

# The Allocation of Aggregate Risk, Secondary Market Trades and Financial Boom-Bust Cycles\*

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## Abstract

The financial crisis of 2007-2008 started with the collapse of the market for Collateralized Debt Obligations backed by sub-prime mortgages. In this paper we present a mechanism aimed at explaining how a freeze in a secondary debt market can be amplified and propagated to the real economy, and thereby cause a recession. Moreover, we show why such a process is likely to be especially strong after a prolonged expansion based on the growth of consumer credit and endogenously low risk premia. Hence our model offers a new perspective on the links between the real and financial sectors, and we show how it can help make sense of several macroeconomic features of the 2001-2009 period. The key elements of the model are heterogeneity across agents in terms of risk tolerance, a financial sector that allocates systematic risk efficiently across agents, and real decisions that depend on the price of risk.

**Key Words:** Financial crisis, leverage, risk allocation

**JEL Class:** E3, E4

## 1 Introduction

The beginning of the financial crisis of 2008 is generally associated with a collapse of trading in the market for Collateralized Debt Obligations (CDOs). Up until the crisis, many of the standard macroeconomic indicators, such as consumer demand, firm balance sheets, productivity growth and firm profits painted a rather healthy picture of much of the economic landscape. The one weak spot was the housing market which had started a correction. The lack of obvious weak pre-crisis

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fundamentals outside the housing sector suggests that the collapse of the CDO market may itself have been a causal factor in the subsequent widespread recession. If this is the case, one needs to explain why a freeze in a secondary asset market can induce a major contraction of economic activity. The goal of this paper is to propose such a mechanism.

As a starting point, it is important to recognize that in a representative agent setting the continuous trading of existing assets is a redundant activity and therefore a freeze in a secondary asset market would generally have no effect on economic activity. Hence, to explain why limited trade in existing assets can derail economic activity, one needs to clarify how heterogeneity across agents interacts with secondary market asset trading and influences real decisions. The price of risk plays a central role in the mechanism we will present as it links the real and financial decisions. Given our assumptions on production, when the price of risk is high, it is not profitable for firms to invest in risky endeavors and this causes low economic activity. This mechanism is rather straightforward and easily understood. The key and more novel elements in our model come from the determination and dynamics of the price of risk due to interactions in the asset market between agents with different degrees of risk tolerance.

There are two main elements that directly affect the price of risk in our model. On the one hand, the wealth distribution across agents affects the price of risk as it is the wealth distribution that determines whether the most risk tolerant individuals can bear a large fraction of the aggregate risk in the economy. For example, if the most risk tolerant individuals have sufficient wealth, they can bear almost all the aggregate risk in the economy and this will favor a low price of risk. However, the price of risk depends not only on the wealth of the most risk tolerant individuals but also on the capacity of these risk tolerant individuals to trade their existing assets in asset markets so as to leverage their positions. It is only by leveraging their positions that risk tolerant individuals can fully take on their role as the major risk bearers for the aggregate economy.

In our setup, protracted expansions arise when risk tolerant individuals take on leveraged risky positions and have good outcomes. In this case, the wealth of the risk tolerant individuals increases faster than that of the general population, which allows them to bear a greater and growing share of aggregate risk. As their wealth increases and they bear a larger share of aggregate risk, the price of risk falls which stimulates economic activity. These expansions – with a falling price of risk and greater activity – tend to persist in our model.<sup>1</sup> We refer to such an expansion as being driven by

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<sup>1</sup>The mechanism at play in our model relies on the price of risk varying sufficiently over time. It is well known that generating quantitatively large variations in the price of risk in standard calibrated macroeconomic models is difficult. This difficulty is closely related to the challenge of producing a high equity premium in such models. Recent literature has suggested different routes that can potentially generate such features. For example, the use of Epstein-Zin preferences instead of standard time-separable preferences has been shown to be able to produce high and variable prices of risk. Since the goal of this paper is to highlight as simply as possible a particular mechanism, we do not take on the challenge of presenting a serious quantitative version of the model. If we were to do so, we

an increase in the risk taking capacity of the economy. This risk taking capacity expands because the wealth distribution tilts toward the risk tolerant individuals.

Such an expansion can come to an end for two reasons. First, the economy could get a bad draw in terms of fundamentals, such as a low productivity outcome. In this case, it is the risk tolerant individuals who lose the most as they bear a disproportionate share of the aggregate risk. Hence, when returns on previous investments turn sour, the risk taking capacity of the economy is severely reduced, causing the price of risk to increase and subsequent economic activity to fall. Even if the fundamental disturbance is short lived, the contraction in economic activity can be long lived as the negative effect on the wealth of the risk tolerant individuals – and thereby on the risk taking capacity of the economy – is persistent.

However, a bad draw in terms of fundamentals is not the only type disturbance that can stop such an expansion. If trade in secondary debt markets comes to a halt, then risk tolerant individuals may no longer be able to fulfill their role as major bearers of aggregate risk. For example, if risk tolerant individuals lend to risk averse households, they will want to use the resulting debt as collateral to take on risky leveraged positions. If the market for such assets becomes inactive, they will not be able to create such leveraged portfolios. This will reduce the risk taking capacity of the economy, leading to a rise in the risk premium and a fall in economic activity. Hence, in this case, it is the disturbance in a secondary market which creates a bust. Furthermore, by reducing the risk taking capacity of the economy, a temporary interruption of trade in the secondary market can generate low activity for an extended period through its persistent effect on the price of risk. This second scenario, we believe, may be relevant to understanding how the collapse of trade in secondary markets in mid 2008 contributed to the recession of late 2008 and early 2009.

In the above scenario, the economy can enter a recession simply because the secondary market for assets becomes inactive. But why might this happen? A simple and common explanation, and the one on which we build, is based on adverse selection. If the secondary market pools different assets with different degrees of riskiness, then such a market may be subject to multiple equilibria. In the good equilibrium, this market works well as it pools assets of all types. In the bad equilibrium, it breaks down, and there is no trade (or limited trade) because of the lemons problem. In our model, we use this mechanism to explain why the secondary market for assets may shut down rather unexpectedly. However, on this dimension, we are not providing any particularly novel insight.<sup>2</sup>

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would most likely need to extend the model to include elements known to help generate large prices of risk such as the use of Epstein-Zin preferences.

<sup>2</sup>In presenting the model, we begin by examining a situation where there is no adverse selection problem. Then we introduce asymmetric information and the lemons problem emerges giving rise to potential market freezes. While the resulting multiplicity of equilibria is simply an add-on to our basic model, we could make it more interrelated by

We should reiterate that the main contribution of the paper is not to explain a freeze in a secondary market. Rather, the primary contribution of the paper is to formalize and describe the propagation mechanism that transmits shocks in a secondary asset market to the rest of the economy. In particular, our work tries to explain why the collapse of a secondary market can have substantial effects on the real economy and the conditions under which this channel may be especially relevant. We believe this, along with our focus on financier net worth as a key driver of business cycles, to be the novel contributions of our paper.

Our work is related to at least two strands of the literature on finance and macroeconomics. The first is the work on credit market imperfections and the financial accelerator.<sup>3</sup> This literature formalizes environments in which the effects of shocks are amplified through financial frictions. Our model also generates a propagation mechanism for real shocks but the mechanism works differently as it does not rely on affecting the borrowing constraints of firms through the pricing of their assets or the value of their internal wealth. Instead our model emphasizes variation in the price of risk induced by changes in the aggregate supply of risk capital available on the market. Notice that in the standard credit friction models such as Bernanke and Gertler (1989) or Kiyotaki and Moore (1997), the key restriction is on firms whose borrowing is restricted by their collateral. In contrast, firms in our model face no restriction on borrowing at all. Instead, productivity shocks get propagated and amplified by the financial sector due to their effects on aggregate risk allocation in the economy.<sup>4</sup>

The paper is also related to the literature on behavioral finance which explores the allocations that arise in financial markets that are influenced by investor sentiment.<sup>5</sup> Particularly relevant is the work by Geanakoplos (2010) who explores the effects of investor heterogeneity in beliefs and how it can lead to excessive asset price volatility through leverage cycles. The key difference is that we examine the consequences of heterogeneity in risk aversion while his model examines the consequences of heterogeneity in beliefs. Additionally, in contrast to Geanakoplos, our focus is not just on asset price volatility but also on the link between the real and financial sides of the economy.<sup>6</sup>

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having the multiplicity arise as a sunspot on a state variable along the lines suggested by Manuelli and Peck (1992) or Spear, Srivastava and Woodford (1990). While such a structure could be more elegant, it would complicate matters without clarifying the main message of the paper regarding why and when secondary market freezes can have large real effects.

<sup>3</sup>For work along this line, see Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (2000), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and Williamson (1987).

<sup>4</sup>We should also note that in the Kiyotaki-Moore class of models, a financial crisis with consequent real effects would occur if the assets of firms became frozen since they wouldn't be able to borrow against them. However, in the 2008 episode it was not the assets of firms but rather the assets of financial institutions that became frozen. This latter effect is precisely what our structure can accommodate.

<sup>5</sup>The paper is somewhat related to the work by Holmstrom and Tirole (1998) which also emphasizes the importance of liquidity in secondary markets.

<sup>6</sup>Two other related papers are Brunnermeier and Sannikov (2011) and He and Krishnamurthy (2011). Both these papers study environments with heterogenous agents with the heterogeneity being in the efficiency with agents

The market-freeze aspect of our work is similar in spirit to Acharya et al (2009), Diamond and Rajan (2009) and Kurlat (2012) who all study environments in which credit markets are prone to freezing up. It is worth reiterating that, aside from the differences in details of the mechanism at work, our work differs from most of the related work on this topic in that we try to provide an explanation for the entire boom-bust cycle of 2001-2010 rather than just the bust associated with the financial crisis episode in 2008.<sup>7</sup>

The paper is structured as follows. In the next section we present a set of observations which motivates our modeling choices and our narrative for the 2001-2007 period. Section 3 presents a simple one-period model with heterogeneous agents and aggregate risk to highlight the role of insurance and the interaction of the real and financial sectors in the model. Section 4 extends the one-period model to a dynamic (overlapping generations) setting. In both sections 3 and 4, we abstract from any asymmetry of information and adverse selection problems so as to highlight the key features of our model. In Section 5 we introduce default risk and asymmetric information and show how the resulting adverse selection problem can cause a market freeze. The last section concludes. All proofs are consigned to the Appendix.

## 2 Empirical Patterns

Before presenting the model, we outline some facts about the evolution of the US economy between 2001-2007 since some of the features we build into our narrative rely on these facts. First, as is well known, this period was marked by a rapid build up of debt and leverage ratios. This can be seen in Panel (a) of Figure 1 which shows that the Debt to GDP ratios of households, the private sector as well as the domestic financial sector all went up during this period. Panel (b) of Figure 1 shows that this increase in leverage coincided with corporate profit rate in both the financial and non-financial sectors rising to historic highs during this period.

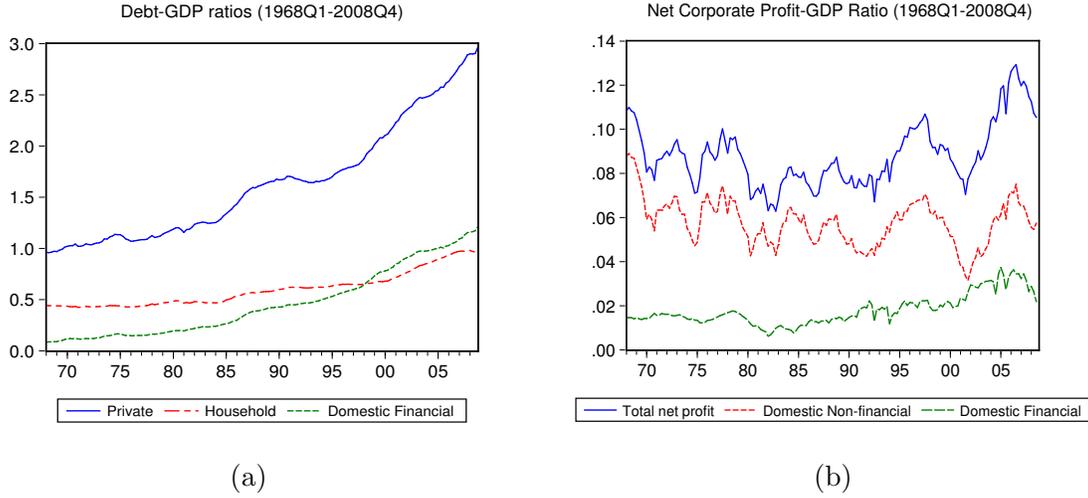
The high profit rate went hand-in-hand with high labor productivity, as Panel (a) of Figure 2 makes clear. With labor productivity being high and profits also rising, did firms start investing more during this period? Panel (b) of Figure 2 shows the rate of non-residential investment during this period. Contrary to what one would expect to see during a high productivity phase, investment

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can use capital. Aside from the difference in the source of agent heterogeneity, agents in those models face equity constraints in raising capital from households due to moral hazard problems. By contrast, in our model there is no moral hazard problem. The risk tolerant agents' ability to provide insurance is limited by the fact that they cannot honor contracts which give them negative consumption.

<sup>7</sup>There is a growing literature which analyzes implications of heterogeneity in terms of risk tolerance. Most of this literature focuses on asset price. See for example Chan and Kogan (2002), or Garleanu and Panageas (2008). Our paper differs from the literature by focusing on interactions between real and financial factors when agents differ in terms of risk tolerance.

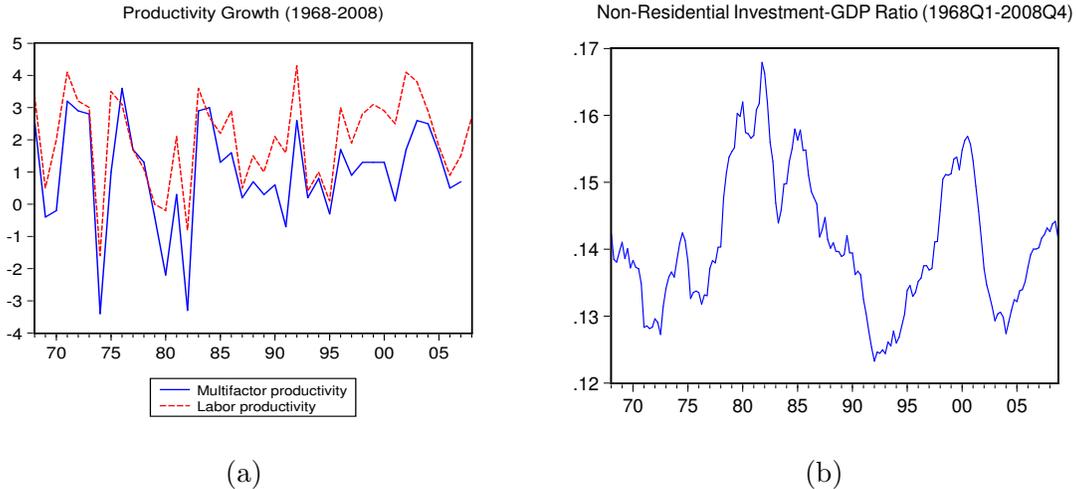
Figure 1: Leverage and Profit rates



Notes: The figures plot the quarterly debt to GDP ratios and the profit to GDP ratios in the US economy between 1968 and 2008.

rates stayed relatively low. In fact, non-residential investment during this phase was lower than during the recession of 1981.

Figure 2: Productivity and investment rates

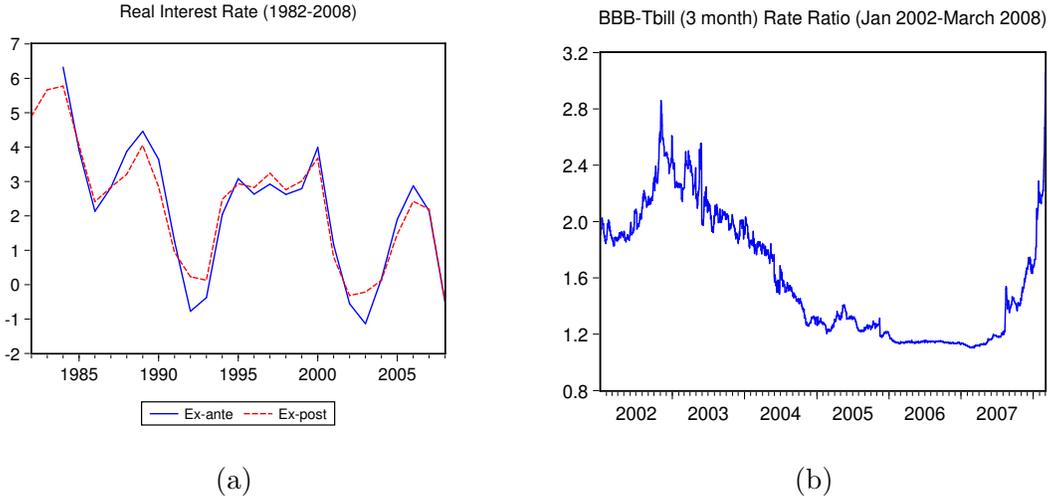


Notes: Panel (a) of this figure presents productivity growth, while panel (b) shows the quarterly non-residential investment rates in the US between 1968 and 2008.

This low investment rate is particularly surprising given the low real rates of interest and the low spread on corporate debt during this period, see Panels (a) and (b) of Figure 3, respectively.

To put this low investment rate in further perspective, Figure 4 reports the ratio of profits to

Figure 3: Real interest rate and the risk premium



Notes: Panel (a) shows the ex-ante and ex-post real interest rates between 1982 and 2008 while while panel (b) reports the monthly risk premium between Jan 2002 and March 2008 as measured by the spread between the BBB and the 3-month Tbill rate.

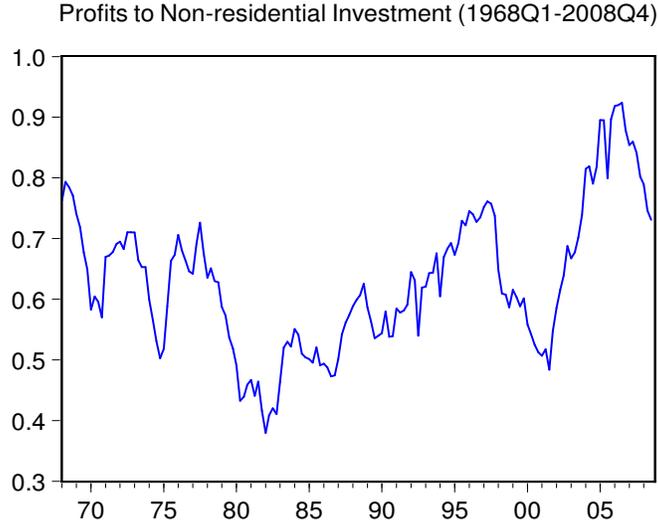
non-residential investment. As can be seen in the figure, the size of profits have historically been around 80% of non-residential investment. However, over the 2002-2007 period they represented more that 100% of non-residential investment. These observations lead us to view the 2002-2007 period as one where productive investments were scarce even though there were substantial profits available for reinvestment.<sup>8</sup>

### 3 A one period model

In order to flesh out the key mechanisms we want to emphasize, we start by presenting a simple one-period model of a closed economy with two types of agents – workers and financiers – who differ in their degree of risk aversion. The framework is designed to illustrate the macroeconomic properties of an economy which aims to efficiently allocate aggregate risk between agents with different degrees of risk tolerance. While the one period structure precludes any discussion of dynamics, it highlights the nature of the interaction between the real and financial sides of the model. We should note that the one-period model takes as given the inherited asset positions of workers and financiers. These positions will be rendered endogenous in the next section.

<sup>8</sup>A much more comprehensive and detailed survey of developments and events during the crisis at the end of this period can be found in Brunnermeier (2009).

Figure 4: Profits to non-residential investment ratio



Consider a one-period economy that produces output using the technology

$$y = Al \tag{1}$$

where  $A$  denotes labor productivity. Productivity is stochastic and follows an i.i.d. binomial process with probability mass function

$$A = \begin{cases} 1 & \text{with probability } q \\ \theta & \text{with probability } 1 - q \end{cases}$$

where  $\theta < 1$ . Productivity is the sole source of uncertainty in this economy.

Events unfold in this economy as follows: at the beginning of the period asset markets and labor markets open. Stocks and bonds are traded in asset markets while employment and wage contracts are struck in labor markets. Thereafter the productivity shock is revealed whereupon output is produced, claims are settled and agents consume.

### 3.1 Firms

Firms hire labor to produce output using the technology given by equation (1). Labor hiring decisions and wage payments need to be made before observing the productivity shock for the period. Without loss of generality, we can assume that firms finance this by issuing shares (risky claims) to output produced by labor. Each share pays one unit of output in the good state (when

$A = 1$ ) and  $\theta$  units of output in the bad state. Thus, firms maximize

$$p^s S - wl$$

subject to the solvency constraint  $S \leq l$ , where  $p^s$  denotes the price of stock,  $S$  denotes the number of shares sold,  $w$  is the wage rate and  $l$  is the level of employment. If  $w > p^s$ , then firms will not hire, if  $w < p^s$ , firms will want to hire an infinite amount. Hence, market equilibrium will require<sup>9</sup>:

$$p^s = w.$$

### 3.2 Workers

The economy is inhabited by a continuum of identical worker-households of measure one. Workers have one unit of labor time that they can allocate to work or leisure. Utility of a representative worker is given by

$$V^w = u(c^w + g(1 - l)), \quad u' >, u'' < 0, g' > 0, g'' \leq 0,$$

where  $c^w$  denotes consumption of the worker and  $l$  is labor supplied to the market.

We assume that each worker inherits a stock of debt  $d$  which needs to be paid off before the period ends. Thus, workers face the state-contingent budget constraints

$$p^s s^w + p^b b^w = wl$$

$$c_g^w = s^w + b^w - d$$

$$c_b^w = \theta s^w + b^w - d$$

where  $c_g^w$  denotes consumption by workers in the good state when  $A = 1$  while  $c_b^w$  denotes consumption in the bad state when  $A = \theta$ .  $s^w$  is stock holdings of workers while  $b^w$  denote their bond holdings. The first equation gives the budget constraint that workers face in their asset market transactions while the last two equations are the budget constraints that define their state contingent consumption allocations.

It is useful at this stage to note that the (percentage) expected excess return to holding stock versus bonds is given by  $\frac{p^b E[A]}{p^s} - 1$ . In the rest of the paper we will refer to  $\frac{p^b}{p^s}$  as the risk premium

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<sup>9</sup>This extremely simple relationship between the price of stock and the wage results from the assumption that there are constant returns to labor in the model. If we allowed for decreasing returns to labor, which is an easy generalization, the price of stock would vary more than wages which would be more empirically reasonable.

as it is just an affine transformation of the excess return.

The workers utility maximization problem leads to two first order conditions:

$$\frac{p^s}{p^b} = g'(1-l),$$

$$\frac{p^s}{p^b} = \frac{qu'(c_g^w + g(1-l)) + \theta(1-q)u'(c_b^w + g(1-l))}{qu'(c_g^w + g(1-l)) + (1-q)u'(c_b^w + g(1-l))}.$$

In this economy hiring labor is a risky activity since labor productivity is unknown prior to the employment decision. The market price of this risk over the risk-free alternative is  $\frac{p^s}{p^b}$ . The first condition shows that at an optimum workers equate the marginal utility from safe, non-market work with  $\frac{p^s}{p^b}$  which is the cost of withholding this labor from the risky market activity. The second condition dictates the optimal mix of stocks and bonds in the worker's portfolio. Note that worker behavior only depends on the ratio  $\frac{p^s}{p^b}$ , not on each price individually.

### 3.3 Financiers

Financiers are also identical and of unit mass. The representative financier maximizes utility  $V^F$  where

$$V^F = c^F$$

where  $c^F$  denotes financier consumption. The linearity of financier preferences implies that they are risk neutral which is a key source of difference relative to workers.

Financiers are born with an initial endowment of assets  $d$ , which we will refer to as their net worth. This is the inherited debt of workers which must be paid back to financiers at the end of the period. Thus, the state contingent budget constraints of the financiers are given by

$$p^s s^F + p^b b^F = 0$$

$$c_g^F = s^F + b^F + d$$

$$c_b^F = \theta s^F + b^F + d$$

where the first equation is the budget constraint on asset market transactions of financiers while the last two define the state-contingent consumption flows. Note that  $s^F$  denotes stock holdings of financiers while  $b^F$  denotes their bond holdings. Combining these constraints yields the following

restriction on all admissible state-contingent consumption allocations:

$$\left(\frac{p^s}{p^b} - \theta\right) c_g^F + \left(1 - \frac{p^s}{p^b}\right) c_b^F = (1 - \theta) d. \quad (2)$$

Equation (2) makes clear that the effective price of wealth in the good state is  $\frac{p^s}{p^b} - \theta$  while the corresponding price in the bad state is  $1 - \frac{p^s}{p^b}$ . Lastly, admissible consumption allocations must satisfy financier solvency which requires that  $c_g^F \geq 0$  and  $c_b^F \geq 0$ .

Financiers have a portfolio problem to solve in order to maximize their expected end of period consumption. The solution for this optimization problem dictates that at an optimum we must have

$$\frac{p^s}{p^b} \leq q + (1 - q)\theta = E[A].$$

If this condition is not satisfied then the marginal utility from  $c_g^F$  would be negative which would lead to a violation of the solvency restrictions  $c_g^F \geq 0$ . Intuitively, if the condition is violated then the risk premium is so low that the relative cost of taking equity positions is greater than the expected returns from stocks which would induce financiers to take short positions in stocks.

Given the linearity of financier preferences, there are two potential cases to be analyzed.

**Case 1:**  $\frac{p^s}{p^b} < E[A]$ . In this case the financier will choose an asset portfolio which results in  $c_b^F = 0$  and  $c_g^F = \frac{(1-\theta)d}{\frac{p^s}{p^b} - \theta}$  where the latter expression follows directly from equation (2). Note that from the financier budget constraint in the bad state (see above),  $c_b^F = 0$  implies that  $b^F = -\theta s^F - d < 0$ , i.e., financiers issue bonds to finance the purchase of a leveraged portfolio of stock.

**Case 2.** If  $\frac{p^s}{p^b} = E[A]$ , then the financier is indifferent between all consumption pairs that satisfy equation (2).

### 3.4 Equilibrium

The equilibrium in this one period economy arises when goods and asset markets clear. The state-contingent goods market clearing conditions are

$$c_g^w + c_g^F = l$$

$$c_b^w + c_b^F = \theta l$$

The corresponding asset market clearing conditions are

$$b^w + b^F = 0$$

$$s^w + s^F = S$$

**Definition:** A Walrasian Equilibrium for this economy consists of prices  $\left\{w, \frac{p^b}{p^s}\right\}$  and allocations  $\{c_g^w, c_b^w, c_g^F, c_b^F, l\}$  such that the allocations are optimal given prices and all markets clear at those prices.

Throughout the paper we shall maintain the following two assumptions:

**Assumption 1:**  $E[A] = q + (1 - q)\theta > g'(1)$

**Assumption 2:**  $g'(0) = \infty$ .

In order for the equilibrium of this economy to potentially involve positive levels of employment, it is necessary to assume that the technology is sufficiently productive relative to workers' value of time. This is captured by Assumption 1. Assumption 2 will ensure that equilibrium levels of employment are interior ( $0 < l < 1$ ). This will eliminate the need to cover potential corner solutions where  $l = 1$ .

### 3.5 Autarky

Before examining equilibrium outcomes for the economy with both workers and financiers, it is useful to first examine how this economy would behave if workers were in financial autarky. In terms of the structure above, this is equivalent to financiers having zero net worth, i.e.,  $d = 0$ . In this case, the equilibrium characterization of worker behavior is very simple. The Walrasian equilibrium is characterized by a time-invariant level of employment denoted by  $l^a$ , and a time invariant risk premium  $\frac{p_a^b}{p_a^s}$ . These two variables are determined by the equations:

$$\frac{p_a^s}{p_a^b} = g'(1 - l^a)$$

$$\frac{p_a^s}{p_a^b} = \frac{qu'(l^a + g(1 - l^a)) + \theta(1 - q)u'(\theta l^a + g(1 - l^a))}{qu'(l^a + g(1 - l^a)) + (1 - q)u'(\theta l^a + g(1 - l^a))}$$

where we have substituted out the state contingent consumption levels of workers by using the market clearing conditions. Output for this economy is given by  $Al^a$ .

The autarkic case makes clear the role that financiers play in this one period economy. Financiers provide insurance to workers and firms by acquiring claims to risky production. This margin is missing in the autarkic case with no financiers. This then leads to the question: How does the presence of individuals with different levels of risk tolerance affect the behavior of an economy when financial markets can allocate aggregate/systemic risk between them?

### 3.6 Comparative Statics

The market clearing conditions along with the household optimality conditions define a system of two equations that describe the effect of the state variable  $d$  on equilibrium employment  $l$  and the risk premium  $p^b/p^s$ . We summarize these effects in two propositions. Proposition 1 gives a description of the relations between employment and financier assets  $d$ , while Proposition 2 expresses the relationship for the risk premium.

**Proposition 1:** The level of employment is a continuous and weakly increasing function of the financier's net worth  $d$ . This function, which we denote by  $l = \phi^l(d)$ , is strictly increasing in  $d$  when  $d \in (0, \tilde{d})$ ,  $\tilde{d} > 0$ , and is constant for all  $d \geq \tilde{d}$ . Moreover,  $\phi^l(d) > l^a$  for all  $d > 0$ . The cutoff level  $\tilde{d}$  is defined by the solution to the following equations, where  $\tilde{l}$  is implicitly defined by  $q + \theta(1 - q) = g'(1 - \tilde{l})$ .

$$g'(1 - \tilde{l}) = \frac{qu' \left( \tilde{l} - \left( \frac{(1-\theta)\tilde{d}}{g'(1-\tilde{l})-\theta} \right) + g(1 - \tilde{l}) \right) + (1 - q)\theta u'(\theta\tilde{l} + g(1 - \tilde{l}))}{qu' \left( \tilde{l} - \left( \frac{(1-\theta)\tilde{d}}{g'(1-\tilde{l})-\theta} \right) + g(1 - \tilde{l}) \right) + (1 - q)u' \left( \theta\tilde{l} + g(1 - \tilde{l}) \right)}.$$

**Proposition 2:** The risk premium, which we denote by  $\frac{p^b}{p^s} = \phi^p(d)$ , is a continuous and weakly decreasing function of financier net worth  $d$ .  $\phi^p(d)$  is strictly decreasing in  $d$  when  $d \in (0, \tilde{d})$ , and  $\phi^p(d) = 1$  for all  $d \geq \tilde{d} > 0$ . Moreover,  $\phi^p(d) < \frac{p_a^b}{p_a^s}$  for all  $d > 0$ .

Propositions 1 and 2 indicate that the equilibrium of our model has the property that when financiers have greater net worth, employment is higher and the risk premium is lower. The easiest way to understand these results is by first noticing that the initial wealth position of financiers (or their net worth) reflects household debt. Since financiers are risk neutral, their initial wealth can be thought of as the amount of funds available for taking on risk. For this reason, it is useful to think of  $d$  as the amount of risk capital available to financiers for providing insurance to firms. When financiers have more wealth, they are able to borrow more in order to acquire more risky claims (stock). This increases the price of risky claims and, consequently, decreases the risk premium. The decline in the risk premium pushes firms to increase employment. As long as the expected value of investing in risky assets is greater than the value of investing in safe assets, financiers will choose a portfolio mix between bonds and stock which leads to positive net pay-outs only in the good state. This has the effect of maximizing the downward pressure on the risk premium and favoring employment. It is only when the risk premium disappears, i.e., when  $d > \tilde{d}$  (and hence  $\frac{p^b}{p^s} = q + (1 - q)\theta$ ), that financier wealth no longer has a positive effect on employment. Any increase in financier net assets beyond this threshold level is used solely to build a portfolio that keeps the

risk premium at zero.<sup>10</sup>

## 4 The model with dynamics

We now extend the model to a multi-period setting in order to illustrate the dynamics that would likely arise in the type of economy we explored in the previous section. While previously we had just started off financiers and worker-households with some inherited asset positions (specifically, their debt positions), we now endogenize those asset positions and trace out the implied equilibrium dynamics.

Our dynamic model economy builds on an overlapping generations setup inhabited by two kinds of agents – workers and financiers. At every date  $t$  new generations of workers and financiers are born who live for two periods. Thus, at each date mature workers overlap with young workers and mature financiers overlap with young financiers. We consider a closed economy with the same production structure as in the one-period model and the same stochastic description for the productivity parameter  $A$ .

We have chosen to adopt a two period overlapping generations setting to discuss dynamics since we believe it is the simplest way to extend our one period model and illustrate its multi-period implications. Moreover, we have chosen a pure consumption-loan version of a dynamic model since it captures, in an extreme form, the scarcity of profitable investment opportunities – a feature that we believe characterized the 2002-2008 period.<sup>11</sup> It also allows for a clear description of the macroeconomic implications of aggregate risk sharing, separate from any role played by collateral and physical capital accumulation.<sup>12</sup>

The timing of events is as follows: at the beginning of every period both asset markets and labor markets open. Mature agents buy and sell risky claims and risk free bonds in asset markets while employment and wage decisions are made in the labor market. We will refer to the risky claim as a stock and the risk free claim as a bond, although the risky claim can alternatively be thought of as a risky bond. After these markets close, a new cohort of agents is born, the productivity shock is revealed and output for the period is produced. After the productivity realization, all outstanding

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<sup>10</sup>It is important to note that the unambiguously positive effect of a decline in the risk premium on employment does rely on our assumption of Greenwood-Hercowitz-Huffman preferences. These preferences eliminate any wealth effects on labor supply. In the presence of wealth effects there would be potentially offsetting effects on labor supply of a falling risk premium.

<sup>11</sup>As we will show at the end of this section, the results from our consumption-loan model extend almost trivially to a situation where consumers can invest in durable goods (such as housing). For this reason, the debt taken on by consumers in the model can be interpreted as being akin to mortgages.

<sup>12</sup>In the following section, we will examine how the results change when we allow for productive but risky investment opportunities. We will show how certain implications of our model are diluted when there is an abundance of risky investment opportunities. This will also help explain why we believe the model is particularly relevant for the 2002-2008 period where such opportunities were scarce.

claims in asset markets are settled including the stock and bond claims contracted at the beginning of the period. At this point young agents may receive transfers from old agents and they can use these proceeds to borrow or lend in the debt market.<sup>13</sup>

#### 4.1 Workers

At every date a continuum of workers of measure one is born. Workers now live for two periods. In the first period of life they receive  $y$  units of the good as an endowment while they have one unit of labor time in the second period of life which they can use for either work or leisure. The lifetime welfare of a representative worker born at date  $t$  is given by

$$V_t^y = E_t [u(c_t^y) + u(c_{t+1}^o + g(1 - l_{t+1}))],$$

where  $c^y$  denotes consumption when young,  $c^o$  is consumption when mature and  $l$  denotes labor supply. We assume throughout that  $u' > 0$ ,  $u'' < 0$ ,  $g' > 0$ ,  $g'' \leq 0$ . We deliberately adopt a utility structure where there are no wealth effects on labor supply as we want to illustrate mechanisms that do not rely on this force. The structure we choose forces worker-households to face non-trivial decisions in terms of saving, labor supply and risk taking.

Workers can access an asset market at the end of the first period of life in order to borrow or lend. In the second period of life, workers supply labor and receive wages that they can invest in stocks and bonds. Stocks and bonds payout at the end of the period after the realization of  $A_t$ .<sup>14</sup> The state contingent flow budget constraints facing the worker in each period of life are

$$c_t^y = y + p_t^d d_t \tag{3}$$

$$p_{t+1}^s s_{t+1}^o + p_{t+1}^b b_{t+1}^o = w_{t+1} l_{t+1} \tag{4}$$

$$c_{gt+1}^o = s_{t+1}^o + b_{t+1}^o - d_t \tag{5}$$

$$c_{bt+1}^o = \theta s_{t+1}^o + b_{t+1}^o - d_t \tag{6}$$

where  $p^d$  denotes the price of debt,  $d$  denotes debt incurred by a young worker,  $p^s$  and  $p^b$  denote the price of stocks and bonds respectively,  $w$  denotes wages, while  $s$  and  $b$  denote stocks and bonds

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<sup>13</sup>In principle, we could allow for the young agents to use the proceeds from debt market transactions to buy a consumer durable (which could be interpreted as housing) which renders consumption services in both the first and second period of life. However, to clarify the mechanisms at work, we choose to focus for now on the case where goods are non-durables and we discuss the extension to the durable goods case later.

<sup>14</sup>We should note that our structure implies complete asset markets since there are two linearly independent securities and two states. Hence, we could equivalently formalize this environment using state-contingent claims markets.

respectively.  $c_{gt+1}^o$  denotes consumption of the mature worker at date  $t + 1$  in the good state when  $A = 1$ . Analogously,  $c_{bt+1}^o$  is consumption of the mature worker at date  $t + 1$  in the bad state when  $A = \theta$ .

The workers maximization problem leads to three optimality conditions:

$$\frac{p_{t+1}^s}{p_{t+1}^b} = g' (1 - l_{t+1}) \quad (7)$$

$$\frac{p_{t+1}^s}{p_{t+1}^b} = \frac{qu' (c_{gt+1}^o + g(1 - l_{t+1})) + \theta(1 - q)u' (c_{bt+1}^o + g(1 - l_{t+1}))}{qu' (c_{gt+1}^o + g(1 - l_{t+1})) + (1 - q)u' (c_{bt+1}^o + g(1 - l_{t+1}))} \quad (8)$$

$$p_t^d u' (c_t^y) = qu' (c_{gt+1}^o + g(1 - l_{t+1})) + (1 - q)u' (c_{bt+1}^o + g(1 - l_{t+1})) \quad (9)$$

The first two conditions are identical to the optimality conditions in the one-period model analyzed above. The third condition, which is new and arises due to the dynamic version of the model, determines optimal borrowing by young agents in the first period of life. It is the standard Euler equation which equates the marginal rate of substitution between current and expected future income with the relative price of current consumption  $1/p^d$ .

## 4.2 Financiers

In every period a new cohort of financiers is born which lives for two periods. Every cohort has a continuum of financiers of measure one. In the first period of life young financiers receive an endowment  $f$  of the good as well as transfers  $T$  from old financiers. They can either consume these resources or use them to lend to other young agents. In the second period of life they begin by transacting in stock and bond markets, which allows them to leverage their inherited asset position and create a risky portfolio. The only constraint on financiers is solvency in that their consumption cannot be negative. At the end of the period, they receive payments on all their claims. Financiers, as before, are assumed to be risk neutral.

We assume that financiers get no utility from consuming in the first period of their life. This simplifies the analysis as it makes the decision of young financiers trivial since they will simply lend all their resources to young worker-households. Allowing for financiers to consume when young can be easily accommodated, but does not provide additional insight. The second important assumption we make about financiers is that they get utility from transferring bequests to the next generation of financiers. In particular, we assume that their objective is to maximize

$$V_t^F = E_t [\min [\gamma c_{t+1}^F, (1 - \gamma) T_{t+1}]] ,$$

where  $c^F$  denotes consumption by financiers and  $\gamma$  controls the utility weight on bequests relative to own consumption.<sup>15</sup> Denoting the end-of-period resources of mature financiers by  $F$ , it is clear that  $T_{t+1} = \gamma F_{t+1}$ .

The objective of financiers born at date  $t$  reduces then to simply maximizing the value of their end-of-life resources. It is worth emphasizing that the assumptions imposed on the financiers make them act like a dynasty that maximizes expected wealth every period and consumes a fixed fraction of its resources. The bequest motive is a very simple way of linking outcomes across time.<sup>16</sup>

Since young financiers can access asset markets at the end of the first period, their budget constraint when young is

$$p_t^d a_t^F = f + T_t$$

where  $a^F$  denotes assets bought by young financiers. Market clearing for assets requires that the total assets of young workers and financiers add to zero, i.e.,  $a^F = d$ , i.e., the debt of young workers must be owed to the young financiers. According we will again refer to  $d_t$  as the net worth of financiers. Using this relationship, the flow budget constraints facing financiers in each state and period of life are given by

$$p_t^d d_t = f + T_t, \tag{10}$$

$$p_{t+1}^s s_{t+1}^F + p_{t+1}^b b_{t+1}^F = 0, \tag{11}$$

$$F_{gt+1} = s_{t+1}^F + b_{t+1}^F + d_t, \tag{12}$$

$$F_{bt+1} = \theta s_{t+1}^F + b_{t+1}^F + d_t, \tag{13}$$

where  $F_{gt+1}$  denotes financier resources at the end of period  $t + 1$  in the good state when  $A = 1$  while  $F_{bt+1}$  denotes resources in the bad state when  $A = \theta$ . The only limit on asset positions is that they must satisfy solvency in that  $F_{gt+1} \geq 0$  and  $F_{bt+1} \geq 0$ . As we showed in the one period model, all admissible choices by financiers must satisfy the consolidated resource constraint for the mature financier in the second period of life given by

$$\left( \frac{p_{t+1}^s}{p_{t+1}^b} - \theta \right) F_{gt+1} + \left( 1 - \frac{p_{t+1}^s}{p_{t+1}^b} \right) F_{bt+1} = (1 - \theta) d_t. \tag{14}$$

The portfolio problem facing mature financiers is identical to the one we solved in the one-period model earlier. Hence, those solutions apply here. The new aspect in this dynamic version of the model is the lending by financiers when young. Since they do not consume in the first period of

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<sup>15</sup>Financiers thereby act partially in a dynastic fashion.

<sup>16</sup>Although we have assumed that workers do not behave in a dynastic fashion, the analysis can be easily extended to include this possibility.

life, they simply lend their resources in the asset market. Using equation (10), the assets bought by financiers is given by

$$d_t = \frac{f + \gamma F_t}{p_t^d}$$

### 4.3 Equilibrium

We now describe the equilibrium for this economy. In equilibrium, transfers received by young financiers must satisfy

$$T_t = \gamma F_t.$$

Moreover, goods markets must clear in both the good and bad states. Thus, we must have

$$c_{gt}^y + c_{gt}^o + (1 - \gamma) F_{gt} = y + f + l_t, \quad (15)$$

$$c_{bt}^y + c_{bt}^o + (1 - \gamma) F_{bt} = y + f + \theta l_t. \quad (16)$$

The first condition describes market clearing in the good state: total consumption of young and mature workers, and financiers must exhaust total output in that state. The second equation is the analogous condition for the bad state.

Lastly, asset market clearing requires that both bond and equity markets must clear

$$b_t^F + b_t^o = 0,$$

$$s_t^F + s_t^o = S_t,$$

**Definition:** A Walrasian Equilibrium for this economy consists of a sequence of prices  $\{w_t, \frac{p_t^b}{p_t^s}, p_t^d\}$  and a sequence of allocations  $\{c_{gt}^y, c_{bt}^y, c_{gt}^o, c_{bt}^o, F_{gt}, F_{bt}, l_t\}$ , such that all agents find their allocations to be optimal given prices, and all markets clear.<sup>17</sup>

In principle, these sequences could depend on the whole history of realizations of  $A_t$ . However, as we shall show, the equilibrium can be represented in recursive form where the level of financier net worth plays the role of a state variable. In particular, we will show the existence of an equilibrium transition equation for financier net worth of the form  $d_t = \phi^d(d_{t-1}, A_t)$ , whereby current financier net worth  $d$  depends only on past net worth and the current realization of the state of nature. Other variables can then be expressed as functions of  $d$ .

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<sup>17</sup>Note that the equilibrium only determines  $\frac{p_t^b}{p_t^s}$ , not each individually. This is because the second period asset market is really only a risk market.

In the rest of the paper we shall impose the following restriction on preferences:

**Assumption 3:**  $u(x) = \log x$ .

Assumption 3 is more restrictive than needed, but is sufficient for demonstrating the mechanisms that we are interested in.

#### 4.4 Equilibrium Characterization

We now discuss the equilibrium properties of our model economy. We start by describing the equilibrium allocations. Combining equations (3) and (10) gives consumption of young workers in each state:

$$c_{gt}^y = y + f + \gamma F_{gt},$$

$$c_{bt}^y = y + f + \gamma F_{bt}.$$

The market clearing conditions then directly yield the state contingent consumptions of mature workers:

$$c_{gt}^o = l_t - F_{gt},$$

$$c_{bt}^o = \theta l_t - F_{bt}.$$

The three key variables of interest in the model are: the level of financier net worth  $d_t$ , the employment rate  $l_t$  and the risk premium captured by  $\frac{p_t^b}{p_t^s}$ . Both employment and the risk premium at date  $t$  are determined within the period as outcomes of the solution to equations (7) and (8). This solution is identical to the solutions for employment and the risk premium in the one-period model that we derived previously. Hence, Propositions 1 and 2 continue to apply here.

The solution for assets,  $d_t$ , however, depends on both the current state of nature as well as the initial level of assets,  $d_{t-1}$ . From equation (10) we have  $p_t^d d_t = f + \gamma F_t$ . From the young workers' first order condition for optimal borrowing we also have

$$p_t^d = \frac{qu'(c_{gt+1}^o + g(1 - l_{t+1})) + (1 - q)u'(c_{bt+1}^o + g(1 - l_{t+1}))}{u'(c_t^y)}.$$

The equilibrium evolution of financier assets in this economy can be derived by combining these two expressions.

In light of Propositions 1 and 2, there are two potential regions of  $d$  to be considered:  $d \leq \tilde{d}$  and  $d > \tilde{d}$ . In each of these regions the dynamic evolution of assets is dependent on the current state of nature (the realization of  $A$ ) aside from the inherited level of assets. This is easy to see from the fact that bequests to young financiers,  $\gamma F_t$ , as well consumption of young workers,  $c_t^y$ , depend

directly on the current state of nature. Hence, both the price of assets  $p^d$  and the level of current assets will depend on the current state of nature as well as the inherited level of assets.

In view of this dependence, the equilibrium dynamic evolution of financier net worth can be summarized by a pair of transition equations:

$$d_t = \begin{cases} \phi^d(d_{t-1}, A_t) & \text{if } d_{t-1} \leq \tilde{d} \\ \tilde{\phi}^d(d_{t-1}, A_t) & \text{if } d_{t-1} > \tilde{d} \end{cases} .$$

We summarize the key dynamic properties of this economy in the following proposition:

**Proposition 3:** The equilibrium evolution of financier net worth is characterized by three key features:

1) For a given state of nature ( $A_t = 1$  or  $A_t = \theta$ ), financier net assets at time  $t$  are a continuous and weakly increasing function of  $d_{t-1}$ . For  $A_t = 1$ ,  $d_t$  is strictly increasing in  $d_{t-1}$ . For  $A_t = \theta$ ,  $d_t = \underline{d}$  for  $d_{t-1} \leq \tilde{d}$  while  $d_t$  is strictly increasing in  $d_{t-1}$  for  $d_{t-1} \geq \tilde{d}$ .

2) There exists a fixed point for  $\phi^d(d, 1)$ , denoted by  $\bar{d} = \phi^d(\bar{d}, 1)$ , and another one for  $\phi^d(d, \theta)$ , denoted by  $\underline{d}$ . Financier assets in this economy will fluctuate within the range  $\bar{d}$  and  $\underline{d}$ .

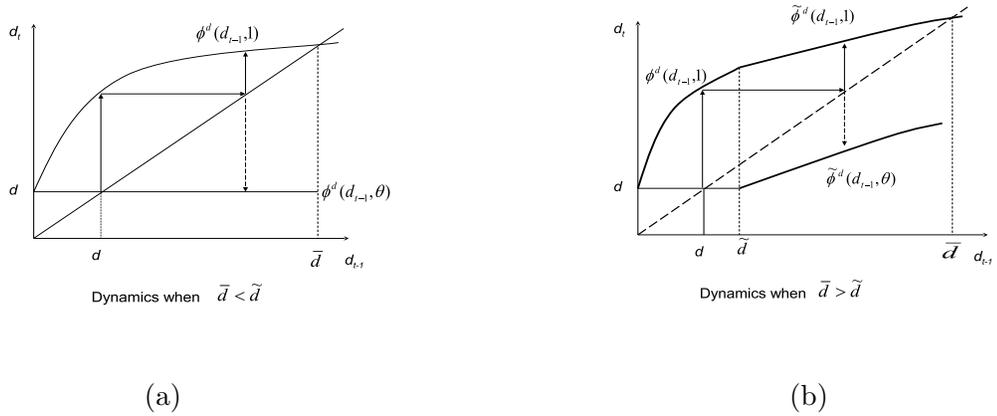
3) For a given level of  $d_{t-1}$ ,  $d_t$  is always greater after a good realization of the state of nature than after a bad realization, i.e.,  $\phi^d(d_{t-1}, 1) > \phi^d(d_{t-1}, \theta)$  and  $\tilde{\phi}^d(d_{t-1}, 1) > \tilde{\phi}^d(d_{t-1}, \theta)$ .

The asset dynamics take the following form: suppose the initial level of financier net worth is some  $d \in [\underline{d}, \bar{d})$ . If the state of nature is good, then their assets will grow. Assets will continue to grow as long as the state is good, and it will gradually approach  $\bar{d}$ . However, if at any date the state of nature is bad, then financier assets fall. If  $\bar{d} < \tilde{d}$  then it falls immediately to  $\underline{d}$ . If  $\bar{d} > \tilde{d}$  then it can take several periods of bad outcomes for financier net worth to converge to  $\underline{d}$ . Panel (a) of Figure 5 depicts the dynamics for the case  $\bar{d} < \tilde{d}$  while Panel (b) of Figure 5 shows the dynamics when  $\bar{d} > \tilde{d}$ . The solid arrows in the two figures depict the dynamic behavior of financier assets in response to good productivity shocks while the dashed arrows show the response of the economy to a low productivity shock.

Given the results expressed in Propositions 1 and 2, the dynamics for employment and the risk premium follow easily from Proposition 3. Thus, employment will continuously rise following a set of good outcomes, but it will drop when a bad state arises. In particular, if  $\bar{d} < \tilde{d}$ , it will drop immediately to its lowest level  $\phi^l(\underline{d})$  following the realization of the bad state. This implies that the fall in employment will be greater the longer an expansion period has been. The behavior of the risk premium is a mirror image of employment.

This highlights a key feature of our model. The presence of debt in financier portfolios links

Figure 5: The dynamic behavior of debt



Notes: The figures show the dynamics of debt implied by the model in different regions of the state variable  $d$ .

periods and thereby acts as a transmission mechanism for shocks. In good states financiers accumulate more claims on households (higher  $d$ ) which raises the level of resources that they have in the next period to provide insurance cover for risky employment (we call this risk capital). The presence of risk capital and its dependence on the current state of nature links adjoining periods and thereby provides a propagation mechanism. A good productivity shock raises resources of financiers which then translates into more insurance, a lower risk premium and greater employment tomorrow. Similarly, a transitory negative productivity shock in any period (say a low  $\theta$ ) translates into low employment in the next period due to its effect on financier balance sheets today. Crucially, the existence of financial markets ends up increasing the volatility in the economy relative to autarky while also raising the mean level of economic activity.

#### 4.5 An Illustrative Example

In order to illustrate the dynamic evolution of the economy we now present a simple example where the disutility from labor (or equivalently, the technology for home production) is linear, that is,

$$g(1-l) = (1-l)g^*, \quad \theta < g^* < 1.$$

Under this specification equilibrium employment<sup>18</sup> is given by

$$l_t = \frac{(1-q)(1-\theta)}{(1-g^*)(g^*-\theta)} d_{t-1} + \frac{[q+(1-q)\theta-g^*]}{(1-g^*)(g^*-\theta)}.$$

Using this solution for employment in the equilibrium difference equation for financier assets gives the following state contingent solutions for  $d_t$ :

1. If  $A = 1$  then

$$d_t = \left( \frac{f + \psi d_{t-1}}{y + 2f + 2\psi d_{t-1}} \right) g^*$$

where  $\psi \equiv \frac{\gamma(1-\theta)}{g^*-\theta}$ . It is easy to check that  $d_t$  is increasing in  $d_{t-1}$  in this case. Moreover, this mapping between current and past assets has a unique positive steady state which is given by

$$\bar{d} = \left( \frac{1}{4\psi} \right) \left[ -(y + 2f - \psi g^*) + \sqrt{(y + 2f - \psi g^*)^2 + 8\psi g^*} \right].$$

Clearly,  $d_t = \left( \frac{f}{y+2f} \right) g^* > 0$  when  $d_{t-1} = 0$  and  $d_t = \frac{g^*}{2} < \infty$  when  $d_{t-1} \rightarrow \infty$ . Thus, for very low levels of  $d_{t-1}$  we must have  $d_t > d_{t-1}$  while  $d_t < d_{t-1}$  for arbitrarily high levels  $d_{t-1}$ . This, along with the fact that  $d_t$  is increasing in  $d_{t-1}$ , is sufficient to establish that the steady state is stable.

2. If  $A = \theta$  then

$$d_t = \underline{d} = \left( \frac{f}{y + 2f} \right) g^*$$

Lastly,

$$d_t|_{A_t=1} > d_t|_{A_t=\theta}$$

which follows directly from the expressions for equilibrium  $d_t$  in the two cases, along with the fact that  $d_t$  is increasing in  $d_{t-1}$  for  $A_t = 1$ . Hence, financier net worth in a good state must be greater than financier net worth in a bad state. Hence, as long as the economy gets good productivity draws, net worth will keep growing along with employment. As soon as a bad productivity shocks hits the economy, financier net worth will jump down to  $\underline{d}$  as will employment in the following period. The process will then start up again.

## 4.6 Discussion of portfolio positions

It is instructive to clarify the evolution of the balance sheet of financiers. When a financier enters the second period of his life, he initially holds only household debt as assets. Therefore his balance sheet looks as follows:

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<sup>18</sup>Since this example does not satisfy the Inada condition that  $g'(0) = \infty$ , it now becomes necessary to check that given the equilibrium solution for  $d_{t-1}$ , employment is bounded between 0 and 1. For the purposes of the example, this can be accomplished by appropriate restrictions on  $g^*$ ,  $q$  and  $\theta$  (as well as the other parameters of the system).

Assets	Liabilities
$d_{t-1}$	Own Equity

In general, a risk neutral financier will not be satisfied with such a balance sheet since his return on own equity is rather low. To increase his expected return on equity, the financier transacts on the second period assets market to buy stocks in the amount:

$$s_t^F = \frac{d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta},$$

and issues bonds in the amount:

$$b_t^F = \frac{p_t^s d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta}.$$

After these transactions, a financier's balance sheet is given by

Assets	Liabilities
$p_t^s s_t^F$	$b_t$
$d_{t-1}$	Own Equity

(Note that in the above the price of bonds is normalized to 1). These transactions in the asset market allow the financier to construct a leveraged portfolio which gives him an expected return on equity which is greater than that associated with his initial asset position. Furthermore, it can be verified that the resulting return on equity is also higher than simply holding a pure equity position, which is directly due to the leveraging. Although this new asset position is more risky, it is preferred by the financier.<sup>19</sup>

In describing the evolution of the financier's asset position, we believe it is informative to keep track of gross positions rather than immediately netting out  $d_{t-1}$  and  $b_t$ . In fact, in our model  $d_{t-1}$  and  $b_t$  are slightly different as  $d_{t-1}$  represents long term financier assets, while  $b_t$  is more akin to short term debt. Obviously, as they have the same maturity date, they eventually become perfect substitutes. One advantage of keeping track of gross positions is that it may better capture actual positions observed in financial markets since many individuals have both short and long positions in assets. If we instead net out the asset position, the financiers balance sheet will look as below, where we see that the financier has created a leveraged position in stocks.

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<sup>19</sup>There are many ways to describe the type of transaction undertaken by financiers. For example, financiers can be seen as performing a swap between their holdings of household debt from the previous period for risky equity claims on firms – a type of “debt-equity” swap.

Assets	Liabilities
$p_t^s s_t^F$	$b_t - d_{t-1} = \frac{\theta d_{t-1}}{\frac{p^s}{p^b} - \theta}$
	Own Equity

## 5 Default Risk and Adverse Selection

Thus far we have focused on an environment where all information is available to all agents. While it was a useful assumption that allowed us to highlight a set of mechanisms, it did not permit us to study environments in which asset markets may freeze up due to adverse selection. We turn to this issue now by introducing information heterogeneity into the model. We should stress that while the idea that asymmetric information may cause markets to freeze up is well established, the value added of this section is to illustrate how adverse selection in an asset market can cause major disruptions in economic activity. We would like to emphasize at the outset that the point of this section is not to assert that adverse selection was the main cause of the market freeze during the financial crisis. Rather, the aim of this section is to illustrate the consequences of a negative shock to financier wealth. The adverse selection based mechanism we are proposing here is but one of many different ways in which financier wealth could be shocked.<sup>20</sup>

In order to simplify the analysis, in the remainder of the paper we shall maintain the following assumption which eliminates the need to consider corner situations where the risk premium is completely eliminated:

**Assumption 4:**  $f$  and  $y$  are such that  $\bar{d} < \tilde{d}$ .

Let us now suppose that workers who takes on debt when young may default on this debt when old. Specifically, let  $\psi^i$  be the probability that a worker  $i$  will have productive market labor at the beginning of the second period life. Hence, with probability  $1 - \psi^i$  worker  $i$ 's labor productivity in market work is zero. Each worker draws a  $\psi \in [0, 1]$  from an i.i.d. distribution with density  $f(\psi)$ . Since there is a continuum of workers of unit mass, the mean of the distribution,  $E[\psi]$ , is also the expected fraction of workers that will have productive market labor. Lastly, assume that the actual productivity of market labor of a prospective worker is revealed to both the worker and firms just before the labor market opens. For this reason, there will not be any information problem in the labor market. The source of asymmetric information will revolve only around asset markets, in particular the market for existing financier assets (or equivalently, worker debt).

<sup>20</sup>However, the secondary market for assets is naturally susceptible to adverse selection problems. Another recent paper that also explores the role of market freezes due to a lemons problem in the market for secondary assets in transmitting aggregate shocks is Kurlat (2012).

If the worker has productive market labor, then he will behave as before in the second period of life. If he does not have productive market labor, he does not get employed, i.e.,  $l = 0$ . In this event, he defaults on his entire debt obligation. His only resource is his time endowment of one unit which he then devotes entirely to home production which produces  $g(1)$  units of consumable goods. Hence, without productive market labor, the worker gets  $u(g(1))$  as utility.<sup>21</sup> Clearly,  $E[\psi]$  is the expected probability of a young worker repaying his debt in the second period of life while  $1 - \psi^i$  is the probability of default of worker  $i$ .

We assume that each financier's portfolio of assets brought into a period is characterized by a specific  $\psi$ . This corresponds to an environment where each financier buys debt from workers with the same  $\psi$ , i.e., each financier buys debt from young workers from a single point in the distribution of  $\psi$ .<sup>22</sup>

Since the quality of outstanding worker debt is heterogeneous, it is helpful to assume that in every period asset markets begin operations with the opening of a market for existing debt. In particular, we assume that intermediaries can set up and offer to buy up the risky assets of individual financiers (which are the risky debt of workers) in order to pool the different default risks. The assets held by such an intermediary will be equivalent to a synthetic package of debt whose payoff has the expected repayment rate of the whole distribution of debt sold to it. Financiers can sell their risky assets to these intermediaries, while the intermediaries finance themselves by issuing bonds to households. Note that the overall quality of the portfolio held by the intermediaries will, in equilibrium, be common knowledge, as all agents will be able to infer who supplied their assets to the market. Hence, these intermediaries will be able to finance themselves by issuing risk free bonds. The potential for an adverse selection problem will arise between financiers and the intermediaries, as we assume that intermediaries cannot directly assess the quality of assets being sold to it by an individual financier. We denote the average payoff per unit of assets held by the intermediary by  $\bar{\psi}$ .

A financier who sells his assets to an intermediary can use the proceeds to build, as before, a new portfolio position. In the following we shall denote the price paid by an intermediary for a unit of worker debt previously held by a financier to be  $p^k$ . The total quantity of financier assets bought by intermediaries will be denoted by  $k$ , and total value of bonds issued to buy them is  $p^k k$ , as we assume free entry in the intermediary sector.

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<sup>21</sup>We are assuming that labor continues to be productive in home production even when its market productivity is zero.

<sup>22</sup>This assumption considerably simplifies the analysis, especially the asymmetric information case we study later. There are alternative ways of setting up the asset portfolio of financiers but they come at the cost of tractability and algebraic complications.

## 5.1 Symmetric Information

We start with the case where there is heterogeneity across quality of financier assets but no asymmetric information. In particular, let us assume for now that no one in the asset market knows the individual  $\psi$  of the assets held by financiers (including the financiers themselves). In this case, we will show that our preceding equilibrium analysis can be carried through with almost no changes. This section is nevertheless useful for understanding the case with asymmetric information.

In the first period of life, young workers will recognize the probability of not having productive market labor when mature and will take this into account when borrowing. The constraint faced by workers in the first period of life is still given by equation (3). The constraints faced by mature workers with productive market labor are also unchanged. Thus, the optimality conditions for the labor-leisure choice as well as the portfolio choice between stocks and bonds, equations (7) and (8) respectively, remain unaltered. The first order condition for optimal borrowing of young workers does change though due to the default probability. The new condition is given by:

$$p_t^d u'(c_t^y) = E[\psi] [q u'(c_{gt+1}^o + g(1 - l_{t+1})) + (1 - q) u'(c_{bt+1}^o + g(1 - l_{t+1}))]. \quad (17)$$

The important new element introduced by the presence of default risk is an additional equilibrium condition for the price of assets bought by financiers. This arbitrage condition associated with free entry into the intermediary sector is given by:

$$\bar{\psi} = \frac{p_{t+1}^k}{p_{t+1}^b} \quad (18)$$

The budget constraint facing young financiers is unaffected by the introduction of default risk. For a mature financier who sells his assets to an intermediary, the constraints in the second period of life are now given by:

$$p_{t+1}^s s_{t+1}^F + p_{t+1}^b b_{t+1}^F = p_{t+1}^k d_t,$$

$$F_{gt+1} = s_{t+1}^F + b_{t+1}^F \geq 0,$$

$$F_{bt+1} = \theta s_{t+1}^F + b_{t+1}^F \geq 0.$$

If the financier does not sell his assets on the market, he has no liquidity to build a new portfolio and therefore must simply hold his existing assets to maturity. Under the current assumption that financiers do not know the  $\psi$  of their own portfolio, no financier has any incentive to not sell their assets to the intermediaries. Since all assets gets sold to intermediaries, the average payoff on the

assets held by intermediaries is

$$\bar{\psi} = \int_0^1 \psi f(\psi) d\psi.$$

Moreover, in equilibrium, we must also have the aggregate relationship

$$k_t = d_{t-1}.$$

The portfolio re-balancing by mature financiers through asset sales implies that their end-of-period resources are now given by

$$F_t^g = \frac{(1-\theta)\bar{\psi}d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta},$$

$$F_t^b = 0,$$

while the stock position of financiers is given by

$$s_t^F = \frac{\bar{\psi}d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta},$$

Lastly, the equilibrium state-contingent consumption levels of mature workers are given by

$$\hat{c}_{gt}^o = l_t - \frac{(1-\theta)\bar{\psi}d_{t-1}}{g'(1-l_t) - \theta},$$

$$\hat{c}_{bt}^o = \theta l_t.$$

with the equilibrium level of employment being determined by the condition:

$$g'(1-l_t) = \frac{qu' \left( l_t - \frac{(1-\theta)\bar{\psi}d_{t-1}}{g'(1-l_t) - \theta} + g(1-l_t) \right) + \theta(1-q)u'(\theta l_t + g(1-l_t))}{qu' \left( l_t - \frac{(1-\theta)\bar{\psi}d_{t-1}}{g'(1-l_t) - \theta} + g(1-l_t) \right) + (1-q)u'(\theta l_t + g(1-l_t))} \quad (19)$$

The transition equation for financier net worth remains unchanged as  $d_t = \frac{f+T_t}{p_t^d}$ , although the actual transfer does change as  $F_{gt}$  changes.

In brief, the equilibrium conditions with default are identical to those without default up to the following two transformations: take the new equilibrium system, and define  $\frac{p_t^d}{E[\psi]}$  as the price of financier assets, and  $E[\psi]d_t$  as financier net worth. The resultant system is identical to the one without default. Hence, Propositions 1 to 3 continue to hold.

## 5.2 Asymmetric Information

Now we introduce private information into the model. In particular, let us suppose that mature financiers only learn the repayment rate  $\psi$  on their own portfolios at the beginning of the period when they enter the asset market. The first thing to note is that when the  $\psi$  characterizing the portfolio of a financier is only known to that financier, there will potentially be an adverse selection problem in the asset market. Financiers with relatively high quality assets (low default probability) may have an incentive not to offer their assets on the secondary market since the market will value their assets at the average default rate, which is higher than the default rate of their portfolio. This implies that only low quality assets (i.e., the highest risk worker debt) may be offered for sale to intermediaries.

Let us denote by  $\hat{\psi}$  the conjectured average repayment rate of the assets that are offered on the market. As we saw in the previous subsection, the zero profit condition for intermediaries implies that the market price of the assets offered on the market is given by

$$\frac{p_t^k}{p_t^b} = \hat{\psi}$$

The problem facing mature workers remains unaffected by this new environment relative to the symmetric information case. For financiers however, the problem changes. Every financier now has to make a choice about whether or not he wants to offer his asset holdings to intermediaries. How will financiers decide whether to sell their assets or to keep them? For a given conjecture  $\hat{\psi}$ , the budget constraint facing a financier  $i$  with asset type  $\psi^i$  who decides to place his assets on the market is

$$p_{t+1}^s s_{t+1}^{Fi} + p_{t+1}^b b_{t+1}^{Fi} = p_t^k d_t$$

with the solvency constraints

$$F_{gt+1}^i = s_{t+1}^{Fi} + b_{t+1}^{Fi} \geq 0$$

$$F_{bt+1}^i = \theta s_{t+1}^{Fi} + b_{t+1}^{Fi} \geq 0$$

With  $\frac{p^s}{p^b} \leq E[A]$ , and under our maintained assumption that  $\bar{d} < \tilde{d}$ , a financier who offers his assets on the market at date  $t$  will use the proceeds to build a risky position which results in the

payoffs:

$$F_{gt}^i = \frac{(1-\theta)\hat{\psi}_t d_t^i}{\frac{p_t^s}{p_t^b} - \theta},$$

$$F_{bt}^i = 0,$$

Since the probability of the good state is  $q$ , the expected payoff from selling his assets on the market is  $\frac{q(1-\theta)\hat{\psi}_t d_t^i}{\frac{p_t^s}{p_t^b} - \theta}$ .<sup>23</sup>

Alternatively, financier  $i$  can choose not to supply his assets to the market and, instead, hold on to it. In this event his expected payoff is  $\psi^i d^i$ . Hence, financier  $i$  will choose to build a risky portfolio if and only if the expected payoff from doing so exceeds the payoff from holding on to it:

$$\frac{q(1-\theta)\hat{\psi}_t}{\frac{p_t^s}{p_t^b} - \theta} > \psi_t^i.$$

Now consider the marginal type who is just indifferent between using his asset holding to build a risky portfolio or simply holding on to his assets. For this marginal type, which we denote by  $\psi^m$ , we must have

$$\frac{q(1-\theta)\hat{\psi}_t}{\frac{p_t^s}{p_t^b} - \theta} = \psi_t^m.$$

Clearly, all financiers with  $\psi^i > \psi^m$  will choose to hold on to their assets while types with  $\psi^i \leq \psi^m$  will offer their asset holdings on the market.

The preceding implies that the average quality of assets offered for sale at time  $t$  is given by

$$\hat{\psi}_t = \frac{\int_0^{\psi_t^m} \psi f(\psi) d\psi}{F(\psi_t^m)}$$

where  $\psi^m$  denotes the highest financier type who sells his assets on the market and  $F(\psi)$  is the cumulative density function of  $f(\psi)$ . Hence, the consistency condition for this behavior to be optimal is that  $\psi^m$  must satisfy

$$\psi_t^m = \frac{q(1-\theta) \int_0^{\psi_t^m} \psi f(\psi) d\psi}{F(\psi_t^m) \left( \frac{p_t^s}{p_t^b} - \theta \right)} \quad (20)$$

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<sup>23</sup>Recall that if  $\frac{p_t^s}{p_t^b} = E[A]$  then financiers are indifferent between all combinations that satisfy their budget constraint. In that special case we shall continue to focus on the equilibrium where  $F_g > 0$  and  $F_b = 0$ .

where  $\frac{p_t^s}{p_t^b} = g'(1 - l_t)$  and  $l_t$  is the solution to:

$$g'(1 - l_t) = \frac{qu'(\tilde{c}_{gt}^o + g(1 - l_t)) + \theta(1 - q)u'(\tilde{c}_{bt}^o + g(1 - l_t))}{qu'(\tilde{c}_{gt}^o + g(1 - l_t)) + (1 - q)u'(\tilde{c}_{bt}^o + g(1 - l_t))} \quad (21)$$

where

$$\begin{aligned} \tilde{c}_{gt}^o &= l_t - d_{t-1} \int_{\psi^m}^1 \psi f(\psi) d\psi - \frac{(1 - \theta)\hat{\psi}_t d_{t-1}}{g'(1 - l_t) - \theta}, \\ \tilde{c}_{bt}^o &= \theta l_t - d_{t-1} \int_{\psi^m}^1 \psi f(\psi) d\psi. \end{aligned}$$

The last two expressions are the optimal consumption levels of mature workers in the good and bad states respectively. Note that the consumption levels of mature workers in this case differ from their corresponding levels under symmetric information by the term  $\left(\int_{\psi^m}^1 \psi f(\psi) d\psi\right) d_{t-1}$ . This term simply reflects the fraction of aggregate worker debt that is not placed on the market by financiers and on which they get repaid by mature workers with productive market labor. As  $\psi^m$  goes to one this term becomes vanishingly small and the consumption levels  $\tilde{c}_{gt}^o$  and  $\tilde{c}_{bt}^o$  tend to approach those under symmetric information.

Equations (20) and (21) jointly define the equilibrium for this economy at every date  $t$  in terms of  $\psi_t^m$  and  $l_t$  as a function of the inherited stock of assets  $d_{t-1}$ . The equilibrium values of all other endogenous variables can then be recovered recursively using these two solutions for  $\psi_t^m, l_t$  and the initial level of financier assets  $d_{t-1}$ . For future reference, we denote the solutions for equilibrium employment and the risk premium  $p^s/p^b$  as

$$\begin{aligned} \tilde{l}_t &= \tilde{\phi}^l(d_{t-1}, \psi_t^m) \\ \frac{\tilde{p}_t^s}{\tilde{p}_t^b} &= \tilde{\phi}^p(d_{t-1}, \psi_t^m) \end{aligned}$$

In general, there can be multiple solutions to the equilibrium system of equations (20) and (21). Below we illustrate the potential multiple equilibria by describing alternative equilibria.

### 5.2.1 Pessimistic Equilibrium: $\psi^m = 0$

In this model economy there always exists an equilibrium in which  $\psi^m = 0$ . We call this the ‘‘pessimistic’’ equilibrium, as it corresponds to the case where the market interprets any offer of assets as reflecting the worst type of asset, that is assets with a zero expected repayment rate. Since this equilibrium always exists, we will begin by characterizing it. In the pessimistic equilibrium, the asset market available to mature financiers breaks down in the sense that financiers do not

trade their assets on this market. Thus, they have no liquidity to build a new portfolio. Hence it must be workers that buy all the risky claims supplied by firms looking to finance working capital.

**Proposition 4:** There always exists a pessimistic equilibrium where  $\psi^m = 0$  and the asset holdings  $S$  and  $b$  of financiers are zero.

We next summarize the response of equilibrium employment to changes in initial assets and contrast this equilibrium with that in the autarkic case with no financiers. The main element to notice in Proposition 5 is that the financier asset-employment relationship in the pessimistic equilibrium is almost the exact opposite of the one derived in Proposition 1.

**Proposition 5:** Under the assumption  $u''' > 0$  the pessimistic equilibrium is characterized by employment being a decreasing function of  $d_{t-1}$ . Moreover, for all  $d_{t-1} > 0$ , employment is lower than in autarky in this case.

The reason why employment is low and is a decreasing function of  $d$  in the pessimistic equilibrium, is that now the presence of financier assets  $d$  increases the risk premium. This occurs because workers, who are the agents that determine the risk premium at the margin, dislike risk even more when they are more indebted and no one is willing to insure them against bad aggregate outcomes. Effectively, workers have no insurance against fluctuating labor income but have to repay a larger debt to financiers when  $d_{t-1}$  is higher. Hence, their net risk exposure becomes higher with higher  $d_{t-1}$ . The market response is for employment to decrease as labor time is shifted to the safe home production technology  $g(1-l)$ . The important new aspect is that the equilibrium mapping between financier assets and employment is decreasing, that is more financier assets leads to lower employment. It is important to note that the condition  $u''' > 0$  implies that preferences exhibit Decreasing Absolute Risk Aversion. If this were not true then additional risk exposure could lead to workers undertaking more risky employment.

**Proposition 6:** In the pessimistic equilibrium current financier net worth is an increasing function of past net worth and is independent of the current state. Moreover, there is a unique steady state level of financier net worth in this equilibrium,  $d^p$ , with  $d^p < \bar{d}$ .

A key feature of Proposition 6 is that the evolution of financier net worth is not affected by the current state. This has an important implication for the dependence of current employment on past states of nature. Since mature financiers do not acquire any equity claims, their end-of-period resources are  $E[\psi]d_{t-1}$  in both states. This implies that their bequests to young financiers are also not dependent on the current state. Hence, resources available to young financiers to lend to young workers every period are  $f + \gamma E[\psi]d_{t-1}$  which too is independent of the current state. Thus, in the pessimistic equilibrium, current employment becomes independent of the state in the previous

period. High productivity shocks today do not translate into high employment tomorrow since transitory productivity shocks are no longer propagated across periods. Clearly, if the economy gets stuck in the pessimistic equilibrium with low employment, a sequence of good productivity shocks will not necessarily lift the economy out of it.

To summarize, in the presence of asymmetric information, there always exists a pessimistic equilibrium in which the secondary market for assets breaks down due to an adverse selection induced “market for lemons” problem. Relative to the symmetric information case, employment is lower and the risk premium is higher in the pessimistic equilibrium. Moreover, both the evolution of financier assets and employment become non-state contingent in this equilibrium.

### 5.2.2 Optimistic equilibrium: $\psi^m = 1$

Next we turn to the potential existence of other equilibria. Conditional on existence, one potential equilibrium is particularly interesting. This is the one where the market becomes optimistic and all mature financiers choose to offer their assets on the market, i.e.,  $\psi^m = 1$ . In this subsection we ask whether this equilibrium can exist and, if so, under what conditions.

Recall that a financier with type  $\psi^i$  assets would choose to sell his assets on the market if and only if

$$\frac{q(1-\theta)\hat{\psi}_t}{\frac{\bar{p}_t^s}{\bar{p}_t^b} - \theta} \geq \psi^i,$$

where  $\hat{\psi}$  is the conjectured average quality of assets sold on the market. Combining this with the consistency condition (equation 20) shows that for the holder of the highest quality asset ( $\psi^i = 1$ ) to place his assets on the market it must be that

$$\frac{q(1-\theta)\bar{\psi}}{\frac{\bar{p}_t^s}{\bar{p}_t^b} - \theta} \geq 1,$$

where we have used the fact that when all assets are sold on the market the average quality of assets are given by

$$\hat{\psi}_t \Big|_{\psi_t^m=1} = \int_0^1 \psi f(\psi) d\psi = \bar{\psi}.$$

Recall that in the asymmetric information case  $\frac{\bar{p}_t^s}{\bar{p}_t^b} = \tilde{\phi}^p(d_{t-1}, \psi_t^m)$ .

**Condition 1:**  $\frac{q(1-\theta)\bar{\psi}}{\tilde{\phi}^p(d_{t-1}, 1) - \theta} > 1$ .

**Proposition 7:** As long as Condition 1 holds there will always exist an optimistic equilibrium with  $\psi^m = 1$ .

Clearly, for the optimistic equilibrium with  $\psi^m = 1$  to exist the density function of asset types

$f(\cdot)$  must have sufficient mass concentrated near 1, so that Condition 1 is satisfied. The optimistic equilibrium is characterized by full pooling of default risk by all financiers and the market behaves as if there was no asymmetric information. Hence, the endogenous variables of the model behave exactly as in the previously analyzed case of a given exogenous default probability and symmetric information. We call this the optimistic equilibrium as it resembles the equilibrium without adverse selection.<sup>24</sup>

### 5.2.3 An Example

We now provide a simple example to illustrate the mechanics of the model under asymmetric information. Assume that

$$g(1 - l_t) = (1 - l_t) g^*,$$

where  $g^* > 0$  is a positive constant. This makes the home production technology linear in its labor input (or equivalently, it makes the utility from leisure linear in time). The direct implication of this assumption is that  $p^s/p^b = g^*$  is now a constant as is the risk premium. Hence, the consistency condition (equation 20) is sufficient to pin down the equilibrium  $\psi^m$ . All other endogenous variables at date  $t$  can then be determined recursively as functions of  $\psi_t^m, d_{t-1}$  and  $A_t$ .

It is easy to see that the logic of the multiple equilibria described above continues to apply.  $\psi^m = 0$  is always a solution to equation (20) which implies that the pessimistic equilibrium always exists. The relevant condition for the optimistic equilibrium to exist can now be written as

$$\bar{\psi} \geq \frac{g^* - \theta}{q(1 - \theta)}.$$

This amounts to a restriction on the parameters and the density function since  $\int_0^1 \psi f(\psi) d\psi = \bar{\psi}$ .

As a specific case, assume that  $f(\psi)$  follows a uniform distribution in the interval  $[0, 1]$ . Define  $\frac{q(1-\theta) \int_0^{\psi^m} \psi f(\psi) d\psi}{F(\psi^m)(g^*-\theta)} \equiv G(\psi^m)$ . Under our distributional assumption on  $f$  we have

$$G(\psi^m) = \frac{q(1 - \theta) \psi^m}{(g^* - \theta) 2}.$$

Note that  $G$  is rising in  $\psi^m$  but becomes a constant  $\frac{q(1-\theta)}{g^*-\theta}$  for all  $\psi^m \geq 1$ . Figure 6 plots  $\psi^m$  and  $G(\psi^m)$  against  $\psi^m$ . There are three possibilities:

**Case 1:**  $\frac{q(1-\theta)}{g^*-\theta} < 2$ : In this case  $G(1) = \frac{q(1-\theta)}{2(g^*-\theta)} < 1$ . Hence, the pessimistic equilibrium is the only possible equilibrium. This case is depicted by the  $G^p$  schedule in the figure.

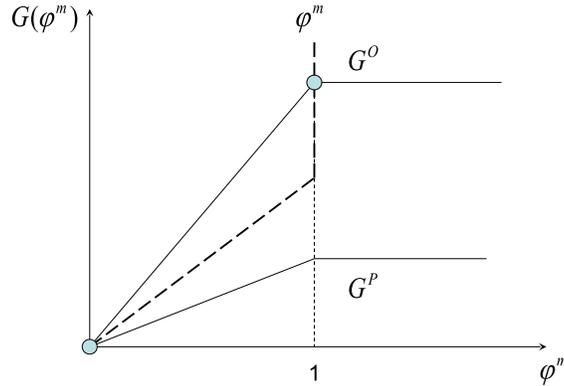
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<sup>24</sup>More generally, under adverse selection there are many possible equilibrium configurations. Whether or not these arise depend on the parameter configuration and the properties of the density function  $f(\cdot)$ .

**Case 2:**  $\frac{q(1-\theta)}{g^*-\theta} > 2$  : In this case,  $G(1) = \frac{q(1-\theta)}{2(g^*-\theta)} > 1$ . There are two equilibria in this case – the pessimistic equilibrium with  $\psi^m = 0$  and the optimistic equilibrium with  $\psi^m = 1$ . This case is depicted by the  $G^o$  schedule in the figure.

**Case 3:**  $\frac{q(1-\theta)}{g^*-\theta} = 2$  : In this case  $G(\psi^m) = \psi^m$ . Hence the  $G$  function coincides with the 45 degree line. There are thus a continuum of equilibria in the interval  $[0, 1]$ .

Figure 6: A multiple equilibria example



Example:  $f$  is uniform in  $[0,1]$

#### 5.2.4 A Scenario: Debt Fueled Expansion and Financial Crisis

We now use our previous results to sketch out one potential scenario that could emerge in the context of our model. Suppose we are in Case 2, i.e.,  $\frac{q(1-\theta)}{g^*-\theta} > 2$ . Hence, there are two equilibria. Recall that in the optimistic equilibrium with complete risk pooling by financiers, good productivity shocks ( $A = 1$ ) will imply greater financier asset creation and hence greater worker debt creation today and higher employment and debt tomorrow. A bad productivity shock would lead to financier net worth (or worker debt) falling to  $\underline{d}$  and then start rising again in response to good states or staying at  $\underline{d}$  in bad states.

Suppose our model economy starts off with some worker debt level  $d_0 < \bar{d}$ . Assume that the economy is initially in the optimistic equilibrium. A sequence of good productivity shocks will thus induce rising financier net worth and worker debt as well as rising employment. Now suppose that at some date  $t$  there is an abrupt switch in expectations to pessimism: financiers conjecture that

the worker debt being offered on the market is of the worst possible type, i.e.,  $\hat{\psi} = 0$ . This will immediately lead to the economy switching to the pessimistic equilibrium in which the market for existing assets will freeze up. Consequently, financiers will simply hold their assets to maturity and there will be no insurance on offer for risky production. The increasing riskiness of market employment will induce a precipitous decline in employment below even the autarky level.

One way of thinking about a map between our model and financial crisis episodes is to view the switch in the equilibrium from the optimistic to the pessimistic as a financial crisis episode. Viewed through this lens, a financial crisis episode in our structure is particularly debilitating because, as long as the economy stays in the pessimistic equilibrium, good productivity shocks are not going to help raise employment and expected output. Moreover, in this equilibrium, creating more debt will make the employment situation even worse as workers will be faced with bigger repayment obligations but no insurance against risky labor income. Thus, a financial crisis in our model can have very large negative effects on employment even if the economy is fortunate enough not to have bad productivity shocks.

## 6 Conclusion

In this paper we have provided a description of an economic environment in which (a) there is systemic aggregate risk; and (b) there is heterogeneity across agents in terms of their risk tolerance. Our model has focused on two key functions of financial markets: they facilitate lending to households for consumption smoothing purposes and they provide insurance to risk averse agents by shifting production risk to agents with greater risk tolerance. We have shown that in this environment, good productivity shocks tend to raise the resources available to financiers. This facilitates greater lending to households and more insurance provisions by these financial intermediaries for risky production and a concomitant decline in the risk premium. Crucially, the resources of financiers act as a form of risk capital which links current states to future economic activity. Hence, financial markets tend to propagate transitory productivity shocks over time through this risk capital. We believe this is a new mechanism linking the financial and real sides of the economy and is an independent contribution of the paper.

We have also shown that in this environment financial markets facilitate not just a higher level of economic activity but also greater volatility. Moreover, the equilibrium level of activity can often be fragile in such economies in that a small amount of asymmetric information can cause asset markets to freeze and induce large real dislocations. We have shown this using a very simple model of default risk in which private information with financial intermediaries regarding the default

rates on their asset portfolios gives rise to an adverse selection problem in asset markets. This can easily lead to multiple equilibria which are sensitive to expectations. Pessimism about the quality of assets on offer in asset markets can lead to a freezing of transactions – a financial crisis – and low employment, while optimism about the average quality of assets in the market can give rise to an equilibrium with high employment that resembles one with no adverse selection at all. Financial crises in our model are thus associated with a switch in expectations from optimism to pessimism. These crises are associated with precipitous falls in employment and a shutting down of insurance markets. Perhaps, most strikingly, once an economy gets into the pessimistic equilibrium, productivity shocks are no longer transmitted across periods. Hence, the economy can be stuck in a low employment phase for long periods of time despite continual good productivity realizations.

It is worth reiterating that the goal of the paper is not to propose a specific view of the cause of the crisis. Rather, our goal was to provide an explanation for why a disruption in one small segment of the economy can have large disruptive effects on the aggregate economy. We have done so by proposing a new mechanism linking the real and financial sides of the economy which focuses on the insurance provision role of financial markets. The specific mechanism inducing the crisis in our model – adverse selection and multiple equilibria – is not fundamental. One could potentially formalize alternative descriptions of financial market disruptions instead without altering the mechanism linking the real and financial sides of the economy. While our model clearly omits many elements relevant during the 2001-2008 episode, we believe the margins isolated in this paper offer an insight into the deeper or more fundamental causes.

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## A Appendix

### A.1 Proof of Proposition 1

Combining equations (7) and (8) gives

$$g'(1-l) = \frac{qu' \left( l - \left( \frac{(1-\theta)d}{g'(1-l)-\theta} \right) + g(1-l) \right) + (1-q)\theta u'(\theta l + g(1-l))}{qu' \left( l - \left( \frac{(1-\theta)d}{g'(1-l)-\theta} \right) + g(1-l) \right) + (1-q)u'(\theta l + g(1-l))}.$$

This expression implicitly defines the equilibrium relationship  $l = \phi^l(d)$ ,  $\frac{\partial \phi^l}{\partial d} > 0$  where the sign of the derivative of the  $\phi^l$  function follows from differentiating the key equation above and applying the implicit function theorem. To derive the cut-off level of debt  $\tilde{d}$ , note that the financier optimality conditions dictate that  $p^s/p^b$  is bounded above by  $E(A) = q + \theta(1-q)$ . From equation (7)  $p^s/p^b = g'(1-l)$  which leads to the solution for  $\tilde{l}$ . The solution for  $\tilde{d}$  then follows directly from the equilibrium relationship  $l = \phi^l(d)$ . The result that  $\phi^l(d) > l^a$  follows trivially from the fact that  $\phi^l$  is increasing in  $d$  and that  $d = 0$  in the autarkic case.

### A.2 Proof of Proposition 2

The proof follows by combining the household optimality condition for optimal labor-leisure  $p^s/p^b = g'(1-l)$  with Proposition 1 which showed that  $\phi^l$  is increasing in  $d$  and  $l$  is bounded above by  $\tilde{l}$ .

### A.3 Proof of Proposition 3

We start by noting that the equilibrium for this economy is described by the solution to the following system of equations:

$$g'(1-l_{t+1}) = \frac{qu' \left( l_{t+1} - \left( \frac{(1-\theta)d_t}{g'(1-l_{t+1})-\theta} \right) + g(1-l_{t+1}) \right) + \theta(1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}{qu' \left( l_{t+1} - \left( \frac{(1-\theta)d_t}{g'(1-l_{t+1})-\theta} \right) + g(1-l_{t+1}) \right) + (1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}$$

$$p_t^d = \frac{qu'(c_{gt+1}^o + g(1-l_{t+1})) + (1-q)u'(c_{bt+1}^o + g(1-l_{t+1}))}{u'(c_t^y)}$$

$$p_t^d d_t = f + \gamma F_t$$

where  $F_t$  and  $c_t^y$  are dependent on the realization of  $A$  at date  $t$ . Combining the last two equations and substituting in the allocations for  $F_t$  and  $c_t^y$  in each of the two states yields four difference equations which describe the state-contingent equilibrium dynamics of this economy:

1. If  $d_{t-1} \leq \tilde{d}$  and  $A_t = 1$ , then

$$d_t = \frac{\left( f + \frac{\gamma(1-\theta)d_{t-1}}{g'(1-\phi^l(d_{t-1}))-\theta} \right) u' \left( f + y + \frac{\gamma(1-\theta)d_{t-1}}{g'(1-\phi^l(d_{t-1}))-\theta} \right)}{qu' \left( \phi^l(d_t) - \frac{(1-\theta)d_t}{g'(1-\phi^l(d_t))-\theta} + g(1-\phi^l(d_t)) \right) + (1-q)u'(\theta\phi^l(d_t) + g(1-\phi^l(d_t)))}$$

2. If  $d_{t-1} \leq \tilde{d}$  and  $A_t = \theta$ , then

$$d_t = \frac{fu'(f+y)}{qu' \left( \phi^l(d_t) - \frac{(1-\theta)d_t}{g'(1-\phi^l(d_t))-\theta} + g(1-\phi^l(d_t)) \right) + (1-q)u'(\theta\phi^l(d_t) + g(1-\phi^l(d_t)))}$$

This expression is independent of  $d_{t-1}$  and depends only on the time independent variables  $f$  and  $y$  and constant parameters. We denote the solution for  $d$  in this case by  $\underline{d}$ .

3. If  $d_{t-1} > \tilde{d}$  and  $A_t = 1$ , then

$$d_t = \frac{\left( f + \gamma \left( d_{t-1} + \frac{(1-q)\tilde{d}}{q} \right) \right) u' \left( f + y + \gamma \left( d_{t-1} + \frac{(1-q)\tilde{d}}{q} \right) \right)}{qu' \left( \phi^l(\tilde{d}) - d_t - \frac{(1-q)\tilde{d}}{q} + g \left( 1 - \phi^l(\tilde{d}) \right) \right) + (1-q)u' \left( \theta\phi^l(\tilde{d}) - (d_t - \tilde{d}) + g \left( 1 - \phi^l(\tilde{d}) \right) \right)}$$

4. If  $d_{t-1} > \tilde{d}$  and  $A_t = \theta$ , then

$$d_t = \frac{\left( f + \gamma(d_{t-1} - \tilde{d}) \right) u' \left( f + y + \gamma(d_{t-1} - \tilde{d}) \right)}{qu' \left( \phi^l(\tilde{d}) - d_t - \frac{(1-q)\tilde{d}}{q} + g \left( 1 - \phi^l(\tilde{d}) \right) \right) + (1-q)u' \left( \theta\phi^l(\tilde{d}) - (d_t - \tilde{d}) + g \left( 1 - \phi^l(\tilde{d}) \right) \right)}$$

The last two expressions follow from the fact that when  $d_{t-1} > \tilde{d}$  the financiers build risky positions by buying stocks using only  $\tilde{d}$  of their initial debt holdings. Beyond this there is no further gain from taking risky positions since the risk premium achieves its lowest feasible level at this point. Hence, financiers hold onto to all debt holdings in excess of  $\tilde{d}$ . Thus,

$$F_{bt} = d_{t-1} - \tilde{d},$$

$$F_{gt} = \frac{(1-\theta)\tilde{d}}{\phi^p(\tilde{d})-\theta} + d_{t-1} - \tilde{d} = d_{t-1} + \frac{(1-q)\tilde{d}}{q},$$

where we have used the fact that  $\phi^p(\tilde{d}) = q + \theta(1-q)$  implies that  $\frac{1-\phi^p(\tilde{d})}{\phi^p(\tilde{d})-\theta} = \frac{1-q}{q}$ .

In order to proceed further it is convenient to use the definition

$$W_{t+1} \equiv \frac{(1-\theta)d_t}{g'(1-l_{t+1})-\theta}$$

and analyze the equilibrium dynamics of the economy in terms of  $W$ . Note that since  $l_t$  is a

function of  $d_{t-1}$ , which is the state variable of the original system,  $W_t$  will be the state variable of the new system. We should also note that

$$\frac{dW_{t+1}}{dd_t} = \left[ \frac{1-\theta}{g'(1-l_{t+1})-\theta} \right] \left[ 1 + \left( \frac{d_t g''(1-l_{t+1})}{g'(1-l_{t+1})-\theta} \right) \frac{\partial l_{t+1}}{\partial d_t} \right] > 0,$$

which implies that the mapping between  $W_t$  and  $d_{t-1}$  is monotone. To see the latter inequality, one can totally differentiate the first equation of this section to get

$$\frac{\partial l_{t+1}}{\partial d_t} = \frac{A(1-\theta)/[g'(1-l_{t+1})-\theta]}{-p_t^d u'(c_t^y) g''(1-l_{t+1}) + A \left[ 1 - g'(1-l_{t+1}) - \frac{(1-\theta)dg''(1-l_{t+1})}{g'(1-l_{t+1})-\theta} \right] + B} > 0$$

where

$$A = -q [1 - g'(1-l_{t+1})] u''(g, t+1) > 0$$

$$B = -(1-q) [g'(1-l_{t+1}) - \theta]^2 u''(b, t+1) > 0$$

where the inequalities follow from the fact that  $\theta < g' < 1$ . Note that in the above we are using the notation  $u(g, t)$  to denote utility in the good state in period  $t$  and  $u(b, t)$  to denote period- $t$  utility in the bad state.

Using this expression for  $\frac{\partial l_{t+1}}{\partial d_t}$  it is easy to check that

$$\left( 1 - g'(1-l_{t+1}) - \frac{(1-\theta)dg''(1-l_{t+1})}{[g'(1-l_{t+1})-\theta]^2} \right) \frac{\partial l_{t+1}}{\partial d_t} < \frac{1-\theta}{g'(1-l_{t+1})-\theta}$$

This can be rearranged to give

$$\frac{dg''(1-l_{t+1})}{[g'(1-l_{t+1})-\theta]} \frac{\partial l_{t+1}}{\partial d_t} + 1 > \frac{[1 - g'(1-l_{t+1})][g'(1-l_{t+1})-\theta]}{1-\theta} \frac{\partial l_{t+1}}{\partial d_t} > 0.$$

Hence,  $\frac{dg''(1-l_{t+1})}{[g'(1-l_{t+1})-\theta]} \frac{\partial l_{t+1}}{\partial d_t} + 1 > 0$  which implies that  $\frac{dW_{t+1}}{dd_t} > 0$ .

Using the change of variable from  $d$  to  $W$ , we can rewrite the equilibrium conditions describing the economy as

$$g'(1-l_{t+1}) = \frac{qu'(l_{t+1} - W_{t+1} + g(1-l_{t+1})) + \theta(1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}{qu'(l_{t+1} - W_{t+1} + g(1-l_{t+1})) + (1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}$$

$$p_t^d = \frac{qu'(l_{t+1} - W_{t+1} + g(1-l_{t+1})) + (1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}{u'(c_t^y)}$$

$$p_t^d d_t = f + \gamma F_t$$

Totally differentiating the first condition gives

$$\frac{\partial l_{t+1}}{\partial W_{t+1}} = \frac{-q(1-g')u''(g, t+1)}{-p_t^d u'(c_t^y) g''(1-l_{t+1}) - q(1-g')^2 u''(g, t+1) - (1-q)(\theta-g')^2 u''(b, t+1)} > 0.$$

For future reference it useful to note that the above also implies that

$$0 < (1-g') \frac{\partial l_{t+1}}{\partial W_{t+1}} < 1.$$

We shall denote this implicit solution for  $l_t$  by  $\tilde{\phi}^l(W_t)$ .

Throughout the following analysis we shall maintain Assumption 3 (see Section 4.3 above) so that preferences are given by

$$u(x) = \log x.$$

First, consider the case where  $A_t = 1$ . In this event one can combine the three equilibrium conditions above along with the equilibrium allocations for  $F_t$  and  $c_t^y$  in the good state to get

$$W_{t+1} = (f + \gamma W_t) \left[ \frac{u'(y + f + \gamma W_t)}{qu'(l_{t+1} - W_{t+1} + g(1 - l_{t+1}))} \right].$$

In deriving this expression we have used the fact that

$$\frac{(1-q)u'(b, t+1)}{qu'(g, t+1)} = \frac{1-g'(1-l_{t+1})}{g'(1-l_{t+1})-\theta}.$$

Under our assumption on preferences the equilibrium difference equation reduces to

$$qW_{t+1} = (f + \gamma W_t) \left[ \frac{l_{t+1} - W_{t+1} + g(1 - l_{t+1})}{y + f + \gamma W_t} \right]. \quad (22)$$

Totally differentiating this expression gives

$$\frac{dW_{t+1}}{dW_t} = \frac{\gamma \left[ \frac{y}{y+f+\gamma W_t} \right] [l_{t+1} - W_{t+1} + g(1 - l_{t+1})]}{\left[ 1 - \left\{ (1-g'(1-l_{t+1})) \frac{\partial l_{t+1}}{\partial W_{t+1}} - 1 \right\} \right] [f + \gamma W_t]}$$

As we showed above,  $(1-g') \frac{\partial l_{t+1}}{\partial W_{t+1}} < 1$ . Hence the denominator in the expression for  $\frac{dW_{t+1}}{dW_t}$  is clearly positive as is the numerator. Thus,  $\frac{dW_{t+1}}{dW_t} > 0$ . Noting that  $\frac{dW_{t+1}}{dd_t} > 0$  then implies that  $d_t$  must be an increasing function of  $d_{t-1}$  when  $A_t = 1$ .

Second, recall that from expression 2 (the equilibrium difference equation when  $A = \theta$ ) above that  $d_t$  is independent of  $d_{t-1}$  when  $A_t = \theta$ .

Next, straight forward differentiation of the equilibrium difference equations governing the sys-

tem when  $d_{t-1} > \tilde{d}$  (the third and fourth equilibrium difference equations described at the beginning of this section above) shows that  $\frac{dd_t}{dd_{t-1}} > 0$  in both cases. This completes the proof of part 1 of Proposition 3. Part 3 follows trivially from part 2.

For part 2 of the Proposition we start by noting that the fixed point for the equilibrium map under  $A = 1$  and  $d_{t-1} < \tilde{d}$  is derived by first solving for  $\bar{W}$  from the expression

$$q\bar{W} = (f + \gamma\bar{W}) \left[ \frac{\tilde{\phi}^l(\bar{W}) - \bar{W} + g(1 - \tilde{\phi}^l(\bar{W}))}{y + f + \gamma\bar{W}} \right]$$

and using the definition of  $W$  to get

$$\bar{d} = \frac{\bar{W} \left[ g' \left( 1 - \tilde{\phi}^l(\bar{W}) \right) - \theta \right]}{1 - \theta}.$$

To see the existence of such a fixed point, first note that from equation (22)  $W_{t+1} > 0$  when  $W_t = 0$ . Next, divide both sides of equation (22) by  $W_t$  and to get

$$q\omega_{t+1} = \left( \frac{f}{W_t} + \gamma \right) \left[ \frac{\frac{l_{t+1} + g(1 - l_{t+1})}{W_t} - \omega_{t+1}}{\frac{y+f}{W_t} + \gamma} \right].$$

Taking the limit of both sides as  $W_t \rightarrow \infty$  gives

$$(1 + q) \lim_{W_t \rightarrow \infty} \omega_{t+1} = 0.$$

Hence,  $\lim_{W_t \rightarrow \infty} \omega_{t+1} = 0$ . Since  $\omega_{t+1} > 0$  around  $W_t = 0$  and  $\omega_{t+1} = 0$  as  $W_t$  goes to infinity,  $\omega_{t+1}$  must equal one somewhere in the interior of this range for  $W_t$  as  $W_{t+1}$  is continuous. Hence, there always exists such a fixed point.

The equilibrium transition equation for  $d_t$  when  $A = \theta$  is independent of  $d_{t-1}$  (see the second difference equation at the beginning of this appendix). The solution to this expression is also the fixed point for this mapping  $\underline{d}$ . The fixed points for the equilibrium mappings from  $d_{t-1}$  to  $d_t$  when  $d_{t-1} > \tilde{d}$  are determined analogously from the corresponding difference equations described above.

#### A.4 Proof of Proposition 4

The consistency condition given by equation (20) is trivially satisfied when  $\psi^m = 0$ . Employment, for a given debt level  $d_{t-1}$ , is now given implicitly by

$$g'(1 - l_t) = \frac{qu'(l_t - \bar{\psi}d_{t-1} + g(1 - l_t)) + (1 - q)\theta u'(\theta l_t - \bar{\psi}d_{t-1} + g(1 - l_t))}{qu'(l_t - \bar{\psi}d_{t-1} + g(1 - l_t)) + (1 - q)u'(\theta l_t - \bar{\psi}d_{t-1} + g(1 - l_t))}. \quad (23)$$

This has a well defined solution  $l^p$  given by

$$l_t^p = \tilde{\phi}^l(d_{t-1}, 0).$$

### A.5 Proof of Proposition 5

The proof is straightforward as it follows from differentiating equation (23) with respect to  $l_t$  and  $d_{t-1}$  and using the implicit function theorem to get  $\frac{\partial l_t^p}{\partial d_{t-1}} < 0$ . Importantly, this holds for preferences where  $u''(x)/u'(x)$  is decreasing in  $x$ , i.e., there is Decreasing Absolute Risk Aversion. The second statement implies that  $l^a \geq l_t^p$  for all  $t$ . This follows directly from the fact that  $\frac{\partial l_t^p}{\partial d_{t-1}} < 0$  and that  $d_{t-1} > 0$  when both financiers and workers coexist while  $d_{t-1} = 0$  corresponds to the autarkic case.

### A.6 Proof of Proposition 6

The dynamics of debt in this case are given by

$$d_t = \frac{f + \gamma\bar{\psi}d_{t-1}}{p_t^d},$$

where  $p_t^d$  is determined by

$$p_t^d u'(y + f + \gamma\bar{\psi}d_{t-1}) = E[\psi] [qu'(l_{t+1} - d_t + g(1 - l_{t+1})) + (1 - q)u'(\theta l_{t+1} - d_t + g(1 - l_{t+1}))].$$

This gives an implicit mapping between  $d_t = \phi_p^d(d_{t-1})$ . From the previous two equations it is clear that this transition equation is not state contingent. Straightforward differentiation shows  $d_t$  is increasing in  $d_{t-1}$ .

### A.7 Proof of Proposition 7

The proof follows directly by combining Condition 1 with the condition under which the financier with the highest quality debt sells his debt on the market  $-q(1 - \theta)\bar{\psi} \geq \frac{p_t^s}{p_t^b} - \theta$ .