-r)\} \qquad \text{differentiating (3)} \\
&\quad + (1/2) \sum_{n=1}^{N} \{[ks_n^{t-1}] + [ks_n^t]\}[\ln y_n^t - \ln y_n^{t-1}] \qquad \text{using (29)} \\
&\quad + (1/2) \sum_{m=1}^{M} \{[S_m^{t-1}] + [S_m^t]\}[\ln w_m^t - \ln w_m^{t-1}] \qquad \text{using (30)} \\
&= -r + k \ln Q_T(p^{t-1},p^t,y^{t-1},y^t) + \ln P_T(w^{t-1},w^t,x^{t-1},x^t)
\end{aligned}
$$

where $Q_T(p^{t-1},p^t,y^{t-1},y^t)$ is the *Törnqvist* (1936) (1937) *quantity index* for output growth between periods t−1 and t and $P_T(w^{t-1},w^t,x^{t-1},x^t)$ is the *Törnqvist input price index* for input price growth between periods t−1 and t. The logarithms of these two indexes are defined as follows:

$$(32)\ \ln Q_T(p^{t-1},p^t,y^{t-1},y^t) \equiv (1/2) \sum_{n=1}^{N} [s_n^{t-1} + s_n^t][\ln y_n^t - \ln y_n^{t-1}]\ ;$$

$$(33)\ \ln P_T(w^{t-1},w^t,x^{t-1},x^t) \equiv (1/2) \sum_{m=1}^{M} [S_m^{t-1} + S_m^t][\ln w_m^t - \ln w_m^{t-1}].$$

The Törnqvist input price index between periods t−1 and t, $P_T(w^{t-1},w^t,x^{t-1},x^t)$, can be used in order to define the *implicit Törnqvist input quantity index* between periods t−1 and t as follows:

$$(34)\ Q_T^*(w^{t-1},w^t,x^{t-1},x^t) \equiv w^t \cdot x^t / w^{t-1} \cdot x^{t-1}\ P_T(w^{t-1},w^t,x^{t-1},x^t).$$

Note that the Törnqvist output quantity index and implicit input quantity indexes, $Q_T(p^{t-1},p^t,y^{t-1},y^t)$ and $Q_T^*(w^{t-1},w^t,x^{t-1},x^t)$ respectively, can be calculated using observable data on output and input prices and quantities for periods t−1 and t. Using definitions (21) and (33), we can derive the same sort of estimating equation that Nakajima, Nakamura and Yoshioka (1998) did by rewriting (31) as follows:

$$(35)\ \ln Q_T^*(w^{t-1},w^t,x^{t-1},x^t) = -r + k \ln Q_T(p^{t-1},p^t,y^{t-1},y^t)\ ; \qquad t = 1,2,\ldots,T.$$

If $T \geq 2$, then the technical change parameter r and the returns to scale parameter k can be estimated by running a linear regression using equations (35) after appending error terms. If there is positive technical progress, then $r > 0$ while if there are increasing returns to scale, then $k < 1$. Hence, a combination of technical progress and increasing returns to scale will cause input growth to be less than output growth. Equation (35) enables us to assess the contribution of each factor in a very simple regression model that has eliminated all of the nuisance parameters that are in the translog cost function that was defined earlier by (3). This is a rather remarkable result which is valid even if M and N are extremely large so that traditional econometric methods for estimating r and k fail.[8]

Equation (35) can be compared with two more familiar methods. A simple rearrangement yields a production-function-type model, with the log of an output aggregate on the left-hand side and the log of an input aggregate on the right-hand side. The difference here is

[8] Nakajima, Nakamura and Yoshioka (1998) derived this result starting from a revenue-function framework, rather than a cost-function framework, as was also done in Nakajima, Nakamura and Nakamura (2002), and in Diewert's (2002) alternative derivation of their result. In this revenue function set up, the indirect input quantity index in (35) is replaced by an indirect output index, and the direct output quantity

that (35) allows for the existence of multiple outputs, and the inputs are aggregated using a Törnqvist index, rather than the usual Cobb-Douglas index.[9] It is also easy to relate (35) to a simple index-number approach to estimating total-factor-productivity growth. Assuming constant returns to scale (k=1), taking exponents and re-arranging, (35) can be expressed as an output index divided by an input index, or total-factor-productivity growth. This is a very interesting observation, as we have derived this result without assuming perfect competition. Thus, the use of index-number techniques is consistent with the existence of monopolistic behaviour. While we have assumed that monopolistic markups are the same for each output within each period, this is not a restriction if there is only one (aggregate) output.

In addition, as the output and input indexes in (35) are in the form of one-plus-the-growth-rate between periods t-1 and t, the logarithm of each approximates the growth rate of output and input, respectively. Hence, (35) can be interpreted as (approximately) an equation relating the growth rate of input to the growth rate of output.

Once an estimate for the returns to scale parameter k has been obtained from the estimation of (35), an estimate for the period t markup factor M^t can be obtained using equations (25) and assumptions (28):

$$(36)\quad M^t = k(w^t \cdot x^t)/(p^t \cdot y^t)\ ; \qquad t = 0,1,\ldots,T.$$

We conclude this section by spelling out in more detail how the parameters r and k could be estimated along with all of the other parameters in the definition of the translog joint cost function defined by (3)-(11). In a traditional econometric specification, there would be 1 + (N−1) + (M−1) estimating equations. The first estimating equation is the definition of the cost function itself, for t = 0,1,…,T:

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index is replaced by a direct input quantity index. The coefficients have a correspondingly different interpretation, with the resulting equation being in the more familiar form of a production function.

[9] In a growth accounting approach, the shares in the Cobb-Douglas index are often set to an average of the shares. This is equivalent to using the Törnqvist index if the averaged shares are only those of the periods in each bilateral comparison. This approach is often described as a “Divisia” approach, but the Divisia

$$\text{(37)}\ \ln w^t\cdot x^t = -rt + \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln w_m^t + \sum_{n=1}^{N} \beta_n \ln y_n^t + (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \ln y_i^t \ln y_j^t$$

$$+ (1/2) \sum_{k=1}^{M}\sum_{m=1}^{M} \delta_{km} \ln w_k^t \ln w_m^t + \sum_{m=1}^{M}\sum_{n=1}^{N} \phi_{mn} \ln w_m^t \ln y_n^t .$$

The second set of estimating equations is based on the use of equations (30) and differentiating (3) with respect to input prices:[10]

$$\text{(38)}\ S_m^t = \alpha_m + \sum_{k=1}^{M} \delta_{km} \ln w_k^t + \sum_{n=1}^{N} \phi_{mn} \ln y_n^t ; \qquad m = 1,\ldots,M ; t = 0,1,\ldots,T.$$

The third set of estimating equations is based on the use of equations (29) and differentiating (3) with respect to output quantities:[11]

$$\text{(39)}\ s_n^t = k^{-1}[\beta_n + \sum_{j=1}^{N} \gamma_{nj} \ln y_j^t + \sum_{m=1}^{M} \phi_{mn} \ln w_m^t] ; \qquad n = 1,\ldots,N ; t = 0,1,\ldots,T.$$

It can be seen that a major problem with the system of estimating equations (37)-(39) is that the technical progress parameter r appears only in equations (37), which contains all of the parameters and hence is subject to multicollinearity problems.[12] In any case, it can be seen that if our focus is on obtaining estimates of the degree of returns to scale along with estimates of the rate of technical change, then it will generally be much more convenient to use equations (35) in place of the system of equations (37)-(39).

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index is a continuous-time index to which the Törnqvist index is only one of the many possible discrete approximations.

[10] Since, for each t, the sum over the m input cost shares, S_m^t, equals 1, one of the share equations must be dropped from the estimating equations.

[11] Since, for each t, the sum over the n output revenue shares, s_n^t, equals 1, one of the share equations must be dropped from the estimating equations. We have not used all of the restrictions (4)-(11) to eliminate some of the parameters in these estimating equations.

[12] In addition, if a seemingly unrelated regression model specification is used, some algorithms require that the number of degrees of freedom exceed the number of parameters in any single equation, leading to

4. Productivity Estimation Assuming Constant Markups Across Periods

In this section, we allow for the possibility of different markups for different outputs but we assume that these commodity-specific markups are constant across time. Thus we assume that the markup factors M_n^t satisfy the following restrictions:

$$(40)\ M_n^t = M_n\ ; \qquad n = 1,\ldots,N\ ;\ t = 0,1,\ldots,T.$$

Substituting (40) into (26) leads to the following equations for the logarithmic derivatives of the period t cost function with respect to outputs:

$$(41)\ \partial \ln C(w^t,y^t,t)/\partial \ln y_n = p_n^t y_n^t\, M_n/w^t\cdot x^t\ ; \qquad n = 1,\ldots,N\ ;\ t = 0,1,\ldots,T.$$

We can again apply the Quadratic Identity and obtain the following counterparts to equations (31):

$$\begin{aligned}
(42)\ & \ln C(w^t,y^t,t) - \ln C(w^{t-1},y^{t-1},t-1) \qquad t = 1,2,\ldots,T \\
&= (1/2)\{[\partial \ln C(w^{t-1},y^{t-1},t-1)/\partial t] + [\partial \ln C(w^t,y^t,t)/\partial t]\}[(t)-(t-1)] \\
&\quad +(1/2)\sum_{n=1}^{N} \{[\partial \ln C(w^{t-1},y^{t-1},t-1)/\partial \ln y_n] + [\partial \ln C(w^t,y^t,t)/\partial \ln y_n]\}[\ln y_n^t - \ln y_n^{t-1}] \\
&\quad +(1/2)\sum_{m=1}^{M} \{[\partial \ln C(w^{t-1},y^{t-1},t-1)/\partial \ln w_m] + [\partial \ln C(w^t,y^t,t)/\partial \ln w_m]\}[\ln w_m^t - \ln w_m^{t-1}] \\
&= (1/2)\{(-r) + (-r)\} \qquad \text{differentiating (3)} \\
&\quad + (1/2)\sum_{n=1}^{N} \{[p_n^{t-1}y_n^{t-1}/w^{t-1}\cdot x^{t-1}] + [p_n^t y_n^t /w^t\cdot x^t]\}[\ln y_n^t - \ln y_n^{t-1}]M_n \qquad \text{using (41)} \\
&\quad + (1/2)\sum_{m=1}^{M} \{[S_m^{t-1}] + [S_m^t]\}[\ln w_m^t - \ln w_m^{t-1}] \qquad \text{using (30)} \\
&= -r + (1/2)\sum_{n=1}^{N} \{[p_n^{t-1}y_n^{t-1}/w^{t-1}\cdot x^{t-1}] + [p_n^t y_n^t/w^t\cdot x^t]\}[\ln y_n^t - \ln y_n^{t-1}]M_n \\
&\quad + \ln P_T(w^{t-1},w^t,x^{t-1},x^t).
\end{aligned}$$

---

further difficulties. One way out of these difficulties may be to drop equations (37) as estimating

Using definitions (21) and (33), we can rewrite (42) as follows: for t = 1,2,…,T:

$$(43)\ \ln Q_T^*(w^{t-1},w^t,x^{t-1},x^t)$$
$$= -r + (1/2)\sum_{n=1}^{N} \{[p_n^{t-1}y_n^{t-1}/w^{t-1}\cdot x^{t-1}] + [p_n^t y_n^t/w^t\cdot x^t]\}[\ln y_n^t - \ln y_n^{t-1}]M_n.$$

Note that the right hand side of (43) is linear in the unknown technical progress parameter r and the N unknown markup parameters $M_1,\ldots,M_N$ and hence *linear regression techniques can be used in order to estimate these parameters* if $T \geq 1 + N$.

Unfortunately, the linear regression equation (43) is not as satisfactory as the linear regression equation (35) derived in the previous section. In (35), the dependent variables involved the input price and quantity vectors while the independent variable involved the output price and quantity vectors and thus there was no logical problem in conditioning on the independent variables and applying a linear regression. In the present model (43), the dependent variable for period t again involves the input price and quantity vectors for periods t−1 and t, w^{t-1},w^t,x^{t-1},x^t, but now the independent variables also depend on these cost variables; i.e., note the terms $w^{t-1}\cdot x^{t-1}$ and $w^t\cdot x^t$ in the right hand side independent variables in (43). However, instrumental variable techniques could be used in order to estimate the unknown parameters in (43). Once estimates for the markups have been obtained (call these estimates M_n^*), then equations (25) may be used in order to obtain an estimator for the returns to scale parameter k as follows:

$$(44)\ k^* \equiv \sum_{t=0}^{T} [1/(T+1)] \sum_{n=1}^{N} p_n^t y_n^t M_n^*/w^t\cdot x^t .$$

However, if it is known that the firm behaves competitively (i.e., takes the output price in each period as a fixed parameter beyond its control) in one or more output markets, then an entirely different estimation strategy can be used in order to obtain estimates for r, k and the remaining markup factors, M_n. Suppose that it is known that the firm behaved

equations, replacing them by the much simpler equations (35).

competitively with respect to output N.[13] Then we can set $M_N = 1$ and using assumptions (40), equations (25) become:

$$(45)\ w^t \cdot x^t k = \sum_{n=1}^{N-1} p_n^t y_n^t M_n + p_N^t y_N^t \qquad t = 0,1,\ldots,T.$$

Equations (45) can be rearranged in a more convenient form as follows:[14]

$$(46)\ p_N^t y_N^t / w^t \cdot x^t = k - \sum_{n=1}^{N-1} [p_n^t y_n^t / w^t \cdot x^t] M_n \qquad t = 0,1,\ldots,T.$$

Note that equations (46) are linear in the unknown parameters k, $M_1,\ldots,M_{N-1}$ and hence linear regression techniques can be used to obtain estimates. A stochastic specification for (46) is not so problematic as it was for equations (43): we need only assume that y_N^t is a random variable and perform the regression, conditioning on the other variables. Since y_N^t does not appear in the right-hand-side variables for (46), there are at least no logical difficulties with this assumption. Once estimates for the markup factors $M_1,\ldots,M_{N-1}$ have been obtained (call these estimates $M_1^*,\ldots,M_{N-1}^*$), then equations (43) can be rearranged as follows in order to obtain an estimate for the technological change parameter r:

$$(47)\ r^* \equiv \sum_{t=1}^{T} (1/T)[(1/2) \sum_{n=1}^{N-1} \{[p_n^{t-1} y_n^{t-1}/w^{t-1} \cdot x^{t-1}] + [p_n^t y_n^t / w^t \cdot x^t]\}[\ln y_n^t - \ln y_n^{t-1}] M_n^*$$
$$+ (1/2)\{[p_N^{t-1} y_N^{t-1}/w^{t-1} \cdot x^{t-1}] + [p_N^t y_N^t / w^t \cdot x^t]\}[\ln y_N^t - \ln y_N^{t-1}]$$
$$- \ln Q_T^*(w^{t-1}, w^t, x^{t-1}, x^t)].$$

---

[13] In the following 2 sections, we shall consider value added cost function models as opposed to the (gross output) joint cost function model of this section. In the value added cost function framework, intermediate inputs are moved out of the x variables and treated as (negative) y variables and it is natural to assume that the firm behaves competitively with respect to these new y variables so that the corresponding markup factors can be set equal to unity.

[14] When we add error terms to (46), these errors are more likely to be homoskedastic. Note also that (46) has a constant term on the right hand side.

As an alternative to this two stage estimation procedure, we could substitute $M_N = 1$ into equations (43), rearrange terms and obtain the following system of estimating equations:

$$(48)\ \ln Q_T^*(w^{t-1},w^t,x^{t-1},x^t) - (1/2)\{[p_N^{t-1}y_N^{t-1}/w^{t-1}\cdot x^{t-1}] + [p_N^t y_N^t/w^t\cdot x^t]\}[\ln y_N^t - \ln y_N^{t-1}]$$
$$= -r + (1/2) \sum_{n=1}^{N-1} \{[p_n^{t-1}y_n^{t-1}/w^{t-1}\cdot x^{t-1}] + [p_n^t y_n^t/w^t\cdot x^t]\}[\ln y_n^t - \ln y_n^{t-1}]M_n\ ;$$
$$t = 1,\ldots,T.$$

Note that the right hand side of each of the equations in (48) is linear in the unknown parameters r, $M_1,\ldots,M_{N-1}$. The T equations in (48) along with the T+1 equations in (46) could be used as a system of two simultaneous estimating equations in order to estimate the unknown parameters k, r, $M_1,\ldots,M_{N-1}$, provided that $2T+1 \geq N+1$. However, the stochastic specification for the simultaneous system (46) and (48) is not straightforward.[15]

It is straightforward to generalize the model defined by (46) and (48) to cover the case where the firm behaves competitively in more than one output market.

In both this section and the previous section, it can be seen that the use of index number techniques can dramatically simplify the econometric estimation of a firm's returns to scale parameter along with other parameters of major interest like monopolistic markups and technical progress. The use of the translog functional form and the quadratic identity led to estimating equations that eliminated a tremendous number of elasticity parameters, which conserved degrees of freedom. In many cases, our suggested procedures will make estimation of a firm's degree of returns to scale possible in situations where the number of inputs and outputs is large relative to the number of observations on the firm.

[15] This is somewhat unfortunate since different econometricians will make differing stochastic specifications, leading to differing estimates for the parameters and thus there may be reproducibility problems.

In the following 2 sections, we shall address a number of issues in our multiple output framework that were raised by Basu and Fernald (1997) (2002) in the context of their one output model.

5. Value Added versus Gross Output Estimates of Returns to Scale

In a very interesting paper, Basu and Fernald (1997; 261-262) compared estimates of returns to scale using a value added concept of output compared to those obtained using a gross output formulation. Basu and Fernald (1997; 255) also provided a theoretical proof that the value added estimate of returns to scale should be greater than the corresponding gross output estimate if there were increasing returns to scale in the gross output model and vice versa for decreasing returns to scale in the gross output model. To prove this magnification result, they used a Divisia index concept for outputs, inputs and real value added. In this section, we will essentially replicate their results using our multiple output joint cost function framework.[16]

In the previous sections, we have used a gross output model for the firm and have assumed competitive behavior with respect to inputs and allowed for the possibility of monopolistic markups on each output. Dropping the superscript t, the key equation relating the returns to scale parameter k, the markup factors M_n, output prices and quantities, p_n and y_n, and input prices and quantities, w_m and x_m, was equation (25), which we rewrite as follows:

$$(49)\ k = \sum_{n=1}^{N} p_n y_n M_n / \sum_{m=1}^{M} w_m x_m .$$

Suppose that the number of inputs M is equal to or greater than 2 and that the first K of these inputs are intermediate inputs (i.e., inputs that are produced by other production sectors in the region of interest), where $1 \leq K < M$. Then these intermediate inputs may

[16] We note that there are problems in translating the continuous time Divisia indexes into unique discrete time counterparts; i.e., there are many ways of approximating the Divisia indexes in discrete time. We also note that Basu and Fernald modeled only a single output firm using production function techniques.

be treated as negative outputs and hence, we could consider a new production model where the number of outputs (including intermediate inputs) was equal to N + K and where the inputs would only include the M − K primary inputs. We could assume a new translog joint cost function similar to that defined earlier by (3), but of course, the number of inputs and outputs would be changed. Assuming that the markups remained the same, the new estimate for the degree of (inverse) returns to scale for this value added joint cost function, k_v, would be as follows:[17]

(50) $k_v \equiv [\sum_{n=1}^{N} p_n y_n M_n - \sum_{k=1}^{K} w_k x_k] / \sum_{m=K+1}^{M} w_m x_m$.

Proposition 1: If $k < 1$ (so that there are decreasing costs or *increasing returns to scale* in the gross output model), then $k_v < k < 1$ (so that the degree of decreasing costs in the value added model is smaller or the degree of increasing returns to scale is *bigger*). If $k > 1$ (so that there are increasing costs or *decreasing returns to scale* in the gross output model), then $k_v > k > 1$ (so that the degree of increasing costs in the value added model is bigger or the degree of decreasing returns to scale is *smaller*). If $k = 1$ (so that there are *constant returns to scale* in the gross output model), then $k_v = k = 1$ (so that the value added model also exhibits *constant returns to scale*).

Proofs of the Propositions are in an Appendix.

Proposition 1 says that moving to a value added framework for measuring the degree of returns to scale will lead to *magnified estimates* for the degree of returns to scale, except for the case where the technology is subject to constant returns to scale. In other words, if there are increasing returns to scale using the gross output framework, then the degree of increasing returns to scale will be even greater in the value added framework.[18]

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[17] Of course, we assume that the markup factors that correspond to the intermediate inputs are all equal to 1.

[18] It should be noted that both the gross output joint cost function and the value added cost function can approximate an arbitrary constant returns to scale technology to the second order and so both approximations are equally good in our present nonconstant returns to scale framework. It should also be noted that the cost reduction parameter r that occurred in the original gross output framework will also be subject to a magnification effect when we move to the translog value added joint cost function; i.e., if the

Basu and Fernald (1997; 262) note that Hall's (1990) estimates of markups in two digit manufacturing using a value added framework seemed to be much larger than those obtained by Basu and Fernald (1997) using a gross output formulation. Again, we will use our joint cost function framework to see if we can cast some light on this matter.

Recall equation (50) above. For the following proposition, we will assume that all of the markup factors M_n are the same. Thus this common markup factor M satisfies the following equation:

$$(51)\ k_v \equiv [\sum_{n=1}^{N} p_n y_n M - \sum_{k=1}^{K} w_k x_k] / \sum_{m=K+1}^{M} w_m x_m .$$

Now suppose that we have an estimate for the value-added returns-to-scale parameter, k_v, but instead of applying the markup factor to just the outputs, we apply it to value added. Call this value added markup factor M_v. It is defined by the following equation:

$$(52)\ k_v \equiv [\sum_{n=1}^{N} p_n y_n - \sum_{k=1}^{K} w_k x_k] M_v / \sum_{m=K+1}^{M} w_m x_m .$$

The following proposition relates the true gross output markup factor M to the (erroneous) value added markup factor M_v.

Proposition 2: If the gross output markup factor M is less than 1 (so that the firm is behaving monopolistically), then the value added markup factor M_v is less than M, indicating a bigger markup. If M = 1, indicating that the production unit is behaving competitively, then M_v equals 1 also.

---

original r is positive, then the corresponding r* say in the value added cost function will generally be bigger and if r is negative, then r* will generally be more negative so that r* < r. For a discussion of these issues and numerical examples, see Schreyer (2001).

Proposition 2 shows that moving to a value added framework for markups will tend to lead to larger markups; i.e., again, a magnification effect is at work when one uses a value added framework.

Basu and Fernald (1997; 262) also noted that estimates of returns to scale using data that aggregated over sectors tended to be higher than a weighted average of the sectoral estimates of returns to scale. This may be a reflection of the problem noted in Proposition 1 above. That is, as we consolidate over sectors, gross output moves toward aggregate value added and hence estimates of the degree of increasing returns to scale will tend to rise. But there are other factors at work as well as we shall see in the following section.

6. Returns to Scale and Aggregation over Sectors

We consider a simple framework where there are only two sectors or production units and we want to aggregate over these two sectors using a value-added framework. We shall use a variant of equation (50) to describe the technology in each sector. Thus let the (inverse) returns to scale parameter in sector 1, k_1, be defined in a value-added framework as follows:

$$(53)\ k_1 \equiv [\sum_{n=1}^{N} p_n^1 y_n^1 M_n^1 - \sum_{n=1}^{N} p_n^2 z_n^1] / \sum_{m=1}^{M} w_m^1 x_m^1$$

where p_n^1 is the market price of output n produced by sector 1, y_n^1 is the total gross output of commodity n produced by sector 1, M_n^1 is the markup factor for commodity n in sector 1, p_n^2 is the market price of output n produced by sector 2 (this is the price sector 1 must pay for commodity n if it uses it as an intermediate input) and z_n^1 is the amount of commodity n used as an intermediate input by sector 1. The price w_m^1 is the price sector 1 must pay for a unit of primary input m and x_m^1 is the corresponding quantity utilized by sector 1. It will usually be the case that if sector 1 is producing finally demanded output n so that $y_n^1 > 0$, then it will not be demanding this commodity as an intermediate input so that $z_n^1 = 0$. Conversely, if sector 1 is demanding commodity n so that $z_n^1 > 0$, then it will not be producing this commodity as a final output so that $y_n^1 = 0$. In a similar fashion, let the (inverse) returns to scale parameter in sector 2, k_2, be defined as follows:

$$(54)\ k_2 \equiv [\sum_{n=1}^{N} p_n^2 y_n^2 M_n^2 - \sum_{n=1}^{N} p_n^1 z_n^2] / \sum_{m=1}^{M} w_m^2 x_m^2$$

where p_n^2 is the market price of output n produced by sector 1, y_n^2 is the total gross output of commodity n produced by sector 2, M_n^2 is the markup factor for commodity n in sector 2, and z_n^2 is the amount of commodity n used as an intermediate input by sector 1. The price w_m^2 is the price sector 2 must pay for a unit of primary input m and x_m^1 is the corresponding quantity utilized by sector 2.

An average (inverse) returns to scale parameter for the entire economy, k_a can be defined by just treating each output and input in each sector as a contributing output or input to the economy. This leads to the following definition:

$$(55)\ k_a \equiv [\sum_{n=1}^{N} p_n^1 y_n^1 M_n^1 - \sum_{n=1}^{N} p_n^2 z_n^1 + \sum_{n=1}^{N} p_n^2 y_n^2 M_n^2 - \sum_{n=1}^{N} p_n^1 z_n^2] / [\sum_{m=1}^{M} w_m^1 x_m^1 + \sum_{m=1}^{M} w_m^2 x_m^2]$$

$$= s_p^1 k_1 + s_p^2 k_2$$

where the sector i share of primary input is defined as:

$$(56)\ s_p^i \equiv [\sum_{m=1}^{M} w_m^i x_m^i] / [\sum_{m=1}^{M} w_m^1 x_m^1 + \sum_{m=1}^{M} w_m^2 x_m^2] ; \qquad i = 1,2.$$

Thus the economy wide (value added framework) returns to scale parameter, k_a, turns out to be a primary input share weighted average of the sectoral value added returns to scale parameters, k_1 and k_2.

However, equation (55) is not the only way that an economy wide average could be formed. In particular, using national accounts data, we may be tempted to form a consolidated average of the returns to scale parameters (call it k_c) by netting out intermediate input transactions. This leads to the following definition for an economy wide (consolidated) returns to scale estimate:

$$(57)\ k_c \equiv$$

$$[\sum_{n=1}^{N} p_n^1(y_n^1 - z_n^2)M_n^1 + \sum_{n=1}^{N} p_n^2(y_n^2 - z_n^1)M_n^2]/[\sum_{m=1}^{M} w_m^1 x_m^1 + \sum_{m=1}^{M} w_m^2 x_m^2]$$

$$= k_a + [p_n^1 z_n^2(1 - M_n^1) + p_n^2 z_n^1(1 - M_n^2)]/[\sum_{m=1}^{M} w_m^1 x_m^1 + \sum_{m=1}^{M} w_m^2 x_m^2]$$

$$\geq k_a$$

where the inequality follows assuming that $M_n^i \leq 1$ for each n and i, which are natural assumptions on the markup factors. Hence, in general, the consolidated economy estimate for the (inverse) returns to scale parameter k_c will be greater than the simple weighted average of the sectoral parameters if the correct markup factors are used.

The important point that is illustrated by (57) is that while it is satisfactory to work in a value added framework, when there are monopoly elements in the economy, it is not satisfactory to consolidate out intermediate input transactions, because there are little bits of deadweight loss that are lost in the consolidation process.[19] This poses problems for the usual national income accounting treatment of the sectoral consolidation process; i.e., monopolistic markup wedges on intermediate input transactions will be ignored in the usual accounting consolidation process. We note along with Basu and Fernald (2002)[20] that these ignored wedges on intermediate input transactions will generally lead to an aggregate output allocation that is not on the economy's production frontier; i.e., there

[19] This effect was noticed by Basu and Fernald (2002; 974-979).
[20] "Firm level value added is useful for national accounting, regardless of technology or market structure. But with imperfect competition, the construction of value added does not subtract off the full marginal product of intermediate inputs, since the marginal product of these goods exceeds their cost; there is a wedge between each firm's measured real value added and productive value added, R_M, which equals the sum of these wedges, represents real goods and services, and hence affects aggregate output and productivity." Susanto Basu and John G. Fernald (2002; 980-981).

will be productive inefficiency in this monopolistic economy that is analogous to the productive inefficiency due to taxes on intermediate inputs that was noted by Diamond and Mirrlees (1971) and Diewert (1983).

7. Productivity, Technical Progress and Returns to Scale in U.S. Manufacturing

We illustrate the usefulness of the proposed methods by using data from the U.S. Bureau of Labour Statistics (BLS), directly available from their web site (www.bls.gov/data/home.htm). The BLS provides input, output, and "multifactor" productivity data for the major manufacturing industries/sectors of the economy (Bureau of Labor Statistics, 2002a). The (gross) output and input quantity indices are constructed using the Törnqvist index formula, and the multifactor productivity results are calculated as the ratio of these indices.[21] The input quantity index is an aggregate of capital, labor, energy, materials and purchased business services inputs. Data for twenty-one industries/sectors, including the aggregate manufacturing sector, are available, 1949-2000. For simplicity, we will refer to all levels of aggregation as "sectors". Summary statistics for year-on-year productivity growth indexes are presented in Table 1, for each of the sectors, for 1950-2000 as we lose one observation in calculating growth rates. We label these results using the more familiar terminology of Total Factor Productivity (TFP) growth in order to distinguish these numbers from the accumulated growth numbers published by the BLS.

There are a number of additional points to note about this data. First, Leather and Leather Products (SIC 31) is missing. This is "because of the small size of the industry and data limitations" (Bureau of Labor Statistics, 2002b). Thus it is excluded from the BLS productivity data set. Second, the total factor productivity measures by industry "are not directly comparable to measures for aggregate manufacturing because industry measures

[21] Although we do not use the implicit Törnqvist quantity index for inputs, as in equation (35), the direct and implicit indexes should approximate each other quite closely (Allen and Diewert, 1978).

exclude transactions only within the specific industry while the aggregate manufacturing measures also exclude transactions between all manufacturing industries" (Bureau of Labor Statistics, 2002b). That is, a consolidation process of the type described in Section 6 is applied. Hence, if the markup factors are different from one then the consolidated economy estimate for the returns to scale will differ from the simple weighted average of sectoral parameters. However, our final observation on the data is that the BLS calculates the cost of capital to ensure that costs equal revenue so that the markup factors all equal one.

This last observation restricts the ability to use the available data to implement several of the methods discussed in this paper. Hence, we focus on estimation of equation (35) of Section 3, and illustrate its usefulness in generating estimates of returns to scale and technical progress very easily using only officially published data. However, we do consider using supplementary data from the Bureau of Economic Analysis (BEA) to enable us to get some estimates of markups in this framework. This is discussed in section 7.3 below.

As estimates of returns to scale and technical progress are intended as explanations for movements in total-factor productivity growth, we begin by observing the summary statistics in Table 1, and the plotted series in Figure 1. The figure plots TFP growth for each of the sectors in the sample. This reveals a considerably different pattern of productivity growth pre- and post-1974. Specifically, there appears to be more dispersion in the productivity estimates between the sectors post-1974. Table 1 reveals that the majority of sectors had lower average productivity growth in the period 1974-2000 compared with 1950-1973. The sectors which, notably, had higher productivity growth in the latter period were 'Industrial Machinery, Computer Equipment' (SIC 35) and 'Electric & Electrical Equipment' (SIC 36).

7.1 Estimation of Equation (35)

Equation (35) was estimated using ordinary least squares (OLS). Some authors have suggested using instrumental variables in estimating production-function models in similar contexts, due to potential simultaneity bias; e.g., Hall (1998)(1990) and Burnside (1996). The instruments which are typically suggested include the world price of oil, government defense spending, and a dummy variable for the political party of the president. However, we follow Basu and Fernald (1997) and Roeger (1995) in emphasizing OLS results. These authors note that the instruments may not be completely exogenous, and are relatively weakly correlated with the inputs for some industries.[22] In addition, it is unclear that the "standard" input instruments are appropriate instruments for the output index in equation (35) anymore than they are appropriate instruments for inputs. Hence, instrumental variable estimates may be more biased than OLS estimates. Burnside (1996) shows that results can vary markedly depending on the set of instruments used, and he argues that "alternative instrument sets should be sought which contain industry-specific components". Thus, while such issues may be pursued in future research, by using OLS we are focusing on presenting results which are easily reproducible by other researchers, with the interpretation (consistent with the theory of an input requirements function) that these are conditioning regressions.

Estimates for the (inverse) returns to scale parameter, k, are reported in Table 2, while estimates for the technical progress parameter are reported in Table 3. Table 2 reveals that most industries exhibit increasing returns to scale ($k<1$), and that the null hypothesis of constant returns can generally be rejected at the standard levels of significance. It is particularly of interest to note the size of the deviations from constant returns to scale. For example, the aggregate manufacturing sector the estimate of k is 0.657, and for 'Paper & Allied Paper Products' it is 0.485, with the null hypothesis of constant returns

[22] "Hall then goes on to estimate the extent of scale economies using a straightforward scale equation and his three instruments. The results are an embarrassment to the theory, with scale factors of 33 in food products and 138 in chemicals. The results are completely inconsistent with engineering production function, technological studies of scholars like Scherer, and common sense. The nonsensical findings are probably the result of using poor instruments." (Nordhaus, 1990, p. 151). "Another important innovation of this research program has been the introduction of some rather whacky variables as instruments.".... "In general, it is problematic to insert instrumental variables without a model to guide how to interpret what is found." (Baily, 1990, p. 147).

to scale easily rejected in each case.[23] Basu and Fernald (1997, Table 1, p. 259) similarly find statistically significant evidence of increasing returns to scale. For example, for aggregate manufacturing sector over the period 1959-1989, they report an estimate which equates to a k of 0.71 (=1/1.41).

In contrast to our returns to scale estimates, the estimates of technical progress in Table 3 are very modest, and most are insignificantly different from zero at standard levels of significance. Those that are significant are often negative, implying technical regress. The highest significant estimate is 2.2% technical progress for 'Textile Mills Products' For the aggregate Manufacturing sector, the null hypothesis of no technical progress cannot be rejected.

These results can be viewed as somewhat unexpected. However, the very simple approach of estimating equation (35) leaves not much room for ambiguities concerning the methodology. In what follows, we deviate from this simple model slightly, in order to see if the results are robust to other specifications.

We first consider relaxing the restrictive assumption of a linear time trend, and adopt the quadratic time trend discussed in footnote 3 of Section 2. The returns to scale estimates are reported in Table 4. These results are very similar to those found for the linear-time-trend case, both in terms of the magnitude of the estimates and their statistical significance. Hence, again there appears to be evidence of strongly increasing returns to scale in U.S. manufacturing industries. Also, the corresponding estimates of the technical progress parameters in Table 5 are consistent with those reported in Table 3 for the

[23] As there was little evidence of first- or second-order autocorrelation in the OLS regressions, we followed Burnside's (1996) suggestion to use industry-specific instruments by using two lags of the dependent and independent variable as instruments and estimating each equation by two-stage least squares. Three of the estimated returns-to-scale coefficients were greater than one ('Textile Mills Products', 'Apparel & Related Products', and 'Instruments'), but insignificantly different from one at standard significance levels, consistent with the OLS results in Table 2. Thirteen industries exhibited statistically significant increasing returns to scale, including aggregate Manufacturing.

linear-time-trend case; for the majority of industries, technical progress has been small or statistically indistinguishable from zero.[24]

From tables 2 to 5, it is interesting to observe some contrasting results. For ‘Textile Mill Products’ and ‘Apparel & Related Products’ the null hypothesis of constant returns to scale cannot be rejected, and the estimates of technical progress are positive and statistically significant. For ‘Industrial Machinery, Computer Equipment’ and ‘Electric & Electrical Equipment’ there are significant increasing returns to scale *and* technical progress parameters. This implies that the sources of TFP growth for the latter two sectors have come from both increasing returns to scale and positive technical progress, whereas the former two sectors have not been able to exploit scale economies and have relied upon positive technical progress.

As technical regress ($r<0$) was found for several sectors in the previously estimated models, and this may seem an unreasonable finding over the time period considered, we imposed positive technical progress on the linear-time-trend model by squaring the technical progress parameter and performing a non-linear regression. The results are reported in tables 6 and 7. Of course, the results for the industries that had positive technical progress in the earlier model are identical in this case.[25] For the industries which exhibited technical regress, the results in Table 7 show that the null hypothesis of zero technical change cannot be rejected. The results on returns to scale in Table 6 are reassuringly consistent with those of Table 2, implying that the observation of increasing returns to scale over most industries is robust to these alternative specifications of the estimated model.

[24] Piecewise-linear time trends were also investigated, which in the current set up simply requires the addition of a dummy variable to equation (35) for part of the sample. A dummy for 1974-2000 is statistically significant for some sectors, but the qualitative results do not change.

[25] Any (minor) differences are due to the different estimation procedures.

7.2 Reverse Regressions

Bartelsman (1995) criticized Hall (1988)(1990) for estimating the inverse of his derived production relationship in his empirical work, as "the estimate of the inverse of a parameter in a linear relationship is not equal to the inverse of the estimate of the parameter" (p. 60). Our equation (35) can be viewed as an inverse production relationship, but is not subject to Bartelsman's critique as it is consistent with our theoretical model, i.e. it is the actual relationship we have derived, rather than its inverse. An advantage of estimating this relationship is that variations in utilization of inputs do not affect the magnitude of coefficient estimates, only their standard errors as the omitted utilization terms will be captured by the error term.

However, we also estimated the reverse regression of equation (35). That is, we regressed the index of output on the index of input. While this is not consistent with our theoretical model, it is of interest for (at least) two reasons. The first is that we could have derived the model in this reverse form, and it could be argued that inputs are more exogenous than outputs.[26] The second is that we know from theory that if the dependent and independent variables are positively correlated, the coefficients from inverse regression will be larger (Bartelsman, 1995, p. 60). Thus, such regressions can act as a kind of sensitivity analysis, where we would have more confidence in our results if similar conclusions can be drawn from either the direct or reverse regressions.

The returns to scale and technical change parameters from the reverse regressions (with a linear time trend) are reported in tables 8 and 9, along with the corresponding estimates from the direct regressions of equation (35). From Table 8, we see that the returns to scale estimates, represented as 1/k, are now smaller (i.e., the estimates of k are larger). Testing the appropriate null hypothesis at the 5% level of significance, the direct regressions suggested that only four sectors exhibited constant returns to scale, while the reverse regressions suggest that seven sectors have constant returns. Two of these sectors are the same across the estimation approaches ('Apparel & Related Products', and 'Instruments'). The 'Food & Kindred Products' sector has gone from having significant

[26] This may depend on the industry, specifically the extent to which contracts determine output.

increasing returns to decreasing returns to having significant decreasing returns to scale. However, the R^2 is low for this sector. Significant decreasing returns to scale are also found for another two sectors ('Textile Mills Products', and 'Lumber & Wood Products') whereas the direct regressions suggested that they exhibited constant returns.

Although the estimates from the reverse regressions are lower, the findings are broadly consistent in the sense that the finding of increasing returns is common (eleven out of the twenty-one sectors). Perhaps of most interest from the macroeconomic point of view is that the aggregate Manufacturing sector exhibits increasing returns to scale under either specification.

From Table 9 we see that, consistent with the results from Table 3, technical progress estimates are again very modest, although there are now a few more positive numbers. The sector for which technical progress is highest is 'Food & Kindred Products' with a healthy 3.4%, but the regression has a low R^2 of .251. The aggregate Manufacturing sector estimate is 0.6%, consistent with the previous results of low overall technical progress.

7.3 Markups

As noted earlier, the BLS calculates the cost of capital to ensure that costs equal revenue. This ensures that markup factors all equal one. In this section, we use supplementary data for the cost of capital, so that we can find some estimates that do not equal one by definition. This data comes from the BEA web site (www.bea.doc.gov), Table 3.45 Current-Cost Depreciation of Private Structures by Industry. This is combined with cost data for other inputs from the BLS to give total cost.

Using either the estimates of returns to scale from the direct regression of equation (35) or the reverse regression, we can then use equation (36) to find estimates of the monopolistic markups for each sector. Summary statistics for these estimates are given in tables 10 and 11, respectively, for the case of a linear time trend. These are estimates of the inverse of the "markup factors" of equation (36) ($M^t=k(w^t \cdot x^t)/(p^t \cdot y^t)$), so that a value greater than one indicates a positive markup of output price over marginal cost.

The only difference between the results reported in tables 10 and 11 is through the way in which an estimate of k is found. Thus, the higher returns to scale estimates from the direct regression relative to the reverse regression (see Table 8) lead to higher markup estimates in Table 10 than in Table 11. It is worthwhile noting that the ratio of revenue to cost is greater than one on average for each sector, meaning that any estimates of markups which are less than one (as in Table 11) come from small estimates of returns to scale.

Using the results on statistical significance of the returns to scale estimates, and the reported regression diagnostics, the estimates in tables 10 and 11 indicate that there is evidence of positive markups in the aggregate manufacturing sector and several other key sectors. While some of the estimates in Table 10 are quite high, they are for sectors for which we have unreliable estimates of returns to scale; see Table 2. Also, we do not have any markups of the extremely large magnitude report by some other authors, such as Hall (1988). For example, Hall reports a markup of 20.112 for Chemicals and Allied Products whereas our estimate in Table 10 is 2.692. We would not put much faith in even this lower estimate due to the fit of the regression for this sector being so poor (R^2=0.476). Morrison (1992, Table 1, p. 387) also reports some large markups for the aggregate manufacturing sector, for example 1.286 for the U.S. in 1970 and 1.427 for Japan in 1981. The data used by Basu and Fernald (1997) were constructed such that revenue (almost) equals cost, so that their estimates of returns to scale can also be interpreted as estimates of markups. Again, they find evidence of large and statistically significant markups, particularly for the "Manfacturing Durables" and "Private Economy" sectors where estimates of up to 1.72 are found. (tables 1 and 2, p. 259).

The markups over time for the aggregate sectors of manufacturing, nondurable goods and durable goods are plotted in figures 2 and 3, using the results as summarized in tables 10 and 11 respectively, and reveal some interesting patterns. There appears to be a dip in the markups from the early 1970s until there is a noticeable pickup in the late 1980s. The large increase in the markup for the Electrical and Electrical Equipment is especially notable, and corresponds with evidence of high profitability in this sector for much of the 1990s; the dramatic fall in the markup in the late 1990s also corresponds with evidence of reduced profitability in this sector.

8. Conclusion

This paper has presented a number of theoretical results in the context of estimating returns to scale, technical progress and monopolistic markups when there are multiple outputs and/or multiple inputs. The results are of particular practical use when there are a large number of outputs and/or inputs, implying a typical lack of degrees of freedom using standard econometric methods and aggregate annual data.

Similar results could have been derived starting from a revenue function, rather than a cost function, framework, yielding more familiar production-function-type estimating equations (Nakajima, Nakamura and Yoshioka, 1998; and the follow-up papers of Nakajima, Nakamura and Nakamura, 2002; and Diewert, 2002). Therefore, there are several different ways in which the theoretical results of this paper can be re-expressed in order to allow simple estimation of parameters of interest. This observation is of use when data is only available in a form that suits estimation by one of the potential expressions.

Two propositions on the choice between value added versus gross output in the estimation of returns to scale were proved, showing that the use of value added lead to magnified estimates of returns to scale and monopolistic markups. Consideration was also given to problems that arise in averages of returns to scale across sectors of an economy.

To illustrate the usefulness of the derived methods, we estimated a simple equation which allows us to obtain estimates of returns to scale and technical progress, starting with only an aggregate index of output and an aggregate index of input. We used US data on manufacturing at the aggregate and sectoral levels, and found evidence of strong increasing returns to scale and positive monopolistic markups for most sectors. This estimation used data for 1949-2000 that is freely downloadable from the Bureau of Labor Statistics web site, and used ordinary least squares. Hence, by avoiding the usual problems of differences in data sets and choice of instrumental variables, the results are easily reproducible by other researchers and can be updated as more data becomes available. This use of officially published data contrasts with other studies which have relied on less recent data, constructed by the respective authors or by other researchers (e.g., Basu and Fernald, 1997; Burnside, 1996).

Technical progress is typically found to be insignificant implying that, contrary to previous results, US economic growth has been driven by increasing returns to scale rather than technical progress. Although somewhat rare, similar findings have appeared in the past (Denison, 1974; Berndt and Khaled, 1979; Hall, 1990; Morrison, 1992; Morrison and Siegel, 1997). It is important to note that the finding of increasing returns to scale at a sectoral level does not necessarily imply that there are increasing returns at the level of individual establishments (Nakajima, et al., 1998). However, for many purposes there is still interest in whether the expansion in size of a sector leads to increased output due to economies of scale, whether from positive externalities, increasing returns in the technologies used by individual establishments, or for some other reason.

The empirical findings of this paper have important implications for the macroeconomic modeling of economic fluctuations. In particular, they can cast light on the sources of U.S. manufacturing growth and provide an explanation for the procyclicality of productivity growth.

Appendix: Proofs of Propositions

Proof of Proposition 1:

Define a ≡ $\sum_{n=1}^{N}$ $p_n y_n M_n$; b ≡ $\sum_{m=1}^{M}$ $w_m x_m$ and c ≡ $\sum_{k=1}^{K}$ $w_k x_k$. We assume that a, b and c are all positive and that a − c > 0 and b − c > 0. Then the gross output degree of returns to scale is defined as:

(A1) $k \equiv a/b$

and the corresponding value added degree of returns to scale is defined as

(A2) $k_v \equiv [a - c]/[b - c]$.

Using the above definitions, it can be shown that

(A3) $k_v = k + c[a - b]/b[b - c]$.

All of the terms on the right hand side of (A3) are positive except for the term a − b. If $k = a/b = 1$, then a = b and $k_v = k$. If k < 1, then a < b and $k_v < k$. If k > 1, then a > b and $k_v > k$. Q.E.D.

Proof of Proposition 2:

Define a ≡ $\sum_{n=1}^{N}$ $p_n y_n$; b ≡ $\sum_{m=1}^{M}$ $w_m x_m$ and c ≡ $\sum_{k=1}^{K}$ $w_k x_k$. Then equations (51) and (52) can be rewritten as follows:

(A4) $k_v = [aM - c]/[b - c]$ and

(A5) $k_v = [a - c]M_v /[b - c]$.

Equating (A4) to (A5) and solving for M_v in terms of M leads to the following equation:

(A6) $M_v = [aM - c]/[a - c]$.

Hence if $M < 1$, then using (A6), $M_v < M$ and if $M = 1$, then $M_v = 1$ as well. Q.E.D.

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Table 1: Total Factor Productivity Growth, Törnqvist Index Approach

Sector	1950-00		1950-73		1974-00	
	Mean	s.d.	Mean	s.d.	Mean	s.d.
Manufacturing	1.012	0.020	1.015	0.019	1.010	0.021
Nondur. Goods (SIC 20-23, 26-31)	1.007	0.016	1.013	0.012	1.002	0.018
Food & Kindred Prod. (SIC 20)	1.005	0.022	1.007	0.016	1.003	0.026
Textile Mill Prod. (SIC 22)	1.023	0.020	1.023	0.017	1.023	0.022
Apparel & Related Prod. (SIC 23)	1.009	0.013	1.007	0.014	1.010	0.012
Paper & Allied Prod. (SIC 26)	1.007	0.034	1.016	0.037	0.999	0.028
Printing & Publishing (SIC 27)	0.999	0.017	1.005	0.017	0.993	0.015
Chem. & Allied Prod. (SIC 28)	1.011	0.039	1.026	0.033	0.998	0.040
Petroleum Refining (SIC 29)	1.004	0.011	1.008	0.009	1.000	0.012
Rubber & Plastic Prod. (SIC 30)	1.008	0.026	1.010	0.030	1.007	0.023
Durable Goods (SIC 24-25, 32-39)	1.016	0.025	1.015	0.027	1.016	0.024
Lumber & Wood Prod. (SIC 24)	1.012	0.032	1.017	0.034	1.007	0.030
Furniture & Fixtures (SIC 25)	1.007	0.019	1.006	0.023	1.007	0.015
Stone, Clay & Glass (SIC 32)	1.008	0.024	1.011	0.025	1.005	0.024
Primary Metal Ind. (SIC 33)	1.002	0.034	1.005	0.038	0.999	0.030
Fabricated Metal Prod. (SIC 34)	1.004	0.017	1.005	0.014	1.002	0.020
Ind. Machinery, Comp.Eq. (SIC 35)	1.019	0.032	1.007	0.027	1.030	0.034
Electric & Electr. Eq. (SIC 36)	1.031	0.034	1.022	0.026	1.040	0.037
Transportation Equip. (SIC 37)	1.008	0.033	1.016	0.040	1.002	0.025
Instruments (SIC 38)	1.014	0.025	1.018	0.031	1.010	0.017
Misc. Manufacturing (SIC 39)	1.010	0.032	1.016	0.013	1.005	0.041

Note: Arithmetic means. A mean value greater than one implies positive TFP growth, while a value less than one implies negative TFP growth. The mean value less one times a hundred gives the average percentage growth in TFP. "s.d." denotes standard deviation.

Table 2: Estimates of Returns to Scale, Equation (35), Linear Time Trend

Sector	$\hat{k}$	Ho: $k=1$ t-ratio	R^2	DW
Manufacturing	0.657	-7.802	0.820	1.514
Nondur. Goods	0.593	-5.632	0.579	1.825
Food & Kindred Prod.	0.580	-2.926	0.251	2.572
Textile Mill Prod.	0.972	-0.503	0.865	2.018
Apparel & Related Prod.	0.963	-0.799	0.899	1.817
Paper & Allied Prod.	0.485	-8.646	0.576	2.099
Printing & Publishing	0.617	-7.495	0.749	2.113
Chem. & Allied Prod.	0.460	-7.839	0.476	1.729
Petroleum Refining	0.744	-10.227	0.948	1.454
Rubber & Plastic Prod.	0.795	-4.621	0.868	1.863
Durable Goods	0.723	-8.333	0.906	1.546
Lumber & Wood Prod.	0.867	-1.763	0.731	1.830
Furniture & Fixtures	0.824	-5.292	0.926	2.480
Stone, Clay & Glass	0.712	-7.050	0.861	1.879
Primary Metal Ind.	0.763	-8.064	0.932	1.702
Fabricated Metal Prod.	0.860	-4.727	0.945	2.025
Ind. Machinery, Comp.Eq.	0.767	-5.756	0.880	1.298
Electric & Electr. Eq.	0.720	-7.532	0.884	1.821
Transportation Equip.	0.785	-6.663	0.924	1.648
Instruments	0.898	-1.546	0.791	2.428
Misc. Manufacturing	0.663	-5.247	0.685	2.194

Note: Ho: $k = 1$ is a test of constant returns to scale, and $k < 1$ implies increasing returns to scale. SIC codes are as in Table 1. DW denotes the Durbin-Watson statistic for first-order autocorrelation.

Table 3: Estimates of r, Equation (35), Linear Time Trend

Sector	$\hat{r}$	Ho: $r = 0$ t-ratio
Manufacturing	0.001	0.393
Nondur. Goods	-0.004	-1.362
Food & Kindred Prod.	-0.005	-1.093
Textile Mill Prod.	0.022	7.215
Apparel & Related Prod.	0.008	3.949
Paper & Allied Prod.	-0.010	-2.754
Printing & Publishing	-0.012	-5.540
Chem. & Allied Prod.	-0.011	-2.459
Petroleum Refining	-0.002	-1.902
Rubber & Plastic Prod.	-0.002	-0.522
Durable Goods	0.005	1.774
Lumber & Wood Prod.	0.008	1.794
Furniture & Fixtures	0.001	0.549
Stone, Clay & Glass	0.001	0.413
Primary Metal Ind.	-0.001	-0.412
Fabricated Metal Prod.	0.000	-0.070
Ind. Machinery, Comp.Eq.	0.007	1.636
Electric & Electr. Eq.	0.010	2.471
Transportation Equip.	0.000	0.116
Instruments	0.008	1.619
Misc. Manufacturing	0.001	0.346

Note: $r > 0$ represents positive technical change. Multiplying by 100 gives percentage growth rates. SIC codes are as in Table 1.

Table 4: Estimates of Returns to Scale, Equation (35), Quadratic Time Trend

Sector	$\hat{k}$	Ho: $k = 1$ t-ratio	R^2	DW
Manufacturing	0.655	-7.771	0.822	1.526
Nondur. Goods	0.609	-5.211	0.585	1.842
Food & Kindred Prod.	0.592	-2.786	0.255	2.588
Textile Mill Prod.	0.977	-0.411	0.865	2.019
Apparel & Related Prod.	0.962	-0.809	0.899	1.818
Paper & Allied Prod.	0.474	-8.436	0.580	2.140
Printing & Publishing	0.629	-6.932	0.752	2.150
Chem. & Allied Prod.	0.453	-7.434	0.476	1.734
Petroleum Refining	0.720	-10.643	0.953	1.598
Rubber & Plastic Prod.	0.791	-4.715	0.871	1.925
Durable Goods	0.722	-8.629	0.913	1.676
Lumber & Wood Prod.	0.877	-1.657	0.744	1.934
Furniture & Fixtures	0.824	-5.258	0.927	2.492
Stone, Clay & Glass	0.708	-6.970	0.862	1.890
Primary Metal Ind.	0.760	-8.124	0.934	1.740
Fabricated Metal Prod.	0.857	-4.736	0.946	2.042
Ind. Machinery, Comp.Eq.	0.789	-6.057	0.914	1.818
Electric & Electr. Eq.	0.746	-7.562	0.912	2.281
Transportation Equip.	0.788	-6.526	0.925	1.673
Instruments	0.916	-1.187	0.793	2.464
Misc. Manufacturing	0.665	-5.121	0.685	2.200

Note: Ho: $k = 1$ is a test of constant returns to scale, and $k < 1$ implies increasing returns to scale. SIC codes are as in Table 1. DW denotes the Durbin-Watson statistic for first-order autocorrelation.

Table 5: Estimates of r and s, Equation (35), Quadratic Time Trend

Sector	$\hat{r}$	Ho: $r = 0$ t-ratio	$\hat{s}$	Ho: $s = 0$ t-ratio	Ho: $r = s = 0$ p-value
Manufacturing	-0.001	-0.270	0.000	0.583	0.783
Nondur. Goods	0.000	-0.048	0.000	-0.839	0.289
Food & Kindred Prod.	-0.002	-0.215	0.000	-0.525	0.489
Textile Mill Prod.	0.024	3.859	0.000	-0.382	0.000
Apparel & Related Prod.	0.007	1.791	0.000	0.179	0.001
Paper & Allied Prod.	-0.014	-1.924	0.000	0.661	0.025
Printing & Publishing	-0.009	-2.169	0.000	-0.785	0.000
Chem. & Allied Prod.	-0.013	-1.433	0.000	0.259	0.059
Petroleum Refining	-0.007	-2.923	0.000	2.264	0.016
Rubber & Plastic Prod.	-0.009	-1.251	0.000	1.150	0.456
Durable Goods	-0.003	-0.736	0.000	2.008	0.032
Lumber & Wood Prod.	0.021	2.286	0.000	-1.590	0.063
Furniture & Fixtures	-0.001	-0.125	0.000	0.476	0.771
Stone, Clay & Glass	-0.001	-0.248	0.000	0.511	0.808
Primary Metal Ind.	-0.007	-1.082	0.000	1.008	0.557
Fabricated Metal Prod.	-0.003	-0.577	0.000	0.622	0.823
Ind. Machinery, Comp.Eq.	-0.016	-2.612	0.001	4.399	0.000
Electric & Electr. Eq.	-0.008	-1.337	0.001	3.858	0.000
Transportation Equip.	0.005	0.748	0.000	-0.794	0.726
Instruments	0.014	1.426	0.000	-0.688	0.226
Misc. Manufacturing	0.004	0.468	0.000	-0.342	0.890

Note: $r + st > 0$ represents positive technical change. SIC codes are as in Table 1.

Table 6: Estimates of Returns to Scale, Equation (35), Linear Time Trend, Positive Technical Progress Imposed

Sector	$\hat{k}$	Ho: $k=1$ t-ratio
Manufacturing	0.657	-8.161
Nondur. Goods	0.664	-6.712
Food & Kindred Prod.	0.696	-3.152
Textile Mill Prod.	0.972	-0.514
Apparel & Related Prod.	0.963	-0.807
Paper & Allied Prod.	0.574	-8.049
Printing & Publishing	0.797	-4.068
Chem. & Allied Prod.	0.562	-7.657
Petroleum Refining	0.771	-10.794
Rubber & Plastic Prod.	0.808	-5.192
Durable Goods	0.723	-8.562
Lumber & Wood Prod.	0.867	-1.826
Furniture & Fixtures	0.824	-5.375
Stone, Clay & Glass	0.712	-7.212
Primary Metal Ind.	0.764	-9.107
Fabricated Metal Prod.	0.861	-5.174
Ind. Machinery, Comp.Eq.	0.767	-5.840
Electric & Electr. Eq.	0.720	-7.831
Transportation Equip.	0.785	-7.311
Instruments	0.898	-1.589
Misc. Manufacturing	0.663	-5.300

Note: Ho: $k=1$ is a test of constant returns to scale, and $k<1$ implies increasing returns to scale. SIC codes are as in Table 1.

Table 7: Estimates of $r = g^2$, Equation (35), Linear Time Trend

Sector	$\hat{g}$	Ho: $g = 0$ t-ratio	$\hat{r}$
Manufacturing	-0.030	-0.803	0.001
Nondur. Goods	0.000	0.000	0.000
Food & Kindred Prod.	0.000	0.000	0.000
Textile Mill Prod.	-0.148	-14.733	0.022
Apparel & Related Prod.	-0.090	-8.052	0.008
Paper & Allied Prod.	0.000	0.000	0.000
Printing & Publishing	0.000	0.000	0.000
Chem. & Allied Prod.	0.000	0.000	0.000
Petroleum Refining	0.000	0.000	0.000
Rubber & Plastic Prod.	0.000	0.000	0.000
Durable Goods	-0.068	-3.624	0.005
Lumber & Wood Prod.	-0.091	-3.707	0.008
Furniture & Fixtures	-0.036	-1.027	0.001
Stone, Clay & Glass	-0.033	-0.844	0.001
Primary Metal Ind.	0.000	0.000	0.000
Fabricated Metal Prod.	0.000	0.000	0.000
Ind. Machinery, Comp.Eq.	-0.082	-3.334	0.007
Electric & Electr. Eq.	-0.101	-5.075	0.010
Transportation Equip.	-0.020	-0.252	0.000
Instruments	-0.089	-3.307	0.008
Misc. Manufacturing	-0.037	-0.704	0.001

Note: SIC codes are as in Table 1.

Table 8: Estimates of Returns to Scale, Reverse Regression of Equation (35), Linear Time Trend

Sector	$1/\hat{k}$	$\widehat{1/k}$	Ho: $1/k = 1$ t-ratio	R^2	DW
Manufacturing	1.522	1.248	2.974	0.820	1.536
Nondur. Goods	1.686	0.977†	-0.197	0.579	1.603
Food & Kindred Prod.	1.724	0.432	-5.328	0.251	2.756
Textile Mill Prod.	1.029†	0.890	-2.199	0.865	1.836
Apparel & Related Prod.	1.038†	0.934†	-1.487	0.899	1.876
Paper & Allied Prod.	2.062	1.186†	1.281	0.576	1.782
Printing & Publishing	1.621	1.213	2.125	0.749	2.054
Chem. & Allied Prod.	2.174	1.035†	0.225	0.476	1.687
Petroleum Refining	1.344	1.273	6.387	0.948	1.484
Rubber & Plastic Prod.	1.258	1.091†	1.499	0.868	1.614
Durable Goods	1.383	1.253	4.396	0.906	1.580
Lumber & Wood Prod.	1.153†	0.843	-2.156	0.731	1.955
Furniture & Fixtures	1.214	1.124	2.735	0.926	2.471
Stone, Clay & Glass	1.404	1.209	3.016	0.861	1.839
Primary Metal Ind.	1.311	1.222	4.713	0.932	1.738
Fabricated Metal Prod.	1.163	1.099	2.616	0.945	2.001
Ind. Machinery, Comp.Eq.	1.304	1.147	2.432	0.880	1.305
Electric & Electr. Eq.	1.389	1.228	3.601	0.884	1.492
Transportation Equip.	1.274	1.176	3.656	0.924	1.770
Instruments	1.114†	0.880†	-1.847	0.791	2.273
Misc. Manufacturing	1.508	1.033†	0.328	0.685	2.274

Note: $1/\hat{k}$ is from inverting the estimates of k from Table 2 and is included here for comparison purposes. $\widehat{1/k}$ is the estimate of $1/k$ from estimating the reverse regression $\ln Q_T(\cdot) = r/k + (1/k)\ln Q_T^*(\cdot)$. Ho: $1/k = 1$ is a test of constant returns to scale, and $1/k > 1$ implies increasing returns to scale. A † indicates that constant returns to scale cannot be rejected at the 5% level of significance. SIC codes are as in Table 1. DW denotes the Durbin-Watson statistic for first-order autocorrelation.

Table 9: Estimates of r, Reverse Regression of Equation (35), Linear Time Trend

Sector	$\hat{r}$	$\tilde{r}$	Ho: $r = 0$ t-ratio
Manufacturing	0.001	0.006	2.064
Nondur. Goods	-0.004	0.008	1.894
Food & Kindred Prod.	-0.005	0.034	2.420
Textile Mill Prod.	0.022	0.026	7.753
Apparel & Related Prod.	0.008	0.010	4.648
Paper & Allied Prod.	-0.010	0.002	0.338
Printing & Publishing	-0.012	-0.006	-2.402
Chem. & Allied Prod.	-0.011	0.009	1.149
Petroleum Refining	-0.002	-0.001	-0.961
Rubber & Plastic Prod.	-0.002	0.004	0.941
Durable Goods	0.005	0.008	2.720
Lumber & Wood Prod.	0.008	0.015	2.786
Furniture & Fixtures	0.001	0.003	1.357
Stone, Clay & Glass	0.001	0.004	1.343
Primary Metal Ind.	-0.001	-0.001	-0.216
Fabricated Metal Prod.	0.000	0.001	0.527
Ind. Machinery, Comp.Eq.	0.007	0.012	2.738
Electric & Electr. Eq.	0.010	0.017	3.764
Transportation Equip.	0.000	0.003	0.717
Instruments	0.008	0.021	3.518
Misc. Manufacturing	0.001	0.009	1.854

Note: $\hat{r}$ is the estimate of r from Table 3 and is included here for comparison purposes. $\tilde{r}$ is the implied r from estimating the coefficient r/k in the reverse regression. $r > 0$ represents positive technical change. Multiplying by 100 gives percentage growth rates. SIC codes are as in Table 1.

Table 10: Markups, Using Returns to Scale Estimates from Table 2

Sector	Mean	St. Dev.	Minimum	Maximum
Manufacturing	1.753	0.040	1.675	1.841
Nondur. Goods	1.930	0.034	1.875	2.016
Food & Kindred Prod.	1.855	0.053	1.760	1.971
Textile Mill Prod.	1.103	0.031	1.019	1.174
Apparel & Related Prod.	1.100	0.018	1.060	1.128
Paper & Allied Prod.	2.350	0.100	2.184	2.633
Printing & Publishing	1.807	0.025	1.720	1.864
Chem. & Allied Prod.	2.692	0.119	2.420	2.862
Petroleum Refining	1.579	0.031	1.504	1.657
Rubber & Plastic Prod.	1.339	0.028	1.281	1.404
Durable Goods	1.569	0.055	1.463	1.684
Lumber & Wood Prod.	1.330	0.041	1.224	1.418
Furniture & Fixtures	1.288	0.018	1.253	1.333
Stone, Clay & Glass	1.561	0.079	1.355	1.711
Primary Metal Ind.	1.414	0.076	1.258	1.544
Fabricated Metal Prod.	1.272	0.029	1.228	1.361
Ind. Machinery, Comp.Eq.	1.452	0.055	1.342	1.543
Electric & Electr. Eq.	1.594	0.089	1.466	1.826
Transportation Equip.	1.380	0.073	1.228	1.528
Instruments	1.173	0.051	1.079	1.270
Misc. Manufacturing	1.690	0.070	1.600	1.834

Note: These markups are the inverse of M^t in equation (36) ($M^t = k(w^t \cdot x^t)/(p^t \cdot y^t)$), so that a value greater than one indicates a positive markup. The estimate of k comes from the estimation of equation (35), with a linear time trend.

Table 11: Markups, Using Reverse Regression Returns to Scale Estimates from Table 8

Sector	Mean	St. Dev.	Minimum	Maximum
Manufacturing	1.436	0.033	1.374	1.510
Nondur. Goods	1.118	0.020	1.086	1.168
Food & Kindred Prod.	0.465	0.013	0.441	0.494
Textile Mill Prod.	0.954	0.026	0.881	1.015
Apparel & Related Prod.	0.989	0.016	0.953	1.014
Paper & Allied Prod.	1.353	0.058	1.258	1.516
Printing & Publishing	1.354	0.018	1.288	1.396
Chem. & Allied Prod.	1.281	0.056	1.151	1.361
Petroleum Refining	1.497	0.030	1.425	1.570
Rubber & Plastic Prod.	1.162	0.024	1.111	1.218
Durable Goods	1.422	0.050	1.326	1.526
Lumber & Wood Prod.	0.972	0.030	0.894	1.036
Furniture & Fixtures	1.193	0.017	1.161	1.234
Stone, Clay & Glass	1.344	0.068	1.166	1.473
Primary Metal Ind.	1.318	0.071	1.172	1.440
Fabricated Metal Prod.	1.202	0.028	1.160	1.286
Ind. Machinery, Comp.Eq.	1.278	0.048	1.181	1.358
Electric & Electr. Eq.	1.409	0.079	1.297	1.615
Transportation Equip.	1.275	0.067	1.135	1.411
Instruments	0.928	0.040	0.853	1.004
Misc. Manufacturing	1.157	0.048	1.095	1.256

Note: These markups are the inverse of M^t in equation (36) ($M^t = k(w^t \cdot x^t)/(p^t \cdot y^t)$), so that a value greater than one indicates a positive markup. The estimate of k comes from the estimation of the reverse regression of equation (35), with a linear time trend.

Figure 1: TFP Growth for US Manufacturing Sectors

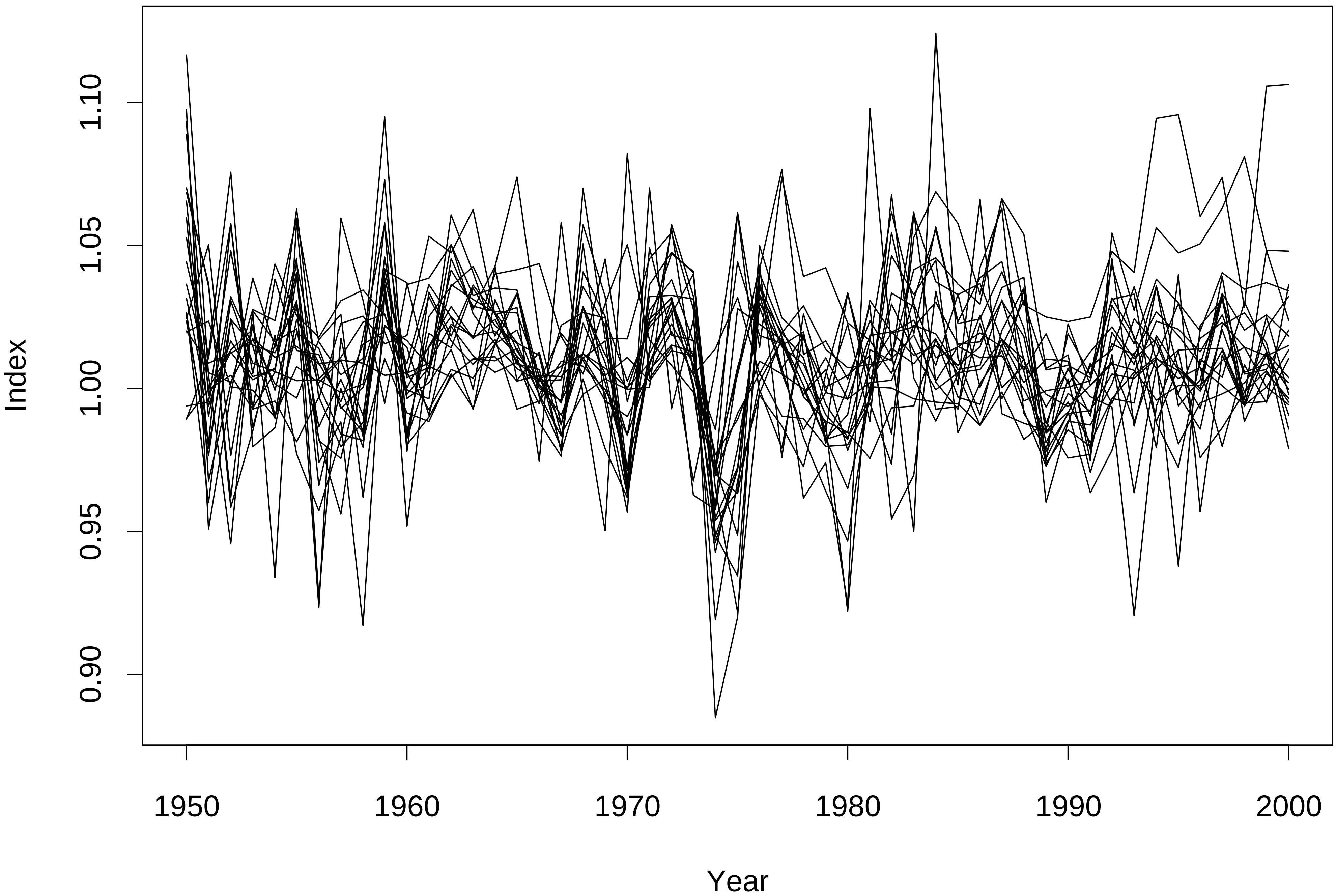

Figure 2: Selected Markups from Table 10

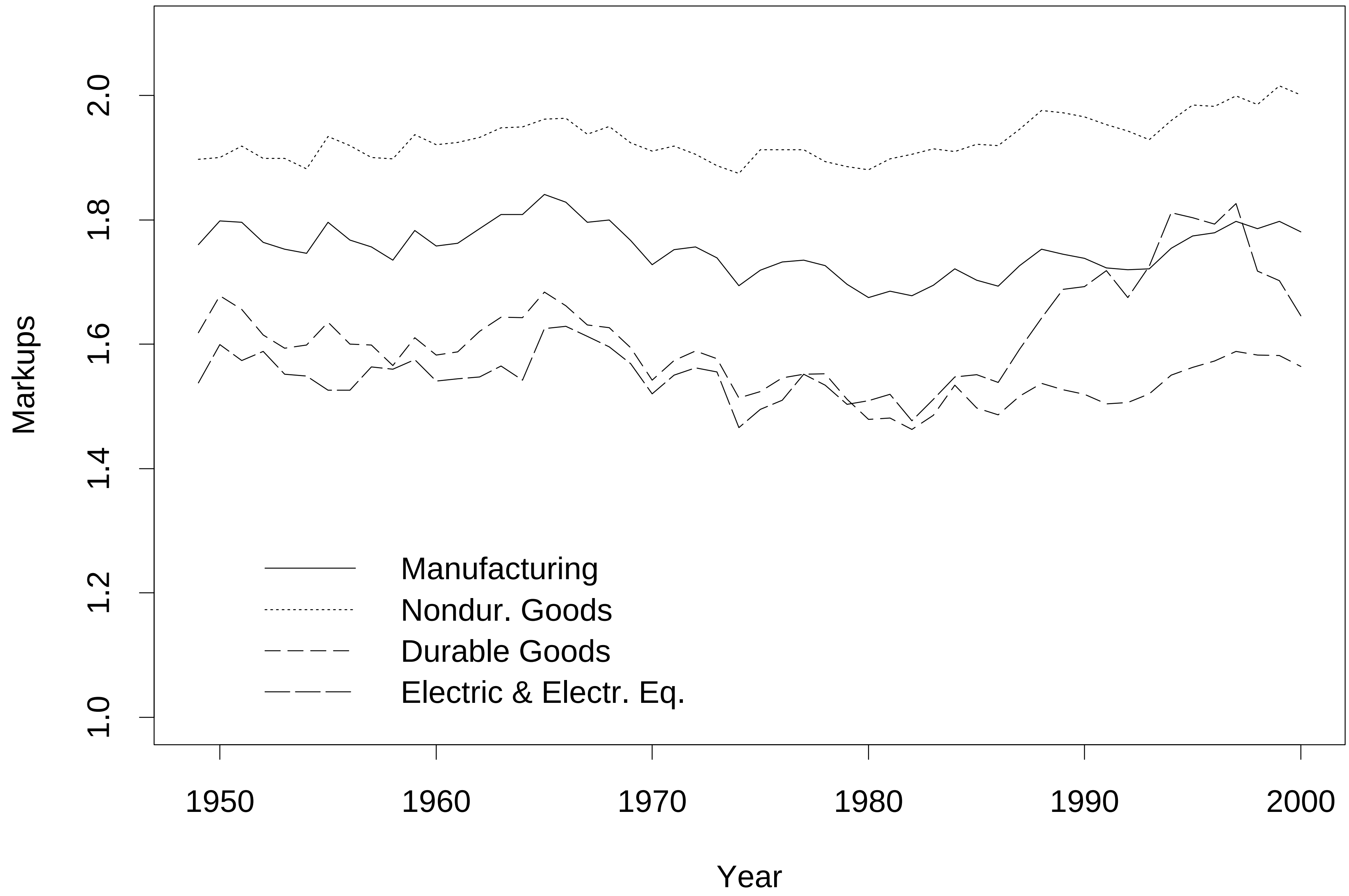

Figure 3: Selected Markups from Table 11