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Abstract

This note considers the Lowe consumer price index as an approximation to a true cost of living index. A simple example, based on systematic long run trends in prices, is used to obtain some idea of the magnitude of the substitution bias.

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1 Introduction

Usually the substitution bias of an official CPI is assessed under the assumption that such an index is an estimator of a Laspeyres price index. The generic form of the Laspeyres price index is

$$P_L(p^0, p^t, q^0) \equiv \frac{\sum_{n=1}^N p_n^t q_n^0}{\sum_{n=1}^N p_n^0 q_n^0}, \tag{1}$$

where p^t (p^0) is the current (reference) period price vector and q^0 is the reference period quantity vector. The question then is how this index relates to its true cost of living counterpart.

Indeed, many statistical agencies are employing a Laspeyres price index as their conceptual target. For example, the Netherlands' CPI is modelled this way, where currently the reference period is the year 2000, and t is any month from January 2001 onwards. Nevertheless, the headline inflation figure is obtained as the percentage change between the current month and the corresponding month of the previous year. Put otherwise, the really interesting index number is the one given by

$$\frac{P_L(p^0, p^t, q^0)}{P_L(p^0, p^{t-12}, q^0)} = \frac{\sum_{n=1}^N p_n^t q_n^0}{\sum_{n=1}^N p_n^{t-12} q_n^0}, \tag{2}$$

which is a fixed basket price index, but definitely not a Laspeyres index. Actually, the right hand side of this expression is an instance of the Lowe price index[1], and will be denoted by $P_{Lo}(p^{t-12}, p^t, q^0)$. Thus it makes much more sense to inquire after the substitution bias of this index than the Laspeyres index.

There are also many statistical agencies which employ as conceptual target for their CPI a so-called modified Laspeyres index. This concept measures the price change between reference month 0 and current month t as a weighted average of price relatives

[1]Named after Lowe (1823). See Diewert (1993a) for Lowe's place in the history of index number theory.

$$\sum_{n=1}^{N} w_n(p_n^t/p_n^0) \text{ where } \sum_{n=1}^{N} w_n = 1. \tag{3}$$

The weights are then derived from consumer expenditures of some year b prior to month 0, which are price-updated to month 0. Formally written,

$$w_n \equiv \frac{p_n^b q_n^b (p_n^0/p_n^b)}{\sum_{n=1}^{N} p_n^b q_n^b (p_n^0/p_n^b)} \ (n = 1, ..., N). \tag{4}$$

But this means that

$$\sum_{n=1}^{N} w_n(p_n^t/p_n^0) = \frac{\sum_{n=1}^{N} p_n^t q_n^b}{\sum_{n=1}^{N} p_n^0 q_n^b} = P_{Lo}(p^0, p^t, q^b), \tag{5}$$

that is, the target index is a Lowe index.

The foregoing is sufficient to motivate the question to be addressed in this paper: can the Lowe index $P_{Lo}(p^0, p^t, q^b)$, where typically $b \leq 0 < t$, be related to one derived from the economic approach to index number theory? Note that when $b = 0$, the Lowe index reduces to the Laspeyres index. Thus our question is more general than the usual one.

The lay-out of this paper is as follows. Section 2 considers in a very general way the Lowe index as an approximation to a true cost of living index. Sections 3 and 4 respectively pursue first and second order approximations to its substitution bias. Section 5 concludes.

2 The Lowe index as an approximation to a true cost of living index

Assume that the consumer has preferences defined over consumption vectors $q \equiv (q_1, ..., q_N)$ that can be represented by the continuous increasing utility function $f(q)$. Thus if $f(q^1) > f(q^0)$, then the consumer prefers the consumption vector q^1 to q^0. Let q^b be the annual consumption vector for the consumer in the base year b. Define the base year utility level u^b as the utility level that corresponds to $f(q)$ evaluated at q^b:

$$u^b \equiv f(q^b). \tag{6}$$

For any vector of positive commodity prices $p \equiv (p_1, ..., p_N)$ and for any feasible utility level u, the consumer's cost function, $C(u, p)$, can be defined in the usual way as the minimum expenditure required to achieve the utility level u when facing the prices p:

$$C(u,p) \equiv \min_q \{\sum_{n=1}^{N} p_n q_n \mid f(q_1,...,q_N) = u\}. \quad (7)$$

Let $p^b \equiv (p_1^b,...,p_N^b)$ be the vector of annual prices that the consumer faced in the base year b. Assume that the observed base year consumption vector $q^b \equiv (q_1^b,...,q_N^b)$ solves the following base year cost minimization problem:

$$C(u^b,p^b) = \sum_{n=1}^{N} p_n^b q_n^b. \quad (8)$$

The cost function will be used below in order to define the consumer's cost of living index.

Let p^0 and p^t be the monthly price vectors that the consumer faces in months 0 and t. Then the Konüs true cost of living index, $P_K(p^0,p^t,q^b)$, between months 0 and t, using the base year utility level $u^b = f(q^b)$ as the reference standard of living, is defined as the following ratio of minimum monthly costs of achieving the utility level u^b:

$$P_K(p^0,p^t,q^b) \equiv \frac{C(f(q^b),p^t)}{C(f(q^b),p^0)}. \quad (9)$$

Using the definition of the monthly cost minimization problem that corresponds to the cost $C(f(q^b),p^t)$, it can be seen that the following inequality holds:

$$C(f(q^b),p^t) \leq \sum_{n=1}^{N} p_n^t q_n^b \quad (10)$$

since the base year quantity vector q^b is feasible for the cost minimization problem. Similarly, using the definition of the monthly cost minimization problem that corresponds to the cost $C(f(q^b),p^0)$, it can be seen that the following inequality holds:

$$C(f(q^b),p^0) \leq \sum_{n=1}^{N} p_n^0 q_n^b \quad (11)$$

since the base year quantity vector q^b is feasible for the cost minimization problem.

It will prove useful to rewrite the two inequalities (10) and (11) as equalities. This can be done if nonnegative substitution bias terms, e^t and e^0, are subtracted from the right hand sides of these two inequalities. Thus (10) and (11) can be rewritten as follows:

$$C(f(q^b), p^t) = \sum_{n=1}^{N} p_n^t q_n^b - e^t \tag{12}$$

$$C(f(q^b), p^0) = \sum_{n=1}^{N} p_n^0 q_n^b - e^0. \tag{13}$$

Using (12) and (13) and the definition of the Lowe index, the following approximate equality results:

$$\begin{aligned} P_{Lo}(p^0, p^t, q^b) &\equiv \frac{\sum_{n=1}^{N} p_n^t q_n^b}{\sum_{n=1}^{N} p_n^0 q_n^b} \\ &= \frac{C(u^b, p^t) + e^t}{C(u^b, p^0) + e^0} \\ &\approx \frac{C(u^b, p^t)}{C(u^b, p^0)} \\ &= P_K(p^0, p^t, q^b). \end{aligned} \tag{14}$$

Thus if the nonnegative substitution bias terms e^0 and e^t are small, then the Lowe index between months 0 and t, $P_{Lo}(p^0, p^t, q^b)$, will be an adequate approximation to the true cost of living index between months 0 and t, $P_K(p^0, p^t, q^b)$.

A bit of algebraic manipulation shows that the Lowe index will be exactly equal to its cost of living counterpart if the substitution bias terms satisfy the following relationship:[2]

$$\frac{e^t}{e^0} = \frac{C(u^b, p^t)}{C(u^b, p^0)} = P_K(p^0, p^t, q^b). \tag{15}$$

Equations (14) and (15) can be interpreted as follows: if the rate of growth in the amount of substitution bias between months 0 and t is equal to the rate of growth in the minimum cost of achieving the base year utility level u^b

[2]This assumes that e^0 is greater than zero. If e^0 is equal to zero, then to have equality of P_K and P_{Lo}, it must also be the case that e^t is equal to zero.

between months 0 and t, then the observable Lowe index, $P_{Lo}(p^0, p^t, q^b)$, will be exactly equal to its true cost of living index counterpart, $P_K(p^0, p^t, q^b)$.[3]

It is difficult to know whether condition (15) will hold or whether the substitution bias terms e^0 and e^t will be small. Thus in the following two sections, first and second order Taylor series approximations to these substitution bias terms will be developed.

3 A first order approximation to the substitution bias of the Lowe index

The true cost of living index between months 0 and t, using the base year utility level u^b as the reference utility level, is the ratio of two unobservable costs, $C(u^b, p^t)/C(u^b, p^0)$. However, both of these hypothetical costs can be approximated by first order Taylor series approximations that can be evaluated using observable information on prices and base year quantities. The first order Taylor series approximation to $C(u^b, p^t)$ around the annual base year price vector p^b is given by the following approximate equation:[4]

$$\begin{aligned} C(u^b, p^t) &\approx C(u^b, p^b) + \sum_{n=1}^{N} \frac{\partial C(u^b, p^b)}{\partial p_n}(p_n^t - p_n^b) \\ &= C(u^b, p^b) + \sum_{n=1}^{N} q_n^b(p_n^t - p_n^b) \\ &= \sum_{n=1}^{N} p_n^b q_n^b + \sum_{n=1}^{N} q_n^b(p_n^t - p_n^b) \\ &= \sum_{n=1}^{N} p_n^t q_n^b, \end{aligned} \tag{16}$$

where Shephard's Lemma and assumption (8) have been used. Similarly, the first order Taylor series approximation to $C(u^b, p^0)$ around the annual base year price vector p^b is given by the following approximate equation:

[3]It can be seen that when month t is set equal to month 0, $e^t = e^0$ and $C(u^b, p^t) = C(u^b, p^0)$ and thus (15) is satisfied and $P_{Lo} = P_K$. This is not surprising since both indices are equal to unity when $t = 0$.

[4]This type of Taylor series approximation was used in Schultze and Mackie (2002; 91) in the cost of living index context but it essentially dates back to Hicks (1941-42; 134) in the consumer surplus context. See also Diewert (1992; 568).

$$
\begin{aligned}
C(u^b, p^0) &\approx C(u^b, p^b) + \sum_{n=1}^{N} \frac{\partial C(u^b, p^b)}{\partial p_n}(p_n^0 - p_n^b) \\
&= C(u^b, p^b) + \sum_{n=1}^{N} q_n^b (p_n^0 - p_n^b) \\
&= \sum_{n=1}^{N} p_n^b q_n^b + \sum_{n=1}^{N} q_n^b (p_n^0 - p_n^b) \\
&= \sum_{n=1}^{N} p_n^0 q_n^b,
\end{aligned} \tag{17}
$$

Comparing (16) to (12), and (17) to (13), it can be seen that to the accuracy of the first order the substitution bias terms e^t and e^0 will be zero. Using these results to reinterpret (14), it can be seen that if the month 0 and month t price vectors, p^0 and p^t, are not too different from the base year vector of prices p^b, then the Lowe index $P_{Lo}(p^0, p^t, q^b)$ will approximate the true cost of living index $P_K(p^0, p^t, q^b)$ to the accuracy of the first order. This result is quite useful, since it indicates that if the monthly price vectors p^0 and p^t are just randomly fluctuating around the base year prices p^b (with modest variances), then the Lowe index will serve as an adequate approximation to a theoretical cost of living index. However, if there are systematic long term trends in prices and month t is fairly distant from month 0 (or the end of year b is quite distant from month 0), then the first order approximations given by (16) and (17) may no longer be adequate and the Lowe index may have a considerable bias relative to its cost of living counterpart. The hypothesis of long run trends in prices will be explored in the following section.

4 A second order approximation to the substitution bias of the Lowe index

A second order Taylor series approximation to $C(u^b, p^t)$ around the base year price vector p^b is given by the following approximate equation:

$$C(u^b, p^t)$$

$$
\begin{aligned}
&\approx \; C(u^b,p^b) + \sum_{n=1}^{N} \frac{\partial C(u^b,p^b)}{\partial p_n}(p_n^t - p_n^b) \\
&\quad +(1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}(p_n^t - p_n^b)(p_{n'}^t - p_{n'}^b) \\
&= \; \sum_{n=1}^{N} p_n^t q_n^b + (1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}(p_n^t - p_n^b)(p_{n'}^t - p_{n'}^b), \qquad (18)
\end{aligned}
$$

where the last equality follows using (16).[5] Similarly, a second order Taylor series approximation to $C(u^b, p^0)$ around the base year price vector p^b is given by the following approximate equation:

$$
\begin{aligned}
&C(u^b,p^0) \\
&\approx \; C(u^b,p^b) + \sum_{n=1}^{N} \frac{\partial C(u^b,p^b)}{\partial p_n}(p_n^0 - p_n^b) \\
&\quad +(1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}(p_n^0 - p_n^b)(p_{n'}^0 - p_{n'}^b) \\
&= \; \sum_{n=1}^{N} p_n^0 q_n^b + (1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}(p_n^0 - p_n^b)(p_{n'}^0 - p_{n'}^b), \qquad (19)
\end{aligned}
$$

where the last equality follows using (17).

Comparing (18) to (12), and (19) to (13), it can be seen that to the accuracy of the second order, the month 0 and month t substitution bias terms, e^0 and e^t, will be equal to the following expressions involving the second order partial derivatives of the consumer's cost function evaluated at the base year standard of living u^b and the base year prices p^b:

$$e^0 \approx -(1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}(p_n^0 - p_n^b)(p_{n'}^0 - p_{n'}^b) \qquad (20)$$

$$e^t \approx -(1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}(p_n^t - p_n^b)(p_{n'}^t - p_{n'}^b). \qquad (21)$$

[5] This type of second order approximation is due to Hicks (1941-42; 133-134) (1946; 331). See also Diewert (1992; 568) and Schultze and Mackie (2002; 91).

Since the consumer's cost function $C(u,p)$ is a concave function in the components of the price vector p,[6] it is known[7] that the $N \times N$ (symmetric) matrix of second order partial derivatives is negative semidefinite.[8] Hence, for arbitrary price vectors p^b, p^0 and p^t, the right hand sides of (20) and (21) will be nonnegative. Thus to the accuracy of the second order, the substitution bias terms e^0 and e^t will be nonnegative.

Now assume that there are systematic long run trends in prices. Assume that the last month of the base year for quantities occurs M months prior to month 0, the base month for prices, and assume that prices trend linearly with time, starting with the last month of the base year for quantities. Thus assume the existence of constants α_n $(n = 1, ..., N)$ such that the price of commodity n in month t is given by:

$$p_n^t = p_n^b + \alpha_n(M+t) \; (n = 1, ..., N; t = 0, 1, ..., T). \tag{22}$$

Substituting (22) into (20) and (21) leads to the following second order approximations to the two substitution bias terms:

$$e^0 \approx \gamma M^2 \tag{23}$$

$$e^t \approx \gamma (M+t)^2, \tag{24}$$

where γ is defined as

$$\gamma \equiv -(1/2)\sum_{n=1}^{N}\sum_{n'=1}^{N}\frac{\partial^2 C(u^b,p^b)}{\partial p_n \partial p_{n'}}\alpha_n\alpha_{n'} \geq 0. \tag{25}$$

It should be noted that the parameter γ will be zero under two sets of conditions:[9]

- All of the second order partial derivatives of the consumer's cost function are equal to zero.

[6] See Diewert (1993b; 109-110).

[7] See Diewert (1993b; 149).

[8] A symmetric $N \times N$ matrix A with nn'-th element equal to $a_{nn'}$ is negative semidefinite if and only if for every vector $z \equiv (z_1, ..., z_N)$ it is the case that $\sum_{n=1}^{N}\sum_{n'=1}^{N} a_{nn'} z_n z_{n'} \leq 0$.

[9] A more general condition that ensures the positivity of γ is that the vector $(\alpha_1, ..., \alpha_N)$ is not an eigenvector of the matrix of second order partial derivatives $\partial^2 C(u^b,p^b)/\partial p_n \partial p_{n'}$ that corresponds to a zero eigenvalue.

- Each commodity price change parameter α_n is proportional to the corresponding commodity n base year price p_n^b.[10]

The first condition is empirically unlikely since it implies that the consumer will not substitute away from commodities whose relative price has increased. The second condition is also empirically unlikely, since it implies that the structure of relative prices remains unchanged over time. Thus in what follows, it will be assumed that γ is a positive number.

In order to simplify the notation in what follows, define the denominator and numerator of the month t Lowe index, $P_{Lo}(p^0, p^t, q^b)$, as a and b respectively; i.e., define

$$a \equiv \sum_{n=1}^{N} p_n^0 q_n^b \tag{26}$$

$$b \equiv \sum_{n=1}^{N} p_n^t q_n^b. \tag{27}$$

Using equations (22) to eliminate the month 0 prices p_n^0 from (26) and the month t prices p_n^t from (27) leads to the following expressions for a and b:

$$a = \sum_{n=1}^{N} p_n^b q_n^b + \sum_{n=1}^{N} \alpha_n q_n^b M \tag{28}$$

$$b = \sum_{n=1}^{N} p_n^b q_n^b + \sum_{n=1}^{N} \alpha_n q_n^b (M + t). \tag{29}$$

It is assumed that

$$\sum_{n=1}^{N} \alpha_n q_n^b \geq 0, \tag{30}$$

which rules out a general decrease of prices. It is also assumed that $a - \gamma M^2$ is positive.

[10]It is known that $C(u, p)$ is linearly homogeneous in the components of the price vector p; see Diewert (1993b; 109) for example. Hence, using Euler's Theorem on homogeneous functions, it can be shown that p^b is an eigenvector of the matrix of second order partial derivatives $\partial^2 C(u^b, p^b)/\partial p_n \partial p_{n'}$ that corresponds to a zero eigenvalue and thus $\sum_{n=1}^{N} \sum_{n'=1}^{N} [\partial^2 C(u^b, p^b)/\partial p_n \partial p_{n'}] p_n^b p_{n'}^b = 0$; see Diewert (1993b; 149) for a detailed proof of this result.

Define the bias in the month t Lowe index, B^t, as the difference between the true cost of living index $P_K(p^0, p^t, q^b)$ defined by (9) and the corresponding Lowe index $P_{Lo}(p^0, p^t, q^b)$. Then,

$$
\begin{aligned}
B^t &\equiv P_K(p^0, p^t, q^b) - P_{Lo}(p^0, p^t, q^b) \\
&= \frac{C(u^b, p^t)}{C(u^b, p^0)} - \frac{b}{a} \\
&= \frac{b - e^t}{a - e^0} - \frac{b}{a} \\
&\approx \frac{b - \gamma(M + t)^2}{a - \gamma M^2} - \frac{b}{a} \\
&= \gamma \frac{(b - a)M^2 - 2aMt - at^2}{a(a - \gamma M^2)} \\
&= \gamma \frac{(\sum_{n=1}^N \alpha_n q_n^b) M^2 t - 2(\sum_{n=1}^N p_n^b q_n^b + \sum_{n=1}^N \alpha_n q_n^b M) Mt - at^2}{a(a - \gamma M^2)} \\
&= -\gamma \frac{(\sum_{n=1}^N \alpha_n q_n^b) M^2 t + 2(\sum_{n=1}^N p_n^b q_n^b) Mt + at^2}{a(a - \gamma M^2)} \\
&< 0,
\end{aligned}
\tag{31}
$$

using respectively (26) and (27), (12) and (13), (23) and (24), (28) and (29), and (30).

Thus for $t \geq 1$, the Lowe index will have an upward bias (to the accuracy of a second order Taylor series) relative to the corresponding true cost of living index, since the approximate bias defined by the last expression in (31) is the sum of one nonpositive and two negative terms. Moreover this approximate bias will grow quadratically in time t.[11]

In order to give the reader some idea of the magnitude of the approximate bias B^t defined by the last line of (31), a simple special case will be considered at this point. Suppose there are only 2 commodities and at the base year, all prices and quantities are equal to 1. Thus $p_n^b = q_n^b = 1$ for $n = 1, 2$ and $\sum_{n=1}^N p_n^b q_n^b = 2$. Assume that $M = 24$ so that the base year data on quantities take 2 years to process before the Lowe index can be implemented. Assume that the monthly rate of growth in price for

[11] If M is large relative to t, then it can be seen that the first two terms in the last equation of (31) can dominate the last term, which is the quadratic in t term.

commodity 1 is $\alpha_1 = 0.002$ so that after 1 year, the price of commodity 1 rises 0.024 or 2.4%. Assume that commodity 2 falls in price each month with $\alpha_2 = -0.002$ so that the price of commodity 2 falls 2.4% in the first year after the base year for quantities. Thus the relative price of the two commodities is steadily diverging by about 5 percent per year. Finally, assume that $\partial^2 C(u^b, p^b)/\partial p_1 \partial p_1 = \partial^2 C(u^b, p^b)/\partial p_2 \partial p_2 = -1$ and $\partial^2 C(u^b, p^b)/\partial p_1 \partial p_2 = \partial^2 C(u^b, p^b)/\partial p_2 \partial p_1 = 1$. These assumptions imply that the own elasticity of demand for each commodity is -1 at the base year consumer equilibrium. Making all of these assumptions means that:

$$2 = \sum_{n=1}^{N} p_n^b q_n^b = a = b; \; \sum_{n=1}^{N} \alpha_n q_n^b = 0; \; M = 24; \; \gamma = 0.000008. \tag{32}$$

Thus the Lowe index keeps for all months t the value 1. Substituting the parameter values given in (32) into (31) leads to the following formula for the approximate amount that the Lowe index will exceed the corresponding true cost of living index at month t:

$$-B^t = 0.000008 \frac{96t + 2t^2}{2(2 - 0.004608)}. \tag{33}$$

Evaluating (33) at $t = 12, t = 24, t = 36, t = 48$ and $t = 60$ leads to the following estimates for $-B^t$: 0.0029 (the approximate bias in the Lowe index at the end of the first year of operation); 0.0069 (the bias after 2 years); 0.0121 (3 years); 0.0185 (4 years); 0.0260 (5 years). Thus at the end of the first year of the operation of the Lowe index, it will only be above the corresponding true cost of living index by approximately a third of a percentage point but by the end of the fifth year of operation, it will exceed the corresponding cost of living index by about 2.6 percentage points, which is no longer a negligible amount.[12]

The numerical results in the previous paragraph are only indicative of the approximate magnitude of the difference between a Lowe index and the corresponding cost of living index. The important point to note is that to the accuracy of the second order, the Lowe index will generally exceed its cost of living counterpart. However, the results also indicate that this difference can be reduced to a negligible amount if:

- the lag in obtaining the base year quantity weights is minimized, and

[12] Note that the relatively large magnitude of M compared to t leads to a bias that grows approximately linearly with t rather than quadratically.

- the base year is changed as frequently as possible.[13]

It also should be noted that the numerical results depend on the assumption that long run trends in prices exist, which may not be true,[14] and on elasticity assumptions that may not be justified.[15] Thus statistical agencies should prepare their own carefully constructed estimates of the differences between a Lowe index and a cost of living index in the light of their own particular circumstances.

5 Conclusion

The conceptual target for measuring consumer price change appears to be a Lowe price index rather than a Laspeyres price index. In this paper we derived first and second order approximations to the substitution bias of the Lowe index. A simple, but not unreasonable, example was used to get some idea of the magnitude of this bias. The bias is seen to crucially depend on the time span between the year to which the quantities refer and the price reference month.

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[13] In our example, if $M = 0$, which means that the Lowe index reduces to the Laspeyres index, the approximate bias at $t = 12$ turns out to be 0.0006.

[14] For mathematical convenience, the trends in prices were assumed to be linear rather than the more natural assumption of geometric.

[15] Another key assumption that was used to derive the numerical results is the magnitude of the divergent trends in prices. If the price divergence vector is doubled to $\alpha_1 = 0.004$ and $\alpha_2 = -0.004$, then the parameter γ quadruples and the approximate bias will also quadruple.

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