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A characterization of the Törnqvist price index[☆]

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Abstract

This note provides an axiomatic characterization of the Törnqvist price index.

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1. Introduction

In his survey of the axiomatic approach to index number theory, Balk (1995) listed among the topics for further research the problem of finding a characterization of the Törnqvist (1936) price index.

The axiomatic approach, also referred to as the test approach, consists of formulating 'desirable' properties which price and quantity indices should satisfy. Prices and quantities of commodities are thereby regarded as separate variables. The properties are formalized as functional equations, and a price (or quantity) index is said to be characterized by a set of functional equations if it is the unique solution to this set.

The economic approach assumes optimization, such as cost minimization or revenue maximization, which implies a relation between prices and quantities. Within the economic approach, Diewert (1976) obtained a characterization of the Törnqvist price index, namely as being the economic price index corresponding to a linearly homogeneous translog unit cost or revenue function.

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This note provides a characterization of the Törnqvist price index from the axiomatic approach. We consider a rather broad class of aggregated price relatives and show that the imposition of two rather natural requirements reduces this class to a single element, being the Törnqvist price index.

2. The main result

We consider two time periods, a base period, denoted by the label 0, and a comparison period, denoted by the label 1, and a set of commodities, labelled from 1 to N. The vectors of quantities and prices of these commodities will be denoted by $x^t \equiv (x_1^t, \ldots, x_N^t)$ and $p^t \equiv (p_1^t, \ldots, p_N^t)$ respectively ($t = 0, 1$). It will be assumed that $x^t, p^t \in \Re_{++}^N$. The value shares of the commodities are defined by $s_n^t \equiv p_n^t x_n^t / \Sigma_{n=1}^N p_n^t x_n^t$ ($n = 1, \ldots, N$; $t = 0, 1$). It is clear that

$$\sum_{n=1}^{N} s_n^t = 1 \ (t = 0, 1) \tag{1}$$

that is, the base and comparison period value shares add up to 1.

We consider here the class of aggregated price relatives $P(p^1, x^1, p^0, x^0)$ defined by

$$\ln P(p^1, x^1, p^0, x^0) \equiv \sum_{n=1}^{N} m_n(s_n^0, s_n^1) \ln(p_n^1/p_n^0), \tag{2}$$

where $m_n\colon [0,1] \times [0,1] \to \Re_+$ such that $m_n(0, 0) = 0$ ($n = 1, \ldots, N$). It will also be assumed that $N > 2$. Thus, $P(p^1, x^1, p^0, x^0)$ is a weighted product of the individual price relatives p_n^1/p_n^0 ($n = 1, \ldots, N$), whereby each weight is a function of the base and comparison period value shares of the commodity. Note that these functions might be different across commodities.

An important requirement is that a price index be linearly homogeneous in comparison period prices, that is

$$P(\lambda p^1, x^1, p^0, x^0) = \lambda P(p^1, x^1, p^0, x^0) \ (\lambda > 0). \tag{3}$$

Put otherwise, if the comparison period prices were to be λ times as high as they actually are, then the price index number should also be λ times as high. For the class of aggregated price relatives defined by (2), one verifies immediately that the requirement of linear homogeneity implies that

$$\sum_{n=1}^{N} m_n(s_n^0, s_n^1) = 1, \tag{4}$$

that is, the weights must add up to 1.

Another important requirement is that a price index satisfies the time reversal test, that is

$$P(p^1, x^1, p^0, x^0) = 1/P(p^0, x^0, p^1, x^1). \tag{5}$$

Put otherwise, the price index number for period 1 relative to period 0 and the price index number for period 0 relative to period 1 are reciprocal to each other. For the class of aggregated price relatives defined by (2), one verifies immediately that the time reversal test implies that

$$m_n(s_n^0, s_n^1) = m_n(s_n^1, s_n^0) \; (n = 1, \ldots, N), \tag{6}$$

that is, the functions $m_n(.,.)$ must be symmetric.

Our main result is now obtained by using Theorem 2 of Chapter 1 of Aczél (1987). This theorem states that the general solution of the functional Eqs. (1) and (4) is given by

$$m_n(s_n^0, s_n^1) = m(s_n^0, s_n^1) = \alpha_0 s_n^0 + \alpha_1 s_n^1, \; \alpha_0 + \alpha_1 = 1, \; \alpha_0, \alpha_1 \geq 0, \tag{7}$$

that is, the functions $m_n(.,.)$ are identical, and linear with positive coefficients adding up to 1. Immediate consequences are that the function $m(.,.)$ exhibits the mean property (that is, $m(a, a) = a$), is continuous, is increasing in its components, and exhibits the homogeneity property (that is, $m(\lambda a, \lambda b) = \lambda m(a, b)$ for all $\lambda > 0$). Moreover, the requirement of symmetry, (6), implies that

$$\alpha_0 = \alpha_1 = 1/2. \tag{8}$$

Thus, imposing linear homogeneity in comparison period prices and the time reversal test on the class of aggregated price relatives defined by (2) reduces this class to the Törnqvist price index, which is defined by

$$\ln P^T(p^1, x^1, p^0, x^0) \equiv \sum_{n=1}^{N} \frac{s_n^0 + s_n^1}{2} \ln (p_n^1 / p_n^0). \tag{9}$$

Reversely, it is straightforward to verify that the Törnqvist price index is linearly homogeneous in comparison period prices and satisfies the time reversal test.

We summarize the foregoing in a theorem.

Theorem 1. *The Törnqvist price index* (9) *is the unique member of the class of aggregated price relatives defined by* (2) *that has the property of being linearly homogeneous in comparison period prices*, (3), *and satisfies the time reversal test*, (5).

It goes without saying that a similar result could be established for the Törnqvist quantity index, by considering the similarly defined class of aggregated quantity relatives, linear homogeneity in comparison period quantities, and the time reversal test for quantity indices.

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