HEDONIC PRODUCER PRICE INDEXES AND QUALITY ADJUSTMENT

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Abstract

The paper considers the problem of quality adjustment in the context of a producer's output price index. The paper provides a formal justification for the "user valuation" point of view in making quality adjustments. The paper sets up a revenue maximization problem for a producer who has a choice in each period of what type of model to produce but once the model is chosen, produces only a single model. However, the chosen model can change as the period changes and hence the problem of measuring the real output of the firm across periods in consistent units arises. The family of revenue functions is used in order to define a family of producer output price indexes between two periods. Two special cases of this family of price indexes are defined and "observable" bounds for them are provided. The paper concludes with a discussion of the user value versus resource cost controversy to the theory of quality adjustment in the context of the national accounts.

Key Words

Quality adjustment, hedonic regressions, output price index, economic approach to index number theory, user versus resource cost valuations, national accounts.

Journal of Economic Literature Classification Codes

C32, C43, D20, D57, E31.

1. Introduction

This note considers the problem of quality adjustment in the context of a producer's output price index. We provide a formal justification for the "user valuation" point of view in making quality adjustments.[2]

[1] The author is indebted to Johannes Hoffman, Alice Nakamura and Mick Silver for helpful comments and to the Social Sciences and Humanities Research Council of Canada for financial support.
[2] See Triplett (1983) for a thorough discussion of this point of view and the opposing "resource cost" point of view. However, in this paper, we follow the example of Griliches (1971b; 14), who argued that it is the user value or utility of a model that is the "right" characteristic for government statisticians to attempt to measure.

Section 2 below sets up the revenue maximization problem of a producer who has a choice in each period of what type of model to produce but once the model is chosen, produces only a single model. However, the chosen model can change as the period changes and hence the problem of measuring the real output of the firm across periods in consistent units arises. A key assumption is that the producer in each period faces a *hedonic price function*, $\Pi^t(z)$, which gives the market price in period t for a model with the vector of characteristics z.

Section 3 uses the family of revenue functions defined in section 2 in order to define a family of producer output price indexes between two periods. Two special cases of this family of price indexes are defined and "observable" bounds for them are provided.

Section 4 derives a theoretical output price index that lies between the observable bounds found in section 3.

Section 5 concludes with a brief discussion of some of the weaknesses of our suggested output price index. Section 5 also discusses the user value versus resource cost controversy to the theory of quality adjustment in the context of the national accounts.

2. The Producer's Revenue Maximization Problem

We consider a producer's revenue maximization problem assuming that it produces a single output but in each period, it has a choice of what type of model it could produce. Let the model be identified by a K dimensional vector of characteristics, $z \equiv [z_1,...,z_K]$.

Before we proceed to the firm's revenue maximization problem, we need to characterize the set of output prices that it faces in period t as a function of the characteristics of the model that the firm might produce. We assume that in period t, the demanders of the output of the firm (or establishment) have a separable cardinal utility function[3], $f^t(z)$, that enables each demander to determine that the value of a model with the vector of characteristics $z^1 \equiv [z_1^1,...,z_K^1]$ compared to a model with characteristics vector $z^2 \equiv [z_1^2,...,z_K^2]$ is $f^t(z^1)/f^t(z^2)$. Thus in period t, demanders are *willing to pay* the amount of money $\Pi^t(z)$ for a model with the vector of characteristics z where

(1) $\Pi^t(z) \equiv \rho^t f^t(z)$; $\qquad t = 0,1.$

The scalar ρ^t is inserted into the willingness to pay function because under certain restrictions, ρ^t can be interpreted as a period t price for the entire family of hedonic models that might be produced in period t. These restrictions are:

(2) $f^0 = f^1$;

[3] These strong assumptions are not required: we need only assume that the firm faces a model price schedule defined by (1) in each period t; i.e., if the firm produces a model with characteristics z in period t, then it can sell an unlimited number of units of this model at the market price $\Pi^t(z)$. However, the strong assumptions on demander's preferences help "explain" the nature of the model price schedule $\Pi^t(z)$.

i.e., the *model relative utility functions* f^t are identical for the two periods under consideration. We will make use of the specific assumption (2) later.

In what follows, we assume that econometric estimates for the period 0 and 1 *hedonic model price functions*, Π^0 and Π^1 are available although we will also consider the case where only an estimate for Π^0 is available.[4]

We now consider an establishment or firm that produces a *single model* in each period in the marketplace that is characterized by the hedonic model price functions, $\Pi^t(z)$, for periods t = 0,1. However, the single model can change as we go from one period to the next and it is this change in model that causes problems for statistical agencies attempting to measure the real output of the firm. We suppose that in period t, the establishment has the *production function* F^t where

(3) $q = F^t(z,v)$

is the number of models, each with vector of characteristics z, that can be produced if the vector of inputs v is available for use by the firm in period t. As is usual in the economic approach to index numbers[5], we assume a competitive model, where each firm takes output prices as fixed parameters beyond its control. In this case, there is an entire schedule of model prices that the firm takes as given instead of just a single price in each period. Thus we assume that if the firm decides to produce a model with the vector of characteristics z, then it can sell any number of units of this model in period t at the price $\Pi^t(z) = \rho^t f^t(z)$. Again, note that we allow the firm to choose what model type to produce in each period.

We now define the firm's *revenue function*, R, assuming that the firm is facing the period s hedonic price function $\Pi^s = \rho^s f^s$ and is using the vector of inputs v and has access to the period t production function F^t:

(4) $$\begin{aligned} R(\rho^s f^s, F^t, Z^t, v) &\equiv \max_{q,z} \{\rho^s f^s(z)q : q = F^t(z,v) \text{ ; z belongs to } Z^t\} \\ &= \max_{z} \{\rho^s f^s(z)F^t(z,v) : \text{z belongs to } Z^t\} \end{aligned}$$

where Z^t is a *technologically feasible set of model characteristics* that can be produced in period t and the second line follows from the line above by substituting the production function constraint into the objective function.

---

[4] Of course, we will need some identifying restrictions in order to identify the parameters of f^0 and f^1 along with ρ^0 and ρ^1. One common model sets $\rho^0 = 1$ and $f^0 = f^1$. This two period time dummy variable hedonic regression was first considered explicitly by Court (1939; 109-111) as his hedonic suggestion number two. If the two periods being compared are consecutive periods, Griliches (1971b; 7) coined the term "adjacent year regression" to describe this dummy variable hedonic regression model. A more general model sets $\rho^0 = 1$ and $f^0(z^*) = f^1(z^*)$ for a reference characteristics vector, $z^* \equiv [z_1^*,...,z_K^*]$. The restriction $f^0(z^*) = f^1(z^*)$ essentially allows the utility of the various models to be compared across the two periods.

[5] For material on the economic approach to output price indexes, see Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055), Triplett (1983; 287-293) and Balk (1998; 83-89).

The actual period t revenue maximization problem that the firm faces is defined by the revenue function (4), except that we replace the period s hedonic price function $\rho^s f^s$ by the period t hedonic price function $\rho^t f^t$ and we replace the generic input quantity vector v by the observed period t input quantity vector used by the firm, v^t. We further assume that the firm produces q^t units of a single model with characteristics vector z^t and that $[q^t,z^t]$ solves the period t revenue maximization problem; i.e., $[q^t,z^t]$ is a solution to:[6]

$$(5)\quad R(\rho^t f^t, F^t, Z^t, v^t) \equiv \max_{q,z} \{\rho^t f^t(z)q : q = F^t(z,v^t) \text{ ; z belongs to } Z^t\}; \qquad t = 0,1$$
$$= \rho^t f^t(z^t)q^t$$

where the period t firm output q^t is equal to

$$(6)\quad q^t = F^t(z^t,v^t) \text{ ;} \qquad t = 0,1.$$

3. Konüs Type Hedonic Output Price Indexes

Now we are ready to define a family of *Konüs(1924) type hedonic output price indexes* P between periods 0 and 1 for the firm under consideration as follows:

$$(7)\quad P(\rho^0 f^0, \rho^1 f^1, F^t, Z^t, v) \equiv R(\rho^1 f^1, F^t, Z^t, v)/R(\rho^0 f^0, F^t, Z^t, v).$$

Thus a particular member of the above family of indexes is equal to the firm's revenue ratio, where the revenue in the numerator of (7) uses the hedonic model price function for period 1 and the revenue in the denominator of (7) uses the hedonic model price function for period 0 but for both revenues, the technology of period t is used (i.e., F^t and Z^t are used in both revenue maximization problems) and the same input quantity vector v is used. This is the usual definition for an economic output price index, except that instead of a single price facing the producer in each period, *we have a whole family of model prices facing the firm in each period*. Note that the only variables that are different in the numerator and denominator of (7) are the two hedonic model price functions facing the firm in periods 0 and 1.[7]

The right hand side of (7) looks a bit complex. However, if assumption (2) holds (i.e., the period 0 and 1 hedonic model price functions are identical except for the multiplicative scalars ρ^0 and ρ^1), then (7) reduces to the very simple ratio, ρ^1/ρ^0. To see this, use definitions (7) and (5) as follows:

$$(8)\quad P(\rho^0 f^0, \rho^1 f^1, F^t, Z^t, v) \equiv R(\rho^1 f^1, F^t, Z^t, v)/R(\rho^0 f^0, F^t, Z^t, v)$$
$$= \max_z \{\rho^1 f^1(z)F^t(z,v^t) \text{ ; z belongs to } Z^t\}/\max_z \{\rho^0 f^0(z)F^t(z,v^t) \text{ ; z belongs to } Z^t\}$$

---

[6] If the firm is competitively optimizing with respect to its choice of inputs as well, then the period t input vector v^t, along with q^t and z^t, are a solution to the following period t profit maximization problem for the firm: $\max_{q,z,v} \{\rho^t f^t(z)q - w^t\cdot v : q = F^t(z,v) \text{ ; z belongs to } Z^t\}$ where w^t is a vector of input prices that the firm faces in period t and $w^t\cdot v$ denotes the inner product of the vectors w^t and v. It is possible to rework our analysis presented below, conditioning on an input price vector rather than on an input quantity vector.

[7] This is why we term the price index a Konüs (1924) type index since his true cost of living index has the analogous form $C(u,p^1)/C(u,p^0)$ where u is a reference utility level, p^0 and p^1 are vectors of prices faced by the consumer in the two periods and C is the cost or expenditure function which represents the consumer's preferences; i.e., only *prices* change in the numerator and denominator of the definition. This is the defining characteristic of an economic price index.

$= \max_z \{\rho^1 f^0(z)F^t(z,v^t)$; z belongs to $Z^t\}/\max_z \{\rho^0 f^0(z)F^t(z,v^t)$; z belongs to $Z^t\}$ using (2)
$= [\rho^1/\rho^0]\max_z \{\rho^0 f^0(z)F^t(z,v^t)$; z belongs to $Z^t\}/\max_z \{\rho^0 f^0(z)F^t(z,v^t)$; z belongs to $Z^t\}$ assuming ρ^0 and ρ^1 are positive
$= \rho^1/\rho^0$ canceling terms.

This is a very useful result since many hedonic regression models have been successfully estimated using assumption (2). Under this assumption, *all* of the theoretical hedonic firm or establishment output price indexes reduce to the observable ratio, ρ^1/ρ^0.

We return to the general case where we do not make assumption (2). As usual, it is always of interest to specialize (7) to the special cases where the conditioning variables that are held constant in the numerator and denominator of (7), F^t, Z^t, and v, are equal to the period 0 and 1 values for these variables, namely, F^0, Z^0, and v^0 and F^1, Z^1, and v^1. Thus define the *Laspeyres type hedonic output price index* between periods 0 and 1 for our firm as follows:

(9) $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0) \equiv R(\rho^1 f^1, F^0, Z^0, v^0)/R(\rho^0 f^0, F^0, Z^0, v^0)$
$= R(\rho^1 f^1, F^0, Z^0, v^0)/\rho^0 f^0(z^0)q^0$ using definition (5) for t = 0
$= \max_z \{\rho^1 f^1(z)F^0(z,v^0)$; z belongs to $Z^0\}/\rho^0 f^0(z^0)q^0$ using definition (4)
$\geq \rho^1 f^1(z^0)F^0(z^0,v^0)/\rho^0 f^0(z^0)q^0$ since z^0 is feasible for the maximization problem
$= \rho^1 f^1(z^0)q^0/\rho^0 f^0(z^0)q^0$ using (6) for t = 0
$= \rho^1 f^1(z^0)/\rho^0 f^0(z^0)$
$\equiv P_{HL}$

where we have defined the *observable hedonic Laspeyres output price index* P_{HL} as[8]

(10) $P_{HL} \equiv \rho^1 f^1(z^0)/\rho^0 f^0(z^0)$.

Thus the inequality (9) says that the unobservable theoretical Laspeyres type hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$ is bounded from below by the observable (assuming that we have estimates for ρ^0, ρ^1, f^0 and f^1) hedonic Laspeyres output price index P_{HL}. The inequality (9) is the hedonic counterpart to a standard Laspeyres type inequality for a theoretical output price index.

It is of interest to rewrite P_{HL} in terms of the observable model prices for the firm in periods 0 and 1. Denote these prices by P^0 and P^1 respectively. Using (1), we have:

(11) $P^0 = \rho^0 f^0(z^0)$ and $P^1 = \rho^1 f^1(z^1)$.

Now rewrite (10) as follows:

(12) $P_{HL} \equiv \rho^1 f^1(z^0)/\rho^0 f^0(z^0)$
$= \rho^1 f^1(z^1)[f^1(z^0)/f^1(z^1)]/\rho^0 f^0(z^0)$
$= P^1[f^1(z^0)/f^1(z^1)]/P^0$ using definitions (11)

[8] The index defined by (10) is "observable" under the assumption that we have run hedonic regressions for both periods under consideration.

$= [P^1/f^1(z^1)]/[P^0/f^1(z^0)]$.

The prices $P^1/f^1(z^1)$ and $P^0/f^1(z^0)$ can be interpreted as *quality adjusted model prices* for the establishment in periods 1 and 0 respectively, using the hedonic regression pertaining to period 1 to do the quality adjustment.[9]

In the theoretical hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$ defined by (9) above, we conditioned on F^0 (the base period production function), Z^0 (the base period set of models that were technologically feasible in period 0) and v^0 (the establishment's base period input vector). We now define a companion period 1 theoretical hedonic output price that conditions on the period 1 variables, F^1, Z^1, v^1. Thus define the *Paasche type hedonic output price index* between periods 0 and 1 for our establishment as follows:[10]

(13) $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1) \equiv R(\rho^1 f^1, F^1, Z^1, v^1)/R(\rho^0 f^0, F^1, Z^1, v^1)$
$= \rho^1 f^1(z^1)q^1/R(\rho^0 f^0, F^1, Z^1, v^1)$ using definition (5) for t = 1
$= \rho^1 f^1(z^1)q^1/\max_z \{\rho^0 f^0(z)F^1(z,v^1) ; z \text{ belongs to } Z^1\}$ using definition (4)
$\leq \rho^1 f^1(z^1)q^1/\rho^0 f^0(z^1)F^1(z^1,v^1)$ since z^1 is feasible for the maximization problem
$= \rho^1 f^1(z^1)q^1/\rho^0 f^0(z^1)q^1$ using (6) for t = 1
$= \rho^1 f^1(z^1)/\rho^0 f^0(z^1)$
$\equiv P_{HP}$

where we have defined the *observable hedonic Paasche output price index* P_{HP} as

(14) $P_{HP} \equiv \rho^1 f^1(z^1)/\rho^0 f^0(z^1)$.

Thus the inequality (13) says that the unobservable theoretical Paasche type hedonic output price index $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$ is bounded from above by the observable (assuming that we have estimates for ρ^0, ρ^1, f^0 and f^1) hedonic Paasche output price index P_{HP}. The inequality (13) is the hedonic counterpart to a standard Paasche type inequality for a theoretical output price index.

Again, it is of interest to rewrite P_{HP} in terms of the observable model prices for the firm in periods 0 and 1. Rewrite (14) as follows:

(15) $P_{HP} \equiv \rho^1 f^1(z^1)/\rho^0 f^0(z^1)$
$= \rho^1 f^1(z^1)/\{\rho^0 f^0(z^0)[f^0(z^1)/f^0(z^0)]\}$
$= P^1/\{P^0[f^0(z^1)/f^0(z^0)]\}$ using definitions (11)
$= [P^1/f^0(z^1)]/[P^0/f^0(z^0)]$.

The prices $P^1/f^0(z^1)$ and $P^0/f^0(z^0)$ can be interpreted as *quality adjusted model prices* for the firm in periods 1 and 0 respectively, using the hedonic regression pertaining to period 0 to do the quality adjustment.

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[9] This type of constant utility interpretation of the quality adjusted prices dates back to Court (1939; 108) as his hedonic suggestion number one. The idea was explicitly laid out in Griliches (1971a; 59-60) (1971b; 6). It was implemented in a statistical agency sampling context by Triplett and McDonald (1977; 144).

[10] We assume that all ρ^t, $f^t(z)$ and $F^t(z,v^t)$ are positive for t = 0,1.

4. Finding an Output Price Index that is between the Observable Bounds

It is possible to adapt a technique due originally to Konüs (1924) and obtain a theoretical hedonic output price index that lies between the observable Laspeyres and Paasche bounding indexes, P_{HL} and P_{HP}, defined above. Recall the definition of the revenue function, $R(\rho^s f^s, F^t, Z^t, v)$, defined by (4) above. Instead of using either F^0, Z^0, v^0 or F^1, Z^1, v^1 as reference production functions, feasible characteristics sets and input vectors for the establishment in definition (7), let us use a *convex combination* or *weighted average* of these variables in our definition of a theoretical hedonic output price index. Thus for each scalar λ between 0 and 1, define the theoretical hedonic output price index between periods 0 and 1, $P(\lambda)$, as follows:

$$
\begin{aligned}
(16)\quad & P(\lambda) \\
& \equiv R(\rho^1 f^1,(1-\lambda)F^0+\lambda F^1,(1-\lambda)Z^0+\lambda Z^1,(1-\lambda)v^0+\lambda v^1)/R(\rho^0 f^0,(1-\lambda)F^0+\lambda F^1,(1-\lambda)Z^0+\lambda Z^1,(1-\lambda)v^0+\lambda v^1) \\
& = \max_z\{\rho^1 f^1(z)[(1-\lambda)F^0(z,(1-\lambda)v^0+\lambda v^1) + \lambda F^1(z,(1-\lambda)v^0+\lambda v^1)] : z \text{ belongs to } (1-\lambda)Z^0+\lambda Z^1\}/ \\
& \qquad \max_z\{\rho^0 f^0(z)[(1-\lambda)F^0(z,(1-\lambda)v^0+\lambda v^1) + \lambda F^1(z,(1-\lambda)v^0+\lambda v^1)] : z \text{ belongs to } (1-\lambda)Z^0+\lambda Z^1\}.
\end{aligned}
$$

When $\lambda = 0$, $P(\lambda)$ simplifies to $P(\rho^0 f^0, \rho^1 f^1, F^0, Z^0, v^0)$, the Laspeyres type hedonic output price index defined by (9) above. Thus using the inequality in (9), we have:

$$(17)\ P(0) \geq P_{HL}$$

where P_{HL} is equal to $\rho^1 f^1(z^0)/\rho^0 f^0(z^0)$, the observable Laspeyres hedonic output price index defined by (10) above. When $\lambda = 1$, $P(\lambda)$ simplifies to $P(\rho^0 f^0, \rho^1 f^1, F^1, Z^1, v^1)$, the Paasche type hedonic output price index defined by (13) above. Thus using the inequality in (13), we have:

$$(18)\ P(1) \leq P_{HP}$$

where P_{HP} is equal to $\rho^1 f^1(z^1)/\rho^0 f^0(z^1)$, the observable Paasche hedonic output price index defined by (15) above.

If $P(\lambda)$ is a continuous function of λ between 0 and 1, then we can adapt the proof of Diewert (1983; 1060-1061), which in turn is based on a technique of proof due to Konüs (1924), and show that there exists a λ^* such that $0 \leq \lambda^* \leq 1$ and either

$$(19)\quad P_{HL} \leq P(\lambda^*) \leq P_{HP} \text{ or } P_{HP} \leq P(\lambda^*) \leq P_{HL}\ ;$$

i.e., there exists a theoretical hedonic output price index between periods 0 and 1 using a technology that is intermediate to the technology of the firm between periods 0 and 1, $P(\lambda^*)$, that lies *between* the observable[11] Laspeyres and Paasche hedonic output price indexes, P_{HL} and P_{HP}. However, in order to obtain this result, we need conditions on the hedonic model price functions, $\rho^0 f^0(z)$ and $\rho^1 f^1(z)$, on the production functions, $F^0(z,v)$ and $F^1(z,v)$, and on the feasible characteristics sets, Z^0 and Z^1, that will ensure that the maximum functions in the numerator and

[11] Again, we need estimates of the hedonic model price functions for both periods in order to implement these "observable" indexes.

denominator in the last equality of (16) are continuous in λ. Sufficient conditions to guarantee continuity are:[12]

- the production functions $F^0(z,v)$ and $F^1(z,v)$ are positive and jointly continuous in z,v;
- the hedonic model price functions $f^0(z)$ and $f^1(z)$ are positive and continuous in z;
- ρ^0 and ρ^1 are positive; and
- the sets of feasible characteristics Z^0 and Z^1 are convex, closed and bounded sets.

Now we have a theoretical output price index that is bounded by two observable indexes. It is natural to take a symmetric mean of the bounds in order to obtain a "best" single number that will approximate the theoretical index. Thus let m(a,b) be a symmetric homogeneous mean of the two positive numbers a and b. We want to find a "best" $m(P_{HL},P_{HP})$. If we want the resulting index, $m(P_{HL},P_{HP})$, to satisfy the time reversal test, then we can adapt the argument of Diewert (1997; 138) and show that the resulting m(a,b) must be the geometric mean, $a^{1/2}b^{1/2}$. Thus our candidate to best approximate a theoretical hedonic output price index is the following observable *Fisher hedonic output price index*:

$$
\begin{aligned}
(20)\ P_{HF} &\equiv [P_{HL}P_{HP}]^{1/2} \\
&= [\rho^1 f^1(z^0)/\rho^0 f^0(z^0)]^{1/2}[\rho^1 f^1(z^1)/\rho^0 f^0(z^1)]^{1/2} \qquad \text{using (10) and (14)} \\
&= [\rho^1/\rho^0][f^1(z^0)/f^0(z^0)]^{1/2}[f^1(z^1)/f^0(z^1)]^{1/2} .
\end{aligned}
$$

Note that P_{HF} reduces to ρ^1/ρ^0 if $f^0 = f^1$; i.e., if the hedonic model price functions are identical for each of the two periods under consideration, except for the proportional factors, $\rho^1$and ρ^0.

Instead of using (10) and (12) in the first line of (20), we can use (12) and (15). The resulting formula for the Fisher hedonic output price index is:

$$
\begin{aligned}
(21)\ \ P_{HF} &\equiv [P_{HL}P_{HP}]^{1/2} \\
&= \{[P^1/f^1(z^1)]/[P^0/f^1(z^0)]\}^{1/2}\{[P^1/f^0(z^1)]/[P^0/f^0(z^0)]\}^{1/2}.
\end{aligned}
$$

Formula (21) is our preferred one. It is the geometric mean of two sets of quality adjusted model price ratios, using the hedonic regression in each of the two periods to do one of the quality adjustments.

5. Conclusion

The above theory for the quality adjustment of establishment or firm output prices is not perfect. It has a number of weak parts:

- The theory assumes competitive price taking behavior on the part of the firm.
- The theory allows the firm or establishment to produce only one model in each period.[13]

[12] The result follows using Debreu's (1952; 889-890) (1959; 19) Maximum Theorem.

[13] The theory can of course be readily generalized to stand alone establishments or divisions that produce only one product in each period. However, this still does not deal with the usual case where the establishment produces many products in each period and there are shared inputs and overheads.

- The assumptions that are required to obtain the bounding result (19) are somewhat restrictive.[14]
- Our suggested method for converting the upper and lower bounds on the theoretical index into a single point estimator for the index by taking a geometric mean as in (20) may not appeal to all researchers.

Another aspect of our model may not appeal to all constructors of output price and quantity indexes: namely, our theory supports the "user value" approach to quality adjustment in the context of the producer price index rather than the "production cost" view. In our approach, the user valuations of the various models that could be produced in any given period flow through to producers via the period's hedonic function in the same way that output prices are taken as givens in the usual theory of the output price index. Triplett describes the user value and resource cost views of quality adjustment as follows:

The 'user value' approach looks at the output implications of quality change in some productive input; a machine is higher quality if it has higher productivity when used in making something else. On the 'resource cost' view, the cost of making a machine is the proper basis for making quality adjustments, not the productivity of using machines to produce other goods." Jack E. Triplett (1983; 304).

The resource cost view of quality change can be traced back to Denison (1957) and Jaszi (1971) and it has been justified by Triplett (1983) as being the theoretically correct method for doing quality change in the context of the producer output price index.[15] However, with respect to the producer input price index, Triplett (1983) endorsed the user value point of view.[16] Unfortunately, in the context of the national accounts, if the price for a transaction between two producers is given one quality adjustment for the seller and a different quality adjustment for the purchaser, then the resulting system of real accounts will not add up properly.[17] Eurostat has noticed this problem:[18]

"It is important to compile one unique measure of GDP volume growth. Although one may argue whether or not conceptually differences may exist between GDP volume from the output and expenditure approaches, in practice it would be highly undesirable to publish two different GDP growth rates."

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[14] In particular, the continuity assumptions on the hedonic functions rule out the appearance of new dummy variables in either period. Also, it may not be feasible to produce a convex combination of the models produced in periods 0 and 1.

[15] This resource cost point of view seems to be the viewpoint used in the U. S. Producer Price Index and the U. S. System of National Accounts.

[16] Triplett's (1983; 296-300) endorsement of the resource cost point of view for the output price index rests on a rather different hedonic model than the one that we have used (Triplett uses what Pollak (1983) terms the "L-Characteristics Approach") and this may explain why he obtained rather different results than we did.

[17] Thus it would be difficult to reconcile resulting economic accounts with any kind of general equilibrium model where supply equals demand. Triplett (1983; 272) touches on the difficulties involved in getting input output models in real terms to add up: "Even at relatively detailed levels, a price index is still an aggregation, and even if the prices were all measured in the same way, the weights for input and output price indexes for a similarly named commodity would make them different measurements. A price index for the output of steel mills has weights that differ from those of an input price index for steel used in the auto industry." This is a very valid criticism of existing published input output tables in real terms; i.e., because the aggregates in each cell are not homogeneous, we cannot expect rows to add up! However, at a disaggregated conceptual level, we should be able to decompose each value transaction into a unique price and quantity component and these components should be the same to both buyer and seller. Making a different quality adjustment to each side of the same transaction will violate this fundamental property.

[18] Hoffmann (2002) also noted the importance of this problem. He also pointed out some rather anomalous results that can arise in the Producer Price Index if the resource cost point of view is used.

"Compiling one unique measure of GDP volume requires full consistency between the concepts of price and volume used within the output approach and the expenditure approach. For example, adjustments for quality change of products should be made in the same way on both sides of the accounts." Eurostat (2001; 7-8).

Thus the Eurostat view does not seem to be consistent with Triplett's view that different quality adjustments should be done on opposite sides of the same transaction. Our view is that the user value approach to quality adjustment can be theoretically justified on both sides of the transaction and hence if this approach is used consistently, the Eurostat problem will vanish.

A final implication of our paper is that a hedonic regression can be used not only to quality adjust the prices of users of the hedonic family of products but the same hedonic regression can also be used to quality adjust the prices of suppliers.[19]

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[19] There are some additional complications with respect to weighting and the treatment of commodity taxes.

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